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Alternation blindness in the perception of binary sequences

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Authors
Yu, Ru Qi
Osherson, Daniel
Zhao, Jiaying

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Alternation blindness in the perception of binary sequences

Ru Qi Yu (ruqiyu@psych.ubc.ca)
Department of Psychology, University of British Columbia

Daniel Osherson (osherson@princeton.edu)
Department of Psychology, Princeton University

Jiaying Zhao (jiayingz@psych.ubc.ca)
Department of Psychology; Institute for Resources, Environment and Sustainability, University of British Columbia

Abstract

Binary information is prevalent in the environment. In this study, we examined how people process repetition and alternation in binary sequences. Across four paradigms involving estimation, working memory, change detection, and visual search, we found that the number of alternations is under-estimated compared to repetitions (Experiment 1). Moreover, recall for binary sequences deteriorates as the sequence alternates more (Experiment 2). Changes in bits are also harder to detect as the sequence alternates more (Experiment 3). Finally, visual targets superimposed on bits of a binary sequence take longer to process as alternation increases (Experiment 4). Overall, our results indicate that compared to repetition, alternation in a binary sequence is less salient in the sense of requiring more attention for successful encoding. The current study thus reveals the cognitive constraints in the representation of alternation and provides a new explanation for the over-alternation bias in randomness perception.

Keywords: alternation bias, randomness perception, working memory, attention, numerosity perception

Introduction

Perceptually, many events in the world can be interpreted as binary, from the outcomes of coin flips to the daily alternations between the sun and the moon. Past research that examines the perception of binary information has focused on the perception of randomness (Bar-Hillel & Wagenaar, 1991; Nickerson, 2002) and regularities (Julesz, 1962; Lopes & Oden, 1987). Although it is difficult to define randomness (Ayton et al., 1989; Beltrami, 1999; Chater & Vitányi, 2003; Fitelson & Osherson, 2012; Oskarsson et al., 2009), there are systematic biases in people's conception of randomness, such as the gambler's fallacy (Kahneman & Tversky, 1972) and the hot hand fallacy (Gilovich et al., 1985). One particular bias that has received much attention in the past is the over-alternation bias: a binary sequence that alternates more than expected on the basis of random generation tends to be judged as random (Bar-Hillel & Wagenaar, 1991; Falk & Konold, 1997; Lopes & Oden, 1987; Nickerson, 2002), and people tend to produce random sequences that contain too many alternations (Kahneman & Tversky, 1972; Wagenaar, 1972). This bias is robust across different stimulus types, sensory modalities, or presentation modes (Yu et al., in press).

A number of accounts have been proposed to explain the over-alternation bias. One explanation focuses on the limits of working memory (Baddeley, 1966; Kareev, 1992). Since people can only hold a limited number of items in working memory at any given time, the amount of bits being processed is constrained, leading to a biased sample of randomness (Hahn & Warren, 2009; Miller & Sanjuro, 2015; Yu et al., in press). Another prominent account of the over-alternation bias is the idea of local representativeness, which suggests that people assume equal frequency of outcomes within a local sequence (Tversky & Kahneman, 1971). A recent account is offered by Falk and Konold (1997) who proposed an encoding hypothesis that states that the probability that a bit sequence is labelled random varies directly with the time needed to correctly memorize or copy it. However, this account has been challenged by recent work showing a discrepancy between encoding difficulty of the binary sequence and labeling the sequence as random (Zhao et al., 2014). While these explanations have offered valuable insights, there remains a possibility that people have an accurate view of randomness, but the cognitive limitations contribute to a biased conception of randomness (Rapoport & Budescu, 1992).

The current study

We explore a new explanation focusing on a perceptual limitation in the ability to represent alternations vs. repetitions. If alternations are under-represented compared to repetitions, there needs to be more alternations in the sequence in order for people to perceive a 50% alternation rate that is typically assumed in a random sequence.

In order to generate a binary sequence that contains different levels of alternations while maintaining the equal probability of outcomes, we used an algorithm that deviates from stochastic independence by allowing previous bit to influence the next one. Specifically, for each p in the unit interval (from 0 to 1), let $D(p)$ generate a sequence of bits consisting of zeros and ones as follows:

> A fair coin toss determines the 1st bit. Suppose that the n^{th} bit (for $n \geqslant 1$) has been constructed. Then with probability p the $n + 1^{st}$ bit is set equal to the opposite of the n^{th} bit; with probability $1 - p$ the $n + 1^{st}$ bit is set equal to the n^{th} bit. Repeat this process to generate a sequence of any length.

This procedure was first introduced by Zhao, Hahn, and Osherson (2014). *D*(.5) is a genuinely random device. For $p<.5$, D(*p*) tends to repeat itself, resulting in long streaks, whereas for $p>.5$, *D*(*p*) tends to alternate. The expected proportion of each bit is 50% for all $p \in [0, 1]$, although empirically, the output might deviate from 50%; however such deviations should be small and random (Yu et al., in press). For any sequence produced by *D*(*p*), the expected proportion of alternation, called the "switch rate" of the generating process, is *p*. The expected proportion of repetitions, called the generating "repeat rate", is $1 - p$.

We conducted four experiments to examine how people represent alternations vs. repetitions. In Experiment 1, participants viewed a binary sequence and estimated the number of switches or repeats in the sequence. In Experiment 2, participants viewed a binary sequence and recalled the sequence. In Experiment 3, participants viewed two sequences and judged whether the sequences were the same or different. In Experiment 4, participants searched for a target embedded in a binary sequence.

Experiment 1

This experiment examined if there are differences in the estimation of alternation vs. repetition in binary sequences.

Participants

Forty-five undergraduate students (32 female, mean age=19.9 years, SD=2.3) from the University of British Columbia (UBC) participated for course credit. Participants in all experiments provided informed consent. All experiments reported here have been approved by the UBC Behavioral Research Ethics Board. We conducted a power analysis using G*Power (Faul, Erdfelder, Lang, & Buchner, 2007), which showed that given an effect size of 0.53 (based on our prior work, Zhao & Yu, 2016), a minimum of 38 participants would be required to have 95% power to detect the effect in our design.

Stimuli

In each trial, participants viewed a 30-bit sequence. Each sequence contained circles of two colors: green (RGB: 0 255 0) and blue (RGB: 0 0 255). Each circle subtended 0.9° in diameter (Figure 1a). There were five levels of switch rates in *D*(*p*) in generating the sequences, where p = 0.1, 0.3, 0.5, 0.7, and 0.9. Correspondingly, there were five levels of repeat rates $(1 - p)$ = 0.9, 0.7, 0.5, 0.3, and 0.1.

Temporal sequences. For half of the trials, participants viewed a temporal sequence where the 30 circles were presented one after another, making simple visual grouping impossible. Each circle was presented at the center of the screen for 100ms, and the inter-stimulus interval (ISI) was 100ms with a blank screen (Figure 1a).

Spatial sequences. For the other half of the trials, participants viewed a spatial sequence, where the 30 circles were presented on the screen simultaneously. The circles were presented left to right. The space between two adjacent circles in the sequence subtended 0.1°. Each sequence was presented on the screen for 1000ms (Figure 1a).

Procedure

There were 200 trials in total for each participant. In each trial, participants viewed a sequence with one of the five generating switch rates (0.1, 0.3, 0.5, 0.7, or 0.9). Each level of switch rate contained 40 trials, among which 20 trials were temporal sequences and 20 trials were spatial sequences. After viewing the 30-bit sequence, participants were asked to estimate either the number of the color switches (10 trials), or the number of color repeats (10 trials). Specifically, the instruction for estimating color switches was "How many times did a dot have a DIFFERENT color from the previous dot in the sequence?" and the instruction for estimating color repeats was "How many times did a dot have the SAME color as the previous dot in the sequence?". Participants were also told that the range of their estimate was from 0 to 29 (29 was the maximum possible number of switches or repeats in the sequence). Participants typed in their estimate after seeing each sequence. In sum, there were three within-subjects factors: the generating switch rate of the sequence (from 0.1 to 0.9), the presentation of the sequence (temporal vs. spatial), and the estimation type (switches vs. repeats). The order of the trials was randomized for each participant. There was no mention of randomness in all experiments.

Results and Discussion

Estimated switch rate was the derived by dividing the estimated number of switches from the participants by 29 (the maximum possible switches in the sequence). Likewise, *estimated repeat rate* was calculated by dividing the estimated number of repeats from the participants by 29 (the maximum possible repeats in the sequence). *Observed switch rate* was the objective switch rate in the sequence presented to the participants in each trial. Likewise, *observed repeat rate* was the objective repeat rate in the sequence presented in each trial. The *generating switch rate* was the *p* in *D*(*p*) in the algorithm that generated the sequence. The *generating repeat rate* was $1 - p$. To verify that the presented sequence actually exhibited the generating switch rate or repeat rate, we plotted the observed switch rate or repeat rate for each sequence (Figure 1 b to e), which mapped closely to the generating switch rate or repeat rate.

We computed the signed error (estimated – observed switch rate or repeat rate) at each of the five generating levels. For temporal trials (Figure 1 b and d), a 5 (generating rate: 0.1, 0.3, 0.5, 0.7, and 0.9) × 2 (estimation type: switches vs. repeats) repeated-measures ANOVA revealed a main effect of generating rate [$F(4,176)=162.3$, $p<.001$, $\eta_p^2=0.79$] and of estimation type [$F(1,44)=49.34$, $p<.001$, $\eta_p^2=0.53$], and a reliable interaction [$F(4,176)=10.75$, $p<.001$, $\eta_p^2=0.20$]. Pair-wise comparisons at each generating rate showed that participants underestimated the number of switches more than repeats at each of the five generating rates [p's<.01]. For spatial trials (Figure 1 c and e), the same

ANOVA revealed a main effect of generating rate [$F(4,176)=107.2$, $p<.001$, $\eta_p^2=0.71$] and of estimation type [$F(1,44)=114.2$, $p<.001$, $\eta_p^2=0.72$], but no interaction [$F(4,176)=0.07$, $p=.99$, $\eta_p^2<0.01$]. Again, pair-wise comparisons at each generating rate showed that participants underestimated the number of switches more than repeats at each of the five generating rates [p's<.001].

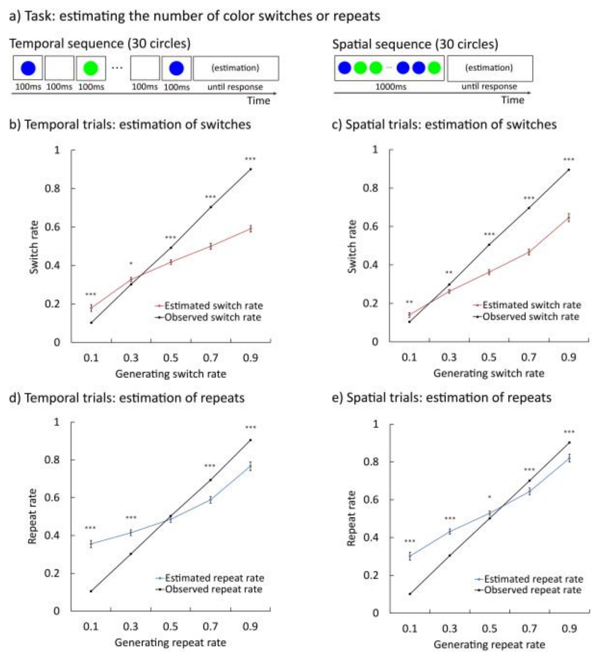

Figure 1. Experiment 1. (a) Participants (N=45) were presented with temporal or spatial sequences, and estimated either the number of circles that had a different color from the previous circle (switch) or the number of circles that had the same color as the previous one (repeat). (b) The estimated switch rate and the observed switch rate were plotted for temporal trials. (c) The estimated switch rate and the observed switch rate were plotted for spatial trials. (d) The estimated repeat rate and the observed repeat rate were plotted for temporal trials. (e) The estimated repeat rate and the observed repeat rate were plotted for spatial trials. (Error bars reflect ± 1 SEM; *$p < .05$, **$p < .01$, ***$p<.001$)

We further compared the estimated switch or repeat rate with the observed switch or repeat rate. For temporal trials (Figure 1b), participants over-estimated the switch rate at 0.1 and 0.3, but under-estimated the switch rate at 0.5, 0.7, and 0.9. They also over-estimated the repeat rate at 0.1 and 0.3, but under-estimated the repeat rate at 0.7 and 0.9 (Figure 1d). For spatial trials (Figure 1c), participants over-estimated the switch rate only at 0.1, and under-estimated the switch rate at 0.3, 0.5, 0.7, and 0.9. They over-estimated the repeat rate at 0.1, 0.3, and 0.5, but under-estimated the repeat rate at 0.7 and 0.9 (Figure 1e).

Interestingly, when estimating the number of repeats, participants were the most accurate around 0.5 where the sequences were truly random. For the same random sequence, participants were significantly under-estimating the number of switches. In fact, for people to perceive a 0.5 switch rate, the sequence must contain more than 50% switches, with a switch rate of around 0.7 (Figure 1 b and c). This perceptual insensitivity to switches may underlie the conceptual over-alternation bias of randomness. Taken together, these results suggest that alternations in a binary sequence were under-represented compared to repetitions.

Experiment 2

One explanation for the under-estimation of switches could involve working memory. Specifically, people may have trouble representing switches accurately in memory, mistaking them for repeating bits, thus leading to under-estimation. To examine this possibility, here participants were asked to recall each sequence.

Participants

Forty-five students (30 female, mean age=19.6 years, SD=1.2) from UBC participated for course credit.

Stimuli and Procedure

The stimuli were the same as those in Experiment 1, except for these differences: there were 10 circles per sequence to circumvent a floor effect in the recall task; each circle was slightly larger, subtending 1.4° in diameter, and the distance between each circle in spatial sequences was 0.2°; and each spatial sequence was presented for 500ms (Figure 2a).

The procedure was identical to Experiment 1, except for one difference: after seeing each sequence, participants were asked to recall the sequence as accurately as they could, by pressing two keys to produce the green circle (the "G" key) or the blue circle (the "B" key). Participants were instructed to recall the dots in the same order as they appeared. After each key press, the corresponding circle was presented on the screen for 100ms, and then disappeared. To recall the spatial sequence, participants pressed one key and the corresponding circle appeared from left to right on the screen, and remained on the screen.

Results and Discussion

Since the observed switch rate of the sequences mapped closely onto the generating switch rates (Experiment 1), for all following experiments task performance was plotted against the five generating switch rates.

To assess the accuracy of participants' recalled sequences, we divided the exact matches between the presented sequence and the recalled sequence by 10. The accuracy was plotted over the five levels of switch rates. For temporal trials (Figure 2b), a one way repeated-measures ANOVA revealed a significant difference in accuracy across the five switch rates [$F(4,176)=75.61$, $p<.001$, $\eta_p^2=0.63$]. Post-hoc Tukey HSD analysis showed all pair-wise comparisons were significant except between 0.7 and 0.9, and 0.5 and 0.9. For spatial trials (Figure 2c), accuracy was different across the switch rates [$F(4,176)=111.5$, $p<.001$, $\eta_p^2=0.72$], and post-hoc Tukey HSD analysis showed that all pair-wise comparisons were significant except between 0.7 and 0.9. These results demonstrate that as the switch rate of the sequence increased, recall accuracy decreased.

To obtain a more fine-grained analysis, from the second bit on, we calculated the recall accuracy of each bit depending on whether the bit repeated or switched from the previous bit. We compared the recall accuracy of switching versus repeating bits. For temporal trials (Figure 2d), a 5 (generating rate: 0.1, 0.3, 0.5, 0.7, and 0.9) × 2 (bit type: repeating vs. switching) repeated-measures ANOVA showed a main effect of generating rate [$F(4,176)= 75.61$, $p<.001$, $\eta_p^2=0.63$] and of bit type [$F(1,44)=206.7$, $p<.001$, $\eta_p^2=0.82$], and a reliable interaction [$F(4,176)=37.4$, $p<.001$, $\eta_p^2=0.46$]. Pair-wise comparisons at each generating rate showed that the recall accuracy of repeating bits was consistently higher than that of switching bits [p's<.01]. For spatial trials (Figure 2e), the same ANOVA showed a main effect of generating rate [$F(4,176)= 111.5$, $p<.001$, $\eta_p^2=0.46$] and of bit type [$F(1,44)=28.84$, $p<.001$, $\eta_p^2=0.40$], and a reliable interaction [$F(4,176)=7.18$, $p<.001$, $\eta_p^2=0.14$]. Pair-wise comparisons at each generating rate showed that the recall accuracy of repeating bits was higher than that of switching bits [p's<.001] at switch rates 0.1, 0.3, and 0.5.

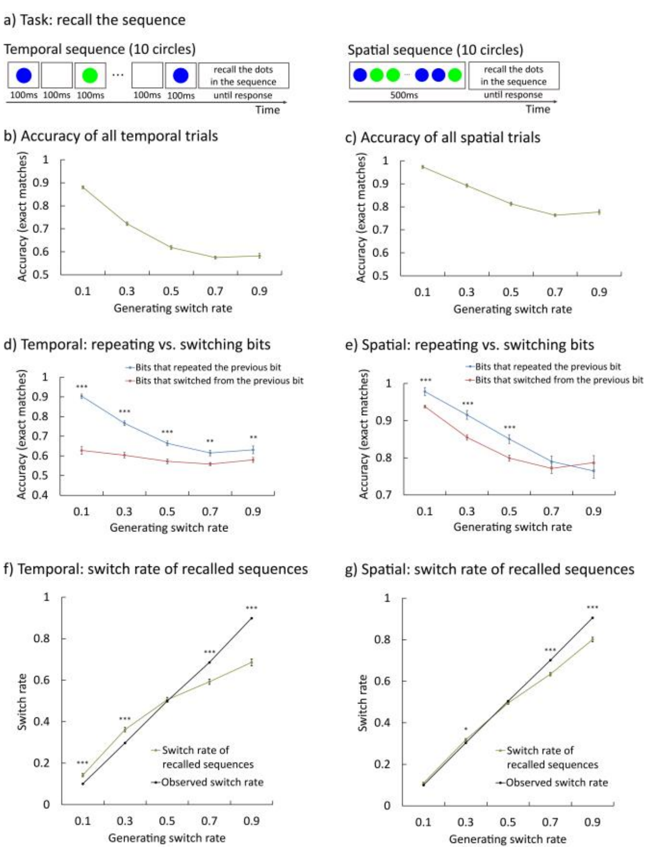

Figure 2. Experiment 2. (a) Participants (N=45) were presented with 10-bit temporal or spatial sequences, and recalled the sequences. Accuracy was calculated as the proportion of exact matches between the presented sequence and the recalled sequence for temporal trials (b) and spatial trials (c). From the second bit on in each sequence, recall accuracy of each bit was calculated depending on whether the bit repeated the previous bit, or switched from the previous bit, for temporal sequences (d) and spatial sequences (e). We also calculated the switch rate of the recalled sequences, plotted with observed switch rate of the presented sequences for temporal trials (f) and spatial trials (g). (Error bars reflect ± 1 SEM; *$p < .05$, **$p < .01$, ***$p<.001$)

One problem with the accuracy measure based on exact matches was that it penalizes cases where participants reversed or misplaced bits but were nonetheless accurate. To circumvent this problem, we conducted another analysis where we calculated the switch rate of the recalled sequence, and compared that to the observed switch rate of the presented sequence (Figure 2 f and g).

We computed signed error (switch rate of the recalled sequences – observed switch rate) separately for temporal and spatial trials. For temporal trials (Figure 2f), a one way repeated-measures ANOVA revealed a significant difference in signed error across the five generating switch rates [$F(4,176)=140.7$, $p<.001$, $\eta_p^2=0.76$]. Post-hoc Tukey HSD analysis showed all pair-wise comparisons were significant except between 0.1 and 0.3, and 0.1 and 0.5, suggesting that errors were greater at higher switch rates. For spatial trials (Figure 2g), the same ANOVA revealed a significant difference in signed error across the five switch rates [$F(4,176)=92.54$, $p<.001$, $\eta_p^2=0.68$]. Post-hoc Tukey HSD analysis showed all pair-wise comparisons were significant except between 0.1 and 0.3, and 0.1 and 0.5, suggesting errors were greater at higher switch rates.

These results showed that as the sequence alternated more, recall accuracy diminished. The greater recall error in switching bits compared to repeating bits suggests that people are more likely to encode switches as repeats, than to encode repeats as switches.

Experiment 3

What explains the encoding difficulty of switching bits? One explanation is that switching bits may be less salient than repeating bits. To examine salience, Experiment 3 used a change detection task where participants detected changes in two sequences that were presented one after another.

Participants

Forty-five students (24 female, mean age=20.6 years, SD=1.8) from UBC participated for course credit.

Stimuli and Procedure

There were 200 trials in total. In each trial, participants were presented with two back-to-back sequences of 15 green and blue circles (Figure 3a). The color and size of the circles were identical to those used in Experiment 2. The sequences were generated with one of the five switch rates (0.1 to 0.9) as before. There were 40 trials per switch rate, 20 of which contained a change where the color of one randomly selected circle was different between the two sequences, and 20 of which contained no change where the two sequences were the same. In each trial, all circles in the first sequence were presented simultaneously at the center of the screen for 500ms, with an ISI of 500ms, followed by the second sequence also presented for 500ms. Participants had to judge whether the two sequences were the same or different by pressing the “Y” key or the “N” key, respectively. The trials were presented in a random order.

Results and Discussion

To examine the performance of the change detection task, we calculated A’ by dividing the average of correct rejection rate and correct hit rate by two, then adding 0.5 to the resultant number (Pollack & Noman, 1964). A’ was plotted across the five generating switch rates (Figure 3b). There was a reliable difference in A’ across the five rates [$F(4,176)=24.64$, $p<.001$, $\eta_p{}^2=0.38$]. Post-hoc Tukey HSD analysis showed all pair-wise comparisons were significant except for between 0.5, 0.7, or 0.9.

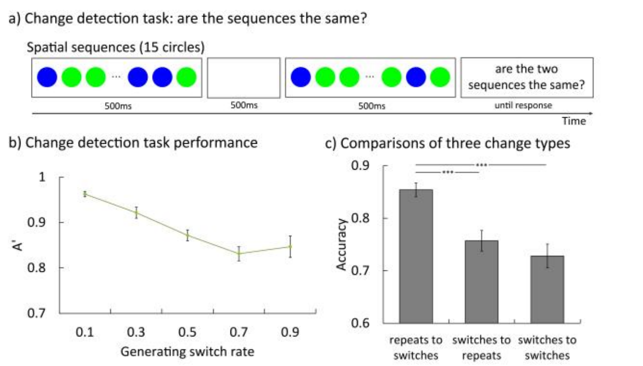

Figure 3. Experiment 3. (a) Participants (N=45) viewed two back-to-back sequences, and judged if the two sequences were the same or different. (b) Performance was assessed using A’. (c) Trials with changes were categorized into three change groups: 1. repeats to switches, 2. switches to repeats, and 3. switches to switches. (Error bars reflect ± 1 SEM; ***$p<.001$)

We also examined change detection accuracy depending on the local environment where the change occurred. For all change trials, we categorized them into three groups: repeats to switches (e.g., 000 to 001, 010, or 100), switches to repeats (e.g., 010, 001, or 100 to 000), and switches to switches (e.g., 001 to 011 or 101, 010 to 110 or 011, 100 to 101 or 110). Since we only considered trials where a change occurred, there was no false alarm. Therefore, we used accuracy as the measure here (Figure 3c). Among the three types changes, there was a reliable difference in accuracy [$F(2,88)=55.95$, $p<.001$, $\eta_p{}^2=0.56$]. Post-hoc Tukey HSD analysis showed that accuracy in the repeats to switches group was reliably higher than that in the switches to repeats and switches to switches groups [p’s<.001].

These results showed that as the sequence became more alternating, a change in the sequence was harder to detect. This suggests that repetitions were more salient than alternations. Moreover, a change was more salient when a streak was interrupted, than when an alternating pattern became streaky or remained alternating. This differential performance suggests that people may have paid more attention to the streak presented in the first sequence, than to the switches presented in the first sequence.

Experiment 4

To provide further support for the salience account, Experiment 4 used a visual search task to measure attention to switching vs. repeating sequences.

Participants

Forty-five students (33 female, mean age=19.6 years, SD=2.1) from UBC participated for course credit.

Stimuli and Procedure

As in Experiment 3, there were 200 trials, and in each trial, a sequence containing 15 colored circles were presented simultaneously on the screen. As before, the sequences were generated with one of the five switch rates, and there were 40 trials per switch rate. For each trial, participants had to search for a target (a red arrow pointing left “<” or right “>”) in one of the randomly selected circles in the sequence. They were asked to identify the direction at which the arrow was pointing as fast and as accurately as they could (Figure 4a). Half of the trials contained an arrow pointing left, and the other half contained an arrow pointing right. Each sequence was presented for 1500ms. The trials were presented in a random order.

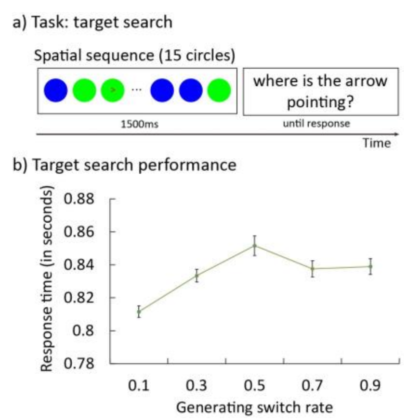

Figure 4. Experiment 4. (a) Participants viewed 15-bit spatial sequences. The target was a small red arrow, pointing to the left or right, in one of the circles. Participants reported the direction of the arrow as fast and as accurately as they could. (b) Response time of correct trials was plotted. (Error bars reflect ± 1 SEM)

Results and Discussion

The accuracy of the target search task was high (mean=97.5%, SD=2%). Thus, we only examined the response times of correct trials as our measure of attention (Figure 4b). There was a reliable difference in response time across the five switch rates [$F(4,176)=2.55$, $p<.05$, $\eta_p{}^2=0.05$]. Post-hoc Tukey HSD analysis showed a reliable difference in response times only between switch rates 0.1 and 0.5. This result showed that participants were faster to find the target in sequences with more repetitions than with more switches. One explanation is that repeating sequences may draw attention more strongly than switching sequences.

General Discussion

The goal of the current study was to examine how people represent alternations vs. repetitions in a binary sequence. Across four experiments using estimation, working memory, change detection, and visual search tasks, we found that the number of alternations was under-estimated more strongly than the number of repetitions (Experiment 1). This under-estimation of switches could be explained by the fact that recall accuracy diminished as the sequence became more alternating (Experiment 2). The greater encoding difficulty of alternations could be explained by the finding that changes were harder to detect as the sequence became more alternating (Experiment 3). Finally, visual targets were slower to be found as the sequence became more alternating, suggesting that alternating sequences draw attention less strongly than repeating sequences (Experiment 4). Overall, these results converge to support the same finding that people are more blind or insensitive to alternations than to repetitions, which suggests that alternations are under-represented compared to repetitions.

The current findings support a new account on the over-alternation bias. Specifically, there is a perceptual limitation in the ability to accurately represent alternations as opposed to repetitions in a binary sequence. This means that for people to perceive a 0.5 switch rate, the sequence must contain more than 50% alternations (in fact around 70%).

Why are alternations under-represented compared to repetitions? We offer two explanations. First, two alternating bits (e.g., 10) may be perceptually more complex than two repeating bits (e.g., 11), and this higher complexity in an alternation could be more difficult to encode. Second, people may implicitly chunk an alternation into a unit (e.g., perceiving 101010 as three chunks of 10, Zhao & Yu, 2016), but rely on numerosity perception for repetitions (e.g., perceiving 111111 as 1 repeating five times).

The current study reveals a perceptual limitation in the representation of alternations. The study is important in several ways: first, it provides a new explanation of the over-alternation bias in randomness perception; second, it reveals new insights on the limits in the perception of binary information; and finally, the same finding was replicated in four different paradigms using different measures. The current findings shed light on how people process binary information, which is fundamental to understanding the limits of the cognitive system.

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