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DIRECTIVE RADIATION FROM A LINE SOURCE IN A METAMATERIAL SLAB WITH LOW PERMITTIVITY

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Abstract

An investigation is made into the directive radiation patterns produced by an electric line source inside of a low-permittivity metamaterial slab over a ground plane. The metamaterial slab is modeled as a homogeneous slab with a plasma-like dispersive permittivity. For low values of the slab permittivity, a very directive beam pointing at broadside can be obtained. Conditions for the maximization of the power density radiated at broadside are given, and it is shown that the directive radiation effect at broadside is due to a leaky mode supported by the slab with small and nearly equal values of the phase and attenuation constants. The frequency bandwidth and the directivity at broadside are also expressed in a simple closed form in terms of the attenuation constant of this leaky wave. Finally, numerical results are obtained for an actual metamaterial slab composed of metallic rods.

I. Introduction and background

It is well known that under suitable conditions, periodic structures can be homogenized and modeled as metamaterials with an effective permittivity and permeability, presenting some interesting features that natural materials do not possess. In particular, a structure composed of a periodic array of metallic cylinders with a spatial period small compared to the wavelength (see Fig. 1(a)) behaves as a homogenous material with an effective permittivity that exhibits a plasma-like behavior [1-2]. Recently, the issue of enhancing the directivity at broadside of a source embedded in an artificial medium and controlling the direction of its radiation has been addressed by several authors, both in the optical and in the microwave ranges [3-5]. Some of these effects have been observed previously in a different context [6].

In this paper it is shown how the extremely directive radiation attainable with such low-permittivity metamaterial structures is related to the excitation of a leaky mode with a sufficiently-low value of the attenuation constant, and we derive simple design formulas to optimize radiation at broadside in such periodic structures, based on a modal analysis of their equivalent homogenous models.

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II. Analysis and optimization

The structure considered here is a grounded plasma slab of height h excited by an electric line source $\boldsymbol{J}_{\mathrm{i}}$ along y, embedded in the plasma medium at a distance h_s from the ground plane (see Fig. 1(b)).

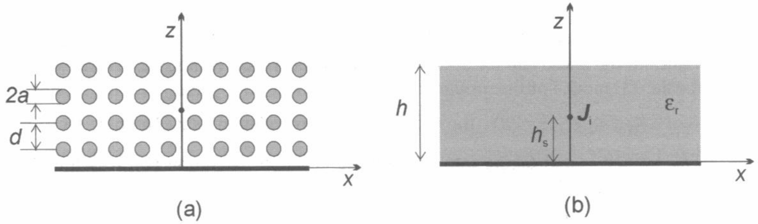


Fig. 1 – (a) Example of EBG material excited by an electric line source: a finite number of periodic rows of metallic cylinders with radius a and spatial period d, small with respect to the wavelength, above a perfectly-conducting ground plane; (b) equivalent homogenized structure, modeled as a grounded plasma slab with relative permittivity ε_r.

The plasma is modeled as homogeneous and isotropic, with relative permeability $\mu_r = 1$ and relative permittivity $\varepsilon_r = 1 - f_p^2 / f^2$, where f_p is the plasma frequency. From the spectral Green's function of the problem, known in a simple closed form, the radiated power density at broadside ($\theta = 0$) can be obtained:

$$P(0) = \frac{k_0\eta_0}{4\pi} \frac{\sin^2\left(k_0 h_s \sqrt{\varepsilon_r}\right)}{\left|\sqrt{\varepsilon_r}\cos\left(k_0 h\sqrt{\varepsilon_r}\right) + j\sin\left(k_0 h \sqrt{\varepsilon_r}\right)\right|^2}, \tag{1}$$

where k_0 and η_0 are the free-space wavenumber and characteristic impedance, respectively. Since $|\varepsilon_r| < 1$, in order to maximize $P(0)$ we require that the sine term in the denominator of (2) be equal to zero, i.e,

$$k_0 h \sqrt{\varepsilon_r(f)} = n\pi \qquad n = 1, 2, \ldots \tag{2}$$

The location of the source should be chosen in order to maximize the numerator in (1), i.e., $h_s = h/2$. From (2), an explicit expression for the optimum frequencies f_{opt} can then be obtained in terms of f_p and h, i.e., $f_{\mathrm{opt}} = \left(f_{\mathrm{p}}^2 + n^2 c^2 / 4h\right)^{1/2}$, where c is the speed of light in free space. Moreover, when (2) holds, at the optimum frequency the power density at broadside is inversely proportional to the relative permittivity:

$$P(0) = P_{\max} = \frac{k_0\eta_0}{4\pi}\frac{1}{\varepsilon_r(f_{\mathrm{opt}})}, \tag{3}$$

and this result clearly shows the importance of metamaterials with *low* values of pemittivity to obtain strong field values at broadside.
The bandwidth for radiation at broadside has also been studied, defined as the frequency range $f_{3\mathrm{dB}}^- < f < f_{3\mathrm{dB}}^+$ where the level of power density $P(0)$ radiated at

broadside is 3 dB or less lower than its maximum P_{max} reached at $f = f_{opt}$. The result for the fractional bandwidth, valid in the limit of small permittivity of the slab, is

$$\text{FBW} = \frac{f_{3dB}^{+} - f_{3dB}^{-}}{f_{opt}} \simeq \frac{c^3}{4\pi f_p^3} \frac{1}{h^3}. \tag{4}$$

The optimum condition (2) implies the presence of a TE_z leaky wave with nearly equal values of the phase and the attenuation constants. To show this, the dispersion equation for the TE modes of the grounded plasma slab is used together with the representation $\hat{k}_x = k_x / k_0 = \hat{\beta} - j\hat{\alpha}$ for the unknown propagation wavenumber. By assuming $|\hat{k}_x| \ll \sqrt{\varepsilon_r}$ and taking into account the optimum condition (2), it is found, in the limit of small ε_r, that

$$\hat{\beta} \simeq \hat{\alpha} \simeq \sqrt{\frac{\varepsilon_r^{3/2}(f_{opt})}{n\pi}} \simeq \sqrt{\frac{n^2 c^3}{8\pi f_p^3 h^3}}. \tag{5}$$

Finally, expressions for P_{max} and FBW can be obtained in terms of the normalized attenuation constant $\hat{\alpha}$ at the optimum frequency f_{opt}. Using (5), the results from (3) and (4) may be expressed as

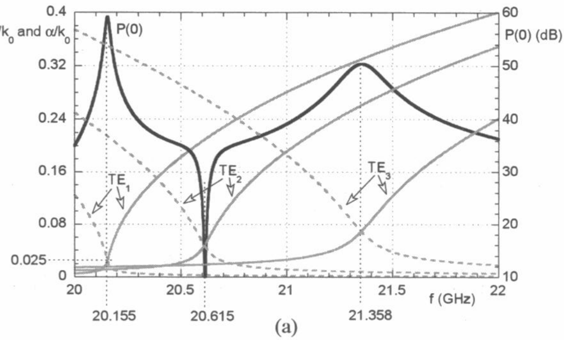
$$P_{max} \simeq \frac{k_0 \eta_0}{4\pi^{5/3}} \frac{1}{\hat{\alpha}^{4/3}}, \qquad \text{FBW} \simeq 2\hat{\alpha}^2. \tag{6}$$

It is interesting to note that the bandwidth limitation is quite severe when using this type of metamaterial leaky-wave antenna.

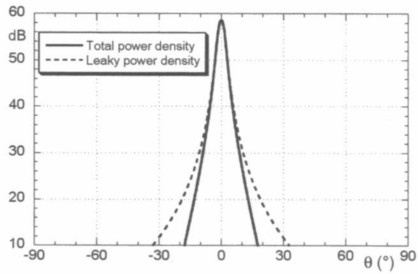
III. Numerical results

In order to validate the above analysis, numerical results for a specific plasma grounded slab with f_p = 20 GHz, h = 60 mm, and $h_s = h/2$ are shown in Fig. 2. In Fig. 2(a), the dispersion curves of the first three TE_z leaky modes are reported together with the power density radiated at broadside, $P(0)$, as a function of frequency. As expected, $P(0)$ has its maxima at the frequencies for which a leaky mode has equal values of its phase and attenuation constants. Moreover, the approximate formulas for both the optimum frequencies and the leaky-mode phase and attenuation constants are seen to be in excellent agreement with the exact ones calculated by solving numerically the TE_z dispersion equation. In Fig. 2(b), the total radiation pattern is compared with the pattern radiated by the leaky mode excited along the structure; their close agreement confirms the dominant role of the leaky mode in producing the high directivity.

Finally, full-wave numerical results have been obtained through a periodic moment-method analysis of the structure in Fig. 1(a) with d = 20 mm, a = 0.5 mm and N = 6 rows of cylinders. By considering the equivalent homogeneous grounded-slab model of the structure, with $h = Nd$ (see [2]) and f_p = 3.841 GHz (according to the approximate formula reported in [7]), it is found that f_{opt} = 4.039 GHz (see equation after (2)), which is slightly shifted from the exact value (f_{opt} = 4.01 GHz) obtained from the full-wave numerical analysis of the periodic structure. The pattern and leaky wave phenomenology observed for the homogeneous material are in good agreement with those obtained from the numerical analysis of the structure in Fig. 1(a).



(a)



(b)

Fig. 2 – Results for a structure as in Fig. 1(b) with f_p = 20 GHz, h = 60 mm, and $h_s = h/2$. (a) Normalized leaky-mode phase (*solid gray lines*) and attenuation (*dashed gray lines*) constants and power density radiated at broadside (*solid black line*) as a function of frequency. (b) Comparison between the total radiated pattern and that due to the dominant leaky mode at f_{opt} = 20.155 GHz.

References

[1] J. Brown, "Artificial dielectrics having refractive indices less than unity," *Proc. IEE*, vol. 100, pp. 51-62, May 1953.

[2] G. Guida, D. Maystre, G. Tayeb, and P. Vincent, "Mean-field theory of two-dimensional metallic photonic crystals," *J. Opt. Soc. Am. B*, vol. 15, pp. 2308-2315, 1998.

[3] A. Fehrembach, S. Enoch, and A. Sentenac, "Highly directive light sources using two-dimensional photonic crystal slabs," *Appl. Phys. Lett.*, vol. 79, pp. 4280-4282, Dec. 2001.

[4] M. Thèvenot, C. Cheype, A. Reineix, and B. Jecko, "Directive photonic bandgap antennas," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 2115-2122, Nov. 1999.

[5] S. Enoch, G. Tayeb, P. Sabouroux, N. Guèrin, and P. Vincent, "A metamaterial for directive emission," *Phys. Rev. Lett.*, vol. 89, pp. 213902-1-213902-4, Nov. 2002.

[6] T. Tamir and A. A. Oliner, "The influence of complex waves on the radiation field of a slot-excited plasma layer," *IRE Trans. Antennas Propagat.*, AP-10, pp. 55–65, Jan. 1962.

[7] P. A. Below, S. A. Tretyakov, and A. J. Viitanen, "Dispersion and reflection properties of artificial media formed by regular lattices of ideally conducting wires," *J. Electromag. Waves Applicat.*, vol. 16, no. 8, pp. 1153-1170, 2002.