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Interpreting logic diagrams: a comparison of two formulations of diagrammatic representations

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Abstract

In the context of the cognitive study of diagrammatic representations for deductive reasoning, the availability of syntactic manipulation of diagrams has played a key role in accounting for their efficacy. Currently, however, little has been known about the interface between such syntactic or proof-theoretical aspects and the corresponding semantic or informational aspects of diagram use. The present paper investigates the cognitive processes of interpreting diagrammatic representations underlying deductive reasoning, combining the insights from both logical and cognitive studies of diagrams. A semantical analysis of two different ways of formalizing logic diagrams is provided. Based on it, a multiple stage model of cognitive processes of extracting information from logic diagrams is proposed, and evidence was found to support this model. A consequence for the way the abstract syntax and semantic of diagrammatic representations are constrained is also explored.

Keywords: Diagrammatic reasoning; Logic diagram; Semantic interpretation of sentences and diagrams; Deductive reasoning; External representation.

Introduction

Over the past few decades, many researchers have shown an interest in logical and cognitive aspects of reasoning with diagrammatic representations (e.g. Glasgow, Narayanan, & Chandrasekaran, 1995). In particular, diagrammatic representations used for deductive reasoning, which were traditionally regarded as an auxiliary device to help understanding of quantificational and set-theoretical formalisms in logic textbook, have been intensively studied using the method of mathematical logic (e.g. Allwein & Barwise, 1996; Shin, 1994). The formal and mathematical study of diagrammatic representations have yielded fruitful insights into the cognitive aspects of the use of diagrammatic representations.

As an illustrative example, let us explain with Euler and Venn diagrams that externally support processes of solving deductive reasoning tasks. What plays a crucial role in accounting for efficacy of diagrams in problem solving is the existence of *syntactic manipulations* of diagrams, whose structure has been studied in the tradition of the logical study of diagrams. When logic diagrams are externally given to reasoners, solving processes of deductive reasoning tasks could be replaced by processes of combining premise diagrams and extracting the relevant information. Thus a syllogism consisting of two premises, *All A are B* and *No C are B*, and a conclusion *No C are A* has a diagrammatic derivation with Euler diagrams, as illustrated to the left in Figure 1. Here the unified diagram D_3^e is obtained by identifying the circle B in the two premises and reading off the relation between the circle A and C, i.e., the disjointness relation, which is *automatically* determined by the unifying process (cf. Shimojima, 1996). Based on experiments on syllogism solving tasks aided by logic diagrams, Sato, Mineshima and Takemura (2010a,b), provided evidence which suggested that the syntactic manipulation of logic diagrams could be available even to untrained users.

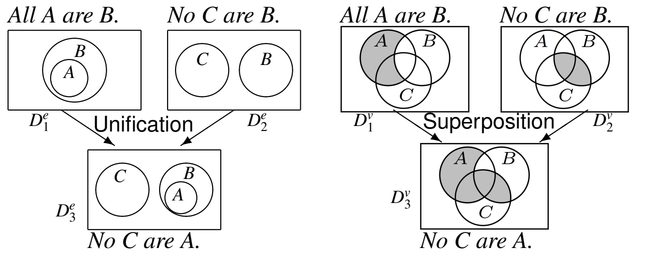

Figure 1: A diagrammatic derivation of a syllosigm *All A are B, No C are B; therefore No C are A* with Euler diagrams (left) and Venn diagrams (right).

In the tradition of the logical and cognitive study of diagrams, particular emphasis has been on comparison between diagrammatic and sentential (linguistic) representation systems. In addition to the availability of concrete manipulations, a number of properties which distinguish diagrammatic from sentential representations have been proposed, seeking to account for what advantages diagrammatic representations in general have over sentential ones (e.g. Stenning, 2000; Shimojima, 2001). By contrast, relatively few attention has been paid to comparison between different diagrammatic representation systems. However, such a comparison might be potentially important to provide a more fine-grained analysis of efficacy of various diagrams in human problem solving.

As a crucial example, consider a solving process of the syllogism we saw above with Venn diagrams. It is shown to the right in Figure 1. In Venn diagrams, every circle partially overlaps each other, and then meaningful relations among circles are expressed by *shading* under the convention that shaded regions denote the empty set. Given this semantics, the process of composing two premise diagrams automatically yields the information corresponding to the correct conclusion, in a similar way to the process in Euler diagrams. Intuitively, however, reasoning with Venn diagrams appears to be relatively more difficult to handle in reasoning. As emphasized in Gurr, Lee and Stenning (1998), what differentiates the two cases is the process of *interpreting* (externally given or internally constructed) diagrammatic representations. In particular, a conventional device such as shading involved might cause complication in interpretation processes of Venn diagrams. This suggests that in order to obtain a more comprehensive account of diagrammatic reasoning, we need to take a closer look at cognitive processes underlying *information extraction* from diagrams.

Traditionally, the notion of "similarity" has played a role in accounting for differences in efficacy of semantic aspects of sentential and diagrammatic representations. Generally speaking, if a certain structural similarity holds between a representation and what it represents, the representation could be effective in interpretation and communication even for users who do not learn conventions governing its use explicitly. In the literature, such a notion of similarity has been specified in various ways: "homomorphism" (Barwise & Etchemendy, 1991), "directness" (Gurr, Lee, & Stenning, 1998; Stenning, 2000); "structural similarity" (Gattis, 2004). However, a precise characterization of the notion of similarity that could be applied to a varieties of diagrammatic representations remains to be explored. In particular, how a cognitive account of interpretation processes could be connected to the formal semantics of diagrams still remains unclear.

The question of information extraction has also been investigated in the study of the cognitive role of relatively simple use of diagrams, such as charts, maps, and graphs (e.g. Ratwani, Trafton, & Boehm-Davis, 2008; Shimojima & Katagiri, 2010). What plays an important role here is the distinction between lower-level and higher-level information. For example, a scatter plot contains the lower-level information about specific data and the overall distribution of the dots delivers the higher-level information about the structural properties of data (Kosslyn, 1994). Currently, however, applications of these findings to an analysis of higher cognitive processes, such as deduction reasoning, were not fully explored.

The present paper aims to investigate the cognitive processes of interpreting diagrammatic representations underlying deductive reasoning, combining the insights from these different research traditions. In particular, based on a semantical analysis of diagrams, we argue that a certain structural correspondence between a diagrammatic representation and its semantic contents plays a crucial role in both interpretation and inference processes with the representations. Our approach can also provide a further *constraint* on the choice of different ways of formalizing the abstract syntax and semantics of diagrammatic representations, motivating a more fruitful way of approaching to the logical study of diagrams. This could contribute to establishing a closer connection between the logical and cognitive approach to human problem solving with diagrammatic representations.

General Hypothesis

A main goal of the present study is to explore the hypothesis that the "matching" relation between the diagrammatic representation used in deductive reasoning and the conveyed information available to users plays an important role in effective diagrammatic reasoning. As a case study, we focus on the use of logic diagram in syllogistic reasoning. In order to make clear what is the relevant structural relationship between logic diagrams used and their semantic contents, it is helpful to first look at what semantic information is carried by syllogistic sentences, using the insight obtained in the semantics of natural language.

1. The relational analysis of quantified sentence. From a general viewpoint, syllogistic inferences as investigated in the psychology literature can be regarded as a special case of inferences with *quantificational* sentences in natural language. According to the standard textbook treatment, such sentences are analyzed using representations in first-order predicate logic, which essentially involve quantification over *individuals* as semantic primitives. In the field of natural language semantics, by contrast, quantifiers in natural language, such as *all*, *some* and *no*, are analyzed as denoting *relations* between sets, i.e., what is called *generalized quantifiers* (Barwise & Cooper, 1981). Thus, a sentence of the form *All A B* is analyzed as $\mathbf{A} \subset \mathbf{B}$, rather than as the first-order representation $\forall x(Ax \rightarrow Bx)$. Similarly, *No A B* can be analyzed as expressing $\mathbf{A} \cap \mathbf{B} = \emptyset$. Here the semantic primitives of quantificational sentences are considered as the relations between sets, such as subset relation and disjointness relation. Interestingly, the modern reconstructions of Aristotelian categorical syllogisms (Łukasiewicz, 1958) and recent development of the so-called *natural logic* (van Benthem, 2008) take as a primitive logical form the relational structure of a quantified sentence, which is schematically represented as *Q(A,B)*. These logical findings suggest that syllogistic inferences can be formulated as *transitive* inferences in a perspicuous way, without reference to individuals terms.

It should be emphasized that such a "relational" formulation of the meaning of a quantified sentence could capture not only the truth condition or logical form of a sentence, as is traditionally assumed, but also *how* speakers mentally represent such a truth condition or logical form (Hackl, 2008; Pietroski et al., 2009). The relational approach to quantificational sentences has also been successfully applied to the psychological study of deduction, resulting in a processing model based on the assumption that inferences with quantifiers are done in terms of sets rather than individuals (Geurts, 2003). For a discussion on the role of relational knowledge in general in higher cognition, see Halford, Wilson, & Phillips (1998, 2010)

In sum, we may plausibly assume that the semantic primitives of quantificational sentences in natural language are *relations* between sets, and that people's inferences with quantified constructions are sensitive to such a relational structure.

2. Relational analysis of Euler diagrams. In the logical study of Euler and Venn diagrams (e.g. Shin, 1994), diagrammatic representations have been given their own formal syntax and semantics, in a similar way as for formulas in mathematical logic. What is remarkable here is that a diagram may have several "equivalent" formalizations. As an illustrative example, consider a simple Euler diagram E in Figure 2. This diagram can be naturally interpreted as denoting the subset relation between sets $\mathbf{A}$ and $\mathbf{B}$, i.e., $\mathbf{A} \subset \mathbf{B}$. But it is also possible to interpret it as expressing that $\mathbf{A} \cap \overline{\mathbf{B}} = \emptyset$, where

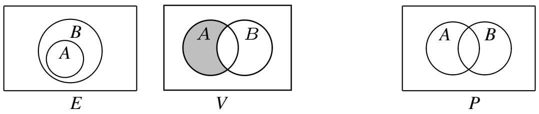

Figure 2: Examples of Euler diagram E and Venn diagram V that correspond to the sentence *All A are B*. The diagrams P is a so-called "plain" diagram, which expresses a tautology.

$\overline{\mathbf{B}}$ denotes the complement of the set $\mathbf{B}$. Correspondingly, there are two ways of formalizing the abstract syntax of Euler diagrams (Mineshima, Okada & Takemura, 2010; 2011). According to what is called a "relational" approach, Euler diagrams are abstractly specified as a set of topological relations holding between objects in the diagrams. For example, the diagram E in Figure 2 is represented as $\{A \sqsubset B\}$, where $A \sqsubset B$ means that circle A is inside circle B. Another approach is a "region-based" approach, which is fairly standard in the logical study of diagram (e.g. Howse, Stapleton, & Taylor, 2005). Here diagrams are abstractly defined in terms of regions and emptiness. Thus the diagram E in Figure 2 can be represented by specifying the region $(A, \overline{B})$, the region inside circle A and outside circle B, as a "missing" region.

Clearly, these two ways of defining Euler diagrams predict, for each diagram, the equivalent truth-condition. The difference consists in the way these truth-conditions are given. Now the question is: which formulation (or possibly others) reflects the way the user represents the semantic content of a given diagram? Here the region-based formulation appears to be more natural for the meaning of the Venn diagram such as the diagram V shown in Figure 2. Given the convention that the shaded region denotes the empty set, $\mathbf{A} \cap \overline{\mathbf{B}} = \emptyset$ has the syntactic reading "the region inside the circle A but outside the circle B denotes the empty set", or more colloquially, "there is nothing which is A but not B". It should be noted here that throughout our discussion, we are assuming that both Euler and Venn diagrams adopt the convention that each *unshaded* region lacks existential imports, i.e., may denote the empty set. Thus the diagram P in Figure 2, where the circles A and B partially overlap each other, conveys semantically tautologous information. In other words, this diagram means that the semantic relationship between A and B is indeterminate. In this respect, our semantics differs from the one discussed in Johnson-Laird, 1983 (for a more detailed discussion of the semantics of Euler and Venn diagrams, see Hammer & Shin, 1998 and Sato, Mineshima, & Takemura, 2010a). The two ways of formulating logic diagrams were then summarized as follows.

	building blocks	meaningful units (semantic premitives)
Relation-based analysis	circles	relations between circles
Region-based analysis	regions	non-emptiness of minimal regions

Our basic hypothesis is that an Euler diagram like E in Figure 2 expresses relational information that could be accounted for by the relation-based analysis; it triggers a relational representation such as $\mathbf{A} \subset \mathbf{B}$ to the users. By contrast, Venn diagrams are subject to a region-based analysis, triggering the semantic information such as $\mathbf{A} \cap \overline{\mathbf{B}} = \emptyset$ to their users.

We saw above that syllogistic sentences are quantificational sentences of the relational form, schematically represented as $Q(A,B)$, and that such sentences force a reasoner to form and operate on relational representations in reasoning. If our basic hypothesis above is correct, Euler diagrams directly express the topological relationship between circles. Thus it is hypothesized that when a reasoner is asked to match a syllogistic sentence with a corresponding Euler diagram (or vise versa), he could appeal to the process of reading off the relational information from an Euler diagram immediately, i.e., without any intermediate steps, and then verifying that it is the same as the information conveyed by the sentence.

In contrast, Venn diagrams have a fixed configuration of circles and represent set relationships *indirectly*, by stipulating that shaded regions denote the empty set. Accordingly, the process of extracting the relevant relational information from a Venn diagram would be expected to proceed in several steps. As a concrete example, consider how the reasoner could extract the correct relational representation from the Venn diagram V in Figure 2 above. Let us call a region inside of some circles and outside of the rest of the circles (possibly none) in a diagram a *minimal* region. Thus the diagrams V has four minimal regions. Firstly, the reasoner needs to check each minimal region whether it is shaded or not, as lower-level information. In this example, only the region $(A, \overline{B})$ is shaded. Next, the reasoner internally builds the *segments* that lumps together the shaded minimal regions continuous with each other, as well as those which lump together the unshaded minimal regions. This step makes it possible for the reasoner to conclude that the diagram delivers the higher-level information "There is nothing which is A but not B". Then the reasoner would be able to paraphrase it as "All A are B", which corresponds to the required information in syllogistic inferences. It is thus hypothesized that such complexities would cause some difficulties in extracting the required information from Venn diagrams.

Experiment 1

As an initial test of our hypothesis, we conducted a "sentence-diagram" matching test, in which participants were presented with a syllogistic sentence and asked to choose the diagram expressing the same information. In Experiment 1, we used a simple form of Venn diagrams consisting of two circles (see the diagram V in Figure 2), rather than three. In order to exclude external factors such as familiarity with presented diagrams, participants were provided with sufficient informal explanation of the semantics of diagrammatic representations.

Method

Participants Twenty-seven undergraduates and graduates (mean age 22.34 $\pm$ 3.27 SD) took part in the experiment. They gave a consent to their cooperation in the experiment, and were given small money after the experiment. All the students were native speakers of Japanese and task sentences and instructions were given in Japanese. The participants were divided into two groups: Euler group (13 students) and Venn groups (14 students).

Materials The syllogistic sentences used in the experiment are divided into *existential* and *non-existential* sentences. They are of the following patterns:

Non-existential sentences	Existential sentences
(1) All A are B.	(5) Some A are B.
(2) All B are A.	(6) Some B are A.
(3) No A are B.	(7) Some A are not B.
(4) No B are A.	(8) Some B are not A.

The participants were presented with one sentence in a PC monitor and required to choose the corresponding diagram (if any). Figure 3 shows templates of tasks for the two groups.

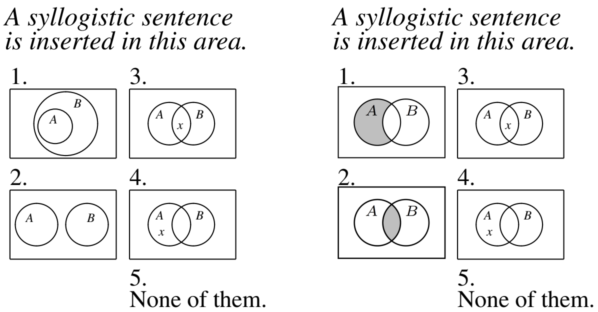

Figure 3: Templates of task sentences with Euler diagrams (left) and Venn diagrams (right) used in the experiment.

Here, in both diagrams, sentence *All A are B* corresponds to Answer 1, *No A are B* and *No B are A* to Answer 2, *Some A are B* and *Some B are A* to Answer 3, and *Some A are not B* to Answer 4. Recall that a diagram in which circles partially overlap each other does not express any specific semantic relationship between them (see the diagram P in Figure 2). In order to express the existence of objects (i.e., the non-emptiness of a set), then, we use the point x. As a consequence, in Euler and Venn diagrasm, existential sentences are represented in the same way as indicated in Figure 3. Note also that the sentences *All B are A* and *Some B are not A* have no corresponding diagram, hence the correct answer is 5 ("None of them"). Stimuli were presented randomly. After a task sentence and four diagrams appeared, the participants were asked to press one of five buttons. There is no time limit to solve the matching tests.

Procedure The experiment was conducted individually.
(1) **Instruction and pretest.** Before the test, the participants were provided with instructions on the meaning of sentences and diagrams used. A pretest to check whether they understand the instructions correctly was conducted; they were presented with ten diagrams of basic forms and asked to choose all the sentences (if any) that have the same meaning as given diagrams. After the pretest, the experimenter told the participants whether they answered to the problems correctly. When an incorrect answer was found, they were asked to reread the instruction and to select the correct answer.
(2) **The matching task.** One task example was displayed in a PC monitor. A total of eight different types of sentences were prepared. The participants were asked to press, as quickly and as accurately as possible, a button with the number representing the answer they reached.

Prediction It is predicted that for non-existential sentences, the response time to choose Euler diagrams would be shorter than the response time to choose Venn diagrams. For existential sentences, Euler and Venn diagrams have the same form (see Figure 3 above), hence it is predicted that there would be no difference between the two cases.

Results and discussion

Among the twenty-seven students, we excluded four students (one in the Euler group and three in the Venn group), who did not answer correctly at all or gave only one correct answer.

Table 1 shows the average response times in the sentence-diagram matching tasks for Euler and Venn diagrams.

Table 1: The response times of the sentence-diagram matching task with Euler and Venn diagrams.

	non-existential sentence	*existential sentence*
Euler diagrams	07.286s	09.298s
Venn diagrams	11.057s	10.127s

These data were subjected to a two-way Analysis of Variance (ANOVA) for a mixed design. There was no significant main effect for the difference between Euler and Venn diagrams (F (1, 21) = 2.203. $p > .10$). There was no main effect for the difference between existential and non-existential sentences (F (1, 21) = 0.484. $p > .10$). There was a significant interaction effect for these two factors (F (1, 21) = 3.575. $p < .10$). Multiple comparison tests were conducted by Ryan's procedure. The results indicated that (i) regarding non-existential sentences, the response times in the sentence-diagram matching task for Euler diagrams were significantly shorter than those for Venn diagrams (F (1, 42) = 4.730, $p < .05$), and that (ii) regarding existential sentences, there was no significant difference in performance between the Euler group and the Venn group (F (1, 42) = 0.270, $p > .10$). We note that the average accuracy rates for both types of diagrams were very high (more than 82 %). Furthermore, no significant difference was shown by changing the order of terms in presented sentences, for example, between *No A are B* and *No B are A*.

The overall results provide partial evidence for our hypothesis that the process of extracting relational information from Euler diagrams to match it with sentence meaning would be simple and immediate, whereas in the case of Venn diagrams it could be more complicated.

Experiment 2

In order to provide a further support for our hypothesis, we conducted a "diagram-sentence" matching test, in which participants were presented with a diagrams and asked to select the sentence conveying the same information. In Experiment 2, we used Euler and Venn diagrams consisting of *three* circles as in Figure 1, which is expected to be more sensitive to the difference in complexity of information-extracting processes for the two types of diagrams.

Method

Participants Twenty-three undergraduates and graduates (mean age 22.73 $\pm$ 2.41 SD) took part in the experiments. They gave a consent to their cooperation in the experiments, and were given small money after the experiments. The subjects were native speakers of Japanese, and the task sentences and instructions were given in Japanese. The participants were divided into two groups: Euler group (12 students) and Venn groups (11 students).

Materials Eleven Euler diagrams and the corresponding eleven Venn diagrams were used in the "diagram-sentence" matching task. They are shown in Figure 4 and Figure 5.

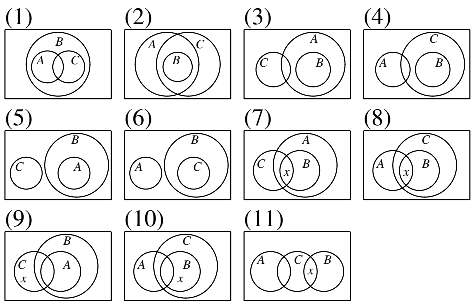

Figure 4: The Euler diagrams used in Experiment 2

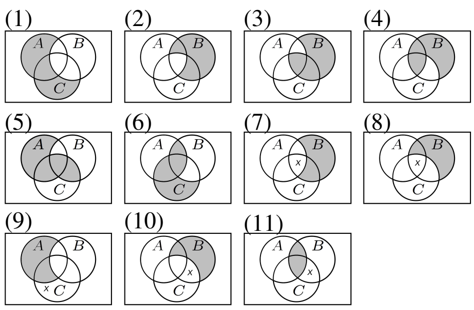

Figure 5: The Venn diagrams used in Experiment 2

Task examples for Euler and Venn groups are shown in Figure 6. The participants were presented with eleven Euler diagrams (Venn diagrams), and were asked to choose all the sentences (if any) that express the same information as a given diagram. There were five answer options: *All-*, *No-*, *Some-*, *Some-not*, and *None of them*, as indicated in Figure 6. Stimuli were presented randomly. When task diagrams and sentences

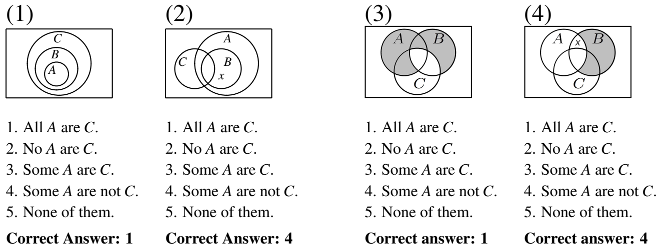

Figure 6: Examples of the diagram-sentence matching task with Euler diagrams (1) (2) and Venn diagrams (3) (4).

appeared, the participants were asked to press, as quickly and as accurately as possible, one of the five buttons with the number corresponding to the answer they chose. There is no time limit to solve the tasks.

Procedure The experiment was conducted in the same manner as Experiment 1. The instructions of sentences and diagrams were provided, pretests were conducted, and then the matching task were imposed.

Prediction It is predicted that when diagrams do not contain a point x, the response time for Euler diagrams would be shorter than that for Venn diagrams. When Venn diagrams contain a point x, what users need to do is just to recognize the relationship between the point x and a relevant circle. In such cases, the relationship between circles is irrelevant, and thus the processes of identifying each minimal region as shaded or unshaded and constructing the relevant segments could be simply omitted. Hence it is expected that the response time for Venn diagrams that contain a point x would be shorter than that for those which do not. By contrast, when an Euler diagram contains a point x, there would be no difference with respect to whether it contains a point x or not, since in both cases what the users need to do is to check the relationship between two objects, i.e., the one between two circles or the one between a circle and a point.

Results and discussion

Table 2 shows the average response times in the "diagram-sentence" matching tasks with Euler and Venn diagrams. In this table, "no-point" stands for diagrams that do not contain point x and "point" stands for those which contain point x. These data were subjected to a two-way Analysis of Variance (ANOVA) for mixed design.

Table 2: The response times of the "diagram-sentence" matching tasks with Euler and Venn diagrams.

	no-point	*point*
Euler diagrams	10.137s	11.946s
Venn diagrams	20.435s	14.022s

There was a significant main effect for the difference between Euler and Venn diagrams, (F (1, 21) = 6.087. $p < .05$). There was no significant main effect for the difference between diagrams which contain points and those which do not, (F (1, 21) = 2.032. $p > .10$). There was a significant interaction effect for these two factors, (F (1, 21) = 6.480. $p < .05$). Multiple comparison tests were conducted by Ryan's procedure. The results indicated that (i) regarding diagrams without point x, the response times for Euler diagrams were significantly shorter than those for Venn diagrams (F (1, 42) = 11.919, $p < .005$), and that (ii) regarding Venn diagrams, the response times for diagrams that contain a point x were significantly shorter than those which do not (F (1, 21) = 7.885, $p < .05$). (iii) regarding Euler diagrams, there was no significant difference in the response between cases that contain a point x and those which do not (F (1, 21) = 0.627, $p > .10$). These results clearly support our predictions. It is noted that the average accuracy rates for both diagrams were very high (more than 83 %).

General discussion

The overall results of the two experiments provide evidence for our hypothesis about the multiple stage view on information-extraction from Euler and Venn diagrams. According to this view, a process of extracting relational information from Euler diagrams consists of a single step, whereas that from Venn diagrams consists of multiple steps, i.e., identifying shaded and unshaded minimal regions and constructing the segments corresponding to the terms in question.

These results are compatible with the findings in Sato, Mineshima and Takemura (2010b), where the performances in syllogisms solving tasks with Euler diagrams were compared with those with Venn diagrams involving three circles, and it was shown that the former was better than the latter. Sato, Mineshima and Takemura (2010b) also reported that the performances in syllogism solving with Venn diagrams involving *two* circles (see Figure 2) were worse than the three-circle case. They argued that this difference could be ascribed to the availability of internal syntactic manipulations of diagrams: in the case of Venn diagrams with two circles, such a syntactic manipulation (i.e. the process of combining two premise diagrams) would be simply unavailable to untrained users. However, the difference in performance between Euler and Venn diagrams with three circles remained to be accounted for, since both types of diagrams could easily trigger syntactic manipulations because of their uniform forms. The present paper provides an account of it based on the difference in information-extraction processes for these diagrams.

Together with these findings, the present study suggests a possibility of providing a model on the overall processes of reasoning with diagrams for deduction, where not only the availability of syntactic manipulations of diagrams but also a subtle difference in processes of extracting information from diagrams could make difference in effective diagrammatic inferences. It should be emphasized here that the present cognitive study could be fruitful in providing a further constraint or data on theorizing about proof theory (syntax) and model theory (semantics) for higher-level diagrammatic representations. Cognitive or procedural differences in semantically equivalent ways of specifying truth-conditions or logical forms of diagrams (and for that matter, sentences) tend to be neglected in theorizing about formal semantics and proof theory for those representations. Even in the study of diagrammatic logic, such theoretical investigations could be conducted independently of any cognitive considerations. The present study suggests a possibility of a more constrained and integrated framework to deal with both logical and cognitive aspects of diagram use.

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