Title
An Introduction to Auctions

Journal
Journal of Industrial Organization Education, 1(1)

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Publication Date
2006

An Introduction to Auctions†

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September 17, 2005

Revised April 21, 2006

The material can be covered in the last part of a first year graduate industrial organization course as well as an introduction to auctions course, in two or three lectures, using the lecture notes attached.

† These sets of lectures are based on lecture notes for the course ARE 202, which is a first year graduate course in the PhD program in the Agricultural and Resource Economics Department at U. C. Berkeley. These notes have greatly benefited from material covered in several industrial organization courses offered at the Economics graduate program at Berkeley and from comments from Jeffrey Perloff, James Dearden, and Jen Brown.

1. INTRODUCTION

An auction is a public sale of property or merchandise defined by explicit rules determining resource allocation and prices on the basis of bids from participants.

Four main types of auctions are considered theoretically and in practice. In the "First-price sealed-bid auction" (FPSB) a bidder does not observe the opponents' bids, and the object is won by the bidder submitting the highest bid. The winner pays her bid value and receives the object, and the losers pay nothing and receive nothing. The second type is the "Second-price sealed-bid auction" (SPSB), also called the Vickrey auction. The SPSB auction is like the FPSB auction, with one exception: the winner pays the second highest bid in the auction. The third auction type is the "English" or "Ascending-price drop-out auction". It works much like the game of chicken; bidders are active in the auction until they decide to drop out. In this auction, the price increases incrementally from 0, and bidders withdraw from the auction when price exceeds their willingness to pay. The auction ends when only one active bidder remains. This winner pays a price equal to the current price when second-to-last person exits. Finally, the fourth type is the "Dutch" or "Descending price auction", which is similar to the game of chicken in reverse. The price starts high and decreases until the first bidder agrees to pay the current price. In each of these auctions, the winner's surplus is her value minus the price she pays. Each loser's surplus is zero.

Most auction studies focus on the FPSB and English auctions, although recent studies have examined online auctions such as eBay. Typically, eBay auctions are modeled as SPSB auctions. Other recent work has studied multi-unit auctions and sequential auctions (see, for example, Cramton, Shoham and Steinberg (2005)).

The text is structured as follows: First, we analyze participation strategies and auction house revenue under several auction formats. Second, we discuss the main issues in auction design. We then discuss the main positive and normative goals of empirical work in auctions, and also study reduced form and structural approaches that have been developed and applied to empirical work. The last part of these notes presents a formal outline of the main issues in structural estimation of auctions in applied work. Here, we provide a primer on structural auction estimation—for two surveys, see Laffont et al. (1991) and Hendricks and Paarsch (1995). We also introduce the issue of the strategic behavior of buyers and collusion in bidding. We conclude with a paper identifying collusion in procurement auctions by Porter and Zona (1993).

Some of the many topics we are not covering formally here are multi-unit issues (see Hansen (1985) for an empirical paper on the topic), the introduction of entry, risk aversion[1], budget constraints (see Klemperer's (1998) Wallet Game), asymmetries

[1] In Vickrey and English auctions with risk aversion, it is still a dominant strategy to bid one's valuation. However, in Dutch and FPSB auctions, bidders may be more aggressive to increase the probability of winning, as in Holt (1979).

(Hendricks and Porter (1988), Wilson (1994) and Porter (1995)), affiliation and common values (Milgrom and Weber (1982), and Laffont (1996)), and auction design. A (far from complete) list of cited and suggested references is provided in the end of the text.

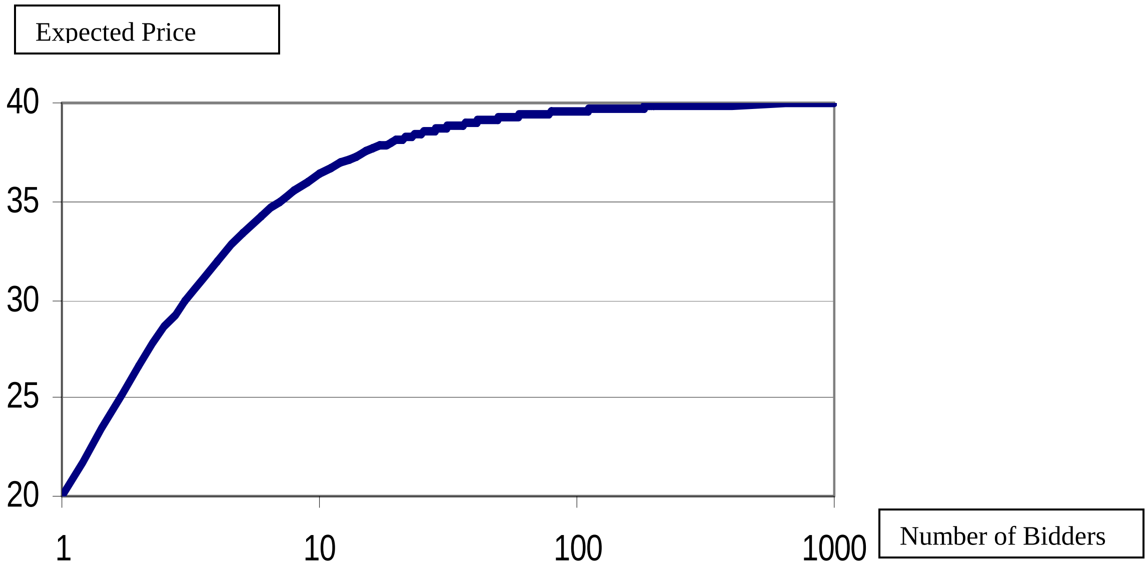
2. PARTICIPATION STRATEGIES AND REVENUE EQUIVALENCE

The Vickrey (Second-Price) Auction is a very popular format, perhaps due to the simple bidding strategy. Bidding one's true valuation is a (weakly) dominant strategy—the amount a bidder pays is independent of her bid and values are also independent among bidders. The winner's surplus is equal to her valuation minus the second-highest bid submitted, and decreases as the number of bidders increases.

In the first-price auction formats, potential bidders face trade-offs—if a bidder bids her valuation then she has no surplus, and lowering the bid from her valuation increases the surplus but reduces the chances of winning. The optimal bidding strategy is more complicated than in the second-price auction and will consist of bid shading. The degree of bid shading depends upon several factors including the number of bidders and information availability.

To motivate auction design, one may ask: Given the optimal bidding strategies of the potential bidders, which type of auction will the seller prefer? For a risk-neutral seller, the answer depends on the expected revenue of each type of auction. Under some assumptions (*i.e.*, if item goes to the person with highest valuation and bidders with lowest possible valuation expect zero surplus), the seller may be indifferent between the four types of auctions because they yield the same expected revenue.

What about competition? Does a seller prefer more bidders? More bidders lead to higher prices and less surplus for winners. Figure 1 plots data generated through simulations to illustrate this relationship. The y-axis presents expected price, with bidders' valuations drawn uniformly from [20,40] and the x-axis indicates numbers of bidders (Source: author's calculations).



We will define another dimension upon which auction theory has developed, namely, the dimension of uncertainty. We will define private and common value auctions. Informally, if you see the signals of the other bidders (or their bids) and it causes you to revise your own valuation of the object, then it is a common value auction. Otherwise, it is a private value auction.

In a private value auction, the object is for personal use. Moreover, the object will not be resold and no bidder is speculating about future value. If there are inherent differences among bidders, then any single bidder can have a valuation that differs from the others' valuations, and each bidder's value will be private. In contrast, in common value auctions, the item has a single, true value which is unknown at time of bidding. An example of common value auctions are offshore oil leases; the value of oil is the same for every participant, no bidder knows her value with certainty, and each bidder has *some* information about the value due to exploratory drilling.

Let r be the reserve price, n is the number of potential bidders, and m be the actual number of bidders. Consider an example in which bidder i's utility function is $U_i = U(\sigma_i, v)$, where v is the value of the good that bidder i may or may not observe, and σ_i is the private signal of bidder i. Let F be the cumulative distribution function of $(\sigma_1, ..., \sigma_N, v)$. Finally, let Y_i be the $\max\{\sigma_j : j \neq i\}$ and W be the winning bid. We make the following assumptions: (i) Each bidder wants only one object; (ii) U is non-negative, continuous and increasing; (iii) the signals are one-dimensional, $\sigma_i \in R$; (iv) F is exchangeable across bidders, because no bidder has better information; (v) $(\sigma_1, ..., \sigma_N, v)$ are positively correlated, and (vi) F, n, U are common knowledge.

Independent Private Values (IPV)

$U_i = \sigma_i$ for each $i=1, \ldots, N$ and σ_i are all independently and identically distributed (iid), so that:

$$F(\sigma_1,...,\sigma_N)=\prod_{i=1}^{N}F_{\sigma_i}(\sigma).$$

Bidder *i*'s signal is her payoff. She receives no signal about the other bidders' values.

<u>Pure Common Value (CV)</u>

$U_i = v$, and the cdf is now $F(\sigma_1,...,\sigma_N,v)$. We can $v=\sum_j \sigma_j$ and there is uncertainty about *v*. Signals of other bidders cause you to revise your valuation, σ_i.

<u>Bidding Equilibria in a Nutshell: Private Values</u>

In a Dutch auction (FPSB), your bid should be lower than your signal—that is, you should shade your bid—and your bid should be increasing in *N*. Let your signal be $\sigma_i=\sigma$. Suppose the valuations have support $[0,\sigma]$ and are uniformly distributed. If we ignore risk aversion, then

E[next highest bid]= $\frac{(N-1)}{N}\sigma$

and the equilibrium bid is equal to

$$w=\frac{(N-1)}{N}\sigma<\sigma.$$

In an English auction with second-price open bidding, each bidder should be willing to pay as much as her valuation. Similarly, in a Vickrey auction (SPSB), the bid should be bidder valuation and the winner pays the second highest price. In this sense, there is no strategic behavior in either of these auctions.

<u>Bidding Equilibria in a nutshell in Common Value Auctions</u>:

To account for the winner's curse, each bidder should reduce her bid. Specifically, each bidder should bid less than $\mathrm{E}[v\,|\,\sigma_i=\sigma]$.

Given this strategy, we can derive some empirical, testable implications to distinguish IPV and CV. In reduced form setting, the comparative-statics on the number of bidders, *N,* vary by auction type. In FPSB auctions, the bid is monotone in *N* with IPV and non-monotone for CV. In SPSB auctions, the bid is independent of *N* with IPV and decreasing in *N* with CV.

With common values, different auction formats are not equivalent. Oral auctions provide information to bidders, while sealed-bid auctions do not. When bids are announced orally, bidders learn about the value of the item from others' bids and may revise their own estimates of object value as other participants drop out. The inability to revise, as in sealed bid auctions, for example, leads to the so-called "Winner's Curse". To avoid the winner's curse, a bidder should always bid as if she has received the highest signal. If she does not having the truly highest signal but bids as though she does, she simply will not win. On the other hand, if she indeed has the highest signal, she'll use the object's true value as the basis for bidding (please look at numerical example in lecture notes). A bidder is more likely to bid high and acquire the item when he overestimates the item's value. From Jensen's inequality (convexity of max function) we have $E\left[\max_i\{\sigma_i\} \mid v\right] \geq \max_i\{E\left[\sigma_i \mid v\right]\} = v$. Intuitively, the person who has the highest expectation is overly-optimistic.

3. Auction Design

Auction design requires a seller to consider some, or all, of the following questions: How many objects are to be auctioned? Is there a reserve price? Is the reserve price known to bidders? How are bids collected? Who is the "winner"? How much does the winner have to pay?

These questions lead to the other decisions about auction bidding rules: Who is allowed to bid? How are bids presented? By how much must bids be beaten? Is bidding anonymous or favored? In addition, a seller must determine the amount of information to be provided to bidders: Are current bids revealed? Are winners identified?

Ideally, the seller aims to design an efficient auction—efficiency implies that the bidder with the highest valuation receives the object. The efficient auction also requires that if the highest valuation is greater than seller's value (i.e. if there are gains from trade), then the sale is consummated. Neither first-price nor second-price auction guarantees that both efficiency conditions are satisfied.

In a first-price auction, the highest valuation may be higher than seller's value, but bid-shading may result in lower bid. Hence, no transaction may occur despite bidder valuations in excess of the seller's value. In a second-price auction, the highest valuation may be higher than the seller's value, but the second-highest value, which determines the price, may not be higher. One potential solution is a reserve price. A reserve price is a "phantom bid" by the seller and, while it does not resolve inefficiencies of first-price auction due to bid-shading, it does resolve the inefficiency in second-price auction. The reserve price guarantees that a sale will occur at or above the seller's value.

Another goal in auction design is to prevent sellers and/or bidders from behaving strategically to affect price. For example, if the seller in a second price auction places his "phantom bid" after knowing all the bids, he will set his reserve slightly below the highest bid. This move by a seller effectively turns a second-price auction into a first-price auction. Bidders also may submit bids strategically to prevent competition and lower winning bid. That is, bidders may try to collude. Collusion in auctions, sometimes called also bid-rigging, must be organized. Bidders must be identifiable, there must be an established process to award the item auctioned to one of the rig members and, finally, the surplus needs to be split among the rig members. Successful collusion is, by definition, hard to detect. However, experiences with identified bid-rigging cases suggest that members use simple rules to rotate bids and award the items. Sealed bids auctions and anonymity guarantees for bidders may discourage collusion.

4. GOALS AND APPROACHES OF EMPIRICAL WORK

The positive goals of applied work in auction literature are to understand how agents bid, whether bidders' valuations are correlated and, if so, how. Moreover economists may ask whether the observed bids are consistent with a model of bidder behavior such as Bayesian Nash Equilibrium (BNE) and whether bidders are colluding, as for example in Porter and Zona (1993). The main normative goals of applied auction work are to identify optimal reserve price and, more generally, to identify revenue maximizing efficient auction formats. Reduced form and structural approaches to auction applied work have aimed to address these goals. In reduced form studies, the authors typically test theoretical predictions from auction behavior models and make inferences about behavior and the bidding environment. Alternatively, in structural work, a certain theory is assumed and estimation of the data generating process is the goal.

5. INDEPENDENT PRIVATE VALUE AUCTIONS, SINGLE OBJECT (IPV)

Let there be N potential bidders with private value for bidder i given by σ_i. The symmetry and independence assumption implies that $\sigma_i \sim_{iid} F(\sigma) on \left[\underline{\sigma}, \overline{\sigma}\right]$, where F is smooth with pdf $f(\sigma)$. Assume the bidder is risk neutral. When she wins the item, her net utility is σ_i-p and, if she does not win, she does not pay and her net utility is 0. There is no reservation price, no minimum bids, and no entry fees. In this context, we can look for symmetric, monotone Bayesian–Nash–Equilibria (BNE).

A strategy (a bid), $\beta(\sigma)$, is strictly monotone and increasing, with an inverse, $\eta(b)$, defined as the valuation associated with the bid. The probability that bidder i wins given that she submits bid b is given by

$$\Pr\{b > b_j, j \neq i\} = \Pr\{b > \beta(\sigma_j), \forall j \neq i\} \underset{monotone}{=}$$

$$\Pr\{\sigma_j < \eta(b) \forall j \neq i\} \underset{iid}{=} F^{N-1}(\eta(b))$$

(Note that the first equality is due to monotonicity)

If $b = \beta(\sigma_i)$ then $\Pr\{\sigma = \sigma_i\} = F^{N-1}(\sigma_i)$. That is, the bidder with the highest value receives the object. If a bidder wins, she pays $p(b_1, b_2, \ldots b_N)$. The expected payoffs are given by

$$\Pi(\sigma_i, b) = \{\sigma_i - E[p(b, \beta(\sigma_j) j \neq i) \mid \sigma_j < \eta(b) \forall j \neq i\} \cdot F^{N-1}(\eta(b)).$$

5.1. ENGLISH (CRY-OUT), VICKREY (SPSB)

Claim: The dominant strategy equilibrium is $\beta(\sigma)=\sigma$, (i.e., to bid one's valuation) .
Proof: (see Vickrey, by contradiction)

Here, the auction strategy is to raise price until only one bidder is willing to raise the standing highest bid. There is no regret if $\beta(\sigma)=\sigma$. The winning bidder is the person with the highest valuation.

Let us rank the people by order statistic permutation: $\sigma_{(1)} > \sigma_{(2)} > \sigma_{(3)} > ... > \sigma_{(N)}$. A person expects to pay $E[\sigma_{(2)} \mid \sigma_i = \sigma_{(1)}]$ in the event that she wins. The seller's expected revenues are $E[\sigma_{(2)}] = E[E[\sigma_{(2)} \mid \sigma_i = \sigma_{(1)}]]$. So, in the SPSB, the expected payment given is

$$E[\sigma_{(2)} \mid \sigma_i = \sigma_{(1)}] = \int_{\underline{\sigma}}^{\sigma_i} s \frac{f(\sigma_{(2)}(s))}{F(\sigma_i)^{N-1}} ds = \int_{\underline{\sigma}}^{\sigma_i} s \frac{(N-1)F(s)^{N-2} f(s)}{F(\sigma_i)^{N-1}} ds .$$

5.2. DUTCH (PRICE GOES DOWN AND YELL STOP), FIRST PRICE SB

Claim: The dominant strategy equilibrium is no longer to bid $b=\beta(\sigma)=\sigma$.
(indirect proof: McAfee and McMillan (1987): $b<\sigma$ and that $b \rightarrow \sigma$ as $N \rightarrow \infty$.

The FPSB format suggest that $p(b_1, b_2, \dots, b_N)=b_i$. Here, the expected payoff is given by $\Pi(\sigma_i, b) = (\sigma_i - b) \bullet F(\eta(b))^{N-1}$. Under what circumstances is it optimal to bid $b=\beta(\sigma_i)=\sigma_i$?

Solving the First Order Condition (FOC) $\frac{\partial \pi(\sigma_i, b)}{\partial b} = 0$, you get the optimal bid given the signal, $\beta(\sigma)$. Let us define $\pi^*(\sigma) = \pi(\sigma, \beta(\sigma))$.

Taking the derivative with respect to the signal yields

$$\frac{\partial \pi^*(\sigma)}{\partial \sigma} = \frac{\partial \pi(.)}{\partial \sigma} + \left. \frac{\partial \pi(.)}{\partial \beta} \right|_{=0} \frac{\partial \beta(.)}{\partial \sigma},$$

by the envelope theorem. So, given that

$$\pi^*(\sigma) = \pi(\sigma, \beta(\sigma)) = (\sigma - \beta(\sigma)) F(\eta(\beta(\sigma)))^{N-1}$$

which is equivalent to

$$\pi^*(\sigma) = (\sigma - \beta(\sigma)) F(\sigma)^{N-1},$$

we can show that

$$\frac{\partial \pi^*(\sigma)}{\partial \sigma} = \frac{\partial \pi(.)}{\partial \sigma} = F(\sigma)^{N-1}.$$

Integrating the above equation gives

$$\pi^*(\sigma) = \int_{\underline{\sigma}}^{\sigma} F(s)^{N-1} ds \ .$$

Finally, solving for optimal bid yields

$$\beta(\sigma) = \sigma - \frac{\int_{\underline{\sigma}}^{\sigma} F(s)^{N-1} ds}{F(\sigma)^{N-1}} < \sigma \ .$$

We already knew to expect a mark-down strategy (bid shading) in FPSB. Now, we have shown this to be an optimal strategy to maximize expected payoff. The bidder, by bid-shading (a mark-down strategy), is trading off the probability of not winning the good with the possibility of positive surplus with a win.

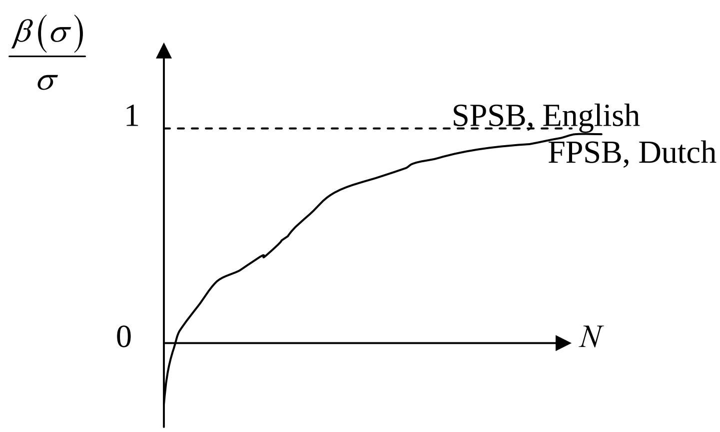
We can also show, by integrating the optimal bid function by parts, that $\beta(\sigma) = E[\sigma_{(2)} \mid \sigma_{(1)} = \sigma]$.

That is,

$$\beta(\sigma) = \frac{\int_{\underline{\sigma}}^{\sigma} s(N-1)F(s)^{N-2} f(s) ds}{F(\sigma)^{N-1}} = E[\sigma_{(2)} \mid \sigma_{(1)} = \sigma] \ ).$$

(see also proof Revenue Equivalence SPSB and FPSB)

The bid function is increasing in *N* and concave in *N*—bids go to the valuations as *N* increases.



5.3. MONEY LEFT ON THE TABLE

$\beta\left(\sigma_{(1)}\right)-\beta\left(\sigma_{(2)}\right)$ is a measure of ex-post regret on part of winner for FPSB, sometimes presented as

$$\frac{\beta\left(\sigma_{(1)}\right)-\beta\left(\sigma_{(2)}\right)}{\beta\left(\sigma_{(1)}\right)}.$$

In oil auctions, the money left on the table is nearly 50 percent! As *N* increases the amount of regret approaches zero.

6. Structural Estimation 101

One of the main objectives in structural estimation in auctions is to estimate the data generation process. Specifically, the primary goal of the procedure is to recover *F*. Why do we care about *F*? The researcher is able to ask the following questions:

Given distribution *F*, is *r* optimal?

Are all of the observed bids determined by the same data generating process? For example, can we find evidence that a group of bidders submitting strategic bids (bid rigging) while another group did not? See Porter and Zona (1993) in the end of these lectures.

Once the researcher has estimated the costs of collusion, how much higher are the bids relative to their no-collusion counterparts?

Given a distribution *F* that has been estimated under a certain auction format, what would happen to the seller's expected revenues under an alternative format?

As in the previous section, let us consider an IPV scenario and maintain the assumptions of risk neutrality, symmetry, privacy, and BNE/dominant strategy behavior. Let there be *N* bidders with valuations $\sigma_i \sim_{iid} F(\sigma)\, on \left[\underline{\sigma},\overline{\sigma}\right]$, where *F* is smooth with pdf $f(\sigma)$. Let there be a minimum bid $r(>\underline{\sigma})$. We will derive the likelihood function for SPSB—we will begin with the easiest format—then derive the likelihood functions for English auctions where we observe only the winning bid, English auction where we observe all bids, and FPSB. In addition, we will briefly mention some extensions allowing for risk aversion, multiple and sequential auctions, and asymmetric information. Three approaches shall be discussed and described here: (i) Simulated Non-Linear Least Squares (NLLS), (ii) Maximum Likelihood Estimation (MLE), and (iii) non-parametric estimation.

6.1. SPSB ("the easiest")

The dominant strategy is to bid $b_i = \sigma_i$ if $\sigma_i \geq r$, otherwise a bidder should not bid at all. The winning bid is given by $w_t = \max[\sigma_{(2:N_t)}, r_t]$.

Suppose we have data on n_t people who submitted bids, among N_t potential bidders $\{b_{it}\}_{i=1}^{n_t}, n_t, N_t, r_t, t = 1,...T,$ where $n_t = \#\{i \mid b_{it} \geq r_t\}$.

The likelihood function is given by $L = \prod_t \{F(r_t)^{N_t - n_t} \cdot \prod_i^{n_t} f(b_{it})\}$ where the first part consists of people who did not submit bids, and the second part represents people who did submit bids. With risk aversion and/or asymmetry, it is still a dominant strategy to bid one's valuation. If there is observable heterogeneity in auction t, $\exists Z_t : \sigma_{it} \sim iid, F(\sigma \mid Z_t)$, this type of heterogeneity may be considered explicit. However, unobserved heterogeneity may cause substantial problems in the estimation. Moreover, if we do not observe the number of potential bidders, N_t , and observe instead only the number of actual bidders, n_t, then we must account for the truncation. If N_t also depends on unobserved heterogeneity, then it is not identified when bidder and bid counts are low—either N_t is low or there it is difficult to consider the unobserved *Z*'s.
(Note that for this first part of Structural estimation, we are estimating parameters for the model we have assumed. That is, we are not testing theory.)

6.2. English Auction where observe only winning bid

The dominant strategy for an English auction with only one observed bid is the same as for SPSB auctions: Bidders should bid $b_i = \sigma_i$ if $\sigma_i \geq r$.

Our data are for the sample where $n_t \geq 1$: {w_t, N_t (debatable), r_t }. Assuming that *F* is smooth and the probability of a tie equals to zero, one can infer n_t on the basis of the winning bid as follows: $n_t = 0$ if no winner; $n_t=1$ if $w_t=r_t$;$n_t>1$ if $w_t>r_t$. The winning bid is given by $w_t = \max[\sigma_{(2:N_t)}, r_t], n_t \geq 1$. To derive the likelihood function, note that $Prob\{n_t=0\}=F(r_t)^{Nt}$; $Prob\{n_t=1\}=N_t\, F(r_t)^{Nt-1}\, [1-F(r_t)]$; and $Prob\{n_t>1\}=h(w_t)$, where the winning bid is $w_t=\sigma_{(2:nt)}$ and $h(w_t)$ is the distribution of the 2nd order statistic given by
$h(w_t) = N_t(N_t - 1)F(w_t)^{N_t - 2} f(w_t)[1 - F(w_t)]$.

Define $D_t=1$ if $n_t=1$ and $D_t=0$ if $n_t>1$. Then, accounting in the denominator for truncation, the likelihood is given by

$$L = \prod_t \left\{ \frac{\Pr\{n_t = 1\}^{D_t} \cdot h(w_t)^{1-D_t}}{1 - \Pr\{n_t = 0\}} \right\}.$$

Rewriting yields:

$$L = \prod_t \left\{ \frac{\{\mathrm{N_t F(r_t)^{Nt\text{-}1}[1\text{-}F(r_t)]}\}^{D_t} \cdot h(w_t)^{1-D_t}}{1 - F(r_t)^{N_t}} \right\}.$$

The seller is not indifferent over the choice of r_t. Suppose σ_0 is the seller's valuation in event the item is not sold. Then the expected revenue of the seller is given by

$$R = \sigma_0 F^N(r) + r\Pr\{n=1\} + \underbrace{\int_r^{\bar{\sigma}} w \cdot h(w)dw}_{E[w]}.$$

The seller wants to maximize this with respect to r.

$\frac{\partial R}{\partial r} = 0 = \ldots \Leftrightarrow (\sigma_0 - r)f + (1-F) = 0$ which is equivalent to choosing $r = \sigma_0 + \frac{(1-F(r))}{f(r)}$.

Once we have estimated F and f, we can choose r. Note that optimal r is independent of N. The formula is similar to a take it or leave it offer of a firm facing just one buyer. Finally, note that IPV is critical here.

6.3. English Auction where all bids are observed

Although we may observe all bids in this third case, we still do not observe N_t. Bidder i may submit many bids or none. Let b_{it} be the highest bid submitted by bidder i in period t. Let b_{it} be equal to 0 if no bid was submitted at t. Suppose the number of potential bidders, N_t , is observed. If not observed, assume that N_t is constant and estimate it, assume $N_t=N=max\{n_t\}$, or assume $N_t \sim$ Poisson truncated below by n_t. We observe the number of bidders $n_t = \#\{i|\ b_{it} > 0\}$. Let us order i bids in t as $b_{1\,t} > b_{2\,t} > \ldots > b_{n\,t}$. Note that if we used the likelihood function from above in section 6.2, we would ignore the non-winning bids and estimate a partial likelihood. By submitting a bid one gets a lower bound on its valuation, so the inferences on these ordered bidders are:

Bidder 1: $\sigma_{1t} \geq b_{1t}$, (A)

Bidders 2, ..., n_t : $\sigma_{it} \in [b_{it}, b_{1t}]$ (B)

Bidders n_t, ... N_t (those that did not bid): $\sigma_{it} \leq b_{1t}$ (C)

Before deriving the likelihood function, note that we cannot infer (B), above, if bidders submit bids in the way described next. Sometimes bidders submit high bids to inflate the price faced by other firm. In the case of the FCC spectrum auctions, firms did this by keeping standing high bids on more than one license in one same area.

So the likelihood function is

$$L = \prod_t \left\{ \underbrace{[1-F(b_{1t})]}_{(A)} \left[\underbrace{\prod_{t=2}^{n_t} (F(b_{1t}) - F(b_{it}))}_{(B)} \right] \underbrace{F^{N_t - n_t}(b_{1t})}_{(C)} \right\}$$

Seller objectives may vary, from efficient allocation to maximum revenue (reading: Cramton, McMillan).

6.4 – FPSB

In FBSB auctions, the strategy is to bid *b=β(б),* where *β* is monotone and increasing with inverse *η*. Let *b : max (б-b) F(η(b))*$^{N-1}$. From the first-order condition with respect to *b,* we get:

$$b : \frac{1}{\eta'(b)} = [\eta(b) - b](N-1)\frac{f(\eta(b))}{F(\eta(b))}. \qquad \text{(D)}$$

From (D), we obtain (see derivation pages 25-26 of lecture notes):

$$\beta(\sigma)\begin{cases} = \sigma - \dfrac{\int_r^{\sigma} F(s)^{N-1} ds}{F(\sigma)^{N-1}}, \sigma \geq r \\ = 0, \text{ otherwise.} \end{cases}$$

We know that $\sigma \sim F$, where F is unknown. Since $b = \beta(\sigma)$, we can apply a change of variable technique to derive the following:
$b \sim G(\sigma) = F(\eta(b)) = \Pr\{\sigma \leq \eta(b)\} = \Pr\{\beta(\sigma) < b\}$.
The support of $b = \{0\} \cup \left[r, \beta(\overline{\sigma})\right]$ depends on parameters, as described in Donald and Paarsch's (1993) pseudo-Maximum Likelihood approach. Suppose we observe a cdf *G*, where *g(b)=G'(b)=f(η(b)) η'(b)*, and wish to identify the distribution *F.*
Then equation (D) becomes

$$1 = [\eta(b) - b](N-1)\frac{g(b)}{G(b)}$$

which implies that the valuation to be infered is equal to $\eta(b) = \underbrace{b}_{bid} + \dfrac{G(b)}{(N-1)g(b)}$.

Given *G*, we are left to ask: Under what conditions *G* implies a unique *F*?

6.5 Extra notes: Risk Aversion

With risk aversion, it is still a dominant strategy to bid $b=\beta(\sigma)$ in SPSB and English auctions. On the other hand, FPSB and Dutch auctions with risk aversion imply

$$\pi(\sigma, b=\beta(\sigma)) = U(\sigma-b)F(\eta(b))^{N-1} + \underbrace{U(0)}_{=0,without\_loss\_gen}[1-F(\eta(b))^{N-1}].$$

Bidders will bid more aggressively to increase the probability of winning (see Holt (1979)). Suppose that U(x)=x^{α} , $\alpha \in$ [0,1], and α=1 for risk neutrality. $\pi(\sigma,b)=(\sigma-b)^{\alpha}F(\eta(b))^{N-1}$. The BNE of this game is equivalent to the BNE of the game with $\pi(\sigma,b)=(\sigma-b)F(\eta(b))^{\frac{N-1}{\alpha}}$ (also risk neutral) where the number of opponents increased by $1/\alpha$. So, if N increases, the bid increases and $\beta_{\alpha}() > \beta_{1}()$.

6.6. K-object Auction

Consider an auction of K objects. Let $K<N$ and assume that each buyer wants only one item. The bidders' valuations for a single item are IPV, and the marginal valuations for additional items are zero.

<u>Theorem</u>: Suppose the auction rules and the BNE are such that the k highest types win and the lowest possible type $(\underline{\sigma})$ has zero expected pay-off, then expected payoff of bidder i with type σ_i is

$$T(\sigma) = \int_{\underline{\sigma}}^{\sigma} s \binom{N-1}{K} K \cdot F(s)^{K-N-1}[1-F(s)]^{K-1} f(s)ds$$

$$\Leftrightarrow T(\sigma) = E[\sigma_{(K+1)} \mid \sigma_i \in \{\sigma_{(1)},...,\sigma_{(K)}\}].$$

Auction types considered are: a discriminatory auction where the k highest bidders win and pay their own bid; the Vickrey auction where the k highest bidders win and pay the $k+1$ highest bid; and the uniform price auction where the k highest bidders win and pay the k^{th} highest bid.

Hansen (1985) provides a reduced form test of revenue equivalence using U.S. Forest Service Timber Auctions (English and FPSB where conducted at same time). In this paper the natural log of revenue over time is regressed on forest characteristics and an auction format dummy, D_t. The OLS coefficient on Dt $(=1$ if FPSB) is estimated at 0.1 and achieves statistical significance. Since D_t is not exogenously determined, this paper uses a two-step Heckman method. First, a probit of D_t on Z_t and w_t is performed. In a second step, the coefficient on D_t is estimated and found to be not significantly different from zero. (See also Haile (2000, AER) and Athey and Levin (JPE, 2001) who argue that a positive coefficient on D_t can be rationalized by risk aversion.)

6.7. Sequential Auctions

Suppose there are N bidders and 2 items. The objects will be allocated in a sequential FPSB auction. We will make two key assumptions: (i) the only information revealed after first item sold is the winning bid (not the other bids) and (ii) each bidder wants only one item.

With IPV, BNE yields the following strategies, for both items:

$$item\_1: \beta_{(1)}(\sigma) = E[\sigma_{(3)} \mid \sigma_{(1)} = \sigma]$$

$$item\_2: \beta_{(2)}(\sigma, b) = E[\sigma_{(3)} \mid \sigma_{(2)} = \sigma, \underbrace{\sigma_{(1)} = \beta^{-1}_{(1)}(b)}_{winner\_1st\_item}]$$

Then the winning bids are

$$item\_1: w_1 = \beta_{(1)}(\sigma_{(1)}) = E[\sigma_{(3)} \mid \sigma_{(1)}]$$

$$E[w_1] = E[\sigma_{(3)}]$$

$item\_2: w_2 = \beta_{(2)}(\sigma_{(2)}\ \underbrace{\beta_{(1)}(\sigma_{(1)})}) = E[\sigma_{(3)} \mid \sigma_{(2)}, \sigma_{(1)}]$, which implies as a result that

$$\mathrm{Re}\,sult: E[w_1] = E[\sigma_{(3)}] = E[w_2] \Rightarrow Martingale$$

$$E[w_2 \mid w_1] = w_1.$$

This result is also true for SPSB.

<u>Empirical Evidence</u>: Ashenfelter (1989) examines identical, sequential wine auctions and finds a “declining price anomaly”. In an affiliated common value context, Milgrom and Weber (1982) show that prices are super martingale (prices increase since there is more information about the object). One conclusion is that revenue equivalence fails when bidders want multiple items.

6.8 Asymmetric auctions

We now examine Common Value auctions where there exist asymmetries of information of bidders and of bidders’ valuations. Consider, for example, auctions for off-shore drilling rights with adjacent and non-adjacent tracts being sold in auction to multiple bidders, some of whom are neighbors to the different tracts. The question is: how can we use observable data to distinguish the “asymmetry of information” from the “asymmetry of valuation”?

Consider the set-up of the FPSB auction with risk neutrality, where the common value is v and one item is being sold. There are two types of bidders: (a) a neighbor that observes a public signal Z and private signal X, and (b) a non-neighbor (NN) who only observes a public signal Z. Let there be a minimum bid R, $E[v]>R$, and let the joint distribution of (v,X,Z,R) be common knowledge.

The claim is that the strategy of a NN is to randomize—that is, he will follow a mixed strategy. To understand this strategy choice, we may ask: Why would a NN bid? If he did not bid, then the neighbors would bid. Why randomize? Note that a NN's payoff is *E[*Payoff for all bids*]*=0 or <0. If a bidder is a neighbor, then his payoff is (*E[V/X]-b*)*Prob{*their bid>NN bid*}=h*, where the neighbor has the realization of *h, NN* forms the *E*[.].

For theoretical treatments of asymmetric auctions, see Wilson (1967) and Milgrom and Weber (1983). For empirical papers, see Hendricks and Porter (1988), Hendricks, Porter and Wilson (1994) and Porter (1995).

6.9. THREE APPROACHES TO ESTIMATION AND IDENTIFICATION ISSUES

Three different estimation techniques are discussed below. The first estimation approach is Simulated Non Linear Least Squares (NLLS) with winning bids, N_t and r_t, $t=1, \ldots T$. The second approach is a "brute force" Maximum Likelihood Estimation (MLE) that needs the same information as NLLS above. Finally, we describe a non parametric techniques that requires data on all bids. For a good survey of "Empirical Work concerning auctions", see Hendricks and Paarsch (1995).

<u>Simulated NLLS (Laffont et al., 1995)</u>

This method consists of fitting a moment condition and exploiting revenue equivalence. In English auctions, $w_t = \max\left\{r_t, \sigma_{(2:N_t)}\right\}$ and a bid is given by

$$\beta(\sigma) = E\left[\max\left\{r_t, \sigma_{(2:N_t)}\right\} \mid \sigma = \sigma_{(1:N_t)}\right].$$

Revenue equivalence implies that, given a winning bid, the following expression holds:

$$\beta\left(\sigma_{(1:N_t)}\right) = E\left[\max\left\{r_t, \sigma_{(2:N_t)}\right\} \mid \sigma = \sigma_{(1:N_t)}\right] = E\left[\max\left\{r_t, \sigma_{(2:N_t)}\right\}\right]$$

Therefore, the moment condition used in the NLLS method is simply

$$E\left[w_t\right] = E\left[\max\left\{r_t, \sigma_{(2:N_t)}\right\}\right].$$

In the data, we observe $\{N_t, w_t, r_t\}$ $t=1, \ldots T$. The simulation consists of positing a cdf F (log-normal). We then perform random draws from this distribution and compute $E[\max\{r_t, \sigma_{(2:Nt)}\}]$. Then, we minimize the weighted average difference of w_t-$E[.]$ to get parameters of F. The main problems with this method are (i) the presumption that r_t is exogenous (when, in fact, r_t may be chosen to maximize the sellers expected profits, for example), (ii) we do not observe N_t, (iii) there exist possible alternative hypotheses for F, and (iv) there are over-identifying assumptions.

This method allows for larger families of distribution functions but still requires specific distributional assumptions. This drawback is overcome in the third non-parametric approach to estimation described below.

MLE (as in Paarsch, 1992) and Piecewise Pseudo-ML (as in Donald and Paarsch, 1993)

Individual bids are given by $b_t \sim F(\eta_t(b))$, conditional on N_t and r_t . The winning bid is given by

$$w_t \sim F(\eta_t(w))^{Nt} = H_t(w_t)$$

$$h_t(w_t) = N_t F^{N_t - 1}(\eta_t(w_t)) \cdot \underbrace{\eta_t\,'(w_t)}_{(*)} f(\eta_t(w_t))$$

where

$$\eta_t\,'(w_t) = \frac{N_t}{N_t - 1} \frac{F^{N_t}(\eta_t(w_t))}{(\eta_t(w_t) - w_t)},$$

and where it is tricky to find a control for the expression (*).

This approach requires that the joint distribution belong to particular families of distributions that admit closed-form solutions to the bid functions that can be solved using numerical methods and, therefore, is more restrictive than the previous method discussed above.

Non-parametric Estimation (as in Elyakime et al., 1994)

This estimation technique requires data on all submitted bids. Consider a sample of bids, {bid}, for t=1,..., T, i=1, ... N_t, r_t, and Z_t (where Z are observed covariates). We get the following inverse bid function from the First Order Conditions, where the valuation to be inferred is given by

$$\eta_t(b) = \underbrace{b}_{bid} + \frac{G_t(b)}{(N-1)g_t(b)},$$

where G and g depend on t because they depend on N_t and r_t.

Consider a sub-sample where N_t is constant and all bidders bid—that is, $r_t \leq \underline{\sigma}$ —which assures that g and G are constant. The approach consists of a two-stage non-parametric estimation. First, non-parametrically estimate g and G. Then, create a data-set where

$$\hat{\sigma}_{it} = \underbrace{\widehat{\eta_t(b)}}_{estimated} = \underbrace{b}_{bid} + \frac{\hat{G}(b)}{(N-1)\hat{g}(b)}.$$

In a second step, non-parametrically estimate the bid function β by $bid=\beta(\hat{\sigma}_{it})$. We can then verify the identification condition—that is, we can check whether the right hand side of $\hat{\sigma}_{it} = \underbrace{\widehat{\eta_t(b)}}_{estimated} = \underbrace{b}_{bid} + \frac{\hat{G}(b)}{(N-1)\hat{g}(b)}$ is monotone increasing ($\lim \eta(\beta) \to r$ as $b \to r$).

Note the limitations to this approach: If r_t is optimally set, then it cannot be treated as exogenous. Furthermore, if there is strategic bidding or collusion, some bids are “not real”.

7. BID RIGGING – AN EMPIRICAL STUDY

Early empirical work on collusion in auctions looks at patterns of identical bidding, unlikely outcome of un-cooperative bidding and simple patterns of bid rotation. For example, players in the GE/Westinghouse conspiracy set their bids according to the phases of the moon.

Porter and Zona (1993) analyze and identify bid rigging in procurement auctions in Long Island highway construction and repair. There is a suspicion of “complementary bidding” and bid rigging by firms participating in meetings, to create the appearance of competition and to manipulate the expectation of the seller. In the procurement auctions, the firm with the lowest bid is awarded the contract. Porter and Zona’s results show that, for participants in meetings, the higher bids look “phony” and were not determined by the same process.

There are other empirical papers on bid rigging, as well. For example, Porter and Zona (1999) examine Ohio school milk markets, while Pesendorfer (1997) studies collusion in first price auctions. The central research question is whether these data exhibit evidence inconsistent with competitive bidding. Bid rigging may be easier in FPSB auctions, where competition is over price only, when quality is pre-specified, the number of firms is stable, and there are trade associations.

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