## Title
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# Independent Recognition of Numerosity Requires Attention

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## Abstract

The overlap of numerical and non-numerical properties in concrete object arrays raises the question of how these input dimensions interact. Two studies were conducted to address this question and showed that changing the object identity (while retaining the numerosity) and changing the numerosity (while retaining the object identity) both resulted in attenuated recognition of object arrays. However, this interference differed across development. In adults interference was asymmetrical (i.e. changing the object identity has greater effect on memory for numerosity than changed numerosity had on object identity). In contrast, children showed a symmetrical pattern of interference. These results imply that for adults, processing numerosity might be an attention-demanding process compared to a spontaneous object perception. Children, however, processed neither the object identity nor numerosity independently.

**Keywords:** Numerical cognition; Interference

Despite early sensitivity to quantity (Brannon et al., 2004; for a review, Cantrell & Smith, 2013; Cordes & Brannon, 2008,) and environmental support, such as frequent exposure to number words and visual stimuli, moving from an initial quantitative ability to a mature concept of numbers is one of the most challenging tasks for children. Although the question of how infants perceive numerosity is hotly debated, there is general consensus that the initial sensitivity to non-symbolic numbers was not enough to support the number concept (Barth et al, 2006; Cary, 2004; Mix, 2002; for a review, Rips et al., 2008). However, exactly what is lacking of the early mechanism dealing with non-symbolic numbers that cannot support the mature number concept, and whether there is any developmental change in that mechanism are not yet understood well. Thus, this study investigated forms of representation of object arrays as a tool to understand non-symbolic numerical perception process from young children to adults.

Perceiving numerosity from a set of concrete objects intrinsically includes a selection problem because numerosity is a property of a set, not a property of individual items. That is, people must attend selectively to the "numerical" dimension from various object arrays, while ignoring other perceptual dimensions such as shape, color, or size. For example, they can recognize "three-ness" by detecting the common quantity between three apples and three cars. Numerical perception from concrete object arrays thus requires people to select one relevant dimension of the set of stimuli from an array of multiple irrelevant dimensions of individual stimuli.

This selection problem, together with the well-known fact that young children's selective attention is immature (Kemler, & Smith 1978; Robinson & Sloutsky, 2004; for a review, Hanania & Smith, 2010), suggests that children might have difficulty in processing a numerical dimension of object arrays in specific situations. Cantlon et al. (2007) reported that when objects in a set were not homogeneous but heterogeneous, 3- to 5-year-olds' perception of cardinality was impaired, supporting this prediction.

Mix (2008a) demonstrated that 3-year-olds failed to perceive equivalence in the number of object arrays, especially when irrelevant dimensional features of object arrays were highly varied between sets and within a set. In a follow-up study, preschoolers' performance of numerical comparison was also impaired when the surface similarity of sets was pitted against numerical equivalence, but not when all sets contained the same objects (Mix, 2008b). These studies showed that irrelevant non-numerical properties interfered with young children's perception of numerical equivalence in specific situations.

However, previous studies have considered the interference as only a failure, instead of as a more general problem of non-symbolic numerical perception. As a result, it remains unclear why an irrelevant perceptual property would affect perception of numerosity, or whether numerosity affects processing of the non-numerical property. Answers to these questions are important for understanding non-symbolic numerical perception, because in many circumstances numerical perception requires people to intentionally integrate non-numerical object properties with numerical properties, such as enumerating subsets of object arrays based on sub-categories of objects (Goldfarb & Treisman, 2013). Therefore, depending on the context, it makes sense to assume that non-numerical information surrounding object arrays could become either relevant or irrelevant features. That possibly drives people to form different forms of representations with the same object arrays. In addition, developmental change in the ability to change a direction of attention flexibly would allow us to observe an age-related difference in the variety of the representation.

To test these predictions, the current study focused on recognition memory and the relative distance between studied stimuli and various types of foils. The recognition memory test allows us to use diverse types of test stimuli, which provide an efficient tool to disentangle the participants' responses to each property of the object arrays. In addition, we investigated the interactions between the numerical and non-numerical properties of object arrays by assessing the memory sensitivity of each dimension using signal detection theory, which permits the estimation of the same performance measure (*d'*) from a variety of experimental paradigms (Macmillan & Creelman, 2005). This allowed us to quantify the dimensional interactions and to directly compare the performance of adults with that of children, without making additional assumptions (Kingston & Macmillan, 1995).

## Experiment 1

The first experiment was designed to develop a new task to investigate whether numerical and object properties of sets of objects are processed interactively or independently of each other. We also investigated developmental changes in this interaction by testing adults and 4- to 5-year-olds performing the same task.

### Method

**Participants.** Participants in Exp. 1 included 25 undergraduates enrolled at The Ohio State University and 36 typically developing 4- and 5-year-olds. The children were recruited from their childcare centers around Columbus, Ohio and their parents provided written consent.

**Materials and Design.** The stimuli were sets of colorful arrays of everyday objects. For a training set of object arrays, pictures of school buses, bicycles, airplanes, alarm clocks, brown chairs, and short houses were used. The numerosity of object arrays ranged from five to ten objects. Thus, six different objects and six different numbers were used to constitute the training set of object arrays (e.g., five school buses and six short houses), resulting in 36 trials.

In the test phase, four different types of stimuli set were generated: completely old arrays from the training set, completely new arrays having new items across the two dimensions, and partially new arrays consisting of combinations of new and old items. For the partially new arrays, the studied and novel items from one dimension were combined with the studied and novel items from the other dimension (e.g., two school buses and six tall houses). New items in object identity were double-decker buses, tricycles, army jets, long case clocks, yellow chairs, and tall houses. The new numbers were 1, 2, 3, 30, 31, and 32. Each type of test stimuli set consisted of 12 different object arrays, so the total number of test stimuli was 48.

To ensure that the distance between studied and novel stimuli was indeed comparable across the two dimensions, we asked a separate groups of adults ($n$ = 13) to perform a discrimination task with a set of concrete objects (for object discrimination) and a set of dots (for numerical discrimination). The task was to determine whether a target and a succeeding test item were exactly the same either in a numerical dimension or in an object dimension. We compared accuracy and response times between each dimension. The results point to virtually equivalent distances between target and test items across the dimensions ($t_{accuracy}(12) < 1, t_{RT}(12) < 1$). These target and test items were used as studied and novel items in Exp. 1 respectively.

**Procedure.** Exp. 1 consisted of a training- and a testing phase. The procedure of Exp. 1 is presented graphically in Figure1. During the training phase,

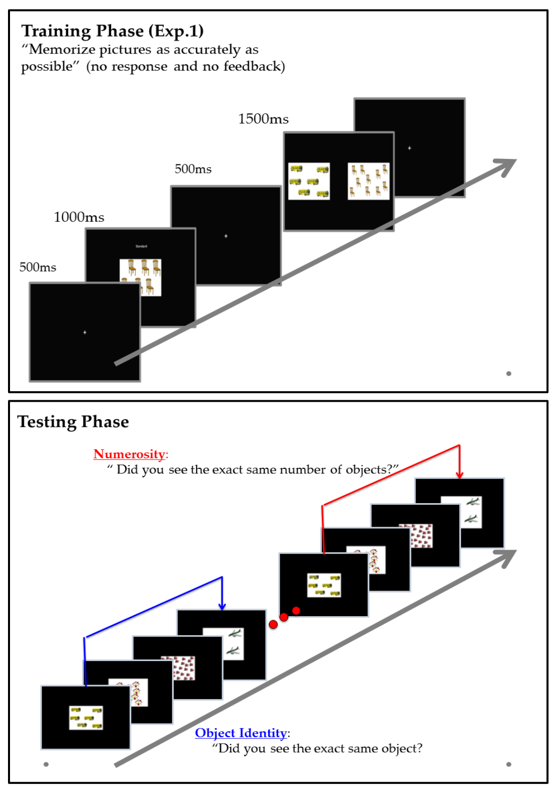


Figure 1. Examples of training and testing stimuli

participants were instructed to study pictures carefully and to remember them as best as they could. They were also informed that they would receive questions about these pictures later. In the test phase, both numerical and object questions were given to the participants in two separate blocks. For example, in the numerical block, participants were asked whether they saw the exact same number of objects (regardless of object). Conversely, in the object block, participants were asked whether they saw the exact same objects (regardless of number). Participants received no feedback. Accuracy rates were recorded.

## Results and Discussion

The test stimuli were categorized into congruent- and incongruent trials. If each dimension of the stimuli conveyed the same information, such as a studied numerosity with studied objects or a novel numerosity with novel objects, the stimuli were labeled as "congruent trials." Similarly, if each dimension of the stimuli conveyed different information, such as a studied numerosity with novel objects or a novel numerosity with studied objects, the stimuli were labeled as "incongruent trials." Thus, in each testing dimension, there were two types of old- and new trials: congruent- and incongruent old stimuli vs. congruent- and incongruent new stimuli. This categorization allowed us to examine the interactions between the two dimensions of the object arrays. In particular, if participants made their decisions based only on a testing dimension, their responses to each type of old stimuli were expected to be the same, as were their responses to each type of new stimuli. However, if participants' decisions were affected by features from a non-testing dimension, congruent- and incongruent trials of either the old- or new stimuli were expected to yield different responses, which would imply an interaction between the two dimensions.

To quantify the extent of the interaction, we applied detection theory and calculated *cumulative d'* from

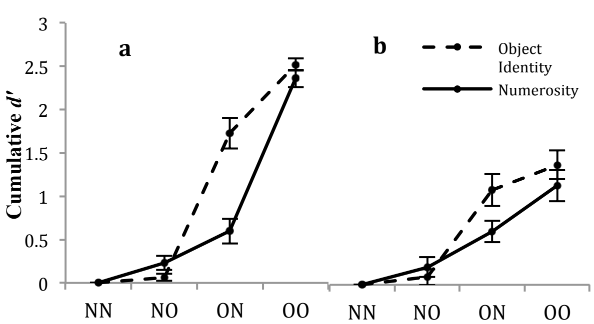


Figure 2. A psychometric function for the data of Exp. 1 in terms of *cumulative d'* (a) for adults and (b) for children. The horizontal axis shows the stimuli types, NN (congruent new stimuli), NO (incongruent new stimuli), ON (incongruent old stimuli), and OO (congruent old stimuli).

congruent new trials (NN) to congruent old trials (OO) by placing the incongruent new trials (NO) and the incongruent old trials (ON) between them. The value of *cumulative d'* can be obtained between any stimulus and the endpoint stimulus if responses come from the same dimension. Since *d'* has the mathematical properties of distance measure, i.e., having true zero and being unique, it can also represent discriminability (Macmillan & Creelman, 2005). Thus, in the current study, by comparing *d'*s from each dimension, we were able to investigate whether two dimensions of stimuli, object identity and numeorosity, were equally discriminable.

Figure 2 shows the *cumulative d'*s for each dimension of object arrays (object identity & numerosity) separately for adults and children. The slope of each dimension between stimuli types tells us how rapidly the perceptual effect grows with stimulus value – that is, how sensitive the participants are to systematic changes in stimuli. The total discriminability of each dimension, total *d'* between NN and OO, was not significantly different for adults, nor for children, $F(1,59) = 1.70, p > .2$ – that is, new numerosities were as equally discriminable as new objects. Furthermore, the total discriminability of adults was close to ($d' = 2.77$), but significantly different from, the discriminability of an ideal observer, $M_{object} = 2.52, T(35) = -2.38, p < .05$; $M_{numerosity} = 2.43, T(35) = -3.47, p < .05$.

To investigate the interaction between two dimensions of object arrays and any developmental changes in that interaction, a mixed-design analysis of variance model (ANOVA) was used. The type of test stimulus (NN, NO, ON, & OO) and the testing dimension (Numerosity & Object Identity) were within-subject variables, and age group was applied as a between-subject factor. The value of *d'*s on the test trials served as the dependent variable.

First, for both age groups, the main effect of the type of the test stimulus was significant, $F(2.13, 125.57) = 186.50, p < .001, \eta^2_{partial} = .76$. The distance between congruent and incongruent trials of stimuli, i.e., NN ~ NO, and ON ~ OO, was significantly different. That is, the participants' responses in each testing dimension were clearly influenced by incongruent information of the non-testing dimension.

Furthermore, the pattern of this interaction changed depending on the testing dimension, $F(2.52, 148.82) = 13.84, p < .001, \eta^2_{partial} = .19$. In particular, the planned comparison analysis on the interaction showed that when the object identity was a testing dimension, the distance between OO and ON was much smaller than that of the numerical dimension, $F(1, 59) = 16.30, p < .001$. That is, recognition of each dimension was significantly impaired by incongruent features from the non-testing dimension, but the magnitude of interference was significantly larger in the numerosity dimension. This asymmetrical pattern was found exclusively in adults' performance, $F(1, 70) = 8.18, p < .005, \eta^2_{partial} = .11$.

The results showed that this novel recognition paradigm was useful for measuring memory sensitivity to each dimension of object arrays and investigating the interactions between them. Both adults and children showed a similar form of integrated representation in the numerical dimension by showing interference.

The presence of integrated representations suggests two possible explanations for the underlying process. Specifically, as in false memory studies (Roediger & McDermott, 1995), the underlying "actual" dimension of the decision in both given dimensions might be uni-dimensional familiarity. However, if the results confirm the uni-dimensional familiarity hypothesis, the pattern of interaction should be the same, regardless of the testing dimension. Indeed, this was not the case for adults. Rather, the distribution of the perceived distance from congruent new trials to each type of trial demonstrated a clear asymmetrical pattern of interaction according to the testing dimension.

Another explanation for the underlying process might be that the multidimensional structure of object arrays would evoke spontaneous association between both dimensions during the training, which resulted in interactive responses between the testing and non-testing dimensions. In particular, since the training did not require a deeper and abstract processing of each dimensional feature, adults may process both features in an integral and perceptually focusing manner, which would affect their memory sensitivity. In this case, adults' perceived decision space would not be unidimensional, but multidimensional, in an integrated manner. Moreover, this latter explanation suggests that the perceived structure of object arrays could change based on the context (Ashby & Maddox, 1990; Cook & Odom, 1992; Goldfarb & Treisman, 2013), such as the testing dimension, as in our current study. Furthermore, this approach provides developmental change in the

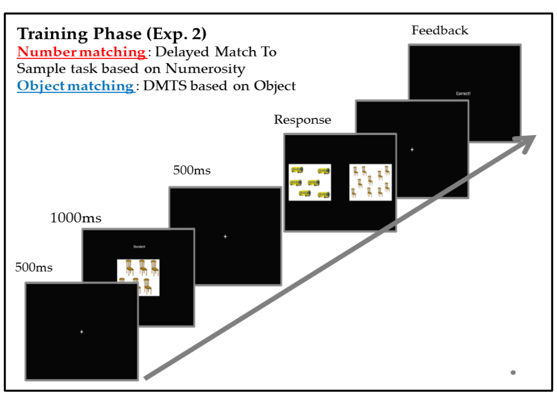


Figure 3. Examples of training stimuli in Experiment2.

process. Shepp et al. (1976) suggest that only adults can easily change their perception pattern in response to task instruction because they have flexible attention control. Thus, we could expect that when we ask adults to shift their focused attention to each dimension of object arrays, their perceived structure of object arrays would change accordingly, but children would not show a similar change in their responses. This hypothesis was tested systemically in Exp. 2.

## Experiment 2

During the training phase of Exp. 2, we instructed participants to focus exclusively on a specific dimension of the stimuli. This manipulation allowed us to examine whether directed attention during training could change the pattern of interactions between the two dimensions of object arrays, and whether there were any developmental changes in the effect of attention manipulation.

### Method

**Participants.** 73 undergraduates who are enrolled at the Ohio State University and 80 typically developing 4- to 5-year-olds took part in this study.

**Procedure.** All procedures were the same as in Exp. 1, except for two conditions in the training session: number and object matching. The procedure of the training phase is presented graphically in Figure 3. Participants were randomly assigned to these conditions, in which they were given different instructions that manipulated the direction of their attention to each stimulus dimension. Specifically, during the training phase, participants in either the number or object condition were given a delayed-match-to-sample task based on either numerosity or object identity of the stimuli, respectively.

In the number condition, when we presented a standard stimulus to the participants, we asked them to choose one of the two stimuli that correctly matched the sample based on numerosity. In this case, later in the testing phase, responses to the numerical test were used to calculate numerical memory sensitivity when it was a target dimension; and responses to object test were used to calculate object memory sensitivity when it was an irrelevant dimension.

In the object condition, we asked the participants to do the same task as in the number condition, but based on object identity. Their memory responses to the numerical question denoted numerical memory when it was an irrelevant dimension; and their memory responses to the object question denoted object memory when it was a target dimension. The test phase was exactly the same as in Exp. 1.

## Results

Memory sensitivity to each dimension for both adults and children is presented graphically in Figure 4. The results from Exp. 1 are presented as baseline performance in Figure 4, but they were not used in the analysis. A mixed-design ANOVA model was used as in Exp. 1.

The type of test stimulus (NN, NO, ON, & OO) and the testing dimension (Numerosity & Object Identity) served as within-subject variables; the age group and training condition (Number matching & Object matching) were between-subjects variables; and the value of *cumulative d'*s on the test trials served as the dependent variable. Because a four-way interaction among all of the independent variables was found to be significant, $F(1.79, 266.98) = 4.23$, $p < .05$, $\eta^2_{partial} = .03$, the results from each age group will be presented separately.

The ANOVA on the adults' responses revealed significant main effects of the training condition, $F(1, 71) = 7.23$, $p < .01$, $\eta^2_{partial} = .09$, and the test stimulus type, $F(1.71, 121.40) = 438.15$, $p < .001$, $\eta^2_{partial} = .86$. However, the main effect of the testing dimension was not significant, whereas the interaction among the stimulus type, testing dimension, and training condition was significant, $F(1.55, 109.88) = 10.91$, $p < .001$, $\eta^2_{partial} = .13$.

In particular, when the testing dimension was object identity, the memory sensitivity pattern between each stimulus type showed a less integrated form, regardless of the direction of attention, $F(1, 71) = .09$. However, when numerosity was the testing dimension, the participants' memory pattern significantly changed by the attention manipulation, $F(1.68, 119.41) = 15.32$, $p < .001$ $\eta^2_{partial} = .18$. Specifically, when numerosity was a target dimension, the perceived distance (*d*') between NN and NO was zero, and *d*' between OO and ON was significantly decreased compared to when it was an irrelevant dimension, i.e., numerical memory sensitivity after object matching training, $F(1, 71) = 17.73$, $p < .001$, $\eta^2_{partial} = .23$.

The children's responses showed quite a different pattern from those of the adults. The training condition did not lead to a significant main effect across the testing dimensions, $F(1, 78) = .19$. More importantly, there were no significant interaction effects involving the training condition. Thus, the external attention manipulation did not cause any changes to the children's behavior. However, the main effect of stimulus type was strongly significant, $F(1.91, 149.06) = 108.18$, $p < .001$ $\eta^2_{partial} = .58$. These results showed that regardless of the testing dimension, the children's responses showed a robust pattern of interaction between numerosity and object identity.

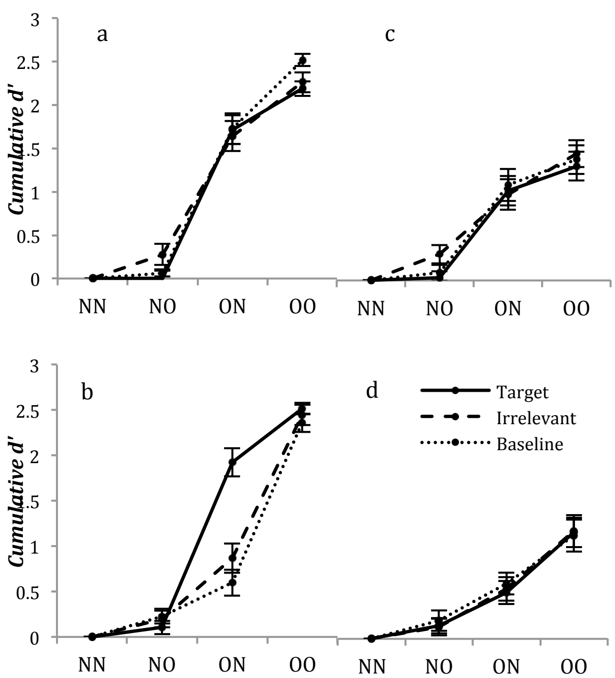


Figure 4. A psychometric function for the data of Exp. 2 in terms of *cumulative d'*: (a) for adults in the object identity dimension, (b) for adults in the numerosity dimension, (c) for children in the object identity dimension, and (d) for children in the numerosity dimension. The results are divided by the condition whether the testing dimension was target during the training: Target (the testing dimension was target), Irrelevant (the testing dimension was irrelevant), Baseline (the results from Exp.1). The horizontal axis shows the stimuli types as in Exp. 1, NN (congruent new stimuli), NO (incongruent new stimuli), ON (incongruent old stimuli), and OO (congruent old stimuli).

## General Discussion

This study was motivated by the finding that children's performance in early numerical comparison tasks is affected by the level of non-numerical similarities between sets and within a set (Cantlon, Fink, Safford & Brannon, 2007; Mix, 1999; 2008a; 2008b). This observation suggests that children may experience difficulty matching a perceived quantity of object arrays with their numerical value, partially due to their less robust representation of numerosities from concrete object arrays. Thus, we examined how robust a representation – of either numerosity or object identity – adults and children formed when they perceived object arrays; and we tested whether external attention manipulation could change the form of each representation.

A striking aspect of the results is that adults' memory sensitivity of numerosity was severely disrupted by incongruent features from object property, compared to the sensitivity of object identity, especially when the test context did not require the participants to process numerical dimension exclusively. However, external manipulation of attention direction successfully changed this integrated form of numerical representation into a more independent one. This change was observed only in the numerical dimension. This finding suggests that, compared to spontaneous object perception, perceiving numerical properties from a set of concrete objects is an attention demanding process. Adults could form an independent and abstract numerical representation from concrete object arrays only when they focused their attention exclusively to numerosity.

In contrast to adults' response, attention manipulation did not change the children's memory sensitivity at all. The children's memory sensitivity denoted a robust integrated form of representation regardless of the testing dimension. These results suggest that even if children were asked to make either object or numerical decision exclusively, they would continue to consider both types of features. This pattern of responses may be automatically driven by their distributed attention processes.

This robust integrated representation provides considerable explanation for young children's dissociable pattern of responses in different numerical tasks (Cantlon et al., 2007; Posid & Cordes, in press). That is, the integrated representation of a studied set of objects would lead to an increase in the numerical dissimilarity of new set of objects when it was presented with heterogeneous objects. That might result in impaired identification of numerical equivalence and spared discrimination of numerical difference. Moreover, the integrated representation also suggests that children may have difficulty identifying the same objects when the objects were presented in a slightly different context like changing the numerosities of items.

Finally, we also considered and rejected an alternative explanation for the robust integrated representations from the children's responses. It could be that the children may have relied exclusively on a specific dimension, and would give the same responses regardless of the testing dimension. If this was the case, the children would have failed to discriminate between incongruent old stimuli and new stimuli. For example, if the children responded to numerical questions as object questions, their responses to old numerosities with new objects would have been the same as those to new numerosities with new objects. However, the children's responses to those two stimuli were significantly different. Thus, we can conclude that the children understood questions in the testing phase, but that their immature selective attention resulted in robust interference, denoting an integrated representation.

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