## Title
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# ON INSTABILITY OF ASYMPTOTIC FREEDOM OF SUPERGAUGE YANG-MILLS THEORY*

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## ABSTRACT

Asymptotic freedom is examined in detail for a simple supergauge symmetry model of Yang-Mills type. Although the model of perfect supergauge symmetry is asymptotically free, the symmetry limit is not realized as a local minimum in every direction in the parameter space of independent coupling constants if one starts with approximate supergauge symmetry. In spite of the unstable nature of the perfect symmetry limit, breaking due to a soft operator explicitly or spontaneously does not ruin the asymptotic freedom. This leads to speculation on a new possibility of achieving the asymptotic freedom without massless particles.

## I. INTRODUCTION

A simple model of supergauge symmetry has been proposed which possesses the ordinary non-Abelian gauge symmetry as well as baryon number conservation (1). The asymptotic freedom is proved for this model. Since the supergauge symmetry requires that not only the Yang-Mills fields, but also scalar, pseudoscalar, and fermion fields

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should be massless, symmetry breaking must occur due to a soft operator in order for the model to apply to the real world.

In this paper the asymptotic freedom of supergauge Yang-Mills theory is examined as the limit of the corresponding theory of approximate symmetry in which several renormalizable couplings are left independent in Lagrangian. It happens that the Yukawa couplings of scalar and pseudoscalar mesons must "choose" unstable fix points to realize the supergauge symmetry. This implies that the asymptotic freedom of the supergauge symmetry cannot be realized if one starts with an approximate supergauge symmetry involving a tiny hard breaking.

In Chapter 2 renormalization group equations are written down for the model Lagrangian that consists of scalar, pseudoscalar, fermion, and the Yang-Mills fields. Seven coupling constants, which are normally independent, become related to each other in the supergauge symmetric limit. Asymptotic stability of the supergauge symmetric limit is examined in the renormalization group. It is shown that if one introduces a soft symmetry breaking, the theory would still possess asymptotic freedom. In Chapter 3 we will report on our unsuccessful search for a model without a massless vector meson that realizes the asymptotic freedom through an unstable, instead of stable, ultraviolet fixed point just as the supergauge Yang-Mills theory.

## II. MODEL AND RENORMALIZATION EQUATION

Ferrara and Zumino and Salam and Strathdee proposed a simple and almost realistic model of supergauge symmetry which incorporates the Yang-Mills fields as well as fermion number conservation (1,2).

The model is given by the Lagrangian

$$L = \mathrm{Tr}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu A)(D^\mu A) + \frac{1}{2}(D_\mu B)(D^\mu B)\right.$$

$$\left. + \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi - g\bar{\psi}\left[A + i\gamma_5 B, \psi\right] + \frac{g^2}{2}\left(\left[A,B\right]\right)^2\right\} , \qquad (2.1)$$

where

$$F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu + ig\left[V_\mu, V_\nu\right] , \qquad (2.2a)$$

$$D_\mu \phi \equiv \partial_\mu \phi + ig\left[V_\mu, \phi\right] , \qquad (2.2b)$$

$V_\mu$, A, B, and $\psi$ represent the Yang-Mills scalar, pseudoscalar, and fermion fields transforming like the adjoint representation of SU(N) written in the $(N \times N)$ matrix. The supergauge symmetry forbids mass terms and relates all the coupling constants. The function $\beta(g)$ of the renormalization equation is written for $g$ as

$$\beta(g) = -\frac{g^3}{8\pi^2} N , \qquad (2.3)$$

which indicates the asymptotic freedom (1).

We raise here the question whether this perfect symmetry limit can be realized asymptotically if one starts with a model in which the various coupling constants (the gauge coupling, the Yukawa couplings and the four-point boson couplings) deviate slightly from the values of the perfect symmetry. The answer would be affirmative if the supergauge symmetric point is ultraviolet stable for all the $\beta$'s of those couplings when they are allowed to vary independently. To examine this question, we start with the most general renormalizable

Lagrangian that consists of $V_\mu$, $A$, $B$, and $\psi$ and still retains the ordinary SU(N) non-Abelian gauge symmetry,

$$L = \mathrm{Tr}\Big\{ -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu A)(D^\mu A) + \frac{1}{2}(D_\mu B)(D^\mu B)$$

$$+ \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{D}_\mu \psi - f_s \bar{\psi}\left[A,\psi\right] - i f_p \bar{\psi}\left[B,\gamma_5\psi\right]$$

$$+ \frac{\lambda}{2}(\left[A,B\right])^2 \Big\} - \frac{\kappa}{4}\mathrm{Tr}\, A^2\, \mathrm{Tr}\, B^2$$

$$- \frac{\sigma}{8}(\mathrm{Tr}\, A^2)^2 - \frac{\rho}{8}(\mathrm{Tr}\, B^2)^2 \quad . \tag{2.4}$$

The coupling constants would be subjected to the constraints

$$f_s = f_p = g \qquad \text{(the Yang-Mills coupling)} \quad , \tag{2.5a}$$

$$\lambda = g^2 \quad , \tag{2.5b}$$

$$\kappa = \sigma = \rho = 0 \quad , \tag{2.5c}$$

in the supergauge symmetric limit. Since the following argument remains true for SU(N) with an arbitrary N, we will present our result in case of $N = 2$.

It is just a matter of labor to obtain the following functions of renormalization group to the order of a single loop,

$$16\pi^2 \frac{dg^2}{dt} = -8g^4 \quad , \tag{2.6a}$$

$$16\pi^2 \frac{df_s^{\,2}}{dt} = 16 f_s^{\,4} - 24 f_s^{\,2} g^2 \quad , \tag{2.6b}$$

$$16\pi^2 \frac{df_p^{\,2}}{dt} = 16 f_p^{\,4} - 24 f_p^{\,2} g^2 \quad , \tag{2.6c}$$

$$16\pi^2 \frac{d\lambda}{dt} = \left\{6\lambda^2 - 24\lambda g^2 + 8({f_s}^2 + {f_p}^2)\lambda - 6g^4\right\}$$

$$+ \left\{8\kappa\lambda + 2\lambda(\sigma + \rho)\right\} , \qquad (2.6d)$$

$$16\pi^2 \frac{d\kappa}{dt} = \left\{8\lambda^2 + 24g^4 - 32{f_p}^2{f_s}^2\right\}$$

$$+ \left\{4\kappa^2 + 4\lambda(\rho + \sigma) + 5\kappa(\rho + \sigma) - 24\kappa g^2 + 8\kappa({f_s}^2 + {f_p}^2)\right\} , \qquad (2.6e)$$

$$16\pi^2 \frac{d\sigma}{dt} = \left\{8\lambda^2 + 24g^4 - 32{f_s}^4\right\}$$

$$+ \left\{11\sigma^2 + 3\kappa^2 + 8\lambda\kappa - 24\sigma g^2 + 16\sigma{f_s}^2\right\} , \qquad (2.6f)$$

$$16\pi^2 \frac{d\rho}{dt} = \left\{8\lambda^2 + 24g^4 - 32{f_p}^4\right\}$$

$$+ \left\{11\rho^2 + 3\kappa^2 + 8\lambda\kappa - 24\rho g^2 + 16\rho{f_p}^2\right\} , \qquad (2.6g)$$

where $t = \ln \xi$ with $\xi$ being a scale of the linear momenta. In the right-hand sides of (2.6a)-(2.6g) the terms which vanish trivially in the symmetric limit are grouped together in the second curly brackets, and suppressed are the terms of the sixth order in $g$, $f_s$, and $f_p$, of the fourth order in $g$, $f_s$, and $f_p$ and the first order in $\lambda$, $\kappa$, $\sigma$, and $\rho$, of the second order in $g$, $f_s$, and $f_p$ and the second order in $\lambda$, $\kappa$, $\sigma$, and $\rho$, of the third order in $\lambda$, $\kappa$, $\sigma$, and $\rho$, and of any higher order. It is because we are looking for an asymptotic solution to (2.6a) to (2.6g) in which $g^2$, ${f_s}^2$, ${f_p}^2$, $\lambda$, $\kappa$, $\sigma$, and $\rho$ decrease like const. $\times$ $t^{-1}$

(including the case of const. = 0). This is, in fact, the only asymptotic behavior consistent with all of (2.6a)-(2.6g). One can observe immediately that when the supergauge constraints (2.5a)-(2.5c) are substituted, (2.6b)-(2.6d) for $f_s^2$, $f_p^2$, and $\lambda$ reduce to (2.6a) for $g^2$, and (2.6e)-(2.6g) are satisfied trivially as $0 = 0$.

Our problem is to look into stability of the fixed point realized in the supergauge Yang-Mills theory as viewed in the seven-dimensional parameter space of $g$, $f_s$, $f_p$, $\lambda$, $\kappa$, $\sigma$, and $\rho$ rather than along the specific direction constrained by the supergauge symmetry. The first equation (2.6a) can be solved as

$$g^2 = \frac{g_0^2}{1 + \frac{g_0^2}{2\pi^2} t} . \tag{2.7}$$

Redefining the remaining couplings as

$$(\bar{f}_s^2, \bar{f}_p^2, \bar{\lambda}, \bar{\kappa}, \bar{\sigma}, \bar{\rho}) \equiv (f_s^2/g^2,\ f_p^2/g^2,\ \lambda/g^2,\ \kappa/g^2,\ \sigma/g^2,\ \rho/g^2) , \tag{2.8}$$

we cast (2.6b)-(2.6g) into

$$16\pi^2 \frac{d\bar{f}_s^2}{dt} = 16\bar{f}_s^2(\bar{f}_s^2 - 1) , \tag{2.9a}$$

$$16\pi^2 \frac{d\bar{f}_p^2}{dt} = 16\bar{f}_p^2(\bar{f}_p^2 - 1) , \tag{2.9b}$$

$$16\pi^2 \frac{d\bar{\lambda}}{dt} = \left\{6\bar{\lambda}^2 - 16\bar{\lambda} + 8\bar{\lambda}(\bar{f}_s^2 + \bar{f}_p^2) - 6\right\} + \left\{8\bar{\lambda}\bar{\kappa} + 2\bar{\lambda}(\bar{\sigma} + \bar{\rho})\right\} , \tag{2.9c}$$

$$16\pi^2 \frac{d\bar{\kappa}}{dt} = \left\{8\bar{\lambda}^2 - 32\bar{f}_s{}^2\bar{f}_p{}^2 + 24\right\}$$

$$+ \left\{4\bar{\kappa}^2 + 4\bar{\lambda}(\bar{\rho} + \bar{\sigma}) + 5\bar{\kappa}(\bar{\rho} + \bar{\sigma}) - 16\bar{\kappa} + 8\bar{\kappa}(\bar{f}_s{}^2 + \bar{f}_p{}^2)\right\}, \tag{2.9d}$$

$$16\pi^2 \frac{d\bar{\sigma}}{dt} = \left\{8\bar{\lambda}^2 - 32\bar{f}_s{}^4 + 24\right\}$$

$$+ \left\{11\bar{\sigma}^2 + 3\bar{\kappa}^2 + 8\bar{\lambda}\bar{\kappa} - 16\bar{\sigma} + 16\bar{\sigma}\bar{f}_s{}^2\right\}, \tag{2.9e}$$

$$16\pi^2 \frac{d\bar{\rho}}{dt} = \left\{8\bar{\lambda}^2 - 32\bar{f}_p{}^4 + 24\right\}$$

$$+ \left\{11\bar{\rho}^2 + 3\bar{\kappa}^2 + 8\bar{\lambda}\bar{\kappa} - 16\bar{\rho} + 16\bar{\rho}\bar{f}_p{}^2\right\}. \tag{2.9f}$$

Equation (2.9a) shows that the supergauge symmetric solution $\bar{f}_s{}^2 = 1$ corresponds to an ultraviolet unstable fixed point. If our model (2.4) possesses only an approximate symmetry such as $\bar{f}_s = 1 + \delta$ at some $t$, this small deviation $\delta$ would amplify itself and either approach a finite constant of a stable fixed point, if any, or else blow up as $t \to \infty$. Only if $\bar{f}_s$ falls sharply on unity, the solution $\bar{f}_s{}^2 = 1$ is realized at $t \to \infty$. As for $\bar{f}_p{}^2$ the fixed point $\bar{f}_p{}^2 = 1$ is again ultraviolet stable. It is interesting to see whether the supergauge symmetric point is stable or not with respect to the other couplings varied independently around it. Since (2.9a) and (2.9b) forbid any deviation from $\bar{f}_s{}^2 = \bar{f}_p{}^2 \equiv 1$, we restrict ourselves onto the intersection of the two hypersurfaces given by $\bar{f}_s{}^2 = 1$ and $\bar{f}_p{}^2 = 1$ to examine the remaining equations in (2.9c)-(2.9f). It turns out that to the first order deviation

$$d(32\Delta\bar{\lambda} + 14\bar{\kappa} + 5\bar{\sigma} + 5\bar{\rho})/dt = 24(32\Delta\bar{\lambda} + 14\bar{\kappa} + 5\bar{\sigma} + 5\bar{\rho}) \quad , \tag{2.10a}$$

$$d(2\Delta\bar{\lambda} - \bar{\sigma} - \bar{\rho})/dt = -4(2\Delta\bar{\lambda} - \bar{\sigma} - \bar{\rho}) \quad , \tag{2.10b}$$

$$d(2\bar{\kappa} - \bar{\sigma} - \bar{\rho})/dt = -8(2\bar{\kappa} - \bar{\sigma} - \bar{\rho}) \quad , \tag{2.10c}$$

$$d(\bar{\sigma} - \bar{\rho})/dt = 0 \quad . \tag{2.10d}$$

Stable deviation is allowed along the normals to the hyperplanes given by $2\Delta\bar{\lambda} - \bar{\sigma} - \bar{\rho} = 0$ and $2\Delta\bar{\kappa} - \bar{\sigma} - \bar{\rho} = 0$. Physical implication of such a stable deviation is not clear since we have already chosen the unstable fixed points in $\bar{f}_s^{\,2}$ and $\bar{f}_p^{\,2}$ prior to this. If one starts with the perfectly symmetric Lagrangian, of course, no deviation occurs in any direction.

The supergauge Yang-Mills theory forbids mass terms for $A$, $B$, and $\psi$ as well as for $V_\mu$. To have relevance to the real physical world, therefore, the supergauge symmetry with the Yang-Mills fields must be broken. Because of the unstable nature of the asymptotic freedom of the supergauge theory, symmetry breaking due to soft operators of canonical dimension less than four either explicitly or spontaneously is of physical interest. When the breaking is soft, the $\beta$ functions for the dimensionless couplings are unaffected, but new super-renormalizable couplings enter into the renormalization group equation. Their $t$ dependence is determined by (3-5)

$$\frac{d\chi}{dt} = -\left(d - \gamma(t)\right) \quad , \tag{2.11}$$

where $d(>0)$ is the naive dimension of the coupling $\chi$ and $\gamma$ is the anomalous dimension of the symmetry breaking operator associated with $\chi$. The coupling $\chi$ includes mass terms and vacuum expectation values of scalar fields. The function $\gamma(t)$, in general, behaves like $c/t$ as $t \to \infty$, thus leading to

$$\chi(t) \sim t^{-c} e^{-dt} . \qquad (2.12)$$

Such soft coupling constants can be accommodated as symmetry breaking into the supergauge theory without ruining the asymptotic freedom for the dimensionless couplings.

## 3. SEARCH FOR ASYMPTOTIC FREEDOM WITHOUT MASSLESS PARTICLES

The foregoing analysis tempts us to speculate that the ultra-violet unstable solutions so far abandoned in the literature (6,7) may have a chance of survival. In order for such a solution to be realized in the physical world, all the dimensionless couplings normalized at any common finite $t$ must be dependent on each other, and there should be no deviation allowed for them. Associated with each of such models there exist a class of Lagrangians that are obtained by adding super-renormalizable terms to it. Although no obvious symmetry exists in general that forces the dimensionless couplings subject to constraints, such a Lagrangian or a class of Lagrangians are certainly asymptotically free.* We have explored in a few simpler cases whether there exists a model of strong interactions that raises all vector meson masses spontaneously through elementary scalar fields without ruining the asymptotic freedom nor the global symmetry incorporated originally in the Lagrangian. We learn from the preceding chapter that such a model must contain a nonvanishing

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* The possibility was discussed in a recent paper by N. P. Chang (8).

Yukawa coupling of scalar fields; if not, it would reduce to one of those that were already examined extensively (5,6). Following the line of reasoning developed by Bardakci and Halpern (9), we find that this excludes many of simpler models. With SU(N) as a global symmetry, one can assign the scalar multiplet S either into the $(N,\bar{N})$ or the $(N^2-1, N^2-1)$ representation of SU(N). When S is like $(N,\bar{N})$, where the physical global symmetry is $SU_1(N) \oplus SU_2(N)$, the simplest choice for fermions is to introduce a pair of fermion multiplets like (N,1) and (1,N). With the Yang-Mills fields $\sim(N^2-1,1)$ added to them, this model is one of the simplest candidates for the model that we search for. After a standard calculation, we have found that there always exists an unstable nonzero fixed point for the ratio of the Yukawa coupling of the scalar fields to the gauge coupling for an arbitrary N ($\geq 2$), but the four-scalar couplings have no fixed point at zero either stable or unstable. We have further looked into the assignment of

$$S \sim (N^2-1,\ N^2-1)\ , \tag{3.1a}$$

$$\psi_1 \sim (N^2-1,\ 1) \text{ and } \psi_2 = (1,\ N^2-1)\ , \tag{3.1b}$$

$$V_\mu \sim (N^2-1,\ 1)\ , \tag{3.1c}$$

as the next simplest possibility, though it contains rather depressingly many elementary fields of the exotic quantum numbers. Because of the same reason as in the preceding model, no desirable model has been discovered in this class, that maintains the asymptotic freedom. If the fermions are assigned into a single multiplet $(N^2-1,\ N^2-1)$, there is no unstable fixed point except at zero for the ratio of the

Yukawa coupling to the gauge coupling. We feel that even if one chooses an unstable fixed point of renormalization group, there is no simple field theory of strong interactions that is asymptotically free and avoids massless vector mesons through elementary scalar fields. Dynamical spontaneous breaking seems to be compelling.

## 4. ACKNOWLEDGMENTS

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