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Publication Date
1988-04-01

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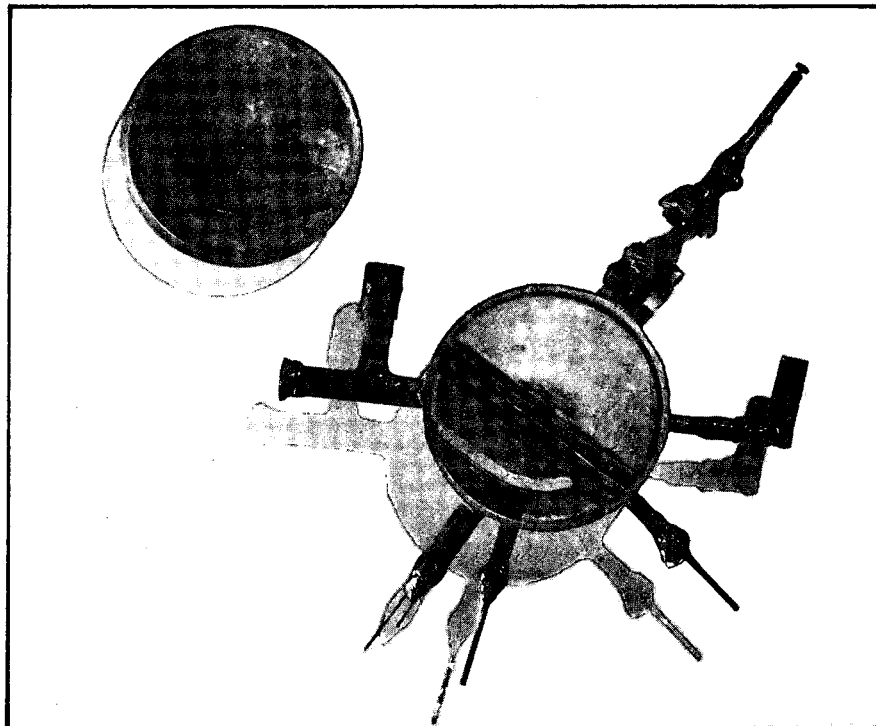
Aperture of Two-Counter Telescopes

W.P. Swanson

April 1988

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APERTURE OF TWO-COUNTER TELESCOPES

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ABSTRACT

Calculation of the geometrical aperture of two-counter telescopes using circular counters is discussed. A FORTRAN program OVERLAP is described that performs such calculations with convenient user input. Representative results are shown and the LBL Berklet detector is calculated as a specific example. A listing of the program is given in an appendix.

April 3, 1988

APERTURE OF TWO-COUNTER TELESCOPES

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INTRODUCTION

The telescope considered consists of two coaxial circular counters (Fig. 1). The "aperture" $A_p \equiv \, <\Delta A \cdot \Delta \Omega>$ is defined as the scalar quantity that satisfies the equation

$$C = A_p \times \phi_{\text{Isotropic}} \qquad (1)$$

where C is the number of particles that pass through both counters ("counts"); and $\phi_{\text{Isotropic}}$ is a broad uniform isotropic fluence as defined by the ICRU (1977). If $\phi_{\text{Isotropic}}$ is in units of $cm^{-2} \, sr^{-1}$, A_p will have units of cm^2 sr. Although the author was assured by experts that this problem had been solved many times over, a specific reference showing practical results could not be found. This note solves only the simple geometrical problem; particle "loss" by scattering or interactions in the telescope are not considered.

METHOD OF CALCULATION

"Pretend you are very far away and shining a flashight on the smaller counter. Its shadow on the plane of the larger counter will be circular. Now move the flashlight around in all possible directions ... ."* Program OVERLAP, written in FORTRAN for the VAX computer, evaluates the integral

$$\int AREA \, (\theta, \phi) \, d\Omega \, . \qquad (2)$$

AREA is the overlap area of the shadow on the disk of the larger counter and $d\Omega = 2\pi \sin\theta \, d\theta$, where θ is the polar angle to the telescope axis (Fig. 1).

For $\theta < \theta_1 \equiv \tan^{-1}[(R_2 - R_1)/S]$, $AREA = \pi(R_1)^2$, and

For $\theta > \theta_2 \equiv \tan^{-1}[(R_2 + R_1)/S]$, $AREA \equiv 0$,

---

(*) D. E. Groom, SSC Central Design Group, Private communication.

where R_1 and R_2 are the radii of the smaller and larger counter, respectively, and S is the counter separation. For the intermediate case, $\theta_1 < \theta < \theta_2$, the overlap area is the sum of two segments defined by the intersection of the two circles in the plane of the larger counter (Fig. 2). In this plane, the center of the smaller circle is at $(0, YC)$, where $YC = S \times \tan\theta$. The two points of intersection, (XI, YI), are solved for by:

$$YI = (R_2^2 - R_1^2 + YC^2)/(2 \times YC) . \tag{3a}$$

Then, XI is found using the equation of the larger circle:

$$XI = \pm\sqrt{(R_2^2 - YI^2)} . \tag{3b}$$

The segment area for the larger circle is found from

$$AREA_2 = R_2^2 \ (\phi_2 - \sin\phi_2) / 2.0 \tag{4}$$

where $\phi_2 = 2 \sin^{-1} | XI / R_2 | < \pi$ is the angle subtended by the segment of the larger circle at the circle's center $(0, 0)$. The same considerations are used to find $AREA_1$ (segment of the smaller circle) but in this case attention must be paid to the quadrant in determining ϕ_1 from the FORTRAN arcsin routine because ϕ_1 can exceed π. Then $AREA = AREA_1 + AREA_2$.

The result of the entire calculation is the summation

$$APERT = \sum (2\pi \sin\theta \ \Delta\theta) \times (AREA \ \times \cos\theta) , \tag{5}$$

where APERT is the desired aperture for the counter telescope and $\cos\theta$ is the obliquity factor. As written, OVERLAP uses 100 equal intervals in θ, so that $\Delta\theta = | \theta_2 - \theta_1 | /100$.

LIMITING CASE: SEPARATION → ZERO

For the two counters in contact $(S = 0)$, we have the equivalent of a single counter of area $\pi (R_1^2)$. All particles passing through the smaller counter are registered but the relevant area will appear smaller for larger values of θ by an obliquity factor equal to $\cos\theta$. The result can be shown to be π multiplied by the area of the smaller counter. In the results shown in Figs. 3 and 4, which involve a smaller counter of area $\pi(1 \ cm)^2$, all curves converge to the value $\pi^2 \ cm^2 \ sr = 9.87 \ cm^2$ sr as the separation goes to zero. The counters need not be circular for this limit to hold.

LIMITING CASE: SEPARATION → LARGE

For very large separations the aperture is given by

$$A_p = A_1 A_2 / S^2 \tag{6}$$

where A_1 and A_2 are the areas of the two counters and S is the separation. Units will be

steradians times the unit of area. This is shown in Fig. 5 where the dimensionless quantity $S^2 A_p$ / $(A_1 A_2)$ is plotted as a function of counter separation and for various ratios of R_2 / R_1. Unity is approached for large separations. The counters need not be circular for this limit to hold.

RUNNING THE PROGRAM OVERLAP

From any VAX terminal, invoke the procedures FORTRAN, LINK and RUN using the source file OVERLAP.FOR. During the RUN procedure OVERLAP will ask for, in turn, R_1 (radius of the smaller counter), R_2 (radius of the larger counter) and S (separation of the two counters). Units are cm. A single line of answers will appear almost immediately, containing:

<center>R1, R2, S and APERT</center>

OVERLAP will then request a new set of input parameters and so on, *ad infinitum*. Exit from this cycle by typing CNTL C.

An example for which this program was designed is the Berklet detector (Llacer *et al*., 1984). For this instrument, used to characterize beams of heavy ions, the parameters are $R_1 = 0.25$ cm, $R_2 = 0.55$ cm and $S = 1.0$ cm. The aperture for these parameters is calculated to be 0.138 cm^2 sr.

Another file, TELE.TOP, is also generated containing pairs of values (S, APERT). Each pair will appear on a separate line in a form easily adapted for input to TOPDRAWER (Chaffee, 1980). To properly invoke TOPDRAWER, some editing on TELE.TOP is required.

Examples of results from OVERLAP, plotted by TOPDRAWER are shown in Figs. 3 - 5. These may be applied to other geometries by simple scaling.

REFERENCES

ICRU (1977). International Commission on Radiological Protection, Recommendations of the International Commission on Radiological Protection, ICRP Publication **26** (Pergamon, Oxford, 1977).

R. B. Chaffee (1980). "Introduction to Top Drawer or A Lot of Things You Always Wanted To Know About Drawing Graphs, but Couldn't Wade Through the Top Drawer Manual to Find Out," SLAC Computation Group, Report No. CGTM-189, Stanford Linear Accelerator Center, Stanford, CA (revised Nov., 1980).

J. Llacer, C. A. Tobias, W. R. Holley and T. Kanai (1984). "On-line characterization of heavy-ion beams with semiconductor detectors," Med. Phys. **11**(3), 266-278 (1984).

FIGURE CAPTIONS

Fig. 1   Diagram defining parameters of a telescope consisting of two circular counters. Values shown are for the Berklet detector.

Fig. 2   Diagram in plane of larger counter showing how the overlap area is calculated from the sum of two segments, one from each circle.

Fig. 3   Output of the program OVERLAP showing behavior of aperture as a function of counter separation S and for various radii of the larger counter. For this graph, radius of the smaller counter is $R_1 = 1$ cm; other lengths are shown in units of R_1. The limit for zero separation, π^2 cm^2 sr, is shown. The single point is the result for the Berklet detector.

Fig. 4   Same as previous figure but with expanded scale.

Fig. 5   The dimensionless quantity $A_p S^2/(A_1 A_2)$ as a function of counter separation S and for various ratios R_2/R_1. For large separations the limit is 1.0.

APPENDIX: LISTING OF FORTRAN PROGRAM OVERLAP

```
C       PROGRAM OVERLAP COMPUTES THE APERTURE OF 2-COUNTER TELESCOPES
C       CONSISTING OF TWO CO-AXIAL CIRCULAR COUNTERS.

C       S IS SEPARATION OF COUNTERS;   UNITS ARE CM.
C       R2, R1 ARE RADII OF LARGER, SMALLER, CIRCLE RESPECTIVELY.
C       R2 MUST BE LARGER THAN R1.

C       TO TEST THE PROGRAM, TRY THE PARAMETERS OF THE BERKLET DETECTOR:
C       R1 = 0.25, R2 = 0.55, S = 1.0 CM;   RESULT:   APERTURE = 0.138 CM2 SR.

        DATA NTHETA / 100 /

C       THE FOLLOWING STATEMENTS CALL IN THE INPUT DATA FROM THE TERMINAL.

  1     TYPE 2
  2     FORMAT(//1X,'PROGRAM OVERLAP:   COMPUTES APERTURE OF 2-COUNTER TELE
       1SCOPE;'
       2      / 1X '                        CIRCULAR COUNTERS ARE ASSUMED.'
       3     // 1X,' TYPE RADIUS OF SMALLER COUNTER [cm] :'            :)
        ACCEPT*, R1
        TYPE 3
  3     FORMAT(/ 1X, ' TYPE RADIUS OF LARGER COUNTER [cm]'     )
        ACCEPT*, R2
        TYPE 4
  4     FORMAT(/ 1X, ' TYPE SEPARATION BETWEEN COUNTERS [cm]'   )
        ACCEPT*, S

C       THETA IS IN RADIANS
        THETA   = 0.0
        THETA2  = ATAN((R2 + R1) / S)
        DTHETA  = THETA2 / NTHETA
        APERT   = 0.0

C       PRINT 1902, R1, R2, S
 1902   FORMAT(/ ' CONFIRM INPUT DATA: ' 12E10.3)
C       PRINT 1903,   THETA2, DTHETA
 1903   FORMAT(  / ' THETA2, DTHETA = ' 2E10.3 / )
 1901   FORMAT(12E10.3 /)

        DO 1000 I = 1, NTHETA

        THETA = THETA + DTHETA
        IF (THETA . GT . THETA2) GO TO 1001

C       COORDINATES OF CENTER OF SMALLER CIRCLE AS PROJECTED ONTO SECOND PLANE:
C       S DENOTES SEPARATION BETWEEN PLANES OF CIRCLES

        YC = S * TAN(THETA)

C       WE NOW HAVE TWO CIRCLES IN THE PLANE OF THE SECOND COUNTER
C       LARGER  CIRCLE IS CENTERED AT (0,  0) WITH RADIUS R2.
C       SMALLER CIRCLE IS CENTERED AT (0, YC) WITH RADIUS R1.

C       FIND POINTS OF INTERSECTION
C       AXIS OF SYMMETRY IS THE Y-AXIS
```

```fortran
      AREA1 = 0.0
      AREA2 = 0.0
      PHI1  = 0.0
      PHI2  = 0.0

C     SOLVE FOR YI:

      YI = (R2**2 - R1**2 + YC**2) / (2.0 * YC)

C     SOLVE FOR XI1, XI2 USING EQUATION OF LARGER CIRCLE

      XI = 0.0
      XSQ = R2**2 - YI**2
      IF (XSQ . LE . 0.0) GO TO 899

      XI = SQRT(XSQ)

C     NOW WE HAVE COORDINATES OF INTERSECTION POINTS: (-XI, YI) AND (XI, YI).

C     AREA OF SEGMENT BOUNDED BY LARGER CIRCLE (2):

      IF (YI . LE . 0.000001) GO TO 899
      PHI2 = 2.0 * ASIN(XI / R2)

      AREA2 = R2 * R2 * (PHI2 - SIN(PHI2)) / 2.0

C     AREA OF SEGMENT BOUNDED BY SMALLER CIRCLE (1):

      AMY = YC - YI
      IF (ABS (AMY) . LE . 0.000001) GO TO 899

C     PHI1 FROM FORTRAN SUBROUTINE ASIN WILL ALWAYS BE POSITIVE.
      PHI1 = 2.0 * ASIN(XI / R1)

C     PUT PHI1 INTO CORRECT QUADRANT.
      IF (AMY. LE . 0.0) PHI1 = 2.0 * 3.1415927 - PHI1

      AREA1 = R1 * R1 * (PHI1 - SIN(PHI1)) / 2.0

C     NOW FIND TOTAL AREA:

      AREA = AREA1 + AREA2

  899 IF (YC + R1 . LE . R2) AREA = 3.14159 * R1 * R1
      IF (YC - R1 . GE . R2) AREA = 0.0

C     USE AREA OF SMALLER CIRCLE AS SEEN BY RAYS INCIDENT AT ANGLE THETA.

  999 IF (AREA . LE . 0.0) GO TO 1001

      DWTAREA = 2.0 * 3.1415927 * SIN(THETA) * DTHETA * AREA
      APERT   = APERT + DWTAREA * COS(THETA)

C     THE FOLLOWING DE-ACTIVATED STATEMENTS WERE USED FOR DEBUGGING.
C     PRINT 1901, THETA, YC, YI, XI,   PHI1,   PHI2,
C    1              AREA1, AREA2, AREA, DWTAREA, APERT
```

```
1000  CONTINUE
1001  CONTINUE

C     INPUT PARAMETERS AND RESULT ARE PRINTED ON THE TERMINAL SCREEN.
      PRINT 1905, R1, R2, S, APERT
1905  FORMAT(' R1, R2, S =' 3E10.3,
     1 ' APERT =' E12.4,' [sr cm2]'  // )

C     A FILE IS WRITTEN THAT CAN BE USED FOR TOPDRAWER INPUT.
      OPEN (UNIT=11, NAME='TELE.TOP', ERR=1, TYPE='NEW')
      WRITE(11, 1906) S, APERT
1906  FORMAT(2E12.4)

      GO TO 1

      END
```
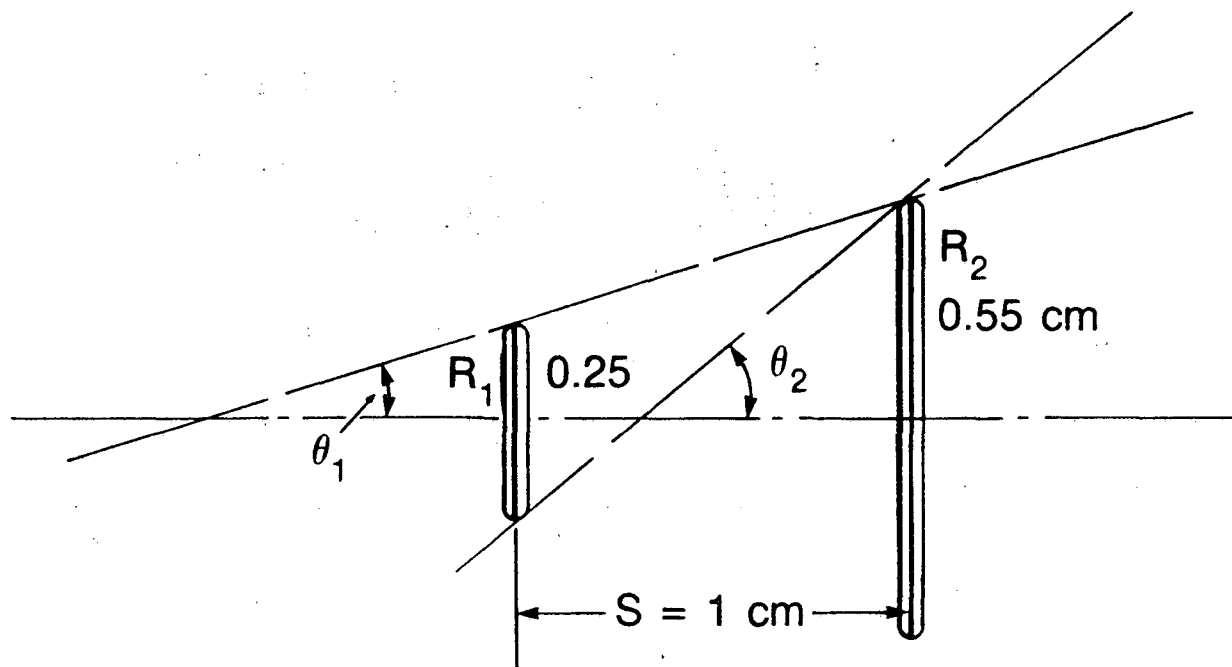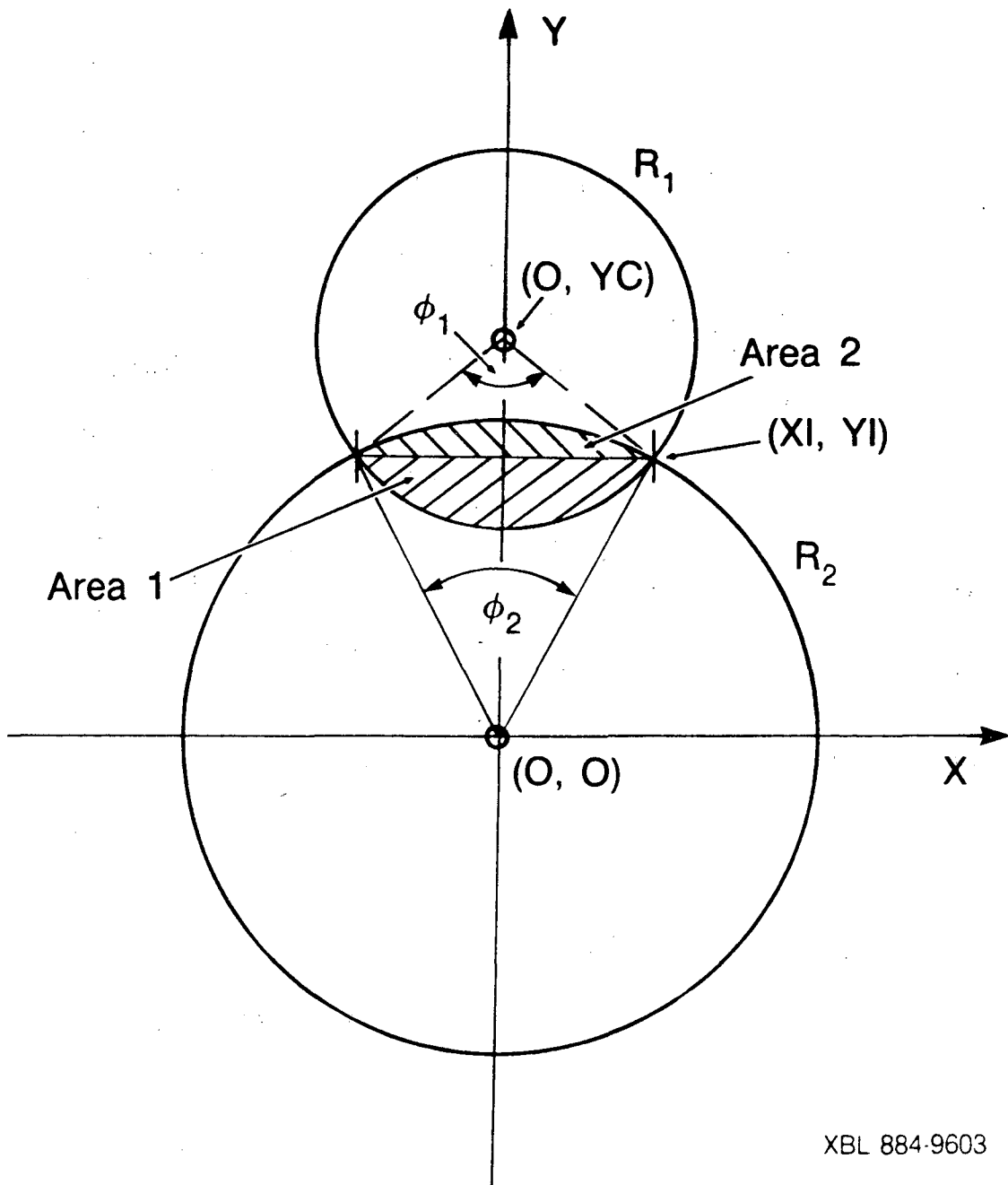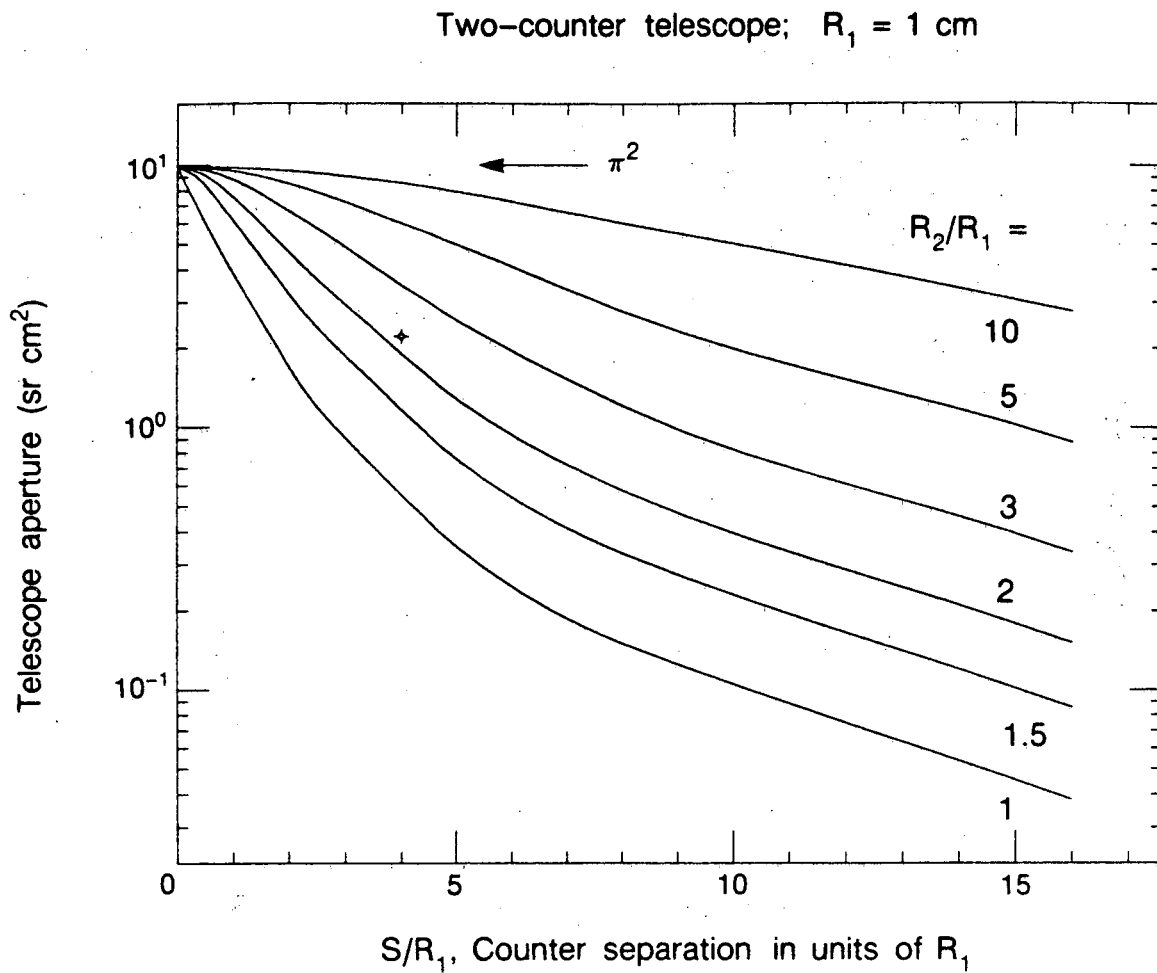
XBL 884-9602

Fig. 1   Diagram defining parameters of a telescope consisting of two circular counters. Values shown are for the Berklet detector.

Fig. 2   Diagram in plane of larger counter showing how the overlap area is calculated from the sum of two segments, one from each circle.

Two–counter telescope; R_1 = 1 cm

(y-axis) Telescope aperture (sr cm²)

π^2

$R_2/R_1 =$

10

5

3

2

1.5

1

(x-axis) S/R_1, Counter separation in units of R_1

XBL 884-9615

Fig. 3 Output of the program OVERLAP showing behavior of aperture as a function of counter separation S and for various radii of the larger counter. For this graph, radius of the smaller counter is $R_1 = 1$ cm; other lengths are shown in units of R_1. The limit for zero separation, π^2 cm² sr, is shown. The single point is the result for the Berklet detector.
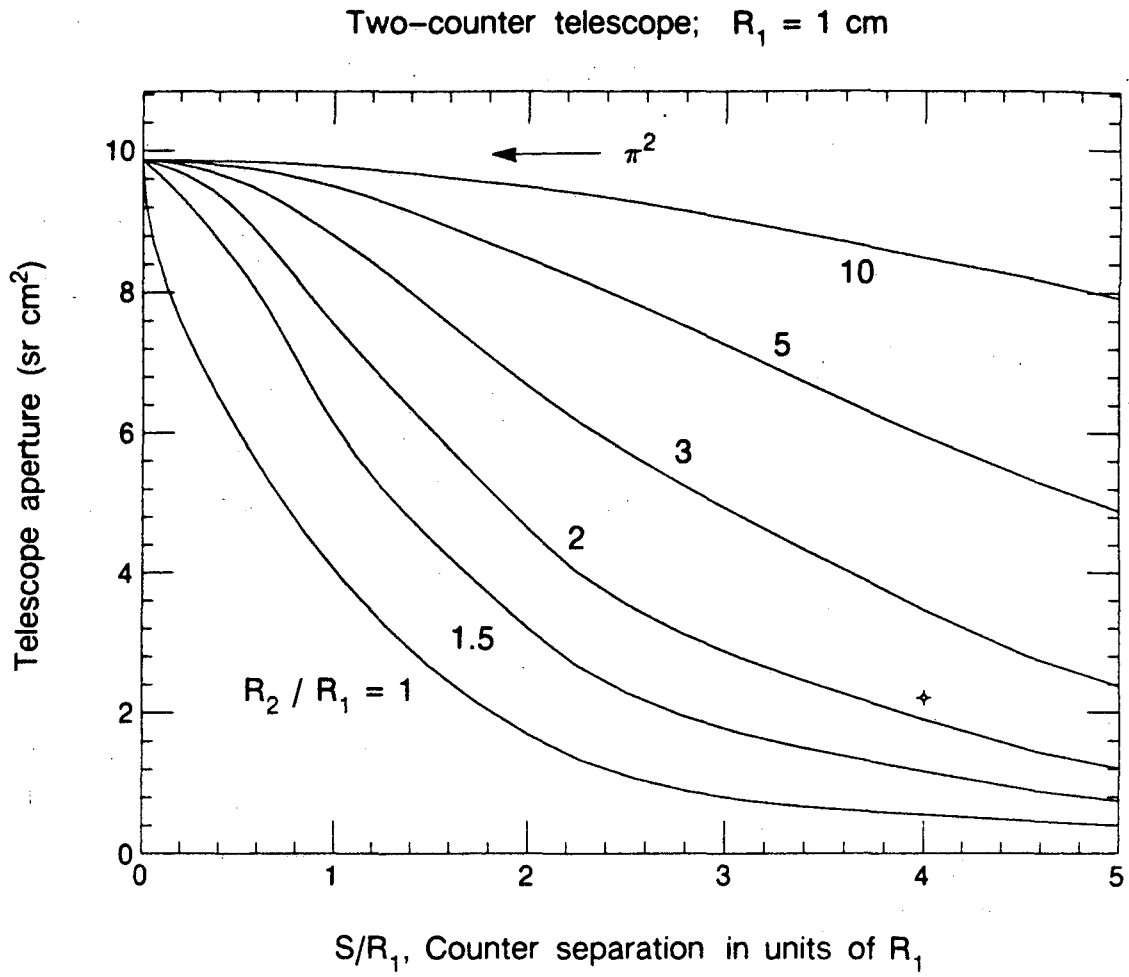
Fig. 4   Same as previous figure but with expanded scale.

Two–counter telescope
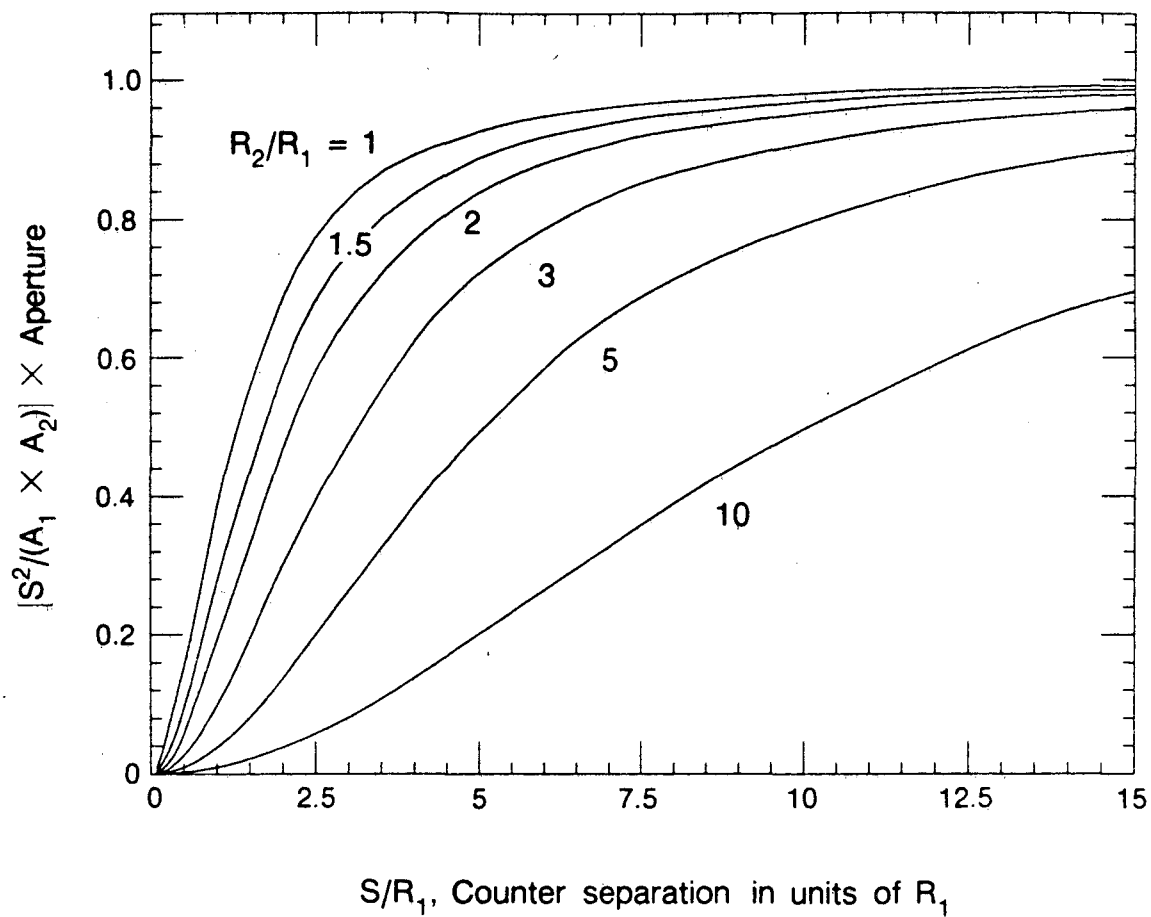
XBL 884-9613

Fig. 5 The dimensionless quantity $A_p S^2/(A_1 A_2)$ as a function of counter separation S and for various ratios R_2/R_1. For large separations the limit is 1.0.

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