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ABSTRACT

Consequences of C invariance, isospin invariance and unitary symmetry for limiting distributions are derived. The same symmetries are used to isolate the nonscaling contributions. Many of the predictions are susceptible to direct experimental test. The relations based on unitary symmetry are expected to be violated and thus furnish extensive challenges to symmetry breaking models. The relations based on isospin and C are expected to be exact (asymptotically) at high energies.

I. INTRODUCTION

The optical theorem for two-body scattering (Fig. 1a) enables one to understand many features of total cross sections, even without a detailed knowledge of how they are built up by multiparticle states. A corresponding optical theorem for three-body scattering is related to the cross section for the production of a single particle of definite momentum, along with anything else--what has been called inclusive production (see Figs. 1b). This application of the three-body optical[1] theorem is not straightforward, involving both crossing symmetry and careful analytic continuation. Still, the result is a conceptual and theoretical simplification similar to the one achieved through the two-body optical theorem.

Many general features of single particle inclusive production experiments can be understood in terms of the three-body forward scattering amplitude. From this point of view, Mueller[2] was able to show that the existence of limiting distributions in those parts of phase space referred to as the fragmentation and pionization regions is a general consequence of the dominance of pomeron exchange. These results, which will be described in Sec. II, had been established much earlier in the context of the multiperipheral model.[3] These would also appear to be satisfied in dual resonance models,[4] although the representation of the pomeron in such models is unsettled.

Charge conjugation and isospin invariance provide useful relations among two-body total cross sections. Furthermore, these invariances allow one to eliminate or isolate crossed channel amplitudes with certain quantum numbers, by selecting appropriate linear combinations of reactions. This is particularly useful when a Regge description is being employed. It is the purpose of this paper to extend such considerations to single particle inclusive production cross sections.[5] We shall also consider some consequences of SU_3 symmetry.

The outline of the paper is as follows: In Sec. II, we briefly review previous results for inclusive reactions in order to establish notation and to put our discussion into context. Those familiar with this standard discussion may pass immediately to Sec. III, in which the consequences of charge conjugation (C) invariance and G-parity are discussed. In Sec. IV, further consequences of isospin invariance are presented. We take up unitary symmetry in Sec. V. In Sec. VI we discuss the simplifications one can expect for the pionization limit. Finally, we conclude in Sec. VII with a summary of results and suggestions for further work.

II. DEFINITIONS AND KINEMATICS

Since spin is unimportant for the discussion of internal symmetries, we will suppress all spin indices. Consider the inclusive process $a + b \to c + x$, for fixed momenta p_a, p_b, p_c. The differential cross section for the production of particle c with momentum p_c will be written as

$$E_c \frac{d\sigma}{d^3p_c} = \left[\lambda(s, m_a^2, m_b^2)\right]^{-\frac{1}{2}} \mathcal{M} \tag{1}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz$ is the usual triangular function. The Lorentz invariant $\mathcal{M}$, which is related by the optical theorem is a discontinuity of the three-body scattering amplitude, is a function of three kinematical invariants which we choose to be

$$s = (p_a + p_b)^2, \qquad t = (p_a - p_c)^2, \qquad u = (p_b - p_c)^2 .$$

In the center of mass frame, we parameterize momenta as

$$p_a = (E_a, 0, 0, p)$$

$$p_b = (E_b, 0, 0, -p)$$

$$p_c = (E_c, p_\perp, 0, p_{||}) .$$

Consider a high-energy limit in which $s \to \infty$, for fixed $p_\perp$ and fixed positive $x = 2p_{||}/(s)^{\frac{1}{2}}$. This limit, which we denote $(a{:}c|b)$ is called the fragmentation of particle a into particle c, since in the rest frame of a, the momentum of c remains finite.[6] Of course in the center of mass, particle c carries away a finite fraction, x, of the center of mass energy. It follows from Mueller's analysis[2] that in this limit (see Fig. 2)

$$\mathcal{M} \to \sum_j \gamma_{b\bar{b}}^{\ j} \, f_{a\bar{c}}^{\ j}(p_\perp, x) \; s^{\alpha_j(0)} \tag{2}$$

where the sum extends over all Regge pole which couple to the crossed channel amplitude $b\bar{b} \to (a\bar{c})(\overline{a\bar{c}})$. In invariant terms, the limit corresponds to $s \to \infty$, for fixed t and for a fixed $x \cong -u/s$. The leading term, corresponding to pomeron exchange, implies that $(E_c/\sigma_T^{ab})(d\sigma/d^3p_c)$ tends to a finite limit (assuming a nonvanishing pomeron coupling) which is independent of particle b, if the pomeron factorizes as indicated. Thus, at sufficiently high energies, one may meaningfully speak of the fragmentation $(a{:}c)$ without reference to particle b. Obviously, one could similarly discuss the fragmentation $(a|c:b)$ by interchanging the roles of particle a and b in the preceding discussion.

Next consider a different high-energy limit in which $s \to \infty$ for fixed $p_\perp$ and for $x = 2p_{||}/(s)^{\frac{1}{2}} \to 0$. This corresponds to the production in the center of mass of a particle which carries away a vanishingly small fraction of the center of mass energy. This limit, denoted $(a|c|b)$ is called the pionization of particle c.[7] In invariant terms the limits corresponds to $|t| \to \infty$, $|u| \to \infty$ for fixed $tu/s = p_\perp{}^2 + m_c{}^2 + \mathcal{O}[E_c/(s)^{\frac{1}{2}}]$. From Mueller's analysis[2] (see Fig. 3), we find

$$\mathcal{M} \to \sum_{i,j} \gamma_{a\bar{a}}{}^{i}\, \gamma_{b\bar{b}}{}^{j}\, f_c{}^{ij}(p_\perp)\,|t|^{\alpha i(0)}\, |u|^{\alpha j(0)} \qquad (3)$$

where, by factorization, $f_c{}^{ij}$ depends only on particle c. Thus, if the double pomeron coupling does not vanish, $(E_c/\sigma_T{}^{ab})(d\sigma/d^3p_c)$ tends to a finite limit independent of both particles a and b. Thus one may speak of the pionization, $(|c|)$, independently of any reference to the beam or target.[8]

The pomeron is presently one of the most mysterious features of the strong interactions. For one thing, it seems to conserve s-channel helicity, which is not a natural property for an exchange mechanism. Secondly, there is the possiblity that the Pomeranchuk theorem may be violated. This would mean that the pomeron does not contribute solely to the $C = +$ crossed channel amplitude. Moreover, the pomeron is almost certainly not a Regge pole. This belief has been reinforced by the preliminary data[9] from the CERN Intersecting Storage Rings (ISR) showing that the pp elastic differential cross-section does not shrink. In the following we shall assume that at high

energies the dominant cross-channel amplitude has C even and $I = 0$ quantum numbers. Although we shall call this dominant contribution the pomeron, we shall assume neither that it is a pole nor that it factorizes.

Similarly, because of our uncertainty about whether a purely Regge pole description is adequate, we prefer not to assume that the parametrization given above for the asymptotic expansion is necessarily correct. It is, however, useful to keep this description in mind.

III. CHARGE CONJUGATION AND G-PARITY

In the following, C will denote charge conjugation; I, isospin; $G = e^{-i\pi I_2}C$, G-parity. As discussed above, we may think of the fragmentation $(a:c|b)$ as a particular discontinuity of the $ba\bar{c} \to ba\bar{c}$ scattering amplitude. Amplitudes even or odd under C in the crossed channel, $b\bar{b} \to a\bar{c}(\overline{a\bar{c}})$, are obtained by forming $(a:c|b) \pm (a:c|\bar{b})$ or equivalently, $(a:c|b) \pm (\bar{a}:\bar{c}|b)$. Similarly, eigenstates of G-parity are obtained by forming $(a:c|b) \pm (Ga:Gc|b)$. Denoting the four eigenamplitudes in the crossed channel by $\mathcal{M}_{GC}$ one finds

$$4\begin{pmatrix}\mathcal{M}_{++}\\ \mathcal{M}_{+-}\\ \mathcal{M}_{-+}\\ \mathcal{M}_{--}\end{pmatrix} = \begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}\begin{pmatrix}\mathcal{M}(a:c)\\ \mathcal{M}(\bar{a}:\bar{c})\\ \mathcal{M}(Ga:Gc)\\ \mathcal{M}(G\bar{a}:G\bar{c})\end{pmatrix} . \qquad (4)$$

(Since the particle b is unchanged throughout these manipulations the b dependence has been suppressed. We have not assumed anything about factorization.) Inversely, we have the relations

$$\begin{pmatrix} \mathcal{M}(a:c) \\ \mathcal{M}(\bar{a}:\bar{c}) \\ \mathcal{M}(Ga:Gc) \\ \mathcal{M}(G\bar{a}:G\bar{c}) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} \mathcal{M}_{++} \\ \mathcal{M}_{+-} \\ \mathcal{M}_{-+} \\ \mathcal{M}_{--} \end{pmatrix} \tag{5}$$

If $\mathcal{M}_{++}$ dominates the other amplitudes as $s \to \infty$, all four fragmentations are asymptotically equal. This leads, for example, to the asymptotical equalities

$$\begin{aligned} (\pi^-:K^-) &= (\pi^+:K^+) = (\pi^-:K^0) = (\pi^+:\bar{K}^0) \\ (K^-:K^-) &= (K^+:K^+) = (K^0:K^0) = (\bar{K}^0:\bar{K}^0) \\ (\pi^-:\eta) &= (\pi^+:\eta) \\ (K^+:\Lambda) &= (K^-:\Lambda) \ . \end{aligned} \tag{6}$$

This description is quite independent of any Regge assumptions; we have done no more than isolate various crossed channel amplitudes. If, however, the expansion in Eq. (2) is used, the dominant trajectories contributing to each amplitude are $\mathcal{M}_{++}$:P,P',f'; $\mathcal{M}_{+-}$:ρ; $\mathcal{M}_{-+}$:A_2; $\mathcal{M}_{--}$:ω,ϕ. Thus if Regge poles suffice to describe the exchanges we have

$$\begin{aligned} \mathcal{M}_{++} &= \gamma^{P}_{b\bar{b}}\, f^{P}_{a\bar{c}}\, s^{\alpha_P} + \gamma^{P'}_{b\bar{b}}\, f^{P'}_{a\bar{c}}\, s^{\alpha_{P'}} + \gamma^{f'}_{b\bar{b}}\, f^{f'}_{a\bar{c}}\, s^{\alpha_{f'}} \\ \mathcal{M}_{+-} &= \gamma^{\rho}_{b\bar{b}}\, f^{\rho}_{a\bar{c}}\, s^{\alpha_\rho} \\ \mathcal{M}_{-+} &= \gamma^{A_2}_{b\bar{b}}\, f^{A_2}_{a\bar{c}}\, s^{\alpha_{A_2}} \\ \mathcal{M}_{--} &= \gamma^{\omega}_{b\bar{b}}\, f^{\omega}_{a\bar{c}}\, s^{\alpha_\omega} + \gamma^{\phi}_{b\bar{b}}\, f^{\phi}_{a\bar{c}}\, s^{\alpha_\phi} \ . \end{aligned} \tag{7}$$

Note that these relations must be satisfied at each value of $p_\perp$ and x, i.e., the functions $f^{j}_{a\bar{c}}(p_\perp,x)$ must satisfy these relations. For $a = \pi^\pm$ and $c = \pi^\pm$ the negative G-parity exchanges are absent, so one can isolate P + P' + f' and ρ. Explicity, the invariant amplitudes $\mathcal{M}$ for $(\pi^+:\pi^-|p)$ and $(\pi^-:\pi^+|p)$ are asymptotically equal, and at finite s their difference is due solely to ρ exchange. Data[10] exist for $(\pi^+:\pi^-|p)$ at 7 GeV/c and for $(\pi^-:\pi^+|p)$ 24.8 GeV/c. Although they are very nearly the same, the clear difference between the two suggest the presence of the P' and/or the ρ secondary. This example illustrates one important use of the relations presented above. Even though data may exist on a reaction at only one energy, one can use data obtained on other reactions related by C or G to determine the importance of secondary trajectories.

IV. ISOSPIN RELATIONS FOR LIMITING DISTRIBUTIONS

So far we have assumed that the dominant amplitude, which we loosely refer to as the pomeron, has $C = +$ and $G = +$. Let us now assume that the pomeron is an isosinglet. Suppose, for a moment, that it were a Regge pole as illustrated in Fig. 2. Then it is clear that there will be relations among the limiting distributions $(a:c|b)$ for different members of the same isospin multiplets, relations which are perfectly analogous to the decomposition of the $a\bar{c}$ elastic scattering amplitude. Applying the Wigner-Eckart theorem, we find for the dominant contribution to the fragmentation the decomposition

$$\mathcal{M}(a:c|b) = \sum_{I,I_z} (\langle II_z|I_aI_{za},I_cI_{zc}\rangle)^2 \mathcal{M}_I \qquad (8)$$

where the $\mathcal{M}_I$ are reduced matrix elements. The total isospin I in the $a\bar{c}$ channel takes on clearer significance in the triple-reggeon limit where the fragmentation may be thought of as a sum of t-channel Regge exchanges (Fig. 4). Now we drop the assumption that the pomeron is a Regge pole; clearly the decomposition holds whenever the dominant amplitude in the $b\bar{b} \to (a\bar{c})(\overline{a\bar{c}})$ channel is an isosinglet.[11]

Just as in two-body scattering, isospin invariance greatly reduces the number of independent reactions. For example, the six fragmentations $(N:\pi|b)$ are described by only two independent amplitudes, and satisfy the relations (again suppressing the dependence on b)

$$(p:\pi^0) = (n:\pi^0) = \tfrac{1}{2}[(p:\pi^+) + (p:\pi^-)]$$

$$(p:\pi^+) = (n:\pi^-) \qquad (9)$$

$$(p:\pi^-) = (n:\pi^+) .$$

(Analogous formulae hold for the fragmentation of any isodoublet into any isotriplet.) These relations depend only on isosinglet, $C = +$ dominance and should be rigorously true as $s \to \infty$. Since they are valid at each value of $p_\perp$ and x, it follows <u>a fortiori</u> that the mean number of neutral pions produced in NN collisions (at sufficiently high energies) is half the total number of charged pions produced.[12] Thus models of the strong interactions differentiate only between the distributions of π^+ and of π^-.[13] Interchanging the roles of the pion and nucleon, we have $(\pi^-:p) = (\pi^+:n)$ and $(\pi^+:p) = (\pi^-:n)$. Similarly, the fragmentation $(\pi:\Sigma)$ satisfy

$$(\pi^+:\Sigma^-) = (\pi^-:\Sigma^+) ,$$

$$(\pi^+:\Sigma^+) = (\pi^-:\Sigma^-) ,$$

$$(\pi^+:\Sigma^0) = (\pi^-:\Sigma^0) .$$

(Because of their experimental inaccessibility, we have omitted relations involving the fragmentation of neutral pions.) Other simple relations are $(K^+:\Lambda) = (K^0:\Lambda)$, $(\pi^+:\Lambda) = (\pi^-:\Lambda)$.

V. SU_3 RELATIONS FOR LIMITING DISTRIBUTIONS

The invariances discussed so far are good symmetries, and the relations presented in Sec. IV, if not exact, are expected to be violated only slightly. Unitary symmetry is certainly not exact and, in spite of efforts over the past decade, no satisfactory theory of symmetry breaking yet exists. Consequently the primary use of SU_3 symmetry has been the pattern it predicts for particle classification. Nevertheless, when combined with a certain amount of lore concerning how to compare predictions with data, SU_3 has provided a semi-quantitative guide to resonance widths.[14] In addition, the predictions for the relations between Regge residues [e.g., $\gamma(\rho\pi\pi) = 2\gamma(\rho K\bar{K})$] have been moderately successful.[15] It has proved essential to use reduced resonances widths and reduced Regge residues in order to obtain reasonable agreement with the predictions of SU_3 invariance. The kinematical correction factor associated with the angular momentum barrier is sensitive to both the spins of the particles involved and their masses. Consequently, the direct application of SU_3 invariance to scattering amplitudes, such as in meson-baryon scattering, involving many partial waves or several Regge poles, has had limited success. In addition, an obvious but severe experimental handicap is the constraint that only nucleons may be used as targets.

In this section, we would like to extend the asymptotic predictions for limiting distributions to a theory having SU_3 symmetry. A common hypothesis, which we shall also make, is that the pomeron is purely a unitary singlet. This is aesthetically appealing since a universal diffraction mechanism ought to have purely vacuum quantum numbers. Experimentally[16] it appears that the $\pi^- p$ and $K^- p$ cross-sections are not asymptotically equal, differing by about 15%.

It is natural to assume that this difference is due to the same symmetry breaking which makes a pion different from a kaon rather than some intrinsic contamination of the pomeron.

Consider the fragmentation of an octet into an octet, such as pseudoscalar mesons (P) into baryons (B). Assuming that the pomeron is a unitary singlet leads to an expression[17] analogous to Eq. (8), obtained from isospin invariance,

$$\mathcal{M}(a:c|b) = \sum_{\mu_\gamma,\mu_{\gamma'},\nu} \begin{pmatrix} 8 & 8 & \mu_\gamma \\ \nu_a & -\nu_c & c \end{pmatrix} \begin{pmatrix} 8 & 8 & \mu_{\gamma'} \\ \nu_a & -\nu_c & \nu \end{pmatrix} \mathcal{M}_{\mu_\gamma,\mu_{\gamma'}} . \tag{10}$$

This is precisely the same as the decomposition of meson-meson or meson-baryon total cross-sections. However, because we do not require meson or hyperon targets, many more reactions are accessible experimentally for the fragmentation processes than for the scattering processes. In general, the nondiagonal terms, $\mathcal{M}_{8_a 8_s}$ and $\mathcal{M}_{8_s 8_a}$ occur. Without loss of generality, we may assume $\mathcal{M}_{8_a 8_s} = \mathcal{M}_{8_s 8_a}$.[18] Thus there are seven invariant amplitudes corresponding to 1, 8_s, 8_a, 10, 10^*, 27, and $8_s 8_a$.

The results for (P:B), (B:B), (B:P), $(P:\bar{B})$ etc. are easily obtained from one another mutatis mutandis. We present the details of the (P:B) case. The 64 reactions are reduced to 26 independent ones by isospin invariance. The decomposition of the 26 fragmentations is presented in Table I. There are thus 19 independent relations which we choose as follows:

$$
\begin{aligned}
(\pi^-:p) &= (K^-:\Sigma^+) = (K^+:\Xi^0) \\
(\pi^+:p) &= (K^-:\Xi^0) = (K^+:\Sigma^+) \\
(\pi^-:\Sigma^+) &= (K^-:p) = (K^+:\Xi^-) \\
(\pi^-:\Sigma^-) &= (K^-:\Xi^-) = (K^+:p) \\
(\pi^-:\Xi^-) &= (K^-:\Sigma^-) = (K^+:n) \\
(\pi^+:\Xi^-) &= (K^-:n) = (K^+:\Sigma^-)
\end{aligned}
\qquad (11)
$$

$$
\begin{aligned}
&2(\pi^-:p) + 2(K^+:n) + 4(\pi^+:\Sigma^0) = (\pi^+:p) + (K^-:n) + 6(K^+:\Lambda) \\
&2(\pi^+:p) + 2(K^-:n) + 4(\pi^+:\Sigma^0) = (\pi^-:p) + (K^+:n) + 6(K^-:\Lambda) \\
&(\pi^+:p) + (\pi^-:p) + (K^-:n) + (K^+:n) = (\pi^+:\Sigma^0) + 3(\pi^-:\Lambda) \\
&(\eta:\Sigma^+) = (\pi^+:\Lambda) \\
&(\pi^+:\Sigma^0) + (\eta:\Lambda) = (\pi^+:\Sigma^+) + (\pi^+:\Sigma^-) \\
&6(\eta:p) + (\pi^-:p) + (\pi^+:p) = 2(K^+:n) + 2(K^-:n) + 4(\pi^+:\Sigma^0) \\
&6(\eta:\Xi^-) + (K^-:p) + (K^+:n) = 2(\pi^-:p) + 2(\pi^+:p) + 4(\pi^+:\Sigma^0) .
\end{aligned}
$$

Of course using C and isospin invariance, the relations given above can be cast in a variety of equivalent forms. With the exception of the last four equations involving η fragmentation, we have chosen forms which should be testable experimentally. An insufficient number of fragmentation reactions have been measured so far to enable a comparison with the predictions above. Recently, however, data[19] became available on $(K^+:\Lambda|p)$ and $(K^+:\bar{\Lambda}|p)$ at 12.7 GeV/c. By C invariance,

the latter reaction equals $(K^-:\Lambda|\bar{p})$. If this reaction is dominated by pomeron exchange at this energy (as it well might be since both the initial channel and missing mass are exotic) and if a Pomeranchuk-like theorem holds so that $(K^-:\Lambda|\bar{p}) = (K^-:\Lambda|p)$ asymptotically, then we may interpret $(K^+:\Lambda|p)$ at 12.7 GeV/c as the value of $(K^-:\Lambda|p)$ at infinite energy. In the SU_3 limit, the fragmentations $(K^+:\Lambda|p)$ and $(K^+:\bar{\Lambda}|p)$ differ only because $\mathcal{M}_{10,10} \neq \mathcal{M}_{10^*,10^*}$ and $\mathcal{M}_{8_a 8_s} \neq 0$. Noting that the $(K^+:\Lambda|p)$ data differ from $(K^+:\bar{\Lambda}|p)$ data, we conclude either that secondaries are still important at these energies in the reaction $(K^+:\Lambda|p)$ or, what seems more likely, that inversion of the baryon octet (R invariance) is a bad symmetry.[14]

Let us turn now to the case of a pseudoscalar fragmenting into a pseudoscalar, (P:P). Assuming the pomeron is C even, one can show that $\mathcal{M}_{8_s 8_a} = 0$ and $\mathcal{M}_{10,10} = \mathcal{M}_{10^*,10^*}$. Thus there are only five SU_3 invariant amplitudes for (P:P) and the general expression, Eq. (10), reduces to

$$
\mathcal{M}^{(a,c|b)} = \sum_{\mu,\gamma,\nu} \begin{pmatrix} 8 & 8 & \mu\gamma \\ \nu_a & -\nu_c & \nu \end{pmatrix}^2 \mathcal{M}_{\mu\gamma,\mu\gamma} . \qquad (12)
$$

Of the 64 reactions for (P:P), only 16 are independent after taking into account C invariance and isospin invariance. It follows immediately from Eq. (12), that $(a:c) = (\bar{c}:\bar{a})$ or, using C invariance again, $(a:c) = (c:a)$. Useful examples of this relation are $(\pi^+:K^+) = (K^+:\pi^+)$ and $(\pi^-:K^+) = (K^+:\pi^-)$. Since there are five invariant amplitudes, there must be seven relations among the twelve remaining fragmentations. They may be chosen to be

$$(\pi^-:\pi^+) = (K^-:K^+)$$

$$(K^-:K^0) = (K^-:\pi^+)$$

$$(K^-:K^-) = (\pi^-:\pi^-)$$

$$(K^-:\pi^-) = (K^-:\bar{K}^0)$$

$$3(K^-:\eta) = 2(\pi^+:\pi^0) + (K^+:\pi^0)$$

$$3(\pi^-:\eta) = 4(K^+:\pi^0) - (\pi^+:\pi^0)$$

$$(\eta:\eta) = (\pi^+:\pi^+) + (\pi^+:\pi^-) - (\pi^+:\pi^0) . \qquad (13)$$

The last relation is written for completeness only. The others are experimentally accessible. Notice that it is unnecessary to observe $(K^-:\pi^0)$ since by isospin and C,

$$(K^-:\pi^0) = (K_L^o:\pi^-) = \frac{1}{2}\left[(K^-:\pi^+) + (K^-:\pi^-)\right] .$$

By similar manipulations, these relations can be written in a variety of other forms.

The fragmentation of an octet of pseudoscalars (P) into a degenerate nonet of vector mesons (V) can also easily be treated. We will summarize the results. After isospin and C invariance are invoked, there are 19 independent fragmentations and seven SU_3 invariant amplitudes. Of the 12 relations thereby implied among the reactions, eight may be chosen which do not involve the SU_3 singlet ω_1 and the $Y = I = 0$ member of the octet ω_8 . These eight are perfectly analogous to the $P \to P$ case:

$$(K^+:\rho^+) = (\pi^+:K^{*+})$$

$$(K^+:\rho^-) = (\pi^-:K^{*+})$$

$$(K^-:K^{*0}) = (K^-:\rho^+)$$

$$(K^-:K^{*-}) = (\pi^-:\rho^-)$$

$$(K^-:\rho^-) = (K^-:\bar{K}^{*0})$$

$$(K^-:K^{*+}) = (\pi^-:\rho^+)$$

$$3(\eta:K^{*+}) = 2(\pi^+:\rho^0) + (K^+:\rho^0)$$

$$3(\eta:\rho^+) = 4(K^+:\rho^0) - (\pi^+:\rho^0) . \qquad (14)$$

Using isospin invariance in conjunction with these relations, one can generate other experimentally testable predictions, such as

$$2(K^-:\rho^0) = (K^-:\bar{K}^{*0}) + (K^-:K^{*0}) .$$

As for the four relations involving ω_1 and ω_8 , even if we assume that the only effect of SU_3 breaking is the determination of the mixing angle between them, we find no prediction which seems even remotely testable.

VI. PIONIZATION

Since all of the preceding equations are expressed at fixed $p_\perp$ and fixed $x > 0$, we are free to consider their limit as x tends to zero. As discussed in Sec. II, we expect the exchanges in the $a\bar{a}$ channel to determine the behavior of the fragmentations $(a:c|b)$ as $x \to 0$. As illustrated in Fig. 3 and seen in Eq. (3), in a Regge pole model the leading contribution to the pionization $(a|c|b)$ comes from double pomeron exchange. Consequently if the pomeron is a $C = +$, isosinglet, f_c^{PP} involves the coupling of the $c\bar{c}$ state to an isosinglet, which implies that each charge state of particle c couples with equal strength.

As we have done previously, the hypotheses may be weakened somewhat by dropping the assumption that the exchange mechanisms are Regge poles. One may instead write the pionization distribution $(a|c|b)$ as a sum of amplitudes labelled by the quantum numbers of the $a\bar{a}$ and $b\bar{b}$ channels. Then assuming that the dominant amplitude has purely vacuum quantum numbers in these channels yields the result that $c\bar{c}$ must couple to a $C = +$ isosinglet. We feel it is important to make this observation; quite generally, the success or failure of the predictions of this paper is neither a confirmation nor a refutation of the Regge pole model.

As noted already by Mueller, one immediate consequence of this analysis is that the pionization distributions for the three-charge states of the pion must be equal. It follows that their mean multiplicities in the pionization limit are equal. In a Regge pole model, the mean multiplicity of c at high energies can be written[2] as $\langle n_c \rangle = A_c \ln s + B_c$, where coefficient A_c of $\ln s$ is determined by the pionization distribution.[13,20]

Similarly in the pionization limit, the production of protons and neutrons must be equal. Invariance under C requires that these equal the antinucleon production. This equality may furnish a useful test of whether protonization has been achieved at the CERN ISR.

Assuming the pomeron to be a unitary singlet and assuming exact SU_3 symmetry leads to the prediction that in the pionization

region, the production of all the pseudoscalar mesons be equal. Similarly, in the baryonization domain, all members of the baryon octet should be produced with identical distributions.

A primitive model for SU_3 breaking in the pionization region can be obtained by assuming the pomeron has a small part which transforms as the eighth member of an octet, that this the dominant symmetry breaking mechanism, and working only to first order in the symmetry breaking. This is precisely analogous to the usual mass-splitting calculation and gives relations which may be read directly from the Gell-Mann-Okubo relations. For any two incident particles a and b one finds

$$\frac{1}{2}\left[(|N|) + (|\Xi|)\right] = \frac{1}{4}\left[(|\Sigma|) + 3(|\Lambda|)\right]$$

$$(|K|) = \frac{1}{4}\left[(|\pi|) + 3(|\eta|)\right]$$

where Σ denotes any one member of the Σ isomultiplet and so forth.[21]

VII. CONCLUSION

Isospin invariance is well established for strong interactions. The Pomeranchuk theorem, on the other hand, is not beyond question experimentally, so it is not clear that the pomeron can be regarded as purely a $C = +$ isosinglet. One would like to use data on limiting fragmentations as further tests of this theorem. Unfortunately, the asymptotic equality $\mathcal{M}(a:c|b) = \mathcal{M}(a:c|\bar{b})$ has not been established starting from assumptions about the three-body scattering amplitude analogous to those made in the proof of the Pomeranchuk theorem for two-body scattering. If it were proved, then the asymptotic equalities such as $(\pi^-:\pi^-) = (\pi^+:\pi^+)$, $(\pi^-:K^-) = (\pi^+:K^+)$, $(p:p) = (\bar{p}:\bar{p})$ could be used to supplement the data on total cross sections.

If the assumptions made in Secs. II, IV, and VI that the pomeron has $C = +$, $I = 0$ is correct, then the relations given there, based on isospin invariance, are to be used rather than to be tested. We have illustrated, for example, how one can use data on different but related reactions to test for the presence of secondaries. We should not forget, however, that Mueller's analysis, summarized in Sec. II, is still somewhat speculative, since our understanding of the analyticity and singularity structure of the three-body scattering amplitude is rather meager. Thus one might regard the experimental verification of these relations as a confirmation that certain crossed channel amplitudes dominate asymptotically.

On the other hand, where SU_3 invariance is concerned, the question is not whether the relations will be satisfied, but rather by how much, and in what fashion will they be broken. Since the predictions given above are for the asymptotic fragmentation distributions as functions of x and $p_\perp$, there is potentially a great wealth of information on symmetry breaking. The ratio, say, of $(\pi^-:\pi^+)$ to $(K^-:K^+)$, predicted to be 1 in an SU_3 invariant world, will probably depend on both x and $p_\perp$. This demands a dynamical theory of symmetry breaking. Can one define "reduced" fragmentations which will bring observations more nearly in line with theory?

Inclusive reactions provide a plethora of symmetry relations which can be tested experimentally. On the one hand there are relations which ought to be exact asymptotically if our understanding of these processes is correct. On the other hand there are relations which one expects to be violated by symmetry breaking. Data are rapidly being accumulated on a wide variety of inclusive processes. Guided by the symmetry relations discussed herein, these experiments should provide both a test of our basic understanding and a challenge to various symmetry breaking models.

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FOOTNOTES AND REFERENCES

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1. H. P. Stapp, Phys. Rev. D3, 3177 (1971); C. Tan, Brown University Preprint NYO-2262TA-240.
2. A. H. Mueller, Phys. Rev. D2, 2963 (1970).
3. See, in particular, the lectures by S. Fubini in Strong Interactions and High Energy Physics, Scottish Universities' Summer School, 1963, R. G. Moorhouse, Ed., Plenum Press, New York. More recent discussions have been given for single particle distributions by D. Silverman and C. Tan, Phys. Rev. D2, 233 (1970). The extension and implications of this model for two particle inclusive production have been presented by S. Pinsky and W. Weisberger, "Final State Spectra in a Multiperipheral Model," University of Utah preprint.
4. M. A. Virasoro, Phys. Rev. D3, 2834 (1971); C. DeTar, et al., MIT preprint, MIT-CTP-180; D. Gordon and G. Veneziano, Phys. Rev. D3, 2116 (1971). These results have also been generalized to two-particle inclusive production by C. Jen et al., Brown University Preprint NYO-2262TA-244. B. Hasslecher et al., "Dual Resonance Model Implications for Two Particle Spectra in Inclusive Reactions," Preprint, Institute for Theoretical Physics, State University of New York at Stony Brook.
5. A few of these results have been pointed out previously by H. D. I. Abarbanel, Phys. Letters 34B, 69 (1971) and by H. M. Chan et al., Phys. Rev. Letters 26, 672 (1971).

6. We propose the following notation for inclusive processes: $a + b \to c_1 + c_2 + c_3 + c_4 + c_5 + X$ where c_1 is a fragment of a, c_5 is a fragment of b, c_2, c_3, and c_4 are pionization products and c_2 is separated from c_3 and c_4 by a large rapidity difference, and c_3 and c_4 are close in rapidity is indicated by $(a{:}c_1|c_2|c_3,c_4|c_5{:}b)$. The corresponding amplitude as defined in Eq. (1) is indicated by an $\mathcal{M}$ preceding the parentheses. Thus the process is designated by a schematic rapidity plot. (For a discussion of rapidity variables see R. P. Feynman's paper in High Energy Collisions, Yang et al., Ed., Gordon and Breach Pub., Inc., New York, 1969.) Each vertical line represents a large spacing in rapidity. Inessential labels can be dropped without confusion. Thus $(a{:}c|b)$ becomes $(a{:}c)$ when particle b is understood to remain fixed. For the sake of brevity, the $\mathcal{M}$ will often be deleted in various relations, but the relations are always for the amplitudes or the asymptotic amplitudes.
7. The terminology stems from the analogue in atomic physics to ionization. It would perhaps be better to call it hadronization. It is awkward to speak of the pionization of the proton, but even more so to talk about the anti-omega-minus-ization.
8. It is common practice to define the pionization limit as $s \to \infty$ for fixed $p_\perp$ and fixed $p_\parallel$. This corresponds to $|t|/\sqrt{s}$ and $|u|/\sqrt{s}$ fixed. The beauty of Mueller's analysis is that the result, Eq. (9), is valid more generally, that is, no matter how $E_c/\sqrt{s}$ tends to zero. Moreover, to the extent the Regge pole description is valid, the dependence on $p_\parallel$ is determined explicitly by the secondary trajectories in Eq. ( ). See H. D. I. Abarbanel, Phys. Rev. D3, 2227 (1971).

9. M. Holder et al., Phys. Letters 35B, 355 (1971).

10. M.-S. Chen et al., Phys. Rev. Letters 26, 1585 (1971).

11. This decomposition can easily be generalized to involve a sum over isospins in the $b\bar{b}$ channel. Since most of the interesting applications have already been dealt with in the preceding section, we have omitted this generalization.

12. See, for example, L. Caneschi and A. Schwimmer, Phys. Rev. D3, 1588 (1971).

13. The precise statement is as follows: As $s \to \infty$, the mean multiplicity of pions of charge c can be written as $\langle n_c \rangle = A_c \ln s + B_c$. The statement is that $2\langle n_0 \rangle = \langle n_+ \rangle + \langle n_- \rangle$. In fact, the coefficient A_c of the logarithm is determined entirely by the pionization region. Consequently, as discussed in Sec. VI, $A_0 = A_+ = A_-$.

14. See the summary in P. Carruthers, Introduction to Unitary Symmetry (Interscience Publishers, New York, 1966).

15. C. Michael, Regge Residues, in Springer Tracts in Modern Physics (Springer-Verlag, New York, 1970), Vol. 55; B. Kayser to be published.

16. J. V. Allaby, et al., Phys. Letters 30B, 500 (1969).

17. Our notation and conventions for SU_3 Clebsch-Gordan coefficients are as in J. J. DeSwart, Rev. Mod. Phys. 35, 916 (1963). For a summary, see Chap. 4 of Carruthers, Ref. (9).

18. This follows from time reversal invariance applied to the non-diagonal processes $a\bar{c}b \to a'\bar{c}'b$

19. S. Stone et al., University of Rochester preprint UR-875-349.

20. This is not to say that most of the pions produced asymptotically are pionization products, since, as Mueller[2] showed, to obtain $\ln s$ growth one must include fragmentation products. The interpretation of the growth of multiplicity depends sensitively on the border between fragmentation and pionization.

21. The relation for the meson case is likely to be broken more than for the baryon case, but perhaps improved agreement will be obtained if the squares of the differential inclusive cross sections are used.

TABLE I. $P \to B = \sum_{\Gamma} c_{\Gamma} \mathcal{M}_{\Gamma}$

	$\mathcal{M}_{27}$	$\mathcal{M}_{10^*}$	$\mathcal{M}_{10}$	$\mathcal{M}_{8D}$	$\mathcal{M}_{8F}$	$\mathcal{M}_{8D8F}$	$\mathcal{M}_{1}$
$\pi^+ \to p$	1/5	1/6	1/6	3/10	1/6	$2\sqrt{5}/10$	0
$\pi^- \to p$	1/2	1/2	0	0	0	0	0
$K^+ \to p$	7/40	1/12	1/12	1/5	1/3	0	1/8
$K^+ \to n$	1/5	1/6	1/6	3/10	1/6	$-2\sqrt{5}/10$	0
$K^- \to p$	1	0	0	0	0	0	0
$K^- \to n$	1/2	0	1/2	0	0	0	0
$\eta \to p$	9/20	0	1/4	1/20	1/4	$-\sqrt{5}/10$	0
$\pi^- \to \Sigma^+$	1	0	0	0	0	0	0
$\pi^+ \to \Sigma^0$	1/2	1/12	1/12	0	1/3	0	0
$\pi^+ \to \Sigma^+$	7/40	1/12	1/12	1/5	1/3	0	1/8
$K^+ \to \Sigma^+$	1/5	1/6	1/6	3/10	1/6	$2\sqrt{5}/10$	0
$K^+ \to \Sigma^-$	1/2	0	1/2	0	0	0	0
$K^- \to \Sigma^+$	1/2	1/2	0	0	0	0	0
$K^- \to \Sigma^-$	1/5	1/6	1/6	3/10	1/6	$-2\sqrt{5}/10$	0
$\eta \to \Sigma^+$	3/10	1/4	1/4	1/5	0	0	0
$\pi^+ \to \Lambda$	3/10	1/4	1/4	1/5	0	0	0
$K^+ \to \Lambda$	9/20	1/4	0	1/20	1/4	$-\sqrt{5}/10$	0
$K^- \to \Lambda$	9/20	0	1/4	1/20	1/4	$\sqrt{5}/10$	0

TABLE I. (Cont.)

	$\mathcal{M}_{27}$	$\mathcal{M}_{10^*}$	$\mathcal{M}_{10}$	$\mathcal{M}_{8D}$	$\mathcal{M}_{8F}$	$\mathcal{M}_{8D8F}$	$\mathcal{M}_{1}$
$\eta \to \Lambda$	27/40	0	0	1/5	0	0	1/8
$\pi^+ \to \Xi^-$	1/2	0	1/2	0	0	0	0
$\pi^- \to \Xi^-$	1/5	1/6	1/6	3/10	1/6	$-2\sqrt{5}/10$	0
$K^+ \to \Xi^-$	1	0	0	0	0	0	0
$K^+ \to \Xi^0$	1/2	1/2	0	0	0	0	0
$K^- \to \Xi^-$	7/40	1/12	1/12	1/5	1/3	0	1/8
$K^- \to \Xi^0$	1/5	1/6	1/6	3/10	1/6	$2\sqrt{5}/10$	0
$\eta \to \Xi^-$	9/20	1/4	0	1/20	1/4	$\sqrt{5}/10$	0

FIGURE CAPTIONS

Fig. 1 (a). The two-body total cross section as a discontinuity of the two-body forward amplitude.

(b). The one particle inclusive cross section as a discontinuity of the three-body forward amplitude.

Fig. 2. A representation of the fragmentation $(a:c|b)$ as a sum over Regge poles j.

Fig. 3. A representation of the pionization $(a|c|b)$ as a sum over Regge poles i and j.

Fig. 4. The limiting fragmentation $(a:c|b)$ in the triple reggeon region. Regge poles (R, R') of isospin I couple to the pomeron (P).

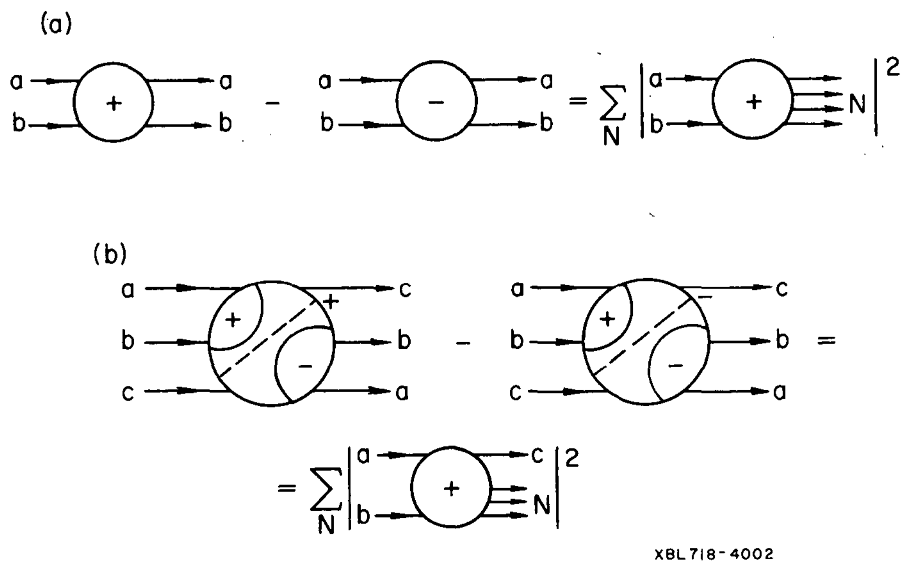

Fig. 1

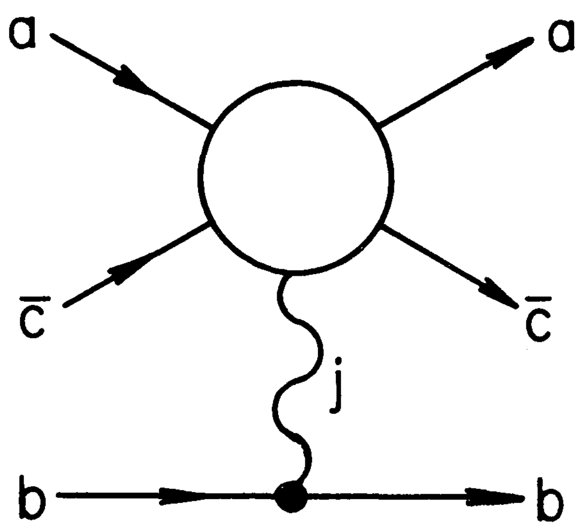

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Fig. 2

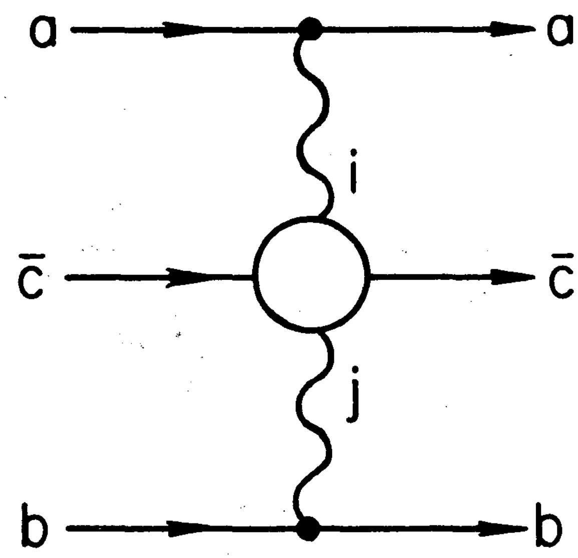

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Fig. 3

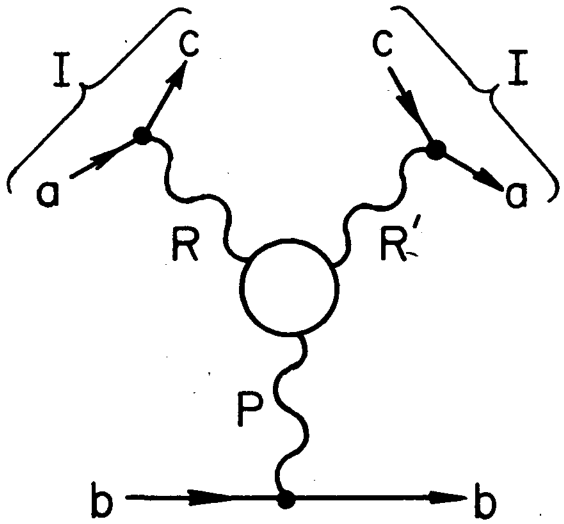

Fig. 4

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