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The Two-Sex Problem and the Marriage Squeeze in an Equilibrium Model of Marriage Markets

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Introduction

This paper develops a model of marriage market equilibrium that can be used to analyze the effects of age structure on marriage patterns. The model clarifies a number of issues in the literature on the "two-sex problem" and the "marriage squeeze." Particular emphasis is placed on the age difference between spouses as an equilibrating mechanism in marriage markets. The model follows the spirit of Becker's application of the Koopmans-Beckman assignment model to marriage markets (Becker, 1973, 1981). We derive conditions for the assignment of marriage partners among men and women born in different cohorts and use the results to analyze the effects of fluctuations in cohort size fluctuations on marriage markets. The results indicate that even large changes in cohort size can be absorbed by relatively modest adjustments in the age difference between spouses, with no necessary adjustments in the proportions of men and women marrying.

There is a large sociological literature on the "marriage squeeze," a reference to an imbalance in the relative numbers of marriageable males and females.[1] The fundamental force behind most marriage squeezes is changes in cohort size over time. If marriages were always between men and women of identical ages, changes in cohort size would have no effect on the relative supplies of husbands and wives.[2] Most populations are characterized by persistent systematic differences in the ages of spouses, however, with men on average marrying women who are two to five years younger. Studies for the U.S. over the last forty years, for example, regularly find that men are on average two to three years older than their wives.[3] Although the magnitude of this age difference varies across countries and time periods, the tendency for men to marry younger women persists across diverse demographic, economic, and social conditions. A study of 29 developing countries by Casterline et al. (1986) found a persistent tendency for men to marry younger women. The median difference between the age of husband and the age of wife was as high as 9.7 years and never less than 2.5 years in the countries studied.

Given a persistent tendency for women to marry older men, cohort size fluctuations will upset the balance in the marriage market. If there is a "baby boom," women from this large cohort will, when they reach maturity, be seeking spouses from among the males in a much smaller cohort. Simple arithmetic guarantees that not all women from the large cohort will find mates from the smaller cohort from which their partners would normally be supplied. Some women could be expected to postpone marriage, waiting for the males from their own large cohort to become

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1 See, for example, Akers (1967), Schoen (1983, 1985), Goldman et al. (1984). For an economic analysis of marriage squeeze effects see Ermisch (1981).

2 Discrepancies in the relative numbers could still arise through differential mortality rates between sexes, an effect that has been argued to be an important determinant of black marriage patterns in the U.S. (Spanier and Glick, 1980). Mortality differentials between men and women might vary over time due to changing health conditions or wars, but they should not be directly related to changing sizes of birth cohorts.

3 See, for example, Glick and Landau (1950), Presser (1975), Carter and Glick (1976), Atkinson and Glass (1985).

available as potential partners. Others, unable to find a suitable partner at reasonable terms, might choose to remain unmarried. The terms of marriage might also be expected to become less favorable for the sex and cohort that is in excess supply, an effect emphasized in recent literature by Heer and Grossbard-Schechtman (1981) and Guttentag and Secord (1983).

In the following sections we outline a theory of marriage assignments that can be applied to test a number of hypotheses regarding the effects of age structure on marriage market outcomes. No simple "mating function" can ever capture all properties of a general equilibrium marriage market. Our model of marriage begins in the spirit of Becker's (1974, 1981) analogy between marriage assignments and the linear programming assignment problem analyzed by Koopmans and Beckman (1957). Primary emphasis is on assignments based on age (or cohort), the essential issue in analyzing marriage squeeze effects.

Age Differences Between Spouses in Stable Populations

One of the fundamental issues to be analyzed in the model of marriage assignments developed below is age differences between spouses and the extent to which those age differences are affected by changes in age structure. A number of basic points regarding the role of the age difference between spouses as an equilibrating mechanism in marriage markets can be demonstrated by analyzing a stable population with a constant proportional age distribution.

Suppose that all men who marry do so at age a_m, all women who marry do so at age a_f, the probability of survival to marriage age is $p_m(a_m)$ for males and $p_f(a_f)$ for females, and the probabilities of marriage for males and females conditional on survival are π_m and π_f respectively. The proportions of male and female births who eventually marry is the product of the survival probability and the conditional marriage probability, and will be denoted by $m = \pi_m p_m(a_m)$ and $f = \pi_f p_f(a_f)$.

If all of these probabilities are to hold in every period, then the fundamental marriage market clearing condition is that in every period $\pi_m M(a_m) = \pi_f F(a_f)$, where $M(a)$ and $F(a)$ are the number of males and females at age a. Equivalently, the condition is that $msB_{t-a_m} = f(1-s)B_{t-a_f}$, where s is the proportion of births that are male and B_t is the number of births at time t. If we assume a stable population growing at annual rate r, then $B_t = B_0 e^{rt}$, and the marriage market equilibrium condition is that in every period t

$$msB_t e^{-ra_m} = f(1-s)B_t e^{-ra_f}. \tag{1}$$

If we assume for simplicity that the sex ratio at birth is unity, (1) implies that

$$\frac{f}{m} = e^{-r(a_m - a_f)}. \tag{2}$$

Two simple special cases are immediately evident from (2). If $f = m$, implying that the same proportion of male and female births eventually marry, then either $r = 0$, in which case the age difference between spouses is irrelevant, or $a_m = a_f$, in which case the population growth rate is irrelevant. Note that m and f represent the product of the probability of surviving to the marriage age for that sex and the probability of marriage conditional on survival. More generally, the condition in (2) describes equilibrium values of the age difference between spouses $a_m - a_f$ and the population growth rate r that will sustain a given ratio of male and female marriage probabilities. From (2) these can be found by simply noting that for $a_m - a_f \neq 0$, marriage market equilibrium requires that

$$r = \frac{-\ln(f/m)}{(a_m - a_f)}. \tag{3}$$

Figure 1 shows contours of constant f/m ratios as a function of r and $a_m - a_f$.[4] Not surprisingly, the figure demonstrates that higher population growth rates require smaller age differences between spouses in order to be sustained as equilibrium rates. The more interesting result demonstrated by the figure is the large range of population growth rates that can be supported without any change in marriage probabilities by varying the age gap between spouses. For example, when 99% as many female births eventually marry as do male births, population growth rates from less than .5% up to 4% can be supported as equilibrium rates by simply varying the age difference between spouses between .3 and 4 years.

While these results apply to stable populations, the ability of relatively modest variations in the age gap between spouses to absorb substantial variations in population growth rates, with no adjustment in the proportions of males and females marrying, has important implications for short-run fluctuations in age structure as well. As will be demonstrated below, even large short-run fluctuations in sex ratios can often be absorbed by relatively small variations in the age gap between husbands and wives. This has important implications for the marriage squeeze, since it suggests that changing sex ratios may have less dramatic effects on proportions marrying or the terms of marriage than are often assumed.

The Linear Programming Assignment Algorithm and the Two-Sex Problem

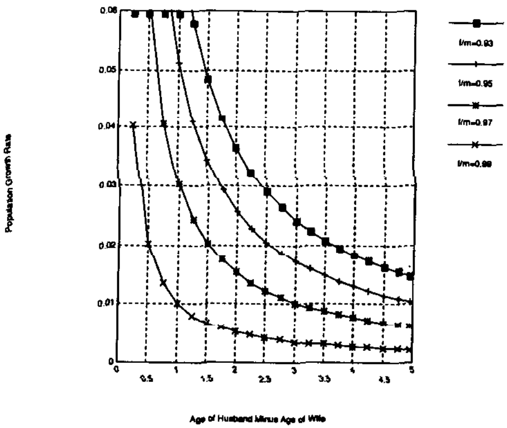
A major puzzle in classical demography is the "two-sex problem." The problem is well set out in papers by Keyfitz (1971), McFarland(1972), Mahsam (1974), Das Gupta (1974), Sanderson (1983), Pollak (1986, 1987) and Caswell and Weeks (1986). These papers point out that "one-sex" models of marriage and reproduction are not proper equilibrium models. The one-sex models attempt to estimate future marriage patterns or birth patterns by projecting age-specific propensities of

[4] It is convenient to look at the ratio f/m, since one simple interpretation of it is the proportion of females that could marry in equilibrium if all males were to marry. The empirically relevant case of $a_m > a_f$ and $r > 0$ requires that $m > f$, so f will attain its upper bound in such cases when $m = 1$.

Figure 1.

Contours of Constant Ratio of Males and Females Marrying As a Function of Stable Population Growth Rate and Age Difference Between Spouses.

(f/m = Ratio of female births eventually marrying to male births eventually marrying.)



a single sex to marry or to have children from earlier cohorts onto the population of that sex in younger cohorts. One evident weakness of this approach is that the male one-sex models predict different numbers of marriages and of births than do the female one-sex models. As economists, we find it natural to think that this state of affairs calls for a general equilibrium model in which the decisions of agents are rational responses to the "market" environment in which they find themselves. In this case we are inclined to think about a marriage market in which supplies and demands are equilibrated by adjustment in the terms of marriage. We would expect that persons of the same sex would be substitute goods, with persons being closer substitutes the closer their proximity in age.

Pollak (1986, 1987) has recently reformulated the "two-sex problem" by replacing the constant age-specific fertility schedule of the classical theory with two more fundamental relationships. These are a "birth matrix" and a "mating rule." The birth matrix postulates an expected number of

births per period from a marriage of an age i male to an age j female. The mating rule is a function that determines the number of marriages of type i males to type j females for all i and j as a function of the vector listing the numbers of males and females of each age and sex in the population. Pollak shows that if these relationships remain constant over time and if the mating rule follows certain natural conditions, the dynamical system so defined will converge to a constant equilibrium growth rate, yielding a constant equilibrium age structure. Pollak imposes only certain very general conditions on the mating function such as nonnegativity, homogeneity, continuity, and that the number of persons of a given sex who marry must not exceed the number of persons of that age and sex in the population.

Pollak's mating rule can be thought of as a "reduced form" description of the dependence on the outcome of a marriage market on supplies and demands of the two sexes from various cohorts. In this paper we look behind this reduced form by posing an explicit structure of payoffs to the possible patterns of mating and to study the marriage market equilibrium under this structure. In particular, we take advantage of an idea proposed by Becker (1973), who suggested that the problem of finding an efficient and stable assignment of marriage partners can be usefully viewed as an application of the linear programming assignment problem. If correct, this is very good news, because the assignment problem is very well-studied and very nice to work with. There are efficient algorithms for solution of empirical assignment problems. Furthermore there are elegant and simple comparative statics for this model.

The assignment problem was originally devised as a model for the efficient assignment of workers to jobs. For each worker, i, in job, j, there is a money value of output a_{ij} which could be produced if worker i is assigned job j. The assignment problem solves for the assignment of workers to jobs that maximizes the total value of output subject to the constraint that each worker has only one job and each job is done by only one worker. The solution to the assignment problem not only reveals an optimal assignment, but it also imputes "shadow prices" to each worker and job in such a way that if each worker were paid his shadow price as a wage and each job received its shadow price as a rent, the optimal assignment would be a competitive equilibrium.

The assignment problem would be a reasonable model of marriage if the "value" of a marriage could be measured by a single number like money income. On the face of it, this seems an outrageous simplification of what marriage is about. In a marriage there are many joint decisions to be made about many matters that are far removed from money. There also may be substantial differences in tastes, in skills, and in initial wealth between potential marriage partners. As it turns out, we have been able to show that the complexity of interaction in a marriage can be quite well modeled by the presence of a large number of shared public goods in the marriage. For a broad and interesting class of preferences over public and private goods, it happens that there is "transferable utility". This means that although many complex joint decisions must be reached about household public goods, the efficient amount of public goods in a marriage is determined independently of the distribution of private goods within the marriage. When this is true, the assignment problem model of marriage as proposed by Becker can be applied directly.[5]

We will discuss here the simplest model that is empirically implementable and yet rich enough to capture much of the character of the marriage squeeze. Suppose that an individual's utility depends only on his or her age at marriage and on consumption of private goods. Of course, if any two people choose to marry each other, they must both marry on the same day. This effect can be nicely modeled by treating the date of the wedding as a (local) public good, entering into both of the potential partners' utility functions. The reason one's wedding date enters one's utility function is because this date determines ones age of marriage.

Let $\bar{A}_i$ be person i's preferred age at marriage, and suppose that individual i has a quadratic loss function for marrying at a less than ideal age. In particular, let C_i be person i's consumption of private goods, let W_i be the year of i's marriage and B_i the year of person i's birth. Then $A_i = W_i - B_i$ is i's age at marriage. Let $\bar{A}_i$ be i's most preferred age of marriage. Then let i's utility function be

$$U_i(A_i, C_i) = C_i - (A_i - \bar{A}_i)^2. \tag{4}$$

Written in terms of the wedding date and birth date, this can be equivalently expressed as

$$U_i(W_i, B_i, C_i) = C_i - [(W_i - B_i) - \bar{A}_i]^2 = C_i - (W_i - \bar{W}_i)^2. \tag{5}$$

where $\bar{W}_i = B_i + \bar{A}_i$ indicates i's most preferred wedding date. Suppose now that male i and female j are contemplating marriage. Their utility functions as given in (5) belong to the class of utility functions shown by Bergstrom and Cornes (1983) to have the property that the optimal choice of public goods is independent of how the private goods are distributed in the marriage. Given these utility functions it is optimal for i and j to maximize joint utility, which given some wedding date W_{ij} will be

$$U_{ij} = U_i + U_j = C_i + C_j - (W_{ij} - \bar{W}_i)^2 - (W_{ij} - \bar{W}_j)^2. \tag{6}$$

Solving (6) for the optimal wedding date W_{ij}, the first order condition is

$$\frac{\partial U_{ij}}{\partial W_{ij}} = -2(W_{ij} - \bar{W}_i) - 2(W_{ij} - \bar{W}_j) = 0,$$

implying that $W_{ij} = (\bar{W}_i + \bar{W}_j)/2$. For these special functions, the optimal time for i and j to marry if they do marry is exactly half way between the favorite wedding date of i and the favorite wedding date of j.

[5] A detailed description of how this works out can be found in Bergstrom (1986) and in Bergstrom and Cornes (1983). See Lam (1988) for an application to the issue of assortative mating in marriage markets.

For example, suppose that male i was born in 1944 and female j was born in 1946. Suppose that i's preferred age at marriage is 25 and j's preferred age at marriage is 21. Then i would prefer to marry in 1969 and j would prefer to marry in 1967. The efficient time for this couple to marry if they do marry would be in 1968. In this case, each person would be missing his or her favorite age of marriage by one year. Because of the special utility function that we have assumed, the model displays transferable utility. That is, for any two people i and j of opposite sexes, there is a number A_{ij} which is the total utility generated by an optimally timed marriage between i and j. Persons i and j can divide this utility in any way that adds up to A_{ij} by redistributing private goods between them.

Suppose that all males preferred to marry at age $\bar{A}_m$ and that all females preferred to marry at age $\bar{A}_f$. Then, given the assumption of a quadratic loss function, we can find the payoff matrix which reports the value A_{ij} of a marriage between a cohort i male and a cohort j female. Given any set of males and females by birth cohort we can solve a linear programming assignment to assign the cohorts to each other in an optimal way. Not only does this give us a prediction of which cohorts of males would marry which cohorts of females in equilibrium, but knowing which cohorts marry, we also know when they marry. Thus we can generate a prediction of the ages at marriage for each cohort.

Suppose, for example, that each person's utility function is of the form given in (5) and that the ideal age at marriage is 24 for females and 26 for males. It follows that the payoff matrix for marriages between males and females of particular cohorts will be of the form given in Figure 2:

One interesting implication of the example described by the payoff matrix in Figure 2 is that the marriage market outcomes observed in equilibrium will give the appearance that individuals care about the age of their spouse or about the age difference between themselves and their spouse even though the example is constructed based on the assumption that individuals care only about their own age at marriage. The payoffs shown in Figure 2 indicate that marriages in which husbands are two years older than their wives are the "best" marriages in the sense that they provide the maximum benefit to the couple. This occurs not because either spouse cares directly about the age of their partner, however. It occurs because marriages of this type will allow both partners to marry at their ideal age. Although a given individual is assumed to be indifferent between spouses of different ages, *given the terms of marriage*, potential spouses of different ages will have different optimal marriage dates. The male born in 1944 in the above example would be equally happy with a wife born in 1950 or a wife born in 1946, as long as each was willing to marry him in 1968 and give him equal levels of consumption C_i. A female born in 1946, however, will view a 1968 marriage much more attractively than will a female born in 1950. The working of the marriage market will

Figure 2
Hypothetical Marriage Payoff Matrix.
Total Utility Cost of a Marriage Between Male Born in Year i and Female Born in Year j Given Quadratic Loss Function and Ideal Age at Marriage of 26 for Males and 24 for Females.

		Female Birth Cohort j						
		1940	1941	1942	1943	1944	1945	...
	1940	-4	-1	0	1	-4	-9	...
Male	1941	-9	-4	-1	0	-1	-4	...
Birth	1942	-16	-9	-4	-1	0	-1	...
Cohort i	1943	-25	-16	-9	-4	-1	0	...
	1944	-36	-25	-1	-9	-4	-1	...
	1945	-49	-36	-25	-16	-9	-4	...
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱

produce a marriage between the 1944 man and 1946 woman with much greater probability than a marriage between the 1944 man and the 1950 woman, even though the age of the spouse has no direct effect on utility from the marriage.

The fact that equilibrium conditions in the marriage market lead to marriage assignments that can be misinterpreted as indicating properties of underlying preferences is one of the important lessons from modeling marriage markets in this way. Casterline, Williams, and McDonald (1986), for example, observe that the joint frequency distribution of husbands' and wives' ages in the countries studied indicates more concentration than would be observed if the same median difference had been generated by random assignments. They interpret this as indicating that the observed distribution results from a true preference for certain age differences, rather than simply from the fact that men and women have different distributions of age at marriage. The example above demonstrates that the outcomes they observe could in fact be generated entirely from differences in ideal marriage ages between men and women, with no preference for the age difference between spouses *per se*.

The procedure outlined above is too rigid and simple as described. For example, we can not expect that everyone in the same cohort and sex will have the same preferred age at marriage. Indeed the matrix of predicted assignments that we obtain by assuming identical preferences predicts that there would be much less variety in the types of matching by age than actually occurs. We propose a method for allowing some variation in "taste". While more elaborate methods could be employed, this device has the virtue of being quite simple to calculate and to contemplate. Suppose

that there is variation in the "maturity" among people of the same calendar age. As potential marriage partners, some 20 year olds may be the equivalent of the average 22 year old and some may be the equivalent of an average 18 year old. This "maturity" that we are concerned with here could be sexual or emotional maturity, but it could also be "economic maturity", in the sense of owning enough assets to be ready for marriage. Let us suppose that there is an exponential distribution of "maturity" around calendar age so that the probability that someone of calendar age A is of maturity, M, is proportional to $|A - M|^{-\alpha}$ where α is a parameter to be estimated.

For a given choice of α, given the distribution of calendar ages in the population at any time, we can also calculate the distribution of the population by sex and maturity level at that time. If we assume that marriage assignments take place according to maturity rather than simply to age, we can follow the same procedure we outlined earlier to generate a prediction of the pattern of marriages by maturity level rather than by age. Using a calculation based on Bayes' theorem, we can determine the estimated matrix of marriage assignments by calendar age from the matrix of assignments by maturity. The fundamental assumption that allows us to perform this "inversion" is the assumption that one's choice of marriage partner and of date of marriage is determined only by maturity levels, so that people of the same sex and maturity will distribute themselves in the same way among marriage partners of the opposite sex even if their calendar ages differ.

We search over a range of the dispersion parameter, α, for the value that best fits the predictions to the data. The greater the value of α, the greater will be the variation in preferred dates of marriage among persons in the same cohort, and the more likely will be assignments of persons of widely differing ages to each other. It is important to remember that our hypothesis is that equilibrium adjustments take place according to the unobserved "maturity." Our strategy is to hypothesize a variance of maturity around calendar age, to "fuzz" this data by assuming a value for α, calculating the implied distribution of maturities and using this distribution and the assignment algorithm to predict the distribution of marriages by maturity. Finally we invert the fuzzing operation to deduce the predicted distribution of marriages by calendar ages implied by the value of the parameter α and the predicted distribution of marriages by calendar ages. More elaborate kinds variation in preferences over the population can be introduced by this same method. For example, it may be that there is greater variation in maturity among people of a given age for one sex or the other. Such elaborations can be introduced quite readily, using the same general procedure we followed above.

Hypothesis Testing and Prediction in the Assignment of Marriage Partners

The interpretation of the "shadow prices" that are found in the assignment problem is especially interesting in the application to marriage assignments. The assignment solution assigns a "price" specific to each cohort and sex such that if the assignment problem assigns a male of cohort i to a female of cohort j, then their respective shadow prices, p_i^m and p_j^f just add up to the value A_{ij} of a marriage between them. Furthermore, neither member of the couple could get a higher payoff by marrying a person from a different cohort, paying his or her price and retaining the residual value from the contemplated alternative marriage. Thus the assignment problem not only is a theory of who marries whom and when they marry, but it also contains a theory of the distribution of benefits within marriages.

If someone belongs to a cohort whose shadow price is unusually low, then our theory suggests that he or she will be offered less attractive terms of marriage than someone who belongs to a cohort whose shadow price is high. As the terms at which a particular cohort and gender can marry become become less attractive, one would expect that a smaller proportion of that group would choose to marry. More specifically, it is reasonable to assume that there is a distribution of "reservation prices" among members of each sex cohort group. Those who are not offered their reservation prices in marriage would choose to remain single. Keyfitz (1971) and others have tested a number of essentially *ad hoc* matching formulas to see how well they predict actual marriage rates. In similar fashion, we can test the predictions of the assignment problem to actual patterns of marriage and compare its performance against that of other methods.

Cohort Size Fluctuations and Marriage Market Equilibrium

The assignment model outlined above can be used to test the effects of cohort size fluctuations on marriage markets. Imagine a population that has been stationary for some time. If the ideal age at marriage for males A_m is two years higher than the ideal age at marriage for females A_f then an efficient marriage market in a stationary population would always have men marrying women two years younger. If we think of assigning cohorts of men to cohorts of women, the payoff matrix will have maximal payoffs along the off-diagonal in which male cohorts are assigned to female cohorts born two years later. According to the specification above, payoffs for less than ideal marriages will drop off quadratically from this "ideal" off-diagonal, exactly as in the example in Figure 2.

Suppose that a long series of identical cohort sizes is interrupted by a temporary increase in cohort sizes, followed by a decrease and eventual return to stationarity. Figure 3 shows such a sequence of hypothetical birth cohort sizes for an initial "baby boom" or larger than normal cohorts followed by a "baby bust" of smaller than normal cohorts. The sequence of cohort sizes relative to an arbitrary (and irrelevant) equilibrium level are shown in the line labeled relative cohort size. The cohort size fluctuations are generated by two six-year sine waves beginning in the tenth year, followed by a return to stationarity. The largest cohort, born in year 13, is about 25% larger than the mean, while the smallest cohort, born in year 19, is about 25% smaller than the mean. The linear programming assignment model described above is used to assign husbands to wives for the twenty-seven years of birth cohorts shown, based on the assumption that the ideal age at marriage for males is three years older than the ideal age at marriage for females.

Figure 3. Simulated Marriage Squeeze With Baby Boom Preceding Baby Bust
Relative Cohort Size, Sex Ratios, and Equilibrium Age Difference Between Husbands and Wives.
Marriage Assignments Based on Linear Programming Assignment Algorithm.
(Based on three year difference between ideal age at marriage for males and females.)

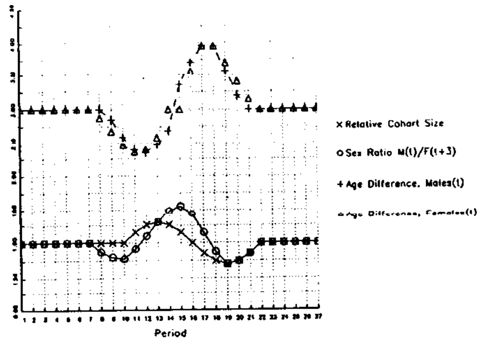
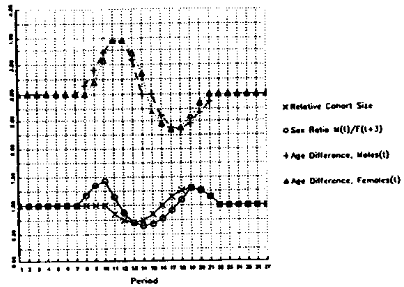


Figure 4. Simulated Marriage Squeeze With Baby Bust Preceding Baby Boom
Relative Cohort Size, Sex Ratios, and Equilibrium Age Difference Between Husbands and Wives.
Marriage Assignments Based on Linear Programming Assignment Algorithm.
(Based on three year difference between ideal age at marriage for males and females.)



The line marked with circles in Figure 3 shows the sex ratios implied by these cohort size fluctuations, where the sex ratio shown is the ratio of males born in year t to females born in year $t+3$, the "ideal" partners for the cohort. The ratio drops below unity in year 8 because the female cohort in year 10 is the first of the larger than normal cohorts. The sex ratio fluctuates significantly, ranging between .7 and 1.5 over the period of cohort size fluctuations.

Figure 3 also shows the average age difference between husbands and wives for each cohort according to the marriage assignments made by our linear programming assignment algorithm when there is a three year difference in the ideal age at marriage for men and women. All men and women from each cohort are assigned a partner, so that none of the "marriage squeeze" shock indicated by the changing sex ratios is absorbed by variations in proportions marrying. The series labeled "Age Difference, Males(t)" shows the average age difference between men born in year t and their wives. The series labeled "Age Difference, Females(t)" shows the average age difference between women born in year t and their husbands. These two series will in general not be identical in a period of fluctuating sex ratios, and the dispersion in the two is of interest in its own right. For now we concentrate on the basic patterns in common between the two series.

The most dramatic result shown in Figure 3 is that we are able to assign marriage partners to *all* men and women from each cohort simply by varying the mean age difference between husbands and wives between 2.3 and 4 years. In fact no couple every marries with an age difference that is greater than 4 years or less than 2 years under the assignments implied by the linear program. The stationary populations that precede and follow the cohort size fluctuations are characterized by all couples having a three year age difference between spouses. As the first large female cohorts begin to enter the marriage market some women are assigned to men only two years older. As the "baby boom" reaches a peak in year 13, the need to assign women to men closer in age to themselves is reduced and the age gap moves back toward three years. The subsequent "baby bust" leads to an increase in the age gap as some women are assigned to men more than three years older. Eventually the gap moves back to its stationary population level of three years as the cohort size fluctuations subside.

The hypothetical sex ratio fluctuations generated in this exercise are in fact similar to those observed in countries like the United States and Sweden in the last fifty years (see Bergstrom and Lam, 1988). What the exercise demonstrates is that even dramatic fluctuations such as these can be absorbed by the marriage market with no change in the proportions of men and women marrying (in the example here 100% of the men and women in every cohort marry). The equilibrating mechanism is the age difference between husbands and wives, with adjustments that are only an average of .7 years lower and 1 year higher than the "ideal" age difference of 3 years capable of absorbing all of the "marriage squeeze."

Figure 4 shows the mirror image series of cohort size fluctuations to those in Figure 3, with

the baby bust preceding the baby boom in this case. This case demonstrates that the relative ease with which adjustments in the age difference between spouses accommodate sex ratio fluctuations in Figure 3 are in no way driven by the fact that cohort sizes initially increase. An initial decrease in cohort sizes can be absorbed in an analogous way, with the age difference between spouses initially *decreasing* rather than increasing. The magnitudes of the adjustments required to absorb this baby boom followed by a baby bust are exactly the same as those required to absorb the same level of cohort size fluctuations in the opposite temporal sequence.

Two other simulations providing additional insights into how a marriage market can absorb sex ratio fluctuations are shown in Figures 5 and 6. The example in Figure 5 shows a "one sided" cohort size shock in which a baby bust occurs and simply returns to some equilibrium level rather than being offset by a subsequent baby boom of above normal cohort sizes. In this case the sex ratio rises to a peak in year 6 of about 1.25 males to every female born two years later and falls to a low of about .7 males to every female born two years later. In this case a difference in the ideal age at marriage for men and women of 2 years is assumed, and a 12 year sine wave of cohort size fluctuations is assumed. As shown by the plots of the mean age differences in Figure 5, we are once again able to assign every man a wife and every woman a husband by relatively modest adjustments in the age difference between spouses. In this case the mean age difference rises to 3.25 years, 1.25 years over the "ideal difference" of 2 years, before returning to the steady state level of 2 years. The mean of 3.25 occurs because some women are assigned husbands who are 4 years older at the trough of the baby bust.

Finally, Figure 6 considers the case of random cohort size fluctuations. The previous examples are characterized by smooth systematic cohort size changes, and it is natural to suspect that the orderliness these changes may make it easier for a marriage market assignment algorithm to accomodate sex ratio fluctuations. In the case presented in Figure 6 cohort sizes are generated as a random walk stochastic process, with each cohort equal in size to the previous cohort plus some pure uncorrelated disturbance. The sequence of relative cohort sizes and resulting sex ratio fluctuations are shown in the bottom two lines of the figure as before. The sex ratio fluctuates fairly erratically, with a high of about 1.25 in years 5 and 12 and a low of about .7 in year 16. A two year difference in the ideal ages at marriage is assumed. The graph of the mean age difference between husbands and wives produced by our assignment algorithm is considerably smoother than the sequence of cohort sizes or sex ratios. This is because the marriage market is "smoothing" the erratic sex ratios across adjacent cohorts in assigning husbands and wives. As above we assign every man a wife and every woman a husband, in this case with age adjustments that stay between two and three years age difference. The mean age difference rises and falls twice over the fourteen years of stochastic cohort sizes considered here, with noone every marrying a spouse more than one year away from the ideal age difference. Although only one case of random walk cohort sizes

Figure 5. Simulated Marriage Squeeze With Long Baby Bust

Relative Cohort Size, Sex Ratios, and Equilibrium Age Difference Between Husbands and Wives.

Marriage Assignments Based on Linear Programming Assignment Algorithm.

(Based on two year difference between ideal age at marriage for males and females.)

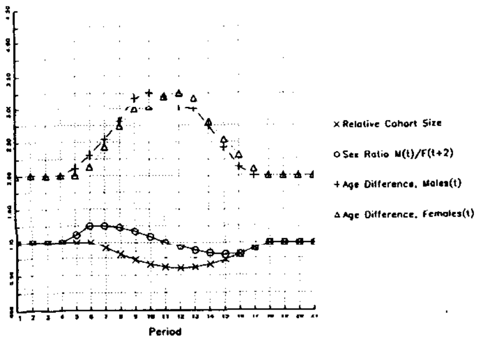
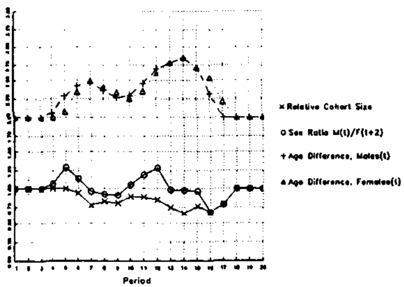


Figure 6. Simulated Marriage Squeeze With Random Cohort Size

Relative Cohort Size, Sex Ratios, and Equilibrium Age Difference Between Husbands and Wives.

Marriage Assignments Based on Linear Programming Assignment Algorithm.

(Based on two year difference between ideal age at marriage for males and females.)



is presented here, the example in Figure 6 is typical of the results of a large number of simulations of this type. The equilibrium marriage market assignments always smooth out even wildly erratic fluctuations in sex ratios, allowing us to assign all individuals to spouses who are very rarely more than two years away from the ideal age difference.

The results summarize in Figures 3 to 6 indicate the value of analyzing marriage squeeze effects in the context of a general equilibrium model of marriage markets. Viewed in isolation, a single year's birth cohort can appear to be extremely disadvantaged in the marriage market. Returning to the sex ratios in Figure 3 as a standard measure, for example, women in the most disadvantaged cohort face only .7 males per woman when three year older men are viewed as the pool of potential husbands. In fact, however, all of the women in this cohort are able to marry men that are either two years older or three years older than themselves when the entire series of cohorts are assigned partners. This implies a surprisingly modest effect of what at first glance is a very large marriage squeeze when compared to marriage squeezes observed in actual populations. The analysis suggests that the effects of sex ratio fluctuations of the type observed in recent decades may be less dramatic than is typically expected.

By viewing the "marriage squeeze"as a general equilibrium assignment problem, we see that what appear to be dramatic shocks to the sex ratio when viewed in isolation can be absorbed relatively easily by the marriage market. The adjustment mechanism is the age difference between husbands and wives, with adjustments in the mean age difference of only one year in either direction capable of absorbing cohort size fluctuations of the magnitude observed in recent decades in industrialized populations. Further empirical research is required to determine whether the marriage market in fact adjusts in this way. Our own empirical analysis of marriage markets in Sweden (Bergstrom and Lam, 1988) suggests that both the direction and magnitude of adjustments in the age difference between spouses are in fact consistent with the simple model we have used in this paper. We expect that theoretical analysis using richer models of marriage market assignments, combined with empirical analysis of actual marriage patterns, will shed further light on the interaction between the age difference between spouses and fluctuations in the age structure of the population.

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