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# Transactions Costs in the Foreign Exchange Market

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# 1 Introduction

One issue in the argument about the merits of pegged and floating exchange rates involves the magnitude of transactions costs in the foreign exchange market under alternative exchange rate regimes. The higher the transactions costs, the greater the deterrence to international trade. Moreover, the higher these costs, the greater the scope for national monetary independence, and more fully the monetary authority in one country could follow policies that might cause the rates of return on assets denominated in its currency to differ from rates of return on comparable assets denominated in other currencies, for any given impact in inducing flows of short-term capital. In contrast, the lower the transactions costs in the foreign exchange market, the more the case for national monetary independence must rest on other deterrents to the shifts of funds among national financial centers, such as uncertainty about changes in exchange rates.

Transactions costs in the foreign exchange market are not explicit, as in the markets with standardized commissions like the home real estate market and organized security and commodity exchanges. Instead, transactions costs are implicit, as in the over-the-counter security market, and are collected by broker-dealers, primarily the large commercial banks, in the spreads between the prices at which they buy and sell foreign exchange. The transactions costs in the foreign exchange market may differ by the pair of currencies involved, by the size of the transaction, by the customer buying the foreign exchange, by the bank selling the foreign exchange, and even by the center in which a particular transaction such as the purchase of dollars with sterling occurs. However, from the point of view providing insights about the scope for monetary independence, the key consideration is the estimate of transactions costs incurred by those who pay the lowest costs – the banks in their transactions with each other.

The next section discusses previous approaches to the measurement of transactions costs in the foreign exchange market. Then a new approach to estimate the transactions costs using futures prices is presented.

## 2 Previous Approaches

### 2.1 Bid-Ask Spreads

One approach to estimation of transactions costs in the foreign exchange market is based on the bid-ask spread quoted by banks to commercial or non-bank customers (See Glassman, 1987 and Boothe, 1988). Let  $S_b$  and  $S_a$  denote the bid and the ask prices expressed in dollars of a unit of foreign currency, say Deutsche Mark, quoted by a bank. Let  $S$  denote the price of one unit of DM if the customer faced no transactions costs. If we let  $t$  denote the proportional transactions costs, then

$$\frac{S - S_b}{S} = t \Leftrightarrow S_b = S(1 - t) \tag{1}$$

$$\frac{S_a - S}{S} = t \Leftrightarrow S_a = S(1 + t). \tag{2}$$

Eliminating  $S$  from the above two equations, we get

$$t = \frac{S_a - S_b}{S_a + S_b}.$$

This approach permits comparisons both of the transactions costs involving different pairs of currencies and the costs of forward contracts of different maturities with each other and with the costs of spot exchange contracts.

There are several problems with this approach, however. First, there is no assurance that the rates quoted are those actually charged by the banks; transactions between banks and non-bank customers may occur at prices within these quotes. Second, the buyers and sellers of foreign exchange may incur various costs in addition to those charged by broker-dealers; these costs might include payments for the expertise deemed necessary to cope with exchange market uncertainties, including the costs of exchange rate forecasting services and exposure management services.

## 2.2 Triangular Arbitrage

A second approach infers transactions costs using a model of triangular arbitrage. In the absence of transactions costs, the following condition must hold for there to be no arbitrage opportunities.

$$S^{\$/\pounds} = S^{\$/DM} S^{DM/\pounds}, \quad (3)$$

where  $S^{x/y}$  denotes the price in terms of currency  $x$  of one unit of currency  $y$ .

If, however, transactions costs are not absent, then, Frenkel and Levich (1975, 1977) suggest, that the transactions costs can be inferred from the upper limit of the absolute discrepancy between the two sides of equation 3. The percentage discrepancy,  $d$ , can be expressed as follows.

$$d = 1 - \frac{S^{\$/DM} S^{DM/\pounds}}{S^{\$/\pounds}}. \quad (4)$$

In order to highlight the problems with this approach, it is useful to understand the mechanics of currency trading. Almost all trading of convertible currencies takes place with respect to the U.S. dollar (Grabbe, 1991). The dollar has a unique role as a vehicle currency. Just as money developed to enhance efficiency in payments, so the development of dollar as an intermediate currency can be attributed to the demand for greater efficiency in international payments and especially to economize on inventories of foreign exchange maintained by broker-dealers. So, for instance, both the  $\pounds$  and the DM are traded with prices quoted in terms of the dollar. If a commercial customer ask for a  $\pounds$  price in terms of the DM, this cross rate is determined from the two dollar rates. The bid and ask prices of  $\pounds$  in terms of the DM then can be expressed as follows<sup>1</sup>:

$$S_b^{DM/\pounds} = S_b^{\$/\pounds} S_b^{DM/\$} = \frac{S_b^{\$/\pounds}}{S_a^{\$/DM}}, \quad (5)$$

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<sup>1</sup>Note that  $S_b^{x/y} = 1/S_a^{y/x}$ .

$$S_a^{DM/\pounds} = S_a^{\$/\pounds} \quad S_a^{DM/\$} = \frac{S_a^{\$/\pounds}}{S_b^{\$/DM}} . \quad (6)$$

Let us now try to implement the Frenkel and Levich approach. The first question is which spot rates - bid or ask - should be used in (4). In Frenkel and Levich (1975) for all rates involving the U.S. dollar, closing *quoted* bid prices are used. For rates not involving the dollar, the rates used are mid-points of the quoted bid and ask rates (see their footnote 2). Thus for their case, we can write

$$d = 1 - \frac{S_b^{\$/DM}}{S_b^{\$/\pounds}} \left[ \frac{S_b^{DM/\pounds} + S_a^{DM/\pounds}}{2} \right] .$$

For simplicity let us assume that transactions costs are identical for all currencies. Substituting from (5), (6), (1) and (2) into above and simplifying, we get:

$$|d| = \frac{2t^2}{1-t^2} \simeq 2t^2 \simeq 0$$

where  $t$  represents one-half of the *quoted* bid-ask spread in percent. Thus the inferences made in Frenkel and Levich (1975) are misleading and incorrect by an order of magnitude as the estimate measures 2 times the *square* of percentage transactions costs as measured by one-half of the quoted bid-ask spreads.

Frenkel and Levich (1977) and McCormick (1979) use mid-points of the bid and ask rates for all three rates in (4). In that case, we can write

$$d = 1 - \frac{S_b^{\$/DM} + S_a^{\$/DM}}{S_b^{\$/\pounds} + S_a^{\$/\pounds}} \left[ \frac{S_b^{DM/\pounds} + S_a^{DM/\pounds}}{2} \right] .$$

Again making the substitutions from (5), (6), (1) and (2) and simplifying, we get

$$|d| = \frac{2t^2}{1-t^2} \simeq 2t^2 \simeq 0 .$$

Here again, the inferences made in these papers about the transactions costs are incorrect by an order of magnitude. It appears to us that the estimates of transactions costs in the foreign exchange market in Frenkel and Levich (1975, 1977) and in a follow up paper McCormick (1979) are simply being caused by errors in data! It is not surprising that Frenkel and Levich (1977) finds that the estimates of transactions costs in what they call the “turbulent period” are higher than the estimates in the “tranquil period” (see Table 1 in Frenkel and Levich, 1977). We suspect that this result is because of the data being non-synchronous which would result in larger estimates in periods with higher volatility of exchange rates because the errors are larger.<sup>2</sup>

<sup>2</sup>The evidence documented in McCormick (1979) in Table 1 also appears to support our conjecture.

Suppose now that we use actual transaction data for spot prices. This will have the advantage that we will be using actual prices faced by traders. If we do not know whether the transaction is a bid or the ask price then we can assume that each observed transaction price has an equal chance of being an *effective* (as opposed to quoted) bid or an ask price. Since there are three different rates in (4), there are eight different possible permutations. Suppose all three prices are bid prices. Then

$$d = 1 - \frac{S_b^{\$/DM} S_b^{DM/\pounds}}{S_b^{\$/\pounds}} . \quad (7)$$

Substituting from (5) in (7) and simplifying, we get

$$d = 1 - \frac{S_b^{\$/DM}}{S_a^{\$/DM}} .$$

For simplicity let us assume that  $t$  represents the effective percentage transactions costs for all currencies. Then, substituting from (1) and (2) in equation above, we get

$$|d| = \frac{2t}{1+t} \approx 2t .$$

We perform similar calculations for all eight permutations. The estimates of  $|d|$  vary from 0 to  $4t$  with a mean of  $2t$ . Thus a mean estimate of  $|d|$ , when actual transaction data for spot prices is used, provides an estimate of effective transactions costs.

### 3 Transactions Costs implicit in Futures Prices

Even when implemented correctly, the above approaches toward measuring transactions costs in foreign exchange market are directed at measuring the transactions costs incurred by the commercial customers of banks and ignore that much of the largest part of foreign exchange transactions, probably 90 to 95 percent, occurs between banks, and involves one bank as a buyer and another bank as a seller of foreign exchange. The costs incurred by banks on transactions undertaken for their own accounts are probably much smaller than any of the estimates of transactions costs suggested by quoted bid-ask spreads. Moreover, while it might seem that the banks would set their bid-ask spreads so that commercial customers would pay the costs of its foreign exchange department, it seems more likely that each bank compares these costs with total income from trading profits from “running the position” as well as from the bid-ask spread.

The approach we use has two features that overcome many of the problems that the previous approaches faced. First, we use prices on foreign currency futures. This has the advantage that we do not have to deal with any bid and ask price *quotes*. Futures contracts are traded on organized exchanges and there is a well defined price. Second, we use deviations from interest rate parity type relationships in estimating the transactions costs. The advantage of that is that we are measuring the transactions costs faced by the marginal investors that set prices in these markets. These marginal investors are likely to be large commercial banks and so the estimates of transactions costs we obtain are likely to be the estimates of the minimum level of transactions costs.

In the absence of arbitrage opportunities the following interest rate parity condition must hold.

$$F(\tau, T) = S(\tau) \frac{1 + i(\tau, T)}{1 + i^*(\tau, T)},$$

where

$F(\tau, T) \equiv$  Forward price at date  $\tau$  for a contract to deliver one unit of foreign currency at date  $T$ .<sup>3</sup>

$S(\tau) \equiv$  Spot price of the currency at date  $\tau$ .

$i(\tau, T) \equiv$  Domestic spot risk-free interest rate for the period from  $\tau$  to  $T$ .

$i^*(\tau, T) \equiv$  Foreign spot risk-free interest rate for the period from  $\tau$  to  $T$ .

Taking natural logarithms we get the following familiar version of interest rate parity in which the percentage forward premium over the spot rate equals the interest rate differential:

$$\ln F(\tau, T) - \ln S(\tau) \simeq i(\tau, T) - i^*(\tau, T) \quad (8)$$

Using (8), we get the following:

$$p(\tau) \equiv \ln F(\tau, T_2) - \ln F(\tau, T_1) \simeq i(\tau, T_1, T_2) - i^*(\tau, T_1, T_2) \equiv \Delta(\tau) \quad (9)$$

where

$i(\tau, T_1, T_2) \equiv i(\tau, T_2) - i(\tau, T_1) \equiv$  Domestic forward risk-free interest rate at date  $\tau$  for the period from  $T_1$  to  $T_2$ .

$i^*(\tau, T_1, T_2) \equiv i^*(\tau, T_2) - i^*(\tau, T_1) \equiv$  Foreign forward risk-free interest rate at date  $\tau$  for the period from  $T_1$  to  $T_2$ .

Thus

$$d(\tau) \equiv p(\tau) - \Delta(\tau) \simeq 0 \quad (10)$$

is a no arbitrage restriction in the absence of any transactions costs where  $p(\tau)$  represents the percentage forward premium for the forward contract maturing at date  $T_2$  over the forward rate for the contract maturing at date  $T_1$  and  $\Delta(\tau)$  represents the forward interest rate differential. Notice that by eliminating spot currency prices, we side-step several potential problems. First, the futures and spot prices may not be observed simultaneously. Second, spot contracts are not traded on a unified exchange and do not have a well-defined price at any moment; we can only obtain bid and ask quotations by various commercial banks or institutions and these could differ across banks and across customers. Third, currency futures contracts for the nearest two maturities, that we use in

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<sup>3</sup>Strictly speaking, the interest rate parity condition holds for forward contracts. The relation between forward and futures prices is discussed in Cox, Ingersoll and Ross (1981), French (1983) and Jarrow and Oldfield (1981). French (1983), however, shows that the difference between the two prices is empirically insignificant. So, in our analysis, we shall assume that futures prices equal forward prices.

our analysis, are heavily traded contracts that are extremely liquid. This reduces the possibility that the prices used for futures contracts with different maturities are non-synchronous.

If there are transactions costs, however, there may be deviations from the parity relationship (10):

$$-k \leq d(\tau) \leq k.$$

Following Frenkel and Levich (1975, 1977) it follows that

$$k = t + t^* + t_{T_1} + t_{T_2}, \quad (11)$$

where

$t \equiv$  Percentage transactions costs in the eurodollar market.

$t^* \equiv$  Percentage transactions costs in the foreign eurocurrency market.

$t_{T_1} \equiv$  Percentage transactions costs in the futures market for a contract with maturity  $T_1$ .

$t_{T_2} \equiv$  Percentage transactions costs in the futures market for a contract with maturity  $T_2$ .<sup>4</sup>

If we make a simplifying assumption that percentage transactions costs are equal in all four markets (denoted  $t$ ), then deviation from the parity relationship must be within a band that can be written as:

$$-4t \leq d(\tau) \leq 4t.$$

Thus  $t$  can be estimated by estimating the range within which observations for  $d(\tau)$  lie. Since, there may be some errors in data, we can estimate the band within which a large percentage (say 95%) of all observations for  $d(\tau)$  fall. This is one approach we follow in our empirical analysis.

The second approach we use follows a simple variant of the approach used in Roll (1984). The precise distribution of  $d(\tau)$  depends on whether each observed price in the parity relationship (9) is a bid or an ask price. Thus,

$$d(\tau) \in \{-4t, -2t, 0, 2t, 4t\},$$

with a probability distribution

$$\left\{ \frac{1}{16}, \frac{1}{4}, \frac{3}{8}, \frac{1}{4}, \frac{1}{16} \right\}.$$

Following Roll (1984), it is easy to show that the changes in  $d(\tau)$  will have negative serial covariance, the estimate of which also provides an estimate of the percentage transaction cost  $t$  according to the following relationship<sup>5</sup> :

$$t = \sqrt{-\text{cov}}/2 \equiv t_{\text{Roll}}.$$

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<sup>4</sup>Deardoff (1979) and Mohsen-Oskooee and Das (1985) argue, and it was pointed out to us by Richard Levich, that one-way arbitrage together with the existence of non-trivial equilibria in the futures markets for contracts with both maturities and in the bond markets in both countries implies a narrower band which could be as low as zero. This is equivalent to the observation that  $d(\tau)$  could equal zero for a particular combination of bid and ask prices for the four prices that appear in the parity relationship. As we shall see below, however, that when we use transaction prices, there will exist some combinations of bid and ask prices that make the band around the parity relation as large as the sum of transactions costs in each of the four markets as is expressed in (11).

<sup>5</sup>A formal proof of the exact derivation of this formula is available from the authors upon request.



### 3.1 Data

All the data were obtained from Datastream. Our sample period is from January 1, 1977 to December 31, 1999. The foreign exchange data consist of daily prices of per unit of four major currencies in the US dollar: the British pound, Deutsche mark, Japanese yen, and Swiss franc. For each currency, we construct two time series of futures prices traded on the Chicago Mercantile Exchange (CME),  $F(\tau, T_1)$  and  $F(\tau, T_2)$ . The first one,  $F(\tau, T_1)$ , is the closest-to-delivery futures contract and the second one,  $F(\tau, T_2)$ , is the next closest-to-delivery contract.<sup>6</sup>

We use the eurocurrency interest rates for the four currencies and the US dollar quoted by London Financial Times. We first obtain the continuously compounded annual yield of these rates with maturities in 1, 3, and 6 months, and then interpolate/extrapolate linearly to get a proxy of the interest rate for any maturity between 10 and 210 days.<sup>7</sup> In this way, we construct two time series of interest rates  $i(\tau, T_1)$  and  $i(\tau, T_2)$  for the domestic currency and  $i^*(\tau, T_1)$  and  $i^*(\tau, T_2)$  for each foreign currency.<sup>8</sup> Our constructed  $i(\tau, T_1, T_2) \equiv i(\tau, T_2) - i(\tau, T_1)$  is basically a synthesized forward rate. We then use  $\Delta(\tau)$  to denote the difference of these synthesized forward rates. One can actually do a better job in approximating the forward rates with the interest futures. The major disadvantage of this approach is the limited availability of the data on interest futures. Another problem is the non-synchronous trading of the interest futures and the foreign exchange futures. Nonetheless, we take the longest possible time series of the futures price of the 3-month interest rates for the four foreign currencies and US dollar. We denote the difference between the domestic and the foreign forward rates using the implied forward rates from the interest futures as  $\hat{\Delta}(\tau)$ .<sup>9</sup> We will use  $\hat{\Delta}(\tau)$  as a robustness check for our results.

### 3.2 Results

Figure 1 shows the time series plots of  $p(\tau)$  and  $\Delta(\tau)$  for the four currencies. As seen in the figure,  $p(\tau)$  and  $\Delta(\tau)$  track each other very closely. However, the plots of  $\Delta(\tau)$  are much smoother than those of  $p(\tau)$  as there are dozens of spikes in  $p(\tau)$  for each currency. These spikes are more evident in Figure 2 which shows the plots of  $d(\tau)$ .

Table 1 reports the summary statistics and percentiles of  $d(\tau)$  for the four foreign currencies for the whole sample period and two sub-sample periods, 1/1977-6/1988 and 7/1988-12/1999. One obvious fact is that the distribution of  $d(\tau)$  is much tighter during the second sub-period, which can also be seen from the graphs of  $d(\tau)$ . The large kurtosis of  $d(\tau)$ 's are mostly caused by the spikes observed in Figure 2. Figure 3 shows the central portions of the histograms of  $d(\tau)$  for the

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<sup>6</sup>We stop using the closest-to-delivery contract to be  $F(\tau, T_1)$  and switch to the next-closest-to-delivery contract on the first day of the delivery month. The foreign exchange futures are on March, June, September, and December cycles. For example, in February, the March and June contracts are used. On the first day of March and thereafter (until September 1), we choose the June and September contracts.

<sup>7</sup>The range for all maturities of the futures contracts is between 13 to 202 days for our sample.

<sup>8</sup>The sample size for Japanese yen is slightly shorter than others since data on the interest rates of Japanese yen was not available until July, 1978.

<sup>9</sup>The data of 3-month interest rate futures are available from 11/1982, 5/1993, 6/1989, 2/1991, and 4/1981 for British pound, Deutsche mark, Japanese yen, Swiss franc, and US dollar respectively. The Deutsche mark series ends in 12/1996.

four currencies.

To check the robustness of our results, we also estimate  $d(\tau)$  using the forward interest rates implied in the interest futures prices, which is denoted by  $\hat{d}(\tau)$ . Table 2 reports the summary statistics of  $\hat{d}(\tau)$  for the currencies for their longest possible sample periods. The sample statistics for Deutsche mark, Japanese yen, and Swiss franc are very close to their corresponding values in Table 1 for the second sub-period. The sample statistics for British pound are comparable but not very close to those in Table 1 for the whole sample period. This can be easily seen from Figure 4 which shows the plot of  $\Delta(\tau)$  and  $\hat{\Delta}(\tau)$ . For all currencies except British pound,  $\Delta(\tau)$  and  $\hat{d}(\tau)$  track each other closely. For British pound,  $\Delta(\tau)$  and  $\hat{\Delta}(\tau)$  are pretty close except in the late 80's when  $\hat{\Delta}(\tau)$  is constantly higher than  $\Delta(\tau)$ .

We further check if the distribution of  $d(\tau)$  is affected by the liquidity of the futures markets. To this end, we examine the trading volume of the two futures contracts from Datastream. Although the closest-to-delivery contract is actively traded most of time, the second-closest-to-delivery contract is not as heavily traded as the first one. We test a number of filter rules which restrict our sample to certain dates when the two contracts are both actively traded.<sup>10</sup> Our estimates (not reported here) are not much different from the those reported in Table 1.

Table 3 reports the serial covariance of the changes in  $d(\tau)$ ,  $\text{cov} \equiv \text{Cov}(\Delta d(\tau + 1), \Delta d(\tau))$ , and its square root,  $\sqrt{-\text{cov}}$ , for the four foreign currencies.

Table 4 reports the estimates of the transaction cost of foreign exchange market for the four foreign currencies considered. The first three measures are based on the sample distribution of  $d(\tau)$ . For example,  $t_{100-\alpha}$  is defined as:

$$t_{100-\alpha} \equiv \frac{1}{4} \left\{ \frac{d_{100-\alpha/2} + (-d_{\alpha/2})}{2} \right\},$$

where  $\alpha \in \{1, 5, 10\}$  and  $d_y$  represents the  $y$  percentile of the sample distribution of  $d(\tau)$ . The last measure is the effective transactions costs estimated using the Roll (1984) method and is defined by  $t_{\text{Roll}} \equiv \sqrt{-\text{cov}}/2$  where  $\text{cov}$  is the serial covariance of the changes in  $d(\tau)$ . All the estimates are in percentage.

Notice that the estimates using the 95% bounds,  $t_{95\%}$ , are remarkably similar to the estimates using the Roll (1984) approach,  $t_{\text{Roll}}$ . These estimates are highlighted in boldface in Table 4. Using these estimates, we can make the following conclusions. For a foreign currency trade valued at \$100,000, the average transaction costs, estimated using the data for the period 1977-1999, indicate that these costs were between \$36 and \$51. The estimates of the transactions costs for a similar sized trade, estimated using the data for the period 1988-1999, indicate that these costs may have been as low as \$18 and perhaps no more than \$35. Thus, not only have we established that transactions costs in the foreign exchange market are extremely small, but also that they have been falling substantially over the years.

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<sup>10</sup>The filters include lower bounds in trading volumes and lower bounds for ratios of trading volumes.

## 4 Conclusion

We argued that previous approaches for estimating the transactions costs in the foreign exchange market had serious methodological flaws. Even if these approaches were to be implemented correctly, these approaches attempt to measure transactions costs in foreign exchange market that are faced by commercial customers of banks. While the estimates of transactions costs useful for judgements about the impacts of alternative exchange rate regimes on the levels of trade, might be those incurred by commercial firms, the estimates of cost relevant for issues of monetary independence, in contrast, are the smaller costs incurred by large commercial banks. The view that there is a unique cost of foreign exchange transactions has no analytic or policy significance. Moreover, for questions of both monetary independence and trade impacts, estimates of the minimum level of transactions costs are more significant than estimates of maximum costs.

The approach we follow has the advantage that we measure transactions costs faced by the marginal investors that set prices in the foreign exchange markets. These marginal investors are likely to be large commercial banks and so the estimates of transactions costs we obtain are likely to be the estimates of the minimum level of transactions costs. We estimate that average transactions costs over the last two decades were no more than one-twentieth of one percent, and in the last decade may have fallen to as low as one-fiftieth of one percent.

Table 1: Sample Statistics for  $d(\tau)$

This table reports the sample statistics and percentiles of  $d(\tau)$  for the four foreign currencies for the whole sample period and two sub-sample periods.

1/1/1977-12/31/1999						
	Mean	S.D.	Skew	Kurt	Max	Min
British Pound	$-2.2 \times 10^{-4}$	$9.9 \times 10^{-4}$	-1.24	92	0.019	-0.023
Deutsche Mark	$-2.5 \times 10^{-4}$	$9.9 \times 10^{-4}$	-2.75	259	0.026	-0.031
Japanese Yen	$-3.5 \times 10^{-4}$	$12.2 \times 10^{-4}$	7.93	217	0.033	-0.018
Swiss Franc	$-2.4 \times 10^{-4}$	$11.7 \times 10^{-4}$	-3.29	190	0.018	-0.036
Percentiles						
	0.5	2.5	5	95	97.5	99.5
British Pound	$-3.54 \times 10^{-3}$	$-2.07 \times 10^{-3}$	$-1.60 \times 10^{-3}$	$0.83 \times 10^{-3}$	$1.40 \times 10^{-3}$	$3.41 \times 10^{-3}$
Deutsche Mark	$-3.30 \times 10^{-3}$	$-2.03 \times 10^{-3}$	$-1.60 \times 10^{-3}$	$0.68 \times 10^{-3}$	$0.97 \times 10^{-3}$	$2.40 \times 10^{-3}$
Japanese Yen	$-3.92 \times 10^{-3}$	$-2.59 \times 10^{-3}$	$-2.05 \times 10^{-3}$	$0.69 \times 10^{-3}$	$1.46 \times 10^{-3}$	$3.34 \times 10^{-3}$
Swiss Franc	$-4.12 \times 10^{-3}$	$-2.37 \times 10^{-3}$	$-1.74 \times 10^{-3}$	$0.83 \times 10^{-3}$	$1.42 \times 10^{-3}$	$3.85 \times 10^{-3}$
1/1/1977-6/30/1988						
	Mean	S.D.	Skew	Kurt	Max	Min
British Pound	$-3.7 \times 10^{-4}$	$12.5 \times 10^{-4}$	-1.36	67	0.019	-0.023
Deutsche Mark	$-5.3 \times 10^{-4}$	$12.4 \times 10^{-4}$	-1.81	207	0.026	-0.031
Japanese Yen	$-5.2 \times 10^{-4}$	$14.5 \times 10^{-4}$	3.66	91	0.029	-0.018
Swiss Franc	$-4.9 \times 10^{-4}$	$14.1 \times 10^{-4}$	-3.91	166	0.018	-0.036
Percentiles						
	0.5	2.5	5	95	97.5	99.5
British Pound	$-4.30 \times 10^{-3}$	$-2.76 \times 10^{-3}$	$-2.03 \times 10^{-3}$	$1.29 \times 10^{-3}$	$1.94 \times 10^{-3}$	$3.59 \times 10^{-3}$
Deutsche Mark	$-3.83 \times 10^{-3}$	$-2.47 \times 10^{-3}$	$-2.00 \times 10^{-3}$	$0.72 \times 10^{-3}$	$1.38 \times 10^{-3}$	$2.68 \times 10^{-3}$
Japanese Yen	$-4.63 \times 10^{-3}$	$-3.11 \times 10^{-3}$	$-2.47 \times 10^{-3}$	$1.47 \times 10^{-3}$	$2.07 \times 10^{-3}$	$4.04 \times 10^{-3}$
Swiss Franc	$-4.41 \times 10^{-3}$	$-2.95 \times 10^{-3}$	$-2.25 \times 10^{-3}$	$1.22 \times 10^{-3}$	$1.84 \times 10^{-3}$	$3.82 \times 10^{-3}$
7/1/1988-12/31/1999						
	Mean	S.D.	Skew	Kurt	Max	Min
British Pound	$-6.6 \times 10^{-5}$	$6.0 \times 10^{-4}$	3.66	87	0.011	-0.007
Deutsche Mark	$-3.3 \times 10^{-5}$	$5.3 \times 10^{-4}$	-5.91	173	0.005	-0.014
Japanese Yen	$-2.1 \times 10^{-4}$	$9.6 \times 10^{-4}$	20.2	650	0.033	-0.005
Swiss Franc	$-1.5 \times 10^{-6}$	$8.0 \times 10^{-4}$	2.54	122	0.017	-0.013
Percentiles						
	0.5	2.5	5	95	97.5	99.5
British Pound	$-1.77 \times 10^{-3}$	$-1.03 \times 10^{-3}$	$-0.79 \times 10^{-3}$	$0.61 \times 10^{-3}$	$0.77 \times 10^{-3}$	$2.09 \times 10^{-3}$
Deutsche Mark	$-1.37 \times 10^{-3}$	$-0.91 \times 10^{-3}$	$-0.70 \times 10^{-3}$	$0.67 \times 10^{-3}$	$0.79 \times 10^{-3}$	$1.26 \times 10^{-3}$
Japanese Yen	$-3.03 \times 10^{-3}$	$-0.97 \times 10^{-3}$	$-0.73 \times 10^{-3}$	$0.42 \times 10^{-3}$	$0.55 \times 10^{-3}$	$0.93 \times 10^{-3}$
Swiss Franc	$-2.22 \times 10^{-3}$	$-0.97 \times 10^{-3}$	$-0.73 \times 10^{-3}$	$0.66 \times 10^{-3}$	$0.83 \times 10^{-3}$	$3.91 \times 10^{-3}$

Table 2: Sample Statistics for  $\hat{d}(\tau)$

This table reports the sample statistics and percentiles of  $\hat{d}(\tau)$  for the four foreign currencies for their longest possible sample periods. The sample periods for British pound, Deutsche mark, Japanese yen, and Swiss franc are 11/4/1982-12/31/1999, 5/6/1993-12/27/1996, 6/30/1989-12/31/1999, and 2/7/1991-12/31/1999 respectively.

	Mean	S.D.	Skew	Kurt	Max	Min
British Pound	$-9.0 \times 10^{-4}$	$10.8 \times 10^{-4}$	0.35	34.9	0.017	-0.016
Deutsche Mark	$9.9 \times 10^{-5}$	$4.1 \times 10^{-4}$	-2.08	67.7	0.005	-0.005
Japanese Yen	$1.1 \times 10^{-5}$	$8.1 \times 10^{-4}$	26.8	1099	0.033	-0.006
Swiss Franc	$1.9 \times 10^{-4}$	$8.2 \times 10^{-4}$	3.92	107	0.017	-0.007
	Percentiles					
	0.5	2.5	5	95	97.5	99.5
British Pound	$-3.47 \times 10^{-3}$	$-3.18 \times 10^{-3}$	$-2.99 \times 10^{-3}$	$0.30 \times 10^{-3}$	$0.65 \times 10^{-3}$	$1.38 \times 10^{-3}$
Deutsche Mark	$-0.67 \times 10^{-3}$	$-0.49 \times 10^{-3}$	$-0.41 \times 10^{-3}$	$0.45 \times 10^{-3}$	$0.48 \times 10^{-3}$	$0.11 \times 10^{-3}$
Japanese Yen	$-1.79 \times 10^{-3}$	$-0.96 \times 10^{-3}$	$-0.70 \times 10^{-3}$	$0.60 \times 10^{-3}$	$0.93 \times 10^{-3}$	$1.08 \times 10^{-3}$
Swiss Franc	$-1.78 \times 10^{-3}$	$-1.02 \times 10^{-3}$	$-0.82 \times 10^{-3}$	$0.74 \times 10^{-3}$	$0.81 \times 10^{-3}$	$3.98 \times 10^{-3}$

Table 3: cov and  $\sqrt{-\text{cov}}$

This table reports cov and  $\sqrt{-\text{cov}}$  for the four foreign currencies for the whole sample period and two sub-periods, where cov is the serial covariance of the changes in  $d(\tau)$ .

	British Pound	Deutsche Mark	Japanese Yen	Swiss Franc
1/1/1977-12/31/1999				
cov	$-5.3 \times 10^{-7}$	$-6.8 \times 10^{-7}$	$-1.0 \times 10^{-6}$	$-8.8 \times 10^{-7}$
$\sqrt{-\text{cov}}$	$0.73 \times 10^{-3}$	$0.83 \times 10^{-3}$	$1.01 \times 10^{-3}$	$0.94 \times 10^{-3}$
1/1/1977-6/30/1988				
cov	$-8.3 \times 10^{-7}$	$-1.2 \times 10^{-6}$	$-1.4 \times 10^{-6}$	$-1.3 \times 10^{-6}$
$\sqrt{-\text{cov}}$	$0.91 \times 10^{-3}$	$1.12 \times 10^{-3}$	$1.20 \times 10^{-3}$	$1.12 \times 10^{-3}$
7/1/1988-12/31/1999				
cov	$-2.4 \times 10^{-7}$	$-1.3 \times 10^{-7}$	$-6.7 \times 10^{-7}$	$-5.0 \times 10^{-7}$
$\sqrt{-\text{cov}}$	$0.49 \times 10^{-3}$	$0.36 \times 10^{-3}$	$0.82 \times 10^{-3}$	$0.70 \times 10^{-3}$

Table 4: Estimates of the Transaction Cost

This table reports the estimates of the transactions costs in the foreign exchange market for the four foreign currencies for the whole sample period and two sub-periods. All estimates are in percentage.

	$t_{99\%}$	$t_{95\%}$	$t_{90\%}$	$t_{Roll}$
1/1/1977-12/31/1999				
British Pound	0.087	<b>0.043</b>	0.030	<b>0.036</b>
Deutsche Mark	0.071	<b>0.038</b>	0.029	<b>0.041</b>
Japanese Yen	0.091	<b>0.051</b>	0.034	<b>0.051</b>
Swiss Franc	0.100	<b>0.047</b>	0.032	<b>0.047</b>
1/1/1977-6/30/1988				
British Pound	0.099	<b>0.059</b>	0.042	<b>0.046</b>
Deutsche Mark	0.081	<b>0.048</b>	0.034	<b>0.056</b>
Japanese Yen	0.108	<b>0.065</b>	0.049	<b>0.060</b>
Swiss Franc	0.103	<b>0.060</b>	0.043	<b>0.056</b>
7/1/1988-12/31/1999				
British Pound	0.048	<b>0.023</b>	0.018	<b>0.024</b>
Deutsche Mark	0.033	<b>0.021</b>	0.017	<b>0.018</b>
Japanese Yen	0.050	<b>0.019</b>	0.014	<b>0.041</b>
Swiss Franc	0.077	<b>0.023</b>	0.017	<b>0.035</b>

Figure 1:  $p(\tau)$  and  $\Delta(\tau)$

The plots of  $p(\tau)$  and  $\Delta(\tau)$  for the four foreign currencies between 1/1/1977 and 12/31/1999. The dotted lines represent  $p(\tau)$ , and the solid lines represent  $\Delta(\tau)$ .

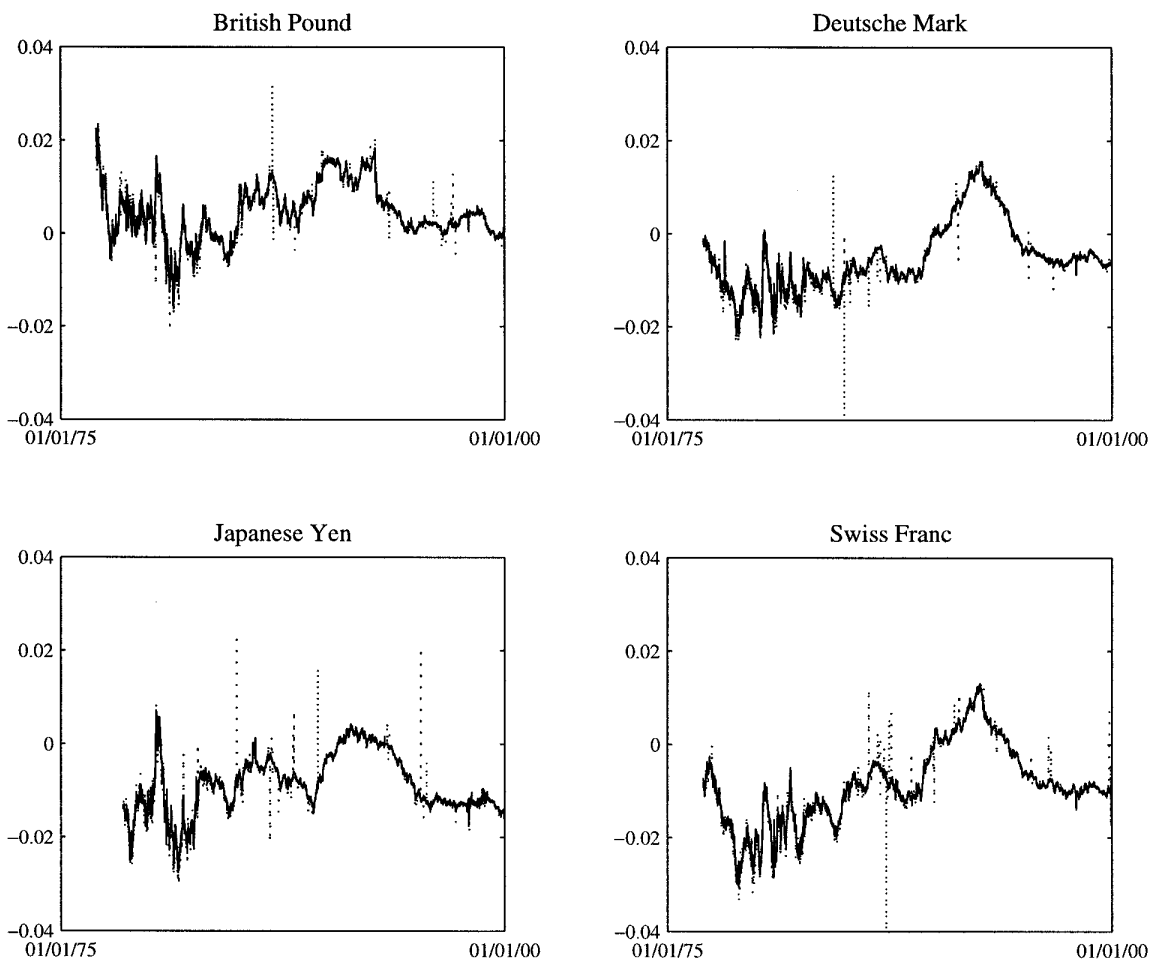




Figure 2:  $d(\tau)$

The plots of  $d(\tau)$  for the four foreign currencies between 1/1/1977 and 12/31/1999.

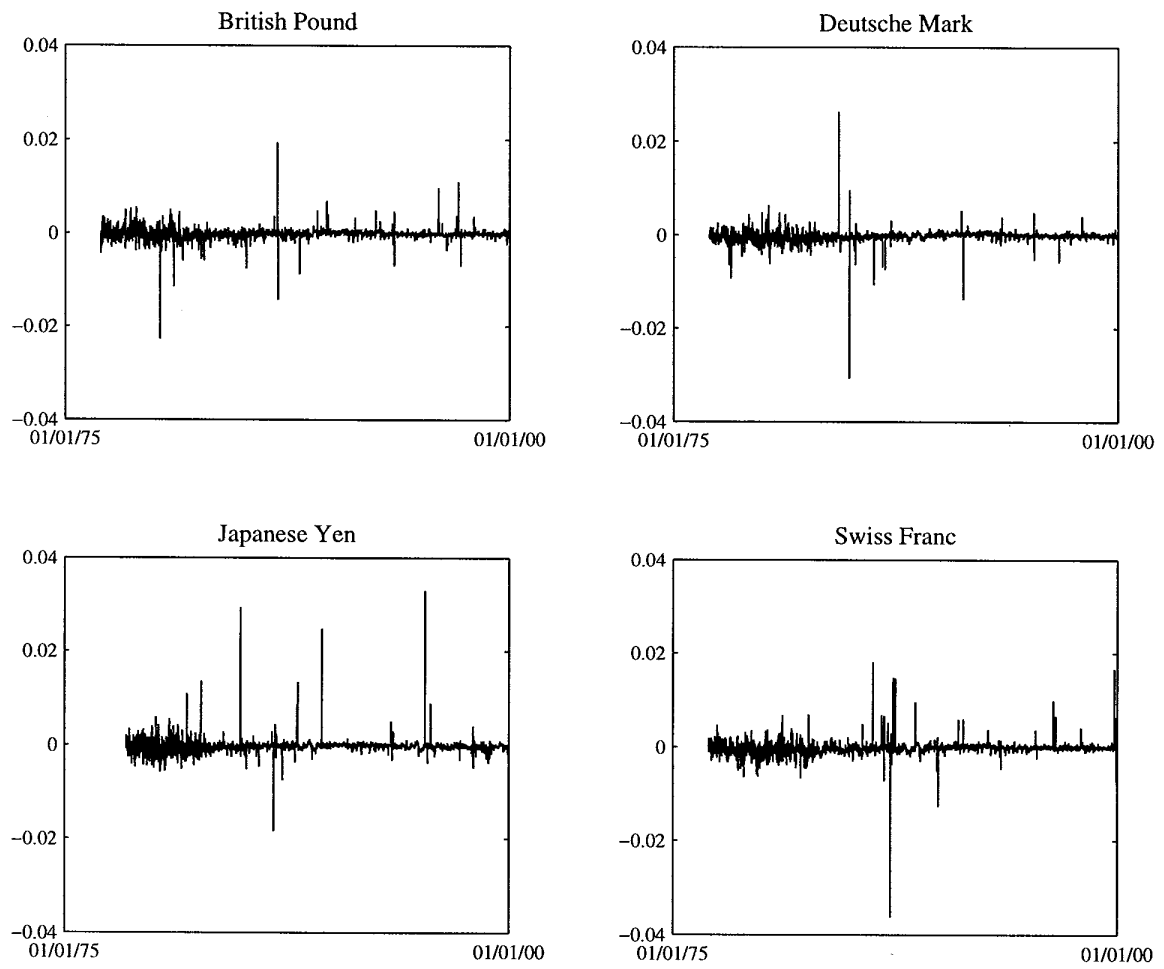


Figure 3: Histogram of  $d(\tau)$

The histograms of  $d(\tau)$  for the four foreign currencies between 1/1/1977 and 12/31/1999.

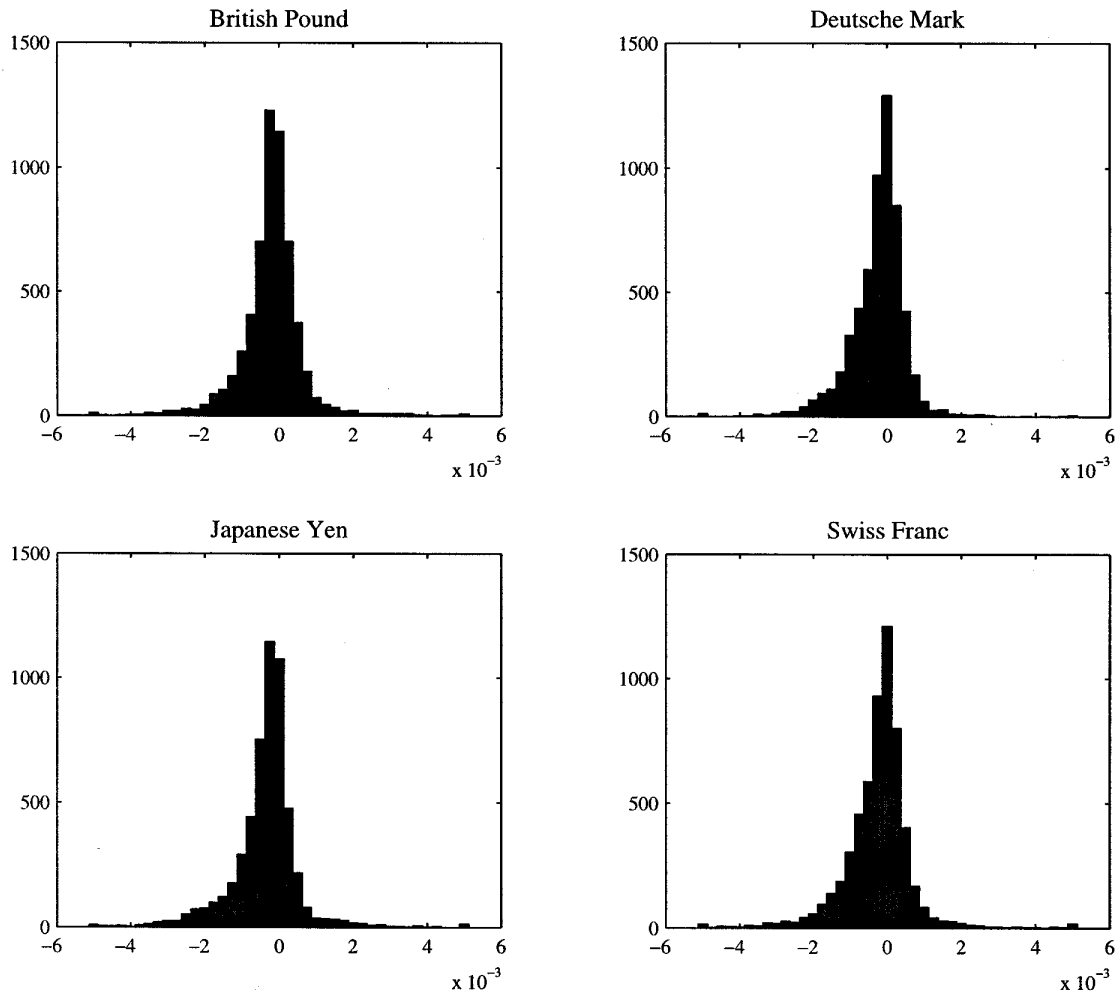
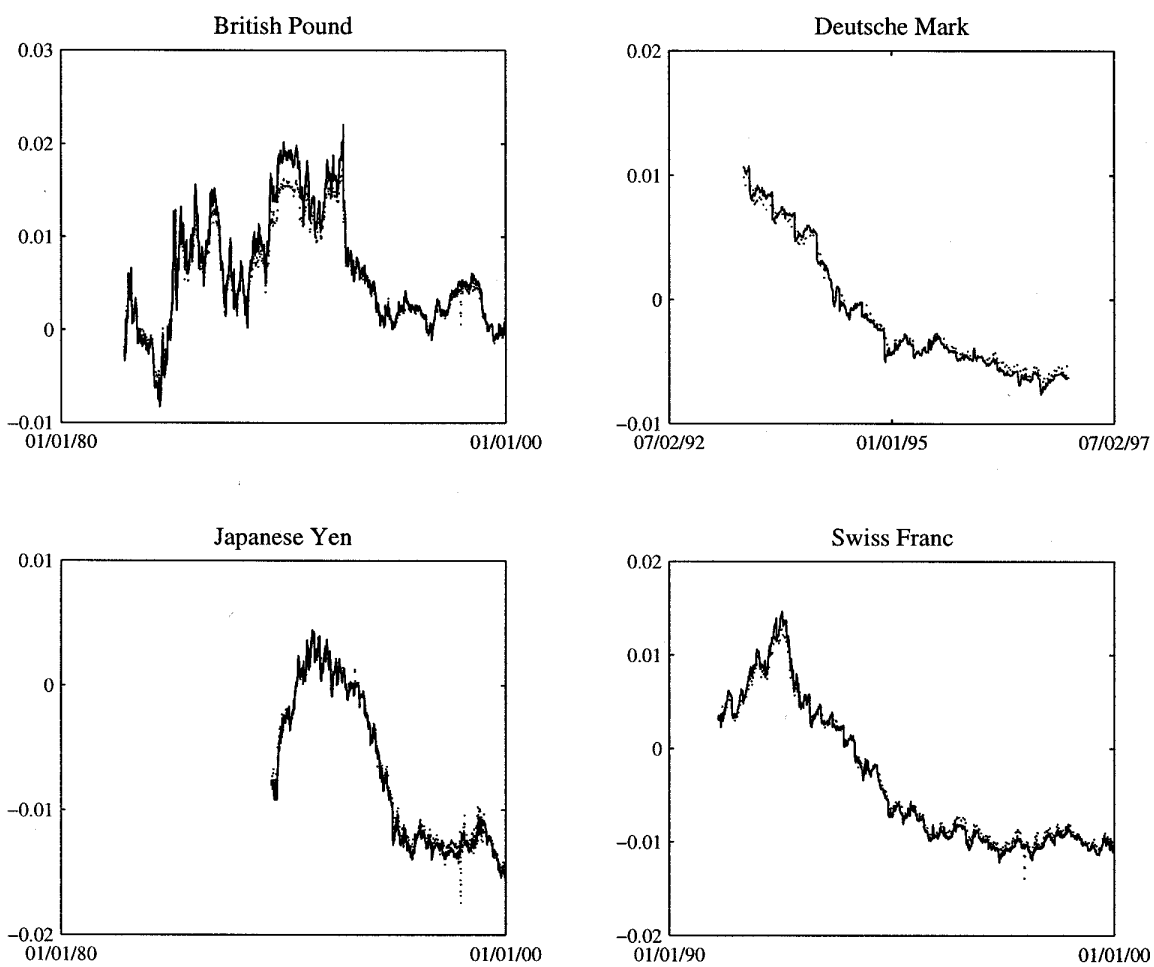


Figure 4:  $\Delta(\tau)$  and  $\hat{\Delta}(\tau)$

The plots of  $\Delta(\tau)$  and  $\hat{\Delta}(\tau)$  for the four foreign currencies, where  $\Delta(\tau)$  is the synthesized 3-month forward rate and  $\hat{\Delta}(\tau)$  is the 3-month forward rate implied in the 3-month interest rate futures. The dotted lines represent  $\Delta(\tau)$ , and the solid lines represent  $\hat{\Delta}(\tau)$ .



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