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SEMICLASSICAL AND QUANTUM MECHANICAL COMPARISON OF ONE AND TWO NUCLEON TRANSFER*

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ABSTRACT

A comparison of semiclassical and quantum mechanical approaches to heavy-ion reactions at energies well above the Coulomb barrier is used to give physical insight into the differential cross sections of one and two neutron pick-up reactions induced by 78 MeV ^{12}C ions on ^{144}Nd.

The criterion for the validity of the semiclassical approach to heavy-ion reactions is usually expressed by the fact that the wavelength of relative motion of the colliding ions is short compared to the interaction radius, and consequently the particles are localized on classical trajectories. Although this criterion is better fulfilled the higher the incident energy, it becomes necessary to take into account the increasing importance of the absorptive nuclear potential by introducing complex trajectories.[1-3] This modification allows a description of interference and diffractive phenomena in almost exact agreement with the quantal treatment,[1-3] but at the expense of the physical insight afforded by real trajectories, which was one of the advantages of the semiclassical approach at sub-Coulomb energies. In this letter we show that a comparison of semiclassical and quantum mechanical approaches for reactions at incident energies well above the Coulomb barrier can lead to a physical understanding of some of the differences observed between one and two neutron transfer reactions in this energy region.

The differential cross sections for the reactions ^{144}Nd(^{12}C,^{13}C)^{143}Nd (g.s.) and ^{144}Nd(^{12}C,^{14}C)^{142}Nd (0^+, 2.98 MeV) at an incident energy of 78 MeV are shown in Fig. 1. These states are known to be formed by direct, one-step reactions.[4] The data were measured at the Berkeley 88-Inch Cyclotron, using the magnetic spectrometer to analyse the reaction products. The angular distribution for one nucleon transfer has an almost symmetrical bell-shaped maximum, whereas in the two nucleon transfer the distribution is much broader and flattened asymmetrically towards forward angles. These features are easily reproduced using standard optical potentials in a DWBA calculation.[5,6] For example, in Fig. 1(a), two theoretical curves, obtained with potentials A and B of Table 1, are almost equally successful in fitting the bell-shaped maximum. This result shows that the ambiguity which is well known in deriving optical potentials from the elastic scattering of heavy-ions[7,8] can persist also for the one nucleon transfer reaction. There is less ambiguity for two nucleon transfer, as illustrated in Fig. 1(b), which compares calculations with the potential of type A, and values of r_w = 1.36 and 1.26 fm. (The results obtained with POT B gave an almost identical distribution to POT A with r_w = 1.36 fm, so the comparison is essentially the same as in 1(a)). We wish to emphasize the sensitivity to the parameter r_w in which a reduction of a few percent increases the cross section by a factor 10 at forward angles. Although in these cases, as in most heavy-ion transfer reactions, the DWBA formalism gives a successful description, the underlying physics is obscured by the complexity of the computer calculation. From an intuitive classical standpoint, the greater sensitivity of two nucleon transfer to details of the optical potential has been ascribed[9] to the sharper fall-off of the form factor which renders the forward cross section more sensitive to close trajectories deflected forward by the attractive nuclear potential. As a result, the two nucleon transfer probes the edge of the potential more closely.[10]

More formally we can develop analytical expressions relating the quantal and semiclassical approaches. In general the scattering amplitude can be written:

$$f(\theta) = \frac{1}{2ik} \sum_{\ell} (2\ell + 1)\eta_{\ell}\, e^{2i\delta_{\ell}}\, P_{\ell}(\cos\theta) \qquad (1)$$

On account of the peripheral nature of heavy-ion collisions [11,12] at high energy, we can paramatrise the magnitude of η_ℓ:

$$\eta_\ell = \eta_{\ell_o} \quad \exp -\left[\frac{(\ell-\ell_o)^2}{(\Delta\ell)^2}\right] \tag{2}$$

The reaction amplitude has a maximum for partial wave ℓ_o = kR, where R is the interaction radius. The spread of contributing ℓ-values $\Delta\ell$ is determined by strong absorption ($\ell<\ell o$) and by the decay of the form factor ($\ell>\ell o$). (This form can be justified from the output of "exact" DWBA calculations; see Fig. 3(a)).

For δ_ℓ, we make a Taylor expansion:

$$\delta_\ell = \delta_{\ell_o} + \left(\frac{d\delta}{d\ell}\right)_{\ell_o} (\ell-\ell_o) + \frac{1}{2}\left(\frac{d^2\delta}{d\ell^2}\right)_{\ell_o} (\ell-\ell_o)^2 + \ldots\ldots \tag{3}$$

Since the WKB formalism relates[13] the phase shift δ_ℓ corresponding to the scattering angle θ_ℓ, associated with partial wave ℓ, according to $\theta_\ell = 2(d\delta_\ell/d\ell)$ we can reduce Eq (3) to the form:

$$\delta_\ell = \delta_\ell \quad \left(\frac{\theta_o}{2}\right) \quad (\ell-\ell_o) + \frac{1}{4}\left(\frac{d\theta_\ell}{d\ell}\right)_{\ell_o} (\ell-\ell_o)^2 \ldots\ldots \tag{4}$$

where θ_o is the angle of deflection of the trajectory corresponding to the grazing partial wave ℓ_o. The scattering amplitude $f(\theta)$in Eq. 1 can be evaluated with the above expressions for δ_ℓ, η_ℓ, together with the asymptotic form of P_ℓ valid for large ℓ, and by replacing the sum over ℓ by an integral. These additional simplifications depend on the conditions $\ell_o \gg \Delta\ell \gg 1$ fairly valid for a peripheral reaction at high incident energy. For the differential cross section we obtain:[14,15]

$$\left(\frac{d\sigma}{d\Omega}\right) = |f(\theta)|^2 \propto \exp\left[\frac{-(\theta-\theta_o)^2}{(\Delta\theta)^2}\right] + \exp\left[\frac{-(\theta+\theta_o)^2}{(\Delta\theta)^2}\right] + \text{(INTERFERENCE TERM)} \tag{5}$$

This equation can be interpreted as the superposition of two "classical" distributions centered around the grazing trajectories at $\pm\,\theta_o$, with an interference term in the region of overlap[14] (e.g. the region forward of 30° in Fig. 1(a)). To simplify the subsequent discussion, we ignore the interference and consider only the first term in Eq. 5, which results in a symmetrical distribution centered at the physical angle θ_o, of width $\Delta\theta$:

$$(\Delta\theta)^2 = \frac{2}{(\Delta\ell)^2} + \frac{1}{2}\left(\frac{d\theta_\ell}{d\ell}\right)^2_{\ell_o}(\Delta\ell)^2 \,. \tag{6}$$

For orientation, we now use the strictly classical result of a Rutherford orbit, that $\ell = \eta \cot(\theta/2)$, where η is the Sommerfeld parameter, and therefore:

$$\left(\frac{d\theta_\ell}{d\ell}\right)_{\ell_o} = -\frac{2}{\eta}\sin^2(\theta_o/2) \tag{7}$$

which takes the value 0.013 for the collision of 78 MeV ^{12}C ions on ^{144}Nd. The curve of $\Delta\theta$ versus $\Delta\ell$ derived from Eq. 6 and 7 is shown in Fig. 2. This curve has a minimum value at:

$$\Delta\ell = \sqrt{2\left(\frac{d\ell}{d\theta_\ell}\right)_{\ell_o}} = \sqrt{\eta}\ \operatorname{cosec}(\theta_o/2) \tag{8}$$

marking the transition between a region of quantal dispersion, $\Delta\ell \ll \sqrt{\eta}$ cosec $(\theta_0/2)$ where $\Delta\theta$ decreases with increasing $\Delta\ell$ and a classically dispersive region $\Delta\ell \gg \sqrt{\eta}$ cosec $(\theta_0/2)$, where $\Delta\theta$ increases with increasing $\Delta\ell$. A discussion of these limits for the validity of quantum and classical approaches by means of explicit DWBA calculations is given in Ref. 16. Also shown in Fig. 2 is the curve obtained by making the expansion in Eq. 3 to first order in ℓ, which removes the "dynamical dispersion" proportional to $(d\theta_\ell/d\ell)$ in Eq. 6 (Ref. 12).

The value of $\Delta\ell$ deduced from the DWBA calculation for one nucleon transfer with POT A is indicated in Fig. 2; the corresponding value of $\Delta\theta = 9.2°$ is almost exactly equal to the observed half-width of the differential cross section at 1/e of the maximum. Since $\Delta\ell$ lies close to the minimum of the curve, the stability of the one nucleon transfer width to variations of $\Delta\ell$ brought about by changes of the potential naturally follows.

The greater sensitivity of the two nucleon transfer cross section can be partly understood from the fact that in this case $\Delta\ell$ lies on the steeply rising portion of the curve, as indicated in Fig. 2 for the calculation with POT B, $r_W = 1.26$ fm. However the corresponding value of $\Delta\theta = 12°$ does not fit the experimental value of 16°. In this case it is necessary to carry the expansion in δ_ℓ to higher than the second order in ℓ, which was adequate for one nucleon transfer. The inclusion of one additional term leads to:

$$\delta_\ell = \delta_{\ell_0} + \ldots\ldots\ldots + \frac{1}{6}\left(\frac{d^3\delta}{d\ell^3}\right)_{\ell_0} (\ell-\ell_0)^3 \qquad (9)$$

and the deflection function develops a parabolic dip:

$$\theta_\ell = 2\left(\frac{d\delta_\ell}{d\ell}\right) = \theta_0 + \left(\frac{d\theta_\ell}{d\ell}\right)_{\ell_0}(\ell-\ell_0) + \left(\frac{d^2\theta_\ell}{d\ell^2}\right)_{\ell_0}(\ell-\ell_0)^2 \qquad (10)$$

The relevance of this expansion for two nucleon transfer was checked by computing an "optical model deflection function" from differentiating the DWBA phase shifts.[17] The results obtained from the two potentials in Fig. 1(b) are compared in Fig. 3(b), together with the pure Coulombic deflection function $\theta_\ell = 2\tan^{-1}(\eta/\ell)$. The corresponding reaction amplitudes are plotted in 3(a). Clearly for the potential which led to the closest agreement with the angular distribution, the deflection function has a pronounced dip for the grazing partial waves in accord with Eq. 10. (No pronounced dip was apparent in the deflection function for one nucleon transfer). The calculated angular distribution using Eq. 9 for δ_ℓ rather than Eq. 3, has been shown[14] to have an asymmetric rise at forward angles, as observed in the experimental data of Fig. 1(b). This asymmetry could not be reproduced using $\Delta\ell$ values taken from Fig. 3(a) and used in Eq. 5.

We have shown that a comparison between quantal and semiclassical approaches to peripheral heavy-ion reactions at energies well above the barrier leads to analytical expressions which can give physical insight into the observed differences between one and two nucleon transfer. This formalism gives a natural definition of the regions of validity of quantum and semiclassical mechanics.[16] A similar approach was recently adopted[18] to trace the evolution from bell-shaped distributions to asymmetrically rising distributions between quasi-elastic and deep inelastic reactions induced by ^{40}Ar or ^{232}Th. However it was not possible to obtain a consistent description of the data, with expansion of δ_ℓ to first order. Since we have shown that even in the case of two nucleon transfer it is essential to carry the expansion to third order, it seems plausible in the above case that even more elaborate parametrizations of reaction amplitudes and phase shifts may be required. Nevertheless, these approaches may prove to be an instructive method of extracting overall physical constraints on values of $\Delta\ell$, ℓ_0, and θ_0, and it will be interesting to see if they agree with the predictions of macroscopic physical theories, e.g. involving frictional and transport phenomena.[19]

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FOOTNOTES AND REFERENCES

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Table I. Optical parameters use in analysis of $^{12}C+^{144}Nd$ at 78 MeV.

	V	W	r_V	r_W	a_V	a_W
POT A Ref.5	-40	-15	1.31	1.31	0.45	0.45
POT B Ref.6	-100	-40	1.22	1.22	0.49	0.60

FIGURE CAPTIONS

Fig. 1. Differential cross sections for (a) the reaction $^{144}Nd(^{12}C,^{13}C)^{143}Nd$ and (b) the reaction $^{144}Nd(^{12}C,^{14}C)^{142}Nd$ at an incident energy of 78 MeV. The theoretical curves in (a) are DWBA predictions normalized to the data, using the optical potentials of Table 1; POT A (solid line) and POT B(dashed line). In (b) both predictions use a potential of type A, with r_w = 1.26 fm (solid line) and 1.36 fm (dashed). The faint solid line is to guide the eye through the data.

Fig. 2. The variation of $\Delta\theta$ with $\Delta\ell$ predicted from Eq. 7. The dot-dash curve represents the limit of Eq. 7, obtained by setting the second dispersive term to zero. The $\Delta\ell$ values derived from fitting Eq. 2 to the DWBA reactions amplitudes for the one and two neutron transfer data of Fig. 1 are marked in the figure.

Fig. 3(a) The reaction amplitudes predicted from DWBA theory for two neutron transfer, using the potentials as in Fig. 1(b); the curves are normalized aribtrarily to the value 10 at the maximum. The corresponding optical model deflection function, calculated as in Eq. 4 by differentiating the DWBA phase shifts are shown in (b) compared with the pure Coulombic deflection $\theta_\ell = 2\tan^{-1}(\eta/\ell)$.

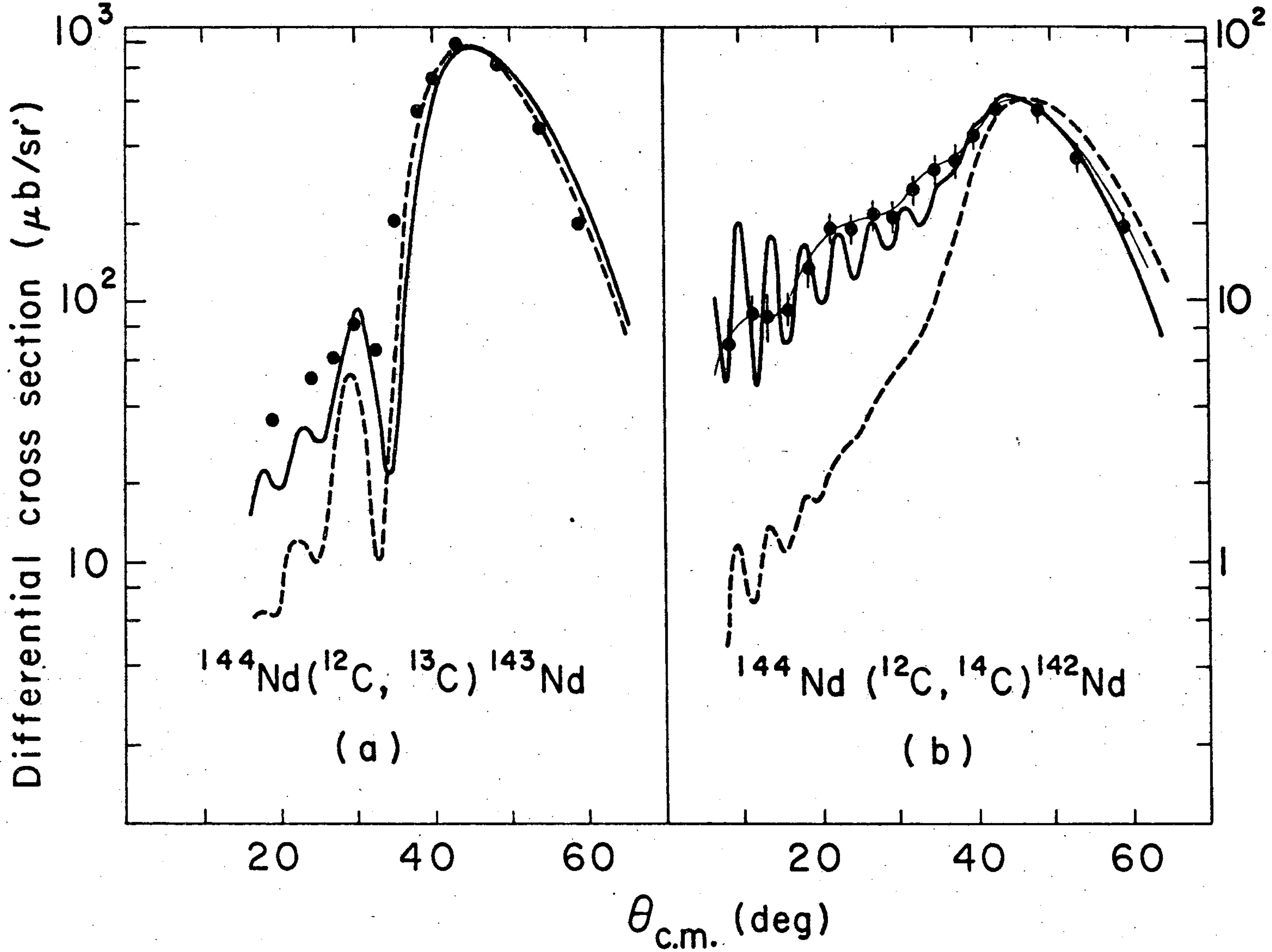

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Figure 1

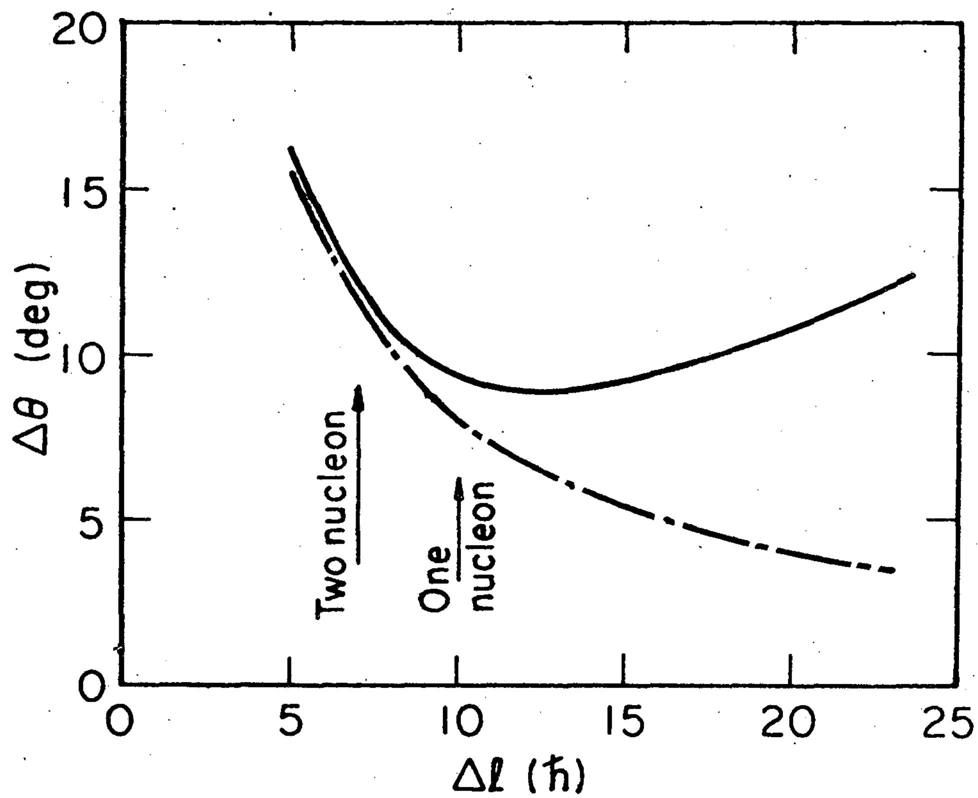

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Figure 2

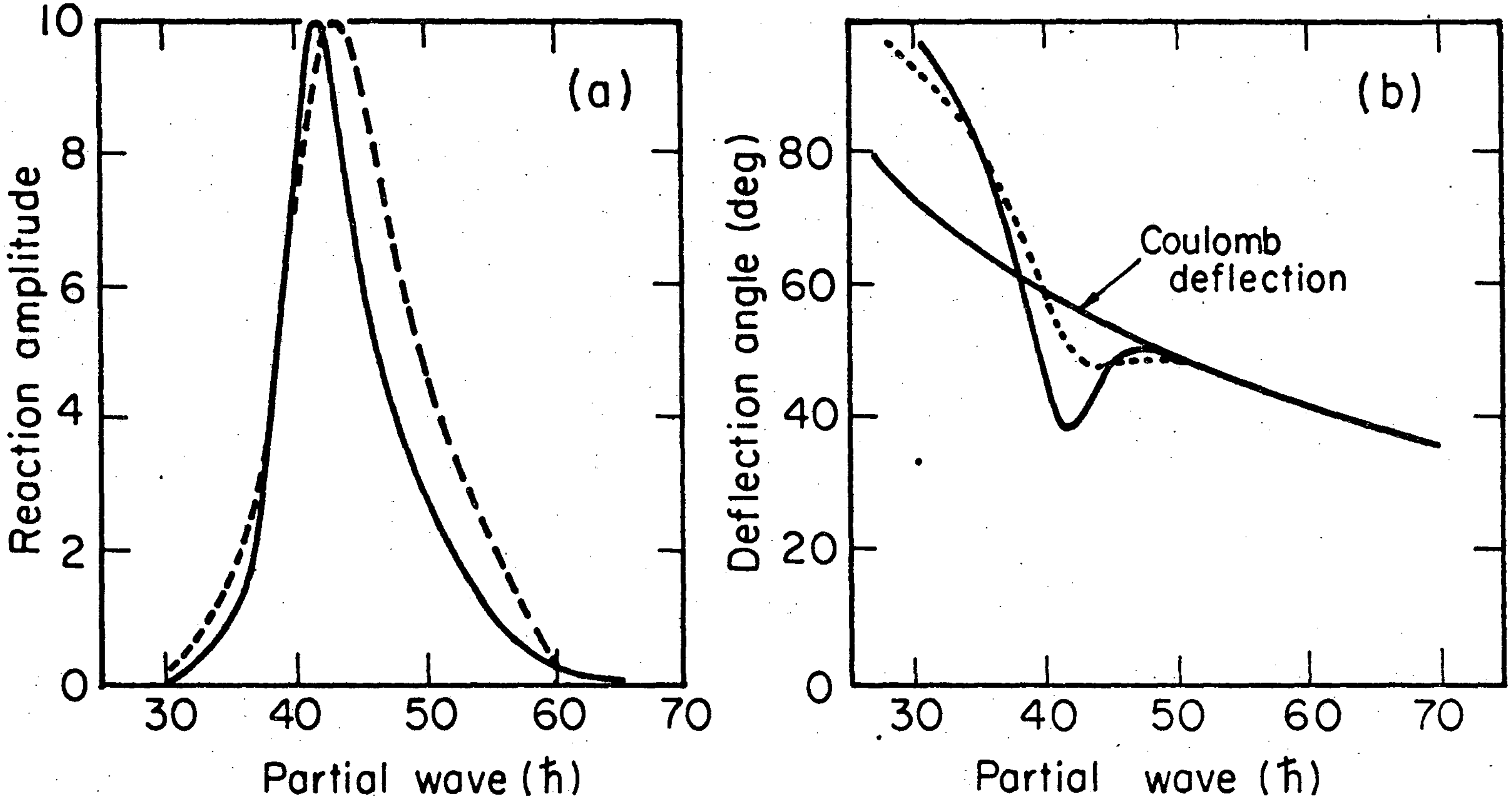

Figure 3

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