Title

COMMENTS CONCERNING THE MAGNETIC FIELD PRODUCED BY AN ION BEAM TRAVERSING A WINDOW-FRAME MAGNET

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Publication Date
1981-11-01

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Accelerator & Fusion Research Division

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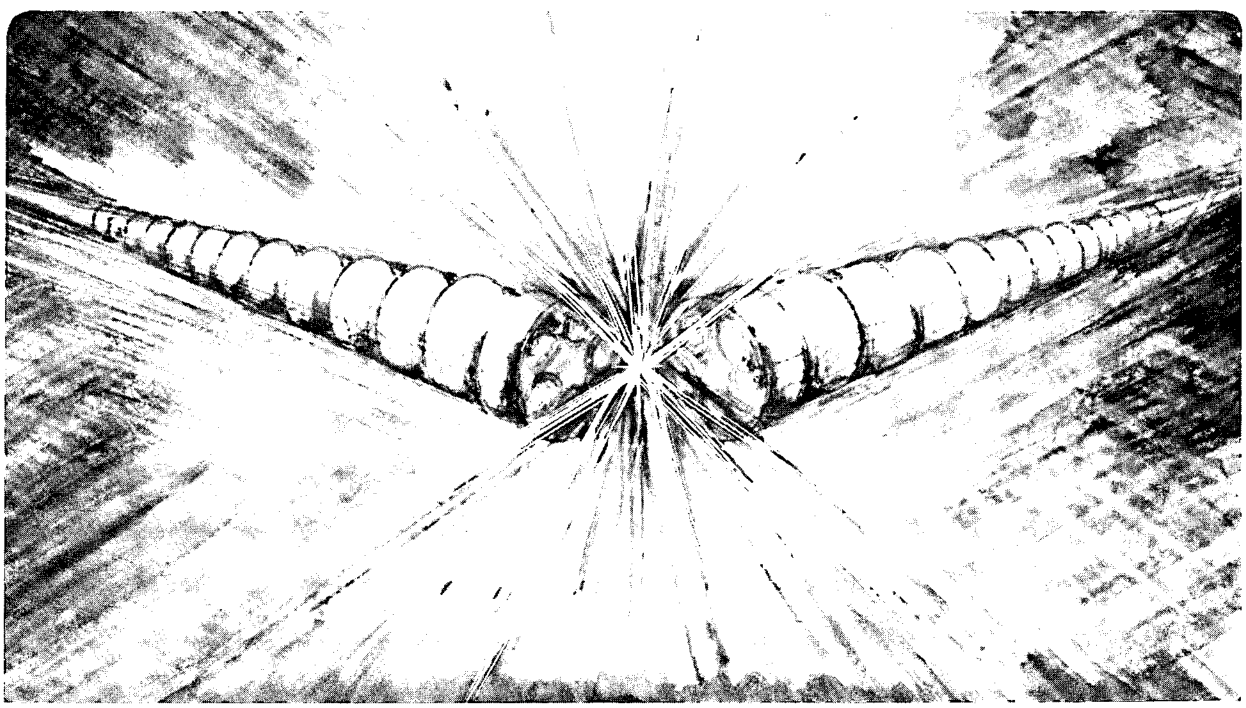

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

LBID-534
BETA-17

Lawrence Berkeley Laboratory

Technical Report of the Betatron Design Study

COMMENTS CONCERNING THE MAGNETIC FIELD PRODUCED BY AN ION BEAM TRAVERSING A WINDOW-FRAME MAGNET*

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16 November 1981

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*Sponsored by Defense Advance Research Projects Agency (DoD)
ARPA Order No. 3718, Amend. 37
Monitored by NSWC under Contract No. N60921-81-LT-W0031

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER BETA-17	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) COMMENTS CONCERNING THE MAGNETIC FIELD PRODUCED BY AN ION BEAM TRAVERSING A WINDOW-FRAME MAGNET		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) L. Jackson Laslett		8. CONTRACT OR GRANT NUMBER(s) N60921-81-LT-W0031
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lawrence Berkeley Laboratory University of California Berkeley, California 94720		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61101E; 0; 0; OR40AA
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Advanced Research Projects Agency 1400 Wilson Boulevard, Arlington, Virginia 22209 Attn: Program Management/MIS		12. REPORT DATE November 1981 13. NUMBER OF PAGES 16
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Surface Weapons Center White Oak, Silver Spring, Maryland 20910 Attn: Code R401		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Magnetic field
image coefficient
ion beam
betatron magnet

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The magnetic field of an ion beam traversing a window-frame magnet is required for the evaluation of the magnetic image coefficient, which is needed to determine the space-charge limit of the beam intensity in circular machines, such as betatrons and synchrotrons. The magnetic field in an elliptical window-frame magnet is obtained analytically by means of a conformal transformation and is verified by a computational technique.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

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I. INTRODUCTION

In 1969 we presented[1] some results concerning "image-field coefficients" for situations that included cases wherein a (centrally situated) ion beam was considered to traverse a picture-frame ferromagnetic structure. One case of this nature concerned a beam centrally situated within an elliptical aperture bounded by high ($\mu \to \infty$) permeability ferromagnetic material.

Analysis of the problem just described required evaluation of the magnetic field produced in the interior region by the central beam current -- subject to boundary conditions consistent with the requirement that $\oint \vec{H} \cdot d\vec{\ell} = 4\pi I$ (unrationalized cgs emu) for any contour that encircles the beam. For the treatment of such a problem it was advocated [cf. Hague[2]] that it would be advisable first to examine the exterior problem (i.e., within the ferromagnetic material), in order to obtain information concerning the values of H_t that then could be employed as boundary conditions for the interior problem.

A solution of the problem was undertaken along the lines outlined above and led to a sketch of magnetic field lines, within the elliptical aperture,

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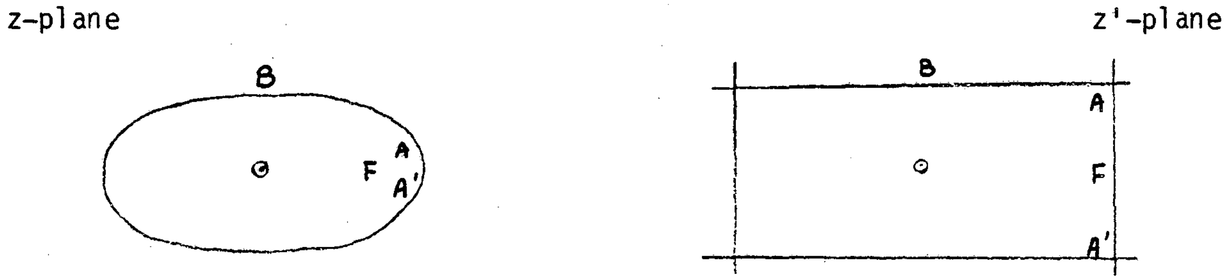
that was presented as Fig. 1 in Ref. 1. Because of a revival of interest in problems of this nature, we now comment on some aspects of the solution and show a similar field-map recently obtained by an independent relaxation computation.

II. REMARKS CONCERNING THE SOLUTION

A conformal transformation was introduced (Ref. 1, Sect. III.1)

$$z' = m \sin^{-1}(z/f)$$

to transform the z-plane region with an elliptical interface (semi-major x axis = w, semi-minor y axis = h, focal length $f = \sqrt{w^2 - h^2}$).



This transformation evidently unfolds the line segment AFA' in the z-plane to assume in the z'-plane the form indicated in the sketch $\left[\text{at } x' = \frac{\pi}{2} m, |y'| \le m \,\mathrm{Tanh}^{-1}(\frac{h}{w})\right]$. It is particularly of interest to mention that the real z-axis, namely OFA extended, in the z'-plane will merely involve an extension of the line FA -- i.e., with x' <u>continuing</u> <u>to</u> <u>assume</u> the value $\frac{\pi}{2}$ m.[4] Because, from symmetry, we can expect lines of force always to be perpendicular to the x-axis in the z-plane ($|x| \ne 0$, $y = 0$), we similarly may assume that in the z'-plane the lines of force will be perpendicular to the vertical line at $x' = \frac{\pi}{2} m$ (and likewise to the line $x' = -\frac{\pi}{2} m$). Thus throughout the high-μ ferromagnetic region, outside of the lines $y' = \pm m \,\mathrm{Tanh}^{-1}(\frac{h}{w})$, we may regard the intense lines of $\vec{B}$ as directed just in the $(\mp)$ x' direction. [The orthogonal function -- representing current flow, in a highly resistive medium, for an analogous (but different) problem in which current must approach and enter into the interior region -- would be characterized (very reasonably) by vertically directed lines in this "exterior" portion of the z'-plane.]

Lines of $\vec{H}$ correspondingly also will be horizontally directed in the exterior region of the z'-plane, with $H_{x'}$, assuming the constant value $\mp 2I/m$. Continuity of H_t at the interfaces $y' = \pm m \, \mathrm{Tanh}^{-1} (\frac{h}{w})$ then provides us with a boundary condition for $H_{x'}$ at these boundaries to the interior region. [The foregoing does not preclude the presence also of a significant normal component, $H_{y'}$ $(= B_{y'})$, on the interior side of such an interface -- and indeed such components are to be expected (for $x' \neq 0$ and $|x'| \neq \frac{\pi}{2}$). Such normal components of $\vec{B}$, being continuous across an interface, will produce no more than an infinitesimal disturbance to the pattern of $\vec{B}$-lines in the ferromagnetic material when $\mu \to \infty$.]

The considerations just presented were employed in Ref. 1 to construct an expression for the vector-potential function, A_z, in the interior region -- either (i), specifically for $x'' = 0$ $(x' = 0)$ and y'' small, by introduction of a second conformal transformation (for the interior), or (ii) by directly constructing, in a series form, an expression for A_z in terms of the z'-plane coordinates x', y'. Such a solution permitted one to construct a sketch of the lines of constant A_z -- lines whose direction coincides with that of the magnetic field -- within the elliptical z-plane aperture.

It will be noticed that this procedure, just described, distinctly (and not unreasonably) results in lines of force approaching the vacuum-ferromagnetic interface neither tangentially nor normally (at all but special points on the interface). It might be wondered whether a new transformation -- say in terms of the elliptic sine function,

$$w = R\sqrt{k}\,\mathrm{sn}\left(\frac{2K}{\pi}\frac{z'}{m}\right)$$

with the modulus k such that $\frac{K'}{K} = \frac{4}{\pi}\,\mathrm{Tanh}^{-1}\left(\frac{h}{w}\right)$, [cf. Ref. 3, p. 177] -- would permit us to transform the inferences we have drawn in the z'-plane into statements applicable to a problem in which the beam current is situated centrally within a circular bounding interface. It would, indeed, be paradoxical if this should prove to be correct, since with the symmetrical circular boundary the field lines surely would be azimuthally directed and would have no component normal to the interface.

The resolution of this matter may be thought to reside in the fact that our previous description of magnetic field lines within the ferromagnetic region of the z'-plane is inapplicable to the situation that would arise from such a transformation of the problem with the circular interface. Specifically, the real axis of the w-plane, when extended well into the ferromagnetic region, would not continue indefinitely upward (e.g., at $x' = \frac{\pi}{2}m$) in the z' plane but would exhibit a kink at $y' = 2m\,\mathrm{Tanh}^{-1}\left(\frac{h}{w}\right)$ [recall the double periodicity, with respect to poles and zeros in the complex plane, of elliptic functions]. With lines of force expected to be perpendicular to the real axis of the w-plane in the problem with a circular interface, the solution we previously adopted for the $\vec{B}$ field within the ferromagnetic portions of the z'-plane thus no longer would be acceptable.

III. COMPUTATIONAL CHECK BY RELAXATION

To seek some verification of the analysis performed in Ref. 1 (and discussed above) for the elliptical window-frame magnetostatic problem, a relaxation run was recently performed for a similar problem, using the Program POISSON (similar to Program TRIM, described in Ref. 5 and in references cited therein. The aspect ratio selected for the elliptical aperture in this work was $h/w = 0.4$, and the ferromagnetic region was bounded by a concentric circular curve of radius 2w along which the vector-potential function was specified as having the fixed value $A_z = 0$. The ferromagnetic region was assigned an arbitrary, but convenient, constant value $\mu = 250$ for the permeability. The zoning (triangular mesh) for the one quadrant for which relaxation computations were performed (LBL CDC-7600) is illustrated in an attached Figure. Resulting flux plots (curves of constant A_z) are shown for the ferromagnetic region in the next Figure. Because the spacing of equipotential curves (A_z = constant) is much different in the interior region (markedly smaller values of the magnetic induction, B), a separate flux plot of this region was required and is shown in the third Figure. The Figure just mentioned may be compared with that presented as Fig. 1 in Reference 1 and reproduced here, as a final Figure, to approximately the same scale as the corresponding plot of the recent results. Direct comparison of these last two figures appears to indicate complete consistency between them in regard to the directions taken by field lines throughout the interior region and, in particular, agreement with respect to the oblique incidence of such lines onto the ferromagnetic interface.

This same computational technique evidently could be applied equally well to other, similar, problems -- involving, for example, non-central or extended ion beams.

IV. ACKNOWLEDGEMENT

It is a pleasure to acknowldge, with thanks, the collaboration of Victor O. Brady of this Laboratory in submitting the relaxation problem to the computer and for preparing the diagrams relating to this work.

V. REFERENCES

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2. B. Hague, "The Principles of Electromagnetism Applied to Electrical Machines" (Dover Publications, Inc., New York, 1962) -- previously "Electromagnetic Problems in Electrical Engineering" (Oxford Univ. Press, 1929).

3. H. Kober, "Dictionary of Conformal Transformations" (Dover Publications, Inc., New York, 1957).

4. The mapping, by an inverse-sine transformation, of a portion of the exterior region of our z'-plane, is suggested by a Figure on p. 97 of Ref. 3.

5. John S. Colonias, "Particle Accelerator Design: Computer Programs" (Academic Press, New York, 1975), Ch. I, Sect. 2, p. 15.

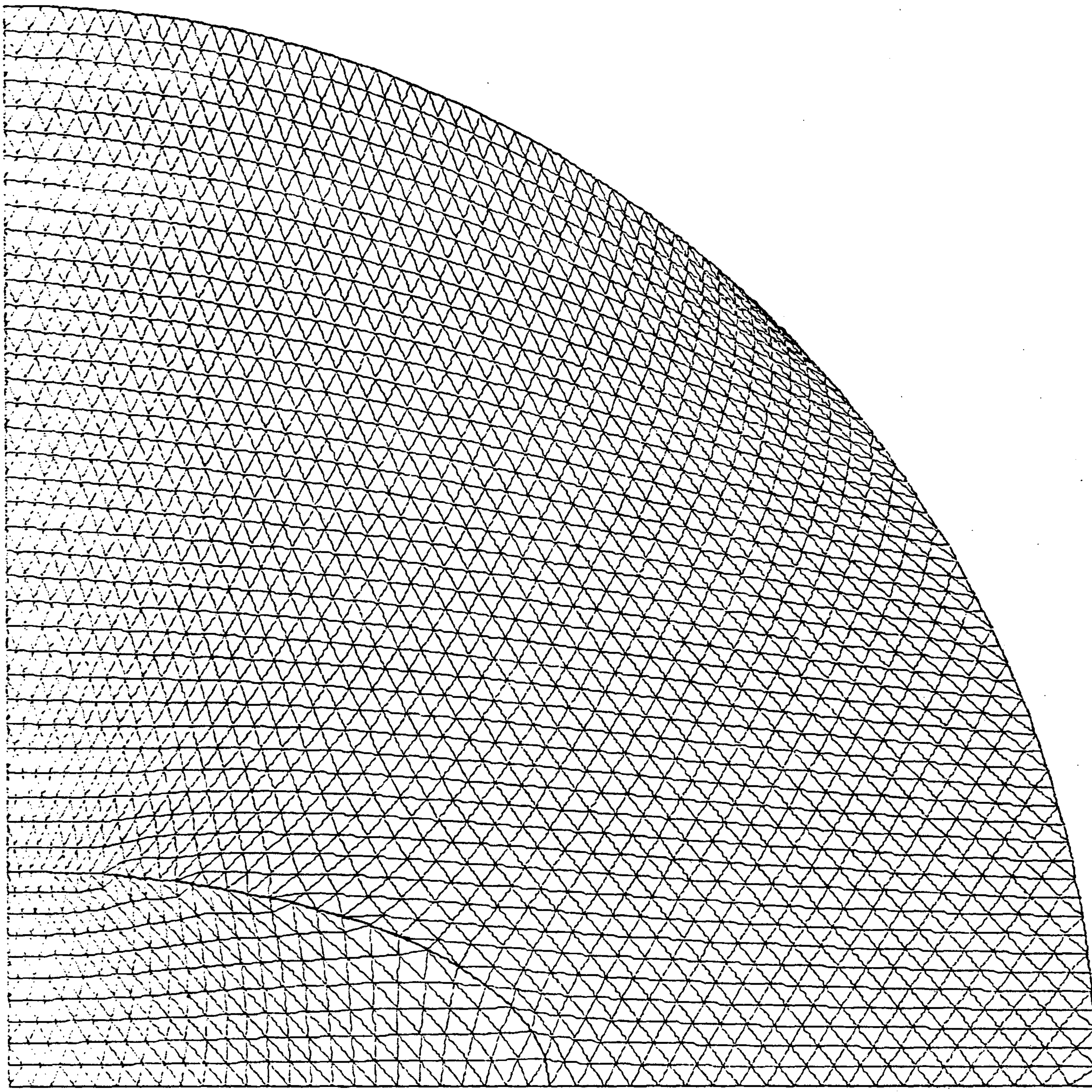

Fig. 1

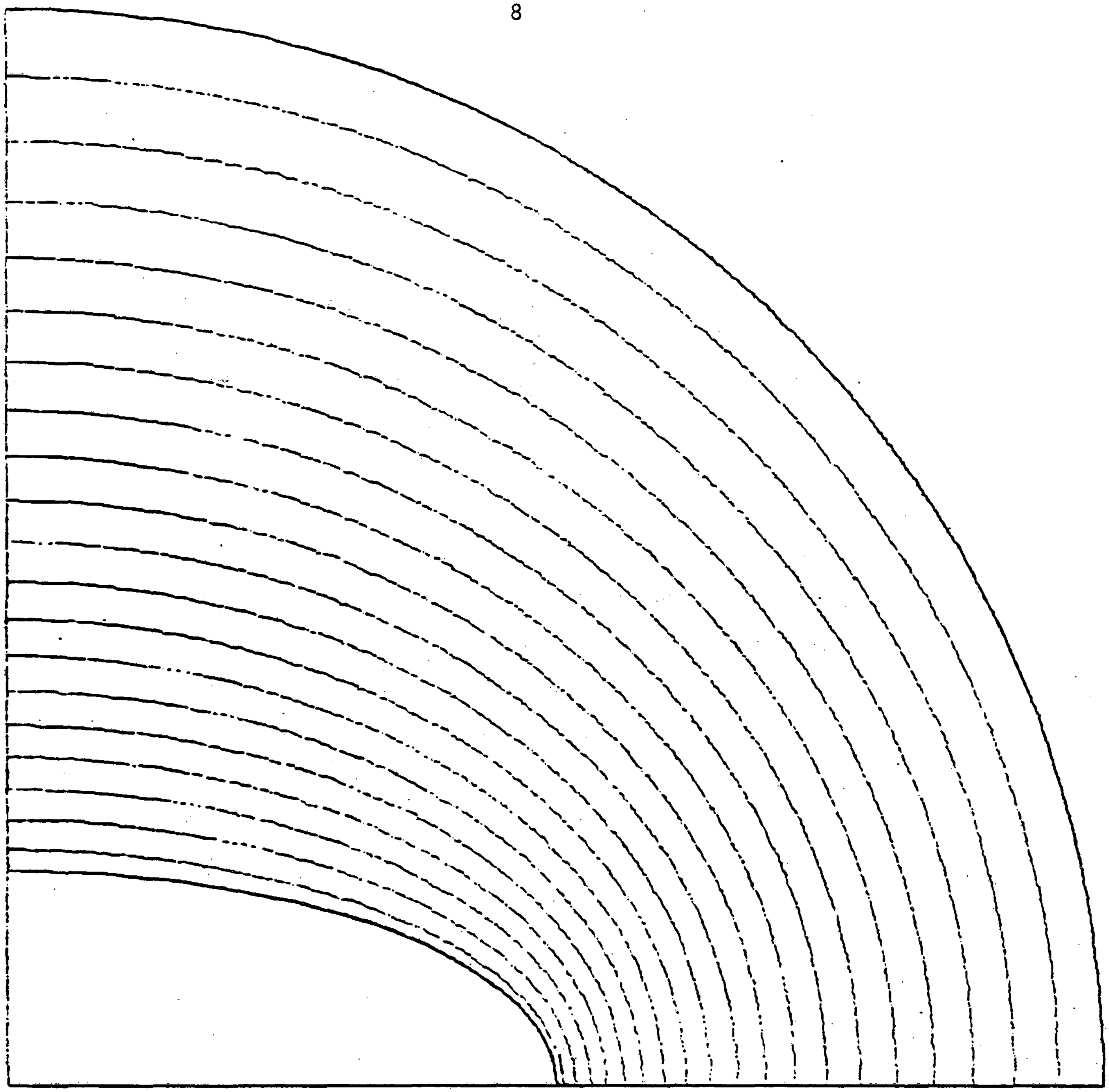

Fig. 2

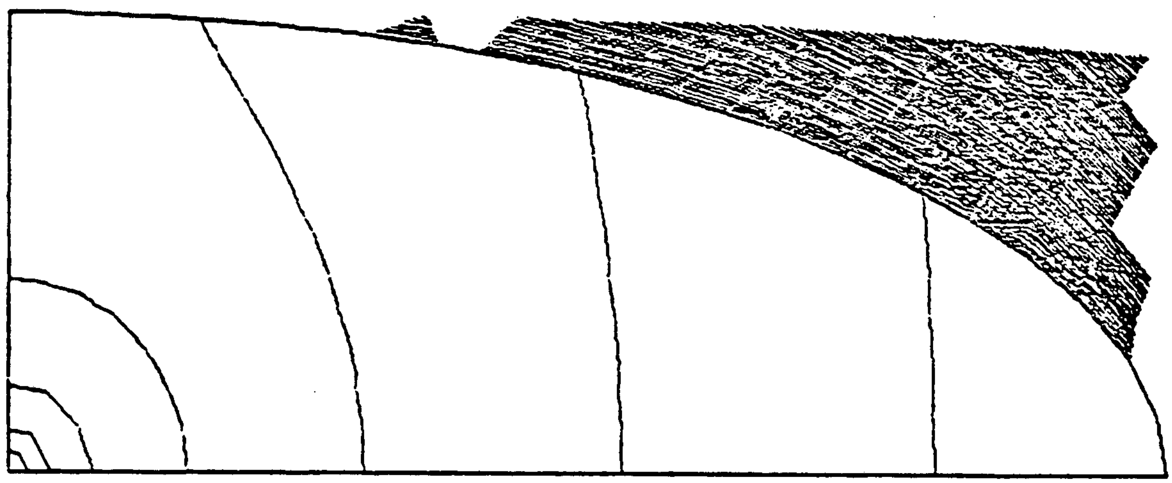

Fig. 3

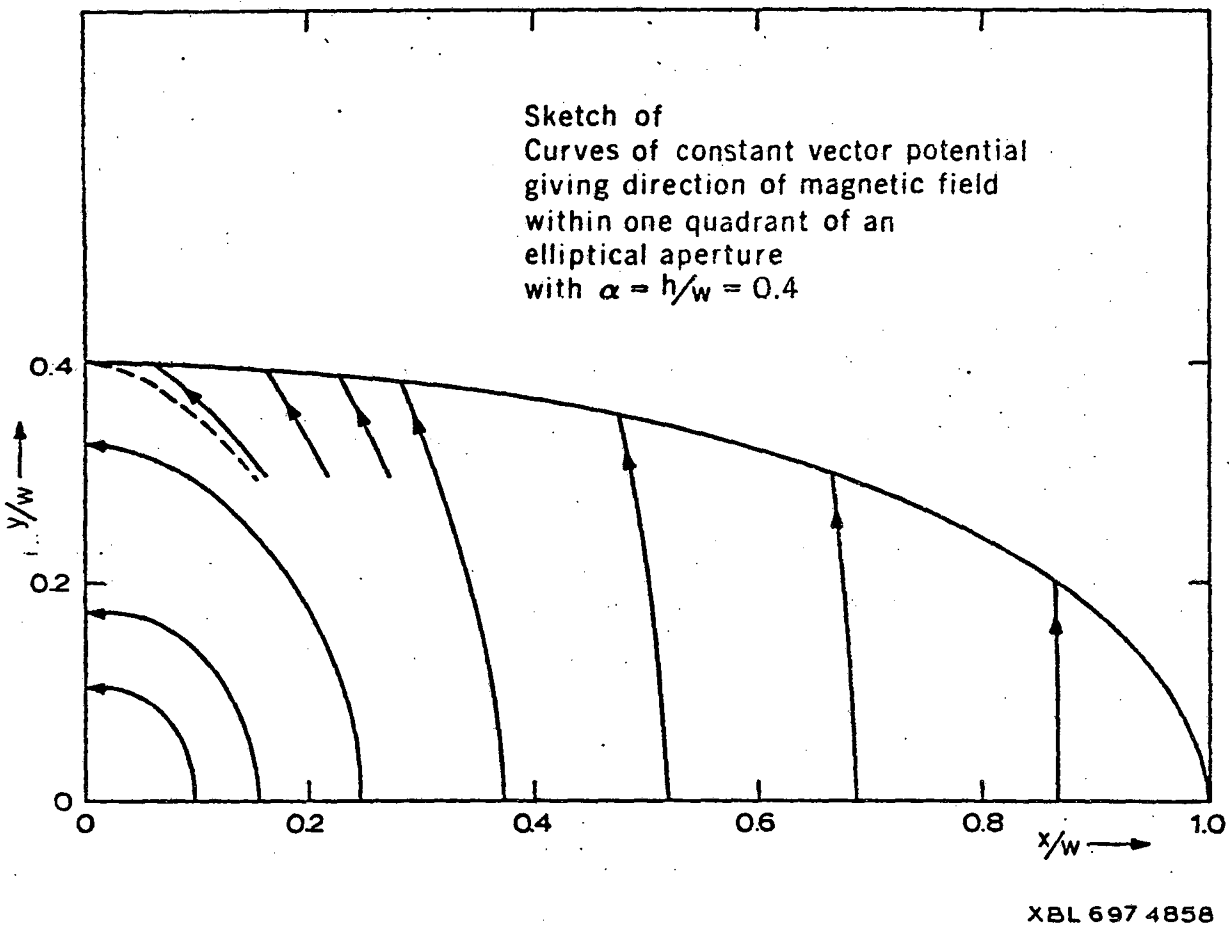
Sketch of
Curves of constant vector potential
giving direction of magnetic field
within one quadrant of an
elliptical aperture
with α = h/w = 0.4
0.4
0.2
0
y/w
0
0.2
0.4
0.6
0.8
1.0
x/w
XBL 697 4858

Fig. 1 of Ref. 1

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