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# Heterogeneity and Trade

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## **Abstract**

Aggregate production functions are a standard feature of the trade theorist's toolbox. While this modeling device has generated some fundamental insights, it presents one obvious shortcoming: it necessarily ignores any effect that the distribution of factor endowments across agents may have on international trade flows. This paper develops a general framework that can shed light on these effects and discusses several applications.

**Keywords:** Heterogeneity, International Trade, Log-Supermodularity, Single Crossing Property

**J.E.L. Classification:** F10

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# 1 Introduction

Aggregate production functions are an implicit starting point of many applied fields in economics, from growth theory to labor economics. The standard theory of international trade—see e.g. DIXIT and NORMAN [1980]—is no exception. While this approach has generated some fundamental insights, it presents one obvious shortcoming: it necessarily ignores any effect that the distribution of factor endowments across agents may have on international trade flows.

Over the last decade, new models have been developed to analyze the role of “heterogeneity” in an open economy. To take a few influential examples, MELITZ [2003], HELPMAN et al. [2004], and ANTRAS and HELPMAN [2004] have shown how productivity heterogeneity across firms may have a profound impact on various dimensions of the organization of production in a global economy. Similarly, GROSSMAN and MAGGI [2000], GROSSMAN [2004] and OHNSORGE and TREFLER [2004] have shown how human capital heterogeneity across workers may determine the pattern of international specialization among countries with similar aggregate factor endowments, thereby offering compelling theories of North-North trade.<sup>2</sup>

The objective of the present paper is to offer a unifying perspective on the relationship between heterogeneity and trade. To do so, we adopt the following strategy. First, we develop a general model highlighting the key features of an environment where heterogeneity matters. Second, we identify “critical sufficient conditions” to predict the cross-sectional variation of aggregate output in such an environment: *log-supermodularity* and the *single crossing property*. Finally, we show that our results are at the heart of many predictions in the previous literature, whether they are concerned with international specialization or the international organization of production.

Log-supermodularity and the single crossing property properties have been used previously in many areas of economics, including optimal taxation, MIRRLEES [1971]; auction theory, MILGROM and WEBER [1982]; monotone comparative statics, MILGROM and SHANNON [1994] and ATHEY [2002]; and matching, SHIMER and SMITH [2000]. One of the main insights of our paper is that these two properties have natural and useful applications for the theory of international trade as well.

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<sup>2</sup>In a related paper, ANTRAS et al. [2006] investigate the relationship between the distribution of human capital across workers and offshoring.

To illustrate potential applications, consider a world economy including multiple countries with characteristic  $\gamma$  (e.g. their number of universities per capita) and multiple sectors with characteristic  $\sigma$  (e.g. their skill intensity). Each country is populated by a continuum of workers with different endowments of an indivisible and non-tradable asset  $\omega$  (e.g. their number of years of education). Workers are immobile across countries and mobile across industries. Technologies are the same around the world and factor price equalization prevails. Countries only differ in terms of their distributions of assets.

In this simple “Heckscher-Ohlin” environment, the central result of our paper can be stated as follows. Suppose that two conditions hold: (i) the number of workers with endowment  $\omega$  in a country with characteristic  $\gamma$  is log-supermodular in  $\omega$  and  $\gamma$ ; and (ii) the revenue of a worker with endowment  $\omega$  in a sector with characteristic  $\sigma$  satisfies the single crossing property in  $\omega$  and  $\sigma$ . Then aggregate output across countries and sectors is log-supermodular in  $\sigma$  and  $\gamma$ . Economically speaking, if conditions (i) and (ii) hold, then high- $\gamma$  countries specialize in high- $\sigma$  sectors.

The basic logic is intuitive. On the one hand, high- $\gamma$  countries have “relatively more” workers with large endowments. On the other hand, workers with large endowments gain “relatively more” by sorting into high- $\sigma$  sectors. Thus, high- $\gamma$  countries shall produce “relatively more” in high- $\sigma$  sectors under free trade. Our paper provides the mathematical apparatus one needs to make these “relatively more” statements precise. This allows us to demonstrate, among other things, that conditions (i) and (ii) are critical sufficient conditions to predict the pattern of international specialization in a “heterogeneous economy”.

Although our results are *not* about monotone comparative statics,<sup>3</sup> our analysis is heavily influenced—in terms of question and method—by the seminal work of MILGROM and SHANNON [1994] and ATHEY [2002]. First, in terms of question, we ask: What are the *weakest* assumptions under which certain qualitative predictions on the pattern of international specialization or the international organization of production will hold? This is very much in line with the literature on monotone comparative statics which “systematically seeks the best possible conditions for robust comparative statics conclusions”. Second, in terms of method,

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<sup>3</sup>We are interested in the cross-sectional variation of aggregate output within a given equilibrium, not changes in aggregate output across equilibria. The fact, for example, that all countries face the same prices within a given free trade equilibrium is crucial for our results.

we rely extensively on basic results in the mathematics of complementarities. In particular, the fact that log-supermodularity is preserved by multiplication and integration is, like in ATHEY [2002], at the core of our analysis.<sup>4</sup>

To us, the broader perspective that our paper offers on the literature on heterogeneity and trade is attractive for three reasons. First, qualitative insights from the previous literature typically rely on strong functional forms which guarantee explicit closed form solutions. For example, distributions of human capital are log-normal in OHNSORGE and TREFLER [2004], whereas distributions of firm productivity are Pareto in HELPMAN et al. [2004]. These assumptions have been made for tractability purposes, but since we do not expect them to hold exactly in practice, it is useful to ask whether they are crucial for a particular conclusion to hold.<sup>5</sup> By showing that log-supermodularity and the single crossing property are critical sufficient conditions for many qualitative insights of the earlier literature to hold, we can formally establish their robustness.

Second, log-supermodularity and the single crossing property are *not* hopelessly abstract mathematical concepts with little economic content. On the contrary, these properties are intimately related to the economic phenomena we are trying to study. As we argue later in the paper, log-supermodularity is the mathematical counterpart to the economic idea of comparative advantage. By focusing on these general properties, our hope also is to deepen our understanding of the common economic forces behind various predictions of the literature on heterogeneity and trade.

Finally, our general approach potentially is rich in new empirical implications. By construction, our results apply to any environment that includes a finite number of “populations” inhabited by a continuum of heterogeneous “agents” sorting across a finite number of “occupations”. Populations may be countries, ethnic groups, villages, or industries; agents may be workers with different levels of human capital, taxpayers with different income, migrants with different information, or firms with different productivity; occupations may be industries, cities, countries, or organizations. If there exist institutional and/or technological features such that conditions (i) and (ii) hold, then our analysis

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<sup>4</sup>By contrast, our focus on the single crossing property is very different from ATHEY [2002], as we discuss in details in Section 4.

<sup>5</sup>HECKMAN and HONORE [1990], for example, have shown that predictions on earnings inequality in a Roy model are sensitive to the log-normality of the talent distribution.

has implications for the cross-sectional variations of aggregate variables across populations and occupations.

The rest of the paper is organized as follows. Section 2 offers some basic definitions and results in the mathematics of complementarities. Section 3 formally defines a heterogeneous economy and presents the restrictions that we impose on this economy. Section 4 derives the implications of these restrictions for the cross-sectional variation of aggregate output. Section 5 discusses the existence of alternative restrictions leading to similar conclusions. Section 6 and 7 relate our theoretical results to the previous literature on heterogeneity and trade. Section 8 offers some concluding remarks. All proofs can be found in the Appendix.

## 2 Preliminary

Our analysis emphasizes two particular forms of complementarities: log-supermodularity and the single crossing property. Since these two concepts are not widely used in the trade literature, we begin with a review of some basic definitions and results. TOPKIS [1998] and ATHEY [2002] offer an excellent overview and additional references.

### 2.1 Log-Supermodularity

For any  $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^p$ , we say that  $\mathbf{x} \geq \mathbf{x}'$  if  $x_i \geq x'_i$  for all  $i = 1, \dots, p$ . We let  $\max(\mathbf{x}, \mathbf{x}')$  be the vector of  $\mathbb{R}^p$  whose  $i$ th component is  $\max(x_i, x'_i)$ , and  $\min(\mathbf{x}, \mathbf{x}')$  be the vector whose  $i$ th component is  $\min(x_i, x'_i)$ . Finally, we denote  $\mathbf{x}_{-i}$  the vector  $\mathbf{x}$  with the  $i$ th component removed and  $\mathbf{x}_{-ij}$  the vector  $\mathbf{x}$  with the  $i$ th and  $j$ th component removed. With the previous notations, log-supermodularity can be defined as follows.

**Definition 1** *A function  $g: \mathbb{R}^p \rightarrow \mathbb{R}^+$  is log-supermodular if for all  $\mathbf{x}, \mathbf{x}' \in \mathbb{R}^p$ ,  $g(\max(\mathbf{x}, \mathbf{x}')) \cdot g(\min(\mathbf{x}, \mathbf{x}')) \geq g(\mathbf{x}) \cdot g(\mathbf{x}')$ .*

If  $g$  is strictly positive, then  $g$  is log-supermodular if and only if  $\ln g$  is supermodular. This means that if  $g$  also is twice differentiable, then  $g$  is log-supermodular in  $(x_i, x_j)$  if and only if  $\frac{\partial^2 \ln g}{\partial x_i \partial x_j} \geq 0$ . To get more intuition about the form of complementarities that log-supermodularity captures, consider  $g(x_i, x_j, \mathbf{x}_{-ij})$ . For every  $x'_i \geq x''_i$ ,  $x'_j \geq x''_j$ , and  $x_{-ij}$ , the log-supermodularity of  $g$  in  $(x_i, x_j)$  implies that

$$g(x'_i, x'_j, \mathbf{x}_{-ij}) \cdot g(x''_i, x''_j, \mathbf{x}_{-ij}) \geq g(x'_i, x''_j, \mathbf{x}_{-ij}) \cdot g(x''_i, x'_j, \mathbf{x}_{-ij})$$

If  $g$  is strictly positive, this can be rearranged as

$$g(x'_i, x'_j, \mathbf{x}_{-ij}) / g(x''_i, x'_j, \mathbf{x}_{-ij}) \geq g(x'_i, x''_j, \mathbf{x}_{-ij}) / g(x''_i, x'_j, \mathbf{x}_{-ij})$$

Thus, the relative returns to increasing the first variable,  $x_i$ , are increasing in the second variable,  $x_j$ . This is equivalent to the monotone likelihood ratio property; see MILGROM [1981]. The core of our analysis relies on two properties of log-supermodular functions:

**Lemma 1** *Suppose that  $g, h : \mathbb{R}^p \rightarrow \mathbb{R}^+$  are log-supermodular functions. Then  $gh$  is log-supermodular.*

**Lemma 2** *Suppose that  $g : \mathbb{R}^p \rightarrow \mathbb{R}^+$  is a log-supermodular function. Then  $G(\mathbf{x}_{-i}) = \int_{-\infty}^{+\infty} g(\mathbf{x}) dx_i$  is log-supermodular.*

In other words, log-supermodularity is preserved by multiplication and integration. Lemma 1 directly derives from Definition 1. Proofs of Lemma 2 can be found in LEHMANN [1955] for the bivariate case, and AHLWEDE and DAYKIN [1978] and KARLIN and RINOTT [1980] for the multivariate case.

## 2.2 Single Crossing Property

The single crossing property is an ordinal version of the concept of complementarity introduced by MILGROM and SHANNON [1994]. According to their definition, a function  $g : \mathbb{R} \times \mathbb{R}^p \rightarrow \mathbb{R}$  satisfies the single crossing property in  $(t, \mathbf{x})$  if, for all  $t' > t''$  and  $x' > x''$ ,  $g(t'', x') > g(t'', x'')$  implies that  $g(t', x') > g(t', x'')$  and  $g(t'', x') \geq g(t'', x'')$  implies that  $g(t', x') \geq g(t', x'')$ . The term “single crossing” comes from the fact that, for any  $\mathbf{x}' > \mathbf{x}''$ ,  $g(\cdot, \mathbf{x}') - g(\cdot, \mathbf{x}'')$  crosses zero only once and from below. In this paper, we use a weak version of the Milgrom and Shannon single crossing property taken from ATHEY [2001].

**Definition 2** *A function  $g : \mathbb{R} \times \mathbb{R}^p \rightarrow \mathbb{R}$  satisfies the weak single crossing property in  $(t, \mathbf{x})$  if, for all  $t' > t''$  and  $\mathbf{x}' > \mathbf{x}''$ ,  $g(t'', \mathbf{x}') > g(t'', \mathbf{x}'')$  implies that  $g(t', \mathbf{x}') \geq g(t', \mathbf{x}'')$ .*

This property captures a weaker form of complementarity between  $t$  and  $\mathbf{x}$  than either supermodularity or log-supermodularity. In particular, we have:

**Lemma 3**  *$g : \mathbb{R} \times \mathbb{R}^p \rightarrow \mathbb{R}$  satisfies the weak single crossing property in  $(t, \mathbf{x})$  if: (i)  $g$  is twice differentiable and  $\frac{\partial^2 g}{\partial t \partial x_i} \geq 0$  for all  $i = 1, \dots, p$ ; or (ii)  $g$  is twice differentiable,  $g > 0$ , and  $\frac{\partial^2 \ln g}{\partial t \partial x_i} \geq 0$  for all  $i = 1, \dots, p$ .*

Conditions (i) and (ii) offer an easy way to check whether the weak single crossing property is satisfied in practice.

### 3 General Framework

For expositional purposes, we use the terminology of the literature on international specialization. In this section, an economy comprises “countries” populated by “workers” sorting across “sectors”. As previously mentioned, our results do not depend on this particular interpretation. For example, our approach applies to an economy where “sectors” are populated by “firms” sorting across “organizations”. We come back to this alternative interpretation in Section 7.

#### 3.1 A Heterogeneous Economy

Consider a world economy with  $c = 1, \dots, C$  countries and  $s = 1, \dots, S$  sectors described by a vector of exogenous characteristics,  $\gamma^c \in \mathbb{R}^n$  and  $\sigma^s \in \mathbb{R}^m$ , respectively. Each country is populated by a continuum of workers with different endowments of an indivisible and non-tradable asset  $\omega \in \mathbb{R}$ .<sup>6</sup> Workers are immobile across countries and mobile across sectors. We denote  $f(\omega, \gamma^c) \geq 0$  the mass of workers with endowment  $\omega$  in country  $c$ . Workers from country  $c$  who use their endowment  $\omega$  in sector  $s$  can produce  $q^c(\omega, \sigma^s) \geq 0$  units of good  $s$  and get revenues  $r^c(\omega, \sigma^s) \geq 0$ . In the spirit of the Heckscher-Ohlin-Vanek literature, we restrict ourselves to environments where workers have access to the same technologies around the world,  $q^c(\omega, \sigma^s) \equiv q(\omega, \sigma^s)$ , and, because of free trade in final goods, factor price equalization prevails,  $r^c(\omega, \sigma^s) \equiv r(\omega, \sigma^s)$ .<sup>7</sup> Therefore, there are no “Ricardian” sources of comparative advantage. Countries only differ in their distributions of assets.

We assume that workers sort across sectors in order to maximize their revenues and that the mass of workers whose maximum revenues are identical in 2 sectors (or more) is negligible and can be set to zero. Under these assumptions, the set of workers in sector  $s$  is given by

$$\Omega(\sigma^s) = \left\{ \omega \in \mathbb{R} \mid r(\omega, \sigma^s) > \max_{s' \neq s} r(\omega, \sigma^{s'}) \right\} \quad (1)$$

In turn, we can express the aggregate output of good  $s$  in country  $c$  as

$$Q(\sigma^s, \gamma^c) = \int_{\Omega(\sigma^s)} q(\omega, \sigma^s) f(\omega, \gamma^c) d\omega \quad (2)$$

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<sup>6</sup>Whereas countries and industries may have multiple characteristics, we restrict the source of worker heterogeneity to be one-dimensional. This does not mean that workers cannot have multiple characteristics, but that—as far as sorting is concerned—their characteristics can be summarized by a one-dimensional attribute. This will always be the case in the applications discussed in Sections 6 and 7.

<sup>7</sup>Our results would not be affected by the introduction of Hicks-neutral technological differences across countries. In our theoretical framework, a Hicks-neutral technological change is equivalent to a change in the total mass of workers.



In the reminder of this paper, we say that:

**Definition 3** *An economy is heterogeneous if, for any  $c = 1, \dots, C$  and  $s = 1, \dots, S$ , there exists  $f(\cdot, \gamma^c)$ ,  $q(\cdot, \sigma^s)$ , and  $r(\cdot, \sigma^s)$  such that  $Q(\sigma^s, \gamma^c)$  is simultaneously determined by Equations (1) and (2).*

A few comments are in order. First, our general framework is admittedly reduced form. The main focus of our analysis is to derive general predictions on the cross-sectional variation of aggregate output *whenever* Equations (1) and (2) are satisfied. It is not to explain *why* these equations are satisfied. The models presented in Sections 6 and 7—GROSSMAN and MAGGI [2000], OHNSORGE and TREFLER [2004], HELPMAN et al. [2004] and ANTRAS and HELPMAN [2004]—offer these microfoundations. In particular applications,  $f$  may derive from the prior sorting of workers across other unspecified sectors;  $q$  may reflect the optimal demand for other factors of productions; and, of course,  $r$  is related to  $q$  through a particular price mechanism.

Second, it should be clear that a heterogeneous economy in the sense of Definition 3 is equivalent to an economy with a continuum of perfectly substitutable factors whose marginal productivity may vary across sectors. The key features of this economy, however, are that: (i) the sorting rule is the same in all countries,  $\Omega(\sigma^s)$  is independent of the characteristics of the distribution of assets; and (ii) the mass of factors whose returns are equalized across sectors is equal to zero. This typically is not the case in a standard environment with only a discrete number of factors of production. Features (i) and (ii) create a tight connection between the cross-sectional variation of  $f$  and the cross-sectional variation of  $Q$  which we exploit in Section 4.

## 3.2 Restrictions

In order to make predictions, we need assumptions. The first restriction that we impose is that:

**R1**  *$f$  is log-supermodular in  $(\omega, \gamma_i)$  for all  $i = 1, \dots, n$ .*

Broadly speaking, R1 states that high- $\gamma$  countries tend to have relatively more workers with large endowments. According to R1, if two countries can be ranked in terms of their characteristics  $\gamma$ , then their distribution of assets can be ranked in terms of monotone likelihood ratio dominance. MILGROM [1981] offers many examples of density functions satisfying R1, including the normal (with mean  $\gamma$ ) and the uniform (on  $[0, \gamma]$ ).

The second restriction imposed on our economy is that:

**R2**  $r$  satisfies the weak single crossing property in  $(\omega, \sigma)$ .

Restriction  $R2$  states that if a worker with a small endowment earns more in a high- $\sigma$  sector than a low- $\sigma$  sector, then a worker with a larger endowment cannot earn less in the high- $\sigma$  sector. This captures the idea that there exists a ranking of sector characteristics  $\sigma$  such that the high- $\omega$  workers are relatively better at producing in the high- $\sigma$  sectors.

Throughout this paper, we say that:

**Definition 4** A heterogeneous economy is regular if  $f$  and  $r$  satisfy Restrictions  $R1$  and  $R2$ .

It is worth emphasizing that we do not impose any restriction on the complementarity between pairs of country characteristics or pairs of sector characteristics. Restrictions  $R1$  and  $R2$  only are concerned with the interaction between  $\omega$  and  $\gamma$ , on the one hand, and  $\omega$  and  $\sigma$ , on the other hand. This reflects the fact that our ultimate objective is to explain trade flows, i.e. why countries with characteristics  $\gamma$  tend to specialize in sectors with characteristics  $\sigma$ , not why countries and sectors have certain characteristics.

Similarly, we do not impose any restriction on  $q$ . Of course, this does not mean that the shape of the production function is irrelevant for our theory. Rather this means that  $q$  only matters *indirectly* for the cross-sectional variation of aggregate output, through its impact on the revenue function  $r$ .<sup>8</sup> In the next subsection, we come back to this relationship and provide an example of primitive assumptions on  $q$  and the market structure such that  $r$  satisfies  $R2$ .

Finally, we have chosen to state our restrictions on  $f$  and  $r$  for *all*  $\omega \in \mathbb{R}$ ,  $\gamma \in \mathbb{R}^n$ , and  $\sigma \in \mathbb{R}^m$ . This is mainly for expositional purposes. Instead, we could require  $R1$  and  $R2$  to hold for all  $\omega$ ,  $\gamma$  and  $\sigma$  in an arbitrary sublattice. Our predictions would be the same, though limited to characteristics in that particular sublattice.<sup>9</sup>

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<sup>8</sup>Since  $q$  *directly* affects the levels of aggregate output in each country and sector, this is somewhat surprising.

<sup>9</sup>This observation is particularly relevant if one is interested in changes in the second moment of the distributions of assets. A distribution with a higher (resp. lower) variance may only dominate—in terms of monotone likelihood ratio—a distribution with a lower (resp. higher) variance for the highest (resp. lowest) values of  $\omega$ .

### 3.3 An Example of Primitive Assumptions

Treating the revenue function  $r$  as a “primitive” of our theoretical framework has one important benefit. It allows us to describe in a transparent manner the selection of heterogeneous workers across sectors without having to commit to a particular market structure. The obvious cost of this approach is that  $r$  is an endogenous function which depends, among other things, on the equilibrium prices. Hence, one may wonder what the “true” primitive assumptions behind  $R2$  are, or worse, whether such assumptions even exist. We now offer a short answer to these questions.

To do so, we need to specify a market structure. Suppose that in each country and sector, there is a large number of firms taking the world price of good  $s$ ,  $p(\sigma^s) \geq 0$ , as given. Because of constant returns to scale at the sectoral level,<sup>10</sup> the revenue of a worker with endowment  $\omega$  employed in sector  $s$  is then given by:

$$r(\omega, \sigma^s) = p(\sigma^s)q(\omega, \sigma^s) \quad (3)$$

As just argued, the main difficulty associated with finding “true” primitive assumptions behind  $R2$  is that  $p(\sigma^s)$  is an endogenous variable which may vary in complex ways with demand conditions, technologies, and the world distribution of assets. A simple way to circumvent that problem is to find a necessary and sufficient condition for  $R2$  to hold for *all* price schedules. Theorem 1 offers such a condition.

**Theorem 1** *Suppose that Equation (3) holds. Then  $r$  satisfies  $R2$  for all  $p : \mathbb{R}^m \rightarrow \mathbb{R}^+$  if and only if  $q$  is log-supermodular in  $(\omega, \sigma_i)$  for all  $i = 1, \dots, m$ .*

## 4 Testable Implications

Restrictions  $R1$  and  $R2$  have testable implications for the cross-sectional variation of aggregate output in a heterogeneous economy. The central result of our paper can be stated as follows.

**Theorem 2** *In a regular heterogeneous economy, for any  $\sigma \geq \sigma'$  and  $\gamma \geq \gamma'$ , aggregate output  $Q$  satisfies*

$$Q(\sigma, \gamma)Q(\sigma', \gamma') \geq Q(\sigma, \gamma')Q(\sigma', \gamma)$$

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<sup>10</sup>According to Equation (2), the output of good  $s$  doubles when the mass of workers in sector  $s$  doubles.

To understand the logic of Theorem 2, start from a situation where sectors and countries only differ in one characteristic,  $\sigma_i$  and  $\gamma_j$ , respectively. If  $r$  satisfies *R2*, then high- $\omega$  workers sort into high- $\sigma_i$  sectors. If, in addition,  $f$  satisfies *R1*, then a high value of  $\gamma_j$  raises the likelihood of high values of  $\omega$  relative to low values of  $\omega$ , which raises the likelihood that a given worker produces the high- $\sigma_i$  good, and in turn, raises the relative output of this sector. This, in a nutshell, explains why  $Q$  is log-supermodular in  $(\sigma_i, \gamma_j)$ . We conclude our proof by noting that if  $\boldsymbol{\sigma} \geq \boldsymbol{\sigma}'$  and  $\boldsymbol{\gamma} \geq \boldsymbol{\gamma}'$ , the complementarities between pairs of country and sector characteristics reinforce each other.

Our theoretical analysis is closely related to ATHEY [2002] who also emphasizes log-supermodularity and the single crossing property. Our focus on the weak single crossing property, however, is very different. Here, *R2* plays the same role as supermodularity in matching models (see e.g. LEGROS and NEWMAN [1997]): it guarantees the monotonic sorting of workers across sectors. We then use this property to show that  $h(\omega, \boldsymbol{\sigma}) \equiv \mathbb{I}_{\Omega(\boldsymbol{\sigma})}(\omega) \cdot q(\omega, \boldsymbol{\sigma})$  is log-supermodular, which allows us to invoke Lemmas 1 and 2, and in turn, to establish the log-supermodularity of  $Q$ . By contrast, ATHEY [2002] assumes that the integrand— $h(\omega, \boldsymbol{\sigma})$  with the present notations—satisfies the Milgrom and Shannon single crossing property and demonstrates that under this assumption—whether or not log-supermodularity is satisfied—one can do monotone comparative statics under uncertainty. Of course, being able to do monotone comparative statics does not necessarily mean that the integral— $Q(\boldsymbol{\sigma}, \boldsymbol{\gamma})$  with the present notations—is log-supermodular, which is the property we are interested in.

Although our emphasis on log-supermodularity may sound abstract, this property can be found, albeit implicitly, in any theory of comparative advantage. Consider, for example, the Ricardian model with 2 sectors,  $s = 1, 2$ , and 2 countries,  $c = 1, 2$ . This model predicts that

$$\frac{a^{11}}{a^{12}} > \frac{a^{21}}{a^{22}} \Rightarrow \frac{Q^{11}}{Q^{12}} > \frac{Q^{21}}{Q^{22}}$$

where  $a^{sc}$  is the labor productivity of country  $c$  in sector  $s$  and  $Q^{sc}$  is its aggregate output.<sup>11</sup> If country 1 is relatively better than country 2 at producing good 1, then it should produce relatively more of that good. This implication can be rearranged as

$$a^{11}a^{22} > a^{12}a^{21} \Rightarrow Q^{11}Q^{22} > Q^{12}Q^{21}$$

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<sup>11</sup>The Ricardian model also predicts something stronger, namely the complete specialization of 1 of the 2 countries:  $\frac{Q^{11}}{Q^{12}} = +\infty$  and/or  $\frac{Q^{21}}{Q^{22}} = 0$ .

Formally, the log-supermodularity of labor productivity in  $s$  and  $c$  implies the log-supermodularity of aggregate output.<sup>12</sup> This is very much in the spirit of Theorem 2, which predicts that, under vertical sorting, the log-supermodularity of  $f$  implies the log-supermodularity of  $Q$ .

Theorem 2 allows us to make clear predictions on the pattern of international specialization in regular heterogeneous economies. To see this, consider a pair of countries,  $c_1$  and  $c_2$ , producing a pair of goods,  $s_1$  and  $s_2$ , with  $\gamma^{c_1} \geq \gamma^{c_2}$  and  $\sigma^{s_1} \geq \sigma^{s_2}$ . Theorem 2 implies  $Q^{s_1 c_1} / Q^{s_1 c_2} \geq Q^{s_2 c_1} / Q^{s_2 c_2}$ , where  $Q^{sc} \equiv Q(\sigma^s, \gamma^c)$ . Still considering the pair of countries,  $c_1$  and  $c_2$ , and applying Theorem 2 to an arbitrary subset of  $K$  goods, we obtain the following Corollary.

**Corollary 1** *Consider a regular heterogeneous economy where 2 countries produce  $K$  goods, with  $\gamma^{c_1} \geq \gamma^{c_2}$  and  $\sigma^{s_1} \geq \dots \geq \sigma^{s_K}$ . Then the high- $\gamma$  country has a comparative advantage in the high- $\sigma$  sectors:*

$$\frac{Q^{s_1 c_1}}{Q^{s_1 c_2}} \geq \dots \geq \frac{Q^{s_K c_1}}{Q^{s_K c_2}}$$

Under factor price equalization, the set of workers who sort into a given sector is the same everywhere; see Equation (1). As a result, cross-country differences in the distribution of factor endowments are mechanically reflected in their patterns of specialization. With identical technologies around the world, a country shall produce relatively more—compared to other countries—in sectors in which a relatively higher share of its population selects; see Equation (2). Corollary 1 operationalizes that idea by showing that Restrictions  $R1$  and  $R2$  are sufficient to characterize the “sectors in which a relatively higher share of [a country’s] population selects”, and in turn, the pattern of comparative advantage.

Note that Theorem 2 holds for any production function  $q$ . In a heterogeneous economy, the log-supermodularity of  $Q$  only derives from restrictions on  $f$  and  $r$ . As a result, if the assumptions of Theorem 2 hold, then aggregate employment,  $L(\sigma, \gamma) \equiv \int_{\Omega(\sigma)} f(\omega, \gamma) d\omega$ , and aggregate revenues,  $R(\sigma, \gamma) \equiv \int_{\Omega(\sigma)} r(\omega, \sigma) f(\omega, \gamma) d\omega$  also are log-supermodular.

**Corollary 2** *Consider a regular heterogeneous economy where 2 countries produce  $K$  goods, with  $\gamma^{c_1} \geq \gamma^{c_2}$  and  $\sigma^{s_1} \geq \dots \geq \sigma^{s_K}$ . Then*

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<sup>12</sup>At a formal level, one can view the recent literature on institutions and trade—see MATSUYAMA [2007] for an overview—as an attempt to provide microfoundations for the log-supermodularity of labor productivity with respect to particular country and industry characteristics.

aggregate employment and aggregate revenues follow the same pattern of comparative advantage as aggregate output:

$$\frac{L^{s_1c_1}}{L^{s_1c_2}} \geq \dots \geq \frac{L^{s_Kc_1}}{L^{s_Kc_2}} \text{ and } \frac{R^{s_1c_1}}{R^{s_1c_2}} \geq \dots \geq \frac{R^{s_Kc_1}}{R^{s_Kc_2}}$$

Corollary 2 is attractive from an empirical standpoint. In order to test our predictions, one is free to use aggregate data on either output, employment, or revenues. Moreover, these predictions all are ordinal in nature. This means that one does not need to observe the “true” country and sector characteristics to confront them with the data, any monotonic transformation of  $\gamma$  and  $\sigma$  will do.

## 5 Alternative Restrictions?

In Section 4, we have shown that  $R1$  and  $R2$  are sufficient conditions to make predictions on the cross-sectional variation of aggregate output. This raises one obvious question: Are there weaker properties on  $f$  and  $r$  that may also impose restrictions on the pattern of international specialization? The short answer is that  $R1$  and  $R2$  are necessary if one wants to make predictions in *all* heterogeneous economies.

### 5.1 Minimal Sufficient Conditions

To address the previous question formally, we follow the strategy of ATHEY [2002] and say that:

**Definition 5**  $H_1$  and  $H_2$  are a minimal pair of sufficient conditions for a given conclusion  $C$  if: (i)  $C$  holds whenever  $H_2$  does, if and only if  $H_1$  holds; and (ii)  $C$  holds whenever  $H_1$  does, if and only if  $H_2$  holds.

Definition 5 states that if  $H_1$  and  $H_2$  are a minimal pair of sufficient conditions, then one cannot weaken either  $H_1$  or  $H_2$  without imposing further assumptions on the model. Note that this does not mean that a given conclusion  $C$  holds if and only if  $H_1$  and  $H_2$  are satisfied. It simply means that, without one or the other, the conclusion  $C$  may not hold in all environments. In the next Theorem, we show that log-supermodularity and the single crossing property are a minimal pair of sufficient conditions to predict the cross-sectional variation of aggregate output in all heterogeneous economies.

**Theorem 3** *In a heterogeneous economy,  $R1$  and  $R2$  are a minimal pair of sufficient conditions for the following conclusion to hold:*

$$Q(\sigma, \gamma)Q(\sigma', \gamma') \geq Q(\sigma, \gamma')Q(\sigma', \gamma) \text{ for any } \sigma \geq \sigma' \text{ and } \gamma \geq \gamma' \quad (4)$$

From Theorem 2, we already know that  $R1$  and  $R2$  are a pair of sufficient conditions. In order to establish that this pair is minimal, we need to find heterogeneous economies in which  $R2$  (resp.  $R1$ ) and Conclusion (4) imply  $R1$  (resp.  $R2$ ). First, we consider an economy with “sector-specific” workers, i.e. an economy where high- $\omega$  workers can only produce in one high- $\sigma$  sector. In this environment, if Conclusion (4) holds for  $Q$ , then it must hold  $f$ , and so,  $R1$  must be satisfied. Second, we consider a 2-sector economy with “country-specific” workers, i.e. an economy where high- $\omega$  workers can only be found in one high- $\gamma$  country. Using the same logic, we show that if Conclusion (4) holds for  $Q$ , then  $r$  must satisfy  $R2$ .

Admittedly, asking Conclusion (4) to hold *whenever*  $R1$  or  $R2$  does is a strong requirement. The fact that  $R1$  and  $R2$  are a minimal pair of sufficient conditions does not preclude, in principle, the existence of interesting economies in which both  $R2$  and Conclusion (4) hold, and yet  $R1$  does not. The next subsection describes such an economy.

## 5.2 A Simple Heterogeneous Economy

Consider a heterogeneous economy with 2 countries with characteristics  $\gamma^1 > \gamma^2$ , and 2 sectors with characteristics  $\sigma^1 > \sigma^2$ . The first restriction imposed on this economy is that:

**R1'** For all  $\omega \in \mathbb{R}$ ,  $f$  satisfies

$$\int_{-\infty}^{\omega} f(\omega', \gamma^1) d\omega' / \int_{-\infty}^{+\infty} f(\omega', \gamma^1) d\omega' \leq \int_{-\infty}^{\omega} f(\omega', \gamma^2) d\omega' / \int_{-\infty}^{+\infty} f(\omega', \gamma^2) d\omega'$$

Restriction  $R1'$  states that for any  $\omega$ , the proportion of individuals with endowments lower than  $\omega$  is smaller in country 1 than in country 2. This has the same flavor as  $R1$ : there are more high- $\omega$  workers in the high- $\gamma$  countries. However,  $R1'$  is a *weaker* restriction than  $R1$ . Formally,  $R1'$  requires that the distribution of assets in country 1 first-order stochastically dominates the distribution of assets in country 2, whereas  $R1$  requires that the distribution of assets in country 1 dominates that in country 2 in terms of monotone likelihood ratio dominance.<sup>13</sup>

The second restriction imposed on this economy is that:

**R2'** There exists  $\omega_0 \in \mathbb{R}$  such that  $r(\omega, \sigma^1) < r(\omega, \sigma^2)$  for  $\omega < \omega_0$  and  $r(\omega, \sigma^1) > r(\omega, \sigma^2)$  for  $\omega > \omega_0$ .

$R2'$  is a slightly stronger version of the single crossing property introduced in Section 3. Like  $R2$ , it guarantees that if a worker with a small

<sup>13</sup>KRISHNA [2002] offers a review of the different concepts of stochastic dominance.

endowment prefers the high- $\sigma$  sector to the low- $\sigma$  sector, then a worker with a larger endowment must prefer the high- $\sigma$  sector as well. In addition,  $R2'$  requires that both sectors be non-empty. In other words, this economy is a non-trivial 2-sector economy.

Since the restriction imposed on  $f$  has been weakened, we know from Theorem 3 that further assumptions are needed to characterize the pattern of international specialization. One possible candidate is:

**R3'**  $q(\cdot, \sigma^2)$  is weakly decreasing for almost all  $\omega < \omega_0$  and  $q(\cdot, \sigma^1)$  is weakly increasing for almost all  $\omega > \omega_0$ .

$R3'$  captures in a stark manner the idea that large endowments of the indivisible assets are more useful in the high- $\sigma$  sector. According to  $R3'$ , the indivisible asset increases workers' productivity in sector 1, and decreases it in sector 2. This includes the particular case where  $q(\omega, \sigma^2) \equiv q$  for all  $\omega$ , that is the indivisible asset only matters in sector 1.<sup>14</sup> Theorem 4 shows that  $R3'$  is necessary and sufficient to make predictions on the cross-sectional variation of aggregate output in all economies where  $R1'$  and  $R2'$  hold.

**Theorem 4** *In a 2-country 2-sector heterogeneous economy where  $\gamma^1 > \gamma^2$  and  $\sigma^1 > \sigma^2$ ,  $Q$  satisfies  $Q(\sigma^1, \gamma^1)Q(\sigma^2, \gamma^2) \geq Q(\sigma^2, \gamma^1)Q(\sigma^1, \gamma^2)$  whenever  $R1'$  and  $R2'$  hold, if and only if  $R3'$  holds.*

The above inequality can be rearranged as  $Q(\sigma^1, \gamma^1)/Q(\sigma^1, \gamma^2) \geq Q(\sigma^2, \gamma^1)/Q(\sigma^2, \gamma^2)$ . Thus, the country with “more” indivisible and non-tradable assets shall specialize in the good whose production relies “more” on that asset. This is reminiscent of Corollary 1. However, Theorem 4 shows that the cost of relaxing the log-supermodularity of  $f$  is quite high. In order to use  $R1'$  instead of  $R1$ —formally, in order to use first-order rather than monotone likelihood ratio dominance—we have restricted ourselves to a 2-sector economy and added one strong assumption: larger endowments of the indivisible asset cannot increase workers' productivity in the low- $\sigma$  sector. To us, this demonstrates that the log-supermodularity of  $f$  is the natural assumption to predict the pattern of international specialization in heterogeneous economies.

## 6 Application: Human Capital Heterogeneity and the Pattern of International Specialization

The mechanisms analyzed in Sections 4 and 5 play a central role in recent works on human capital heterogeneity and trade. In GROSSMAN

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<sup>14</sup>LUCAS [1978] is one well-known example of sorting models using that assumption.



and MAGGI [2000] and OHNSORGE and TREFLER [2004]—like in our general framework—the sorting of workers across industries only depends on factor prices and these prices are identical across countries under free trade. As a result, any change in the distribution of human capital is mechanically reflected in the composition of output across industries.<sup>15</sup>

## 6.1 Example 1: Ohnsorge and Trefler [2004]

Consider multiple countries populated by workers with two skills  $S$  and  $L$ . Skills are drawn from a bivariate normal:

$$\begin{bmatrix} S \\ L \end{bmatrix} \sim N \left( \begin{bmatrix} \mu \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2 & \rho\lambda\sigma^2 \\ \rho\lambda\sigma^2 & \lambda^2\sigma^2 \end{bmatrix} \right) \quad (5)$$

where  $\mu$ ,  $\sigma$ ,  $\rho$  and  $\lambda$  are country-specific parameters. Workers are free to sort across a discrete number of industries.<sup>16</sup> The aggregate output of industry  $i$  is given by

$$Y_i = \int_{\Omega_S} \int_{\Omega_L} A_i \exp(L + \beta_i S) f_{SL}(S, L) dS dL \quad (6)$$

where  $\Omega_S \times \Omega_L$  is the set of workers choosing to work in industry  $i$ ;  $A_i \exp(L + \beta_i S)$  is the output of each of these workers; and  $f_{SL}(L, S)$  is the number of  $(L, S)$  types in the economy. Under perfect competition and constant returns to scale, their wages are given by  $q_i A_i \exp(L + \beta_i S)$ , where  $q_i$  is the producer price in industry  $i$ .

The previous environment is a heterogeneous economy in the sense of Definition 3. To see this, let  $\omega \equiv S$ ,  $\boldsymbol{\sigma} \equiv (\beta, A, q)$ , and  $\boldsymbol{\gamma} \equiv (\mu, \sigma, \rho, \lambda)$ . Define  $f(\omega, \boldsymbol{\gamma}) \equiv \int_{-\infty}^{+\infty} \exp(L) f_{SL}(\omega, L) dL$ . By Equation (5), we have

$$f(\omega, \boldsymbol{\gamma}) = \frac{1}{2\pi\sigma} \exp \left[ \rho\lambda(\omega - \mu) + \frac{(1 - \rho^2)(\lambda\sigma)^2}{2} - \left( \frac{\omega - \mu}{\sigma} \right)^2 \right]$$

Similarly, let  $q(\omega, \boldsymbol{\sigma}) \equiv A \exp(\beta\omega)$  and  $r(\omega, \boldsymbol{\sigma}) \equiv qA \exp(\beta\omega)$ . Using Equation (6) and noting that  $\Omega_L = \mathbb{R}$ , we can express aggregate output in country  $c$  and sector  $s$  as

$$Q(\boldsymbol{\sigma}^s, \boldsymbol{\gamma}^c) = \int_{\Omega(\boldsymbol{\sigma}^s)} q(\omega, \boldsymbol{\sigma}^s) f(\omega, \boldsymbol{\gamma}^c) d\omega$$

<sup>15</sup>GROSSMAN [2004] follows the opposite route. He considers a model where changes in the distribution of talent *do* affect sorting rules across countries, but restricts himself to uniform distributions. In this case, changes in the distribution of human capital do not affect the relative mass of workers across industries. In turn, the pattern of international specialization solely derive from changes in the sorting rule.

<sup>16</sup>OHNSORGE and TREFLER [2007] focus on the continuum case.

where

$$\Omega(\boldsymbol{\sigma}^s) = \left\{ \omega \in \mathbb{R} \mid r(\omega, \boldsymbol{\sigma}^s) > \max_{s' \neq s} r(\omega, \boldsymbol{\sigma}^{s'}) \right\}$$

Having established that OHNSORGE and TREFLER [2004] is a heterogeneous economy, it is a simple matter of algebra to check that: (i)  $\frac{\partial^2 \ln f}{\partial \omega \partial \mu} \geq 0$ ,  $\frac{\partial^2 \ln f}{\partial \omega \partial \sigma} \geq (\leq) 0$  for  $\omega \geq (\leq) \mu$ ,  $\frac{\partial^2 \ln f}{\partial \omega \partial \rho} \geq 0$ , and  $\frac{\partial^2 \ln f}{\partial \omega \partial \lambda} \geq 0$ ; and (ii)  $\frac{\partial^2 \ln r}{\partial \omega \partial \beta} \geq 0$ ,  $\frac{\partial^2 \ln r}{\partial \omega \partial A} = 0$ ,  $\frac{\partial^2 \ln r}{\partial \omega \partial q} = 0$ . Corollary 1 then explains why:

1. Countries with a higher  $\mu$  produce relatively more in sectors with higher returns to skill  $\beta$  (Theorem 5 p. 27);
2. Countries with a higher  $\sigma$  produce relatively more in sectors with the highest and lowest returns to skill  $\beta$  (Theorem 3 p. 19);<sup>17</sup>
3. Countries with a higher  $\rho$  produce relatively more in sectors with higher returns to skill  $\beta$  (Theorem 1 p. 15)

Note that our general approach can also generate new predictions in this environment. Since  $f$  is log-supermodular in  $\omega$  and  $\lambda$ , Corollary 1 implies that countries with a higher  $\lambda$  shall produce relatively more in sectors with higher returns to skill  $\beta$ . Empirically, this means that countries with more intra-industry wage dispersion shall have a comparative advantage in sectors associated with higher average wages.

## 6.2 Example 2: Grossman and Maggi [2000]

Consider 2 countries, Home and Foreign, populated by a mass  $L$  and  $L^*$  of workers, respectively. The proportion of workers with talent below  $t$  in the two countries are given by  $\Phi(t)$  and  $\Phi^*(t)$ . The associated density functions,  $\phi \equiv d\Phi/dt$  and  $\phi^* \equiv d\Phi^*/dt$ , are symmetric about their means,  $\bar{t}$  and  $\bar{t}^*$ . There are 2 sectors indexed by  $i = c, s$ . Production in each sector requires exactly 2 workers, each performing a different task. The aggregate output of industry  $i$  is given by

$$Y_i = L \int_{\Omega_i} F^i(t, m^i(t)) \phi(t) dt \quad (7)$$

where  $\Omega_i$  is the set of workers with talent  $t$  sorting in sector  $i$ ;  $m^i(t)$  is the talent of workers paired with workers of talent  $t$  in sector  $i$ ; and

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<sup>17</sup>In this case, one needs to consider separately the workers who sort into the sectors with high returns to skill ( $\omega \geq \mu$ ), from those who sort into the sectors with low returns to skill ( $\omega < \mu$ ).  $f$  is log-supermodular in  $(\omega, \sigma)$  for the first group of workers, and log-supermodular in  $(\omega, -\sigma)$  for the second one.

$F^i(t_A, t_B)$  is the output of a pair workers with talent  $t_A$  and  $t_B$ . Production functions are symmetric and homogeneous of degree one; supermodular in sector  $C$ ,  $F_{12}^c > 0$ ; and submodular in sector  $S$ ,  $F_{12}^s < 0$ . This implies self-matching in sector  $C$ ,  $m^c(t) = t$ ; and maximal cross-matching in sector  $S$ ,  $m^s(t) = 2\bar{t} - t$ . Under perfect competition and constant returns to scale, total wages in a firm with a worker of talent  $t$  in sector  $i$  are equal to  $p^i F^i(t, m^i(t))$ , where  $p^i$  is the price of good  $i$ .

If  $\bar{t} = \bar{t}^*$ , the previous environment also is a heterogeneous economy in the sense of Definition 3. To see this, let  $\omega \equiv |t - \bar{t}|$ . Then, take  $\gamma^1 > \gamma^2$  and define  $f$  such that  $f(\omega, \gamma^1) \equiv 2L\phi(\bar{t} + \omega)$  and  $f(\omega, \gamma^2) \equiv 2L^*\phi^*(\bar{t} + \omega)$ . Now, take  $\sigma^1 > \sigma^2$  and let  $q(\omega, \sigma^1) \equiv F^s(\bar{t} - \omega, \bar{t} + \omega)$  and  $q(\omega, \sigma^2) \equiv 1/2 [F^c(\bar{t} - \omega, \bar{t} - \omega) + F^c(\bar{t} + \omega, \bar{t} + \omega)]$ . Similarly, let  $r(\omega, \sigma^1) \equiv p^s q(\omega, \sigma^1)$  and  $r(\omega, \sigma^2) \equiv p^c q(\omega, \sigma^2)$ . By Equation (7), aggregate output is then given by

$$Q(\sigma^s, \gamma^c) = \int_{\Omega(\sigma^s)} q(\omega, \sigma^s) f(\omega, \gamma^c) d\omega$$

with

$$\Omega(\sigma^s) = \left\{ \omega \in \mathbb{R}^+ \mid r(\omega, \sigma^s) > \max_{\sigma \neq \sigma^s} r(\omega, \sigma) \right\}$$

for all  $c = 1, 2$  and  $s = 1, 2$ . It is easy to check that if  $\Phi$  is a mean preserving spread of  $\Phi^*$ , then  $f$  satisfies  $R1'$ . One can also check that  $F^c$  homogeneous of degree 1 implies  $q(\omega, \sigma^2) = F^c(\bar{t}, \bar{t})$  for all  $\omega$ ; and that  $F^s$  homogeneous of degree 1 and  $F_{12}^s < 0$  imply  $q(\omega, \sigma^1)$  increasing in  $\omega$ . Thus,  $r$  and  $q$  satisfy  $R2'$  and  $R3'$  as well. Theorem 4 then helps explain one of the main predictions of GROSSMAN and MAGGI [2000]: if  $\bar{t} = \bar{t}^*$  and  $\Phi$  is a mean preserving spread of  $\Phi^*$ , then Home produces relatively more in sector  $S$  (Proposition 4 p. 1265).<sup>18</sup>

## 7 Application: Firm Heterogeneity and the Organization of Production in a Global Economy

The logic of Theorem 2 can be found in various trade models with firm-level heterogeneity à la MELITZ [2003].<sup>19</sup> To see this, let us reinterpret the “workers”, “countries”, and “sectors” of Section 2 as “firms”,

<sup>18</sup>Our analysis also shows that in order to extend the predictions of GROSSMAN and MAGGI [2000] to an economy with more than 2 sectors—which is necessary to confront them with the data—one would need to strengthen the concept of stochastic dominance to monotone likelihood ratio dominance.

<sup>19</sup>ANTRAS and HELPMAN [2004] also recognize the existence of a connection between the mechanism at work in their model and MELITZ [2003] and HELPMAN et al. [2004]; see footnote 10 p. 571. However, they do not discuss the critical assumptions on which this logic depends. This is one of the main insights of our paper.

“sectors”, and “organizations”, respectively. In this context, firms are assumed to be immobile across sectors, but free to choose their organizations. The indivisible and non-tradable asset  $\omega$  captures differences in technological know-how: firms with larger endowments are more productive, independently of the organizational form they select. Under this new interpretation, we refer to  $Q(\boldsymbol{\sigma}, \boldsymbol{\gamma})$  as the total sales by firms with a “ $\boldsymbol{\sigma}$ -organization” in a “ $\boldsymbol{\gamma}$ -sector”.<sup>20</sup>

### 7.1 Example 1: Helpman, Melitz, Yeaple [2004]

Consider 2 symmetric countries  $i$  and  $j$  and multiple sectors, each with a continuum of firms producing a distinct variety. Firms differ in productivity levels,  $1/a$ , which are drawn from a Pareto with shape  $k$

$$G(1/a) = 1 - (ab)^k$$

for  $1/a \geq b > 0$ . Both  $b$  and  $k$  are sector-specific. On the basis of productivity differences, firms in country  $i$  choose whether to become domestic producers ( $D$ ) or to serve country  $j$  via exports ( $X$ ) or FDI ( $I$ ). With CES preferences, linear production functions, and unit wages in both countries, the foreign sales of a firm with organization  $O \in \{D, X, I\}$  are given by  $q_O = (a\tau_O)^{1-\varepsilon} B$ , where  $B$  measures the demand level in each country;  $\varepsilon > 1$  is the elasticity of substitution; and  $\tau_O$  is an organization-specific transport cost. By definition,

$$\tau_I^{1-\varepsilon} = 1 > \tau_X^{1-\varepsilon} > \tau_D^{1-\varepsilon} = 0$$

Multinational firms do not pay any transport costs, whereas domestic firms do not sell abroad. The total profits of a firm with organization  $O \in \{D, X, I\}$  are given by  $\pi_O = a^{1-\varepsilon} B (1 + \tau_O^{1-\varepsilon}) - f_O$ , where  $f_O$  is an organization-specific setup cost. By assumption,

$$f_I > f_X > f_D$$

Compared to exporters and domestic producers, firms engaged in FDI have to pay a higher fixed costs.

The previous environment also is a heterogeneous economy in the sense of Definition 3. Let  $\omega \equiv 1/a$ ,  $\boldsymbol{\sigma} \equiv (\tau^{1-\varepsilon}, f)$ ,  $\boldsymbol{\gamma} \equiv (b, -k)$ , and  $f$  be

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<sup>20</sup>This alternative interpretation of our theoretical framework also is related to recent work in corporate finance by CHAMPONNOIS [2006]. In his model—which satisfies the assumptions of Section 3—heterogeneous firms may choose 2 financing options: market or intermediary. In line with Theorem 2, he finds that industries with larger/more productive firms, in the sense of monotone likelihood ratio dominance, have a higher share of market financing, which is the financing option preferred by larger firms.

the density of a Pareto,  $f(\omega, \gamma) \equiv b^k \omega^{-(k+1)}$ . Similarly, let  $q(\omega, \sigma) \equiv \omega^{\varepsilon-1} \tau^{1-\varepsilon} B$  and  $r(\omega, \sigma) \equiv \omega^{\varepsilon-1} B(1 + \tau^{1-\varepsilon}) - f$ . Total foreign sales by a “ $\sigma$ -organization” in a “ $\gamma$ -sector” can then be expressed as

$$Q(\sigma, \gamma) = \int_{\Omega(\sigma)} q(\omega, \sigma) f(\omega, \gamma) d\omega$$

where

$$\Omega(\sigma) = \left\{ \omega \geq b \mid r(\omega, \sigma) > \max_{\sigma' \neq \sigma} r(\omega, \sigma') \right\}$$

We have: (i)  $\frac{\partial^2 \ln f}{\partial \omega \partial b} = 0$  and  $-\frac{\partial^2 \ln f}{\partial \omega \partial k} \geq 0$ ; and (ii)  $\frac{\partial^2 r}{\partial \omega \partial \tau^{1-\varepsilon}} \geq 0$  and  $\frac{\partial^2 r}{\partial \omega \partial f} = 0$ . Since  $\sigma_I > \sigma_X$ , Corollary 1 explains one major prediction of HELPMAN et al. [2004]: a decline in  $k$ , i.e. an increase in the dispersion of productivity across firms, increases the ratio of FDI versus export sales (p. 305).

## 7.2 Example 2: Antras and Helpman [2004]

Consider 2 countries, North and South, and multiple sectors, each with a continuum of firms producing a distinct variety. Firms differ in their productivity levels  $\theta$  and the distribution of productivity levels is a Pareto with shape  $z$

$$G(\theta) = 1 - \left( \frac{b}{\theta} \right)^z$$

for  $\theta \geq b > 0$ . As before, both  $b$  and  $z$  are sector-specific. Production of any variety requires 2 specific inputs: headquarter services, which can only be supplied by final good producers in the North, and manufactured components, which can be produced by intermediate suppliers in the North or in the South. On the basis of productivity differences (and sectoral characteristics), firms choose their ownership structure—vertical integration ( $V$ ) or outsourcing ( $O$ )—and the location of production—North ( $N$ ) or South ( $S$ )—in order to maximize their profits. Adopting the property rights approach to the theory of the firm, the authors show that, under CES preferences and Cobb-Douglas production functions, the profits associated with each organization  $(k, l) \in \{V, O\} \times \{N, S\}$  can be written as  $\pi_k^l = X^{(\mu-\alpha)/(1-\alpha)} \theta^{\alpha/(1-\alpha)} \psi_k^l - w^N f_k^l$ , where  $X$  is a consumption index;  $\mu < \alpha < 1$  are preference parameters; and  $w^N$  is the wage rate in the North. The parameters  $\psi_k^l$  and  $f_k^l$  capture the “variable” and “fixed” costs associated with each organization. In their benchmark case, variable and fixed organizational costs are ranked as follows:

$$\psi_V^S > \psi_0^S > \psi_V^N > \psi_0^N$$

and

$$f_V^S > f_0^S > f_V^N > f_0^N$$

Let  $\omega \equiv \theta$ ,  $\sigma \equiv (\psi, f)$ ,  $\gamma \equiv (b, -z)$ , and  $f$  be the density of a Pareto,  $f(\omega, \gamma) \equiv b^z \omega^{-(z+1)}$ . Similarly, let  $r(\omega, \sigma)$  be the firm's profits under the  $\sigma$ -organization,  $r(\omega, \sigma) \equiv X^{(\mu-\alpha)/(1-\alpha)} \omega^{\alpha/(1-\alpha)} \psi - w^N f$ . The prevalence of a given organizational form can then be expressed as

$$L(\sigma, \gamma) = \left[ \int_{\Omega(\sigma)} f(\omega, \gamma) d\omega \right] / \left[ \int_{\cup_{k,l} \Omega(\sigma_k^l)} f(\omega, \gamma) d\omega \right]$$

where

$$\Omega(\sigma) = \left\{ \omega \geq b \mid r(\omega, \sigma) > \max_{\sigma' \neq \sigma} r(\omega, \sigma') \right\}$$

Again, we have: (i)  $\frac{\partial^2 \ln f}{\partial \omega \partial b} = 0$  and  $-\frac{\partial^2 \ln f}{\partial \omega \partial z} \geq 0$ ; and (ii)  $\frac{\partial^2 r}{\partial \omega \partial \psi} \geq 0$  and  $\frac{\partial^2 r}{\partial \omega \partial f} = 0$ . Thus, Corollary 2 and  $\sigma_V^S > \sigma_0^S > \sigma_V^N > \sigma_0^N$  imply that  $\frac{L(\sigma_V^S, \gamma)}{L(\sigma_0^S, \gamma)} \geq \frac{L(\sigma_0^S, \gamma)}{L(\sigma_0^N, \gamma)} \geq \frac{L(\sigma_V^N, \gamma)}{L(\sigma_0^N, \gamma)} \geq \frac{L(\sigma_0^N, \gamma)}{L(\sigma_0^N, \gamma)}$  for any  $\gamma \geq \gamma'$ . The previous chain of inequalities and the fact that  $\sum_{k,l} L(\sigma_k^l, \gamma) = \sum_{k,l} L(\sigma_k^l, \gamma') = 1$  explain why an increase in the dispersion of productivity across firms, i.e. a decline in  $z$ :

1. reduces the fraction of firms that outsource in the North (p. 573);
2. increases the fraction of firms that insource in the South (p. 573);
3. increases the prevalence of offshoring (p. 573);
4. increases the prevalence of vertical integration (p. 573).

## 8 Concluding remarks

Aggregate production functions are an implicit starting point of many applied fields in economics, including international trade. While often useful, this approach presents one obvious shortcoming: it necessarily ignores any effect that the distribution of factor endowments across agents may have on international trade flows. This paper develops a general framework that can shed light on these effects.

Our analysis demonstrates that log-supermodularity and the single crossing property are critical sufficient conditions to make predictions in the pattern of internationalization and the organization of production in heterogeneous economies. In Section 6 and 7, we have shown that our general results are at the heart of many predictions in the previous

literature. As mentioned in the introduction, another attractive feature of our approach is that it potentially applies to any environment including a finite number of “populations” inhabited by a continuum of heterogeneous “agents” sorting across a finite number of “occupations”. We conclude by describing in more details such an environment.

Consider an economy à la BANERJEE and NEWMAN [1993] including a continuum of individuals with different wealth, multiple sectors with different financial requirements, and multiple countries with different wealth distributions. Now suppose that these distributions are Pareto<sup>21</sup> and that—because of liquidity constraints—wealthier individuals earn relatively more in sectors with higher financial requirements. In this environment, Corollary 1 directly implies that more unequal countries specialize in sectors with higher financial requirements; and that holding inequality constant, average wealth has no effect on the pattern of international specialization. Using data from the Luxembourg Wealth Study, COSTINOT [2007] offers evidence consistent with these two predictions.

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<sup>21</sup>Evidence that wealth distributions are indeed Pareto can be found in the (very) early work of PARETO [1897] and more recently in KLASS et al. [2006].

## 9 Appendix

**Proof of Theorem 1.** ( $\Leftarrow$ ) If  $q$  is log-supermodular in  $(\omega, \sigma_i)$  for all  $i = 1, \dots, m$ , then  $r(\omega, \boldsymbol{\sigma}) = p(\boldsymbol{\sigma})q(\omega, \boldsymbol{\sigma})$  is log-supermodular in  $(\omega, \sigma_i)$  for all  $i = 1, \dots, m$ , by Lemma 1. Hence,  $r$  satisfies *R2* by Lemma 3.

( $\Rightarrow$ ) Suppose that  $q$  is not log-supermodular in  $(\omega, \sigma_i)$  for all  $i = 1, \dots, m$ . Then, there exist  $\omega > \omega'$  and  $\boldsymbol{\sigma} > \boldsymbol{\sigma}'$  such that

$$q(\omega, \boldsymbol{\sigma}')q(\omega', \boldsymbol{\sigma}) > q(\omega, \boldsymbol{\sigma})q(\omega', \boldsymbol{\sigma}') \quad (8)$$

Since  $q \geq 0$ , Inequality (8) implies  $q(\omega, \boldsymbol{\sigma}') > 0$  and  $q(\omega', \boldsymbol{\sigma}) > 0$ .

**Case (i):**  $q(\omega', \boldsymbol{\sigma}') > 0$

By Inequality (8), there exist  $p(\boldsymbol{\sigma}) > 0$  and  $p(\boldsymbol{\sigma}') > 0$  such that

$$\frac{q(\omega', \boldsymbol{\sigma})}{q(\omega', \boldsymbol{\sigma}')} > \frac{p(\boldsymbol{\sigma}')}{p(\boldsymbol{\sigma})} > \frac{q(\omega, \boldsymbol{\sigma})}{q(\omega, \boldsymbol{\sigma}')}$$

Thus,  $r(\omega', \boldsymbol{\sigma}) > r(\omega', \boldsymbol{\sigma}')$  and  $r(\omega, \boldsymbol{\sigma}') > r(\omega, \boldsymbol{\sigma})$ , which contradicts *R2*.

**Case (ii):**  $q(\omega', \boldsymbol{\sigma}') = 0$

Take  $p(\boldsymbol{\sigma}) > 0$  and  $p(\boldsymbol{\sigma}') > 0$  such that

$$\frac{p(\boldsymbol{\sigma}')}{p(\boldsymbol{\sigma})} > \frac{q(\omega, \boldsymbol{\sigma})}{q(\omega, \boldsymbol{\sigma}')}$$

By construction,  $r(\omega, \boldsymbol{\sigma}') > r(\omega, \boldsymbol{\sigma})$ . Since  $q(\omega', \boldsymbol{\sigma}') = 0$ ,  $q(\omega', \boldsymbol{\sigma}) > 0$ , and  $p(\boldsymbol{\sigma}) > 0$ , we also have  $r(\omega', \boldsymbol{\sigma}) > r(\omega', \boldsymbol{\sigma}')$ . This contradicts *R2*. ■

**Proof of Theorem 2.** Theorem 2 is proved by 3 Lemmas.

**Lemma 4** Let  $h(\omega, \boldsymbol{\sigma}) \equiv \mathbb{I}_{\Omega(\boldsymbol{\sigma})}(\omega) \cdot q(\omega, \boldsymbol{\sigma})$ . Suppose that *R2* holds, then  $h$  is log-supermodular in  $(\omega, \sigma_i)$  for all  $i = 1, \dots, m$ .

**Proof.** We proceed by contradiction. Consider  $\omega \geq \omega'$  and  $\sigma_i \geq \sigma'_i$ . Suppose that  $h(\omega, \sigma'_i, \boldsymbol{\sigma}_{-i})h(\omega', \sigma_i, \boldsymbol{\sigma}_{-i}) > h(\omega, \sigma_i, \boldsymbol{\sigma}_{-i})h(\omega', \sigma'_i, \boldsymbol{\sigma}_{-i})$ . This implies  $\omega \in \Omega(\sigma'_i, \boldsymbol{\sigma}_{-i})$  and  $\omega' \in \Omega(\sigma_i, \boldsymbol{\sigma}_{-i})$  with  $\omega \neq \omega'$  and  $\sigma_i \neq \sigma'_i$ . Using Equation (1), we then get  $r(\omega, \sigma'_i, \boldsymbol{\sigma}_{-i}) > r(\omega, \sigma_i, \boldsymbol{\sigma}_{-i})$  and  $r(\omega', \sigma_i, \boldsymbol{\sigma}_{-i}) > r(\omega', \sigma'_i, \boldsymbol{\sigma}_{-i})$ . This contradicts *R2*. ■

**Lemma 5** Suppose that *R1* and *R2* hold, then  $Q$  is log-supermodular in  $(\sigma_i, \gamma_j)$  for all  $i = 1, \dots, m$  and  $j = 1, \dots, n$ .

**Proof.** By Equation (2), we have  $Q(\boldsymbol{\sigma}, \boldsymbol{\gamma}) = \int h(\omega, \boldsymbol{\sigma})f(\omega, \boldsymbol{\gamma})d\omega$ . We know from Lemma 4 that  $h$  is log-supermodular in  $(\omega, \sigma_i)$  for all  $i = 1, \dots, m$ . By Restriction *R1*, we also know that  $f$  is log-supermodular in  $(\omega, \gamma_j)$  for all  $j = 1, \dots, n$ . Lemma 5 derives from these 2 observations and the fact that log-supermodularity is preserved by multiplication and integration, by Lemmas 1 and 2. ■



**Lemma 6** *Suppose that  $Q$  is log-supermodular in  $(\sigma_i, \gamma_j)$  for all  $i = 1, \dots, m$  and  $j = 1, \dots, n$ , then the following implication holds*

$$\sigma \geq \sigma' \text{ and } \gamma \geq \gamma' \Rightarrow Q(\sigma, \gamma)Q(\sigma', \gamma') \geq Q(\sigma, \gamma')Q(\sigma', \gamma)$$

**Proof.** We proceed by iteration. If  $m = n = 1$ , then Lemma 6 is true by Definition 1. Now suppose that Lemma 6 is true for  $m_0 \geq m \geq 1$  and  $n_0 \geq n \geq 1$ . Consider  $\sigma \equiv (\sigma_i, \sigma_{-i}) \geq \sigma' \equiv (\sigma'_i, \sigma'_{-i})$  and  $\gamma \geq \gamma'$ , where  $\sigma_{-i}, \sigma'_{-i} \in \mathbb{R}^{m_0}$  and  $\gamma, \gamma' \in \mathbb{R}^{n_0}$ . If  $Q$  is log-supermodular in  $(\sigma_i, \gamma_j)$  for all  $i = 1, \dots, m_0 + 1$  and  $j = 1, \dots, n_0$ , then we have

$$\begin{aligned} Q(\sigma_i, \sigma_{-i}, \gamma)Q(\sigma_i, \sigma'_{-i}, \gamma') &\geq Q(\sigma_i, \sigma'_{-i}, \gamma)Q(\sigma_i, \sigma_{-i}, \gamma') \\ Q(\sigma'_i, \sigma_{-i}, \gamma)Q(\sigma'_i, \sigma'_{-i}, \gamma') &\geq Q(\sigma'_i, \sigma'_{-i}, \gamma)Q(\sigma'_i, \sigma_{-i}, \gamma') \\ Q(\sigma_i, \sigma_{-i}, \gamma)Q(\sigma'_i, \sigma_{-i}, \gamma') &\geq Q(\sigma'_i, \sigma_{-i}, \gamma)Q(\sigma_i, \sigma_{-i}, \gamma') \\ Q(\sigma_i, \sigma'_{-i}, \gamma)Q(\sigma'_i, \sigma'_{-i}, \gamma') &\geq Q(\sigma'_i, \sigma'_{-i}, \gamma)Q(\sigma_i, \sigma'_{-i}, \gamma') \end{aligned}$$

After multiplying the 4 previous inequalities and simplifying, we obtain

$$Q(\sigma, \gamma)Q(\sigma', \gamma') \geq Q(\sigma, \gamma')Q(\sigma', \gamma)$$

Thus, Lemma 6 is true for  $m \leq m_0 + 1$  and  $n \leq n_0$ . We can show that it is true for  $m \leq m_0 + 1$  and  $n \leq n_0 + 1$  in a similar manner. Iterating the previous argument implies that Lemma 6 is true for all  $m, n \geq 1$ . ■

**Proof of Theorem 3.** Theorem 2 shows that  $R1$  and  $R2$  are sufficient conditions. That they are a minimal pair is proved by 2 Lemmas.

**Lemma 7** *If  $\sigma \geq \sigma'$  and  $\gamma \geq \gamma' \Rightarrow Q(\sigma, \gamma)Q(\sigma', \gamma') \geq Q(\sigma, \gamma')Q(\sigma', \gamma)$  in any heterogeneous economy where  $R2$  holds, then  $R1$  holds.*

**Proof.** Consider  $\omega' > \omega''$ ,  $\sigma' > \sigma''$ , and  $\gamma' \geq \gamma''$ . Take  $h_1 : \mathbb{R}^m \rightarrow \mathbb{R}$  strictly increasing such that  $h_1(\sigma') = \omega'$  and  $h_1(\sigma'') = \omega''$ . Set  $q(\omega, \sigma) = r(\omega, \sigma) \equiv \mathbb{1}_{\{h_1(\sigma)\}}(\omega)$  for all  $\omega, \sigma$ . By construction,  $R2$  is satisfied. This implies

$$Q(\sigma', \gamma')Q(\sigma'', \gamma'') \geq Q(\sigma', \gamma'')Q(\sigma'', \gamma')$$

which simplifies into

$$f(\omega', \gamma')f(\omega'', \gamma'') \geq f(\omega', \gamma'')f(\omega'', \gamma')$$

Thus,  $R1$  holds. ■

**Lemma 8** *If  $\sigma \geq \sigma'$  and  $\gamma \geq \gamma' \Rightarrow Q(\sigma, \gamma)Q(\sigma', \gamma') \geq Q(\sigma, \gamma')Q(\sigma', \gamma)$  in any heterogeneous economy where  $R1$  holds, then  $R2$  holds.*

**Proof.** We proceed by contradiction in a 2-sector economy. Since the mass of workers whose maximum revenues are identical in 2 sectors is equal to zero, we must have  $r(\omega; \boldsymbol{\sigma}') > r(\omega; \boldsymbol{\sigma}'')$  or  $r(\omega; \boldsymbol{\sigma}') < r(\omega; \boldsymbol{\sigma}'')$  for any  $\omega$  and  $\boldsymbol{\sigma}' \neq \boldsymbol{\sigma}''$ . Now suppose that  $R2$  does not hold. Then there must be  $\omega' > \omega''$  and  $\boldsymbol{\sigma}' > \boldsymbol{\sigma}''$  such that  $r(\omega', \boldsymbol{\sigma}') < r(\omega', \boldsymbol{\sigma}'')$  and  $r(\omega'', \boldsymbol{\sigma}') > r(\omega'', \boldsymbol{\sigma}'')$ . This implies  $\omega' \in \Omega(\boldsymbol{\sigma}'')$  and  $\omega'' \in \Omega(\boldsymbol{\sigma}')$ , and in turn,

$$\mathbb{I}_{\Omega(\boldsymbol{\sigma}'')}(\omega') \mathbb{I}_{\Omega(\boldsymbol{\sigma}')}(\omega'') > \mathbb{I}_{\Omega(\boldsymbol{\sigma}')}(\omega') \mathbb{I}_{\Omega(\boldsymbol{\sigma}'')}(\omega'')$$

Take  $\gamma' > \gamma''$  and  $h_2 : \mathbb{R}^n \rightarrow \mathbb{R}$  strictly increasing such that  $h_2(\gamma') = \omega'$  and  $h_2(\gamma'') = \omega''$ . Set  $f(\omega, \gamma) = \mathbb{I}_{\{h_2(\gamma)\}}(\omega)$  for all  $\omega, \gamma$ . By construction,  $f$  is log-supermodular in  $(\omega, \gamma_i)$  for all  $i = 1, \dots, n$ . So,  $R1$  is satisfied, which implies

$$Q(\boldsymbol{\sigma}', \gamma') Q(\boldsymbol{\sigma}'', \gamma'') \geq Q(\boldsymbol{\sigma}', \gamma'') Q(\boldsymbol{\sigma}'', \gamma')$$

This can be rearranged as

$$\begin{aligned} & [q(\omega', \boldsymbol{\sigma}') q(\omega'', \boldsymbol{\sigma}'')] \cdot [\mathbb{I}_{\Omega(\boldsymbol{\sigma}')}(\omega') \mathbb{I}_{\Omega(\boldsymbol{\sigma}'')}(\omega'')] \\ & \geq [q(\omega', \boldsymbol{\sigma}'') q(\omega'', \boldsymbol{\sigma}')] \cdot [\mathbb{I}_{\Omega(\boldsymbol{\sigma}')}(\omega') \mathbb{I}_{\Omega(\boldsymbol{\sigma}'')}(\omega'')] \end{aligned}$$

By taking  $q$  such that  $q(\omega', \boldsymbol{\sigma}') q(\omega'', \boldsymbol{\sigma}'') < q(\omega', \boldsymbol{\sigma}'') q(\omega'', \boldsymbol{\sigma}')$ , we get

$$\mathbb{I}_{\Omega(\boldsymbol{\sigma}')}(\omega') \mathbb{I}_{\Omega(\boldsymbol{\sigma}'')}(\omega'') \geq \mathbb{I}_{\Omega(\boldsymbol{\sigma}'')}(\omega') \mathbb{I}_{\Omega(\boldsymbol{\sigma}')}(\omega'')$$

A contradiction. ■

**Proof of Theorem 4.** ( $\Leftarrow$ ) First, note that  $R2'$  implies  $Q(\boldsymbol{\sigma}^1, \gamma^2) = \int_{\omega_0}^{+\infty} q(\omega, \boldsymbol{\sigma}^1) f(\omega, \gamma^2) d\omega$  and  $Q(\boldsymbol{\sigma}^1, \gamma^1) = \int_{\omega_0}^{+\infty} q(\omega, \boldsymbol{\sigma}^1) f(\omega, \gamma^1) d\omega$ . Let  $h_1(\omega) \equiv \mathbb{I}_{[\omega_0, +\infty)}(\omega) q(\omega, \boldsymbol{\sigma}^1)$ . By  $R3'$ ,  $h_1$  is increasing in  $\omega$ . So,  $R1'$  implies

$$\begin{aligned} & \int_{-\infty}^{+\infty} h_1(\omega) f(\omega, \gamma^1) d\omega / \int_{-\infty}^{+\infty} f(\omega, \gamma^1) d\omega \\ & \geq \int_{-\infty}^{+\infty} h_1(\omega) f(\omega, \gamma^2) d\omega / \int_{-\infty}^{+\infty} f(\omega, \gamma^2) d\omega \end{aligned}$$

This can be rearranged as

$$Q(\boldsymbol{\sigma}^1, \gamma^1) / \int_{-\infty}^{+\infty} f(\omega, \gamma^1) d\omega \geq Q(\boldsymbol{\sigma}^1, \gamma^2) / \int_{-\infty}^{+\infty} f(\omega, \gamma^2) d\omega \quad (9)$$

Second, note that  $R2'$  implies  $Q(\boldsymbol{\sigma}^2, \gamma^2) = \int_{-\infty}^{\omega_0} q(\omega, \boldsymbol{\sigma}^2) f(\omega, \gamma^2) d\omega$  and  $Q(\boldsymbol{\sigma}^2, \gamma^1) = \int_{-\infty}^{\omega_0} q(\omega, \boldsymbol{\sigma}^2) f(\omega, \gamma^1) d\omega$ . Let  $h_2(\omega) \equiv \mathbb{I}_{(-\infty, \omega_0]}(\omega) q(\omega, \boldsymbol{\sigma}^2)$ . By  $R3'$ ,  $h_2$  is decreasing in  $\omega$ . So,  $R1'$  implies

$$\begin{aligned} & \int_{-\infty}^{+\infty} h_2(\omega) f(\omega, \gamma^2) d\omega / \int_{-\infty}^{+\infty} f(\omega, \gamma^2) d\omega \\ & \geq \int_{-\infty}^{+\infty} h_2(\omega) f(\omega, \gamma^1) d\omega / \int_{-\infty}^{+\infty} f(\omega, \gamma^1) d\omega \end{aligned}$$

This can be rearranged as

$$Q(\boldsymbol{\sigma}^2, \gamma^2) / \int_{-\infty}^{+\infty} f(\omega, \gamma^2) d\omega \geq Q(\boldsymbol{\sigma}^2, \gamma^1) / \int_{-\infty}^{+\infty} f(\omega, \gamma^1) d\omega \quad (10)$$

By multiplying Inequalities (9) and (10), we get

$$Q(\boldsymbol{\sigma}^1, \gamma^1) Q(\boldsymbol{\sigma}^2, \gamma^2) \geq Q(\boldsymbol{\sigma}^1, \gamma^2) Q(\boldsymbol{\sigma}^2, \gamma^1)$$

( $\Rightarrow$ ) We proceed by contradiction. Suppose that  $q(\cdot, \boldsymbol{\sigma}^1)$  is not weakly increasing for almost all  $\omega > \omega_0$ . Thus, there exist  $\omega_0 < \underline{\omega} < \bar{\omega}$  such that  $q(\cdot, \boldsymbol{\sigma}^1)$  is strictly decreasing for all  $\omega \in (\underline{\omega}, \bar{\omega})$ . Let  $f(\omega, \gamma^1) > 0$  be a density function with full support. Define  $f(\omega, \gamma^2)$  such that:

$$\begin{cases} f(\omega, \gamma^2) = f(\omega, \gamma^1) \text{ for all } \omega \notin (\underline{\omega}, \bar{\omega}) \\ f(\omega, \gamma^2) = f(\omega, \gamma^1) + \varepsilon \text{ for all } \omega \in (\underline{\omega}, \frac{\underline{\omega} + \bar{\omega}}{2}) \\ f(\omega, \gamma^2) = f(\omega, \gamma^1) - \varepsilon \text{ for all } \omega \in (\frac{\underline{\omega} + \bar{\omega}}{2}, \bar{\omega}) \end{cases}$$

with  $\varepsilon > 0$  small enough. By construction, we have

$$\int_{-\infty}^{\omega_0} q(\omega, \boldsymbol{\sigma}^2) f(\omega, \gamma^2) d\omega = \int_{-\infty}^{\omega_0} q(\omega, \boldsymbol{\sigma}^2) f(\omega, \gamma^1) d\omega \quad (11)$$

and

$$\int_{\omega_0}^{+\infty} q(\omega, \boldsymbol{\sigma}^1) f(\omega, \gamma^2) d\omega = \int_{\omega_0}^{+\infty} q(\omega, \boldsymbol{\sigma}^1) f(\omega, \gamma^1) d\omega + \delta\varepsilon \quad (12)$$

where  $\delta = \int_{\underline{\omega}}^{\frac{\underline{\omega} + \bar{\omega}}{2}} q(\omega, \boldsymbol{\sigma}^1) d\omega - \int_{\frac{\underline{\omega} + \bar{\omega}}{2}}^{\bar{\omega}} q(\omega, \boldsymbol{\sigma}^1) d\omega$ . Since  $q(\cdot, \boldsymbol{\sigma}^1)$  is strictly decreasing for all  $\omega \in (\underline{\omega}, \bar{\omega})$ , we have

$$\int_{\underline{\omega}}^{\frac{\underline{\omega} + \bar{\omega}}{2}} q(\omega, \boldsymbol{\sigma}^1) d\omega > \left( \frac{\bar{\omega} - \underline{\omega}}{2} \right) \cdot q\left(\frac{\underline{\omega} + \bar{\omega}}{2}, \boldsymbol{\sigma}^1\right) > \int_{\frac{\underline{\omega} + \bar{\omega}}{2}}^{\bar{\omega}} q(\omega, \boldsymbol{\sigma}^1) d\omega$$

which implies  $\delta > 0$ . Combining Equations (11) and (12), we then get

$$\begin{aligned} & \left[ \int_{\omega_0}^{+\infty} q(\omega, \boldsymbol{\sigma}^1) f(\omega, \gamma^2) d\omega \right] \cdot \left[ \int_{-\infty}^{\omega_0} q(\omega, \boldsymbol{\sigma}^2) f(\omega, \gamma^1) d\omega \right] \\ & > \left[ \int_{-\infty}^{\omega_0} q(\omega, \boldsymbol{\sigma}^2) f(\omega, \gamma^2) d\omega \right] \cdot \left[ \int_{\omega_0}^{+\infty} q(\omega, \boldsymbol{\sigma}^1) f(\omega, \gamma^1) d\omega \right] \end{aligned}$$

For any  $r$  such that  $R2'$  holds, the previous inequality becomes

$$Q(\boldsymbol{\sigma}^1, \gamma^2) Q(\boldsymbol{\sigma}^2, \gamma^1) > Q(\boldsymbol{\sigma}^1, \gamma^1) Q(\boldsymbol{\sigma}^2, \gamma^2)$$

By construction, for all  $\omega \in \mathbb{R}$ , we have

$$\int_{-\infty}^{\omega} f(\omega', \gamma^1) d\omega' / \int_{-\infty}^{+\infty} f(\omega', \gamma^1) d\omega' \leq \int_{-\infty}^{\omega} f(\omega', \gamma^2) d\omega' / \int_{-\infty}^{+\infty} f(\omega', \gamma^2) d\omega'$$

So,  $R1'$  holds as well. A contradiction. The case  $q(\cdot, \boldsymbol{\sigma}^2)$  not decreasing for almost all  $\omega < \omega_0$  can be treated in a similar manner. ■

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