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Results are presented concerning the τ decay mode of a homogeneous sample of 1347 K^- decaying in flight in the 15-inch Lawrence Radiation Laboratory Hydrogen bubble chamber.

If weak interactions are invariant under time reversal, the spectra for the c.m. kinetic energies of the pions emitted in τ decay will be identical for both charged states of the K meson. A comparison of our data with the total available statistics on τ^+ decay indicates good agreement between the two. For this reason, the data for τ^+ and τ^- have been combined, and the best fits to straight lines for the relevant spectra are given. Using the equations of Khuri and Treiman[1] to relate the combined data to the difference between the I = 2 and I = 0 S-wave $\pi\pi$ scattering lengths, we find $a_2 - a_0 = (0.9 \pm .1)(\hbar/m_\pi c)$.

The events studied are the decays of K^- mesons that varied in laboratory momentum P_K between 300 and 850 Mev/c.[2] Over this momentum range no appreciable scanning biases were observed. For reliable measurements, however, decay events were accepted only if they occurred within a preselected fiducial volume.

The measured track variables for 1440 "candidates" were fitted by our computer program, KICK, to conserve 4-momentum. The χ^2 distribution differs from that expected in two respects:

(a). There is a flat tail of 93 events ($\approx 6\%$). Most of these events are probably not τ decays at all, but events such as $K_{\pi 2}$ with a Dalitz pair.

The remainder represent poor fits due to unnoticed plural scatterings or mismeasurements.

(b) Apart from this background, the χ^2 distribution has the expected shape but is still too wide by a scale factor of about 1.8, implying that our input errors (computed to allow for Coulomb scattering and estimated measurement accuracy) are too small by about $\sqrt{1.8}$, because of such factors as optical distortion and turbulence. We used 1347 events whose values of $\chi^2/1.8$ corresponded to better than the 1% probability level for a 4-constraint fit. The calculated errors δT_π in the fitted c. m. kinetic energies are an increasing function of P_K. Typical values of δT_π (increased by $\sqrt{1.8}$) vary from $\pm$ 3 Mev for the lower momentum K decays, up to $\pm$ 6 Mev for the higher ones.

Figure 1 is a Dalitz plot of the 1347 events.

We first examine the x-dependence of the density of these events,

$$W^-(x) = \frac{1}{N\rho(x)} \frac{\Delta N}{\Delta x} \quad . \qquad (1)$$

where N is 1347, $\rho(x)$ is the Lorentz-invariant phase space, and $\Delta N/\Delta x$ is the vertical projection of the events in the interval Δx. If the matrix element for τ decay is called $M^-(x, y)$, then the spectrum $W^-(x)$ represents $|M|^2$ averaged over the allowed values of y. The symmetry of the two like pions forces $W^-(x)$ to be of the form $1 + bx^2 + \cdots$. Figure 2a gives the data for $W^-(x)$. Within statistics, we see no evidence for a quadratic dependence, i. e.,

$$W^-(x) = 1 \qquad (2)$$

gives a χ^2 probability of $\approx$ 60%. Therefore $|M(x, y)|^2$ depends only on y (i. e., on the energy of the unlike pion).

To examine the y-dependence of $|M(x, y)|^2$, we form the analogue to Eq. (1), $W^-(y)$. Parametrizing it to the form $1 + ay + by^2 + \cdots$, we find

$$W^-(y) = 1 + (0.28 \pm .045)y, \qquad (3)$$

with no evidence for quadratic terms.

The results of Eqs. (2) and (3) can be summarized by the statement that $|M(x, y)|^2$ is well described by the plane

$$|M(x, y)|^2 = 1 + (0.28 \pm .045)y. \tag{4}$$

Although the variables x and y seem appropriate, it has become conventional in the case of positive τ decays (with which we must compare our data) to use different variables, namely ϵ_u (ϵ_ℓ), given by the c. m. kinetic energy of the unlike (like) pion divided by the maximum allowed kinetic energy, 48.3 Mev. Here ϵ_u and y differ only by a scale factor of 2 $[(-1 \leq y \leq 1), (0 \leq \epsilon_u \leq 1)]$, but ϵ_ℓ is quite different from x. Figure 2b shows our data for $W^-(\epsilon_u)$ and the best straight line fit [Eq. (3)] in this variable.

$$W^-(\epsilon_u) = 1 + (0.56 \pm .09)(\epsilon_u - 1/2). \tag{5}$$

To the extent that $|M|^2$ is just the plane as in Eq. (3), it is easily shown[3] that our distribution in ϵ_ℓ must have -1/2 the slope of Eq. (4). We find

$$W^-(\epsilon_\ell) = 1 - (0.26 \pm .09)(\epsilon_\ell - 1/2). \tag{6}$$

Equation (6) is written mainly for comparison with τ^+ data, but also constitutes a check on the form [Eq. (3)] of $|M|^2$.

Our results, Eqs. (5) and (6), may be compared with the equivalent fits to 899 published τ^+ decays,[4]

$$W^+(\epsilon_u) = 1 + (0.48 \pm .11)(\epsilon_u - 1/2) \tag{7}$$

and

$$W^+(\epsilon_\ell) = 1 - (0.25 \pm .12)(\epsilon_\ell - 1/2). \tag{8}$$

We see that the slopes of $W^+(\epsilon_u)$ and $W^+(\epsilon_\ell)$ are again related by the factor -1/2, as required by Eq. (3).

The previous data indicate that, within statistics, the spectrum for τ^+ and τ^- decays are identical as predicted by time-reversal invariance.[5] Therefore, in Fig. 3 we present the combined $\tau^+ - \tau^-$ spectrum. A straight line fit gives

$$W^{\pm}(\epsilon_u) = 1 + (0.53 \pm .07)(\epsilon_u - 1/2). \tag{9}$$

Khuri and Treiman[1] have used dispersion theoretic methods in an attempt to relate the τ-decay matrix element to the S-wave $\pi\pi$ scattering amplitudes. With approximations suitable to small scattering lengths, they find

$$\frac{1}{4}|M|^2 \approx 1 + \frac{5}{3\pi} \frac{2\rho^2}{(1+\frac{1}{2}\rho^2)^{1/2}} (a_2 - a_0)(\epsilon_u - \frac{1}{2}), \qquad (10)$$

where ρ^2 is a kinematical factor equal to 0.64, and $a_2 - a_0$ is the difference in the I = 2 and I = 0 S-wave $\pi\pi$ scattering lengths. Comparison with Eq. (9) indicates

$$a_2 - a_0 = (0.9 \pm .1)(\hbar/m_\pi c). \qquad (11)$$

Apart from final state interactions, another–perhaps more reasonable–interpretation of the observed structure in τ decay is the possibility of introducing a small admixture of I = 1 states of intermediate symmetry, thereby producing a dependence of the matrix element on y.

The fit to the combined data, Eq. (9), may also be compared with the equivalent fit for τ'^+ decays. From the total available statistics of 119 τ'^+ decays summarized by Bøggild et al.,[6] we compute

$$W'^+(\epsilon_u) = 1 - (1.0 \pm .4)(\epsilon_u - \frac{1}{2}). \qquad (12)$$

Weinberg[3] has pointed out that the $|\Delta I| = 1/2$ rule requires that the coefficients of the linear terms in Eqs. (9) and (12) be in the ratio 1 to -2. This has already been checked by Bjorklund et al.[7] on the basis of 899 τ^+ and 72 τ'^+ events. We see that with slightly improved statistics $|\Delta I| = 1/2$ is still well satisfied.[8]

The decay of a τ in its center of mass can be completely specified by two independent kinematical quantities. In addition to the variables ϵ_u and ϵ_ℓ, W has frequently been expressed in terms of two angles, which for τ^- decays we will call $\theta_{neutral}$ and θ_{--}. To discuss $\theta_{neutral}$ we assume that the K^- decays into a π^- and a neutral dipion. In the rest frame of the dipion, $\theta_{neutral}$ is the angle between its direction of motion and the relative momentum of the π^+ and π^- (see sketch beside Fig. 4). The square of the matrix element $W^-(\cos\theta_{neutral})$ is normalized by the definition

$W^-(\cos\theta_{neutral}) = 1/N\ (dn/d\cos\theta_{neutral})$.

Figure 4 displays the $W^-(\cos\theta_{neutral})$ data and a straight line fitted thereto:

$$W^-(\cos\theta_{neutral}) \propto 1 \pm (0.2 \pm .04) \cos\theta_{neutral}. \qquad (13)$$

Each τ^- decay yields two possible neutral dipions, but the two values of $\theta_{neutral}$ are correlated, so the errors have been calculated on the basis of 1347 events.

In the case of θ_{--}, there is a unique (--) dipion, whose decay must be symmetric around $\cos\theta_{--} = 0$. The folded data give

$$W^-(\cos\theta_{--}) = 1 + (0.0 \pm .1) \cos^2\theta_{--}. \qquad (14)$$

Insufficient data are published to calculate W^+ for these angles.

In contrast to Khuri and Treiman,[1] Thomas and Holladay[9] have used the final state interaction formalism to estimate the effects of $\pi\pi$ scattering on the τ-decay spectrum, assuming that only the I = 2 S-wave scattering is important. Their curve for $W(\cos\theta)$, calculated for $a_2 = 1.0\ \hbar/m_\pi c$, is given in Fig.4b as a dashed line. It fits well.

It is a pleasure to acknowledge the collaboration of all the others who are working on this K^- experiment: P. Bastien, J. P. Berge, O. Dahl, J. Kirz, and M. Watson. We thank Professor L. W. Alvarez and all his group for their interest and support, and Professor Treiman for theoretical advice.

FOOTNOTES AND REFERENCES

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†National Academy of Sciences Fellow.

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5. This is however not a sensitive test of time-reversal invariance; even if time-reversal invariance fails the spectra could still agree by accident or because 1) the "primitive" (i. e. with the pion-pion interaction turned off) τ^+ and τ^- matrix elements may be so symmetric that the spectrum is dominated by final-state interactions, or 2) final-state interactions could be so weak as barely to affect the spectrum.
6. J. K. Bøggild, K. H. Hansen, J. E. Hooper, M. Schaerf, and P. K. Aditya, Nuovo cimento 19, 621 (1961).
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8. This again is an insensitive test if the "primitive" matrix element is highly symmetric.
9. B. S. Thomas and W. G. Holladay, Phys. Rev. 115, 1329 (1959).
10. R. H. Dalitz, Phil. Mag. 44, 1068 (1953).

FIGURE CAPTIONS

Fig. 1. Dalitz plot of the distribution of pion energies for 1347 τ^- decays. The coordinates are those of Dalitz (Ref. 10): T_1, T_2, and T_3 are respectively the c. m. kinetic energies of the π^-, π^- and π^+; and Q = 75.11 Mev.

Fig. 2. Energy dependence of the matrix element for 1347 τ^- decays on:

(a) the like pion energy [through the coordinate $x = \frac{\sqrt{3}}{Q}(T_1 - T_2)$].

(b) the unlike pion energy ($\epsilon_u = T_3/T_{max}$).

Fig. 3. Energy dependence of the matrix element for 2246 combined τ^+ and τ^- decays on the unlike pion.

Fig. 4. Angular dependence of the matrix element for 1347 τ^- decays:

(a) angle with respect to the $\pi^-\pi^-$ rest frame.

(b) angle with respect to the $\pi^+\pi^-$ rest frame.

The dashed line is from Ref. 9.

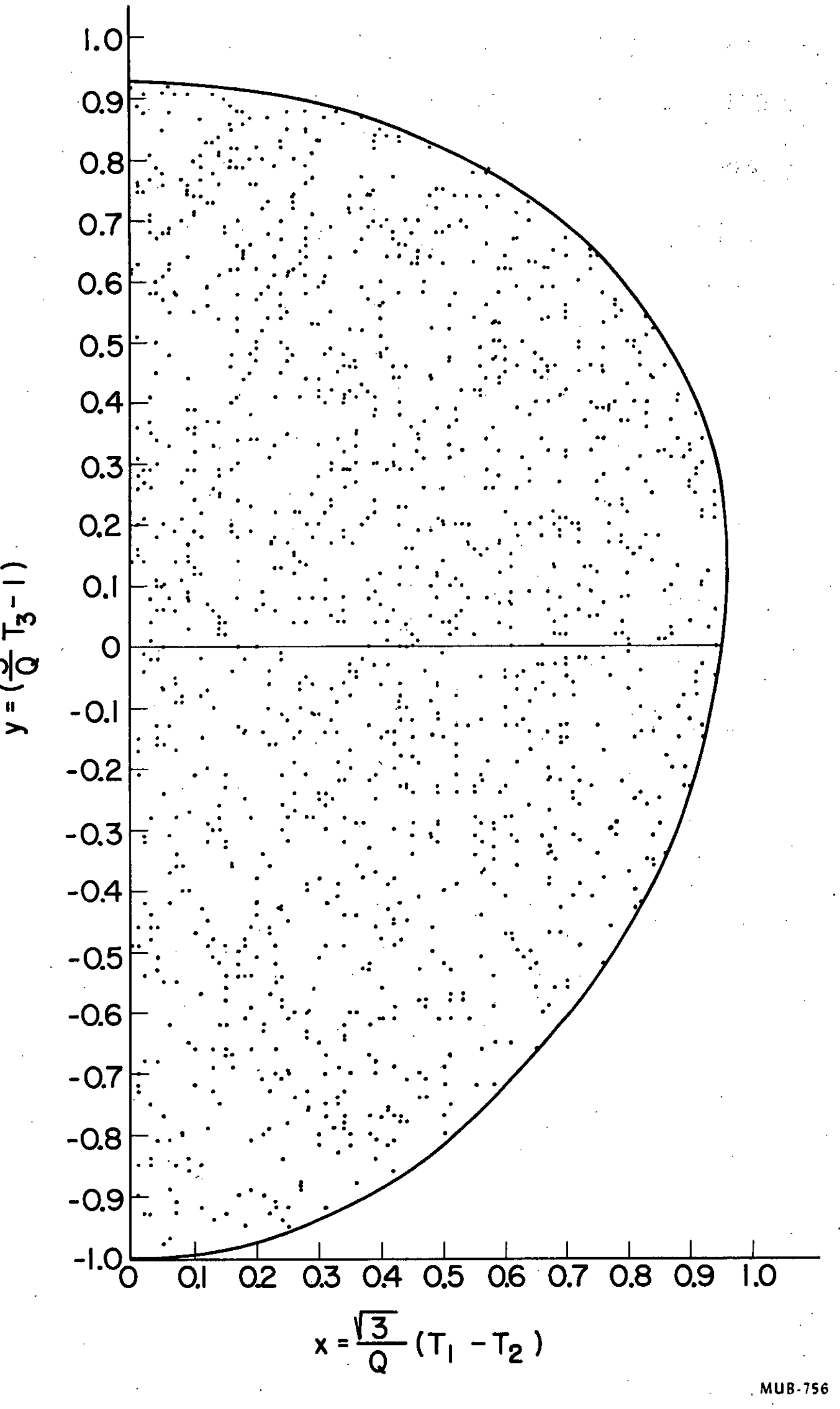
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
y = (3/Q T3 - 1)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
x = √3/Q (T1 - T2)
MUB-756

Fig. 1

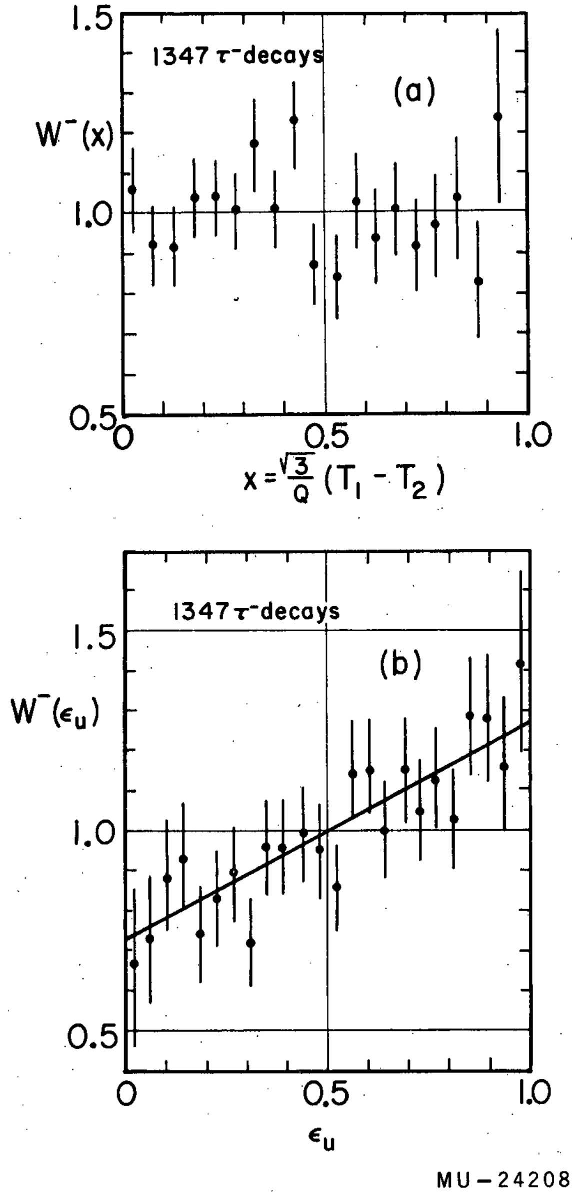
1347 τ⁻-decays
(a)
W⁻(x)
1.5
1.0
0.5
0
0.5
1.0
x = √3/Q (T1 - T2)
1347 τ⁻-decays
(b)
W⁻(εu)
1.5
1.0
0.5
0
0.5
1.0
εu
MU-24208

Fig. 2

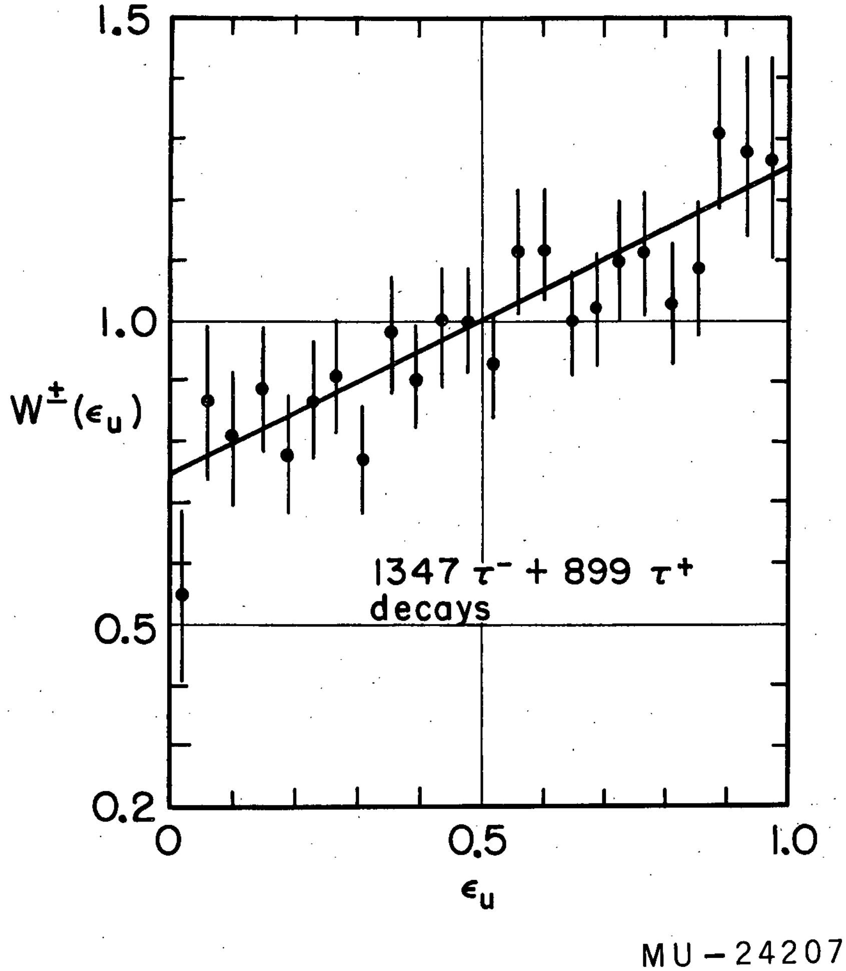

Fig. 3

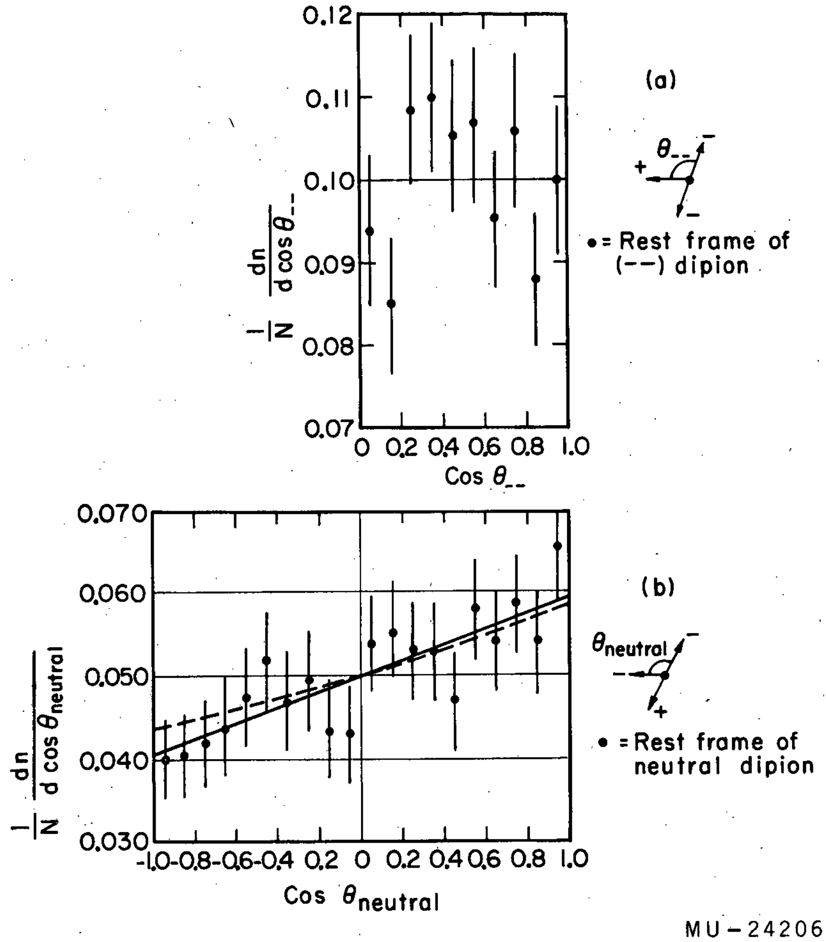
(a)
θ--
= Rest frame of (--) dipion
Cos θ--
(b)
θneutral
= Rest frame of neutral dipion
Cos θneutral
MU-24206

Fig. 4.