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Three Essays on Mortgage Default

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by

Hrishikesh Singhania

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Three Essays on Mortgage Default

To Dadu

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Conferences and Seminar Presentations

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Abstract

Three Essays on Mortgage Default

Hrishikesh Singhania

This dissertation consists of three essays on mortgage default. The first essay discusses the determinants of mortgage default. The financial crisis of 2007-2008 was precipitated by default in subprime mortgages. This episode spurred a lot of research on mortgage default. The essay surveys this research with a focus on what determines mortgage default. It emphasizes that the market value of home equity determines default, not the book value of equity.

The second essay discusses the valuation of mortgage backed securities in an equilibrium framework that explicitly incorporates default decisions of homeowners, along with essential contractual features of these securities. The analysis begins by valuing Collateralized Mortgage Obligations (CMOs), which are securities created by dividing a pool of mortgages into senior and residual tranches. A major finding is that bonds issued on the senior tranche can be risk free, low risk, or high risk in equilibrium, depending on the relative size of the tranche. For house price data from the Case-Shiller house price index between 2006 and 2011, model implied senior bond values decline by 10% and residual bond values decline by 60%. The essay also discusses the valuation of CMO-squared and Credit Default Swaps, which are both derivative securities created from CMOs.

The third essay discusses valuation of mortgages with coupon resets, when homeowners optimally exercise the option to default on their mortgage. The analysis shows that the optimal default boundary is discontinuous at the reset date when the coupon after the reset is large, compared to the coupon prior to the reset. The model connects equilibrium yield spreads on these mortgages to initial loan-to-value ratios, coupon structure, time remaining until the reset, expected growth rate in house prices, and the volatility of house prices. Conditional on the initial loan-to-value ratio, mortgages with low initial payments followed by high payments after the reset have higher default risk than the corresponding fixed rate mortgage. The essay also discusses the valuation of balloon payment mortgages.

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Chapter 1

A Survey of the Determinants of Mortgage Default

1.1 Introduction

Individuals in the United States usually finance home purchases using mortgage loans. A mortgage loan transaction involves a homeowner (borrower) making an initial payment towards the purchase of the property, while a bank (lender) supplies the remaining funds. The borrower pays off the loan over time by making regular coupon payments, which are applied towards principal and interest on the loan. At any time during the life of the loan, the borrower can prepay his mortgage by paying off the remaining loan balance. Alternatively, he can stop paying the coupon and default on his mortgage. Since mortgage loans are collateralized by the underlying property, the ownership of the property is transferred to the lender if the borrower defaults.

Understanding the borrower's incentives to exercise the default option is important for lenders in order to price the mortgages correctly, and to manage the risk of their mortgage portfolios. It is important for policy makers because mortgage default may affect the broader economy. The wider consequences of mortgage default were apparent in the global financial crisis of 2007-2008, in which default in the subprime segment of the U.S. housing market triggered the worst recession since the Great Depression. The unanticipated increase in mortgage defaults has raised a lot of questions regarding mortgage defaults. What drives borrowers to default? What role did subprime mortgages play in the crisis? What was the role of "exotic" mortgages in the crisis? Did lenders make unaffordable mortgage loans in the buildup to the crisis? If so, why? How does securitization of mortgages affect default? How do foreclosures affect local house prices? Are mortgage modification programs effective in reducing foreclosures? In response, research in this area has expanded rapidly. This paper provides a survey of recent research on the question: What determines mortgage default?[1]

Mortgage market analysts usually classify default into "strategic" and "non-strategic" default. Mortgage default is considered strategic if the borrower stops paying the coupon on his mortgage, even though he has the financial resources to do so. For example, a borrower might default strategically if the outstanding balance on his mortgage significantly exceeds the value of the underlying property. Default is non-strategic or involuntary if the borrower stops paying the coupon on his mortgage because he does not have the financial resources to make the payment due to adverse life events. For example, an unemployed borrower who is

[1] Quercia and Stegman (1992) provides a survey of early research on determinants of mortgage default.

underwater on his mortgage, has no savings, and is currently credit constrained. Involuntary default is often called the "double trigger" theory of mortgage default because default is induced by two triggers: negative book equity and an adverse life event.

Following the terminology in the literature on mortgage default, this survey is organized loosely into studies that focus on strategic versus non-strategic default. I classify studies that primarily focus on the role of home equity in default under strategic default. This method of classification results in a large number of studies on the determinants of mortgage default falling under the category of strategic default. Therefore the bulk of this paper, Sections 1.2 to 1.9, focuses on strategic default, while Section 1.10 is devoted to non-strategic default. Section 1.2 presents a simple model of strategic default. This section shows the theoretical relationship between strategic default, economic value of home equity, and book value of home equity. The connection between strategic default and book value of equity is important because empirical studies use book equity as the measure of home equity; economic equity is unobservable. Section 1.3 discusses empirical studies that focus on the importance of negative book equity as a determinant of default. Section 1.4 discusses estimates of the levels of book equity at which defaults occur in the data. Section 1.5 presents evidence on how payment size affects mortgage default. Section 1.6 connects mortgage yields at origination to mortgage default. Section 1.7 shows what survey data reveal about moral and social attitudes towards strategic default. Section 1.8 discusses default in second mortgage liens. Section 1.9 shows how lender recourse impacts mortgage default.

Section 1.10 focuses on studies that emphasize the role of adverse life events on default. Section 1.11 concludes.

1.2 A Simple Model of Mortgage Default

Studies that model strategic default adapt the classic paper of Merton (1974) to the mortgage market. The primary insight from these studies is that default is optimal only when a borrower is significantly underwater on his mortgage. (A borrower is underwater on his mortgage when the value of the underlying property minus the mortgage balance — the book value of equity — is negative.) The studies emphasize that the economic value of home equity matters for the default decision. In Section 1.2.3 I relate the economic value of equity at the time of default to the book value of equity. This relationship is implicit in models of mortgage default. I present it explicitly because it tightens the connection between theoretical and empirical work on mortgage default; empirical work in this area uses book equity because it is easily measured. Kau, Keenan, and Kim (1994) is an early paper that studies strategic default and its implications. Vandell (1995) provides an overview of the early literature on strategic default. This section presents a simple model of strategic mortgage default in fixed rate mortgages (FRMs); see Krainer, LeRoy, and O (2009) for a recent exposition. The exposition here draws heavily upon Singhania (2013).

1.2.1 Model Setup

A house provides a stochastic flow of services. Housing services $x(t)$ are exogenous and follow a geometric Brownian motion:

$$dx(t) = \alpha x(t)dt + \sigma x(t)dw(t), \tag{1.1}$$

where α is the expected proportional growth rate, σ is the volatility parameter, and $w(t)$ is standard Brownian motion. Initial housing services are normalized to one, $x(0) = 1$.

House prices are the expected discounted value of future services:

$$P(x(t)) \equiv \int_{z=t}^{\infty} e^{-\rho(z-t)} \mathbb{E}_t\big(x(z)\big)dz = \frac{x(t)}{\rho - \alpha}. \tag{1.2}$$

The operator $\mathbb{E}_t$ denotes the mathematical expectation conditional on information available at time t. The discount rate ρ is exogenous and constant. House prices can be represented by (1.2) if housing services follow a geometric Brownian motion under the risk neutral probability measure or if agents are risk neutral. This specification of house prices rules out bubbles. By (1.2), house prices also follow a geometric Brownian motion, with α as the expected proportional growth rate and σ as the volatility parameter. Since geometric Brownian motion is a Markov process, the current house price summarizes relevant past and future information. The best forecast for house prices is that they grow at the rate α.

Under the normalization $x(0) = 1$, the purchase price of the house is $P(1) = 1/(\rho{-}\alpha)$. An infinitely lived risk neutral borrower buys the house using a mortgage loan. The difference in the size of the mortgage loan and the purchase price is financed from the borrower's personal wealth, which is not modeled. The mortgage contract requires the borrower to make perpetual coupon payments to the lender in exchange for the flow of services from the property. The coupon c is exogenous. The modeling assumption that mortgages are perpetuities does not lead to major distortions because, in practice, most mortgage defaults occur within the first few years of origination, when mortgage payments are mostly interest payments. The borrower has the option to default on his mortgage, subject to a cost, at any time by paying the lender the current market value of the house. This assumption allows the market value of the house at the time of default to be less than the size of the mortgage loan. The assumption corresponds in reality to the ability of the borrower to turn over the keys and walk away from the house.

In practice, borrowers who choose to default have to bear relocation expenses, loss of future credit access, and loss of tax benefits. These costs are incorporated into the model by assuming that borrowers faces positive default costs. The existence of mortgages with initial loan-to-value (LTV) ratios greater than 100% in practice provides further evidence in favor of positive borrower default costs. Research has also shown that default behavior observed in the data is difficult to reconcile with the behavior implied by a model of costless default; see Deng, Quigley, and Van Order (2000). Default costs, however, need not be positive for all borrowers. The popular press has reported instances of borrowers living rent free in their houses after defaulting on their mortgage. Such cases are incorporated into the analysis by allowing borrower default costs to be negative. Borrower default costs are denoted k_b. These costs are exogenous and proportional to the purchase price of the property.

Default is also costly for lenders. Once borrowers default, lenders gain possession of the property. The cost of maintaining, repairing, and reselling the property is borne by lenders. Usually there is a lag, of a year or more, between the default date and the date at which lenders can repossess and sell the property. During this lag, lenders also lose income from coupon payments. These costs are modeled as lender default costs, denoted k_ℓ. Lender default costs are exogenous, proportional to the purchase price of the property, and identical across mortgages. Default costs paid by borrowers and lenders are deadweight loss to the society. This modeling choice is motivated by the notion that mortgage default is inefficient, rather than a costless transfer of ownership of the property from the borrower back to the lender.

Borrowers are prohibited from prepaying their mortgage. Thus credit risk is the only risk faced by lenders. The mortgage market features free entry and exit, implying that lenders make zero expected profits. Under zero expected profits, the size of the mortgage loan must equal the expected discounted value of borrowers' payments. The latter depends upon the default behavior of the borrower. Therefore the size of the loan is determined as part of the equilibrium. Consequently the initial LTV ratio and the mortgage yield are also determined in equilibrium.

1.2.2 Equilibrium

The borrower chooses the threshold of housing services at which to default so as to maximize home equity, or equivalently minimize mortgage liability. The solution to this problem presented here employs boundary crossing properties of geometric Brownian motion. This method reduces the valuation of home equity to a simple present value calculation. Alternatively, one could use dynamic programming; see Krainer, LeRoy, and O (2009) for this approach.

The home equity maximization problem at origination is

$$\max_{d} \quad \left\{ P(1) - \mathbb{E}_0\left[\int_0^{\tau(d)} c\, e^{-\rho t} dt \right] - \mathbb{E}_0\left[e^{-\rho\tau(d)} \big(P(d) + k_b\big) \right] \right\}, \tag{1.3}$$

where d denotes a generic default threshold and $\tau(d)$ denotes the time at which housing services hit the default threshold. The time of default is random because housing services are random. All mathematical expectations in the maximization problem above are conditional on information available at origination. The random variable in all the expectations above is the default time $\tau(d)$. The term inside the first expectation is the total discounted present value of the coupons paid by the borrower until default. The term inside the second expectation is the discounted present value of the borrower's payments on default: the market value of the house $P(d)$, and default costs k_b. Home equity equals house price minus the expected discounted value of all mortgage payments.

The objective function in (1.3) can be simplified by noting that c, $P(d)$, and k_b can be moved out of the expectation. After evaluating the integral within the first expectation, the problem in (1.3) becomes

$$P(1) - \frac{c}{\rho} + \max_{d} \left\{ \mathbb{E}_0\left[e^{-\rho\tau(d)} \right] \left(\frac{c}{\rho} - P(d) - k_b \right) \right\}. \tag{1.4}$$

Calculating the mathematical expectation of $e^{-\rho\tau(d)}$ at origination is the key to solving the equity maximization problem. This expectation is the moment generating function of the random default time $\tau(d)$ evaluated at $-\rho$. It equals d^m, where $m > 0$ depends on the parameters of the process followed by housing services and the discount rate ρ

$$m = \frac{(\alpha - \sigma^2/2) + \sqrt{\left(\alpha - \sigma^2/2\right)^2 + 2\rho\sigma^2}}{\sigma^2}. \tag{1.5}$$

By the strong Markov property of geometric Brownian motion, the moment generating function of $\tau(d)$ conditional on information available at time t is $(d/x(t))^m$; see Karatzas and Shreve (1991) for a discussion.

After substituting for $\mathbb{E}_0[e^{-\rho\tau(d)}]$, home equity is a function only of the default threshold d. Standard optimization techniques apply. The optimal default threshold is

$$\delta = \left(\frac{m}{m+1}\right)\left(\frac{c/\rho - k_b}{P(1)}\right). \tag{1.6}$$

The optimal threshold is strictly increasing in the mortgage coupon c, and strictly decreasing in borrower default costs k_b. The borrower defaults when $x(t) = \delta$ for the first time. From here on I drop the word optimal and refer to δ as the default threshold. I also suppress the dependence of the optimal default time on δ and indicate the default time by τ.

Let $E(x(t))$ denote the value function of the equity maximization problem. The expression for home equity is

$$E(x(t)) = P(x(t)) - \frac{c}{\rho} + \left(\frac{c}{\rho} - P(\delta) - k_b\right)\left(\frac{\delta}{x(t)}\right)^m \tag{1.7}$$

Home equity equals the current house price minus the present value of mortgage coupon payments plus an adjustment for default. The adjustment reflects the fact that on defaulting the borrower gains the present value of remaining coupon payments c/ρ, loses the house worth $P(\delta)$, and pays the default cost k_b. When $x(t) = \delta$ home equity equals $-k_b$, or equivalently $E(x(t)) + k_b = 0$. The economic value of borrower's equity, not the book value of equity, is zero at the optimal default threshold. This finding has important implications for empirical work on mortgage default. These implications will be discussed in Section 1.2.3.

Let $M(x(t))$ denote the value of the mortgage to the lender when housing services equal $x(t)$. The zero expected profit condition implies that $M(x(t))$ equals the expected discounted value of the borrower's payments,

$$M(x(t)) = \frac{c}{\rho} - \left(\frac{c}{\rho} + k_\ell - P(\delta)\right)\left(\frac{\delta}{x(t)}\right)^m. \tag{1.8}$$

The first term on the right hand side of (1.8) is the value of the mortgage in the absence of default. The second term is the adjustment for default. On default, the lender loses all future coupon payments c/ρ, pays the default cost k_ℓ, and gains the market value of the house at the time of default $P(\delta)$. The value of the mortgage at origination is $M(1)$. The equilibrium asset value of the mortgage is increasing in housing services $x(t)$ because default in the near future becomes less likely as $x(t)$ increases. As $x(t)$ approaches infinity, the value of the mortgage approaches c/ρ. At the default threshold δ, the value of the mortgage equals the net recovery, $M(\delta) = P(\delta) - k_\ell$.

The zero expected profit condition also implies that the size of the mortgage loan equals $M(1)$. Therefore the initial LTV ratio is $M(1)/P(1)$. The initial yield on the mortgage is $c/M(1)$. The difference between the initial yield and ρ — the yield spread — reflects the expected loss due to mortgage default. The recovery rate on the mortgage is $M(\delta)/M(1)$.

1.2.3 Discussion

In the model presented above the borrower always has enough financial resources to pay the coupon. Therefore default is strategic. Why does the borrower default? The borrower defaults in order to maximize his wealth. A mortgage in the model is a financial security that provides the borrower with a stochastic dividend flow $x(t)$ in exchange for the coupon c. The security comes with an option that allows the borrower to free himself from the debt obligation by relinquishing his claim to the dividend flow, and by paying k_b. It is optimal for the borrower to exercise this option when the dividend flow reaches the level at which the net present value of the security is zero. Conversely the borrower holds the security as long as its net present value is positive. The real options literature provides many other examples of such net present value calculations; see Dixit and Pindyck (1994) for an overview.

Certain analysts of mortgage default purport that when the exercise of the default option is costless, $k_b = 0$, a borrower who wants to maximize his wealth should default as soon as the book value of equity is zero. After defaulting the borrower could take out another mortgage to repurchase the house at the lower price, thereby increasing his wealth. According to this view, borrowers with positive default costs should wait until the sum of the book value of equity and $k_b$equals zero. Analysts that hold this view measure borrower default costs in the data as the amount by which the book value of equity is below zero at the time of default. This view of default, and so the measurement methodology, is in error because the book value of equity does not incorporate the value of the default option. The optimal default rule is characterized by the economic value of equity being zero, not book equity.

It is interesting to compare the zero book equity default rule to the optimal default rule according to the model presented here. I compare the two default rules using numerical examples. The parametrization is from Krainer, LeRoy, and O (2009). The discount rate is $\rho = 7\%$, implying that the average real proportional gain on mortgages and home equity is 7%. The parameters for the geometric Brownian motion followed by housing services are $\alpha = 3\%$, and $\sigma = 15\%$. Under the normalization $x(0) = 1$, the purchase price of a house is $P(1) = 25$. The implied price-to-rent ratio in the model is 25.[2] The chosen value of $\sigma = 15\%$ for the standard deviation of housing services is consistent with estimates of individual house price volatility in the literature.[3] Lender default

[2] Price-to-rent ratios in the data are closer to 10 or 15. This discrepancy between the model and the data is appropriate because the model abstracts from operating costs such as maintainence and utilities expenses.

[3] For example, Flavin and Yamashita (2002) estimated the standard deviation of the real return on housing to be 14%. Similarly, Case and Shiller (1989) estimated the return on individual houses to be around 14-15%. Values of σ closer to 10% maybe more appropriate for houses located in certain geographical areas of the United States.

costs are zero. The numerical examples will vary the mortgage coupon c and borrower default costs k_b.

Figure 1.1 shows the optimal default threshold as a function of initial LTV, for borrower default costs ranging from zero to forty percent of the purchase price. The initial LTV ratio was varied by changing the mortgage coupon c. To allow comparison with the zero book equity default rule, the optimal default threshold is expressed in terms of the corresponding book value of equity normalized by house price at time of default. In the model the book value of equity at the time of default is given by $P(\delta) - M(1)$; recall that mortgages are modeled as perpetuities and $M(1)$ is the size of the mortgage loan. The book value of equity is normalized by the current house price to maintain consistency with empirical literature on mortgage default. Therefore the vertical axis in Figure 1.1 corresponds in the model to

$$\left(\frac{P(\delta) - M(1)}{P(\delta)}\right) \times 100. \tag{1.9}$$

The figure highlights that rational borrowers wait to default until they are significantly underwater on their mortgage, as measured by book equity. For example, a borrower with $k_b = 0$ who has a mortgage with an initial LTV of 80% defaults when normalized book value of equity is -22%, not when normalized book equity is zero. As noted earlier, the key reason for the difference is that the zero book equity default rule fails to account for the value of the default option; the option becomes more valuable as house prices decline. The zero book equity default rule and the optimal default rule are identical only when borrower default costs are zero and the initial LTV ratio is 100%.

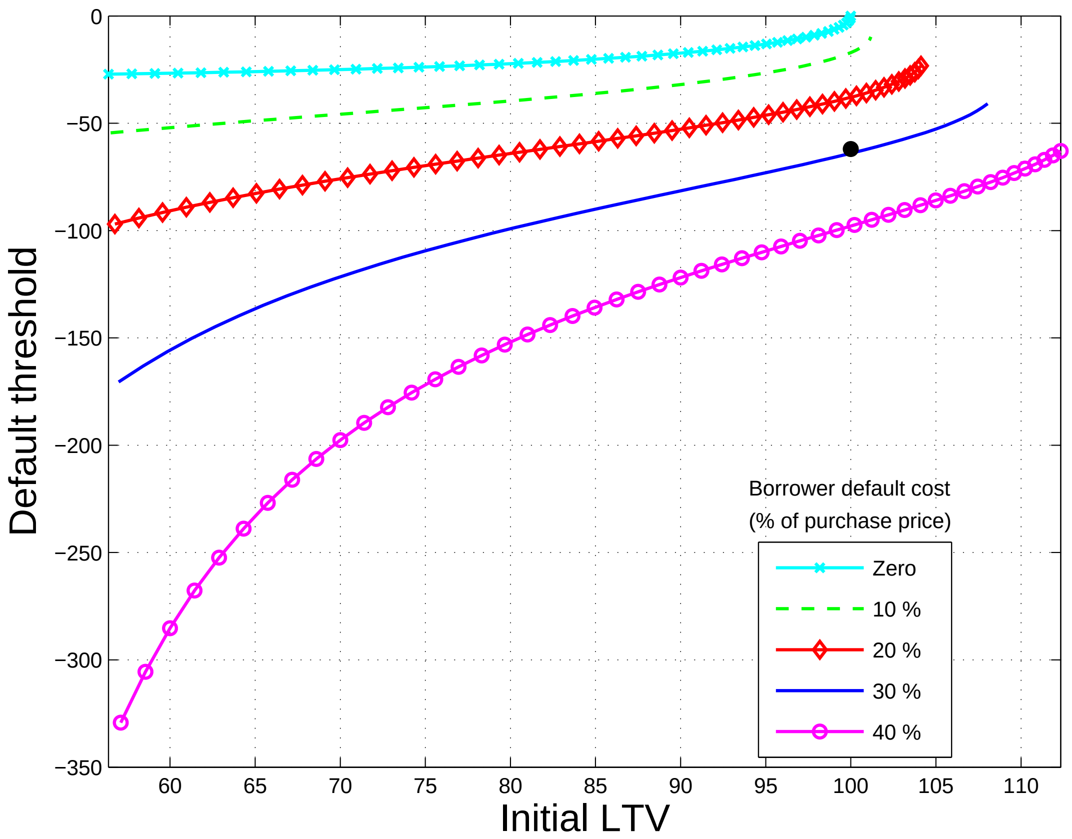


Figure 1.1: Default thresholds as a function of initial LTV ratio when borrowers default strategically on their fixed rate mortgage. The thresholds are expressed in terms of book equity normalized by the house price at the time of default. Default thresholds are shown for various values of borrower default costs, expressed as a percentage of the purchase price. Lender default costs are set to zero. The solid black dot corresponds to the value of default threshold estimated by Bhutta, Dokko, and Shan (2010).

1.3 Negative Book Equity

Figure 1.1 is useful for understanding and interpreting empirical work on mortgage default because empirical studies measure borrowers' home equity using the book value of equity. The figure shows that, according the model of section 1.2, default is optimal only for borrowers who are significantly underwater, as measured by the book value of equity. This implication of the model has been tested in the empirical literature as the hypothesis that being underwater is a necessary but not a sufficient condition for default. The necessity of negative book value of equity follows from the fact that a borrower with positive book equity who wants to terminate the mortgage would prefer to sell the house, pay back the mortgage loan, and pocket the difference.

An empirical study by Foote, Gerardi, and Willen (2008a) confirms that a majority of borrowers with negative book equity do not default on their mortgages. The authors use an extensive dataset compiled from the Massachussetts Registry of Deeds by the Warren Group. The dataset contains information about every house purchase and mortgage origination in the state of Massachussetts from 1987 to 2007, including the purchase price of each house, the size of the mortgage loan, and information on additional liens.

In order to estimate the current value of book equity for a borrower, the authors require an estimate of the current house price and the outstanding mortgage balance. The authors estimate individual house prices using methodology that is standard in the empirical mortgage default literature. Starting from the purchase price, they assume that the property appreciates at the same rate as a house price index for the relevant geographical area. Papers in the literature usually calculate the growth rate of house prices using the Federal Housing Finance Agency (FHFA) county level house price index or the Case-Shiller MSA level house price index. The Massachussetts Registry of Deeds, however, is unique in that it tracks the price of each property over time. Foote, Gerardi, and Willen (2008a) exploit this feature and construct local house price indexes for various cities and towns using the Case-Shiller repeat sales methodology. The advantage of this approach is that it allows them to obtain house price estimates that are relatively more precise. Estimates of the outstanding mortgage balance are unavailable to the authors because the dataset does not contain information on mortgage coupon rates. Therefore the authors calculate the current value of book equity as the current house price minus the size of the mortgage loan, normalized by the loan size. Note that book equity is normalized by house price at the time of default on the vertical axis of Figure 1.1. Normalizing book equity by the size of the mortgage loan instead would raise the scale on the vertical axis because $P(\delta) < M(1)$.

The estimate of book equity employed by Foote, Gerardi, and Willen (2008a) implies that 100,288 borrowers who had purchased their house after the first quarter of 1987 were underwater on their mortgages by the fourth quarter of 1991. Only 6.4 percent of these borrowers defaulted in the subsequent three years.[4] The fact that only a small fraction of mortgages with negative book equity default questions its relevance in driving default in the data, among the proponents of the zero book equity default rule.

[4]In the study by Foote, Gerardi, and Willen (2008a), a mortgage defaults once the lender initiates foreclosure proceedings. Other papers in the literature define mortgage default as the borrower being 60+ or 90+ days delinquent.

Foote, Gerardi, and Willen (2008a) estimate the effect of negative book equity on default using a statistical duration model. The model characterizes hazard functions, which are the probabilities that a borrower living in a particular house with certain mortgage characteristics will terminate his mortgage t quarters from the origination date, conditional on not having terminated it already. According to the empirical model, mortgages can be terminated either by default or by sale of the underlying property. Therefore the model has two hazard functions. The model features a proportional hazard specification, meaning that each hazard function has a baseline hazard and independent variables have a proportional effect on the baseline. The hazard functions depend on borrower characteristics, mortgage characteristics, and local economic conditions. The independent variables include the borrower's book equity, town level unemployment rates, the contemporaneous six month LIBOR, median household income at the zipcode level, fraction of minorites in the zipcode, property type, and subprime status.[5] In the empirical specification, book equity is modeled as a linear spline with seven intervals: $(-\infty, -20\%)$, $[-20\%, -10\%]$, $[-10\%, 0\%]$, $[0, 10\%]$, $[10\%, 25\%)$, $[25\%, \infty)$. The linear spline specification captures the effect of negative book equity on the default hazard flexibly, by allowing book equity to have different slopes in each interval. This specification is common among papers that estimate the effect of negative book equity on default; see for example Deng, Quigley, and Van Order (2000) and Pennington-Cross and Ho (2010).

[5] Town level unemployment rates are from the Bureau of Labor Statistics. LIBOR stands for the London InterBank Offered Rate. Coupon payments on adjustable-rate mortgages are usually indexed to LIBOR. Median household income at the zipcode level and fraction of minorities in a zipcode are obtained from the 2000 Census.

In contrast to the implication of the zero book equity default rule, the authors find that negative book equity is an important determinant of the default hazard. The baseline hazard function gives the probability of default when all variables are evaluated at their sample means and the initial LTV is set to 80%. The estimates show that a borrower with book equity between -10% and 0% is approximately 3 times more likely to default than the baseline borrower. The default hazard rises as book equity drops: borrowers with book equity less than -20% are 5 times more likely to default than the baseline.

The effects of other independent variables on the baseline default hazard are intuitive. The default hazard is increases with LIBOR, and with town level unemployment rate. Subprime borrowers have higher default hazards than prime borrowers.[6] Borrowers in single-family homes have lower default hazards than borrowers in condominiums and multi-family properties. The default hazard is higher for borrowers who live in zipcodes with lower median incomes, and for those who live in zipcodes with a larger fraction of minorities. These findings are consistent with other empirical studies on mortgage default.

While the previous study focuses primarily on negative book equity, the study by Bajari et. al. (2008) analyzes how default is affected by four factors — current house prices, expectations of future house prices, contract interest rates relative to market interest rates, and liquidity constraints — in a unified framework. In their empirical framework, default is induced either by financial incentives or by the inability of a borrower to pay his mortgage coupon. The financial incentives for default depend on the current value-to-loan ratio; the present value of the

[6]Homeowners who borrow from subprime lenders are labeled subprime borrowers.

remaining payments computed using the market interest rate minus the present value computed using the mortgage interest rate; months remaining until a payment reset; and interactions of the value-to-loan ratio with the expectation and variance of future house price growth. In the benchmark specification borrowers expect house prices next period to grow at same rate as house prices in the current period. The authors consider two other specifications. In the first, borrowers have perfect foresight about house prices one period into the future. In the second, borrowers' house price expectations are imputed from the user cost of housing; see Himmelberg, Mayer, and Sinai (2005) for a discussion of user costs.

A borrower's ability to pay his mortgage coupon depends on the borrower's payment-to-income (PI) ratio; a vector of variables that are likely to covary with the borrower's budget and credit constraints (FICO score, employment status, other assets, presence of other liens, initial LTV, level of income documentation during mortgage origination); and the interaction between the PI ratio and the covariates. The empirical setup estimates the role of each factor in mortgage default. Since the econometrician cannot observe whether financial incentives or the inability to pay led the borrower to default, the authors model the default decision as a bivariate probit model with partial observability.

Bajari, Chu, and Park (2008) use LoanPerformance data, a division of First American CoreLogic, on mortgages originated between 2000 and 2007. The dataset only covers securitized subprime and Alt-A mortgages. The authors focus on first liens of fixed and adjustable-rate mortgages only. They supplement LoanPerformance data with house price data from the Case-Shiller house price index at the level of Metropolitan Statistical Areas (MSAs). Loan level data are also matched with data from the 2000 Census to control for demographic characteristics at the zipcode level. County level monthly unemployment rates reported by the Bureau of Labor Statistics (BLS) are used as proxies for individual employment status.

The authors find that a one standard deviation increase in the value-to-loan ratio decreases the default hazard by 7.55%, after controlling for house price expectations, volatility, and MSA and year fixed effects. A one standard deviation increase in the monthly PI ratio increases the default hazard by 17.15%. This effect is stronger for households with FICO scores less than 700; borrowing constraints are more likely to bind for these households. Low documentation on a mortgage loan increases the probability of default by 40%. A second lien on a mortgage increases the probability of default by 125%. A one standard deviation increase in FICO scores decreases the default hazard by 77%. A one standard deviation increase in initial LTV and in the county unemployment rate increases the default hazard by 21% and 10%, respectively. A borrower who expects house prices to appreciate at a rate of 10% above the sample average has a default hazard 4.22% lower than his counterpart in the average housing market. Surprisingly, borrowers are more likely to default when the forward looking and user cost measures suggest strong house price appreciation. The authors conclude that these two measures are ill-suited for measuring borrowers' expectations of house price growth. The interaction term between housing market volatility and the value-to-loan ratio is also significant: a one standard deviation increase in the volatility at the average value-to-loan ratio lowers the default hazard by 2.77%.

The authors also compare the relative contribution of each factor to the differential default performances of various vintages. They ask: how much of the increase in the default rate of the 2006 vintage, relative to the 2004 vintage, can be attributed to changes in its observable characteristics? The findings suggest that the large decrease in house prices coupled with declining credit quality account for the higher default rate of the 2006 vintage.[7] Declining home equity increases the default hazard for the 2006 vintage, relative to the 2004 vintage, by 56%; lower FICO scores in the 2006 vintage increase the relative default hazard by 70%; lower expectations of house price growth increase the relative default hazard by 40%.

1.4 Default Thresholds

Recent work by Bhutta, Dokko, and Shan (2010) estimates default thresholds in mortgage data directly. In their study, a borrower is said to default when he is 90+ days delinquent for more than two months. The time of default is defined to be 3 months prior to the month of 90+ day delinquency. The default threshold is the book equity normalized by the house price, see (1.9), at the time of default. The authors find that the median borrower defaults strategically when normalized book equity equals −62%. The study uses data from LoanPerformance. The authors focus on first liens of purchase nonprime mortgages with combined loan-to-value (CLTV) ratios of 100%, originated in 2006 in Arizona, California, Florida and Nevada.

[7] Mayer, Pence, and Sherlund (2009) and Demyanyk and Van Hemert (2011) come to a similar conclusion.

The study employs a two-step maximum likelihood procedure that accounts for censoring in the sample; 22% of the sample does not default. The two-step procedure allows separation of defaults that are caused by adverse life events from defaults that are strategic. In the first step, the authors estimate a logit model in which default depends on changes in the mortgage interest rate (including two lags), four-quarter change in county unemployment rates (including a quadratic term), four-quarter change in the 60+ day credit card delinquencies for a county (including a quadratic term), along with loan-age and time dummies. The logit model also includes dummies for various levels of book equity; excluding book equity dummies would overestimate the role of liquidity shocks. Book equity is measured using zipcode level repeat sales house price indexes by First American CoreLogic. Once the logit model is estimated the authors use the fitted values of the regression to construct the probability of a household defaulting due to a liquidity shock, based on observable characteristics. This probability is incorporated into the maximum likelihood estimation in the second step. The authors assume that the absolute value of normalized book equity follows a Gamma distribution. The second step involves the maximum likelihood estimation of the shape and scale parameters of the Gamma distribution.

The estimates of the shape and scale parameters are 1.68 and 45. The estimates imply that the median borrower walks away when normalized book equity falls to -62%. The standard deviation of the distribution is 58%. The authors also find that the median borrower in recourse states (Florida and Nevada) defaults when normalized book equity is 20-30% lower (more negative) than the median borrower in non-recourse states. Borrowers with higher FICO scores default at lower levels of normalized book equity. The median default threshold for borrowers with FICO scores between 620-680 is -51%, as compared to -68% for borrowers with FICO scores greater than 720. Normalized book equity also differs across mortgage types. The median borrower with a short-term hybrid mortgage defaults when normalized book equity is -50%, which is 30% higher than normalized book equity for the median borrower with a fixed rate mortgage.

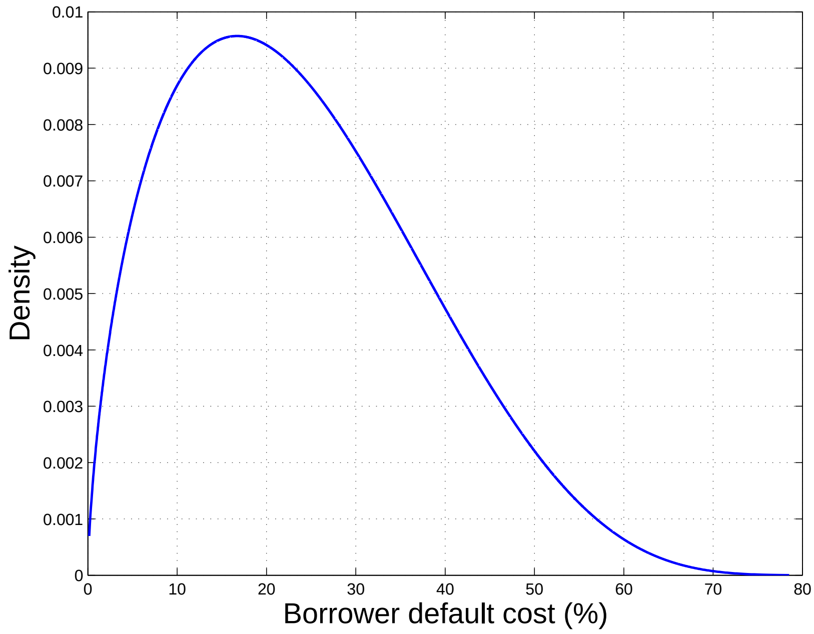


Figure 1.2: The probability density function of the distribution of borrower default costs implied by a model of strategic default with zero lender default costs for default thresholds estimated by Bhutta, Dokko, and Shan (2010). Borrower default costs are expressed as a percentage of the purchase price.

The findings of Bhutta, Dokko, and Shan (2010) can be used to inform the choice of borrower default costs in the model presented in Section 1.2. Figure 1.1

shows that a borrower with 100% initial LTV ratio who defaults when normalized book equity equals -62% faces default costs equity to 30% of the purchase price of the property; the point (100, -62) is marked with a solid black dot. Figure 1.2 shows the entire distribution of borrower default costs k_b implied by the work of Bhutta, Dokko, and Shan (2010); recall that k_b is measured as a percentage of the purchase price. The standard deviation of k_b is 15.58%.

1.5 Payment Size

The size of mortgage payments is an important determinant of default according to theories of both strategic and non-strategic default. Evaluating the importance of payment size empirically, however, has been difficult because borrowers usually refinance their mortgage when payments increase. This behavior confounds estimates of payment increases because it leads to borrower selection. Fuster and Willen (2012) overcome the selection problem by studying the effect of payment size in a group of borrowers who experienced a drop in payment, making refinancing unattractive. They find that reducing a borrower's payment in half reduces the probability of becoming delinquent by 55%.

The study exploits resets on Alt-A hybrid adjustable-rate mortgages (ARMs) with a 10-year interest-only period. Hybrid ARMs feature a fixed coupon payment for an initial period of 3, 5, 7, or 10 years. The mortgage coupon rate adjusts semi annually or annually after the initial period. The new rate depends on the level of market interest rates, as measured by LIBOR or one year Treasury Bill rates. The 10-year interest-only feature implies that the size of the payment for mortgages with an initial period of 3, 5, or 7 years increases (decreases) at the first reset date with an increase (decrease) in the market interest rate. In contrast payment size for a mortgage with an initial period of 10 years usually increases at the first reset date, even if the market rate decreases, because the mortgage starts amortizing at this date. The authors focus on first liens of Alt-A hybrid ARMs originated between January 1, 2005 and June 30, 2006; the data are from LoanPerformance. The focus is on 10-year interest-only hybrid ARMs originated during this time period because, in this sample, hybrid ARMs with 3- and 5-year initial periods experienced a large drop in payments when they first reset; the mean reduction in the interest rate for mortgages with a 5-year initial period was about 3%.

The empirical specification uses a Cox proportional hazard framework; this specification is standard in the mortgage default literature, see Section 1.3. Since the study focuses on changes in mortgage interest rates, the effect of initial and current mortgage interest rates is captured flexibly using indicator variables for bins of various interest rate values. The estimates indicate that a 2% (3%) drop in the payment at the first reset date lowers the baseline default probability by 40% (55%). The reduction in default probability from the 2% (3%) drop in payments corresponds to a reduction in baseline CLTV from 135 to 105 (145 to 95), holding payment fixed. The coefficients on other control variables are consistent with prior studies.

1.6 Mortgage Yields

Many analysts purport that mortgage default risk was systematically underpriced prior to the financial crisis of 2008. Krainer, LeRoy, and O (2009) ask whether the data support this view. They calibrate the model of strategic default presented in Section 1.2 and compare the model implied mortgage yield spreads to those in the data. Contrary to the claim of systematic underpricing, the authors find that the model generates mortgage yield spreads that are consistent with the data for empirically realistic parameter values.

The study uses data from LPS Applied Analytics. It focuses on purchase FRMs and ARMs originated between 2000 and 2007 on California properties. The authors note that default in the model depends only on initial LTV and house price changes. In practice, however, other risk characteristics that are correlated with initial LTV also matter for default; examples include location of property, borrower credit score, mortgage type, subprime status etc. The authors separate the contribution of initial LTV to the initial yield spread from that of other risk factors by running an OLS regression of the yield spread on all risk characteristics. Following standard practice in the mortgage default literature the authors model initial LTV as a linear spline. The adopted specification uses 5% buckets with 45-50% being the first and 95-100% being the last. They find that the coefficient on the 95-100% LTV bucket corresponds to a spread of 40 basis points for FRMs, and 90 basis points for ARMs. The interpretation being that, conditional on other risk characteristics, raising the initial LTV on a FRM (ARM) from 45-50% to 95-100% raises initial yield spreads by 40 (90) basis points.

Focusing on FRMs, the authors calibrate the model to see if empirically realistic parameter values generate an initial yield spread of approximately 40 basis points. The discount rate is $\rho = 7\%$ and the expected proportional gain in housing services is $\alpha = 3\%$. These parameters generate a reasonable price to rent ratio, once maintainence and utilities expenses incurred in practice are taken into account; see footnote 2 earlier. Two sets of parameter values generate yield spreads close to 40 basis points for mortgages with an initial LTV around 95%. The first has the volatility parameter at $\sigma = 15\%$, borrower default costs equal to 16% of the purchase price, and lender default costs equal to zero. These parameter values along with $c = 1.75$ generate a mortgage with an initial LTV of 92% and an initial yield spread of 60 basis points. The second set of parameters has $\sigma = 10\%$, and borrower and lender default costs equal to 16% of the purchase price. In this case, $c = 1.75$ generates a mortgage with an initial LTV of 95% and an initial yield spread of 36 basis points. The authors note that the lender default costs are unrealistically zero in the first parametrization, and that the volatility parameter might be unrealistically low in the second; see footnote 3 earlier.

1.7 Moral and Social Attitudes

Guiso, Sapienza, and Zingales (2009) complement existing work on strategic default by using survey data to study the willingness of American households to default strategically. Motivated by discussions in the popular press regarding the morality of strategic default, the authors try to determine the moral and social attitudes of homeowners towards strategic default. The study uses data from the December 2008 and March 2009 waves of the Chicago Booth Kellogg School Financial Trust Index Survey, in which a representative sample of 1000 households were surveyed. The authors only include the respondents who reported being in charge of the household financials, possibly along with their spouse. Only the responses of homeowners are reported.

The survey elicited the willingness to strategically default by asking the following question: "If the value of your mortgage exceeded the value of your house by 50K would you walk away from your house (that is, default on your mortgage) even if you could afford to pay your monthly mortgage?" Homeowners could answer "yes", "no", or "I don't know". Those who answered no were asked if they would default when the mortgage value exceeded the value of the house by 100K. Homeowners who responded negatively to the previous question were asked the same question with the 100K replaced by 300K (December wave) or 200K (March wave). The responses are qualitatively consistent with theory: 9% of homeowners are willing to walk away at 50K, 26% at 100K, 41% at 200K, and 45% at 300K. These numbers correspond to total defaults of 9% at 50K, 32% at 100K, 61% at 200K (March wave), and 63% at 300K (December wave).

Homeowners' views regarding the morality of strategic default were obtained by asking: "Do you think that it is morally wrong to walk away from a house when one can afford to pay the mortgage?" 81% responded yes. People who consider strategic default immoral are less likely to indicate their willingness to strategically default, conditional on their mortgage value exceeding the value of their house by a specific amount. For example, only 7% of yes-respondents are willing to strategically default when the difference between the two values is 50K, as compared to 20% of no-respondents. In order to determine social attitudes towards strategic default, the survey asked the participants if they knew a strategic defaulter. Nine percent of homeowners knew one, and 26% of these homeowners perceived that default as strategic. Conditional on morality, homeowners who know a defaulter, and perceived that default as strategic, are 82% more likely to declare their intention to default strategically.

1.8 Second Liens

For institutional reasons home purchases with initial LTV ratios greater than 80% are usually financed using two separate mortgage liens. The first lien has an 80% LTV ratio, and the rest of the loan is financed by the second lien. For example, the first and second liens on a home purchase with a 5% downpayment will have LTV ratios of 80% and 15%. Second lien mortgages allow borrowers to have lower equity in their homes. New borrowers can use second liens to buy a house with little or no downpayment. Existing borrowers can take out second liens after the origination date and use them to finance consumption by borrowing against built up equity in the house. Second mortgages fall into one of two categories: closed end home equity loans (HELOANs) or revolving home equity lines of credit (HELOCs). Lee, Mayer, and Tracy (2012) provide an overview of the market for second lien mortgages. These authors note that understanding the default behavior of borrowers with multiple liens is important because second liens grew in importance rapidly prior to the crisis: the total balances outstanding on second lien mortgages was $1.1 trillion in 2006. They also point out that most second liens are not securitized; they are held on lenders' balance sheets instead. Therefore default in this sector of the mortgage market directly affects the financial health of banks.

Lee, Mayer, and Tracy (2012) find that 21% of HELOANs (31% of HELOCs) remain current four quarters after the first mortgage becomes 90 days delinquent. Their findings are similar to Jagtiani and Lang (2010) who find that 31% of borrowers in their sample are 90+ days delinquent on their first lien while staying current on the second (24% for HELOANs and 34% for HELOCs). The second study also finds that, surprisingly, 20% of borrowers stay current on their second mortgage even after the first lender forecloses on the property. Qualitatively, it is not clear that defaulting on a particular lien dominates. On the one hand, if borrowers with HELOCs default on their first lien and stay current on their second, they maintain access to a line of credit until the first lender forecloses. Access to such credit maybe very valuable during times of financial distress. On the other hand, borrowers might want to default on their second lien and stay current on their first lien because the second lender maybe less inclined to initiate foreclosure proceedings. The reluctance of the second lender to foreclose stems from the fact that he must acquire the first lien before he initiates foreclosure proceedings, thereby exposing himself to additional default risk.

Jagtiani and Lang (2010) merge credit data from the Federal Reserve Board Consumer Credit Panel (Equifax database) with mortgage data from LPS Applied Analytics; Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) describe how the two datasets are merged. In addition, they use state-level home price indexes from the FHFA. The focus is on homeowners who have a first mortgage and at least one second mortgage between 2004:Q4 and 2010:Q2. The authors run a logistic regression in which the probability of default (90+ days delinquent) depends on explanatory variables that include log monthly payment on the first mortgage; monthly payment on the second mortgage relative to the first; log of unused home equity credit; dummy for mortgage type (prime, Alt-A, subprime); credit card utilization rate; credit score; along with dummies for current LTV greater than 90, current CLTV greater than 90, loan modification, jumbo, option ARM, HELOC, and time. Consistent with prior studies, the most important determinants of the probability of default are the monthly payment on the first mortgage (positive coefficient), credit score (negative coefficient), negative book equity (positive coefficient on LTV dummy), and loan modification (positive coefficient). Borrower credit score and negative book equity are the most important determinants of the probability of default on second mortgages. Loan modifications increase HELOAN default probability, but not HELOC default probability.

Borrowers who default on their first mortgage while keeping their second lien current tend to have larger first lien payments, smaller ratio of second-to-first lien payments, negative book equity, lower credit scores, and subprime classification. Borrowers with HELOCs are more likely to default this way. On further investigation, the authors find that second lenders do not reduce the line of credit for 90% of the borrowers who default on their first mortgage. The average borrower in this position continues to maintain his pre-default utilization rate of around 90% on his HELOC, even three quarters after defaulting on the first mortgage. Borrowers who increase their HELOC utilization rates do so in small amounts, 2.6% on average. The average borrower who decreases his utilization rate does so by 4.2%.

The fact that borrowers do not change their HELOC utilization rates much after defaulting on their first mortgage suggests that maintainence of a credit line may not be the primary reason why distressed borrowers keep their second mortgages current.

1.9 Lender Recourse

Mortgages in the United States are perceived to be non-recourse loans. This is inspite of the fact that mortgage lenders in many states have legal recourse in the form of deficiency judgements against defaulting borrowers. Analysts point out that mortgage recourse laws are ineffective because deficiency judgements have little value: they are costly, defaulting borrowers have little non-housing wealth, and there are many ways to defeat the judgement. The perception is corroborated by the observation that deficiency judgements are rarely observed in practice. Ghent and Kudlyak (2011) find that conventional wisdom regarding recourse is incorrect. Recourse deters borrowers from defaulting: the probability of default in non-recourse states is 32% higher, at the average value of negative book equity for defaulted loans.

These authors use data on mortgages originated between August 1997 and December 2008, provided by LPS Applied Analytics. They note that the effect of recourse on default is not evident in unconditional default rates across states; unconditional default rates across states are statistically indistinguishable. It is not evident in the conditional default rates from a probit model either. The authors regress the probability of default on the value of the default option (linear and squared), controls for loan and borrower characteristics across states, and a dummy variable for recourse.[8] The controls include divorce and unemployment rate for the state (not seasonally adjusted), a categorical variable for the value of the prepayment option, age of the loan in months, LTV at origination, the borrower's FICO score at origination, dummies for mortgage type (interest-only, adjustable-rate, jumbo, or non-purchase), and a dummy for LTV ratio being exactly 80% (also interacted with the default option value and the value squared). The estimates show that conditional default rates across states are statistically indistinguishable; the coefficient on the recourse dummy is statistically insignificant at the 10% level.

Even though the unconditional and conditional default rates are statistically indistinguishable across recourse and non-recourse states, the presence of recourse does affect default behavior. The effect is captured by the coefficient on the interaction between recourse and the value of the default option, which is negative and significant at the 5% level — borrowers in non-recourse states are more sensitive to negative book equity than borrowers in recourse states. Ceteris paribus, a 1% drop in book equity increases the probability of default more in non-recourse states, as compared to recourse states. The effect is nonlinear: the coefficient on the interaction between recourse and the quadratic default option term is positive and significant at the 5% level. The effect is stronger for houses with higher

[8] The authors measure the value of the default option as the probability that the book value of equity, divided by the imputed value of the house, is less than zero. They calculate the book value of equity as the imputed value of the house minus the principal outstanding on the mortgage. The imputed value of the house at a particular date is the purchase price of the house multiplied by the gross return on the Office of Federal Housing Enterprise Oversight (OFHEO) house price index for the state at that date, calculated from the date of mortgage origination. The house price index is assumed to follow a log-normal distribution.

appraised values at the mortgage origination date. The authors note that these findings are consistent with borrower default thresholds in non-recourse states being higher than default thresholds in recourse states.

Even though deficiency judgements are infrequent in the data, they do influence default behavior. The threat of a deficiency judgement induces defaulting borrowers to cooperate with the lender. Borrowers in recourse states are 10% less likely to default using a foreclosure, as opposed to a short sale or a deed in lieu.[9] Borrowers prefer cooperative methods of default because they affect access to future credit less severely than a forcible eviction. These findings show that the conventional wisdom that deficiency judgements are rarely observed because they are not valuable is incorrect. In fact, deficiency judgements are rarely observed because they pose a credible threat to borrowers, who respond by cooperating with the lender when they default.

[9]A short sale occurs when the borrower sells the house for less than the debt outstanding and turns over the proceeds to the lender, who waives his right to deficiency. A deed in lieu occus when the borrower turns over the house keys to the lender, who forgives the outstanding debt. The transfer of the deed is risky. It maybe deemed improper by a bankruptcy court if the borrower files for bankruptcy within a year of the transfer. Moreover, unlike a foreclosure, the transfer does not eliminate any subordinate liens on the property.

Borrowers in recourse states may also default using a friendly foreclosure, in which the borrower waives his right to contest the foreclosure and submits to the court's ruling. A friendly foreclosure cuts off subordinate liens on the property and protects the lender if the borrower files for bankruptcy. Friendly foreclosures take longer than deed transfers, but not as long as contested foreclosures. The authors cannot observe whether a foreclosure is friendly or not, so they cannot identify the effect of recourse on friendly foreclosures.

1.10 Insolvency, Credit Constraints, and Unemployment

The model of default presented in Section 1.2 assumes that the borrower has enough financial resources to make his mortgage payment at all times. This assumption rules out default due to job loss, divorce, medical bills, and other adverse life events that affect the borrower's ability to meet his mortgage obligation. The adopted model specification is, of course, unrealistic. A borrower's ability to meet his mortgage payment does matter for default, implying a role for adverse life events. For example, a borrower who does not have enough cash, either because he has no savings or because he cannot borrow from other sources, will not be able to pay his mortgage. This section discusses papers that focus on how factors that affect borrowers' ability to pay impact mortgage default. Even though the emphasis is on life events and credit constraints, negative book equity continues to play an important role in mortgage default. Borrowers with positive book equity would prefer to sell the house and prepay the mortgage loan instead. Since borrower's who default because of adverse life events also have negative book equity, studies often refer to these defaults as being induced by two "triggers" or "double trigger" default.

A major difficulty faced by analysts who study the relationship between adverse life events or credit constraints and mortgage default is that existing mortgage datasets do not usually provide detailed information on the financial position of borrowers. The empirical studies discussed here overcome this difficulty by combining two separate data sources. Elul, Souleles, Chomsisengphet, Glennon, and

Hunt (2010) take this approach and ask whether negative book equity or credit constraints faced by borrowers are relatively more important for mortgage default. These authors combine mortgage data from LPS Applied Analytics and credit data from Equifax.[10] They focus on FRMs originated in 2005-2006 for purchase of single-family owner occupied properties, with maturities of 15, 30, or 40 years. The credit data allows them to observe key characteristics of different types of debts held by individuals. In addition, the authors employ MSA-level house price indexes from the FHFA and county level unemployment rates from the BLS.

Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) estimate a dynamic logit model with a dummy for default (60+ day delinquency) as the dependent variable. The independent variables control for various borrower and mortgage characteristics including initial LTV, FICO score, current CLTV, bank card utilization rate, total second mortgage balance, change in county unemployment rate over the previous twelve months, and interactions of current CLTV with bank card utilization rate and with change in unemployment. Current CLTV, card utilization rate, and twelve-month change in unemployment are all modeled as linear splines in order to allow for flexibile estimation.

Consistent with other studies, the authors find that current CLTV is an important determinant of mortgage default. The default probability is monotonically increasing in current CLTV. For example, increasing current CLTV from 50 to above 120 increases the probability of default 1.3 percentage points per quarter (pp/q). Default probabilities also increase with bank card utilization rates, even

[10] Borrowers in the two datasets were linked based on uniquely matched open date, initial balance, and zip code of the first mortgage.

after controlling for CLTV and other risk factors. For example, increasing the utilization rate from below 70% to above 80% raises the default probability by 0.9 pp/q. Both the interaction terms are statistically significant and quantitatively important. The default probability of borrowers with current CLTV between 90-100% and high card utilization rates (greater than 80%) is 1.5 pp/q larger than that of borrowers with low utilization rates. Similarly, the default probability of borrowers with current CLTV above 120 who live in counties that experienced large increases (greater than 1.25 pp) in the twelve-month unemployment rate is 1.1 pp/q larger than that of borrowers in counties that experienced small increases in the twelve-month unemployment rate.

Gerardi, Herkenhoff, Ohanian, and Willen (2013) note that most studies that focus on the "double trigger" hypothesis of mortgage default do not have data on the employment status of borrowers. These studies use regional unemployment rates as a proxy instead. The studies usually find that regional unemployment, by itself, is not a quantitatively important determinant of default. Gerardi, Herkenhoff, Ohanian, and Willen (2013) show that regional unemployment rates do not proxy well for individual employment status. The authors use data from the Panel Study of Income Dynamics (PSID), which contains information on mortgage characteristics, employment status of borrowers, and borrowers' balance sheets. Contrary to many other studies, these authors find that individual employment status is the strongest predictor of default; Gyourko and Tracy (2013) arrive at a similar conclusion using simulated data. The default probability of an individual who loses his job increases by 5-13 pp; for comparison the average default rate in the sample is 3.9%.

The authors estimate a logit model. They regress a dummy for default (60+ day delinquency) on mortgage characteristics, borrower characteristics, and variables that proxy for local economic conditions and economic environment. Mortgage characteristics include CLTV (linear spline), mortgage type, mortgage interest rate, and dummies for maturity greater than 15 years, presence of second lien, and refinance. Borrower characteristics include total household income, liquid and illiquid assets owned, unsecured debt outstanding, hospital bills outstanding, and employment status of the head of the household. Variables for local economic conditions include recent house price appreciation as measured by FHFA state-level indices, state-level unemployment rates, and dummies for recourse, judicial foreclosure, and properties located in Arizona, California, Florida, or Nevada. The results show that CLTV is an important determinant of default. The default probability for households with CLTV ≥ 120 is 6 pp larger than that for households with positive book equity. The default probability for households who have been unemployed 6+ months is 9 to 10 pp larger than that for employed households. On interacting the dummy for employment status with CLTV dummies, the authors find that near-negative book equity combined with job loss increases the default probability by 8 pp.

Most papers discussed in this section so far have been empirical. The theoretical model presented in Section 1.2 abstracts from the role played by adverse life events in mortgage default. Campbell and Cocco (2011) address this gap in the literature by studying the mortgage default decision of a household in an economic environment that features four different sources of risk: labor income, inflation, interest rates, and house prices. The authors consider default in FRMs, ARMs, and interest-only balloon payment mortgages.

Households in the model are active for a finite number of periods. Household preferences are separable over housing, non-durable consumption, and terminal wealth; house size is fixed for simplicity. In each period households choose non-durable consumption and savings. Households save in one-period bonds and housing. The housing decision requires them to choose between making their mortgage payment, defaulting, or selling the house and prepaying the mortgage. Households that default or sell must rent thereafter. Rent equals the user cost of housing multiplied by the value of the house. Home sales are subject to realtor fees. Motivated by the existence of social welfare programs in practice, households are assumed to always have access to a subsistence level of cash. This assumption rules out defaults caused by high marginal utilities of the non-durable consumption good.

Nominal interest rates in the model are stochastic because both the expected inflation rate and the ex ante real interest rate are stochastic. Expected inflation follows an AR(1) process. Log real returns follow a random walk. These processes are calibrated using data from the Federal Reserve. Labor income cannot be traded or collateralized. It depends on age, individual characteristics, a permanent shock, and a transitory shock. Income is taxed at a constant rate. Real house prices follow a random walk with positive drift. Homeownership serves as a hedge against inflation. Homeowners pay property taxes and maintainence fees that are proportional to house prices each period; the tax rate and the fee rate differ from each other and do not change over time. House prices and the permanent component of labor income are correlated. The processes for labor income and house prices are calibrated using PSID data from 1970-2005; the two processes turn out to be positively correlated.

The analysis considers three different types of mortgages: FRMs, ARMs, and interest-only (I/O) balloon payment mortgages. Nominal mortgage interest payments are income tax deductible. The credit risk premium on mortgages is exogenously specified.

The authors find that most defaults occur between between two and eight years after the origination date. As in other studies on mortgage default, the analysis shows that negative book equity is a necessary but not a sufficient condition for default. Households with negative book equity between 0 and -20% of the outstanding mortgage balance tend to default when their current mortgage payment-to-income ratio (MTI) is large. The authors compare default behavior across mortgage types and find that cash flow considerations are relatively more important for ARMs, whereas wealth reasons are relatively more important for FRMs and I/O balloon payment mortgages. The analysis shows that borrowers with ARMs tend to default when inflation and nominal interest rates are high, resulting in higher current mortgage payments. In contrast, borrowers with FRMs tend to default when inflation and nominal interest rates are low, implying a higher debt burden in real terms. Interest rates are not a major determinant of default in aggregate states with large house price declines; negative book equity is the primary determinant of default instead.

The analysis also shows how initial LTV and initial loan-to-income (LTI) ratios affect mortgage default. Campbell and Cocco (2011) decompose the probability of default into the product of the probability of negative book equity and the probability of default conditional of negative book equity. Conditional on initial LTI, higher initial LTV ratios increase the probability of negative book equity unambiguously. The effect on the probability of default conditional on negative book equity, however, is ambiguous: higher initial LTV implies that the borrower will face negative book equity earlier in the life of the mortgage, when the value of the option to wait is the highest. Conditional on initial LTV, higher initial LTI ratios increase the probability of default unambiguously; the effect is nonlinear.

Schelkle (2012) complements the work of Campbell and Cocco (2011) by comparing default rates generated by a similar model of mortgage default to the data. He finds that predictions of a calibrated version of the model are broadly consistent with default rates in the data. The study also compares aggregate default rates in the data to default rates implied by reduced form versions of the frictionless model of strategic default and of the double trigger theory of default. The analysis uses data provided by LPS Applied Analytics on prime 30-year fixed-rate first mortgages originated between 2002 and 2008. The two models are compared based on out of sample predictions of default rates, after estimating the parameters for each model using default rates from the 2002 cohort. The findings indicate that the default rates implied by the double trigger theory are closer to the data.

1.11 Conclusion

The financial crisis of 2007-2008 has raised a lot of questions about mortgage default and the interactions of the mortgage market with the broader economy. This paper surveys recent research on what determines mortgage default. The focus on the determinants of mortgage default necessitated the omission of many interesting papers that are related to the housing and mortgage market, but do not address the question at hand directly. For example, Haughwout, Okah, and Tracy (2009) study the effectiveness of mortgage modification programs; Campbell, Giglio, and Pathak (2011) study negative externalities from foreclosures; Krainer and Laderman (2009), and Elul (2011) study whether mortgages that are securitized are different, in initial characteristics and default performance, from those that are held on bank balance sheets; Chatterjee and Eyigungor (2011), and Corbae and Quintin (2013) develop quantitative equilibrium models of the housing market that focus on aggregate homeownership rates or foreclosure rates.

Even though significant progress has been made in understanding the determinants of mortgage default, there are still many challenges and open questions. In particular, the debate on the relative contribution of negative book equity and adverse life events to mortgage default is still unsettled. Many studies on mortgage default try to inform the debate by distinguishing between defaults that are primarily a result of negative book equity — strategic default — and defaults due to adverse life events — non-strategic default. In practice, however, the distinction between strategic and non-strategic default is not clear. For example, a borrower who loses his job will not default on his mortgage unless he also has negative book equity. Is default by this borrower strategic because negative book equity is necessary for default? Is it non-strategic because he experienced an adverse life event before default? Would the borrower have defaulted on his mortgage even if he had not lost his job? These are some of the questions that arise when one attempts to classify default as strategic or non-strategic.

An alternative approach is to avoid the distinction between strategic and non-strategic default altogether and focus on how borrowers' home equity affects mortgage default decisions. Krainer, LeRoy, and O (2009) take this approach; the model developed by these authors was discussed in Section 1.2.[11] The advantage of the alternative approach is that it results in a clear analysis of mortgage default: Borrowers default when the economic value of equity is zero.

This dissertation builds on the analysis in Krainer, LeRoy, and O (2009) in order to address two important questions on mortgage default. Chapter 2 studies how optimal mortgage default by homeowners affects the valuation of mortgage backed securities. The importance of the connection between mortgage default and valuation of mortgage backed securities is highlighted by the global financial crisis of 2007-2008. The crisis was precipitated by a rise in mortage defaults that led to financial institutions facing the prospect of large losses on mortgage backed securities.

Chapter 3 studies the optimal default decision of borrowers with mortgages that feature payment resets. These mortgages grew in popularity in the buildup to the financial crisis. Some analysts of mortgage default attribute the rise in mortgage defaults to unanticipated payment increases at the reset date. Other analysts, however, point out that default rates in the data did not spike at the reset date. The analysis in Chapter 3 informs this debate by providing conditions under which optimal default boundaries in these mortgages feature a jump at the reset date. The analysis shows that, under certain conditions, the default

[11] In the alternative approach, adverse life events would affect default through the economic value of home equity. Krainer, LeRoy, and O (2009) abstract from the role of adverse life events in order to maintain tractability.

boundary is discontinuous at the reset date even if the size of the payment reset is known at the mortgage origination date.

Chapter 2

Pricing Default Risk in Mortgage Backed Securities

2.1 Introduction

Mortgage Backed Securities (MBS) were at the center of the financial crisis of 2007-2008. Large unexpected drops in house prices resulted in mortgage defaults, which in turn impaired associated MBS. Large unprecedented losses on MBS, which were thought to be virtually immune to default risk and rated so, led analysts to conclude that these securities were "toxic" and difficult to value. Banks and other financial institutions that held large quantities of these assets were adversely affected. The crisis began.

The events leading up to the financial crisis highlight the need for a framework for MBS valuation that explicitly incorporates mortgage default. The goal of this paper is to provide such a framework. The analysis is conducted in two stages.

First, I model the mortgage lending market as in Krainer, LeRoy, and O (2009).[1] Housing services follow a geometric Brownian motion, implying that changes in housing services are exogenous and unforecastable. House prices equal the expected discounted value of services so they are also unforecastable. Borrowers buy houses using mortgages. They have the option to default on their mortgages, subject to a cost. They exercise this option so as to maximize equity. Mortgage lenders make zero expected profits. I solve for equilibrium yield spreads in this setting. The equilibrium is characterized using boundary crossing properties of geometric Brownian motion. To the best of my knowledge, this solution method is new to the mortgage valuation literature. The key advantage of this method is that it reduces the valuation of mortgages to a simple present value calculation.

Second, I model the MBS market. In practice, the term mortgage backed security is applied to a large class of mortgage bonds and their derivatives. To begin with, the analysis focuses on Collateralized Mortgage Obligations (CMOs).[2] These securities are created by combining mortgages into a pool, dividing the pool into senior and subordinate tranches, structuring the pool's cash flows so as to protect the senior tranche from default risk, and selling claims to cash flows on each tranche as bonds. I assume that CMOs are created either from pools that contain one type of mortgage only — homogeneous pools — or pools that contain two types of mortgages — heterogeneous pools.

[1] Krainer, LeRoy, and O (2009) were the first to connect optimal mortgage default to equilibrium yield spreads. They did so by adapting Merton (1974) to the housing market and using the zero expected profit condition to close Merton's model.

[2] In practice, CMOs can be broadly divided into "agency" CMOs, which are insured against default risk by Government Sponsored Enterprises, and "non-agency" CMOs, which are not insured by them. The analysis here focuses on default risk so it applies primarily to non-agency CMOs.

For homogeneous pools, I find that senior bonds are risk-free or risky in equilibrium, depending on the relative size of the senior tranche. Risk-free senior bonds do not experience principal or coupon shortfalls, whereas risky senior bonds may experience principal shortfalls. The equilibrium initial yield on risky senior bonds is a linearly decreasing function of the recovery rate of the tranche. As an application, I consider the valuation of mortgages with two liens. In practice, these mortgages usually consist of a first lien with an 80% loan-to-value ratio and a smaller second lien with a 10-15% loan-to-value ratio. I provide conditions under which the first lien is risk-free in the presence of the second lien, but not otherwise.

The heterogeneous pool contains two mortgages that differ in terms of borrower default costs only; these are referred to as low default cost and high default cost mortgages. This pool experiences up to two default events. I find that, depending on the relative size of the senior tranche, senior bonds can be risk-free, low-risk, or high-risk in equilibrium. Risk-free senior bonds receive both principal and coupon payments even if both the underlying mortgages default. Low-risk senior bonds receive all coupon payments until both mortgages default, but do not recover their entire principal. High-risk senior bonds experience both principal and coupon shortfalls if the underlying mortgages default. The equilibrium initial yield on senior bonds increases as the relative size of the senior tranche increases. I also find that an increase in the fraction of high default cost mortgages has two opposing effects on equilibrium initial yields of senior bonds: it decreases the initial yield by reducing the likelihood of default and it increases the initial yield by reducing the net recovery on default.

As a quantitative exercise, I calculate model implied bond prices when data from the Case-Shiller composite-20 index is fed through the model; the data is from July 2006 to July 2011. I find that senior bonds lose about 10% of their value and residual bonds lose about 60% of their value during this time period.

Next I turn to valuation of CMO-squared and other higher order CMOs.[3] The standard models used by practitioners to value CMO-squared have been widely criticized because they do not connect valuation to mortgage default explicitly.[4] A contribution of my paper is to make this connection and provide an alternative to these models. A CMO-squared is created using a pool made from the residual tranche of the CMO, dividing this pool again into senior and residual tranches, and restructuring the pool's cash flows so as to protect the senior CMO-squared tranche from default risk. I find that the default risk of senior CMO-squared bonds is higher than that of senior CMO bonds, when the relative sizes of senior CMO and senior CMO-squared tranches are identical. The analysis in this section extends to higher order CMOs. I present an example in which the senior tranche of a CMO-cubed suffers a hundred percent principal write down when low default cost mortgages are terminated. When the Case-Shiller house price index is fed through the model, prices of senior CMO-squared bonds drop to 50% of their par values. Prices of residual CMO-squared bonds drop 100%, rendering these bonds worthless. The quantitative findings are roughly consistent with the empirical work of Cordell, Huang, and Williams (2012). These authors report that, in the data, the average principal write down on senior AAA-rated CMO-squared

[3]CMO-squared are often called Collateralized Debt Obligations in practice.

[4]Foote, Gerardi, and Willen (2008b) discuss how analysts of CMO-squared modeled correlation between mortgage defaults directly and estimated this correlation using historical data.

tranches was 67% in 2006 and 76% in 2007; the average write-down for other tranches was 93%.

Finally, I consider the valuation of Credit Default Swaps (CDSs). A CDS is insurance against default. The quantitative exercise in this section suggests that default risk is a major factor in the pricing of CDS written on the residual CMO tranche, but not for CDS on the senior CMO tranche. Comparing these findings to the price declines observed in the ABX.HE CDS index, the model suggests that default risk by itself cannot account for the large price declines observed in the senior AAA-rated index.[5] It may, however, account for a large fraction of the price declines observed in the lower-rated indexes. These findings are consistent with recent research on the ABX.HE index: Stanton and Wallace (2011) show that the observed price declines in the ABX.AAA-HE index cannot be accounted for, by any reasonable expectation regarding defaults and recovery rates on the underlying mortgages. In contrast, Fender and Scheicher (2009) conclude that default risk was an important factor in the pricing of lower-rated ABX.HE indexes.

[5] The ABX.HE index was launched in January 2006 by Markit Group Ltd. in consortium with fifteen investment banks (these banks are usually CDS sellers). The ABX.HE index tracks the price of a single CDS written on a fixed basket of 20 equally weighted CMO pools. Every CMO comprising the index must meet certain criteria; see Markit (2008) for details. The CMOs are classified based on their ratings at the origination date of the index. For example, AAA rated bonds from all the 20 pools comprise the AAA ABX.HE index. These credit ratings are the ratings agencies' assessment of the CMOs at the date of index origination. A new series of the index was scheduled for release every six months. However, the decline in house prices significantly reduced the availability of subprime CMOs, so no new series were released for vintages after 2007. The four vintages that were released according to the six month schedule are 2006-1, 2006-2, 2007-1, and 2007-2.

2.2 Mortgage Market

Setup.– The setup of the mortgage market is as in Krainer, LeRoy, and O (2009, KLO from here on). A house provides a stochastic flow of services. Housing services $x(t)$ are exogenous and follow a geometric Brownian motion:

$$dx(t) = \alpha x(t)dt + \sigma x(t)dw(t), \tag{2.1}$$

where α is the expected proportional growth rate, σ is the volatility parameter, and $w(t)$ is standard Brownian motion. I normalize initial housing services to one, $x(0) = 1$.

House prices are the expected discounted value of future services:

$$P(x(t)) \equiv \int_{z=t}^{\infty} e^{-r(z-t)} \mathbb{E}_t\big(x(z)\big)dz = \frac{x(t)}{r-\alpha}. \tag{2.2}$$

The operator $\mathbb{E}_t$ denotes the mathematical expectation conditional on information available at time t. The discount rate r is exogenous and constant. House prices can be represented by (2.2) if housing services follow a geometric Brownian motion under the risk neutral probability measure or if agents are risk neutral. This specification of house prices rules out bubbles. By (2.1) and Ito's formula, house prices also follow a geometric Brownian motion, with α as the expected proportional growth rate and σ as the volatility parameter. Since geometric Brownian motion is a Markov process, the current house price summarizes relevant past and future information. The best forecast for house prices is that they grow at the rate α.

Under the normalization $x(0) = 1$, the purchase price of the house is $P(1) = 1/(r - \alpha)$. An infinitely lived borrower buys the house for its service flow $x(t)$. He does so using a mortgage. According to the mortgage contract, the lender supplies funds that are applied towards the purchase. In return the borrower pays the coupon c to the lender. The size of the mortgage equals the total amount of funds supplied by the lender. It is exogenous. Any difference between the purchase price and the size of the mortgage comes from the borrower's personal wealth, which I do not model. Once the mortgage size is specified the loan-to-value (LTV) ratio follows. The borrower has the option to default on his mortgage, subject to a cost, at any time by paying the lender the current market value of the house. This assumption allows the market value of the house at the time of default to be less than the mortgage size. The assumption corresponds in reality to the ability of the borrower to turn over the keys and walk away from the house.

In practice, borrowers who choose to default have to bear relocation expenses, loss of future credit access, and loss of tax benefits. I incorporate these costs into the model by assuming that default is costly for the borrower. Borrower default costs are denoted k_β. These costs are exogenous, and modeled as a one-time cost paid at the time of default; the cost of default is the same regardless of the time of default. Borrower default costs are proportional to the purchase price of the house. The existence of negative equity mortgages in practice provides evidence in favor of positive borrower default costs.[6] Default costs, however, need not be positive for all borrowers. The popular press has reported instances of borrowers

[6] As noted in KLO, if borrower default costs were zero and lenders were to supply negative equity mortgages, then borrowers would default at the date of origination. Futhermore, the default behavior observed in the data is difficult to reconcile with the behavior implied by a model of costless default; see Deng, Quigley, and Van Order (2000).

living rent free in their houses after defaulting on their mortgage. Such cases are incorporated into the analysis by allowing borrower default costs to be negative.

In practice, default is also costly for lenders. Once borrowers default, lenders gain possession of the property. The cost of maintaining, repairing, and reselling the property is borne by lenders. Usually there is a lag, of a year or more, between the default date and the date at which lenders can repossess and sell the property. During this lag, lenders also lose income from coupon payments. I model these costs as lender default costs, denoted k_λ. These costs are exogenous and identical across mortgages. Lender default costs are also a one-time cost that is paid at the time of default; the cost is the same regardless of the time of default. Lender default costs are proportional to the purchase price of the house.

Default costs paid by borrowers and lenders are modeled as deadweight losses to the society. This modeling choice abstracts from the fact that some of the costs paid by the lender (borrower) might benefit the other party. For example, the lender's loss of income from coupon payments will benefit the borrower. The analysis can be modified to incorporate such transfers from the lender to the borrower, or vice versa.

I assume that the borrower cannot prepay his mortgage when its fair value exceeds its book value, even though he would like to do so. Thus credit risk is the only relevant risk faced by lenders. The mortgage market is perfectly competitive so lenders make zero expected profits.

Equilibrium.– The borrower chooses the threshold of housing services at which to default so as to minimize his mortgage liability, or equivalently maximize his equity. I solve this problem using boundary crossing properties of geometric

Brownian motion. This method reduces the valuation of mortgages to a simple present value calculation. Alternatively, one could use dynamic programming; see KLO for this approach.

The mortgage liability minimization problem is

$$\min_{d} \quad \Big\{ \mathbb{E}_0 \Big[\int_0^{w(d)} c\, e^{-rt} dt \Big] + \mathbb{E}_0 \Big[e^{-rw(d)} \big(P(d) + k_\beta \big) \Big] \Big\}, \tag{2.3}$$

where d denotes a generic default threshold and $w(d)$ denotes the time at which housing services hit the default threshold. The time of default is random because housing services are random. All mathematical expectations in the minimization problem above are conditional on information available at origination. The random variable in all the expectations above is the default time $w(d)$. The term inside the first expectation is the discounted present value of the coupons paid by the borrower until default. The term inside the second expectation is the discounted present value of the borrower's payments on default: the market value of the house $P(d)$, and default costs k_β.

The objective function in (2.3) can be simplified by noting that c, $P(d)$, and k_β can be moved out of the expectation. After evaluating the integral within the first expectation, the problem in (2.3) becomes

$$\min_{d} \quad \Big\{ \frac{c}{r} \Big(1 - \mathbb{E}_0 \Big[e^{-rw(d)} \Big] \Big) + \big(P(d) + k_\beta \big) \mathbb{E}_0 \Big[e^{-rw(d)} \Big] \Big\}. \tag{2.4}$$

Calculating the mathematical expectation of $e^{-rw(d)}$ at origination is the key to solving the minimization problem. This expectation is the moment generating function of the random default time $w(d)$ evaluated at $-r$. It equals d^m, where

$m > 0$ is the following constant

$$m = \frac{(\alpha - \sigma^2/2) + \sqrt{\left(\alpha - \sigma^2/2\right)^2 + 2r\sigma^2}}{\sigma^2}. \tag{2.5}$$

By the strong Markov property of geometric Brownian motion, the moment generating function of $w(d)$ conditional on information available at time t is $(d/x(t))^m$; see Karatzas and Shreve (1991) for a discussion.

After substituting for $\mathbb{E}_0[e^{-rw(d)}]$, the objective function is entirely in terms of the default threshold d. Standard optimization techniques apply. The optimal default threshold is denoted δ. The threshold is

$$\delta = \frac{m(r - \alpha)}{m + 1}\left(\frac{c}{r} - k_\beta\right). \tag{2.6}$$

It is strictly increasing in the mortgage coupon c, and strictly decreasing in borrower default costs k_β. From here on I drop the word optimal and refer to δ as the default threshold. I denote the optimal default time by τ; the dependence of the optimal default time on δ is supressed.

Let $M(x(t))$ denote the value of the mortgage to the lender when housing services equal $x(t)$. The value of the mortgage at origination is $M(1)$. The lender makes zero expected profits so $M(1)$ equals the size of the mortgage. The zero expected profit condition also implies that the equilibrium mortgage coupon c satisfies,

$$M(1) = \mathbb{E}_0\left[\int_0^\tau c\, e^{-rt} dt\right] + \mathbb{E}_0\left[e^{-r\tau}\left(P(\delta) - k_\lambda\right)\right]. \tag{2.7}$$

That is, c must be such that the value of the mortgage at origination $M(1)$ equals the total expected discounted value of mortgage coupons plus the expected discounted value of the lender's net recovery on default. After evaluating the mathematical expectations, I solve (2.7) for the coupon c to obtain

$$c = \frac{r\left[M(1) - (P(\delta) - k_\lambda)\delta^m\right]}{1 - \delta^m} \tag{2.8}$$

The equilibrium mortgage coupon c is strictly increasing in the size of the mortgage $M(1)$, and strictly decreasing in the lender's net recovery on default $P(\delta) - k_\lambda$.

The equilibrium values of δ and c are found by jointly solving equations (2.6) and (2.8). Once the equilibrium coupon is calculated, the initial yield on the mortgage $c/M(1)$ follows. The equilibrium asset value of the mortgage at any time $t > 0$ can be calculated using an expression similar to (2.7),

$$M(x(t)) = \frac{c}{r} - \left(\frac{c}{r} + k_\lambda - P(\delta)\right)\left(\frac{\delta}{x(t)}\right)^m. \tag{2.9}$$

The first term on the right hand side of (2.9) is the value of the mortgage in the absence of default. The second term is the adjustment for default. On default, the lender loses all future coupon payments c/r, pays the default cost k_λ, and gains the market value of the house at the time of default $P(\delta)$. The equilibrium asset value of the mortgage is increasing in housing services $x(t)$ because default in the near future becomes less likely as $x(t)$ increases. As $x(t)$ approaches infinity, the value of the mortgage approaches c/r, its value in the absence of default. At the default threshold δ, the value of the mortgage equals its net recovery, $M(\delta) = P(\delta) - k_\lambda$. The recovery rate of the mortgage is $M(\delta)/M(1)$.

2.3 Collateralized Mortgage Obligations

The analysis here focuses on the impact of default risk on valuation of MBS so I consider a simple institutional structure in which the lender originates, restructures, and services the MBS. The adopted simplification abstracts away from informational asymmetries between various entities involved in the mortgage securitization process in practice; see Ashcraft and Schuermann (2008) for a discussion. Even though CMOs usually have many tranches, the analysis only considers pools that are divided into two tranches. This simplification captures the essential feature of CMOs: the disproportionate division of default risk among senior and subordinate tranches. None of the substantive conclusions, however, rely on the assumption of two tranches. The analysis proceeds in two stages. First I consider a CMO created from a pool that contains one type mortgage only — a homogeneous pool. Then I analyze CMOs created from a pool that contains two different types of mortgages — a heterogeneous pool. Pools with many mortgages can be studied by modifying the two-mortgage pool analysis appropriately. Before discussing the valuation of CMOs, I discuss how these securities are created in practice.

The creation of MBS in practice involves many different entities. The originator of the mortgage usually sells it to a servicer, a financial institution that is responsible for collecting coupon and recovery payments from the underlying mortgage. The servicer buys many different mortgages, combines them into a pool, and sells the pool to a trust.[7] The trust sells mortgage backed bonds to

[7]A trust is a "Special Purpose Vehicle" (SPV) that is legally separate from the servicer, even though it might be one of its subsidiaries. The legal separation ensures that the assets of the

investors. These bonds may simply represent pro rata claims to the cash flows of the pool; such bonds are called mortgage pass-throughs. Alternatively the pool maybe divided into tranches and its cash flows restructured so as to divide default risk disproportionately among the tranches. The cash flows are structured such that each tranche is protected from default by its subordinate tranches. The resulting bonds are called Collateralized Mortgage Obligations (CMOs).

The disproportionate division of default risk is achieved by making the subordinate tranches absorb all losses first. The senior-most bondholders do not lose principal or coupon payments until the losses are so large that the all subordinate bonds have been wiped out. If any of the underlying mortgages default, the recovery from these mortgages is used to pay back the senior-most bondholders, while the losses on these mortgages are applied to the subordinate bonds. Similarly, any prepayments are applied to the senior-most bonds first. The bond administration is carried out by a trustee, who oversees the entire transaction on behalf of the investors and forwards all payments to them. The rating agencies rate the bonds. The senior-most bonds have the highest ratings because they have the lowest exposure to default risk; these bonds were usually rated AAA prior to the crisis. The ratings decline as the level of subordination decreases.

2.3.1 Homogeneous Pool

A homogeneous pool contains a unit measure of one type of mortgage only. The characteristics of the underlying mortgage and the pool are identical. For

SPV do not belong to the servicer. Therefore if the servicer declares bankruptcy, its creditors cannot claim the assets of the SPV; see Gorton (2010) for further discussion of SPVs.

example the origination value, coupon, and recovery on the underlying mortgage and the pool are identical; the recovery on the pool is the payout to pool when the underlying mortgage defaults. Denote the value of the pool by $V_p(x(t))$, its coupon by c_p, and its recovery on default by R_p. The pool receives its coupon c_p until default and then it recovers R_p. The initial yield on the pool is $c_p/V_p(1)$. The recovery rate of the pool is $R_p/V_p(1)$.

Tranches.– The lender divides the pool by value into two tranches — a senior tranche and a residual tranche. Variables associated with the senior tranche are indexed by s and those associated with the residual tranche are indexed by j. The lender sells bonds that are pro rata claims to cash flows on each tranche. The proceeds from the bond sale finance the initial loan to borrowers. The cash flows to the pool are divided among the tranches so as to make the senior tranche relatively safe, and the residual tranche relatively risky. The disproportionate division of default risk is obtained by giving the senior tranche first claims to cash flows, thereby making the residual tranche absorb all losses first.

The proportional value of the senior tranche at origination is exogenous. It is denoted by $0 \leq \theta \leq 1$. Let $V_s(x(t))$ denote the value of the senior tranche when housing services equal $x(t)$. The value of the senior tranche at origination is $V_s(1) = \theta V_p(1)$. The senior tranche receives its coupon c_s until the mortgage underlying the pool defaults. The recovery on the senior tranche is the payout to the tranche when the underlying mortgage defaults. The recovery is

$$R_s = \min \left\{ V_s(1), R_p \right\}. \tag{2.10}$$

On default, the lender attempts to pay the senior tranche its entire principal $V_s(1)$. If he cannot do so, then he applies the entire recovery on the pool R_p to the senior tranche; the senior tranche has first claims to the recovery on the pool. If the senior tranche recovers its entire principal then the recovery on the pool is adequate, $R_s = V_s(1)$. Otherwise, the recovery on the pool is inadequate. By the definition of $V_s(1)$, the recovery is adequate when the proportional value of the senior tranche at origination is less than the recovery rate of the pool, $\theta \leq R_p/V_p(1)$. Conversely, the recovery is inadequate when $\theta > R_p/V_p(1)$.

The recovery rate of the senior tranche is $R_s/V_s(1)$. The recovery rate of the tranche is always greater than the recovery rate of the pool. When the recovery on the pool is adequate, the recovery rate of the tranche is one. When the recovery on the pool is inadequate, the recovery on the tranche equals the recovery on the pool, $R_s = R_p$, and the value of the tranche at origination is less than the value of the pool, $V_s(1) \leq V_p(1)$. Therefore the recovery rate of the tranche is greater than that of the pool, $R_s/V_s(1) \geq R_p/V_p(1)$. The recovery rate of the tranche and the pool equal each other when $\theta = 1$.

The value of the residual tranche is $V_j(x(t)) = V_p(x(t)) - V_s(x(t))$. Its coupon is $c_j = c - c_s$ and its recovery is $R_j = R_p - R_s$. The recovery on the residual tranche is strictly positive when the recovery on the pool is adequate, and $R_p \neq V_s(1)$. It is zero when either the recovery on the pool is inadequate, or $R_p = V_s(1)$. The recovery rate of this tranche is less than the recovery rate of the pool. It is so because the recovery rate of the pool is the proportional weighted average of the recovery rate of the tranches, and the recovery rate of the senior tranche is greater than that of the pool.

The coupons c_s and c_j are endogenous. They reflect the default risk of the associated tranche. The recovery on the tranches admits the following interpretation: On default the bond manager first buys back outstanding senior bonds at their market value. Any cash left over after the senior bond buyback is used to buy back residual bonds at their market value. This interpretation will be important for the analysis of heterogeneous pools below.

Equilibrium.– To find a CMO market equilibrium, I need to find the coupon at which the senior tranche is issued at par; the coupon on the residual tranche follows. The equilibrium condition is

$$V_s(1) = \mathbb{E}_0 \left[\int_0^{\tau} e^{-rt} c_s dt \right] + \mathbb{E}_0 \left[e^{-r\tau} R_s \right]. \tag{2.11}$$

The left hand side of (2.11) is the value of the senior tranche at origination and the right hand side is the expected discounted value of the payments to this tranche. The mathematical expectations in (2.11) are evaluated using the moment generating function of τ; the senior coupon c_s follows. The implied initial yield on the senior tranche is

$$\frac{c_s}{V_s(1)} = \frac{r \left[1 - (R_s/V_s(1))\delta^m\right]}{1 - \delta^m}. \tag{2.12}$$

The initial yield on the senior tranche is a linear function of its recovery rate, $R_s/V_s(1)$. When the recovery on the pool is adequate, the recovery rate of the tranche is one and the initial yield equals r. The intuition behind this result is the following: When the recovery is adequate, the senior tranche does not face default risk. Therefore the tranche must earn the risk-free rate r in equilibrium.

When the recovery on the pool is inadequate, the initial yield on the senior tranche is greater than r. The difference between the initial yield on the tranche and r reflects the higher default risk of the tranche; this difference is the yield spread on senior bonds.

The equilibrium initial yields on the pool and the residual tranche are given by expressions analogous to (2.12). The recovery rate of the senior tranche, the pool, and the residual tranche can be ordered as $R_s/V_s(1) \geq R_p/V_p(1) \geq R_j/V_j(1)$. Thus initial yields on the tranches and the pool can be ordered as $c_s/V_s(1) \leq c/V_p(1) \leq c_j/V_j(1)$. The ordering of the recovery rates shows how the default risk is divided disproportionately among the tranches. The resulting ordering of the initial yields shows how the division of risk affects equilibrium yields.

Table 2.1: Benchmark parametrization

Description	Symbol	Value
Expected return on mortgages	r	7%
Drift	α	3%
Volatility	σ	15%
Borrower default costs	k_β	0
Lender default costs	k_λ	2
Mortgage size	M(1)	20

Numerical examples.– I present a numerical example of senior and residual tranche yields implied by the model. Table 2.1 shows the benchmark parameterization. The choice of parameters follows KLO. These authors calibrated the mortgage valuation model to data on California mortgages and found that model implied yield spreads at origination were close to the spreads observed in the data, for empirically plausible parameter values. The risk-free rate is $r = 7\%$; this value generates empirically realistic average real proportional gains of 7% on mortgages and home equity. The drift parameter is $\alpha = 3\%$. These values also imply that the price-to-rent ratio in the model is 25. Price-to-rent ratios in data are closer to 10 or 15. This discrepancy between the model and the data is appropriate because the model abstracts from operating costs such as maintenence and utilities expenses. The chosen value of $\sigma = 15\%$ for the standard deviation of housing services is consistent with estimates of individual house price volatitility in the literature.[8] The mortgage size is chosen to obtain an initial LTV ratio of 80%, $M(1) = 20$. Borrower default costs are set to zero, $k_\beta = 0$. Lender default costs are set to ten percent of the mortgage size, $k_\lambda = 2$.

Under the normalization $x(0) = 1$, the implied purchase price of the house is $P(1) = 25$. The equilibrium default threshold δ and mortgage coupon c are obtained by numerically solving the system of nonlinear equations formed by (2.6) and (2.8). In equilibrium, borrowers terminate their mortgage when house prices drop to 67.57% of their original value. The equilibrium mortgage coupon is $c = 1.524$. The equilibrium initial yield on the mortgage is $c/M(1) = 7.62\%$. At the default date, the lender's net recovery is $M(\delta) = 14.89$, so the recovery rate of the mortgage is $M(\delta)/M(1) = 74.46\%$. The pool inherits all the mortgage characteristics. The initial value of the pool is $V_p(1) = 20$, initial yield is $c/V_p(1) = 7.62\%$, and the recovery rate is $R_p/V_p(1) = 74.46\%$. The recovery on the pool is adequate provided $\theta \leq 0.7446$. In this case, the initial yield on senior bonds is $r = 7\%$.

[8]For example, Flavin and Yamashita (2002) estimated the standard deviation of the real return on housing to be 14%. Similarly, Case and Shiller (1989) estimated the return on individual houses to be around 14-15%.

When $\theta = 0.80$, the value of the senior tranche at origination is $V_s(1) = 16$. Since the recovery on the senior tranche is $R_s = R_p = 14.89 < 16$, the recovery on the pool is inadequate. The recovery rate of the senior tranche is $R_s/V_s(1) = 93.07\%$. The equilibrium senior coupon is $c_s = 1.147$ and the equilibrium initial yield on senior bonds is $c_s/V_s(1) = 7.17\%$. The equilibrium characteristics of the residual tranche follow from those of the senior tranche. The initial value of the residual tranche is $R_j(1) = 4$, its coupon is $c_j = 0.377$, and its initial yield is $c_j/R_j(1) = 9.42\%$. The recovery on this tranche is $R_j = 0$.

The disproportionate division of default risk is reflected in the recovery rates. The recovery rates of the senior tranche, the pool, and the residual tranche are 93.07%, 74.46% and 0%. The residual tranche is wiped out on default because the entire recovery on the pool is applied to the senior tranche. The initial yield spreads on the senior tranche, the pool, and the residual tranche are 0.17%, 0.62%, and 2.42%. The yield spread on the residual tranche is higher, reflecting the fact that the default risk of residual bonds is higher. The example reiterates how the credit risk of the pool is divided disproportionately among the tranches to create a relatively safe senior tranche and a relatively risky residual tranche.

To gain further insights into the model solution, I relax the assumption that default is costless for the borrower. An increase in borrower default costs k_β decreases the default threshold; see (2.6). The decrease in the default threshold has two opposing effects: It lowers the probability of default, and it lowers the lender's net recovery on default. A lower default probability decreases the initial yield on the mortgage, but a lower net recovery increases the initial yield. The

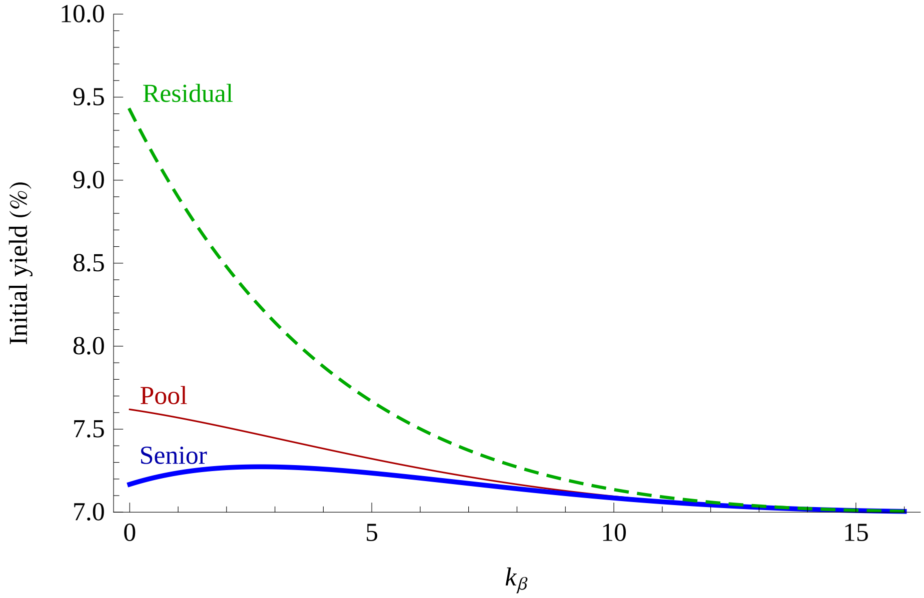


Figure 2.1: Initial yield as a function of borrower default costs k_β

equilibrium initial yield on the pool, the senior tranche, and the residual tranche is the net of these two effects.

Figure 2.1 shows equilibrium initial yields as a function of borrower default costs; all other parameters equal their benchmark values. The solid curve in the middle represents the pool, the bold solid curve represents the senior tranche, and the dashed curve represents the residual tranche. The figure shows that the initial yield on the senior tranche is always less than that on the pool, and the initial yield on the residual tranche is always greater than that on the pool. The initial yield on the pool is monotonically decreasing in k_β, indicating that the probability effect dominates. The initial yield on the senior tranche first increases with k_β and then starts declining, indicating that the recovery effect dominates initially but is eventually overtaken by the probability effect. Since the recovery on the residual tranche is zero when $k_\beta = 0$, an increase in k_β does not lower the net recovery; it only lowers the probability of default. Hence the initial yield on the residual tranche is decreasing in k_β. As k_β increases unboundedly, all three initial yields approach r. This finding is intuitive: if $k_\beta \to \infty$, then the exercise of the default option becomes extremely costly. Therefore the underlying mortgage becomes risk-free and all initial yields approach the risk-free rate.

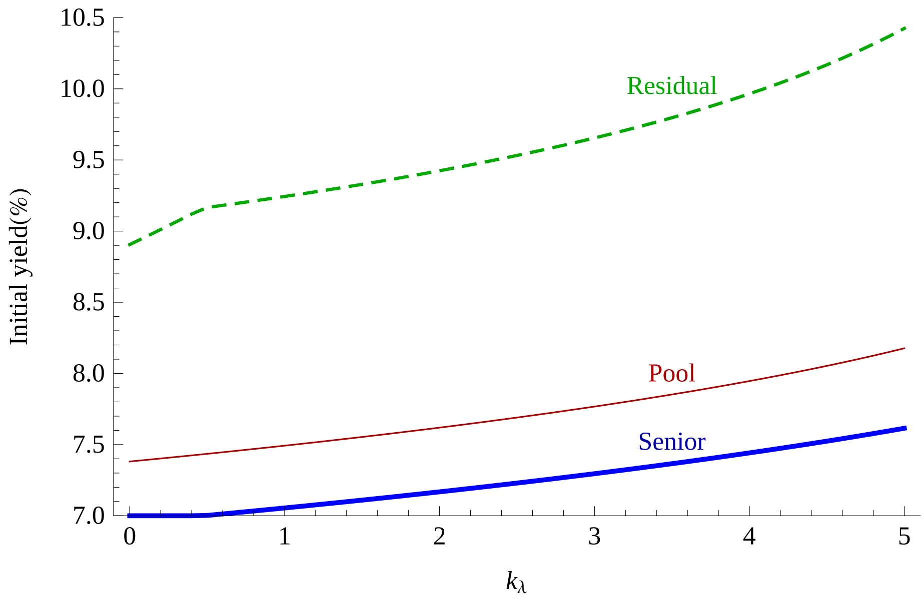


Figure 2.2: Initial yield as a function of lender default costs k_λ

Figure 2.2 shows equilibrium initial yields as a function of lender default costs k_λ. (All other parameters are equal to their benchmark values. In particular, $k_\beta = 0$.) Once again, the solid curve represents the pool, the bold solid curve represents the senior tranche, and the dashed curve represents the residual tranche. An increase in lender default costs k_λ reduces the net recovery on the pool and the tranches without changing the default probability; recall $R_p = P(\delta) - k_\lambda$.

Therefore all three initial yield curves increase in k_λ. The kinks on the yield curve for the tranches indicate the value of k_λ at which the recovery on the pool switches from being adequate to inadequate. The senior tranche is risk-free to the left of the kink and risky to the right of it. The recovery on the residual tranche is positive to the left of the kink and zero to the right of it.

In practice, for institutional reasons, mortgages with LTV ratios greater than 80% usually consist of two different liens. For example, a 92% mortgage usually consists of a first lien with an LTV of 80% and a second lien of 12%. If the borrower defaults on the mortgage, then the first lien has first claims on the recovery. The recovery on the second lien is positive only if the recovery on the underlying mortgage is greater than 80%. The disproportionate division of recovery among the two liens corresponds exactly to the division of recovery between the tranches in the model. The first lien corresponds to the senior tranche and the second lien corresponds to the residual tranche. Therefore the analysis of CMOs created from homogeneous pools applies directly to the valuation of two-liened mortgages.

Consider the two-liened mortgage with a total LTV of 92%. (All other parameters equal their benchmark values.) The recovery rate of the mortgage is 87.17%. A pool containing this mortgage inherits all the characteristics of the mortgage. If the proportional value of the senior tranche at origination is 0.87, then the tranche corresponds to the first lien of the mortgage. The proportional value of the tranche is less than the recovery rate of the pool, so the recovery on the pool is adequate and the senior tranche is risk-free. Correspondingly, the first lien of the mortgage is also risk-free. On default, the entire loss of principal is borne by the residual tranche, which correponds to the second lien of the mortgage. The recovery rate of the second lien is only 1.67%. Therefore the first lien (senior tranche) is risk-free when the second lien (residual tranche) is large enough to absorb the loss of principal on default.

2.3.2 Heterogeneous Pool

This section extends the analysis to heterogeneous pools, which contain two different types of mortgages. After appropriate modification, the analysis here also applies to pools with more than two types of mortgages. The two mortgages differ in borrower default costs. The default costs faced by the borrower of the first mortgage are lower. The other exogenous characteristics of the two mortgages are identical. In particular, one aggregate geometric Brownian motion governs the evolution of housing services for both the mortgaged properties. The cost of exercising the default option is lower for the first borrower, so his default threshold is higher. Therefore, contingent on default, the first mortgage is always terminated earlier. I refer to the first and second mortgages as early default and late default mortgages. Variables associated with early and late default mortgages are indexed by e and l. Since the early default mortgage has a higher default threshold, the value of the property when the mortgage is terminated is higher. Therefore the lender's net recovery on this mortgage is higher; recall that lender default costs are identical across mortgages.

Consider a pool created by combining early and late default mortgages; normalize the total number of mortgages in the pool to one. The proportion of early default mortgages in the pool is exogenous; it is denoted by $\eta \in (0, 1)$. The proportion of late default mortgages is $1 - \eta$. Denote the value of the pool by

$V_p(t, x(t))$. Time is a state variable when the pool is heterogeneous because the composition of the pool changes over time, if borrowers default. The value of the pool is the weighted average of the value of the underlying mortgages that have not defaulted. It equals the weighted average of both the mortgage values before the early default event; the value of the late default mortgage weighted by $1 - \eta$ after the early default event; zero after the late default event.

$$V_p(t, x(t)) = \begin{cases} \eta M_e(x(t)) + (1-\eta) M_l(x(t)) & \text{if } 0 \leq t < \tau_e \\ (1-\eta) M_l(x(t)) & \text{if } \tau_e \leq t < \tau_l \\ 0 & \text{if } \tau_l \leq t. \end{cases} \tag{2.13}$$

Denote the coupon on the pool by $c_p(t)$. It is the weighted average of the coupons on the underlying mortgages that have not defaulted. It equals $\eta c_e + (1-\eta)c_l$ before the early default event; $(1-\eta)c_l$ after the early default event; zero after the late default event. The initial yield on the pool is $c_p(0)/V_p(0, 1)$, and the yield at the time of early default is $c_p(\tau_e)/V_p(\tau_e, \delta_e)$. The yield on the pool can also be expressed as the weighted average of the underlying mortgage yields; the same holds for yields spreads on the pool. At the time of early default, the recovery on the pool is the recovery on early default mortgages weighted by their proportion, $R_{pe} = \eta M_e(\delta_e)$; this is the early recovery on the pool. Similarly the late recovery on the pool is $R_{pl} = (1 - \eta)M_l(\delta_l)$. The sum of the early and late recoveries on the pool, $R_p = R_{pe} + R_{pl}$, is the total recovery on the pool. The recovery rate of the pool is its total recovery divided by its value at origination, $R_p/V_p(0, 1)$.

Mortgage pass-throughs are bonds that represent pro rata claims to the cash flows of the pool. The total value of pass-throughs at origination equals $V_p(0, 1)$. These bonds receive the initial coupon $c_p(0)$ until the early default event. The lender uses the early recovery on the pool to buy back bonds at their market value. Bondholders are indifferent to selling their bonds at this value. The total value of the bonds that remain outstanding is $V_p(\tau_e, \delta_e)$. These bonds receive $c_p(\tau_e)$ until the late default event. The lender uses the late recovery on the pool to buy back the remaining bonds at their market value. The initial yield on pass-throughs is $c_p(0)/V_p(0, 1)$ and the yield at the time of early default is $c_p(\tau_e)/V_p(\tau_e, \delta_e)$. The yield on pass-throughs is always greater than the risk-free rate r because these bonds carry default risk.

Tranches.– As in the case of a homogeneous pool, the lender divides the pool by value into a senior tranche and a residual tranche. He then sells bonds that are pro rata claims to cash flows on each tranche. The senior tranche has first claims to all cash flows on the pool, whereas the residual tranche is the first to bear all losses. This division of cash flows is done so as to make the senior tranche relatively safe and the residual tranche relatively risky.

The proportional value of the senior tranche at origination exogenous; it is denoted by $0 \leq \theta \leq 1$. Let $V_s(t, x(t))$ denote the value of the senior tranche. The initial value of the senior tranche is $V_s(0, 1) = \theta V_p(0, 1)$. Let $c_s(t)$ denote the coupon on the senior tranche. The cash flows to the tranche are as follows. The tranche receives its original coupon $c_s(0)$ until the early default event. The lender uses the early recovery on the pool to buy back senior bonds at their market value. The amount allocated towards the buyback is referred to as the early recovery on the senior tranche; it is denoted R_{se}. The fraction of senior bonds outstanding after the buyback is

$$q_s \equiv \frac{V_s(\tau_e, \delta_e)}{V_s(\tau_e, \delta_e) + R_{se}}, \tag{2.14}$$

where $V_s(\tau_e, \delta_e)$ is the market value of the senior tranche after the buyback. The lender attempts to pay the outstanding senior bonds their original coupon. If he cannot do so, then he forwards the entire coupon on the pool to the tranche. Therefore, the coupon on the senior tranche at the time of early default is

$$c_s(\tau_e) = \min\{q_s c_s(0), c_p(\tau_e)\}. \tag{2.15}$$

Senior bonds receive this coupon until the late default event. The lender uses the late recovery on the pool to buy back the remaining senior bonds at their market value. The amount allocated towards the buyback is termed the late recovery on the senior tranche; it is denoted R_{sl}.

The total recovery on the senior tranche is defined as the sum of the early and late recoveries; it is denoted R_s. The senior tranche has first claims on the cash flows to the pool. Therefore the entire early recovery on the pool is used to buy back senior bonds, unless the recovery exceeds the par value of the senior tranche. The early recovery on the senior tranche is $R_{se} = \min\{V_s(0,1),\ R_{pe}\}$. The entire late recovery on the pool is used to buy back senior bonds, unless the recovery exceeds the par value of the outstanding senior bonds. The late recovery on the tranche is $R_{sl} = \min\{V_s(0,1) - R_{se},\ R_{pl}\}$. In both cases, the minimum operator ensures that the total recovery on the senior tranche does not exceed its par value.

The recovery rate of the senior tranche is its total recovery divided by the par value, $R_s/V_s(0,1)$.

As in the case of the homogeneous pool, the recovery on the pool is adequate if it is large enough buy back the senior tranche at its par value, $R_s = V_s(0,1)$. Otherwise, the recovery is inadequate and $R_s = R_p$. The recovery rate of the tranche is one when the recovery on the pool is adequate, and less than one when the recovery is inadequate. In both cases, the recovery rate of the tranche is greater than the recovery rate of the pool. The coupon on the pool is adequate if it is large enough to continue paying the outstanding senior bonds their original coupon after the early buyback, $c_s(\tau_e) = q_s c_s(0)$. Otherwise, if $q_s c_s(0) > c_p(\tau_e)$ then the coupon is inadequate and $c_s(\tau_e) = c_p(\tau_e)$.

The value of the residual tranche is $V_j(t, x(t)) = V_p(t, x(t)) - V_s(t, x(t))$. Its coupon is $c_j(t) = c_p(t) - c_s(t)$. The coupon on the residual tranche after the early default event is positive when the coupon on the pool is adequate, and zero when the coupon is inadequate. The early recovery on the tranche is $R_{je} = R_{pe} - R_{se}$ and the late recovery is $R_{jl} = R_{pl} - R_{sl}$. The total recovery is $R_j = R_p - R_s$. The total recovery on the residual tranche is positive when the recovery on the pool is adequate, and zero when the recovery is inadequate. The recovery rate of the residual tranche is less than the recovery rate of the pool.

Equilibrium.– The CMO market is perfectly competitive, so the lender makes zero expected profits in equilibrium. The equilibrium coupon schedule on the senior tranche is such that this tranche is issued at par; the coupon schedule on the residual tranche follows. The equilibrium senior coupons $c_s(0)$ and $c_s(\tau_e)$

satisfy

$$V_s(0,1) = \mathbb{E}_0\left[\int_0^{\tau_e} e^{-rt}c_s(0)dt\right] + \mathbb{E}_0\left[e^{-r\tau_e}R_{se}\right] + \mathbb{E}_0\left[e^{-r\tau_e}V_s(\tau_e,\delta_e)\right], \quad (2.16)$$

$$V_s(\tau_e,\delta_e) = \mathbb{E}_{\tau_e}\left[\int_{\tau_e}^{\tau_l} e^{-r(t-\tau_e)}c_s(\tau_e)dt\right] + \mathbb{E}_{\tau_e}\left[e^{-r(\tau_l-\tau_e)}R_{sl}\right]. \quad (2.17)$$

where $c_s(\tau_e)$ is given by (2.15). According to (2.16) the market value of the senior tranche at origination is the expected discounted value of its initial coupon payments, its early recovery R_{se}, and its market value at the time of early default $V_s(\tau_e,\delta_e)$. Similarly (2.17) states that the market value of the senior tranche at the time of early default is the expected discounted value of its remaining coupon payments and the late recovery payment R_{sl}.

The permutations of adequate coupon and adequate recovery on the pool suggest four types of equilibria: adequate coupon, adequate recovery; inadequate coupon, adequate recovery; adequate coupon, inadequate recovery; and inadequate coupon, inadequate recovery. Next I discuss when each type of equilibrium arises, if at all, and characterize the coupon schedule of the senior tranche in that equilibrium.

First I focus on cases in which the recovery on the pool is adequate. Consider the case in which the senior tranche is so small that the entire tranche is bought back at its par value when early default mortgages are terminated. The par value of this tranche must be less than the early recovery on the pool, $V_s(0,1) \leq R_{pe}$. Equivalently, let θ_1 denote the threshold at which the proportional value of the senior tranche is such that $V_s(0,1) = R_{pe}$; the threshold is $\theta_1 = R_{pe}/V_p(0,1)$. The entire senior tranche is bought back at the time of early default at its par value when $\theta \leq \theta_1$.

Now consider a senior tranche whose proportional value is slightly larger than θ_1. The par value of this tranche is slightly larger than R_{pe}. Therefore the entire tranche is not bought back at the time of early default. Instead the fraction $R_{pe}/V_s(0,1)$ is bought back at the time of early default, while the fraction remaining is bought back at its par value at the time of late default. The early and late recoveries on the senior tranche are $R_{se} = R_{pe}$ and $R_{sl} = V_s(0,1) - R_{se}$. The tranche continues to be bought back at its par value at the two default events as long as the late recovery on the pool is large enough to do so, $R_{pl} \geq V_s(0,1) - R_{pe}$. Rearranging terms in the inequality, the recovery on the pool is adequate as long as the par value of the senior tranche is less than the total recovery on the pool. Let θ_2 denote the threshold at which the proportional value of the senior tranche is such that $V_s(0,1) = R_p$; the threshold is $\theta_2 = R_p/V_p(0,1)$. The recovery on the pool is adequate for all $\theta \leq \theta_2$.

Motivated by my finding for one-mortgage pools, I conjecture that the yield on senior bonds equals the risk-free rate r when the recovery on the pool is adequate. The conjecture is correct; see Appendix A. The intuition is identical to the one-mortgage case: If the recovery is adequate, then senior bonds carry no default risk because the total recovery on these bonds equals their initial principal. Therefore the equilibrium yield on senior bonds must equal the risk free rate r at all times. This equilibrium is the risk-free equilibrium. (Note that the equilibrium regions are labeled based on the characteristics of the senior tranche only.) Next I determine whether the coupon on the pool is adequate or inadequate in the risk-free equilibrium. Recall that the yield on the pool is always greater than r. The value of the senior tranche is always less than the value of the pool, $V_s(t, x(t)) \leq V_p(t, x(t))$. Together these two observations imply $c_p(t) \geq rV_p(t, x(t)) \geq rV_s(t, x(t))$. Therefore the coupon on the pool is adequate in the risk-free equilibrium.

To establish the uniqueness of the risk-free equilibrium for $\theta \in [0, \theta_2]$, note that the definition of $c_s(\tau_e)$ in (2.15) implies that the coupon on outstanding senior bonds cannot rise at the early default date. The coupon either falls or remains constant at the early default date. Suppose that the coupon falls, then $c_s(\tau_e) = c_p(\tau_e)$ by definition. This case cannot be an equilibrium when $\theta \in [0, \theta_2]$ because it allows arbitrage: an investor can borrow $V_s(0, 1) - R_{se}$ at the risk-free rate r, purchase the senior tranche, and earn a rate of return greater than r until the late default event (Recall that the yield on the pool is always greater than r and $V_s(t, x(t)) \leq V_p(t, x(t))$, implying that the yield on senior bonds is greater than r when $c_s(\tau_e) = c_p(\tau_e)$.) At the time of late default he will receive a recovery of $V_s(0, 1) - R_{se}$, which he can use to pay back his debt. Similarly, if the coupon on senior bonds is constant, then the yield must equal r to rule out arbitrage. The uniqueness of the risk-free equilibrium for $\theta \in [0, \theta_2]$ implies that there are no equilibria for which the recovery on the pool is adequate while the coupon is inadequate.

If $\theta > \theta_2$, then the recovery on the pool is inadequate. To complete the classification of equilibria, I need to divide the interval $(\theta_2, 1]$ into a region in which the coupon on the pool is adequate, and another region in which the coupon is inadequate. Consider a senior tranche whose proportional value is slightly larger than θ_2. The initial coupon on this tranche is slightly larger than r. After the early default event, the coupon on the pool is large enough to pay the outstanding senior bonds their initial coupon; the coupon on the pool is adequate. As the proportional size of the senior tranche increases, its recovery rate decreases and so the required coupon on the tranche increases. The largest value of θ for which the coupon on the pool is adequate is found by solving $q_s c_s(0) = c_p(\tau_e)$. The resulting threshold, denoted θ_3, is

$$\theta_3 = 1 - \frac{(1-\delta_e^m)\eta c_e/r}{M_e(1)} \left(1 - \frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)}\right). \tag{2.18}$$

The threshold (2.18) lies within the interval $(\theta_2, 1)$ provided $c_e/M_e(\delta_e) \geq c_l/M_l(\delta_e)$; see Appendix A. Since $M_e(1) = M_l(1)$, the required condition on the underlying mortgages holds when the initial yield on the early default mortgage is greater than that on the late default mortgage, $c_e/M_e(1) \geq c_l/M_l(1)$, and the net recovery on the early default mortgage is less than the value of late default mortgage at the same date, $M_e(\delta_e) \leq M_l(\delta_e)$. From here on, I assume that $c_e/M_e(\delta_e) \geq c_l/M_l(\delta_e)$. Therefore, when $\theta \in (\theta_2, \theta_3]$, the proportional value of the senior tranche at origination is small enough to leave the senior coupon unchanged after the early default event; the coupon on the pool is adequate. When $\theta \in (\theta_3, 1]$, however, the proportional value of the senior tranche at origination is so large that the entire coupon on the pool goes to this tranche after the early default event; the coupon on the pool is inadequate.

In summary, all thresholds lie within the interval $(0, 1)$ and satisfy $\theta_1 \leq \theta_2 < \theta_3$. They divide the unit interval into four equilibrium regions. When $\theta \in [0, \theta_1]$, the senior tranche is so small that all the principal on this tranche is repaid at the early default date. This region is risk-free region I. When $\theta \in (\theta_1, \theta_2]$, the senior tranche is still small enough that all the principal on this tranche is repaid on default. In this case, however, part of the principal remains outstanding after the early default date. The outstanding principal is repaid at the late default date. This region is risk-free region II. When $\theta \in (\theta_2, \theta_3]$, the senior tranche is so large that its entire principal cannot be repaid on default. It is, however, small enough that the senior bonds outstanding after the early default event continue to receive their initial coupon. This region is the low-risk region. When $\theta \in (\theta_3, 1]$, the senior tranche is so large that its entire principal cannot be repaid on default, and the coupon on outstanding senior bonds drops after the early default event. This region is the high-risk region.

Now that the classification of equilibria is complete, I solve for the equilibrium coupons in the low-risk and the high-risk equilibria; recall that the coupons for the risk-free equilibrium are such that the yield on the senior tranche equals r. In the low-risk equilibrium, the early and late recoveries on the senior tranche and the pool are identical. The coupon on this tranche at the time of early default is

$$c_s(\tau_e) = q_s c_s(0). \tag{2.19}$$

Equations (2.16), (2.17), and (2.19) form a system of nonlinear equations in $c_s(0)$, $c_s(\tau_e)$, and $V_s(\tau_e, \delta_e)$. The equilibrium senior coupon schedule is obtained by solving this system numerically.

The early and late recoveries on the senior tranche and the pool are also identical in the high-risk equilibrium. In this equilibrium, however, the coupon on the pool after the early default event is distributed pro rata among the outstanding senior bonds, $c_s(\tau_e) = c_p(\tau_e)$. Since $c_s(\tau_e)$ is known, I find $V_s(\tau_e, \delta_e)$ using (2.17) and then solve (2.16) for $c_s(0)$ to obtain

$$c_s(0) = \frac{r}{1-\delta_e^m}\left[V_s(0,1) - \left(R_{pe} + \frac{c_p(\tau_e)}{r}\right)\delta_e^m - \left(R_{pl} - \frac{c_p(\tau_e)}{r}\right)\delta_l^m\right]. \quad (2.20)$$

The equilibrium characteristics of the residual tranche follow from those of the senior tranche. In risk-free region I, the early recovery on the residual tranche is strictly positive, except when $\theta = \theta_1$. After the early default event, the residual tranche mimics the pool: its coupon equals $c_p(\tau_e)$ and its late recovery equals R_{pl}. In risk-free region II, the early recovery on the residual tranche is zero. The late recovery is strictly positive, except when $\theta = \theta_2$. The coupon on this tranche is strictly positive for $t \leq \tau_l$. It does, however, drop after the early default event. In the low-risk region, the early and late recoveries on the residual tranche are both zero. In this region, the coupon on the residual tranche is strictly positive for $t \leq \tau_l$, except when $\theta = \theta_3$. The coupon in this region also drops after the early default event. In the high-risk region, the early and late recoveries on the residual tranche are zero, and its coupon drops to zero after the early default event.

Numerical Examples.– This subsection illustrates various characteristics of the model using numerical examples. It presents examples of yields on low-risk and high-risk senior bonds. It shows how the equilibrium thresholds θ_1, θ_2, and θ_3 change with the composition of the pool η . Finally, it shows how model implied

	Early default mortgage ($k_{\beta e} = 0$)	Late default mortgage ($k_{\beta l} = 4$)
LTV	80%	80%
Default threshold	67.57%	53.06%
Coupon	1.524	1.477
Initial yield	7.62%	7.38%
Recovery rate	74.46%	56.32%

Table 2.2: Benchmark mortgage characteristics

yields for low-risk senior bonds change with the securitization parameters θ and η. The parameter values are as in the numerical example for the homogeneous pool; see Table 2.1. Borrower default costs for early and late default borrowers are set to zero and twenty percent of the mortgage size, $k_{\beta e} = 0$ and $k_{\beta l} = 4$. Lender default costs are set to ten percent of the mortgage size, $k_\lambda = 2$.

Table 2.2 summarizes the implied characteristics of early and late default mortgages. In equilibrium, borrowers with early default mortgages terminate their mortgage when house prices drop to 67.57% of the purchase price. Borrowers with late default mortgages terminate their mortgage when house prices drop to 53.06% of the purchase price. The recovery rates on early and late default mortgages are 74.46% and 56.32%; the equilibrium coupons are $c_e = 1.524$ and $c_l = 1.477$; the equilibrium initial yields are 7.62% and 7.38%. The higher initial yield on the early default mortgage reflects the fact that the default risk of this mortgage is higher.

In the benchmark, the proportion of early and late default mortgages in the pool are equal, $\eta = 0.5$. Once the composition of the pool is set, other characteristics of the pool are implied by those of the underlying mortgages. The implied origination value is $V_p(0, 1) = 20$. The initial coupon is $c_p(0) = 1.500$. The initial yield on the pool is 7.50%. The value at the early default event is $V_p(\tau_e, \delta_e) = 8.42$. The coupon after the early default event is $c_p(\tau_e) = 0.738$, so the yield at the early default date is 8.77%. The early and late recoveries are $R_{pe} = 7.45$ and $R_{pl} = 5.63$. The total recovery is $R_p = 13.08$. The recovery rate is 65.39%.

Once the composition of the pool and the underlying mortgage parameters are chosen, the threshold for each type of equilibrium can be determined. The first threshold is $\theta_1 = 0.3723$. All senior bonds are bought back at the early default event if the initial value of the senior tranche, in proportion to that of the pool, is less than 37.23%. The second threshold is $\theta_2 = 0.6539$. Senior bonds are risk-free if the proportional value of the senior tranche at origination is less than 65.39%. When the proportional value is between 37.23% and 65.39% some risk-free senior bonds remain outstanding after the early default event; these bonds are bought back at par at the time of late default. The third threshold is $\theta_3 = 0.9422$. If the proportional value of the senior tranche at origination is strictly larger than 65.39%, but less than 94.22%, then senior bonds are low-risk in equilibrium. On the other hand, if this value is strictly larger than 94.22%, then senior bonds are high-risk in equilibrium.

Table 2.3 presents examples of each type of equilibrium. The senior tranche is risk-free in equilibrium if its initial value is 40% of the pool's initial value. The implied initial value of the senior tranche is $V_s(0, 1) = 8$. The coupons on the senior tranche are $c_s(0) = 0.560$ and $c_s(\tau_e) = 0.039$. Regardless of the evolution of house prices the yield on senior bonds is $r = 7\%$ until the late default event. The early recovery on the senior tranche equals the early recovery on the pool,

$R_{se} = 7.45$. The late recovery on the senior tranche is $R_{sl} = 0.55$. The recovery rate of this tranche is 100%.

The residual tranche is 60% of the pool at origination, $V_j(0,1) = 12$. The implied initial coupon on residual bonds is $c_j(0) = 0.940$. In contrast to senior bond yields, residual bond yields depend on the evolution of house prices. Prior to the late default event, residual bond yields decrease if house prices increase and vice versa. This decrease in the residual bond yield reflects the decline in the default probability due to the increase in house prices. The initial yield on residual bonds is 7.84%. The initial yield spread on residual bonds is higher than the spread on the pool because all the default risk has been directed to the residual tranche. The coupon on the residual tranche at the early default event is $c_j(\tau_e) = 0.700$. The residual bond yield at the early default event increases to 8.89%. This increase in the residual bond yield reflects the increased likelihood of the late default event. Since the residual tranche absorbs all the losses due to default, the recovery rate of this tranche is only 42.32%.

The low-risk equilibrium occurs when $\theta = 0.80$. The implied initial value of the senior tranche is $V_s(0,1) = 16$. Unlike risk-free senior bonds, the yield on low-risk senior bonds depends on the evolution of house prices; the yield and house prices are inversely related. The yield on low-risk senior bonds approaches $r = 7\%$ only when house prices rise unboundedly. The initial coupon on the senior tranche is $c_s(0) = 1.158$. The initial yield on senior bonds is 7.24%; the yield spread of 24 basis points reflects the fact that low-risk senior bonds carry default risk. The yield increases to 8.03% at the early default event. The early and late recoveries on the senior tranche and the pool are identical. The recovery rates, however, are not. The recovery rate of the pool is 65.39%, whereas the recovery rate of the senior tranche is 81.74%. The recovery rate of the senior tranche is higher because the residual tranche is the first to absorb losses.

Once the characteristics of the senior tranche in the low-risk equilibrium are determined, those of the residual tranche follow. The initial value of the residual tranche is $V_j(0,1) = 4$. The initial yield on residual bonds is 8.55%, and the early default yield on these bonds is 12.34%. The recovery rate of the residual tranche is zero in the low-risk equilibrium.

The high-risk equilibrium occurs when $\theta = 0.95$. The initial value of the senior tranche is $V_s(0,1) = 19$. Since senior bonds are high-risk, the coupon on these bonds after the early default event equals the coupon on the pool, $c_s(\tau_e) = 0.738$. The value of the senior tranche at the early default event is $V_s(\tau_e, \delta_e) = 8.42$. The initial coupon on the senior tranche is $c_s(0) = 1.406$. The initial and early default yields on senior bonds are 7.40% and 8.77%. The early and late recoveries on the senior tranche and the pool are identical. The recovery rates on the pool and the senior tranche are 65.39% and 68.83%. The equilibrium characteristics of the senior tranche approach those of the pool as θ increases. When $\theta = 1$, the senior tranche and the pool are identical.

The initial value of the residual tranche in the high-risk equilibrium is $V_j(0,1) = 1$. The coupon on residual bonds is $c_j(0) = 0.094$. The initial yield is 9.42%. In the high-risk equilibrium, the residual tranche stops receiving payments after the early default event.

Next I show how the model solution changes as the composition of the pool changes; the composition is determined by η. Figure 2.3 shows the equilibrium

	Pool	Senior			Residual		
		Risk-free ($\theta = 0.40$)	Low-risk ($\theta = 0.80$)	High-risk ($\theta = 0.95$)	Risk-free ($\theta = 0.40$)	Low-risk ($\theta = 0.80$)	High-risk ($\theta = 0.95$)
Origination							
Value	20	8	16	19	12	4	1
Coupon	1.500	0.56	1.158	1.406	0.940	0.342	0.094
Yield	7.50%	7%	7.24%	7.40%	7.84%	8.55%	9.42%
Early default event							
Value	8.42	0.55	6.98	8.42	7.87	1.44	0
Coupon	0.738	0.039	0.560	0.738	0.700	0.178	0
Yield	8.77%	7%	8.03%	8.77%	8.89%	12.34%	-
Total recovery	13.08	8	13.08	13.08	5.08	0	0
Recovery rate	65.39%	100%	81.74%	68.83%	42.32%	0%	0%

Table 2.3: Equilibrium tranche characteristics for $\eta = 0.50$

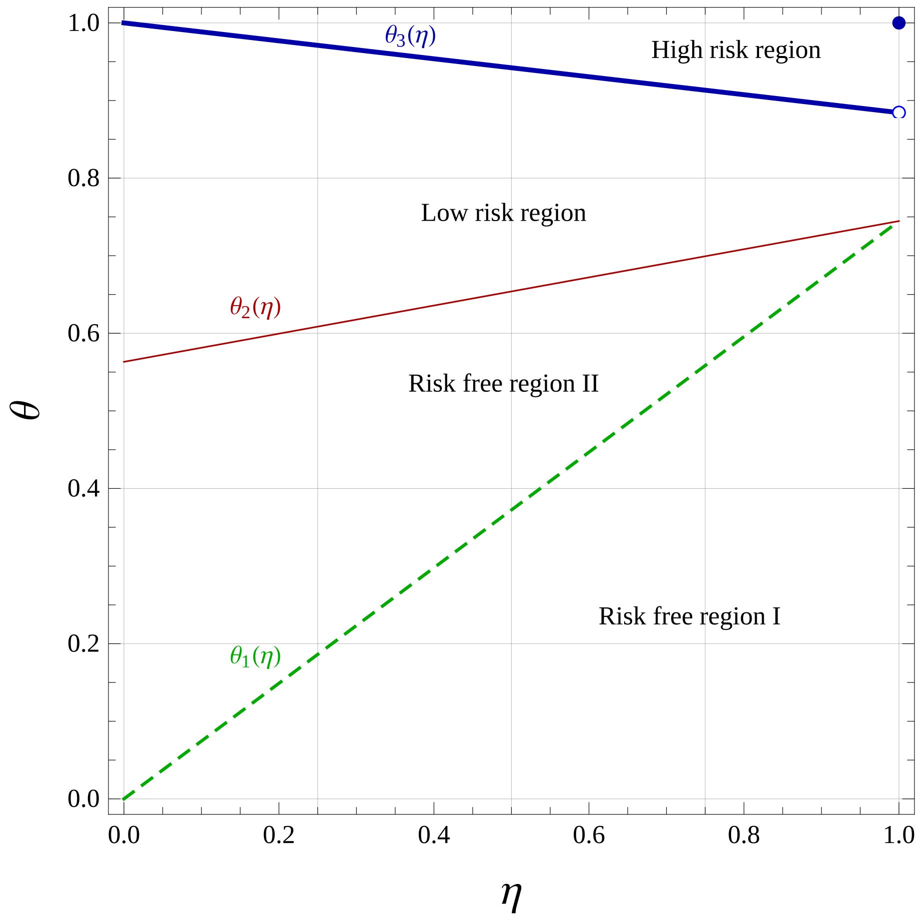


Figure 2.3: Equilibrium regions for admissible values of η and θ.

thresholds θ_1, θ_2, and θ_3 as functions of η. This figure partitions the unit square formed by admissible values of η and θ into four equilibrium regions: risk-free region I, risk-free region II, low-risk region, and high-risk region. To begin with, I restrict attention to $\eta \in (0,1)$. The dashed line is the threshold $\theta_1 = R_{pe}/V_p(0,1)$, where $R_{pe} = \eta M_e(\delta_e)$. Since $M_e(1) = M_l(1)$, the initial value of the pool does not change with η, $V_p(0,1) = M_e(1)$. Therefore θ_1 is linear in η with slope equal to the recovery rate of the early default mortgage $M_e(\delta_e)/M_e(1)$. As η increases, the early recovery on the pool increases so more cash is available to buy back senior bonds at the time of early default. The size of the senior tranche that can be bought back at par at the time of early default increases. Since $V_p(0,1)$ does not change with η, the proportional size of the senior tranche that can be bought back at the time of early default increases with η.

Similarly, $\theta_2 = R_p/V_p(0,1)$ is linear in η, and its slope equals the difference between the recovery rate of early and late default mortgages, $M_e(\delta_e)/M_e(1) - M_l(\delta_l)/M_l(1)$. This difference is positive because the default threshold, and so the recovery, of early default mortgages is higher. As η increases, the weight of early default mortgages in the pool increases so the total recovery on the pool increases. Therefore, the size of the senior tranche that can be bought back at its par value increases.

Equation (2.18) shows that the threshold θ_3 is linearly decreasing in η; recall that $c_e/M_e(\delta_e) \geq c_l/M_l(\delta_e)$ by assumption. An increase in η has two opposing effects — it increases the early recovery on the pool R_{pe}, and it decreases the coupon on the pool after the early default event $c_p(\tau_e)$. The increase in the early recovery implies a higher θ_3 because more senior bonds can be bought back at the time of early default. The decrease in the $c_p(\tau_e)$, however, implies a lower θ_3 because the senior tranche should be smaller for outstanding senior bonds to continue receiving their initial coupon after the early default event. The second effect dominates in the example considered.

The analysis so far has been restricted to $\eta \in (0, 1)$. Now I extend it to include the endpoints of the interval. When η equals zero or one the pool contains one type of mortgage only, so the analysis in section 2.3.1 applies. The analysis in that section can be imbedded into the analysis here by setting the early and late default mortgage variables equal to each other. For example, $c_e = c_l$ and $M_e(\delta_e) = M_l(\delta_e)$ implies that $\theta_3 = 1$ at the end points. To maintain consistency with the two-mortgage pool framework, I assume that the pool experiences two default events when η equals zero or one, with one default event being inconsequential. If $\eta = 0$, then the pool contains late default mortgages only. Therefore the early default event is inconsequential: The coupon on the pool is unchanged, the early recovery on the pool is zero, and no senior bonds are bought back. All senior bonds continue to receive their initial coupon after the early default event. The coupon on the pool is adequate for all $\theta \in [0, 1]$; see Figure 2.3. When $\eta = 1$ the pool contains early default mortgages only; the late default event is inconsequential. In this case, the early recovery on the pool R_{pe} equals the total recovery R_p. Therefore the thresholds θ_1 and θ_2 coincide. When $\eta = 1$, the threshold θ_3 also equals one; set $c_l = c_e$ and $M_l(\delta_e) = M_e(\delta_e)$ in (2.18).

Figure 2.4 shows equilibrium initial yields on the senior tranche as a function of η, for various values of θ. The kink on an initial yield curve indicates the value of η at which the senior tranches switches equilibrium regions. For example, when

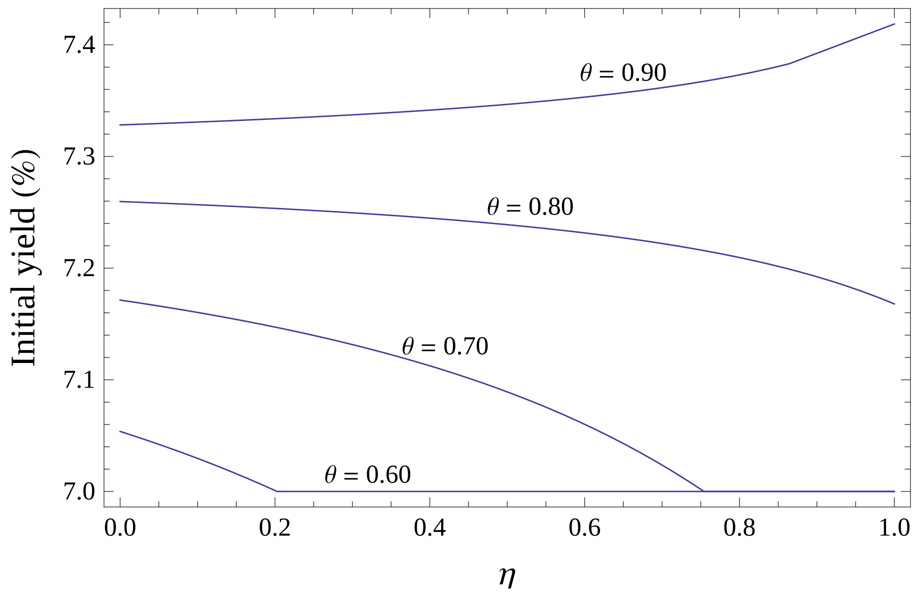


Figure 2.4: Equilibrium initial yields as a function of η.

$\theta = 0.60$, the senior tranche switches from the low-risk equilibrium to the risk-free equilibrium at the kink. When $\theta = 0.90$, the senior tranches switches from the low-risk equilibrium to the high-risk equilibrium at the kink. Depending on the value of θ, an increase in η can either increase or decrease the initial yield on senior bonds. To understand the effect of an increase in η on the yield, calculate the yield at the end points $\eta = 0$ and $\eta = 1$; the yield for $\eta \in (0, 1)$ is a weighted average of the yield at the end points. The analysis of homogeneous pools applies at the endpoints. When $\eta = 0$, the pool consists of late default mortgages only. When $\eta = 1$, the pool consists of early default mortgages only. An increase in η from zero to one is equivalent to a decrease in borrower default costs for a homogeneous pool; borrower default costs decrease from $k_{\beta l} = 4$ to $k_{\beta e} = 0$. As discussed earlier, a decrease in borrower default costs has two opposing effects: it increases the probability of default which raises the initial yield, and it increases the lender's net recovery which lowers the initial yield. The equilibrium initial yield at $\eta = 1$ maybe less than or greater than the yield at $\eta = 0$, depending on which effect dominates. The figure shows that the initial yield on the senior tranche is increasing in η for some θ, and decreasing in η for others.

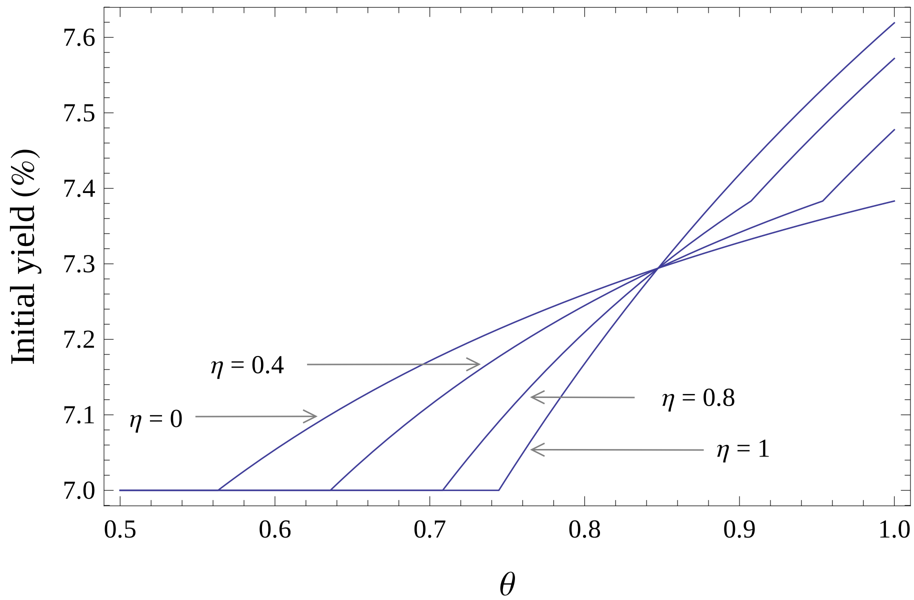


Figure 2.5: Equilibrium initial yields as a function of θ.

Figure 2.5 shows equilibrium initial yields on senior bonds as a function of θ, for various η.[9] For a given η, the initial yield on senior bonds is unambiguously increasing in θ. An increase in θ reduces the size of the residual tranche, so the senior tranche's buffer against default losses is decreased and its losses on default increase. The initial yield increases with θ to reflect the increase in the default

[9] I only show the initial yield for $\theta \in [0.5, 1]$; initial yield equals r for all $\theta \in [0, 0.5)$

risk of senior bonds. The kink on each initial yield curve shows the value of θ at which the senior tranche switches equilibrium regions. The first kink shows the switch from the risk-free region to the low-risk region. The second kink shows the switch from the low-risk region to the high-risk region. The high-risk equilibrium is ruled out when there is only one type of mortgage in the pool, so the initial yield curves for $\eta = 0$ and $\eta = 1$ only have one kink.

This graph also provides a different perspective on how the interaction between the two securitization parameters θ and η affects the initial yield on senior bonds. Consider $\theta = 0.70$. In this case, an increase in η lowers the initial yield because the recovery effect dominates the probability effect; the senior tranche is risk-free when $\eta = 1$. Now consider $\theta = 0.90$. In this case, as increase in η increases the initial yield because the probability effect dominates the recovery effect. The initial yield curves cross at the value of θ at which the two opposing effects cancel each other out. The crossing point is found by equating the initial yield on senior bonds when $\eta = 0$ to the yield when $\eta = 1$ and solving for θ; the initial yield on senior bonds is given by (2.12). In the benchmark parametrization, the initial yield curves cross when $\theta = 0.848$.[10]

2.4 Quantitative Exercises

This section shows model implied security prices, yields, and net monthly returns for house prices observed in the data between July 2006 and July 2011. I

[10]The initial yield curves for $\eta = 0$ and $\eta = 1$ cross provided the initial yield on the early default mortgage is higher than the initial yield on the late default mortgage. This condition is satisfied in the numerical example considered; the initial yield on the early default mortgage is 7.62% and the yield on the late default mortgage is 7.38%.

conduct this exercise using the Case-Shiller house price index. I present the findings for three different indexes: the composite-20 index, the Las Vegas metropolitan area index, and the Denver metropolitan area index. These indexes were chosen because the default experience of the benchmark pool is different for each index — only early default mortgages are terminated according to the composite-20 index; both early and late default mortgages are terminated according to the Las Vegas index; none of the mortgages are terminated according to the Denver index.

Using a hand collected dataset on subprime MBS, Park (2010) showed that the average LTV for non-agency securitizations during 2004-2007 was about 78%. Park (2010) also showed that the subordination for AAA-rated tranches during 2004-2007 ranged from 16.6% to 22.8%, with an average of 20.8%. Usually senior tranches of a CMO were rated AAA, so I assume that these tranches correspond to the senior tranche in the model. Motivated by the data, the low-risk equilibrium with $\theta = 0.80$ is the preferred specification for the quantitative exercise; all other parameters equal their values in the numerical example presented earlier.

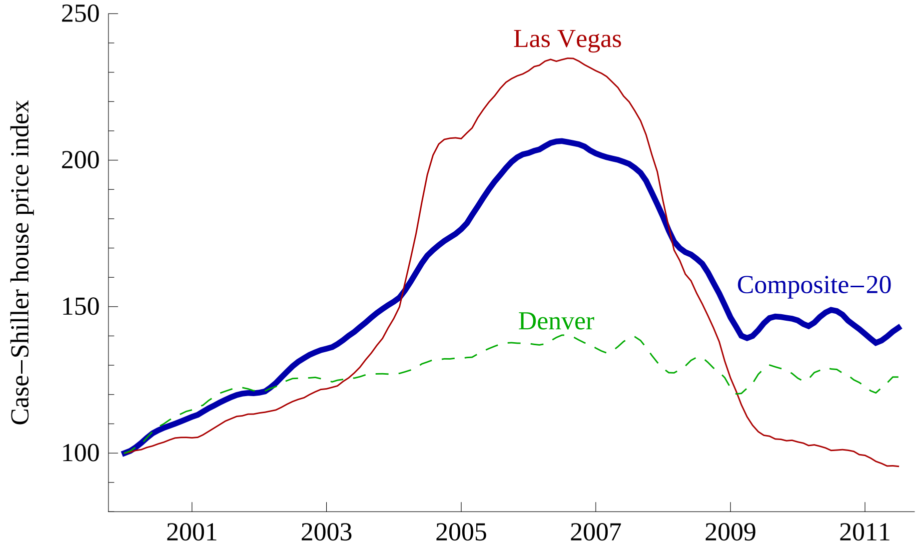
Figure 2.6a shows the composite-20 index (bold solid line), the Las Vegas metropolitan area index (solid line), and the Denver metropolitan area index (dashed line) from January 2000 to July 2011; the composite-20 index aggregates house price information from twenty metropolitan areas. The composite-20 index displays rapid house price appreciation until July 2006. According to this index, house prices doubled between January 2000 and July 2006. House prices in the Las Vegas metropolitan area more than doubled during the same time period; house prices in July 2006 were approximately 2.4 times their January 2001 values.

House price increases in the Denver metropolitan area were comparatively modest; they increased by about 30% in this time period. After reaching their peak in July 2006, house prices declined according to all three indexes. According to the composite-20 index, house prices declined at an average rate of 0.61% per month. By July 2011, the composite-20 index was 30.87% lower than its peak value. The Las Vegas index declined from its peak at an average rate of 1.47% per month, and was 59.25% lower than its peak value. The Denver index declined at a rate of 0.17% per month, and was 10.19% lower than its peak.

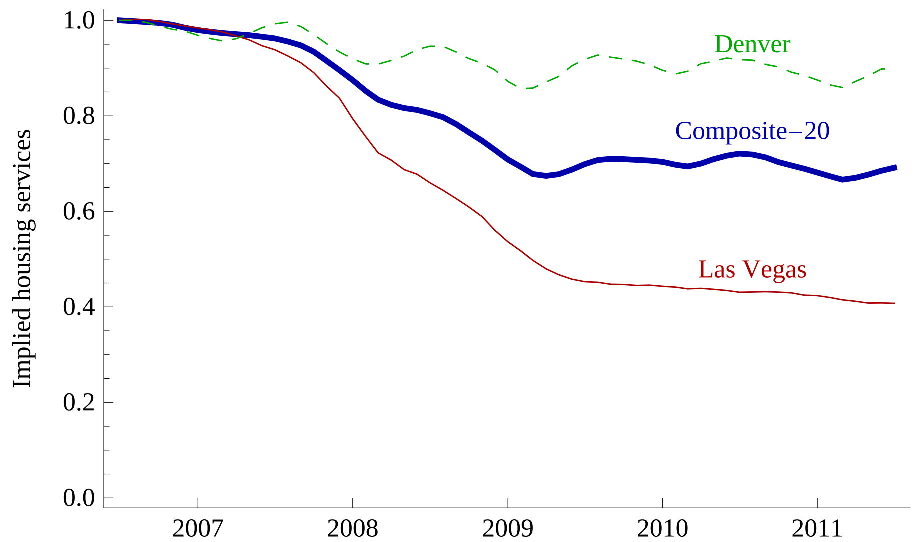
In order to get the house price index data within the model framework, I calculate the housing service flow implied by the data; see Figure 2.6b. I normalize the flow of housing services to be one on July 2006, the date when housing services peak. This date will be the origination date for both mortgages in the pool.

As noted earlier, the realization of the composite-20 index is such that only early default mortgages are terminated. Figure 2.7a shows the yield on the pool (solid line) and the senior tranche (bold solid line), as implied by this index. The figure also shows the early default date. As mentioned earlier, the bonds on the pool can be thought of as mortgage pass-through bonds. The yield on both mortgage pass-throughs and senior bonds rises as house prices fall because the likelihood of mortgage default increases. The yield on pass-throughs drops discontinuously at the early default date because the coupon on these bonds drops at this date. Since senior bonds are low-risk, the yield on these bonds is unchanged at the early default date.

Figure 2.7b shows the market value of a mortgage pass-through, a senior bond, and a residual bond for the composite-20 index. The original bond value has

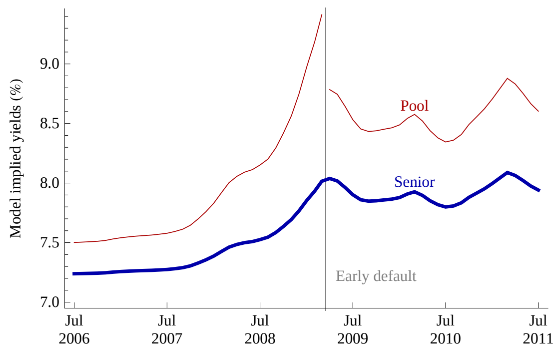


(a) Case-Shiller house price index

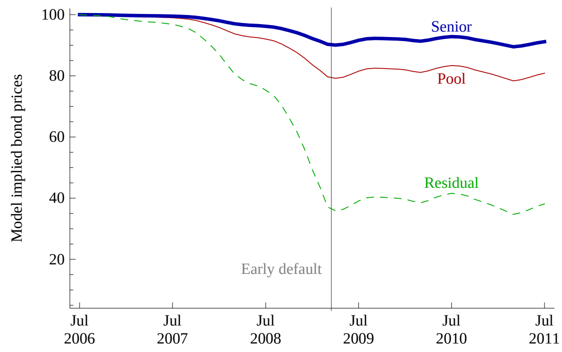


(b) Case-Shiller implied housing services

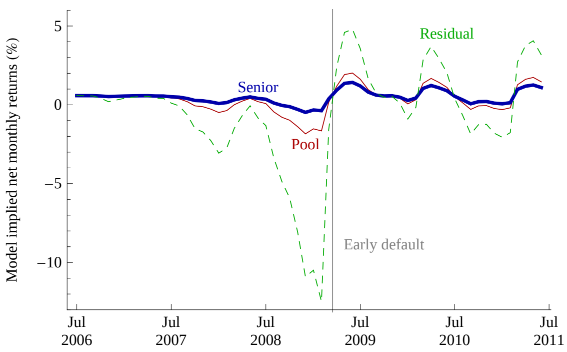
Figure 2.6: Case-Shiller house price index 2006-2011



(a) Model implied bond yields



(b) Model implied bond prices



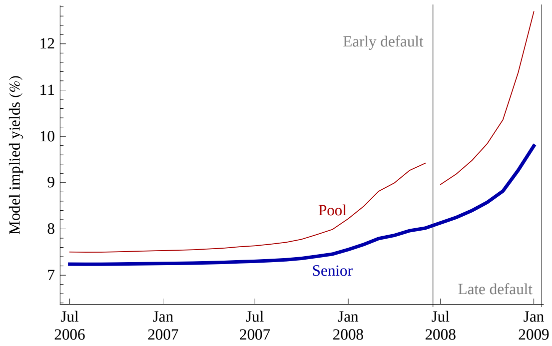
(c) Model implied net monthly returns

Figure 2.7: Composite-20 index

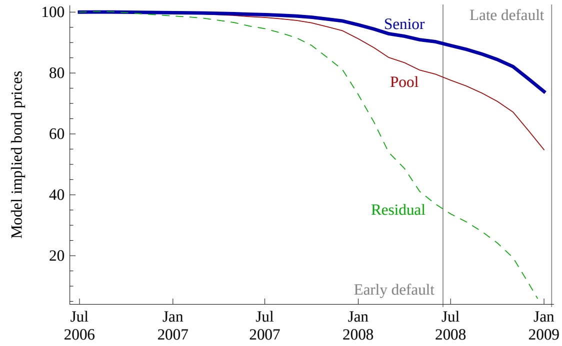
been normalized to 100. Notice that the value of each bond is continuous. As expected, the values decline as house prices fall. The default risk of the pool is divided disproportionately among the tranches to create relatively safe senior bonds, and relatively risky residual bonds. The bond values reflects this division — senior bond values are always above, and residual bond values are always below, the pass-through values. The model implied values of pass-throughs declined by 19.14% between July 2006 and July 2011. During the same time period, senior and residual bond values declined by 8.84% and 61.85%.

Figure 2.7c shows the net monthly return on all three bonds according to the composite-20 index. The net return was calculated as the sum of the monthly coupon payments and capital gains divided by the bond price last month. As the figure shows, the variability of monthly returns is highest for the residual tranche. The net monthly returns on this tranche range from −12% to 5%. In contrast, the net monthly return on senior bonds stays around 1%.

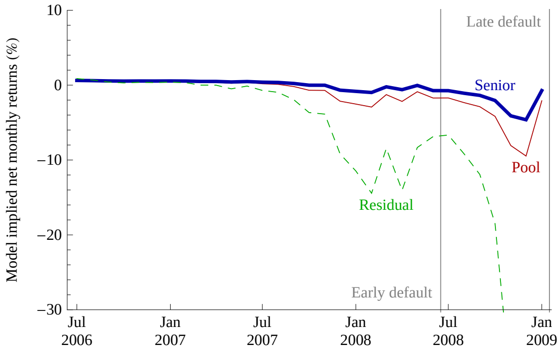
Figure 2.8 shows model implied yields, bond prices, and net monthly returns for housing services realized in the Las Vegas metropolitan area. The key difference between the Las Vegas index and the composite-20 index is that both early and late default mortgages are terminated according to the Las Vegas index. As expected, the realized pass-through and senior bond yields increase over time for this metropolitan area. The initial yield on pass-throughs is 7.50%, while the realized yield on these bonds one month before to the late default event is 12.69%. Similarly, the initial yield on senior bonds is 7.24%, and the realized yield on these bonds one month before to the late default event is 9.79%. The market value of all three bonds declines monotonically over time; bond values are normalized to



(a) Model implied bond yields



(b) Model implied bond prices

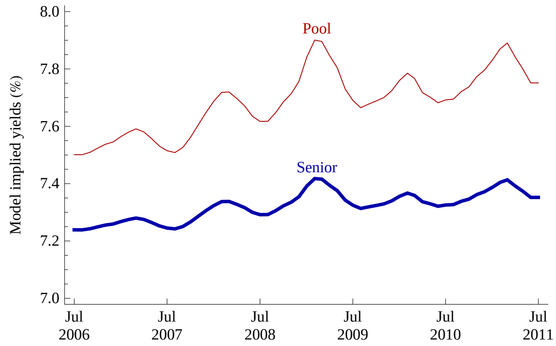


(c) Model implied net monthly returns

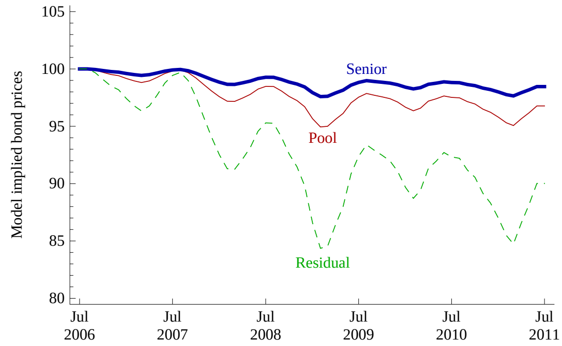
Figure 2.8: Las Vegas metropolitan area index

100 at the origination date. The mortgae pass-through is worth 79.62 at the early default event, and 54.79 at the late default event. The senior and residual bonds are worth 90.29 and 36.98 at the time of early default. The senior bond is worth 73.81 one month before late default event. Since the recovery on the residual bond is zero, it is only worth 2.37 one month before the late default event. According to the Las Vegas index, the net monthly returns on residual bonds are negative throughout the time period studied. In contrast, the net monthly return on senior bonds stays around 0% throughout, reaching its lowest value around −4% before the late default event.

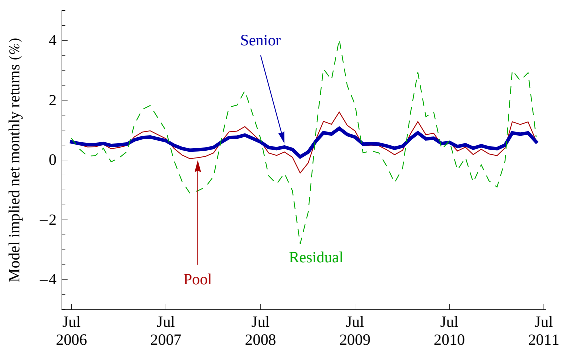
Figure 2.9 shows yields, bonds prices, and net monthly returns for the Denver metropolitan area. Denver's index differs from the composite-20 and the Las Vegas index because neither the early default nor the late default mortgages are terminated according to this index. As with the other two indexes, realized bond yields for this region increase over time. However, the size of the increase is smaller. The yield on pass-throughs increases from 7.50% to 7.75%, and the yield on senior bonds increases from 7.24% to 7.35%. Figure 2.9b shows that senior bond prices do not respond much to house price changes. In contrast, residual bond prices are very sensitive to house price changes. According to the Denver index, the net monthly returns fluctuate around 0.5% for all three bonds. As expected, the net return on senior bonds is close to 0.5%. However, the net return on residual bonds is more volatile; the lowest return is around −3% and the highest is around 4%.



(a) Model implied bond yields



(b) Model implied bond prices



(c) Model implied net monthly returns

Figure 2.9: Denver metropolitan area index

2.5 CMO-squared

In practice, tranches from various CMOs are often combined together into a new pool. The pool is again divided into various tranches, and bonds on these tranches are sold in capital markets. Since the underlying assets of the pool are tranches of an existing CMO, the resulting CMO is called a CMO-squared. Prior to the crisis, CMO-squared were used extensively as collateral in the shadow banking system; the total notional amount of CMO-squared issued between 2005-2007 was about $1.25 trillion.[11] CMO-squared were usually created from subordinate tranches of CMOs.[12] The basic principle behind creating CMO-squared was also to divide default risk disproportionately among the tranches; see Gorton (2010) for an overview. In this section I study the valuation of CMO-squared. I also repeat the quantitative exercise of the previous section using the composite-20 index and calculate model implied CMO-squared yields, prices, and monthly returns.

Even though, in practice, tranches from different CMOs are combined to create the CMO-squared pool, I assume that CMO-squared are created either from the senior or from the residual tranche of a single CMO. This assumption allows me to analyze the interaction between the default risk of mortgages and CMO-squared in a simple setting. Throughout this section, I focus on the benchmark low-risk CMO; the analysis with risk-free and high-risk CMOs is similar.

[11] Securities Industry and Financial Markets Association, "Global CDO Issuance and Outstanding" (April 2013). <www.sifma.org/research/statistics.aspx>.

[12] For example, Park (2013b) reports that only 1% of the value of tranches originally rated AAA was either placed in CMO-squared issued during 2005-2007. In contrast, during the same time period, this fraction was 47.03% for AA-rated, 68.38% for A-rated, 65.80% for BBB-rated tranches.

Consider a pool created from the low-risk senior tranche. The characteristics of the resulting pool are identical to the low-risk senior tranche: its initial value is 16, initial coupon is 1.158, early recovery is 7.45, value at the early default date is 6.98, coupon after early default is 0.560, and late recovery is 5.63; see Table 2.3.

The cash flows to the CMO-squared pool are re-structured so as to protect the senior tranche from default risk; the structure of cash flows is as in section 2.3.2. As earlier, the proportional size of the senior tranche of the CMO-squared created from the new pool is exogenous. In this case, the senior tranche of the CMO-squared can either be risk-free or low-risk in equilibrium. The high-risk equilibrium is ruled out because the coupon on the pool does not drop after the early default event. Consequently, all outstanding senior bonds also continue to receive their initial coupon after the early default event.

Following the analysis in section 2.3.2, I calculate the thresholds for the equilibrium regions. The first and the second thresholds are 0.4656 and 0.8174. The senior tranche of the CMO-squared is risk-free when its proportional value at origination is less than 81.74%. When the proportional value is less than 46.56%, the senior tranche is so small that all of it is bought back at the early default date. When the proportional value is greater than 46.56%, and less than equal to 81.74%, the senior tranche still recovers its entire principal. In this case, however, some senior bonds remain outstanding after the buy back at the time of early default. These senior bonds are bought back at their par value if late default mortgages are terminated. If the proportional value is greater than 81.74%, then senior CMO-squared bonds are low-risk in equilibrium.

As a example, consider the case in which the proportional value is 0.80. The resulting senior CMO-squared tranche is risk-free in equilibrium. The initial value of this tranche is 12.8. Its coupons are 0.896 at origination, and 0.375 after the early default event. The early recovery is 7.45 and the late recovery is 5.35. The implied yield is $r = 7\%$ regardless of the evolution of housing services. In this case, re-tranching has created a senior CMO-squared tranche that is risk-free even though the CMO used to create it is risky. Note that the proportional value of the senior CMO-squared tranche is equal to that of its CMO counterpart.

The implied characteristics of the residual tranche of the CMO-squared follow. The initial value of this tranche is 3.2. Its coupon is 0.262 at origination. The coupon after the early default event is 0.185. Its value at the early default date is 1.63. The early recovery is zero, and the late recovery is 0.28. The recovery rate is 8.75%. The initial yield is 8.19%, and the yield after the early default event is 11.35%. The yield spread at origination for the residual tranche of the CMO-squared is 1.19%, whereas the spread for the residual tranche of the CMO is 1.55%. The lower spread on the residual CMO-squared tranche indicates that the default risk of this tranche is lower than that of its CMO counterpart.

Now consider a CMO-squared created from the residual tranche of the low-risk CMO. The characteristics of the resulting pool are identical to the residual tranche of the low-risk CMO: Its initial value is 4, initial coupon is 0.342, value at the early default date is 1.44, coupon after early default is 0.178, and total recovery is 0; see Table 2.3. Since the total recovery on the pool is zero, the resulting senior tranche cannot be risk-free in equilibrium, except in the trivial case in which the proportional value of this tranche is zero. Since the early recovery on the pool is zero, no senior bonds are bought back at the time of early default. The senior CMO-squared tranche is low-risk in equilibrium as long as its coupon at origination is less than 0.178, the coupon on the pool after the early default event. The threshold at which the equilibrium switches from low-risk to high-risk is found by setting the coupon on the senior tranche equal to 0.178 in (2.16) and (2.17). The resulting value of the threshold is 0.565. The senior CMO-squared tranche is low-risk in equilibrium when its proportional value is strictly greater than zero and less than 56.50%, and high-risk when its proportional value is strictly greater than 56.50%.

In particular, when the proportional value is 0.80, the resulting senior tranche is high-risk in equilibrium. The coupon on this tranche is 0.267 at origination and 0.178 after the early default event. The recovery rate of the senior CMO-squared tranche is zero, whereas the recovery rate of the senior CMO tranche is 81.74%. The implied initial yield is 8.34%, and the yield at the early default date is 12.34%. The yield spread at origination on the senior CMO-squared tranche is 1.34%, whereas the yield spread at origination on the senior tranche of the CMO is 0.24%. The yield spreads reflect the fact that the senior CMO-squared tranche has higher default risk than the senior tranche of the CMO. The default risk of the resulting residual tranche is also higher than that of the residual tranche of the CMO; the yield spread on the residual CMO-squared tranche is 2.42%, whereas the yield spread on its CMO counterpart is 1.55%.

The analysis in this section highlights that default risk for a CMO tranche maybe very different from default risk of the corresponding CMO-squared tranche, even though the relative size of the tranches are identical. In the numerical examples presented, the senior CMO-squared tranche created from a low-risk CMO was risk-free or high-risk in equilibrium, depending on whether the senior or residual tranche of the CMO was used to create the CMO-squared. It is worth emphasizing that the different default risk profiles of the CMO-squared bonds were not generated by differences in the characteristics of the underlying mortgages. Instead, the different risk profiles were generated solely by the structure of cash flows at various levels of tranching.

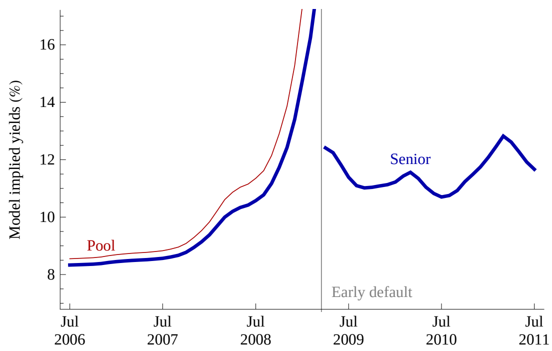
Figure 2.10 shows the yield, bond prices, and net monthly returns on the CMO-squared created from the residual tranche of the benchmark CMO; I used the composite-20 index for this exercise. Since the resulting senior tranche is high-risk, its yield drops discontinuously at the early default date. The yield on the pool and the senior tranche are identical after early default mortgages are terminated, so the solid line and the bold solid line overlap in Figure 2.10a. Figure 2.10b shows that the prices of all bonds decline monotonically. By the early default date, senior bonds are only worth half of their origination value, and residual bonds are worthless.

In practice, most CMO-squared are created from CMO tranches that have not been rated AAA; these tranches together correspond roughly to the residual tranche of the model CMO. For CMO-squared created from the residual tranche, the model implies that the senior tranches of CMO-squared have higher default risk than the senior tranches of CMOs. This prediction of the model seems to be consistent with average losses observed in the data. Cordell, Huang, and Williams (2012) report that the average principal write down on publicly traded CMO-squared issued in 2006 and 2007 was above 93% for all tranches, except the Senior

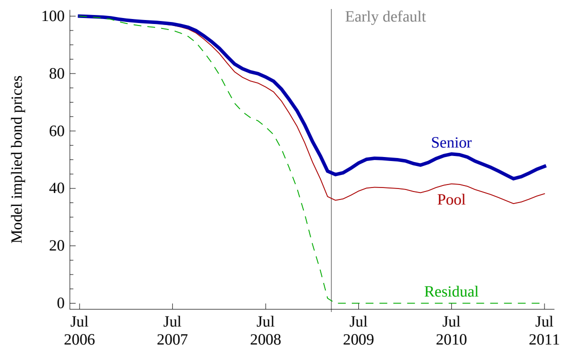
AAA tranche which suffered an average write down of 67% in 2006 and 76% in 2007.[13] (For comparison, note that the model implied senior CMO-squared bond values dropped by 50%, and residual bond values declined 100%.) In contrast, Park (2013a) reports that the average write down on AAA-rated tranches, for subprime CMOs issued during 2004-2007, was only 0.17%; the average write down on the lowest-rated BBB tranches was 56.97%. Similarly, Foote, Gerardi, and Willen (2008b) report that only 10% of AAA-rated CMOs issued in 2006-2007 suffered losses, whereas 90% of CMO-squared issued during the same time period suffered losses.

Even though the analysis in this section involves considerable simplifications, it seems to capture how default risk of the underlying mortgages affects valuation of CMO-squared. Data support the prediction of the model that losses on senior tranches of CMO-squared created from residual tranches of CMOs maybe quite large, even though the losses on senior tranches of the same CMOs are small. The analysis so far has been limited to valuation of CMO-squared. However, it can easily be extended to incorporate valuation of higher order CMOs. For example, the analysis implies that a CMO-cubed created from the residual tranche of the high-risk CMO-squared is such that the entire pool, and so the tranches, becomes worthless as soon as early default mortgages are terminated.

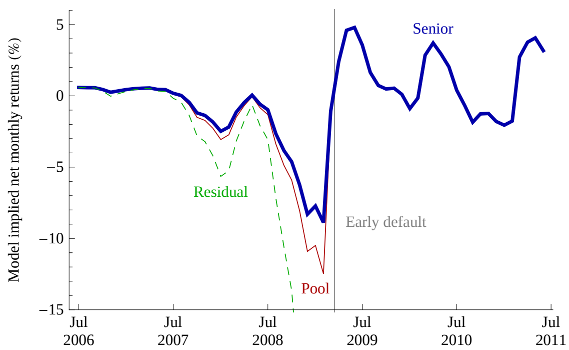
[13] A Senior AAA tranche or super senior tranche usually refers to tranches that have subordinate tranches which are AAA rated. By construction, the Senior AAA tranches had the lowest exposure to default risk.



(a) Model implied bond yields



(b) Model implied bond prices



(c) Model implied net monthly returns

Figure 2.10: Composite-20 index

2.6 Credit Default Swaps

A Credit Default Swap (CDS) is insurance against default. The CDS buyer pays insurance premiums to the CDS seller in exchange for payments contingent on some pre-specified credit events. CDS were a major asset class before the financial crisis. According to the annual market survey of the International Swaps and Derivatives Association (ISDA), the total amount of CDS outstanding in 2007 was $62.2 trillion. Beginning in 2005, CDS allowed market participants to take short positions on subprime MBS for the first time.[14] The launch of the ABX.HE index CDS aggregated and revealed the views of market participants on subprime MBS for the first time. Gorton (2009) argues that this information regarding the subprime market, along with inadequate information regarding the location of subprime risk, began the financial crisis of 2007-2008. This section extends the analysis to the valuation of CDS on mortgage bonds. It also shows the model implied CDS prices for the Case-Shiller house price index.

In practice, the ABX.HE index is traded as follows. The buyer pays a one time upfront fee and a fixed index-specific monthly premium to the seller in exchange for payments contingent on default. CDS prices are quoted as a percentage of par value. They equal the par value, normalized to 100 at origination, minus the upfront payment. For example, a price of 60 means that the upfront fee is 40. Since the insurance premium is fixed, it is the price that changes in response to market conditions so as to reflect the price of insurance against default. The CDS contract in the model looks similar. Consider a CDS written on senior bonds. These bonds

[14]The ISDA standardized its documentation, and successfully launched single-named asset backed CDS contracts in 2005; see Fender and Scheicher (2009).

are scheduled to pay $c_s(0)$ perpetually. The realized payments, however, depend on the realization of housing services. The seller of the CDS insures the buyer against any shortfall in scheduled payments. In return, the buyer pays the seller a one time upfront fee $I_s(t, x(t))$, and an insurance premium i_s; the premium is paid until the late default date.

The following thought experiment shows how to value a CDS contract. Suppose that the buyer of the CDS holds a senior bond, which he turns over to the seller at the time of the purchase along with the upfront fee; the buyer also pays the insurance premium until the late default date. In return, the seller pays the buyer $c_s(0)$ until the late default date. The CDS contract is terminated at this date with the seller giving the buyer an insurance payout of $c_s(0)/r$. The profits of a CDS seller from insuring one senior bond at some $t < \tau_l$ are

$$I_s(t, x(t)) + V_s(t, x(t)) + \mathbb{E}_t \left[\int_t^{\tau_l} e^{-r(\tau_l - z)} i_s dz \right] - \frac{c_s(0)}{r}, \tag{2.21}$$

where $I_s(t, x(t))$ is the upfront fee, $V_s(t, x(t))$ is the market value of the senior bond, i_s is the insurance premium, and $c_s(0)/r$ is the present value of the insurance payout. I assume that the insurance is actuarially fair, so CDS sellers make zero expected profits. The insurance premium is such that the upfront fee is zero at origination. Once determined, the premium is fixed over lifetime of the CDS. As default probabilities change, the upfront fee fluctuates so as to keep the insurance fairly priced.

Even though the discussion so far has been restricted to senior bonds, it carries over to CDS written on mortgage pass-throughs and residual bonds. As a numerical example, consider CDS written on each bond of the benchmark low-risk CMO separately. The insurance premium for mortgage pass-throughs is 0.113, senior bonds is 0.043, and residual bonds is 0.070. As a percentage of the insured amount, the premium on pass-throughs is 0.56%, senior bonds is 0.26%, and residual bonds is 1.74%. Figure 2.11 shows CDS prices on all three securities when the composite-20 index is fed through the model. The implied prices of all three CDSs decrease as house prices decrease and default in the near future becomes more likely.

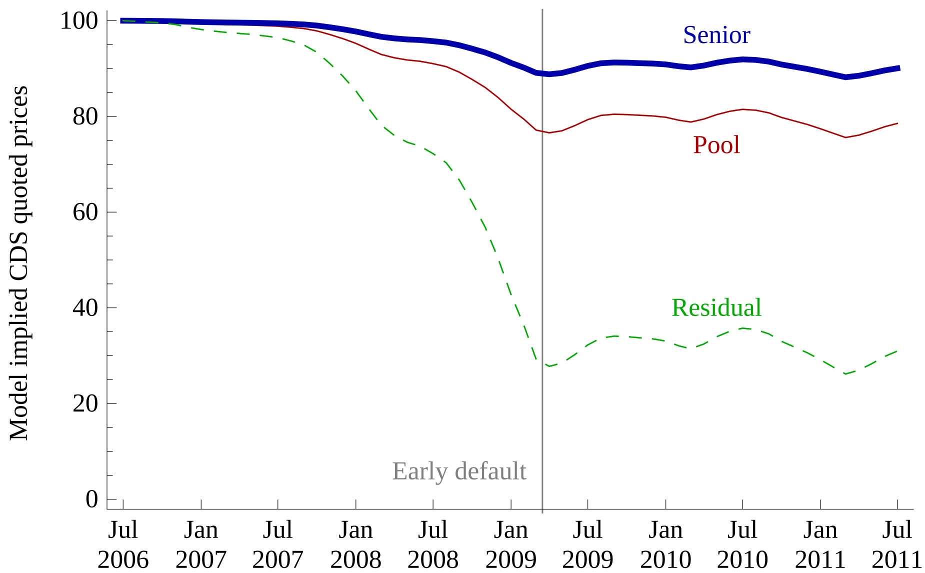


Figure 2.11: Composite-20 index

In practice, prior to the decline in house prices, senior bonds usually carried a AAA rating at origination. Therefore model implied CDS prices for senior bonds correspond approximately to the ABX.HE-AAA index. The correspondence is not exact because the AAA tranches referenced by the corresponding ABX indices were not the senior most tranches in their CMOs. Figure 2.11 suggests that prices of CDS on senior bonds do not fall significantly below the par value; the lowest model implied price for this CDS is 88.18. In the data, however, the ABX.HE-AAA indexes were trading significantly below par; see Figure 1 in Stanton and Wallace (2011). For example, prices of both the 2007 vintages declined steadily and bottomed out around 20, before recovering steadily to around 40 by July 2010. The quantitative exercise suggests that replicating the steep decline in the ABX.HE-AAA indices for reasonable parameter values might be difficult. This finding is in line with recent research on the ABX.AAA-HE index. For example, Stanton and Wallace (2011) conclude that no reasonable expectation regarding defaults and recovery rates on mortgages underlying the ABX.AAA-HE index can account for the observed decline in prices.[15]

CDS written on residual bonds in the model correspond approximately to ABX.HE index on bonds that were rated AA, A, BBB, BBB-. The index on these bonds experienced price declines that were larger than those experienced by the index on AAA-rated bonds. In fact, ABX.HE indexes for some of the lowest-rated bonds experienced 100% principal writedowns, and were trading on an interest-only basis. Figure 2.11 suggests that mortgage default may account for a large fraction of the price decline in the ABX.HE indexes that were rated below AAA; the lowest model implied price for CDS written on residual bonds is 26.14.

[15]Stanton and Wallace (2011) collected detailed data on the individual loans underlying the ABX.HE index, and calculated the default rates implied by the observed prices. They found that a prepayment rate of 25% and a recovery rate of 34% implied default rates of 100% at the observed prices for the ABX.HE-AAA index; the assumed prepayment rate is roughly consistent with historical prepayment rates on the underlying pools, and the recovery rate is below anything observed in U.S. mortgage markets. An expected recovery rate greater than 34% implies that the observed prices are inconsistent with reasonable assumptions regarding default behavior.

This finding is consistent with the empirical work of Fender and Scheicher (2009). Using regression analysis, these authors found that indicators of housing market activity were important for subordinate ABX.HE indexes, but not for AAA and AA-rated indexes.

2.7 Conclusion

This paper provides a structural model for pricing Mortgage Backed Securities (MBS) in the presence of mortgage default risk. I model the mortgage default decision of homeowners, along with essential contractual features of MBSs. The analysis begins by valuing Collateralized Mortgage Obligations (CMOs). For CMOs made from pools containing two types of mortgages, I find that senior bondholders may experience no principal or coupon shortfalls, principal shortfalls only, or both principal and coupon shortfalls; the type of equilibrium depends on the relative size of the senior tranche. The initial yield on senior bonds increases as the relative size of the senior tranche increases. In the quantitative exercise I find that senior bonds lose about 10% of their value and residual bonds lose about 60% of their value when housing services implied by the composite-20 Case-Shiller index, from July 2006 to July 2011, are fed through the model.

I extend the model to study CMO-squared. Conditional on relative size, I find that a senior CMO-squared tranche has higher default risk than the senior tranche of the CMO, when the CMO-squared is created from the residual tranche of the CMO. According to the quantitative exercise, senior CMO-squared bonds lose half their value and residual bonds became worthless when the composite-20 index is fed through the model. I also extend the model to price Credit Default Swaps on mortgage bonds. The model implied prices for CDS on residual bonds suggest that default risk was a major driver of the price declines for the ABX.HE indexes rated below AAA.

The quantitative predictions are not the outcome of a calibrated version of the model. Calibrating the model directly is challenging because of the presence of unobservable default cost parameters. The estimation of these parameters from realized recovery rates along with a calibration of the model is a useful direction for future research. The calibrated version of the model can be used to analyze whether mortgage bonds were "mispriced" prior to, or during, the financial crisis. It would also serve as a benchmark with which to compare the rating agencys' assessments of MBS. Narratives of the crisis argue, with a considerable element of hindsight of course, that mispricing and inflated rating both exacerbated, if not caused, the financial crisis of 2007. The calibrated model can shed light on the role, or the lack thereof, of these distortions in the financial crisis.

Why lenders choose a particular capital structure for MBS is another important area for future research. The Miller-Modigliani theorem applies to the environment laid out here, so lenders are indifferent between all capital structures for MBS. In practice, however, lenders were particular about the capital structure of the MBS; see Park (2010) for some evidence that CMO pools that consisted of mortgages with higher default risk had higher subordination levels. The analysis also abstracted from informational frictions in the MBS market. Ashcraft and Schuermann (2008) discuss seven key sources of informational frictions in the market for subprime MBS. Analyzing how these frictions affect MBS contractual features, yields, and equilibrium prices is also an important area for future work.

Chapter 3

Default Risk and Valuation of Mortgages with Coupon Resets

3.1 Introduction

The buildup to the financial crisis of 2007-2008 saw a large increase in the popularity of mortgages that featured low coupon payments for the first few years followed by a reset to a higher coupon payment. Narratives of the crisis often attribute the high default rates observed during the crisis to the increase in the mortgage payment after the reset. Even though the effects of payment resets on mortgage defaults are widely discussed, there are few studies that formally model borrowers' incentives to default on mortgages with payment resets. This paper fills this gap in the literature by studying optimal default in mortgages with payment resets. It also connects equilibrium yield spreads on these mortgages to initial loan-to-value ratios, time until the reset, expected growth rate in house prices, and house price volatility.

I study optimal default in an environment in which the flow of services from a house are exogenous and stochastic. Changes in housing services are unforecastable. House prices equal the expected discounted value of housing services. Therefore they are also exogenous and unforecastable. Houses are purchased using a mortgage loan and a downpayment. The loans feature a coupon reset. The reset date and the coupons, before and after the reset, are known at the mortgage origination date; the analysis abstracts from interest rate uncertainty. Borrowers have the option to default on their mortgage. They exercise this option so as to maximize home equity. Lenders make zero expected profits. The environment here adapts Merton (1974) to the analysis of mortgage default. This approach is standard in the mortgage default literature. For example, Krainer, LeRoy, and O (2009, KLO from here on) study optimal default by borrowers with fixed rate mortgages in this environment.

I provide conditions under which the optimal default boundary for a mortgage with a payment reset has a discontinuity at the reset date. The discontinuity at the reset date arises when the coupon after the reset is large, compared to the coupon prior to the reset. The intuition behind this finding can be understood by considering the incentives of a borrower with a mortgage in which coupon prior to the reset is zero, and the coupon after the reset is strictly positive. The borrower would never find it optimal to default before the reset. Default after the reset, however, is optimal if house prices are sufficiently low. Therefore the default boundary features a discontinuity at the reset date. The discussion on coupon resets usually attributes any jumps in the default boundary at the reset date to unanticipated payment increases. The model presented here, however, shows that the default boundary jumps at the reset date even when the post-reset coupon is known in advance.

Conversely, the default boundary is continuous at the reset date if the post-reset coupon is not much larger than the pre-reset coupon. This finding shows that an option based model of mortgage default is qualitatively consistent with empirical findings on how coupon resets impact mortgage default behavior. For example, Foote, Gerardi, and Willen (2008b) find that delinquencies on subprime mortgages originated in January 2005 did not spike when the coupon reset, suggesting that the default boundary is continuous in the data.

The analysis also shows that, in addition to the initial loan-to-value ratio, the coupons before and after the reset are important determinants of the initial yield on the mortgage. Conditional on the initial loan-to-value ratio, mortgages with low initial payments followed by high payments after the reset have higher equilibrium initial yield spreads than mortgages in which the increase in coupon at the reset date is smaller. The difference in initial yield spreads reflects the fact that that the expected loss on the former mortgage is greater than the expected loss on the latter. It should be emphasized that this difference is not due to changes in the ability of the borrower to make mortgage payments, which is the channel usually emphasized by analysts. Rather the difference is due to the timing of payments and the associated change in default probabilities, which reflects the borrower's unwillingness to make payments even if he had the ability to do so; analysts often use the term "strategic default" to describe such behavior. The difference between the yield spreads predicted by the model is consistent with empirical work by KLO. The authors find that adjustable-rate mortgages with high initial loan-to-value ratios are more prone to default, and have higher initial yield spreads, than fixed rate mortgages with the same initial loan-to-value ratio.

3.2 Benchmark Model

The model is set in continuous time. Agents discount the future at a constant rate ρ. A house provides a stochastic flow of services. Housing services $x(t)$ follow a geometric Brownian motion

$$dx(t) = \alpha x(t)dt + \sigma x(t)dw(t); \tag{3.1}$$

where α is the drift parameter, σ is the volatility parameter, and $w(t)$ is standard Brownian motion. The drift parameter α represents the expected proportional gain in housing services. I normalize initial housing services to one, $x(0) = 1$.

House prices equal the expected discounted value of future services. That is,

$$P(x(t)) = \mathbb{E}_t \left[\int_{z=t}^{\infty} e^{-\rho(z-t)} x(t) dz \right] = \frac{x(t)}{\rho - \alpha}. \tag{3.2}$$

The mathematical expectation $\mathbb{E}_t$ is conditional on information available at time t. The pricing specification (3.2) is valid if agents are risk neutral, or if housing services follow (3.1) under the risk neutral pricing measure. The adopted specification rules out bubbles. By the Ito-Doeblin formula, house prices also follow a geometric Brownian motion with drift parameter α and volatility parameter σ.

Therefore the best prediction for house prices is that they grow at the constant rate α.

Under the normalization $x(0) = 1$, the purchase price of a house is $P(1) = 1/(\rho - \alpha)$. A borrower buys the house using a mortgage loan. The difference in the size of the mortgage loan and the purchase price is financed from the borrower's personal wealth, which I do not model. The mortgage contract requires the borrower to make regular coupon payments to the lender in exchange for the flow of services from the property. The coupon schedule is divided into two regimes. The borrower pays the coupon c_0 for the first T years. The coupon resets and equals c_1 in perpetuity thereafter. The reset date T and the coupons c_0 and c_1 are exogenous. I use the term reset mortgage to refer to such contracts. When $c_0 = c_1$, a reset mortgage corresponds to a fixed rate mortgage. When $c_0 < c_1$ the initial coupon is a teaser coupon. The initial period of low coupon payments is the teaser period. As modeled here, a reset mortgage with a teaser coupon is a stylized version of graduated payment, hybrid adjustable-rate, and interest-only mortgage contracts observed in practice. As with the contracts observed in practice, the coupon payments in a reset mortgage with a teaser coupon are back loaded: they are low initially and then jump at a pre-specified reset date. In Section 3.2.3 the setup here is used to study fixed rate balloon payment mortgages. In practice, these mortgages do not amortize fully over the lifetime of the mortgage loan. Consequently the borrower has to make a lumpsum payment when the loan matures. Balloon payment mortgages are common in commercial real estate transactions.

The borrower has the option to default on his mortgage. Default is costless for both the borrower and the lender; costly default is analyzed in Section 3.3. The analysis assumes that the borrower can always buy the mortgage back from the lender at its market value. This assumption corresponds in practice to the borrower's ability to hand over the house keys to the lender and walk away from the mortgage. In practice, the borrower also has the option to prepay his mortgage. Since the focus is on default, the analysis abstracts away from prepayment.

Lenders make zero expected profits, implying that the size of the mortgage loan must equal the expected discounted value of the borrower's payments. Since the expected value of payments depends on the default behavior of the borrower, the size of the mortgage is determined as part of the equilibrium. Consequently the initial loan-to-value (LTV) ratio and the mortgage yield are also endogenous.

3.2.1 Equilibrium

The borrower takes the coupon schedule as given and chooses a default rule that maximizes his home equity, or equivalently minimizes his mortgage liability. The equity maximization problem is solved in two steps. First I maximize the equity after the coupon reset. Next I take the post-reset equity as given and maximize the equity prior to the coupon reset. Conditional on non-default till the reset date, home equity after the coupon reset is identical to equity in a fixed rate mortgage with coupon c_1. Let $F(x(t))$ denote the borrower's equity in a fixed rate mortgage with coupon c_1. Home equity after the reset is given by the formula,

$$F(x(t)) = \left(P(x(t)) - \frac{c_1}{\rho}\right) \quad + \quad \left(\frac{c_1}{\rho} - P(\delta_1)\right)\left(\frac{\delta_1}{x(t)}\right)^m; \tag{3.3}$$

where δ_1 denotes the optimal default boundary, m is the positive root of the characteristic quadratic, and $x(t)$ is the level of housing services at some $t > T$; see KLO and Singhania (2013) for two alternative derivations of (3.3). The expression in (3.3) shows that home equity after the coupon reset equals the current house price $P(x(t))$ minus the present value of the remaining coupon payments c_1/ρ plus the value of the default option. On default, the borrower gains the present value of the remaining coupon payments c_1/ρ and loses the house, which is worth $P(\delta_1)$.

The formula for the default boundary after the coupon reset is,

$$\delta_1 = \left(\frac{m}{m+1}\right)\left[\frac{c_1/\rho}{P(1)}\right]. \tag{3.4}$$

The root m equals

$$m = \frac{(\alpha - \sigma^2/2) + \sqrt{(\alpha - \sigma^2/2)^2 + 2\rho\sigma^2}}{\sigma^2}.$$

Equation (3.4) shows that the default boundary for a fixed rate mortgage is proportional to the ratio of present value of remaining payments and the purchase price of the house. Since $m > 0$, the term $m/(m+1)$ is strictly less than one, implying that it is not optimal for the borrower to default on his mortgage as soon as the present value of total outstanding mortgage payments exceeds the purchase price. The formula for the default boundary also highlights the importance of distinguishing between economic value and book value of equity, which equals the current house price minus the outstanding principal balance on the mortgage. Default is optimal when the economic value of equity is zero, not the book value.

It is worth noting that home equity after the coupon reset is a function of housing services only. This is a consequence of the assumption that the mortgage is a perpetuity after the coupon reset, implying that the present value of remaining coupon payments equals c_1/ρ regardless of the time elapsed since the reset, conditional on non-default. Home equity prior to the reset, however, is a function of both time elapsed since origination t and current value of housing services $x(t)$. The additional dependence on t follows from the fact that the present value of remaining coupons, conditional on non-default, depends on time remaining until the reset. Given c_0 and c_1, the remaining coupon payments close to the reset date consist mostly of c_1, whereas the remaining coupon payments for a borrower close to the mortgage origination date consist of both c_0 and c_1. Prior to the reset, the present value of remaining coupon payments is lower (higher) than c_1/ρ, depending on whether c_0 is lower (higher) than c_1. The time remaining until the coupon reset determines the amount by which the present value of remaining coupon payments is lower (higher) than c_1/ρ, depending on whether c_0 is lower (higher) than c_1.

Let $E(t, x(t))$ denote home equity of a borrower with a reset mortgage. The discussion above implies $E(t, x(t)) = F(x(t))$ after the coupon reset, $t > T$. Prior to the coupon reset, home equity satisfies the following Bellman equation,

$$E(t, x(t)) = \max\left\{0,\ (x(t) - c_0)dt + e^{-\rho dt}\mathbb{E}_t\left[E(t + dt, x(t) + dx(t))\right]\right\}. \quad (3.5)$$

The Bellman equation shows that at each instant the borrower decides whether or not to exercise his option to default. He continues with the mortgage as long as his equity is positive. The equity from continuation consists of two components: the immediate net payoff from continuation $(x(t) - c_0)dt$ and the expected discounted value of future equity. In the continuation region, home equity in (3.5) satisfies a partial differential equation along with value matching and smooth pasting conditions that are standard in the real options literature; see Dixit and Pindyck (1994). The value matching and smooth pasting conditions require that the level and the slope of home equity equal zero at each point on the default boundary. There are no known closed form solutions to this partial differential equation. I solve for home equity numerically using the binomial tree framework of Cox, Ross, and Rubinstein (1979); see Appendix B.2 for details. The numerical method computes (3.5) for every node in the binomial lattice by backward induction. The optimal default boundary at each date t, denoted $\delta(t)$, is found by looking up the largest value of $x(t)$ at which equity equals zero. The borrower is indifferent between default and continuation along $\delta(t)$.

Once the home equity of the borrower is found, his mortgage liability follows:

$$M(t, x(t)) = P(x(t)) - E(t, x(t)). \tag{3.6}$$

The fact that default is costless for the lender implies that his asset value of the mortgage equals the borrower's mortgage liability. The zero expected profit condition implies that the size of the mortgage loan equals its asset value. The borrower's equity at the time of default equals zero. Therefore, by (3.6), the lender's recovery on the mortgage equals the house price at the time of default.

The mortgage yield is the discount rate at which the present value of the remaining coupon payments equals the market value of the mortgage. It is the value of r that solves

$$M(t, x(t)) = \frac{c_0}{r}\left(1 - e^{-r(\max\{T,t\}-t)}\right) + \frac{c_1}{r} e^{-r(\max\{T,t\}-t)}. \tag{3.7}$$

The difference between the mortgage yield and the expected return on mortgages ρ is the yield spread on the mortgage. The spread reflects the expected losses from mortgage default. The initial mortgage yield is the value of r that equates the right hand side of (3.7) to $M(0, 1)$.

3.2.2 Numerical Example

Since closed form solutions are unavailable, I discuss the properties of the equilibrium using numerical examples. The discount rate is $\rho = 7\%$, implying that the average real proportional gain on mortgages and home equity is 7%. The parameters for the geometric Brownian motion followed by housing services are $\alpha = 3\%$, and $\sigma = 15\%$. Under the normalization $x(0) = 1$, the purchase price of a house is $P(1) = 25$. The implied price-to-rent ratio in the model is 25. Price-to-rent ratios in the data are closer to 10 or 15. This discrepancy between the model and the data is appropriate because the model abstracts from operating costs such as maintainence and utilities expenses. The chosen value of $\sigma = 15\%$ for the standard deviation of housing services is consistent with estimates of individual house price volatility in the literature.[1] In the benchmark parametrization, the

[1] For example, Flavin and Yamashita (2002) estimated the standard deviation of the real return on housing to be 14%. Similarly, Case and Shiller (1989) estimated the return on in-

coupon resets two years from the date of mortgage origination, $T = 2$. The coupon after the reset is $c_1 = 1.75$. The chosen value of c_1 corresponds to the coupon on a fixed rate mortgage with an initial LTV ratio close to 90%. I compute the equilibrium for various c_0. Recall that the equilibrium requires the joint computation of the default boundary and the size of the mortgage loan, such that the borrower maximizes home equity and the lender makes zero expected profits.

First consider the case with $c_0 = 0$. The borrower strictly prefers to continue with the mortgage until the coupon reset because he gets a positive flow of housing services at zero cost. In this case, I define the default boundary during the teaser period to be zero. This definition is consistent with the discussion of the default boundary earlier: The borrower is indifferent between default and continuation if housing services equal zero before the coupon reset. When the mortgage coupon resets to $c_1 = 1.75$ the default boundary jumps to $\delta_1 = 0.776$. After the reset the borrower defaults when housing services are less than or equal to 77.6% of the purchase price of the house. The level of housing services at which the borrower defaults determines the recovery. The default boundary is discontinuous at the reset date T. Therefore the recovery on the mortgage is not known with certainty at the mortgage origination date. It can take any value within the interval $(0, P(\delta_1)]$, if the borrower defaults as soon as the coupon resets. Otherwise, if he defaults some time after the coupon reset, then the recovery equals $P(\delta_1) = 19.40$. The value of the mortgage at origination is its expected discounted date T value, $M(0, 1) = 19.47$. The initial LTV ratio is $M(0, 1)/P(1) = 77.90\%$. The initial mortgage yield is 7.71%.

dividual houses to be around 14-15%. Values of σ closer to 10% maybe more appropriate for houses located in certain geographical areas of the United States.

It should be pointed out that the equilibrium with $c_0 = 0$ relies heavily on the assumption that borrowers cannot refinance their mortgage. If this assumption was relaxed lenders might avoid contracts with $c_0 = 0$ because borrowers are likely to refinance when the coupon resets.

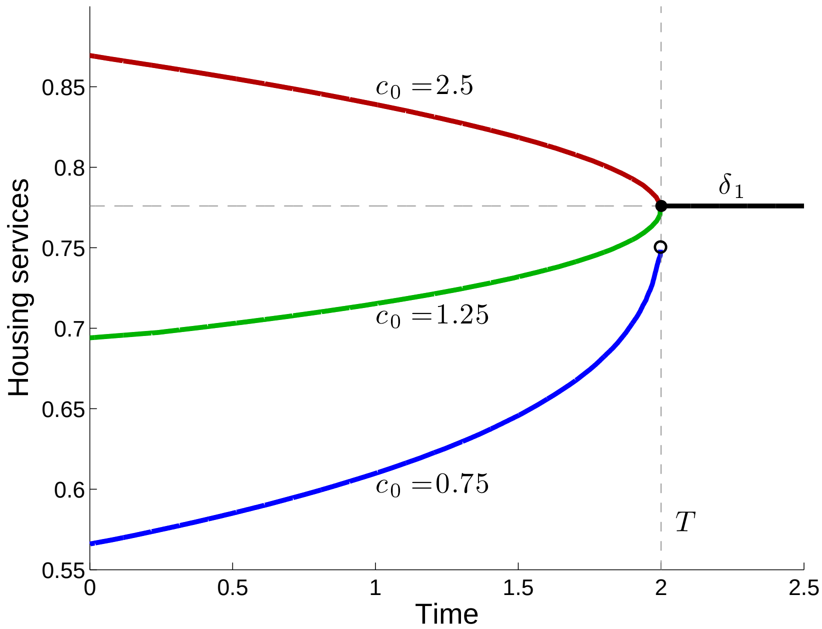


Figure 3.1: The default boundary for various values of the initial coupon c_0. The default boundary is continuous at the reset date T when c_0 is greater than δ_1, the default boundary after the coupon reset.

Now consider the case with $c_0 = 0.75$; this value of c_0 will serve as the benchmark for the rest of the paper. Since $c_0 < c_1$, the mortgage features a teaser coupon. Figure 3.1 shows the default boundary as a function of time; recall that the economic value of equity is zero along the default boundary $\delta(t)$. The figure reiterates the importance of distinguishing between the book value and the economic value of equity — the borrower defaults when the economic value is zero, not the book value. For example, at $t = 1$ the book value of equity is zero when housing services equal 0.835, whereas the economic value of equity is zero when housing services drop to 0.61. These numbers correspond to 83.5% and 61% of the purchase price of the house, respectively. In fact the level of housing services at which book value of equity is zero equals 0.835 for all t, whereas the level curve at which the economic value of equity is zero changes with t.

The figure also shows that the default boundary during the teaser period is below c_0. The borrower strictly prefers to continue with the mortgage when his immediate net payoff from the mortgage is strictly positive, $x(t) - c_0 > 0$. Continuation is optimal even if the immediate payoff is zero, $x(t)-c_0 = 0$. Suppose that the borrower adopted the following rule: default when the immediate payoff is zero. Instead of following the rule above, the borrower could wait and observe the realization of housing services next period. If housing services fall, he can default. If they rise, he strictly prefers to continue because the immediate payoff is now strictly positive. The ability to default when housing services fall implies that the expected cost of waiting is zero. The expected benefit of waiting is strictly positive. Therefore the borrower strictly prefers to continue when the immediate payoff is zero, implying that the adopted default rule cannot be optimal.

If $x(t) - c_0 < 0$ then the borrower compares the immediate net payoff of continuing to the expected discounted value of future net payoffs. This calculation depends on the time remaining until the coupon reset. Conditional on the level of housing services, the net payoff on a reset mortgage is more likely to be positive in the foreseeable future during the teaser period, when the coupon equals c_0, than after the reset, when the coupon equals c_1. Therefore the borrower is willing to accept larger immediate losses when there is more time remaining until the coupon reset, implying that the default boundary is increasing in t.

Conversely, the borrower is less willing to accept immediate losses as the time remaining until the reset decreases. The default point equals δ_1 at the reset date. To prove this result, suppose that the default point is below δ_1 instead. Consider the borrower's decision if housing services at T are between the supposed default point and δ_1. According to the supposed default rule, the borrower should continue making his mortgage payment at this level of housing services. The continuity of geometric Brownian motion implies that the borrower is certain to default after the coupon reset. The immediate payoff from continuing at date T is negative and default next period is imminent. Therefore it is optimal for the borrower to default at date T, implying that the supposed default rule is suboptimal.

The default boundary is discontinuous at the reset date T — it jumps from $c_0 = 0.75$ to $\delta_1 = 0.779$. The discontinuity arises because the post-reset default boundary is greater than the teaser coupon, $\delta_1 > c_0$. It is worth emphasizing that the jump in the default boundary at the reset date is not due to the borrower's inability to meet the coupon payment. It is due to his unwillingness to pay the coupon. The former reason for default has been widely discussed. The latter reason, however, has received little attention. Recognition of the fact that the optimal default boundary can be discontinuous is important for studies that try to distinguish between "strategic" and "liquidity driven" default in mortgages with coupon resets.

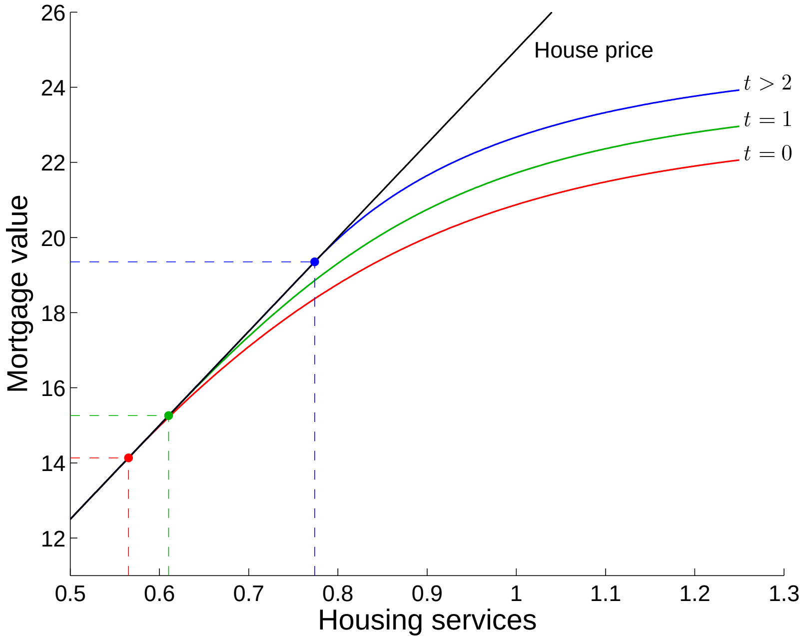


Figure 3.2: Mortgage value as a function of housing services for various t; parameters equal their benchmark values. The default boundary and the recovery at a given t are also shown.

The mortgage value at origination is $M(0,1) = 20.87$. The corresponding LTV ratio is 83.50%. Figure 3.2 shows the level curves of the mortgage value as a function of housing services for $t = 0, 1$ and for $t > T$. The value of the mortgage at every date is an increasing function of housing services $x(t)$ because an increase in $x(t)$ today lowers the likelihood of default in the foreseeable future. The figure also shows the house price function, $P(x(t))$. Optimality of default behavior at date t requires that mortgage value equal the house price at the default point — the value matching condition — and that the slope of the two functions at the default point be identical — the smooth pasting condition. The figure also shows the default point at each date t. The value of the mortgage at the default point is the lender's recovery on the mortgage. Prior to the reset, the recovery rate is an increasing function of time. Since the default boundary is discontinuous at T, the recovery rate at this date is unknown when the mortgage is originated; it lies in the interval $[P(c_0)/M(0,1), P(\delta_1)/M(0,1)] = [89.84\%, 92.96\%]$.

The initial yield on the mortgage is 7.701%. The initial yield spread of about 70 basis points reflects the expected loss on the mortgage.

Next consider the case with $c_0 = 1.25$. Figure 3.1 shows the default boundary. As earlier, the default boundary is an increasing function of t. Now, however, the default boundary is continuous at the reset date T. The continuity of the default boundary at T depends on whether the initial coupon c_0 is less than or greater than δ_1, the default boundary after the reset. The reason for the discontinuity at the reset date when $c_0 < \delta_1$ was discussed earlier; the size of the discontinuity is $\delta_1 - c_0$.

The discussion now turns to the continuity of the default boundary at T for all $c_0 \geq \delta_1$. It proceeds by showing that neither $\delta(T) < \delta_1$ nor $\delta(T) > \delta_1$ is optimal. Therefore $\delta(T)$ must equal δ_1, implying that the default boundary is continuous at T.

Suppose that $\delta(T) < \delta_1$. Consider the default decision of the borrower if housing services lie in the interval $(\delta(T), \delta_1)$ at date T. The continuity of geometric Brownian motion implies that the borrower will default once the coupon resets, almost surely. The immediate net payoff to the borrower from continuing is negative because housing services are below δ_1, which is below c_0. Therefore the borrower prefers to default at date T, implying that $\delta(T) < \delta_1$ is not optimal. Instead, suppose that $\delta(T) > \delta_1$. The continuity of geometric Brownian motion implies that the borrower will have strictly positive equity as soon as the coupon resets. The Taylor series expansion of the borrower's equity after the coupon reset shows that the supposed default rule must violate either the value matching or the smooth pasting condition at date T; see Appendix B.1 for details. Therefore $\delta(T) > \delta_1$ cannot be optimal. The two arguments together imply $\delta(T) = \delta_1$.

The mortgage value at origination is $M(0, 1) = 21.79$. The corresponding LTV ratio is 87.15%. The recovery rate on the mortgage increases from 79.63% to 89.03% as t goes from 0 to T. Since the default boundary is continuous, the recovery at the reset date is no longer unknown when the mortgage is originated. It equals 19.40, implying a recovery rate of 89.03%. The initial yield on the mortgage is 7.704%. The initial yield spread on the reset mortgage with $c_0 = 1.25$ is slightly higher than the spread on the reset mortgage with $c_0 = 0.75$.

When $c_0 = c_1 = 1.75$, the reset mortgage is identical to a fixed rate mortgage, so the default boundary before and after the reset are both equal to $\delta_1 = 0.776$. The mortgage value at origination is $M(0, 1) = 22.67$; initial LTV ratio is 90.69%; recovery rate is 85.56%; and initial mortgage yield is 7.72%.

If c_0 is increased further, the default boundary becomes a decreasing function of time. Figure 3.1 shows the default boundary when $c_0 = 2.50$. The mortgage value is $M(0, 1) = 23.89$; LTV ratio is 95.54%; and initial mortgage yield is 7.78%. The recovery rate decreases from 91% to 81.21% as t goes from 0 to T. When $c_0 > c_1$, the borrower is unwilling to bear large immediate losses close to the mortgage origination date because the immediate payoff is likely to stay negative in the foreseeable future when the coupon equals c_0. However, as the reset date approaches, the borrower's willingness to accept immediate losses increases because the drop in the coupon at the reset date increases the probability of the payoff becoming positive in the foreseeable future. This finding is consistent with empirical work of Fuster and Willen (2012) on payment resets. These authors study the impact of a payment reset on the default probability of a sample of subprime borrowers with hybrid ARMs. The borrowers in the sample experience a drop in their mortgage coupon at reset date. Fuster and Willen find that the default hazard starts decreasing about two months prior to the reset and continues to do so until the reset, after which the hazard stabilizes.

Figure 3.3 shows the tradeoff between initial mortgage yield and initial LTV ratio for reset mortgages. The LTV ratio on the mortgage was varied by increasing the post-reset coupon, starting at $c_1 = 0.75$. The figure shows the tradeoff for reset mortgages with $c_0 = 0.75$. For comparison it also shows the tradeoff for

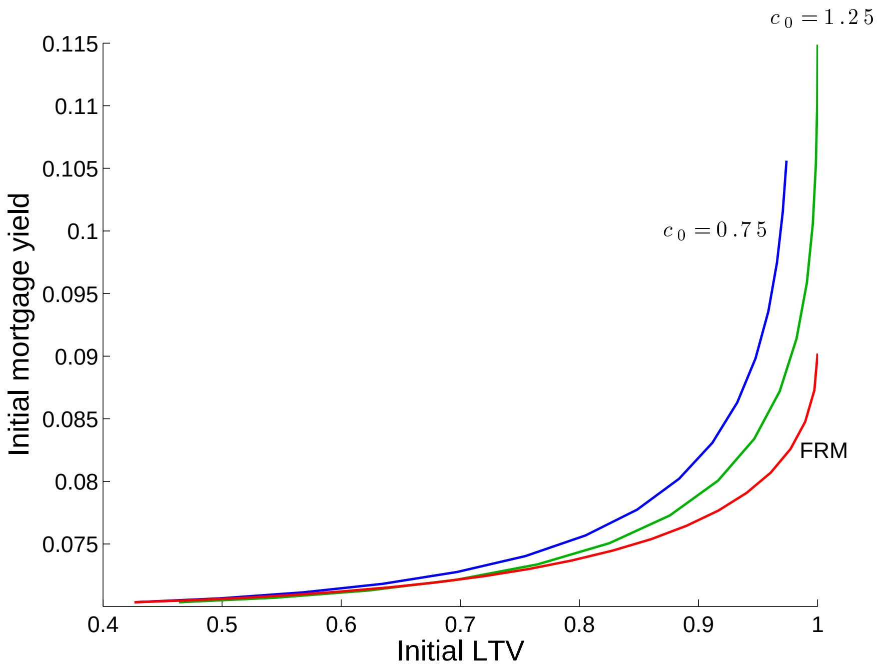


Figure 3.3: The tradeoff between initial yield and initial LTV ratio on reset mortgages and fixed rate mortgages. The LTV ratio was varied by changing c_1, the coupon payment after the reset.

reset mortgages with $c_0 = 1.25$ and for fixed rate mortgages; the coupon for fixed rate mortgages also starts at 0.75. It is worth pointing out the the expected rate of return on all of these mortgages is $\rho = 7\%$, as a consequence of the fact that risk neutral lenders do not require risk compensation. Consider reset mortgages with $c_0 = 0.75$. The figure shows that the expected loss on reset mortgages with $c_0 = 0.75$ is higher than the expected loss on fixed rate mortgages — for a given LTV ratio the initial yield spread on the reset mortgage is higher. For example, the initial yield on the reset mortgage with an initial LTV ratio of 95% is 9.02% whereas the initial yield on a fixed rate mortgage with the same LTV is 7.96%. The difference in initial yields is due to the differences in the schedule of coupon payments. The coupon payments on the reset mortgage during the teaser period are lower than the payments on the corresponding fixed rate mortgage. Since the LTV ratios on the two mortgages are identical, the post-reset coupon on the reset mortgage is higher than the coupon on the fixed rate mortgage.

The default boundary on the teaser mortgage is below the boundary on the fixed rate mortgage during the teaser period, and above the boundary after the coupon reset. For an initial LTV ratio of 95%, the default boundary on the fixed rate mortgage is 0.836 whereas the default boundary on the teaser mortgage jumps from 0.75 to 1.073 at the reset date. The higher initial yield on the reset mortgage, as compared to the fixed rate mortgage, reflects the fact that the increase in default probability due to the higher post-reset default boundary outweighs the reduction in default probability due to the lower coupon during the teaser period.

Now consider the yield-LTV tradeoff on the reset mortgage with $c_0 = 1.25$. To begin with, the initial yield on this mortgage is less than the yield on the corresponding fixed rate mortgage. The lower initial yield reflects the fact that the initial coupon on the reset mortgage is greater than the post-reset coupon, $c_0 > c_1$. The two yield curves cross when $c_0 = c_1 = 1.25$. For higher values of c_1, the initial coupon on the reset mortgage is lower than the post-reset coupon; the mortgage has a teaser payment. Therefore the initial yield on the reset mortgage is higher than the corresponding fixed rate mortgage.

Figure 3.3 also shows the maximum loan amount the lender is willing to supply. The maximum loan size for fixed rate mortgages is found by solving for the smallest coupon at which the LTV ratio at origination is one. Equivalently one can find the coupon at which the default boundary equals one, implying that the borrower defaults at the origination date of the mortgage. The maximum coupon on fixed rate mortgages is 2.26. Consider reset mortgages with $c_0 = 1.25$. The maximum loan amount for these mortgages is found by solving for the post-reset coupon c_1 at which the borrower's default point at origination equals one. The maximum post-reset coupon equals 3.29. Reset mortgages with $c_0 = 0.75$ do not have a well defined maximum post-reset coupon because the default boundary prior to the reset is below 0.75. Since housing services at origination are one, the borrower strictly prefers to continue with the mortgage at $t = 0$, regardless of c_1. He can always default at the reset date if housing services are below the default boundary at that date. Even though maximum post-reset coupon is indeterminate, the maximum loan size and LTV ratio are still well determined. Suppose that $c_1 \to \infty$ and default at the reset date is certain. In this case, the maximum loan size is equal to the sum of the expected discounted value of the coupon payments during the teaser period and the expected recovery at the reset date. When $c_0 = 0.75$ the maximum LTV ratio is 97.9%.

The analysis so far has focused on cases in which the borrower defaults at the reset date when house prices are at some level below $P(1)$, the purchase price of the house. Next I present an example in which the borrower defaults at the reset date even if house prices are above $P(1)$. This example shows that reset mortgages can end up in default when house price growth, although positive, is not high enough. Some analysts purport that many subprime mortgages underwritten in the buildup to the financial crisis of 2007-2008 fall in this category. Consider the reset mortgage with $c_0 = 0.75$ and $c_1 = 2.42$. The initial LTV ratio on this mortgage is 95%. The default boundary after the coupon reset on this mortgage is 1.073. The boundary is greater than one, the level of housing services at origination. Therefore the borrower defaults at the reset date even if house prices appreciate, as long as the increase is less than 7.3%. The initial yield spread on the mortgage is 2.02%.

Figures 3.4 and 3.5 show how changing the volatility parameter σ affects the equilibrium; all other parameters equal their benchmark values, in particular $c_0 = 0.75$ and $c_1 = 1.75$. Figure 3.4 shows how σ affects the mortgage value at $t = 1$. As in standard options theory, an increase in σ makes the default option more valuable at each date, implying that the mortgage value decreases with σ; recall that the lender is short the default option. The figure also shows the default points at $t = 1$ for each value of σ; higher values of σ are associated with lower default points. Figure 3.5 shows the tradeoff between initial mortgage yield and initial LTV ratio for different values of the volatility parameter σ; the initial LTV ratio

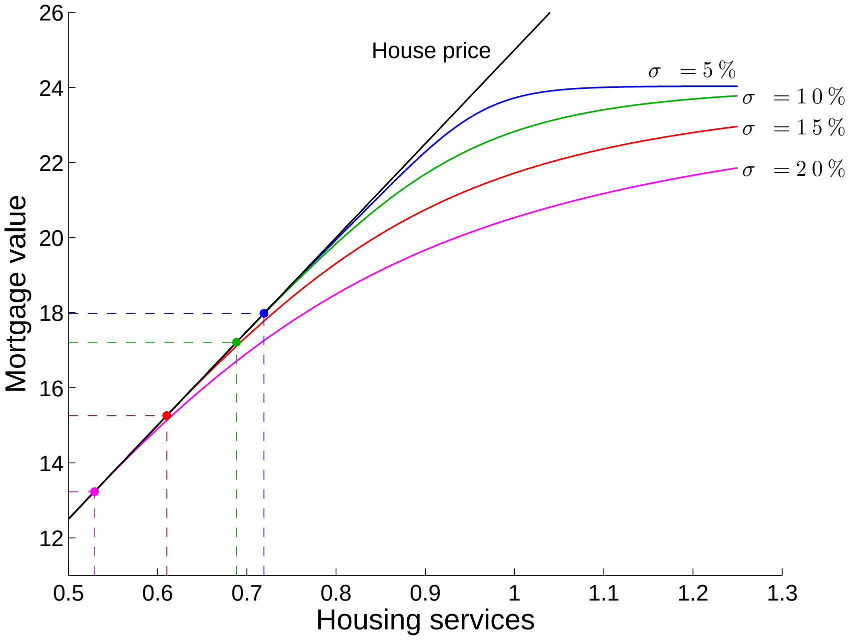


Figure 3.4: Mortgage value at $t = 1$ as a function of housing services, for various values of the volatility parameter σ.

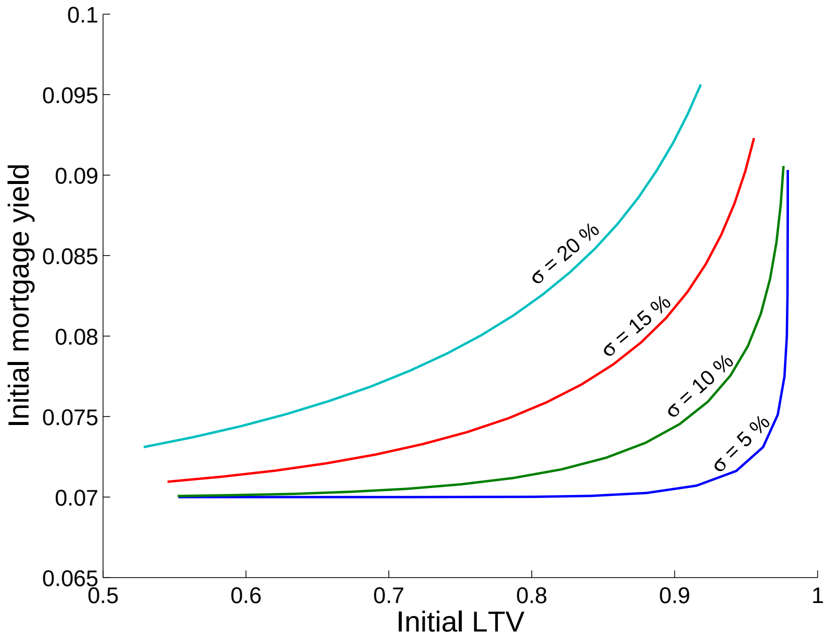


Figure 3.5: The tradeoff between initial mortgage yield and initial LTV ratio, for various values of the volatility parameter σ. The LTV ratio was varied by changing the post-reset coupon c_1.

was varied by changing the coupon c_1. The figure shows that initial mortgage yield is an increasing function of initial LTV ratio. It also shows that, for a given LTV ratio, the equilibrium yield on the mortgage is increasing as housing services become more volatile.

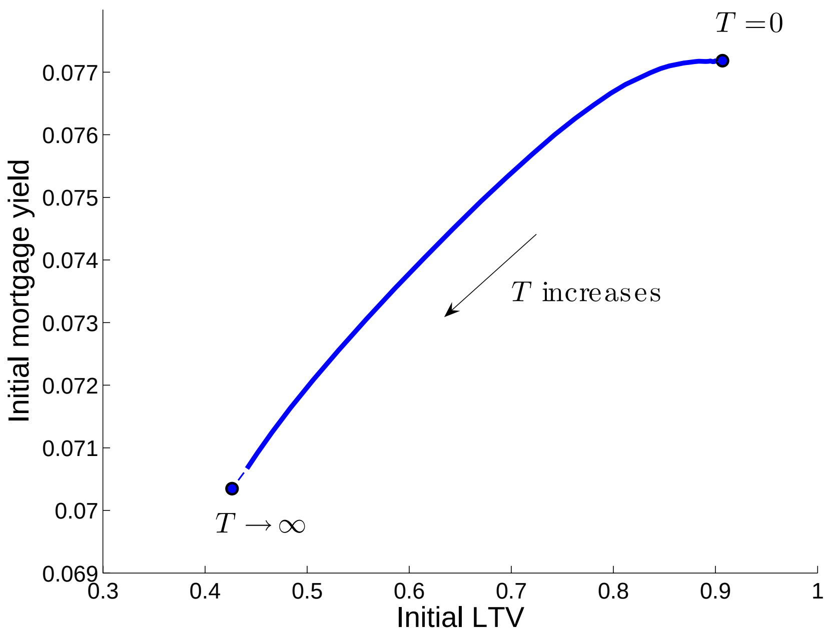


Figure 3.6: The tradeoff between initial mortgage yield and initial LTV as the reset date T changes, but c_0 and c_1 are fixed. When $T = 0$ the reset mortgage contract is identical to a fixed rate mortgage with coupon c_1. When $T \to \infty$ it is identical to a fixed rate mortgage with coupon c_0.

Figure 3.6 shows how increasing the length of the teaser period, while keeping c_0 and c_1 unchanged, affects the tradeoff between initial mortgage yield and initial LTV ratio. When $T = 0$, the reset mortgage is identical to a fixed rate mortgage with the coupon c_1. When $T \to \infty$, the reset mortgage is identical to a fixed rate mortgage with coupon c_0. The yield and the LTV ratio both decline monotonically as the reset date T increases.

Figure 3.7 shows how the tradeoff between initial mortgage yield and initial LTV ratio changes with the reset date T. Unlike figure 3.6, the figure was generated by varying the post-reset coupon c_1, so as to vary initial LTV, for each T; the pre-reset coupon equals $c_0 = 0.75$. The figure shows the tradeoff for $T = 2, 3, 5, 7, 10$. In practice, these reset dates are common for hybrid adjustable-rate mortgages, and interest-only mortgages. In addition to reiterating the role of initial LTV in determining initial mortgage yield, the figure highlights the importance of the coupon schedule in determining initial yield. Conditional on LTV at origination, reset mortgages with longer teaser periods have higher equilibrium initial yields. For example, the mortgage yield at origination on a reset mortgage with a LTV ratio of 85% increases from 7.82% to 9.58% as the length of the teaser period increases from 2 years to 10 years.

3.2.3 Balloon Payment Mortgage

The two-step payment structure developed for reset mortgages can also be used to study mortgages in which the borrower pays off the principal balance on the mortgage in lumpsum at an agreed upon date. The balloon payment may be less than or equal to the original principal balance, depending on the rate at which the mortgage loan amortizes. The analysis here focuses on mortgages that do not amortize, implying that the borrower pays off the entire principal balance when

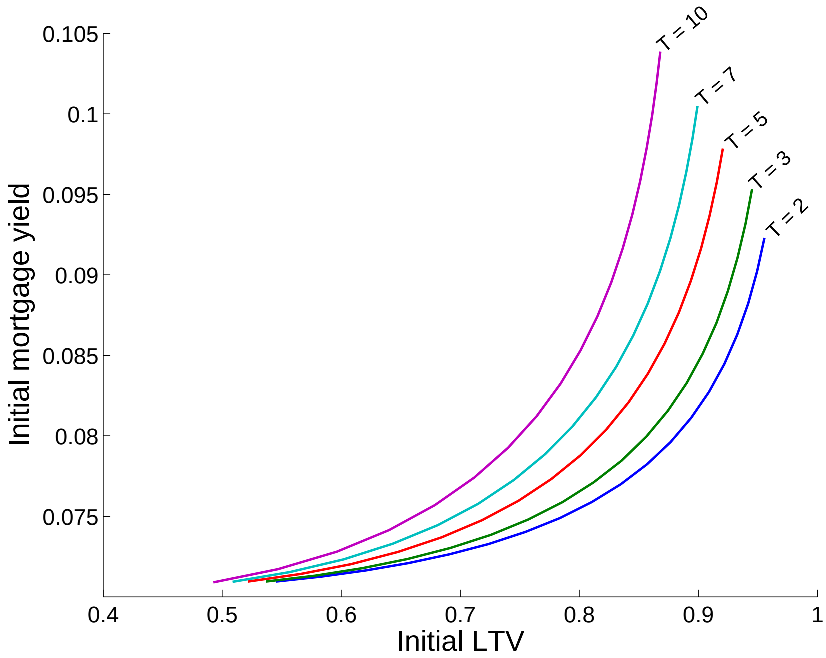


Figure 3.7: The tradeoff between initial mortgage yield and initial LTV ratio for various values of the reset date T. For each T the initial LTV ratio was varied by changing the post-reset coupon c_1.

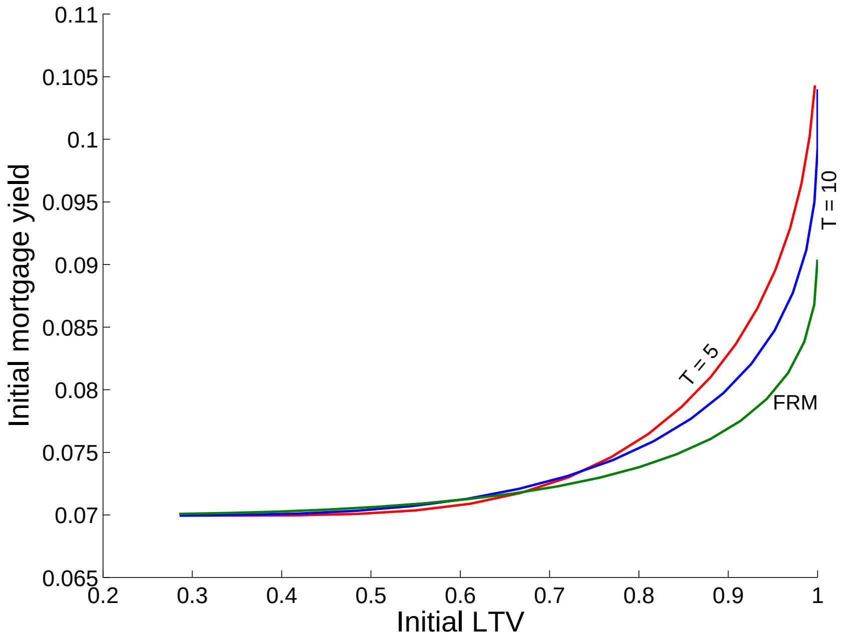


Figure 3.8: The tradeoff between initial mortgage yield and initial LTV ratio for balloon mortgages maturing at various T. A balloon mortgage converges to a fixed rate mortgage as $T \to \infty$.

the mortgage matures. In practice, balloon payment mortgages are commonly observed in commercial real estate transactions.[2]

Suppose that the coupon on the balloon mortgage is c_0, and the maturity date is T; the coupon on the balloon mortgage does not reset. The borrower's equity maximization problem is mapped into the framework developed earlier by noting that, conditional on survival till date T, the borrower pays back the loan provided the value of the house at that date exceeds the size of the mortgage loan. Therefore the default boundary at T equals the level of housing services at which house price equals the size of the mortgage loan. The default boundary for $t < T$ is determined by the coupon c_0. The default boundary jumps at date T if the size of the mortgage loan is such that the default boundary at date T is less than or equal to c_0.

The lender makes zero expected profits, implying that the present value of the borrower's payments equals the size of the mortgage loan. The equilibrium is found by solving for the fixed point at which the borrower maximizes equity and the zero expected profit condition holds.

Figure 3.8 shows the tradeoff between initial yield and initial LTV ratio for balloon payment mortgages. The figure shows the tradeoff for mortgages that mature in 5 years, 10 years, and for fixed rate mortgages; the balloon payment mortgage approaches a fixed rate mortgage with coupon c_0 as $T \to \infty$. For a given maturity T, the LTV ratio at origination was varied by changing the mortgage coupon c_0. Conditional on T, the initial LTV ratio and initial yield increase with

[2]A balloon payment mortgage that matures in 7 years is common in commercial real estate. The mortgage usually amortizes at the same rate as a 30-year fixed rate mortgage. Therefore a lumpsum payment is due when the mortgage matures.

c_0. The probability of default also increases with the mortgage coupon. The probability of default on the mortgage depends upon the size of the mortgage coupon c_0 and the size of balloon payment. The former determines the default probability before maturity, whereas the latter determines the default probability at maturity. Conditional on the time to maturity T, increasing the mortgage coupon increases both probabilities.

Depending on initial LTV, the initial yield on the balloon payment mortgage may increase or decrease with time to maturity T. The effect is more pronounced for high LTV mortgages. For high initial LTV ratios, the initial yield is decreasing in T for two reasons: (i) borrowers in mortgages with longer maturities are willing to accept larger losses initially in hope of a future turnaround in housing services (ii) the positive drift term dominates for larger T, implying that house prices are more likely to exceed the mortgage principal at the maturity date.

3.3 Costly Default

The analysis so far has assumed that default is costless for both borrowers and lenders. In practice, however, default is costly for both parties. A borrower who chooses to default is evicted from the property, and must bear relocation expenses. In addition, the borrower loses access to future credit and tax benefits that come with mortgages. Empirical research on mortgage default confirms that default is costly. For example, the estimates of Bhutta, Dokko, and Shan (2010) suggest that the median non-prime borrower faces default costs equal to approximately 30% of the purchase price of the house; see Deng, Quigley, and Van Order (2000) for a study involving prime borrowers.[3] Moreover, the existence of mortgages with initial LTV ratios greater than 100% in practice provides further evidence in favor of borrower default costs.

Default is also costly for the lender. Usually there is a lag of a year or more between the default date and the date at which the lender can reposses and sell the property. The lender receives no coupon payments from the property during this time period. Moreover, the lender must maintain the property until the resale. The popular press has reported several instances of borrowers damaging the property after defaulting on their mortgage. The lender has to bear the cost of these repairs.

I model borrower and lender default costs as deadweight losses. This modeling choice is motivated by the notion that mortgage default is inefficient: it is not a costless transfer of ownership of the property from the borrower back to the lender. The adopted model specification implies that the deadweight loss from default is the sum of borrower and lender default costs. This approach is standard in the mortgage default literature.

[3]Bhutta, Dokko, and Shan (2010) focus on first liens of purchase mortgages originating in Arizona, California, Florida, and Nevada. They restrict themselves to mortgages with a CLTV of 100%. They use a two-step maximum likelihood estimation procedure to separate default caused by adverse life events from strategic defaults. The median borrower walks away from his house when the value of equity equals -62% of the current house price; where equity equals the difference between the current house price and the principal outstanding on the mortgage. The number reported in the text above is from Singhania (2014).

3.3.1 Costly Borrower Default

Borrower default costs are denoted k_b. The costs are proportional to the purchase price of the property.[4] Lender default costs are set to zero for now. The presence of default costs drives a wedge between the mortgage liability of the borrower and the asset value of the mortgage to the lender. Let $M_b(t, x(t))$ denote the mortgage liability of the borrower, and $M_\ell(t, x(t))$ denote the asset value of the mortgage to the lender. The Bellman equation for the borrower maximizing home equity $E_b(t, x(t))$ now becomes

$$E_b(t, x(t)) = \max\left\{-k_b,\ (x(t) - c_0)dt + e^{-\rho dt}\mathbb{E}_t\left[E_b(t + dt, x(t) + dx(t))\right]\right\}. \quad (3.8)$$

The Bellman equation shows that the borrower defaults when the economic value of his equity equals $-k_b$; therefore the value matching condition requires $E_b(t, \delta(t)) = -k_b$.

The default boundary after the coupon reset equals

$$\delta_1 = \left(\frac{m}{m+1}\right)\left[\frac{c_1/\rho - k_b}{P(1)}\right]. \quad (3.9)$$

where m is given by (3.2.1). The default boundary after the reset is decreasing in borrower default costs. As before, the solution to the home equity maximization

[4] As noted by KLO, the chosen specification does not incorporate the expected discounted present value of default costs into the price of the house, implying that borrowers who purchase a house with mortgages and face positive default costs will find houses overpriced. Incorporating borrower default costs into the house price calculation requires major respecification of the model presented here. To keep the analysis tractable, I do not modify the model to incorporate this calculation.

problem prior to the coupon reset is obtained numerically. The optimal default boundary prior to the reset follows.

The asset value of the mortgage to the lender prior to the reset is calculated by backward induction separately, taking the borrower's default behavior as given. The value of the mortgage to the lender after the reset date is $P(x(t)) - F(x(t))$, with δ_1 in $F(x(t))$ given by (3.9). Since the lender makes zero expected profits, the size of the mortgage loan equals its asset value at origination. The initial mortgage yield is calculated by replacing $M(0,1)$ in (3.7) with $M_\ell(0,1)$.

Figure 3.9 shows the tradeoff between initial yield and initial LTV for the borrower and the lender when $k_b = 8$. For comparison, the figure also shows the tradeoff when default is costless. Conditional on initial LTV, costly default lowers the initial yield. Conversely, with costly default, the borrower can obtain a larger mortgage loan for a given coupon payment. As with fixed rate mortgages, the effect is more pronounced for high LTV mortgages; KLO analyze costly default in fixed rate mortgages.

The initial mortgage yield increases with the borrower's initial LTV ratio. In contrast, the yield-LTV curve for the lender bends backwards. An increase in the post-reset coupon c_1 increases δ_1, the default boundary after the reset. An increase in δ_1 has two opposing effects: it lowers the probability of the lender receiving c_1, and it increases the expected recovery on the mortgage. The former effect lowers the mortgage value, while the latter raises it. The backward bending yield-LTV curve for the lender shows that the former effect outweighs the latter when c_1 is increased past 2.61. The post-reset coupon cannot be greater than 2.61 in equilibrium. If c_1 exceeded this level, the borrower would insist that the

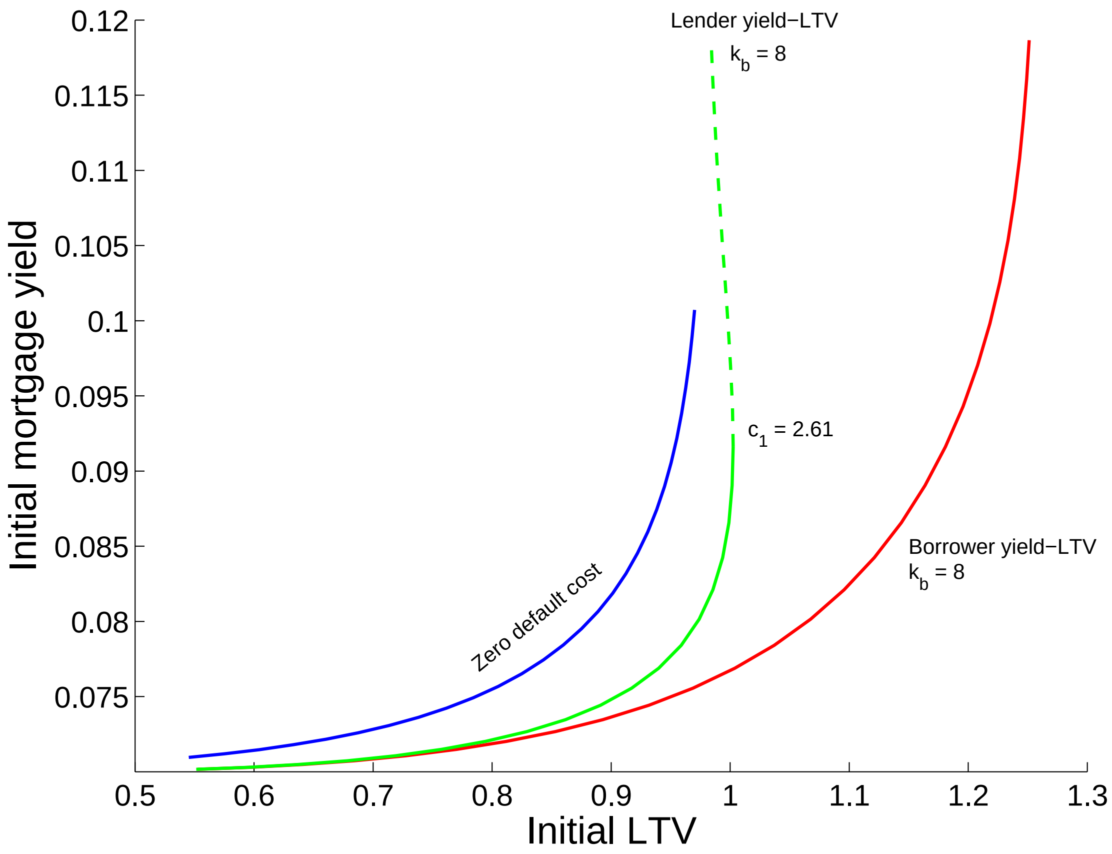


Figure 3.9: The tradeoff between initial yield and initial LTV ratio for the borrower and the lender, when default is costly for the borrower only. The LTV ratio was varied by changing c_1, the coupon payment after the reset.

lender reduce the coupon; doing so would raise the size of the mortgage loan, while reducing the probability of default. The largest equilibrium value of c_1 maximizes lender's LTV. At this coupon, the lender's LTV is 100.22%, the borrower's LTV is 117.13%, and the initial yield on the mortgage is 9.01%.

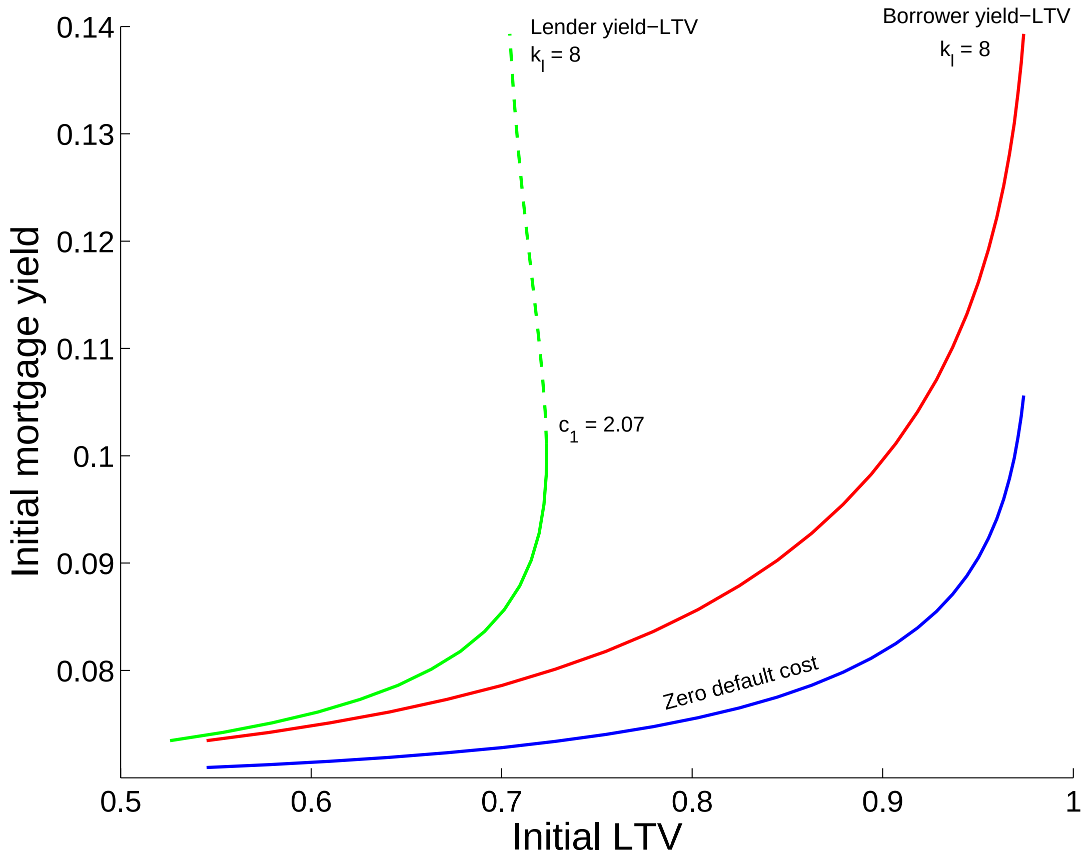


Figure 3.10: The tradeoff between initial yield and initial LTV ratio for the borrower and the lender, when default is costly for the lender only. The LTV ratio was varied by changing c_1, the coupon payment after the reset.

3.3.2 Costly Lender Default

Lender default costs are denoted by k_ℓ. As before, costs are proportional to the purchase price of the property. Borrower default costs are set to zero. The borrower's equity maximization problem is now given by (3.5). The post-reset default boundary is given by (3.4). The lender's value of the mortgage after the reset is

$$\frac{c_1}{\rho} - \left(P(\delta_1) - k_\ell - \frac{c_1}{\rho}\right)\left(\frac{\delta_1}{x(t)}\right)^m$$

where $P(\delta_1) - k_\ell$ is the lender's net recovery on the mortgage. His value prior to the reset is calculated using backward induction.

Figure 3.10 shows the tradeoff between initial yield and initial LTV ratio for the borrower and the lender, when $k_\ell = 8$. For comparison the figure also shows the tradeoff in the benchmark zero default cost case. Conditional on c_0 and c_1, lender default costs do not affect the default behavior of the borrower. The lender's valuation of the mortgage, however, is lower than the benchmark because he must pay k_ℓ if the borrower defaults. Therefore, given c_0 and c_1, the size of mortgage loan is lower in equilibrium. Conversely, the equilibrium yield on the mortgage is higher for a given initial LTV ratio. The lender's yield-LTV curve with $k_\ell = 8$ is backward bending. As with borrower default costs, mortgages with the post-reset coupon c_1 greater than 2.07 are ruled out in equilibrium. When $c_1 = 2.07$, the lender's LTV is 72.35%, the borrower's LTV is 90.69%, and the initial yield on the mortgage is 10.11%.

3.4 Conclusion

This paper studies default in mortgages with coupon resets. It provides conditions under which the optimal default boundary for these mortgages is discontinuous at the coupon reset date. The impact of the increase in payments on default has been debated in policy circles. Certain analysts argue that the payment reset leads to a jump in default probability, while others have presented empirical evidence that suggests otherwise. The analysis here helps reconcile the two views. It shows that payment resets can lead to a jump in the default probability under certain conditions. When these conditions are violated, however, the default probability is continuous at the reset date.

The analysis here also shows that, in addition to the initial loan-to-value ratio, the structure of payments is an important determinant of expected losses due to mortgage default even in option based models. Conditional on the initial loan-to-value ratio, reset mortgages with teaser coupons have higher expected losses than fixed rate mortgages. The higher expected losses on mortgages with teaser coupons are reflected in their higher equilibrium yield spreads. This prediction of the model seems to be substantiated by the empirical work of Krainer, LeRoy, and O (2009). These authors find the same pattern in the data when they compare high loan-to-value adjustable-rate mortgages to fixed rate mortgages with the same loan-to-value ratio.

Narratives of the financial crisis of 2007-2008 purport that lenders underpriced mortgage default risk prior to the crisis. Of course, this claim contains a considerable element of hindsight. The model presented here provides a framework to evaluate the claim from an ex ante perspective because it connects initial yields on reset mortgages to initial loan-to-value ratios. One can ask if a calibrated version of the model generates yield spreads similar to those observed in the data, under reasonable parameter values. Krainer, LeRoy, and O (2009) ask this question for fixed rate mortgages and do not find evidence of drastic mispricing in the data. Conducting a similar exercise for reset mortgages is an important area for future research. In addition, the model could be extended to study the pricing of mortgage backed securities created from reset mortgages; see Singhania (2013) for pricing of default risk in securities created from fixed rate mortgages.

The model presented in this paper abstracted from defaults caused by adverse life events. Empirical research on mortgage default suggests that these events play an important role in precipitating mortgage default; see for example Elul, Souleles, Chomsisengphet, Glennon, and Hunt (2010) and Gerardi, Herkenhoff, Ohanian, and Willen (2013). Adverse life events that affect a borrower's ability to make his mortgage payment, job loss for example, might amplify the effect of coupon resets on mortgage default. Recent work by Campbell and Cocco (2011) and Schelkle (2012) studies mortgage default in an environment that includes adverse life events. Extending the model presented here along these lines is another important area of future research.

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Appendices

Appendix A

Pricing Default Risk in Mortgage Backed Securities

A.1 Omitted Proofs

A.1.1 Risk-free Equilibrium: Guess and Verify

For $\theta \in [0, \theta_2]$, the senior tranche's early recovery equals late recovery is $R_{sl} = V_s(0,1) - R_{se}$. Calculate the senior tranche's value at the early default event, using (2.17), as implied by the guess for $c_s(\tau_e)$; denote the implied value by $V_s'(\tau_e, \delta_e)$.

$$
\begin{aligned}
V_s'(\tau_e, \delta_e) &= \mathbb{E}_{\tau_e}\left[\int_{\tau_e}^{\tau_l} e^{-r(t-\tau_e)} c_s(\tau_e) dt\right] + \mathbb{E}_{\tau_e}\left[e^{-r(\tau_l - \tau_e)} R_{sl}\right] \\
V_s'(\tau_e, \delta_e) &= (V_s(0,1) - R_{se})\left(1 - \left(\frac{\delta_l}{\delta_e}\right)^m\right) + (V_s(0,1) - R_{se})\left(\frac{\delta_l}{\delta_e}\right)^m \\
&= V_s(0,1) - R_{se}.
\end{aligned}
$$

Calculate the senior tranche's value at origination, using the right hand side of (2.16), as implied by the guess for $c_s(0)$ and $V_s'(\tau_e, \delta_e)$ calculated above. If the guessed coupon schedule is an equilibrium the implied origination value should equal $V_s(0,1)$, the actual value of the senior tranche at origination.

$$\begin{aligned} & \mathbb{E}_0 \left[\int_0^{\tau_e} e^{-rt} c_s(0) dt \right] + \mathbb{E}_0 \left[e^{-r\tau_e} R_{se} \right] + \mathbb{E}_0 \left[e^{-r\tau_e} V_s'(\tau_e, \delta_e) \right] \\ = \; & V_s(0,1)(1-\delta_e^m) + R_{se}\delta_e^m + (V_s(0,1) - R_{se})\delta_e^m \\ = \; & V_s(0,1). \end{aligned}$$

Thus the guessed coupon schedule is an equilibrium for $\theta \in [0, \theta_2]$. The uniqueness of this equilibrium was established in the body of the paper.

A.1.2 The Threshold θ_3

In this subsection, we derive the expression for θ_3 along with the conditions necessary for it to lie in the interval $(\theta_2, 1)$.

Implicit differentiation of (2.16) and (2.17) shows that the senior coupon at origination $c_s(0)$ in increasing in θ. So there is a threshold θ_3 such that the coupon on the pool is inadequate for all $\theta > \theta_3$. The threshold is obtained by solving $q_s c_s(0) = c_p(\tau_e)$ for θ, with $c_s(0)$ given by (2.20). The calculation is outlined below.

$$\begin{aligned} q_s c_s(0) &= c_p(\tau_e) \\ V_s(0,1) &= R_{pe}\delta_e^m + R_{pl}\delta_l^m + \frac{c_p(\tau_e)}{r}(\delta_e^m - \delta_l^m) + \frac{c_p(\tau_e)(1-\delta_e^m)}{r q_s} \end{aligned}$$

Add and subtract $(1-\delta_e^m)c_p(0)/r$ to the right hand side and use the equilibrium valuation formula for the pool and the definition of q_s to obtain,

$$V_s(0,1) = V_p(0,1) - \frac{1-\delta_e^m}{r}\left(c_p(0) - c_p(\tau_e)\left(1 + \frac{R_{pe}}{V_p(\tau_e,\delta_e)}\right)\right)$$

Write $V_s(0,1) = \theta_3 V_p(0,1)$, divide both sides by $V_p(0,1)$, and rewrite all pool variables in terms of the underlying mortgage variables; recall that $V_p(0,1) = M_e(1)$ under the normalization $M_e(1) = M_l(1)$. After some algebra, we obtain

$$\theta_3 = 1 - \frac{(1-\delta_e^m)\eta c_e/r}{M_e(1)}\left(1 - \frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)}\right) \tag{A.1}$$

The threshold θ_3 is less than one when the condition $c_l/M_l(\delta_e) < c_e/M_e(\delta_e)$ holds. By (1.8) the leading fraction in the second term of (A.1) is less than one. So the threshold θ_3 is strictly positive when the required condition holds.

Next we verify that $\theta_3 > \theta_2$. By the definition of the thresholds θ_2 and θ_3 the inequality $\theta_3 > \theta_2$ can be written as follows; recall that $V_p(0,1) = M_e(1)$.

$$1 - \frac{(1-\delta_e^m)\eta c_e/r}{M_e(1)}\left(1 - \frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)}\right) > R_p/V_p(0,1)$$

$$V_p(0,1) - (1-\delta_e^m)\eta c_e/r\left(1 - \frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)}\right) > R_p$$

$$\eta(M_e(1) - M_e(\delta_e)) + (1-\eta)(M_l(1) - M_l(\delta_l)) - (1-\delta_e^m)\eta c_e/r\left(1 - \frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)}\right) > 0$$

Since $(1-\eta)(M_l(1)-M_l(\delta_l))>0$, we only need to show that

$$
\begin{aligned}
M_e(1)-M_e(\delta_e)-(1-\delta_e^m)\frac{c_e}{r}\left(1-\frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)}\right) &> 0\\
\left(M_e(1)-(1-\delta_e^m)\frac{c_e}{r}\right)-M_e(\delta_e)+(1-\delta_e^m)\frac{c_e}{r}\frac{c_l/M_l(\delta_e)}{c_e/M_e(\delta_e)} &> 0\\
M_e(\delta_e)(\delta_e^m-1)+(1-\delta_e^m)\frac{c_l/M_l(\delta_e)}{r/M_e(\delta_e)} &> 0\\
(1-\delta_e^m)\frac{c_l/M_l(\delta_e)}{r/M_e(\delta_e)} &> M_e(\delta_e)(1-\delta_e^m)\\
\frac{c_l}{r} &> M_l(\delta_e)
\end{aligned}
$$

The last inequality holds by (1.8). Therefore $\theta_3>\theta_2$.

Appendix B

Default Risk and Valuation of Mortgages with Coupon Resets

B.1 Omitted Proofs

In this section I show that $\delta(T) > \delta_1$ cannot be optimal. The proof is done in discrete time; the continuous time version follows by limit operations. Consider a time step of size Δt. The geometric Brownian motion $x(t)$ steps up with step size $\Delta h = \sigma x \sqrt{\Delta t}$, and steps down with step size $-\Delta h$. Suppose the current value of housing services is $x(t) = \widetilde{x}$. The probability of the geometric Brownian motion taking an up step at $t + \Delta t$ to equal $\widetilde{x} + \Delta h$ is

$$w = \frac{1}{2}\left[1 + \frac{\alpha}{\sigma}\sqrt{\Delta t}\right]. \tag{B.1}$$

First I show that defaulting at any time t is suboptimal if the borrower has strictly positive expected equity at $t+\Delta t$. This result needs to be proven because the negative payoff from continuing today might exceed, in absolute value, the discounted value of expected equity tomorrow. In that case, defaulting would be optimal. The value of equity at time t and housing services $\widetilde{x}$ is

$$E(t,\widetilde{x}) = \max\left\{0, (\widetilde{x}-c_0)\Delta t + \frac{1}{1+\rho\Delta t}\left[wE(t+\Delta t,\widetilde{x}+\Delta h) + (1-w)E(t+\Delta t,\widetilde{x}-\Delta h)\right]\right\}$$

The Taylor series expansion of $E(t+\Delta t,\widetilde{x}\pm\Delta h)$ around $(t,\widetilde{x})$ is

$$E(t+\Delta t,\widetilde{x}+\Delta h) = E(t,\widetilde{x}) \pm E_x(t,\widetilde{x})\Delta h + \text{higher order terms} \tag{B.2}$$

Therefore the Taylor series expansion of $\mathbb{E}_t[E(t+\Delta t, x+\Delta x)]$ is

$$\mathbb{E}_t[E(t+\Delta t,\widetilde{x}+\Delta x)] = E(t,\widetilde{x}) + (2w-1)E_x(t,\widetilde{x})\Delta h + \text{higher order terms}$$

The term $2w-1$ is greater than 0 because $w > 1/2$; see (B.1). If the borrower has strictly positive equity almost surely at $t+\Delta t$, then either $E(t,\widetilde{x}) > 0$ or $E_x(t,\widetilde{x}) > 0$, implying that either the value matching condition or the smooth pasting condition must be violated at $(t,\widetilde{x})$. Therefore defaulting at $(t,\widetilde{x})$ cannot be optimal. The suboptimality of $\delta(T) > \delta_1$ follows from the continuity of housing services, which implies that the expected value of home equity at $T+\Delta t$ is strictly positive.

B.2 Numerical Method

We will use a binomial tree pricing model to compute the mortgage values. Since we have closed form solutions for mortgage values at the reset date, we can compute the mortgage value at origination using backward induction. The detailed steps are as follows.

1. Specify parameters α, σ, ρ, T, c_0, c_1, k_b, k_ℓ. Specify the desired time step-size Δt.

2. Discretize the time interval $[0, T+\varepsilon]$ using step-size Δt to obtain the vector $\mathbf{t} = (0, \Delta t, 2\Delta t, \ldots, T+\varepsilon)$. The value ε is added to T so that we can calculate the default threshold at and near the origination date. We will truncate the vector from $[0, \varepsilon]$ once the calculations are done.

3. Compute the up/down step-size $u = e^{\sigma\sqrt{\Delta t}}$ and $d = e^{-\sigma\sqrt{\Delta t}}$.

4. Compute the vector of lattice nodes at the reset date using the binomial formula $\mathbf{x_T} = (x(0)u^n d^{N-n})$ where $N = (T+\varepsilon)/\Delta t + 1$ and $n = 0, 1, \cdots N$.

5. Compute the mortgage liability at every lattice node at the reset date. That is, $M(T+\varepsilon, x_{Ti}) = P(x_{Ti}) - F(x_{Ti})$, where

$$F(x(t)) = \left(\frac{x(t)}{\rho-\alpha} - \frac{c_1}{\rho}\right) \quad + \quad \left(\frac{c_1}{\rho} - P(\delta_1) - k_b\right)\left(\frac{\delta_1}{x(t)}\right)^m;$$

6. Note that the post-reset default threshold is

$$\delta_1 = \left(\frac{m(\rho-\alpha)}{m+1}\right)\left[\frac{c_1}{\rho} - k_b\right].$$

7. Compute the vector of lattice nodes at date $t = T + \varepsilon - h\Delta t$, where $h = 1, 2, \cdots T/\Delta t$ using the binomial pricing formula $\mathbf{x_t} = (x(0)u^n d^{N-n})$ where $N = (T + \varepsilon)/\Delta t - h + 1$ and $n = 0, 1, \cdots N$.

8. Let p denote the probability of an up-step and q denote the probability of a down-step. These probabilities are

$$p = \frac{e^{\alpha \Delta t} - d}{u - d}$$
$$q = \frac{u - e^{\alpha \Delta t}}{u - d}$$

9. Compute the borrower's mortgage liability

$$M_b(t, x_{ti}) =$$
$$\min \left\{ P(x_{ti}) + k_b, c_0 \Delta t + e^{-\rho \Delta t} \left(pM_b(t + \Delta t, x_{ti}u) + qM_b(t + \Delta t, x_{ti}d) \right) \right\}.$$

10. Calculate the borrower's default threshold at each t from the mortgage liability calculation in the previous step. The default threshold is the first value of x_{ti} at which $M_b(t, x_{ti}) = P(x_{ti}) + k_b$. Save the default threshold in a vector $\delta(t)$. Note that the default threshold may not be defined for t close to zero. This is an artifact of the binomial method, not a characteristic of the problem. This is what necessitates the extension of the time dimension by ε.

11. Repeat steps 9-10 until $t = 0$.

12. Once we have the default threshold $\delta(t)$, we can compute the lender's value of the mortgage using backward induction. The lender's value is a piecewise function that depends on the default threshold $\delta(t)$

$$M_\ell(t, x(t)) = \begin{cases} P(x(t)) - k_\ell & \text{if } x \leq \delta(t) \\ c_0 \Delta t + e^{-\rho \Delta t} \Big(p M_\ell(t + \Delta t, x_{ti} u) + q M_\ell(t + \Delta t, x_{ti} d) \Big) & \text{if } x > \delta(t) \end{cases}$$

13. Truncate the left tail of all variables from $[0, \varepsilon]$ to get the solution from $[0, T]$.