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# UNIVERSITY OF CALIFORNIA, SAN DIEGO

DEPARTMENT OF ECONOMICS

MONETIZING TRADE:  A TATONNEMENT EXAMPLE

BY

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# Monetizing Trade: A Tatonnement Example
(Preliminary)

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Abstract

This paper presents a class of examples where a barter economy develops through agents' optimizing decisions into a monetary economy. A barter economy with m commodities is characterized by m(m-1)/2 commodity pair trading posts for active trade of each good for every other. Monetary equilibrium is characterized by active trade concentrated on m-1 posts, those trading in 'money' versus the m-1 other nonmonetary commodities. Specialization, the concentration of the trading function in a few trading posts in the monetary trade arrangement, reflects the workings of scale economies in transaction costs. As households discover that some pairwise markets (those with high trading volumes) have lower transaction costs, they restructure their trades to take advantage of the low cost. When a trading post's transaction cost is sufficiently low, households find it advantageous to use the low transaction cost goods as intermediary goods in their transactions, rather than trade directly (at high transaction cost trading posts) for the goods they want. The process converges to an equilibrium where only the high volume trade through a single intermediary good ('money') takes place. Monetization of trade results from dynamic adjustment to scale economies in the transaction technology.

## I. Introduction

Walras (1874) describes the setting of trade in a market equilibrium as a complex of trading posts where goods trade pairwise against one another.

> In order to fix our ideas, we shall imagine that the place which serves as a market for the exchange of all the commodities ... for one another is divided into as many sectors as there are pairs of commodities exchanged. We should then have m(m-1)/2 special markets each identified by a signboard indicating the names of the two commodities exchanged there as well as their prices or rates of exchange...

Thus, if there are m goods, Walras envisions a large number, m(m-1)/2, of active trading posts. Buyers of good i for good j meet with buyers of good j for i at the ij trading post.

This picture is in contrast to the practice in actual economies. In a monetary economy, there are no active trading arrangements for most goods directly for one another. Almost all trade is of goods for money, a single distinguished commodity that enters into almost all trades.

1

"Money buys goods. Goods buy money. Goods do not buy goods," Clower (1967). In a monetary economy, most of the $m(m-1)/2$ trading posts Walras posits will be inactive. Active trade will be concentrated on a narrow band of $m-1$ posts, those trading in 'money' versus the $m-1$ other nonmonetary commodities. In a monetary economy, households with supplies of good $i$ and demands for good $j$ trade $i$ for $j$ by first trading $i$ for money and then money for $j$. They do not trade $i$ for $j$ directly.

How does this concentration on trade with a single intermediary good come about? Prof. Tobin (1980) emphasizes scale economy and a positive external effect:

> The use of a particular language or a particular money by one individual increases
> its value to other actual or potential users. Increasing returns to scale, in this
> sense, limits the number of languages or moneys in a society and indeed explains
> the tendency for one basic language or money to monopolize the field.

Einzig (1966, p. 345), writing more from an anthropological perspective suggests "Money tends to develop automatically out of barter, through the fact that favourite means of barter are apt to arise ... object[s] ... widely accepted for direct consumption."

Menger (1892) describes a notion of liquid 'saleable' goods becoming money. "[Call] goods ... *more or less saleable*, according to the ... facility with which they can be disposed of ... at current purchasing prices or with less or more diminution." That is, a good is very saleable (liquid) if it's selling price is very near its buying price. "Men ... exchange goods ... for other goods ... more saleable....[which] become *generally* acceptable media of exchange [emphasis in original]." Hence, Menger suggests that liquid goods, those with narrow spreads between buying and selling prices, become principal media of exchange, money. Einzig notes that these are likely to be those goods with high trading volumes, an observation consistent with Tobin's emphasis on scale economy in the transactions process.

This paper develops a class of examples to formalize this family of observations. The examples describe how specialization of the trading function in a single medium of exchange comes about, starting as Einzig suggests with goods most "widely accepted for direct consumption." With scale economies in the transaction technology, these high volume goods will also be those with the lowest unit transaction cost. Thus they are, in Menger's view, the most saleable, and excellent candidates for "*generally* acceptable media of exchange." Households supplying good $i$ and demanding good $j$ are induced to trade in a monetary fashion, first trading $i$ for 'money' and then 'money' for $j$, by discovering that the transaction costs are lower in this indirect trade than in direct trade of $i$ for $j$. Starting from a barter array consisting (as Walras posits) of $m(m-1)/2$ active trading posts, the allocation evolves through price and quantity adjustments to a monetary array where only $m-1$ trading posts are active. The model portrays a trading post as a firm. The decision to operate a trading post depends on the post's ability to cover costs. To emphasize the pairwise character of trade, the model posits budget constraints enforced at each trade separately: the value of each household's sales to a trading post must equal the value of its purchases from the post. Specialization, the concentration of the trading function in a few trading posts (those specializing in trade that includes the commodity that is endogenously designated as 'money') in the monetary trade arrangement, reflects the workings of scale economies.

To emphasize the role of scale economies at the level of the trading post, this paper will consider only nonconvex transaction technologies and the resultant nonconvex transacation cost functions. Competitive equilibria are hence unlikely to exist and this paper concentrates on

2

average cost pricing equilibria of monopolistic trading posts (the rationale for average cost pricing is potential entry by similarly situated currently inactive trading firms).

The paper then considers the economy's dynamic approach to a monetary structure (an equilibrium where a single good --- the medium of exchange --- is common to virtually all transactions). As households discover that some pairwise markets (those with high trading volumes) have lower transaction costs, they rearrange their trades to take advantage of the low cost. That leads to even higher trading volumes and even lower costs at the most active trading posts[1]. The process converges to an equilibrium where only the high volume trading posts dealing in a single intermediary good ('money') are in use. Under nonconvex transaction costs, this implies a cost saving, since only m-1 trading posts need to operate, incurring significantly lower costs than m(m-1)/2 posts. Monetization of trade results from dynamic adjustment to scale economies in the transaction technology. Scale economies make it cost-saving to concentrate transactions in a few trading posts and one intermediary instrument.

A bibliography of the issues involved in this inquiry appears in Ostroy and Starr (1990). Note particularly Banerjee and Maskin (1996), Iwai (1995), Kiyotaki and Wright (1989), Ostroy and Starr (1974). The treatment of transaction costs in this paper resembles the general equilibrium models with transaction cost developed in Foley (1970), Hahn (1971), and Starrett (1973). The structure of bilateral trade here however is more detailed, with a budget constraint enforced at each trading post separately, so that their results do not immediately translate to the present setting.

This paper displays results similar to those of Iwai (1995) and Kiyotaki and Wright (1989). In Iwai (1995) and Kiyotaki and Wright (1989), traders are induced to concentrate their transactions on high volume markets by the reduced waiting time for a matching trade made possible by high volume. Indivisibility of traders --- a scale economy --- in Iwai (1995) and Kiyotaki and Wright (1989) induces this effect. This paper differs in explicitly emphasizing transaction cost and portraying (in primitive fashion) a dynamic adjustment process that leads from barter to a monetary equilibrium.

In Iwai (1995), Kiyotaki and Wright (1989), and Ostroy and Starr (1974) (and in most of the studies cited in Ostroy and Starr (1990)) the basic unit where economic interaction takes place is the pair of (households) traders coming together to trade. These studies take place without the structure of a specialized transaction function (exchanges, retailers, wholesalers, etc.). The present paper and Starr and Stinchcombe (1993, 1997) emphasize an explicit resource using exchange activity, the trading post for trade of commodities against one another, as the elementary unit of economic interaction. The market-maker there posts a bid-ask spread to cover costs. It is there that supplies and demands are presented and matched. Hence the present paper treats the market-making activity of bringing buyers and sellers together as the fundamental unit of exchange activity.

Starr and Stinchcombe (1993) characterizes monetary trade as the cost minimizing outcome of a centralized programming problem with a nonconvex transaction cost structure. The structure of trade there is modeled as the outcome of cost minimization on the array of possible trading arrangements in a pure exchange economy at general equilibrium prices. In Starr

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[1]     Hahn (1997) describes this situation as "If the number who can gain from trade is ... sufficiently [large] ... , the Pareto improving trade will take place. There is thus an externality induced by set-up costs."

and Stinchcombe (1997) the same monetization of trade is characterized as a decentralized (monopolistically competitive) market equilibrium, rather than as a centralized system solution.

In the present paper, monetization is the outcome of a dynamic adjustment: the most actively traded goods have the the lowest average transaction costs; trade then further concentrates on low cost trading posts increasing their trading volumes (and reducing their average transaction costs) even further. When a trading post's transaction cost is sufficiently low, households find it advantageous to use that post's low transaction cost goods as intermediary goods in their transactions, rather than trade directly (at high transaction cost trading posts) for the goods they want. Trading through an intermediary good may double the number of transactions undertaken, but does so at much lower total transaction cost than would be incurred in direct trade. Eventually a high trading volume good becomes 'money', the single universal intermediary good.

## II. An Economy with Pairwise Trade and Transaction Costs

The population of firms is represented by the finite index set F. The typical firm is denoted $k \in F$. The number of firms is denoted #F. The number of commodities is the positive integer N. Each firm will be thought of as a market-maker for a pair of commodities. The firm is identified with the pair of goods, i and j, $1 \le i, j \le N$, for which it is the unique market maker. The shorthand for this is $k = \{i,j\}$. The typical firm production technology is denoted $Y^k \subset R^{2N}$.

The typical firm production plan $y^k \in Y^k$ is described as $y^k = (y^{kB}, y^{kS})$ where $y^{kB}, y^{kS} \in R^N_+$. $y^{kB}$ represents the vector of firm k's purchases; $y^{kS}$ represents the vector of firm k's sales. This paper concentrates on zero-profit average cost pricing equilibria, so there is no accounting for profits.

We will be dealing with nonconvex transaction (production) technologies, generating a natural monopoly in each pairwise goods market. A competitive equilibrium is not an appropriate solution concept. The equilibrium notion I will use is an average cost pricing equilibrium. The rationale for this choice of equilbrium concept is possible entry (by other similar firms) if any economic rent is actually earned. The presence of potential entrants and their actions is not explicitly modeled.

A pure trade economy with pairwise goods markets

We will confine attention to a pure trade economy with pairwise goods markets, where the only resource using activity is trade. In order to formalize this notion, we assume that $(y^{kB}, y^{kS}) \in Y^k$ implies $y^{kB} \ge y^{kS}$, where the inequality applies co-ordinatewise. Further we identify the set of firms (and markets possibly active) with the set of commodity pairs. Then $F \equiv \{k=\{i,j\} \mid i,j \text{ positive integers} \le N, i \ne j\}$ and for each $\{i,j\} \in F$, $(y^{\{i,j\}B}, y^{\{i,j\}S}) \in Y^{\{i,j\}}$ implies that only the i and j co-ordinates of $y^{\{i,j\}S}$ may be nonnull. This notion admits the possibility that firm $\{i,j\}$ buys inputs to the transaction technology other than goods i or j. The typical firm will be denoted $\{i,j\}$ and we use an unspecified convention so that each i,j pair (independent of order) appears only once. That is, $\{i,j\} = \{j,i\}$. Equilibria will occur with (at most) a single firm operating in any pairwise market, so we ignore the possibility of more than one firm making a

4

market in the pair i,j, though the notion of an average cost pricing equilibrium is best explained as the result of the threat of entry by other firms with access to the same transactions technology. Firm k has prices at which it sells goods (ask prices, retail prices) $p^k \in R^N_+$ and at which it buys goods (bid prices, wholesale prices) $q^k \in R^N_+$ .

There is a finite set H of households, with the typical household denoted $h \in H$. Each household h has an endowment $r^h \in R^N_+$. We denote h's possible consumption set $X^h \subseteq R^N_{+}$. Household h's consumption vector is $x^h \in X^h$. h's consumption preferences are represented by the utility function $u^h: X^h \to R$.

Market conditions facing the typical household are characterized by the buying and selling prices for goods traded in {i,j} (prices for purchases and sales of i and j, and prices for inputs to the transaction technology). Prices are expressed as pure numbers per unit good. This creates no ambiguity since budget constraints depend on price ratios. It is simplest to take the price space for firm k to be $R^{2N}_+$ with a typical element $(p^k, q^k)$. $p^k$ and $q^k$ are each N-dimensional. Zeroes in $p^k$ and $q^k$ will not represent free goods but rather goods where k is inactive. Let $R^{2N\#F}_+$ be the #F-fold Cartesian product of $R^{2N}_+$ with itself; $(p,q) \in R^{2N\#F}_+$ represents the array of prevailing prices.

Given $(p,q) \in R^{2N\#F}_+$, household h then forms its buying and selling plans. $b^{hk}_n > 0$ denotes household h's purchases of good n from firm k; $s^{hk}_n > 0$ denotes h's sales of good n to firm k. $b^{hk}$ is the N-dimensional vector of h's purchases from k; $s^{hk}$ is the N-dimensional vector of h's sales to k.

Household h faces the following constraints on its transaction plans:

(T.i)  $b^{h\{i,j\}}_n > 0$, only if n=i,j; $s^{h\{i,j\}}_n > 0$, only if n=i,j or $q^{\{i,j\}}_n > 0$.

(T.ii)  $p^k \cdot b^{hk} \leq q^k \cdot s^{hk}$ , for each $k \in F$.

(T.iii)  $x^h = r^h + \Sigma_{k \in F} b^{hk} - \Sigma_{k \in F} s^{hk} \in X^h$

Note that condition (T.ii) defines a budget balance requirement at the transaction level, implying the pairwise character of trade. (T.ii) is the source of the demand for a medium of exchange. Since the budget constraint applies to each pairwise transaction separately, there is a demand for a carrier of value to move purchasing power between successive transactions. Household h's behavior is described then as follows. h faces $(p, q) \in R^{2N\#F}_+$. h then chooses $s^{hk}$ and $b^{hk}$, $k \in F$, to maximize $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii). That is, h chooses which firms to transact with --- and hence which pairwise markets to transact in --- and a transaction plan to optimize utility, subject to a multiplicity of pairwise budget constraints. Demand behavior for $h \in H$ is defined as

$D^h(p, q) = \{(s^{h1}, s^{h2}, ..., s^{h\#F}; b^{h1}, b^{h2}, ..., b^{h\#F}) \mid (s^{h1}, s^{h2}, ..., s^{h\#F}; b^{h1}, b^{h2}, ..., b^{h\#F})$ maximizes $u^h(x^h)$ subject to (T.i), (T.ii), (T.iii) at p, q}.

Demand behavior for the economy as a whole is defined as

$$D(p, q) = \Sigma_{h \in H} D^h(p, q)$$

An  average cost pricing equilibrium is defined in the following way.

An average cost pricing equilibrium consists of an array $(p^o, q^o) \in R^{2N\#F}_+$ with the following properties:

- For each $k \in F$ there is $(y^{okB}, y^{okS}) \in Y^k$ so that $q^{ok} \cdot y^{okB} = p^{ok} \cdot y^{okS}$ .
- There is $(s^1, s^2, ..., s^{\#F}; b^1, b^2, ..., b^{\#F}) \in D(p^o, q^o)$ so that,  for all $k \in F$, we have $y^{okB} = s^k$,  $y^{okS} = b^k$.

## Monetary equilibrium

Household h's net trade, denoted $\delta^h$, is the difference between its consumption and its endowment,  $\delta^h = x^h - r^h = \Sigma_k b^{hk} - \Sigma_k s^{hk}$ .

Let $\delta^{h+} = (\max(0, \delta^h_1), \max(0, \delta^h_2), ..., \max(0, \delta^h_n), ..., \max(0, \delta^h_N))$.

Let $\delta^{h-} = (\max(0, -\delta^h_1), \max(0, -\delta^h_2), ..., \max(0, -\delta^h_n), ..., \max(0, -\delta^h_N))$.

An equilibrium is said to be monetary if there is a unique distinguished good $\mu$ with all of the following properties:

- the only firms with nontrivial trade activity are those trading in good $\mu$, $k = \{\mu, j\}$,
- for each $h \in H$, good $\mu$ is the only good so that it may occur that $\Sigma_{k \in F} b^{hk}_\mu > \delta^{h+}_\mu$ or so that it may occur  that $\Sigma_{k \in F} s^{hk}_\mu > \delta^{h-}_\mu$ .

## III.  A Class of  Examples

This section develops a class of examples of convergence from barter to a monetary average cost pricing equilibrium in a pure trade economy with pairwise goods markets and nonconvex technology.

Let N be an even integer, $N \geq 8$.   Without loss of generality, the commodities labeled 1, N, and N/2 play distinct asymmetric roles.  1 will become the endogenously chosen 'money'; N/2 is a high transactions volume good; N is the good in which transaction costs are assessed, an input to the transactions process.

$H = H^1 \cup H^2 \cup H^3 \cup H^4$,  $\#H^1 = (N-1)(N-2)$ , $\#H^2 = 2(N-2)$, $\#H^3 = 2$, $\#H^4 \geq 1$

Each $h \in H^1$ is denoted $h = [m,n]$ where m and n are integers between 1 and N-1 (inclusive).  m denotes the good with which h is endowed. n denotes the good he prefers.  We suppose for each n, m so that  $n \neq m$ with $1 \leq n,m \leq N-1$, there is precisely one $h = [m,n]$.

For typical $h = [m,n]$,  $h \in H^1$,
$\quad r^{[m,n]}_i = 0, i \neq m;  r^{[m,n]}_m =  A.$
$\quad u^{[m,n]}(x) = \Sigma_{i \neq n} x_i + 3x_n$

6

Each $h \in H^2$ is denoted $h = [1,m]$ or $[m, 1]$ where m is an integer between 2 and N-1 (inclusive). $h=[m, 1]$ denotes h endowed with m, preferring 1. $h=[1, m]$ denotes h endowed with 1 preferring m. We suppose for each m with $2 \leq m \leq N-1$, there is precisely one $h = [1, m]$ and one $h=[m,1]$.

For typical $h = [1,m]$, $h \in H^2$,
$$r^{[1,m]}_{i} = 0, i \neq 1; \ r^{[1,m]}_{1} = B.$$
$$u^{[1,m]}(x) = \Sigma_{i \neq m} x_i + 3x_m$$

For typical $h = [m,1]$, $h \in H^2$,
$$r^{[m,1]}_{i} = 0, i \neq m; \ r^{[m,1]}_{m} = B.$$
$$u^{[m,1]}(x) = \Sigma_{i \neq 1} x_i + 3x_1$$

The two elements $h \in H^3$ are denoted $[1, N/2]$ and $[N/2, 1]$.
For $h = [1,N/2]$, $h \in H^3$,
$$r^{[1,N/2]}_{i} = 0, i \neq 1; \ r^{[1,N/2]}_{1} = C.$$
$$u^{[1,N/2]}(x) = \Sigma_{i \neq n} x_i + 3x_{N/2}$$

For $h = [N/2,1]$, $h \in H^3$,
$$r^{[N/2,1]}_{i} = 0, i \neq N/2; \ r^{[N/2,1]}_{N/2} = C.$$
$$u^{[N/2,1]}(x) = \Sigma_{i \neq 1} x_i + 3x_1$$

Each $h \in H^4$ has an endowment of good N only, $r^h_N$ with $\sum_{h \in H^4} r^h_N > \gamma(N-1)(N-2)$, and with utility function $u^h(x) = \sum_{i=1}^{N} x_i$ .

In summary, the endowment and tastes side of the market looks like this. Good N is held by households in $H^4$. Their tastes are very simple: all goods are perfect substitutes. Households in $H^1$ have distinct preferences and endowments. Each is endowed with one good and strongly prefers another. Their tastes and endowments are uniformly distributed among goods 1 through N-1. Households in $H^2$ each individually look like those in $H^1$, but the distribution of their endowments and preferences differ. Half of those in $H^2$ are endowed with good 1 and prefer another good (their preferences being uniformly distributed among $2 \leq n \leq N-1$). The balance of $H^2$ prefer good 1 and are endowed with another good (their endowments being uniformly distributed among $2 \leq m \leq N-1$). This structure of preferences and endowments creates relatively high trading volumes among households trading in good 1. The two households in $H^3$ have the same distinctive preferences, but their tastes and endowments are concentrated on good 1 and good N/2, further enhancing trading volumes in these goods.

Specify a nonconvex transactions technology for pairwise goods markets so that all transaction costs accrue in good N. The typical transactions of firm {i,j} will consist of purchases of $y^B$ and outflows of sales $y^S$.
$$Y^{\{i,j\}} = \{ (y^B, y^S) \mid y^S_n = y^B_n = 0 \text{ for } n \neq i,j,N ; \ y^S_n \leq y^B_n \text{ for } n=i,j; \ y^S_N=0;$$
$$y^B_N \geq \text{med}[0, y^B_i, \gamma] + \text{med}[0,y^B_j, \gamma] \}$$

where $\gamma > 0$, and med represents median. In words, the transaction technology looks like this: The firm {i,j} makes a market in goods i and j, buying each good in order to resell it. It incurs transaction costs in good N. These costs are in the nature of a set-up cost. They vary directly (and expensively) with volume of trade at low volume and then hit a ceiling $\gamma$, after which they do not increase with trading volume. The transaction cost structure is separable in the two principal traded goods. The firm {i,j} buys good N to cover the transaction costs it incurs, paying for N in goods i and j.

To further develop the example, let A>0, $\gamma = .6A$, $.5A=B=C$.

Households formulate their trading plans deciding how much of each good to trade in each pairwise goods market (i.e. with each pairwise market maker). A typical firm (market maker) k={i,j} is denoted by the pair of goods in which it makes a market. A typical household h=[m,n] is denoted by the pair of goods it will typically seek to exchange (m for n). This leads to the rather messy notation

$$b^{[m,n]\{i,j\}}_{\ell} = \text{planned purchase of good } \ell \text{ by household [m,n] on market \{i,j\}}$$
$$s^{[m,n]\{i,j\}}_{\ell} = \text{planned sale of good } \ell \text{ by household [m,n] on market \{i,j\}}$$

A tatonnement adjustment process for average cost pricing:

Prices will be adjusted by an average cost pricing auctioneer. Specify the following adjustment process for prices.

STEP 0: The starting point is somewhat arbitrary, equal bid and ask prices in each pairwise market.

CYCLE 1

STEP 1: Households compute their desired trades and report them for each pairwise market.

STEP 2: Average costs (and average cost prices) are computed for each pairwise market based on the outcome of STEP 1. Average cost prices are announced. In markets with low trading volumes (and hence high bid/ask spreads in average cost prices) a floor on bids, L, and a ceiling on ask prices, H, is imposed to keep values well defined. A market's (market making firm's) nonzero prices are specified only for those goods where the firm has the technical capability of being active in the market; other prices are represented as 0, not denoting free goods, but rather no available trade.

CYCLE 2

Repeat STEP 1 and STEP 2.

CYCLE 3, CYCLE 4, .... repeat until the process converges.

Let's see how this pricing process will work on our example.

STEP 0: For all $1 \leq i,j \leq N-1$, $i \neq j$, $p^{\{i,j\}}_i = p^{\{i,j\}}_j = q^{\{i,j\}}_i = q^{\{i,j\}}_j = q^{\{i,j\}}_N = 1$

CYCLE 1, STEP 1:

● For $[m,n] \in H^1$, $b^{[m,n]\{m,n\}}_n = A$, $s^{[m,n]\{m,n\}}_m = A$; all other purchases and sales are nil.

● For $[m,1] \in H^2$, $b^{[m,1]\{m,1\}}_1 = B = s^{[m,1]\{m,1\}}_m$; all other purchases and sales are nil. For $[1,n] \in H^2$, $b^{[1,n]\{1,n\}}_n = B = s^{[1,n]\{1,n\}}_1$; all other purchases and sales are nil.

8

- For the two elements of $H^3$, [1, N/2] and [N/2, 1], $b^{[1, N/2]\{N/2,1\}}_{N/2}= C = s^{[1, N/2]\{N/2,1\}}_{1}$; $b^{[N/2,1]\{N/2,1\}}_{1}= C = s^{[N/2,1]\{N/2,1\}}_{N/2}$; all other purchases and sales are nil.

- For $h \in H^4$, $b^{hk}=0=s^{hk}$.

STEP 2:

- For $\{m,n\}$ where $m\neq1\neq n$, $p^{\{m,n\}}_{m}=p^{\{m,n\}}_{n}=\dfrac{A}{A-\gamma}=q^{\{m,n\}}_{N}$ , $q^{\{m,n\}}_{m}=q^{\{m,n\}}_{n}=1$.

- For $\{m,1\}$, $m\neq N/2$, $p^{\{m,1\}}_{1}=p^{\{m,1\}}_{m}=\dfrac{A+B}{A+B-\gamma}=q^{\{m,1\}}_{N}$, $q^{\{m,1\}}_{m}=q^{\{m,1\}}_{1}=1$.

- For $\{N/2,1\}$ , $p^{\{N/2,1\}}_{1}=p^{\{N/2,1\}}_{N/2}=\dfrac{A+B+C}{A+B+C-\gamma}=q^{\{N/2,1\}}_{N}$ , $q^{\{N/2,1\}}_{N/2}=q^{\{N/2,1\}}_{1}=1$

CYCLE 2, STEP 1:

- For $[m,n] \in H^1$, $m,n\neq N/2$, $m,n\neq1$,  $s^{[m,n]\{m,n\}}_{m}= A$, $b^{[m,n]\{m,n\}}_{n}= Aq^{\{m,n\}}_{m}/p^{\{m,n\}}_{n}$; all other purchases and sales are nil[2].

For $[m,n] \in H^1$,  $n = N/2$,  $s^{[m,N/2]\{m,1\}}_{m}= A$, $b^{[m,N/2]\{m,1\}}_{1}= Aq^{\{m,1\}}_{m}/p^{\{m,1\}}_{1}$, $s^{[m,N/2]\{1,N/2\}}_{1}=Aq^{\{m,1\}}_{m}/p^{\{m,1\}}_{1}$, $b^{[m,N/2]\{1,N/2\}}_{1}= (Aq^{\{m,1\}}_{m}q^{\{N/2,1\}}_{1})/(p^{\{N/2,1\}}_{N/2}p^{\{m,1\}}_{1})$; all other purchases and sales are nil.

For $[m,n] \in H^1$, $m = N/2$,  $s^{[N/2,n]\{N/2,1\}}_{N/2}= A$, $b^{[N/2,n]\{N/2,1\}}_{1}= Aq^{\{N/2,1\}}_{N/2}/p^{\{N/2,1\}}_{1}$, $s^{[N/2,n]\{1,n\}}_{1}=Aq^{\{N/2,1\}}_{N/2}/p^{\{N/2,1\}}_{1}$, $b^{[N/2,n]\{1,n\}}_{n}= (Aq^{\{N/2,1\}}_{N/2}q^{\{1,n\}}_{1})/(p^{\{1,n\}}_{n}p^{\{N/2,1\}}_{1})$;  all other purchases and sales are nil[3].

For $[m,1] \in H^1$, $s^{[m,1]\{m,1\}}_{m}=A$, $b^{[m,1]\{m,1\}}_{1}= (Aq^{\{m,1\}}_{m})/p^{\{m,1\}}_{1}$; all other purchases and sales are nil. For $[1,n] \in H^1$, $s^{[1,n]\{1,n\}}_{1} =A$, $b^{[1,n]\{1,n\}}_{n}= (Aq^{\{n,1\}}_{1})/p^{\{n,1\}}_{n}$ ; all other purchases and sales are nil.

- For $[m,1] \in H^2$, $s^{[m,1]\{m,1\}}_{m}=B$, $b^{[m,1]\{m,1\}}_{1}= (Bq^{\{m,1\}}_{m})/p^{\{m,1\}}_{1}$ ; all other purchases and sales are nil. For $[1,n] \in H^2$, $s^{[1,n]\{1,n\}}_{1} =B$, $b^{[1,n]\{1,n\}}_{n}= (Bq^{\{n,1\}}_{1})/p^{\{n,1\}}_{n}$ ;  all other purchases and sales are nil.

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[2]    The typical $[m,n]\in H^1$, where $m,n\neq N/2$, $m,n\neq1$, considers, but decides against, using monetary trade with good 1 as the medium of exchange. It tries to decide whether selling m for 1 and then buying n with 1 would be preferable to direct exchange. This depends on whether $q^{\{m,1\}}_{m}q^{\{1,n\}}_{1}/p^{\{m,1\}}_{1}p^{\{1,n\}}_{n}>q^{\{m,n\}}_{m}/p^{\{m,n\}}_{n}$ . At the currently posited values of  A,B,C,and $\gamma$ the answer is "no" --- since   $.16 < .4$ and hence direct trade  is more rewarding.

[3]    The cases $[m,n] \in H^1$,  $n = N/2$,  and  $[m,n] \in H^1$, $m = N/2$ represent the first step where monetization takes place. Household [m,n] considers trading directly at post $\{m,n\}$ versus trading indirectly through 1 as a medium of exchange at $\{m,1\}$ and $\{1,n\}$.  For [m,N/2] this comes down to the calculation whether $(q^{\{m,1\}}_{m}q^{\{N/2,1\}}_{1})/(p^{\{N/2,1\}}_{N/2}p^{\{m,1\}}_{1})> q^{\{m,N/2\}}_{m}/p^{\{m,N/2\}}_{N/2}$ . At the values of A,B,C,and $\gamma$ chosen, this inequality is $0.42>0.4$ , indicating that indirect trade is more rewarding.  Similarly for [N/2,n].

- For the two elements of $H^3$, $[1, N/2]$ and $[N/2, 1]$, $s^{[1, N/2]\{N/2,1\}}_1 = C$ ,

$b^{[1,N/2]\{N/2,1\}}_{N/2} = (Cq^{\{N/2,1\}}_1)/p^{\{N/2,1\}}_{N/2}$ ; $s^{[N/2,1]\{N/2,1\}}_{N/2} = C$, $b^{[N/2,1]\{N/2,1\}}_1 = (Cq^{\{N/2,1\}}_1)/p^{\{N/2,1\}}_1$ ; all other purchases and sales are nil.

- For $h \in H^4$, for each $\{i,j\} \in F$, $\Sigma_{h \in H^4} b^{h\{i,j\}}_i = \gamma = \Sigma_{h \in H^4} s^{h\{i,j\}}_N$, $\Sigma_{h \in H^4} b^{h\{i,j\}}_j = \gamma = \Sigma_{h \in H^4} s^{h\{i,j\}}_N$ .

STEP 2:

- For $\{m,n\}$ where $m \neq 1 \neq n$, $m \neq N/2 \neq n$, $p^{\{m,n\}}_m = p^{\{m,n\}}_n = \dfrac{A}{A-\gamma} = q^{\{m,n\}}_N$ , $q^{\{m,n\}}_m = q^{\{m,n\}}_n = 1$.

- For $\{m, N/2\}$, $p^{\{m, N/2\}}_m = p^{\{m, N/2\}}_{N/2} = H$, $q^{\{m, N/2\}}_N = q^{\{m, N/2\}}_m = q^{\{m, N/2\}}_{N/2} = L$

- For $\{m,1\}$, $m \neq N/2$, $p^{\{m,1\}}_1 = p^{\{m,1\}}_m = q^{\{m,1\}}_N = q^{\{m,1\}}_1 = 1$, $q^{\{m,1\}}_m = \dfrac{2A+B-2\gamma}{2A+B}$ .

- For $\{N/2,1\}$ , $p^{\{N/2,1\}}_1 = p^{\{N/2,1\}}_{N/2} = q^{\{N/2,1\}}_N = q^{\{N/2,1\}}_1 = 1$, $q^{\{N/2,1\}}_{N/2} = \dfrac{(N-2)A+B+C-2\gamma}{(N-2)A+B+C}$ .

CYCLE 3, STEP 1:

- For $[m,n] \in H^1$, $s^{[m,n]\{m,1\}}_m = A$, $b^{[m,n]\{m,1\}}_1 = Aq^{\{m,1\}}_m/p^{\{m,1\}}_1$, $s^{[m,n]\{1,n\}}_1 = Aq^{\{m,1\}}_m/p^{\{m,1\}}_1$,
$b^{[m,n]\{1,n\}}_n = A(q^{\{m,1\}}_m q^{\{n,1\}}_1)/(p^{\{m,1\}}_1 p^{\{1,n\}}_n)$; all other purchases and sales are nil[4].
- For $[m,1] \in H^2$, $s^{[m,1]\{m,1\}}_m = B$, $b^{[m,1]\{m,1\}}_1 = B/p^{\{m,1\}}_1$; all other purchases and sales are nil.

For $[1,n] \in H^2$, $s^{[1,n]\{1,n\}}_1 = B$, $b^{[1,n]\{1,n\}}_n = B/p^{\{1,n\}}_n$ ; all other purchases and sales are nil.

- For the two elements of $H^3$, $[1, N/2]$ and $[N/2, 1]$, $s^{[1, N/2]\{N/2,1\}}_1 = C$, $b^{[1, N/2]\{N/2,1\}}_{N/2} = C/p^{\{N/2,1\}}_{N/2}$; $s^{[N/2,1]\{N/2,1\}}_{N/2} = C$, $b^{[N/2,1]\{N/2,1\}}_1 = C/p^{\{N/2,1\}}_1$ ; all other purchases and sales are nil.

- For $h \in H^4$, for each $\{i,j\} \in F$ with $i \neq 1 \neq j$, all transactions are nil. For $\{1,j\}$, $2 \leq j \leq N-1$,

$\Sigma_{h \in H^4} b^{h\{1,j\}}_j = \gamma = \Sigma_{h \in H^4} s^{h\{i,j\}}_N$ .

STEP 2:

- For $\{m,n\}$ where $m \neq 1 \neq n$, $p^{\{m,n\}}_m = p^{\{m,n\}}_n = H$, $q^{\{m,n\}}_N = q^{\{m,n\}}_m = q^{\{m,n\}}_n = L$.

- For $\{m,1\}$, $m \neq N/2$, $p^{\{m,1\}}_1 = p^{\{m,1\}}_m = q^{\{m,1\}}_N = q^{\{m,1\}}_1 = 1$; $q^{\{m,1\}}_m = \dfrac{(N-2)A+B-2\gamma}{(N-2)A+B}$ .

- For $\{N/2,1\}$ , $p^{\{N/2,1\}}_1 = p^{\{N/2,1\}}_{N/2} = q^{\{N/2,1\}}_N = q^{\{N/2,1\}}_1 = 1$, $q^{\{N/2,1\}}_{N/2} = \dfrac{(N-2)A+B+C-2\gamma}{(N-2)A+B+C}$ .

CYCLE 4, STEP 1:
Repeat Cycle 3, Step 1
STEP 2:
Repeat Cycle 3, Step 2
CONVERGENCE.



What's happening in this example? Preferences and endowments are structured so that at roughly the same prices for all goods, there is a balance between supply and demand. Some pairs of

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[4]      This is the next step where monetization takes place. $[m,n] \in H^1$ decides whether to trade at $\{m,1\}$ followed by $\{1,n\}$ or directly at $\{m,n\}$. This amounts to evaluating whether $(q^{\{m,1\}}_m q^{\{n,1\}}_1)/(p^{\{m,1\}}_1 p^{\{1,n\}}_n) > q^{\{m,n\}}_m/p^{\{m,n\}}_n$ . At the values of A, B, C, $\gamma$ posited this inequality is $.52 > .4$ , indicating that the indirect trade is the more rewarding.

goods are more actively traded than others.  Good 1 has approximately 50% more active demanders (and suppliers) than most other goods.  Good N/2 has slightly more active trade than goods 2,.., N/2-1, N/2+1,...,N-1, and that active trade is concentrated in suppliers who demand good 1 and demanders endowed with good 1.

Here's how trade takes place. The starting point is a barter economy, the full array of m(m-1)/2  trading posts.  For every pair of goods (i,j), where $1 \leq i,j \leq N-1$, there is a post where that pair can be traded at prices as attractive as anywhere else; all prices start out equal.  Then each household computes its demands and supplies at those prices.  It figures out what it wants to buy and sell and to which trading posts it should go to implement the trades.  Since all prices start out equal, each household just goes to the post that trades in the pair of goods that the household wants to exchange for one another;  demanders of good j who are endowed with good i go to {i,j}.  Because of the distribution of demands and supplies, there is 50% higher trading volume on posts {1,j} than on most {i,j} and 100% higher volume on {1,N/2}.

Then the average cost pricing auctioneer responds to the planned transactions.  He prices bid/ask spreads in all markets to cover the costs of the trade on them.  Since there is a scale economy in the transactions technology, this leads to slightly narrower bid/ask spreads on the {1,j} markets and an even narrower spread on the {1, N/2} market.  The auctioneer announces his prices.

Households respond to the new prices.  Households who want to buy or sell good N/2 discover that the bid/ask spread on market {1, N/2} is lower than on any other market trading N/2.  It makes sense to channel transactions through this low cost market, even if the household has to undertake additional transactions to do so.  Ordinarily households [i,N/2] and [N/2,i] would have gone directly to the market {i,N/2} to do their trading.  But the combined transaction costs on {i,1} and on {1,N/2} are lower than those on {i,N/2}.  Households [i,N/2] and [N/2,i] find that they incur lower transaction costs by trading through good 1 as an intermediary.  They exchange i for 1 and 1 for N/2 (or N/2 for 1 and 1 for i) rather than trade directly.  The market makers on the many different {i,1} markets, $2 \leq i \leq N-1$,  find their trading volumes increased as the [i,N/2] and [N/2,i] traders move their trades to {i,1} and {N/2,1}.

The average cost pricing auctioneer responds to the revised trading plans once again. Bid-ask spreads expand on {i,N/2} and they decline on {i,1}, $2 \leq i \leq N-1$.  Now the bid-ask spreads on {i,1} are less than half those on {i,j} for i≠1≠j.  The auctioneer announces his prices.

Households respond to the new prices.  For all households [i,j] now, it is less expensive to trade through good 1 as an intermediary than to trade directly i for j or j for i.  All [i,j] now trade on {i,1} and {j,1}; none trade on {i,j}, for i≠1≠j.  Trade is fully monetized with good 1 as the 'money.'

The average cost pricing auctioneer reprices the markets.  Inactive markets, {i,j} for i≠1≠j, necessarily have high average costs, so he posts slightly arbitrary but large bid-ask spreads on them.  They show an ask price of H and a bid price of L, H >>L.  The active markets {i,1} get posted prices reflecting their high trading volumes, with narrow bid-ask spreads.

Households review the newly posted prices.  The narrow bid-ask spreads on the {i,1} markets reinforce the attractiveness of their previous plans, which called for trading through good 1 as an intermediary.  They leave their monetary trading plans in force.  At current prices, it is much more economical to trade i for j by first trading i for 1 and then 1 for j than to trade i for j directly.  High trading volumes on the {i,1} and {j,1} markets ensure low transaction costs and keep them attractive.

This progression is illustrated in Figure 1. Each numbered node in the figure represents a commodity. The chord connecting nodes i and j represents an active market in the pair i,j. If there is no chord, there is no active market. A broken line chord represents a low volume (eventually high cost) market. A solid chord represents a moderate volume (eventually moderate cost) market. A heavy chord represents a high volume (low cost) market. The progression from barter to money is then the movement from a diffuse array of markets to the concentration on a connected family of high volume (low cost) markets.

IV. Conclusion

The class of examples in section III demonstrates the following conception of the monetization of the transactions process.

Scale economies in the transactions technology mean that high volume markets will be low average cost markets. The transition from barter to monetary exchange is the transition from a complex of many thin markets --- one for trade of each pair of goods for one another to an array of a smaller number of thick markets dealing in each good versus a common medium of exchange. This transition is resource saving if the scale economies in transactions technology are large enough. The example shows that the transition progresses through individually rational decsions when prices reflect the scale economy and the initial condition includes one good (the latent 'money') with a relatively high transaction volume (hence low average transaction cost). Then, as Einzig notes, "favourite means of barter are apt to arise" and a barter economy thus converges incrementally to a monetary economy.

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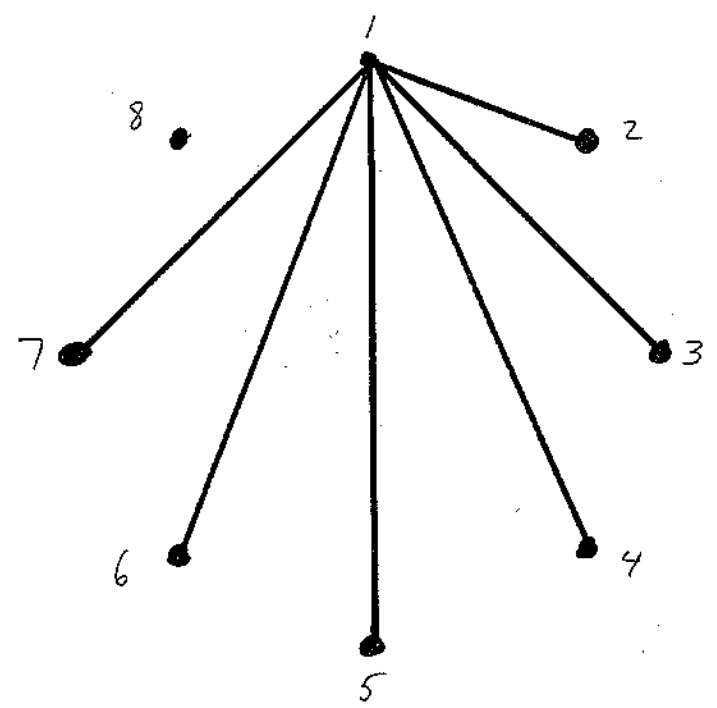Walras, L. (1874), *Elements of Pure Economics*, Jaffe translation (1954), Homewood, Illinois: Irwin.
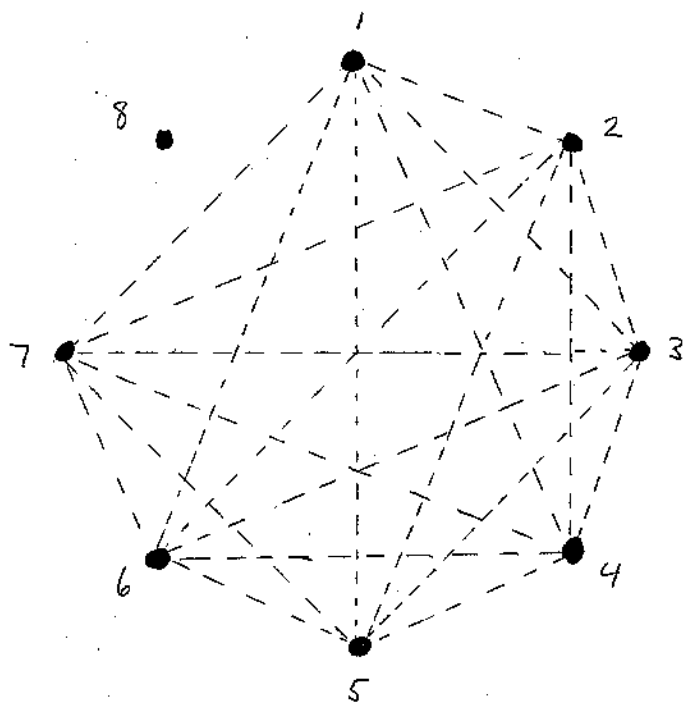
**Barter Economy: Step 0**
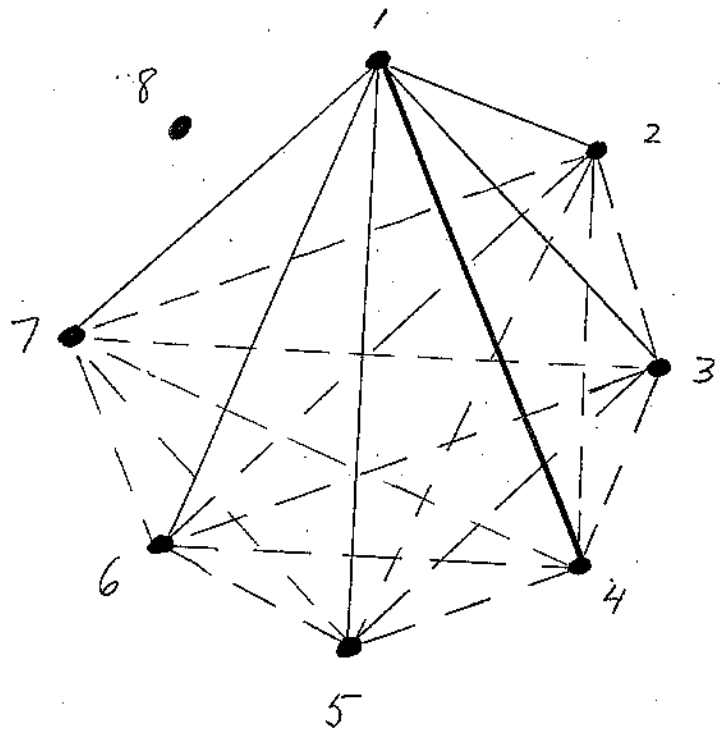
**Barter Trade : Cycle 1**
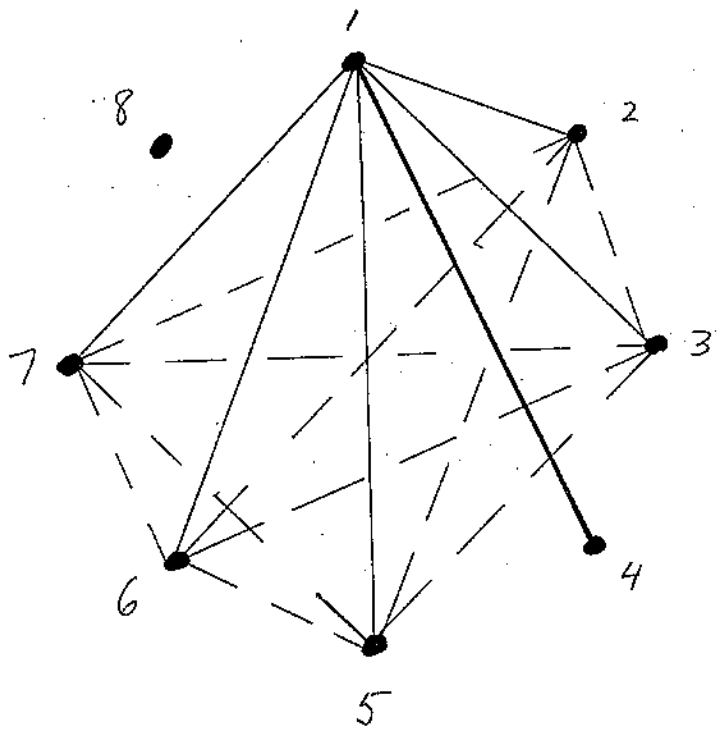
**Partial Monetization: Cycle 2**

**Monetary Economy: Cycle 3**

Heavy chord=High Volume Market
Solid chord=Moderate Volume Market
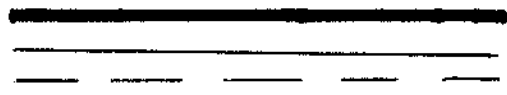Broken chord=Low Volume Market
No chord=Inactive market

**Figure 1**