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Policies in Behavioral Macroeconomics

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To my advisors, Anujit Chakraborty and Athanasios Geromichalos, for their continuous guidance and invaluable suggestions throughout the Ph.D program

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Abstract

Policies in Behavioral Macroeconomics

I explore macroeconomic theory while I adopt recent lessons from behavioral economics. This behavioral-macro exercise not only enriches our description of the economy but also presents a novel perspective on the policy and welfare. Two chapters are devoted to the problem of non-standard preference in the labor market, and one chapter deals with heterogeneous information in the New Keynesian model.

In the first chapter, I investigate how a government should distribute unemployment insurance benefits across time if a job seeker is present biased. Using the one-sided job search model, I first show that an optimal unemployment benefit is a decreasing sequence along with the unemployment spell regardless of the present bias if there's no saving technology. I further present a condition for the optimal policy to be decreasing even if savings are possible. Finally, I use Korean data to find evidence of present bias among job seekers and to estimate the welfare gains of switching to the optimal plan from the current policy.

In the second chapter, I build a New Keynesian model that features heterogeneous awareness. I assume that some consumers are aware of a part of the shocks in the economy, and derive an unawareness augmented IS curve. This augmentation generates a heterogeneous awareness-driven discounting, which provides a resolution for the 'forward guidance puzzle'. I further present 'raising awareness' as a communication policy, and show when it can support the monetary policy.

In the third chapter, I study the value of unemployment insurance as a correctional mechanism for job seekers' temptation. First, it urges workers to search more because the tempted workers search less than the social optimum. Second, lower insurance for the future can correct the tempted consumers by letting them save more. Finally, I show that the optimal insurance level depends on the resistance cost, and present that there is some evidence of the finite resistance cost using Korean data.

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Chapter 1

Optimal Unemployment Insurance for Present Biased Workers

1.1 Introduction

Most countries have unemployment insurance (UI) to protect workers from adverse labor income shocks. UI helps workers to smooth consumption, hence it can improve social welfare. However, too high a level of UI benefit can harm job search incentives. The trade-off becomes larger when the unemployed people's search activities are private information, posing questions about the optimal insurance policy.

With a given amount of total UI budget, how we temporally distribute it also matters because the temporal distribution schedule changes search incentives as well as consumption at each point of time. From the earlier work of Shavell and Weiss [1979] to the more recent Shimer and Werning [2008], many studies on this question assumed a time-consistent preference. Experimental evidence and observational data, however, disagree with the exponential discounting model. The degree of impatience seems to change over time, most significantly when it comes to now. People show temporal reversals[1] in their choices, implying that they put much importance on now. We call this 'present bias'. If such a time preference is common, the policy suggestion should be revisited to accommodate it. This chapter specifically focuses on the job search model with present bias and

[1] An example from Chakraborty [2021] is a choice set \$100 today $\succsim$ \$110 in a week, and \$100 in 4 weeks $\precsim$ \$110 in 5 weeks.

deduces the optimal UI distribution over time.

I adopt the quasi-hyperbolic discounting ($\beta\delta$ discounting) to model present bias.[2] It discounts all periods using the usual exponential discount factor (δ), and discounts additionally by β for any future periods except the present. The overall discounting function is, therefore, discontinuous at 'present'. DellaVigna and Paserman [2005a] was the first to investigate policy implications for job seekers with $\beta\delta$ preferences. Paserman [2008], Cockx et al. [2014], and DellaVigna et al. [2017] also integrate present bias with search theories. Some of these papers compare welfare implications among different policies. However, none of them explicitly pursue the optimal temporal distribution when an agent is present biased. This chapter seeks the optimal UI policy in terms of a temporal distribution of a budget if the workers are present biased.

Other than DellaVigna and Paserman [2005a], another very close paper is Spinnewijn [2015a]. Without modeling underlying primitives, Spinnewijn [2015a] assumes that the agent may have wrong expectations about the probability of getting a job.[3] This chapter complements Spinnewijn [2015a] by adopting a specific time preference and presents an opposite conclusion. While Spinnewijn [2015a] proposes an increasing UI scheme, I suggest a decreasing scheme is also possible.

In section 1.3, I build a sequential search model following McCall [1970] with the quasi-hyperbolic discounting. This baseline model follows the original Shavell and Weiss [1979]'s construction, yet includes present bias. I define an equilibrium concept, perception perfect equilibrium to consider the time inconsistency of the preference. In section 1.4, I build a social planner's problem by assuming that the planner has a long-run preference and

[2]The modeling choice of $\beta\delta$ discounting is deliberate. I will argue the inefficiency caused by the non-stationary preference under the immediate cost and delayed benefit context, which is closely connected to the problem of O'Donoghue and Rabin [1999]. Therefore, formulating the problem using $\beta\delta$ discounting is a natural choice. Further, it is also empirically plausible. DellaVigna and Paserman [2005a] shows that $\beta\delta$ discounting implies that the exit rate from the unemployment state is negatively correlated with impatience. Paserman [2008] directly estimates the short-term discounting β, suggesting the time-inconsistent preference in the job searching domain. One could argue an alternative modeling, for example, a menu-dependent preference. However, it is not very obvious how we define a menu that the job seeker faces when the choice is about search intensity and reservation wage.

[3]There are two types of optimism in the paper. One is baseline optimism which is about the probability itself, and the other is control optimism which is about the additional probability gain of the search activity.

discounts the future exponentially. In the baseline model, the optimal policy features a decreasing UI regardless of the job seeker's present bias or sophistication. Comparative statics show that the direct effect of short-term discounting (β) leads the optimal UI schedule to converge to zero faster. That is, the government should provide a more heavily front-loaded UI program to a present biased job seeker than an exponentially discounting job seeker. However, because β also affects the exit rate, which makes the optimum UI schedule flatter, the overall slope is ambiguous.

In the next subsection, I add a consumption-saving problem to the model and solve it with a specific utility function (CARA), following Shimer and Werning [2008] and Spinnewijn [2015a]. In this modified model, I show that the optimal UI critically depends on the agent's asset liquidation behavior. If the future UI causes larger asset liquidation, then the decreasing UI is optimal. In section 1.5, I estimate key parameters of the model, notably, the short-term discounting factor (β) using Korean labor market data. The estimated β is in the range of 0.5-0.8, indicating the existence of present bias in the job search domain. Based on the estimates, I simulate the optimal policy for each welfare criterion, which confirms the main results in the previous sections.

1.2 Related Literature

Studies on time-inconsistent preference go back to Strotz [1955], who formally introduced the self-control problem and commitments. Phelps and Pollak [1968] proposed a present biased preference. Following these pioneering works, Laibson [1997] investigates the consumption saving decision under quasi-hyperbolic discounting with a commitment technology. Numerous works followed his influential work. Harris and Laibson [2001], Krusell and Smith [2003], Chatterjee and Eyigungor [2016], and Cao and Werning [2018] are examples that study the hyperbolic discounting and equilibrium of a consumption saving problem.

Most present bias models implicitly put assumptions on the agent's perception about her future action. She may correctly foresee her time inconsistency (sophisticated), or she may believe that she will not have such a conflict (naive). O'Donoghue and Rabin [1999] explicitly takes those two extremes into account. It turns out that naifs are influenced solely by the present bias effect, whereas sophisticates are subject to the consideration of their self-control problem as well as the present bias. I will make a clear assumption about the perception in the main text.

Another strand of research this chapter refers to is studies on optimal unemployment insurance. The goal of the UI is to provide insurance for an adverse income shock, but the UI also plagues search incentives and causes a moral hazard. One of the earliest and most influential studies on the optimal unemployment benefit and taxation scheme in the search environment is Shavell and Weiss [1979]. The key message of the paper is that the optimal UI is a decreasing sequence along the unemployment spell if the planner cannot observe the search effort level. Hopenhayn and Nicolini [1997] also considers an optimal UI using a sequential search model. The key difference with Shavell and Weiss [1979] is that they include one more dimension of the policy tool: the labor income tax. They confirm that the optimal UI benefit should decrease over the length of the unemployment as Shavell and Weiss [1979]. On top of that, the optimal tax should increase with the past unemployment spell. Shimer and Werning [2008] also questions optimal UI policy under a similar setting. The critical difference is, however, that they assume the job seeker can save and accumulate assets. With the CARA utility function, they argue that the constant UI can achieve the first best allocation.

In an influential work, Acemoglu and Shimer [1999] studied efficient (in the sense that maximizing output)[4] UI benefit level for the risk-averse workers using the directed job search model. They showed that risk-averse workers prefer low-wage jobs to reduce unemployment risk if there's no UI, hence concluding that a positive UI benefit funded by lump-sum tax from workers can restore the output-maximizing allocation. Golosov et al. [2013] considers firms with heterogeneous productivity.[5] Using the mechanism design approach, they find out that strict positive UI benefits with increasing, regressive labor earning tax (not a lump-sum tax) can achieve the constrained efficient (the second

[4]This level of UI benefit, however, is not the level of maximizing the ex-ante utility of workers.

[5]The heterogeneity generates labor income inequality between a worker at a highly productive firm and a worker at a less productive place. Hence their UI and taxation policy have redistribution-related implications.

best) allocation. Geromichalos [2015] assumes firms that fill their vacancies pay the UI bill, and draw welfare implications that arise with different taxation schemes: lump-sum tax, personalized tax, and “wage-vacancy” contracts. He shows that lump-sum taxation cannot achieve the first best allocation, which comes from the externalities of a job posting.

Three papers are the closest to this chapter. DellaVigna and Paserman [2005a] build a quasi-hyperbolic discounting ($\beta\delta$ discounting) model on the standard McCall model. A present biased worker tends to search less since the job searching cost is immediate while the benefit comes in the future. Also, in the following work, Paserman [2008] empirically reassures the existence of present bias among job seekers and finds out that the degree is larger among medium to low-wage workers. The author further compares different policies and deduces welfare implications. Spinnewijn [2015a] uses a sufficient statistic approach. He derives both the optimal level of UI and temporal distribution when the agent has a wrong belief. Interestingly, he concludes that an increasing UI is optimal when the agent has optimistically wrong beliefs under some conditions.

1.3 Baseline Model

There is a job seeker who is initially unemployed and searches for a job. In period t, she receives a job offer with a probability $\alpha_t \in [0, 1]$. The offer arrival rate α_t is a function of her search effort e_t, which entails an additive disutility $-k(e_t)$ where $k(0) = 0, k'(e) > 0, k''(e) > 0$. For simplicity, I normalize e_t to be identical to α_t.

A job offer promises to pay wage w_t, which is drawn from a known distribution F over a finite support $[\underline{w}, \overline{w}]$. If the job seeker accepts the offer, she gets the proposed wage w_t from the next period and keeps the job for the rest of her life. If she rejects it, she continues in the next period as unemployed. While unemployed, she gets a UI benefit (z_t) which is potentially time-varying. It is natural to think that the UI benefit z_t is also bounded above by $\overline{w}$.

Time is discrete ($T = \{0, 1, 2, ...\}$), and the future flow utilities are discounted with a quasi-hyperbolic discounting ($\beta\delta$) as Laibson [1997], DellaVigna and Paserman [2005a], and Paserman [2008]. δ is the usual long-run discount factor that applies to all periods exponentially. On the other hand, β is a short-run discount factor that separates 'now' and 'later'. If β is strictly less than 1, I call the agent 'present biased'. A present-biased agent cares about the present consumption more than the future one, disproportionally. The discounted sum of utility flows at period t can be described as follows:

$$U_t = u(c_t) - k(\alpha_t) + \beta \sum_{n=1}^{\infty} \delta^n \left(u(c_{t+n}) - k(\alpha_{t+n})\right)$$

The utility function $u(c)$ is assumed to be continuously differentiable, monotonic, strictly concave ($u'(c) > 0, u''(c) < 0$), and finite ($|u(c)| < \infty, \forall c \in [\underline{w}, \overline{w}]$). I can write the utility of being employed ($V_{w,t}$, $W_{w,t}$) and unemployed ($V_{u,t}$, $W_{u,t}$) at time t and the associated continuation values recursively.[6]

$$V_{w,t}(c_t) = u(c_t) + \beta\delta W_{w,t+1}$$
$$W_{w,t+1} = u(c_{t+1}) + \delta W_{w,t+2}$$
$$V_{u,t} = u(c_t) - k(\alpha_t) + \alpha_t \beta\delta \int_{\underline{w}}^{\overline{w}} \max\{W_{w,t+1}, W_{u,t+1}\} dF(w) + \beta\delta(1-\alpha_t) W_{u,t+1}$$
$$W_{u,t+1} = u(c_{t+1}) - k(\alpha_{t+1}) + \alpha_{t+1}\delta \int_{\underline{w}}^{\overline{w}} \max\{W_{w,t+2}, W_{u,t+2}\} dF(w) + \delta(1-\alpha_{t+1}) W_{u,t+2}$$

For simplicity, I assume that there are no savings at this moment. The consumption-saving problem will be considered in the next section. Without savings or borrowing, all of the income is consumed in each period. The consumption of an agent who accepted an offer at period t is w_t ($c_\tau = w_t, \forall \tau > t$) where w_t is the wage drawn at t and promised to be paid from $t+1$. The unemployed can only consume what is given as an unemployment insurance benefit ($c_t = z_t$). Also without job separation, being a worker is an absorbing state, leaving the agent no choice variables in the future. I can simplify the worker's value as follows:

$$W_{w,t+1} = \sum_{n=0}^{\infty} \delta^n u(w_t) = \frac{u(w_t)}{1-\delta}$$
$$V_{w,t} = u(w_{t-1}) + \beta \sum_{n=0}^{\infty} \delta^{n+1} u(w_{t-1}) = \left(\frac{1-\delta+\beta\delta}{1-\delta}\right) u(w_{t-1})$$

[6]The functional form for current value (V) and continuation value (W) should be different because of the short-run discount factor β.

The set of available actions for the unemployed job seeker at period t is a pair $x_t = (\alpha_t, \phi_t)$ where $\alpha_t \in [0,1]$ is the effort level at time t, and $\phi_t : [\underline{w}, \overline{w}] \rightarrow \{\text{Accept}, \text{Reject}\}$ is a decision rule. Note that the continuation value of working ($W_{w,t+1}$) is increasing with w_t, so the job seeker uses the following reservation wage strategy: if an offered wage w_t is larger than the reservation wage R_t, she accepts the offer. Otherwise, she rejects it. The decision rule and the reservation wage can be written as follows:

$$\begin{cases} \phi_t = \text{Reject} \quad \text{if } w_t < R_t \\ \phi_t = \text{Accept} \quad \text{if } w_t \geq R_t, \quad R_t = \{w : W_{w,t+1} = W_{u,t+1}\} \end{cases}$$

The agent also forms a belief on the strategies of the successive selves $\hat{s}^t := \{\hat{x}^t_\tau\}^\infty_{\tau=t+1}$, based on the perceived short run discounting factor ($\hat{\beta}$). I assume that the perceived discount factor is weakly larger than the actual ($\hat{\beta} \in [\beta, 1]$), which means that the workers are optimistic about their future bias. Superscript t means that the belief is formed by the t-self. The agent maximizes her expected discounted sum of utilities given the belief. The equilibrium concept I use is perception perfect equilibrium from O'Donoghue and Rabin [2001a], which imposes the following consistency condition on the beliefs.[7]

(a) $\forall \tau > t$, $\hat{x}^t_\tau \in \arg\max V_{u,\tau}(x_\tau; \hat{s}^t)$, s.t.,
$V_{u,\tau}(x_\tau) := u(z_\tau) - k(\alpha_\tau) + \alpha_\tau \hat{\beta}\delta \int_{\underline{w}}^{\overline{w}} \max\{W_{w,\tau+1}, W_{u,\tau+1}\} dF(w) + \hat{\beta}\delta(1-\alpha_t) W_{u,\tau+1}$

(b) $\forall t < t'$, $\forall \tau > t'$, $\hat{x}^t_\tau = \hat{x}^{t'}_\tau$

Condition (a) implies that the agent believes that her future selves will maximize their value, given the sequence of belief ($\hat{s}^t$), and condition (b) means that the agent does not change her belief on the future actions until the future comes to the present. Given this belief, the agent maximizes her current utility.

$$x_t \in \arg\max V_{u,t}(x_t; \hat{s}^t),$$
$$V_{u,t}(x_t) := u(z_t) - k(\alpha_t) + \alpha_t \beta\delta \int_{\underline{w}}^{\overline{w}} \max\{W_{w,t+1}, W_{u,t+1}\} dF(w) + \beta\delta(1-\alpha_t) W_{u,t+1}$$

[7] See O'Donoghue and Rabin [2001a] for the detailed definition and applications of dynamic consistency and perception perfect equilibrium.

Since our focus is the unemployed agent's problem and the wage draws are independent, the perception perfect strategy exhibits Markov strategy properties. The strategy depends only on the current payoff relevant information. If I impose assumptions for interior solutions[8], then the strategy is an equilibrium if (and only if) the following equations hold for every $t \in T$.

$$k'(\alpha_\tau) = \beta\delta \int_{R_\tau}^{\overline{w}} (W_{w,\tau+1} - W_{u,\tau+1})\, dF(w), \quad \forall \tau \geq t \tag{1.3.1}$$

$$k'(\hat{\alpha}_s) = \hat{\beta}\delta \int_{\hat{R}_s}^{\overline{w}} (W_{w,s+1} - W_{u,s+1})\, dF(w), \quad \forall s \geq t+1 \tag{1.3.2}$$

It is discussed in DellaVigna and Paserman [2005a] that the problem has a unique symmetric equilibrium if the value of the outside option (z_t) doesn't change. The main focus of this chapter is however, the case where the value of outside options varies over time (if necessary for the UI policy to be optimum). I first state that there's an equilibrium in the problem even if the UI is time-varying.

Proposition 1.3.1. *If z_t is not a constant, then there exists a unique, asymmetric equilibrium under a mild condition. Further, if z_t is a decreasing sequence, then the equilibrium $\{R^*_\tau(\hat{s}^\tau_{\tau+1})\}_{\tau=0}^{\infty}$ can not be an increasing sequence. That is, there exist t such that $R^*_t > R^*_{t+1}$.*

Comparing the equilibrium behavior, the present biased agent searches less than the exponential agent. The reservation wage depends on the perception. If the agent is fully naive, the reservation wage is equal to the exponential agent. If the agent is partially naive or sophisticated, the reservation wage is lower than the exponential or full naivete. Figure 1.3 shows the equilibrium behavior graphically.

[8]The current cost of the search is convex whereas the continuation value is linear with respect to α_t. Therefore, if $k'(0) < \beta\delta \int_{R_t}^{\bar{w}} [W_{w,t+1} - W_{u,t+1}] dF(w) < k'(1)$, and $u(\underline{w}) < (1-\delta)W_{u,t+1} < u(\bar{w})$, then it is sufficient to have an interior solution for α_t. A similar condition is sufficient for $\hat{\alpha}_{t+j}$ by replacing β with $\hat{\beta}$. I will impose these conditions for the rest of the chapter.

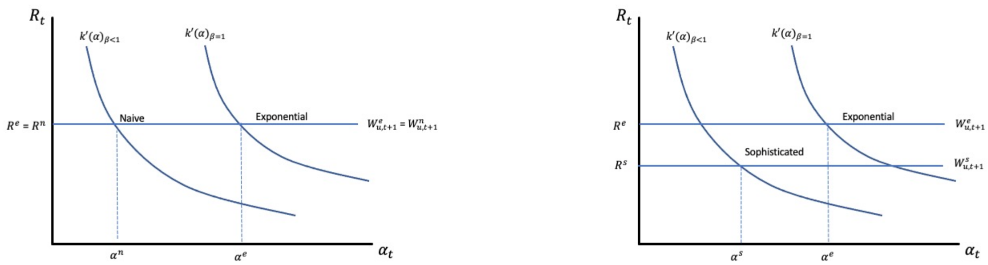

Figure 1.3.1. Equilibrium Search Effort and Reservation Wage

1.4 Optimal Unemployment Insurance Policy

1.4.1 Baseline model

I consider an optimal UI policy for the baseline model. This specification is consistent with Shavell and Weiss [1979] and Hopenhayn and Nicolini [1997]. In the next subsection, I will pursue an optimal policy when people can consume different amounts by saving their income as in Shimer and Werning [2008] and Spinnewijn [2015a].

To derive an optimal policy, I define the social planner's problem. The goal of the planner is to maximize the job seeker's utility given an exogenously specified budget. The objective function of the maximization is not obvious, however, because the environment in this model is a game between different selves due to the time-inconsistent preference. Even with a representative agent, selves at different times may advocate distinct allocations. Following others, I assume that the planner is paternalistic.[9] That is, the planner's utility is identical to the long-term utility ($\beta = 1$), which will deliver an 'Intergenerationally Pareto' allocation in Feng and Ke [2018].[10] The problem of the planner's utility

[9] An alternative social welfare criterion I can consider is a 'sympathetic' planner. A sympathetic planner has the same preference with the current incarnation of self. That is, the planner also has present bias. Since the planner and the agent agree on the value of the current UI benefit, there's no preference gap in the first-order condition. What makes this criterion different from the exponential model is the gap between the discounting factors in the preference and the expenditure function. In the appendix B.1.1, I show that the optimum UI policy is also a decreasing sequence.

[10] The planner's preference$(\succsim_t)_{t\in T}$ is 'Intergenerationally Pareto' if, for any consumption sequence $\{c_\tau\}_{\tau=0}^\infty, \{\tilde{c}_\tau\}_{\tau=0}^\infty$, in each period, $\{c_\tau\}_{\tau=0}^\infty \succsim_{i,s} \{\tilde{c}_\tau\}_{\tau=0}^\infty$ for all i and for all $s \geq t$ implies $\{c_\tau\}_{\tau=0}^\infty \succsim_t \{\tilde{c}_\tau\}_{\tau=0}^\infty$. Adjusting the definition to the environment of this chapter, I can drop subscript i. Completely aligning the planner's preference with the self t's preference achieves current generation Pareto allocation. In that sense, the preference is dictatorial.

maximization can be stated as follows.

$$\max_{z_t} u(z_t) - k(\alpha_t) + \alpha_t\delta\left(\int_{\underline{w}}^{R_t} W_{u,t+1}dF(w) + \int_{R_t}^{\overline{w}} \frac{u(w)}{1-\delta}dF(w)\right) + (1-\alpha_t)\delta W_{u,t+1} \tag{1.4.1}$$

$$\text{s.t., } \overline{E}_t = \sum_{n=0}^{\infty}\left(\delta^n z_{t+n}\prod_{k=0}^{n-1}(1-p_{t+k})\right)$$

where α_t and R_t are the equilibrium objects that come from the individual optimization, p_t $(:= \alpha_t(1-F(R_t))$ is the probability of a successful matching at time t, and $\overline{E}$ is an exogenous UI budget.

If the planner can observe the matching probability p_t, then the planner can implement a constant UI policy and it can be optimal since the planner can condition the policy on the probability. Therefore, the optimal is going to be identical to Shavell and Weiss [1979]. To make the problem interesting, I assume that the planner cannot observe the search effort level nor the reservation wage hence the probability of matching private information, and it is subject to the UI policy. Shavell and Weiss [1979] showed that the constrained efficient UI policy should be a decreasing sequence over the unemployment spell if the search effort and the reservation wage are private information. This conclusion also survives under the present bias.

Proposition 1.4.1. *If the search effort and reservation wage are private information, then the optimal unemployment benefit for a present biased agent is a decreasing sequence.*

Present bias doesn't change the direction of the optimum UI distribution over the unemployment spell. However, the following comparative statics may provide some insights. Consider the following first-order condition of the planner's problem:

$$\frac{u'(z_{t+1})}{u'(z_t)} = \frac{\overbrace{\delta(1-p_t)}^{\text{consumption smoothing}} - \overbrace{\frac{\partial p_t}{\partial z_{t+1}}E_{t+1}}^{\text{UI inefficiency}}}{\underbrace{\delta(1-p_t)}_{\text{consumption smoothing}} - \Omega} \tag{1.4.2}$$

where $\Omega := -\frac{k'(\alpha_t)-\delta\int_{R_t}^{\bar{w}}(W_{w,t+1}-W_{u,t+1})dF(w)}{u'(z_{t+1})}\frac{\partial\alpha_t}{\partial z_{t+1}} \leq 0$.

Observe an exponential problem (as in Shavell and Weiss [1979]) is nested if $\beta = 1$ (hence Ω in equation (1.4.2) is zero), where the trade-off is just between the consumption smoothing ($\delta(1-p_t)$) and the UI inefficiency ($\frac{\partial p_t}{\partial z_{t+1}} E_{t+1}$). UI is intrinsically inefficient even without a present bias because the next period UI harms search incentives ($\frac{\partial p_t}{\partial z_{t+1}} < 0$). A higher UI makes the agent pickier (raising reservation wage) and suppresses job-searching effort. Because of the UI inefficiency, the planner thinks that the future UI benefit is costly. This is the key mechanism of the decreasing sequence in Shavell and Weiss [1979].

What makes the condition different from the exponential case is $\Omega \leq 0$ in the equation (1.4.2). It stems from the gap between the agent's and the planner's valuation of the continuation. The planner disagrees with the agent on the value of additional search effort. $k'(\alpha_t)$ in Ω is the agent's valuation on an additional search effort, whereas $\delta \int_{R_t}^{\bar{w}} (W_{w,t+1} - W_{u,t+1}) dF(w)$ is the planner's.[11] It is obvious that Ω is negative. Therefore, the existence of present bias steepens the optimal UI schedule.

However, other offsetting effects may flatten the schedule because Present bias affects all other parts in equation (1.4.2). For example, consider the job-getting probability (p). Present bias implies lower job finding probability and prolonged unemployment periods, hence the present bias strengthens the planner's consumption smoothing motive. Second, the effect of reservation wage change diminishes with the present bias.

Summing up, the overall direction is ambiguous a priori. The optimal UI should be a decreasing one for both the present biased and exponential agents, but the relative speed of the convergence toward zero is not definitive. I estimate the parameters and simulate the optimal policy to empirically derive the optimal policy in the later section.

1.4.2 With savings

With saving technology and an exponential discounting CARA utility, Shimer and Werning [2008] concluded that the optimal policy is a constant sequence. Spinnewijn [2015a] extended the results further by adding optimistically biased beliefs on the probability of

[11] Note that the preference gap contains two parts. One is from the present bias, and the other one is optimism (naivete) about her bias. If I denote $Q = \beta\delta \int (W_w - W_u) dF$, then the preference gap is $Q(\beta, \hat{\alpha}, \hat{R}) - Q(\beta, \alpha, R) + Q(\beta, \alpha, R) - Q(\beta = 1, \alpha, R)$, and the first difference is the 'optimism' part, and the second difference is the 'present bias' part.

getting a job. Interestingly, the optimal UI with the wrong belief is increasing through the unemployment spell. Now I derive the optimal policy with savings under present bias and compare the results with those of the two papers.

To get a sharper result, I follow the common practice: using the CARA utility function and degenerating the wage distribution. CARA utility makes the problem tractable since consumption policy is linear with the asset holdings. Specifically, the flow utility is $u(c, \alpha) = -\exp(-\gamma(c - \alpha))$. Note that the search cost is integrated into the utility function. A degenerate wage distribution lets us drop the reservation wage from our model.

Denote the probability of getting a job as a generic function of search effort level $p(\alpha)$, which is linear. This is in line with the baseline model ($p(\alpha, R) = \alpha(1 - F(R))$) because now $F(R)$ is dirac in this case. Also as in the previous chapter, I assume an exogenous UI budget $\overline{E}$. Consider a constant UI which can be stated as follows:

$$
\begin{aligned}
V_{u,t}(x_t) =& \max_{\alpha_t, s_t} u(c_t^u, \alpha_t) + \beta\delta p(\alpha_t)\frac{u(c_{t+1}^e, 0)}{1-\delta} + \beta\delta(1 - p(\alpha_t))W_{u,t+1}(x_{t+1}), \\
\text{s.t., } c_t^u =& z_t + \frac{r}{1+r}x_t + s_t \\
c_{t+1}^e =& w + \frac{r}{1+r}x_t - rs_t \\
x_{t+1} =& x_t - (1+r)s_t \\
\overline{E}_t =& \sum_{n=0}^{\infty}\left(\delta^n z_{t+n}\prod_{k=0}^{n-1}(1 - p_{t+k})\right)
\end{aligned}
$$

where $V_{u,t}(x_t)$ is the value of the unemployed state when the job seeker holds asset x_t, s_t is the amount of liquidation of the asset at the unemployed state, and r is the risk-free interest rate. Once she gets the job, she doesn't have to put in the search effort, hence I use $u(c_{t+1}^e, 0)$ for the employed state utility. Note that the difference with the baseline model is now the consumption streams depart from UI benefit and wage. The consumptions come from the following facts. With the asset holdings x_t, she would like to split it across time identically to smooth her consumption. Using $(1 + r)\delta = 1$, the current value of the sum of the splits (Δ) can be stated as follows:

$$\frac{1}{1-\delta}\Delta = x_t$$
$$\Delta = \frac{r}{1+r}x_t.$$

This is in the first two constraints. Next, once Δ is spent, the next period $x_{t+1} = x_t$ because only the interest rx_t is spent at period t. Further, if the unemployed liquidate the asset additionally and use s_t for the current consumption, then it will decrease her asset holdings for the next period as $(1+r)s_t$. This is the third constraint. Assume she is employed in the next period. Then in the employed state, $x_{t+1} = x_t-(1+r)s_t$ multiplying $\frac{r}{1+r}$ gives the second constraint. The continuation value in the equation is defined as

$$\begin{aligned}
W_{u,t+1}(x_{t+1}) = \max_{\hat{\alpha}_{t+1},\hat{s}_{t+1}} \; & u(\hat{c}^u_{t+1}, \hat{\alpha}_{t+1}) + \delta p(\hat{\alpha}_{t+1})\frac{u(\hat{c}^e_{t+2}, 0)}{1-\delta} + \delta(1-p(\hat{\alpha}_{t+1}))W_{u,t+2}(\hat{x}_{t+2}),\\
\text{s.t., } \hat{c}^u_{t+1} =& z_{t+1} + \frac{r}{1+r}x_{t+1} + \hat{s}_{t+1}\\
\hat{c}^e_{t+2} =& w + \frac{r}{1+r}x_{t+1} - r\hat{s}_{t+1}\\
\hat{x}_{t+2} =& x_{t+1} - (1+r)\hat{s}_{t+1}
\end{aligned}$$

where all $\hat{\cdot}$ variables indicate the perceived one by the period t agent. With CARA utility, constant UI is sufficient to have a constant search effort over the unemployment spell. Therefore, the stationary continuation value W_u can be described as following Lemma.

Lemma 1.4.1 (Spinnewijn, 2015)**.** *The stationary continuation value of being unemployed under a constant UI policy can be expressed as follows:*

$$W_u(\hat{\alpha}, \hat{s}; \overline{E}, x) = \frac{u(z+\hat{s}-\hat{\alpha}) + \frac{\delta}{1-\delta}p(\hat{\alpha})u(w-r\hat{s})}{1-\delta(1-p(\hat{\alpha}))\exp(r\gamma\hat{s})}\exp\left(-\frac{r}{1+r}\gamma x\right) \tag{1.4.3}$$

where $u(c,\alpha) = -\exp(-\gamma(c-\alpha))$, and r is the risk free interest rate, x is the asset holdings, s is the liquidation of the asset, and $\overline{E}$ is the exogenous UI budget. All variables with $\hat{\cdot}$ imply perceived values by the current period agent.

The following two first-order conditions for the perceived actions come from the

Lemma 1.4.1.

$$\left(\frac{\partial W_u}{\partial \hat{\alpha}}\right): \ \hat{\beta}\delta \exp(r\gamma\hat{s})p'\left(\frac{u(w)}{1-\delta} - W_u(\hat{\alpha}, \hat{s}; x=0, \overline{E})\right) = u'(z+\hat{s}-\hat{\alpha})$$
$$\left(\frac{\partial W_u}{\partial \hat{s}}\right): \ \frac{u'(z+\hat{s}-\hat{\alpha}) - u'(w)}{u'(z+\hat{s}-\hat{\alpha})} = \frac{1}{p(\hat{\alpha})}\left(1 - \frac{1}{\exp(r\gamma\hat{s})}\right)$$

We move to the current period problem with the above perception. The agent chooses the actual search effort and liquidation for the current period as follows. The current problem with a sufficient[12] initial wealth x_t is as follows:

$$\begin{aligned} V_{u,t}(\alpha_t, s_t; \hat{s}_{t+1}, \overline{E}_t, x_t) =& u\left(\frac{r}{1+r}x_t + (1-\delta(1-p))\overline{E}_t + s_t - \alpha_t\right) \\ &+ \beta\delta\exp(r\gamma s_t)\left(p\frac{u\left(\frac{r}{1+r}x_t + w\right)}{1-\delta} + (1-p)W_u(\hat{\alpha}_{t+1}, \hat{s}_{t+1}; \overline{E}_{t+1}, x_{t+1})\right) \end{aligned}$$

With present bias, I derive how the agent searches for a job and how she liquidates the asset in the following proposition.

Proposition 1.4.2. *An agent's search effort depends on the perceived present bias ($\hat{\beta}$), not the actual present bias (β). Further, the present biased agent spends more of her wealth than an exponentially discounting agent, and because of the optimistic belief about her bias, she expects less liquidation from the next period.*

Now we move our focus to the optimal policy. The planner's problem is again maximizing the social welfare given an exogenous budget. Start with the constant UI described above. Assume further that the level of the constant UI and possibly associated budget are the same as the optimal level for an exponential discounting agent as in Shimer and Werning [2008] or Spinnewijn [2015a], and the job seekers believe no present bias ($\hat{\beta} = 1$) so the constant scheme of the exponential model is optimal in the perceived future problems. I check whether the planner can increase social welfare by changing the level of future UI, keeping the total expected expenditure the same. The social welfare function is the long-term utility as in the previous example. The following equation as well as the decision rules for the perceived and actual choices (equation (1.3.1)) and the individual

[12] $\frac{x_t}{1+r} - \hat{s} \geq s$

rationality constraints (equation (1.3.2)) all together define the planner's problem.

$$\begin{aligned}\max_{\{z_t\}} \mathcal{W}_t &:= u(z_t + s_t - \alpha_t) \\ &\quad + \delta \exp(r\gamma s_t) \left(p(\alpha_t) \frac{u(w)}{1-\delta} + (1 - p(\alpha_t))\mathcal{W}_{t+1}(\hat{\alpha}_{t+1}, \hat{s}_{t+1}, E_{t+1}) \right) \text{ s.t.,} \\ \overline{E}_t &= z_t + \delta(1 - p(\alpha_t))E_{t+1}\end{aligned}$$

The first order condition for an optimal UI is,

$$\begin{aligned}\frac{d\mathcal{W}_t}{dE_{t+1}} =& u'(z_t + s_t - \alpha_t)\frac{\partial z_t}{\partial E_{t+1}} \\ &+ \delta \exp(r\gamma s_t)(1 - p(\alpha_t))\frac{\partial \mathcal{W}_{t+1}}{\partial E_{t+1}} + \frac{\partial \mathcal{W}_t}{\partial \alpha_t}\frac{\partial \alpha_t}{\partial E_{t+1}} + \frac{\partial \mathcal{W}_t}{\partial s_t}\frac{\partial s_t}{\partial E_{t+1}}\end{aligned}$$

Without present bias, the last two terms in the first-order condition are zero with the envelope theorem. The remaining part in the equation, the first order effect of changing $\overline{E}_{t+1}$ is again zero because of the optimality of the UI scheme. With present bias, it is no longer the case.

Proposition 1.4.3. *If the agent is present biased, the optimal UI scheme critically depends on the behavior of asset liquidation. If the future benefit causes a larger asset liquidation, a decreasing UI is optimal. An if and only if condition for the optimum to be decreasing is as follows:*

$$\left(-\frac{\beta p'}{1 - p(\alpha)} + u''_z \right) \frac{\partial z_t}{\partial z_{t+1}} > \beta\delta \exp(r\gamma s_t) p''(\alpha_t) \frac{\partial \alpha_t}{\partial z_{t+1}} \left(\frac{u(w)}{1-\delta} - W_{u,t+1} \right)$$

In the problem we consider where the matching probability p is linear to α, the condition is trivially satisfied, hence the decreasing UI is optimal. Note that this conclusion does not overturn Spinnewijn [2015a]. First, the problem is different. Spinnewijn [2015a] considers budget balanced increase of UI by raising the benefit and tax at the same time. On the other hand, I change the UI benefit only keeping the exogenous budget at the same level. By doing so, I consider the effect of temporal distribution of UI benefit only, which is the original formulation of Shavell and Weiss [1979]. Second, a UI benefit increase in the next period implies a decrease in the current benefit. Because of this, the asset depletion rate can be higher when the planner increases the future benefit. The conclusion in Proposition 1.4.3, conditioned on the liquidating behavior, comes from this fact.

1.5 Simulation

The theoretical results rely on the assumption that people are present biased in the job-searching domain. DellaVigna and Paserman [2005a] and Paserman [2008] show compelling evidence of $\beta < 1$. In this section, I bring Korean labor market data to the baseline model and estimate the discount factor. I then simulate the optimal policy for the present biased workers and compare it with the exponential model. This is interesting because I couldn't find a definitive conclusion about the shape of UI in the previous section because of the forces that move in opposite directions. Both the exponential and present bias support the decreasing policy, but it is not clear which one converges faster to zero. In this section, I present one exemplary answer to the question with a calibration exercise. Finally, I compare the associated welfare and discuss the relevance of the optimality.

1.5.1 Parameter estimation

To match the data to the model in section 1.3, I need a few assumptions on the functional forms. I use log utility and assume further that the wage distribution follows a log-normal distribution as Paserman [2008]. I also adopt Paserman [2008]'s specification for the search cost function $K(\alpha) = k_0 \cdot \alpha^\eta$ with $\eta = 1.4$. The parameter estimation procedure follows other studies such as Flinn and Heckman [1982], Wolpin [1987], and van den Berg [1990].

The data that I use for the estimation is Korean Labor and Income Panel Study (KLIPS) Data. It contains around 5,000 households and 11,000 individuals, and the survey was performed annually from 1998 to 2020. The job history dataset shows that there are 244,560 observations.[13] I compute the length (weeks) of each unemployment spell. Following convention, I treat two distinct unemployment spells of an individual as distinct observations. I consider only working-aged (25-60) observations and dropped wage outliers (top and bottom 1%). This leaves 6,826 unemployment spells from 4,821

[13]This includes a continuation of a job, a new layoff, continuation of unemployment, and getting a new job.

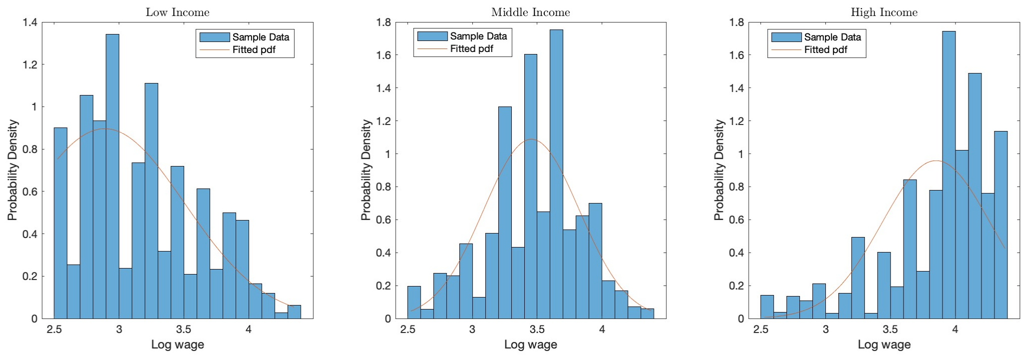

Figure 1.5.1. Wage Distribution Estimation

distinct individuals. Finally, I split the sample into 3 subgroups using the previous job's wage level to control unobservable characteristics.

I make two important identifying assumptions; one is that the agent is fully naive ($\hat{\beta} = 1$). This makes the current period's short-term discount rate identifiable and gives much simpler optimal policy conditions. With this data type, it is generally impossible to separate an actual short-term discount factor from a 'perceived' one. The other assumption that I make is an exclusion assumption. The wage distribution differs among job seekers by the following observed characteristics: marriage status, gender, and whether they get an unemployment insurance benefit. The search cost function parameter, however, depends only on marriage status and gender. Finally, the discounting parameters are identical for all individuals regardless of the individual characteristics. Therefore, the parameters that I shall estimate are two discounting factors (δ, β) that apply to every individual within a wage group, wage distribution (μ, σ), and search effort cost parameters (k_0) for each observable characteristics in a wage group.

The actual implementation of the estimation consists of two steps. First, I estimate the truncated log-normal distribution from the (observed) wage data. Using the first stage estimates, I construct likelihood functions for each wage group and maximize the functions with 6 parameters. The standard errors for the second stage estimate are computed with 50 bootstrap repetitions. Figure 1.5.1 shows the first-stage estimation results. The three groups indicate previous wage levels (low, middle, high), and the red solid line is the fitted log-normal distribution.

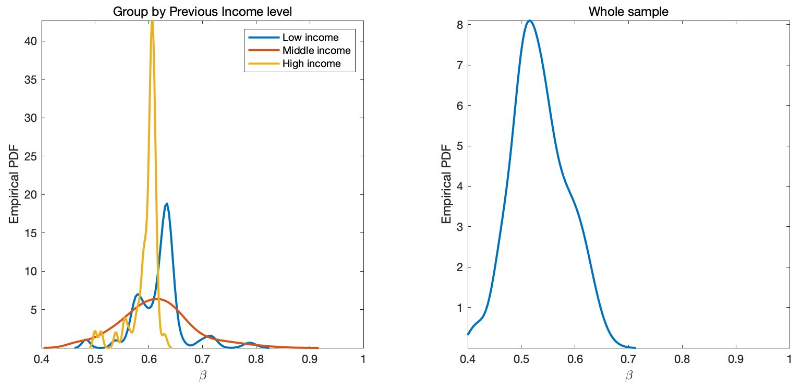

Figure 1.5.2. Kernel Density of Bootstrapped β Estimates

Given the first stage results, I get the second stage estimation result and it is reported in Table 1.5.1. The parameter of our primary interest is β. Although it varies among income groups, the estimates are in the range of 0.5-0.7. Considering the bootstrap standard errors, I can reject a hypothesis of time consistency ($\beta = 1$). The result in the table and the kernel density graph in Figure 1.5.2 shows that the β estimates are not very different among the income groups. The long-term discounting rate is very close to 1 in all income groups. The search cost parameter doesn't show a meaningful tendency across groups.

Table 1.5.1: The Second Stage Estimation Result

	Low	Middle	High	Total
β	0.638	0.582	0.603	0.507
δ	0.999	0.997	0.999	0.984
$k_0(M, U)$	1112.1	465.54	1519.1	91.751
$k_0(M, m)$	1859.7	581.06	1378.7	106.65
$k_0(F, U)$	4-3358.68	592.53	1566.8	71.279
$k_0(F, m)$	1301.1	570.87	982.82	98.566

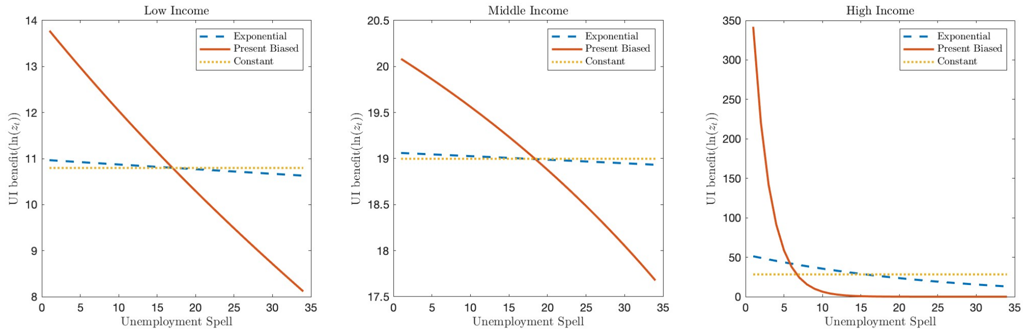

Figure 1.5.3. Optimal UI

1.5.2 Simulation and welfare comparison

Equipped with functional forms and parameter estimates, I simulate the optimal UI for each income group. In Korea, unemployment insurance covers 60% of the previous wage, and the benefit is paid for 120-240 days depending on the length of the previous employment. To operate the simulation, I averaged the search cost parameter estimates (k_0) and assumed that our agents had worked for more than 10 years. Then, the total amount of unemployment benefits for our hypothetical unemployed person is $3,671$ thousand KRW for the low-income, $6,459$ thousand KRW for the middle, and $9,595$ for the high-income groups, respectively.[14]

In Figure 1.5.3, I plot the model implied optimal UIs and compare them with exponential optimal as well as a constant UI policy. The 'constant' is the current policy in Korea, the 'exponential' is the policy that the planner will choose if there's no present bias, and the 'present biased' is the best policy when there is present bias. Each policy is tailored to satisfy the same budget constraint. Therefore, in each comparison, the three UI schemes are expected to spend the same amount of money.[15]

Notice that the two optimal policies (exponential and present biased) exhibit a decreasing sequence as the unemployment spell increases in all income groups. Yet, the exponential optima are very close to the constant, while present biased optima decrease toward zero much faster. This indicates the direct effect of present bias (the preference

[14]For example, the total UI expenditure for the low-income group is the average wage (179.9k KRW)$\times$ UI payment periods 34 weeks $\times$ Replacement rate 60%.

[15]The budget constraint is the expected discounted sum of expenditure which is precisely defined in the previous chapter as an objective function of the minimization problem(equation 1.4.1).

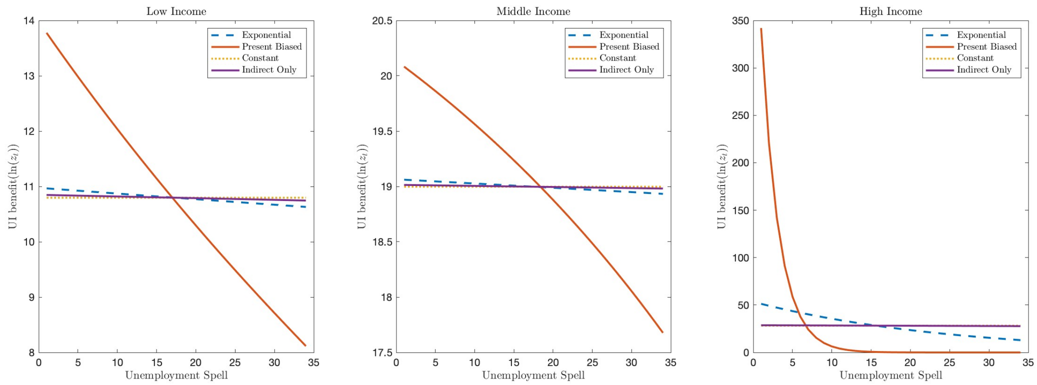

Figure 1.5.4. Decomposition of Effects

gap) in the previous section overwhelms all counteracting indirect effects. I can decompose further by separating the direct and indirect effects (Figure 1.5.4). The purple solid lines are the optimal UIs when I mute the direct effect. They are almost nondistinguishable with the constant scheme. This confirms that most of the action comes from the direct effect.

To better understand the level of β and the UI slope, I plot the optimal UI scheme by changing β using the middle-income group simulation result (Figure 1.5.5). $\beta = 1$ is the exponential case by definition. As I decrease β (hence increase present bias), the slope of optimal UI increases. The slope is the steepest at around $\beta = 0.7$ and flattens as we further go down toward 0.3. The estimated β around 0.6, therefore, generates a fairly steep optimum UI as we have seen in the previous figures.

Now I compare the welfare of each policy option by showing the expected expenditure equivalence to achieve the same level of welfare. That is, I quantify how much more UI budget is necessary to achieve the optimum welfare if the planner sticks to the constant plan. That critically depends on how the optimal policy differs from the constant plan. If the agent doesn't have a present bias, then the optimal policy is fairly similar to the constant, so the welfare gain is not huge. On the other hand, if the agent is present biased, then the optimal policy is quite different with constant as we have seen in Figure 1.5.3. Therefore, if the planner uses the constant policy, he will need much more budget to achieve the optimal welfare level. Table 1.5.2 summarizes the results. For example, for

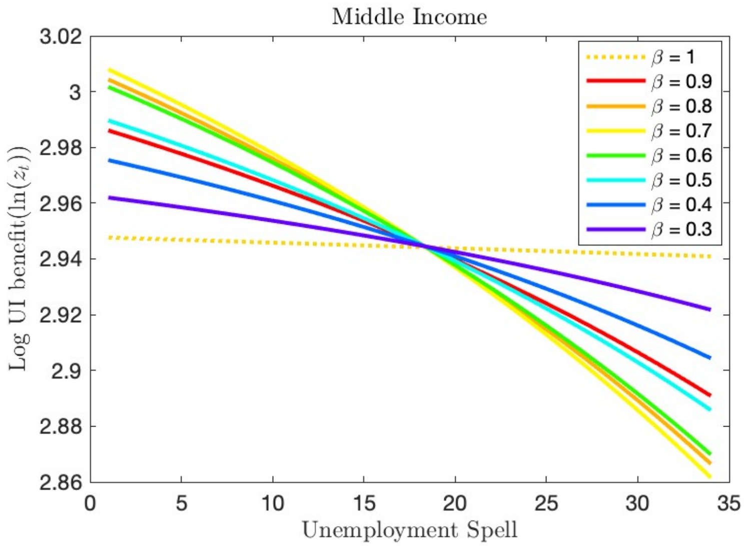

Figure 1.5.5. Comparative Statics

the low-income group, if the agent is present biased, a constant UI requires 5.6% more budget to achieve the welfare of the optimally decreasing UI. If the agent is exponential, the gap is only 0.4%. The high-income group exhibits a striking result. A constant UI will use 47.7% more budget than the optimally decreasing UI to achieve the same welfare level if the agents are present biased as I have estimated.

Table 1.5.2: Additional UI Budget Requirement with a Constant Policy

	Low	Middle	High
Exponential	1.004	1.001	1.154
Present biased	1.056	1.012	1.477

1.6 Concluding Remarks

This chapter derives the optimal policy rules for the present biased agent and compares the welfare of different policies. Intuitively, it is tempting to argue that the present bias leads to a steeper UI. A present biased agent will search inefficiently low, so the fast decreasing UI can be a remedy for the inefficiency. However, the validity of the intuition depends on the level of present bias as well as the possibility of holding an asset. If there are no asset holdings, and if the agent's present bias is moderate, then the optimal policy is a steeper one. It turned out that this is the case for the Korean labor market data. On the other hand, at least in principle, the optimal policy may be flatter than the exponential workers if the workers are extremely present biased. These highly present biased workers experience low wage offer arrivals, so UI inefficiency is not an imminent problem. What matters most is letting them mitigate the negative income shock caused by unemployment. As we have seen in the simulation, the extent of the bias governs the slope of the optimal policy. In addition to that, if the agent can accumulate assets, then the optimal policy critically depends on the asset liquidation behavior. Under a linear matching probability and full naivete, the optimal policy can be a decreasing sequence.

Chapter 2

Forward Guidance and Heterogeneous Awareness

2.1 Introduction

Forward guidance, a central bank suggesting a specific policy rate path, can convert the agent's expectations and affect current output. It is particularly useful if the economy is in the zero lower bound since the nominal interest change can be limited in such a situation (Eggertsson and Woodford [2003]).

As Del Negro et al. [2023] pointed out in their earlier version of the paper, the effectiveness of forward guidance in an estimated DSGE model is remarkable. In fact, the textbook New Keynesian model's prediction is too powerful to be intuitively appealing or to be supported by data.[1] The reason for this 'puzzling' excessive response in the model is as follows: First, any future real interest rate change has the same effect on current consumption. Second, the reaction of the inflation becomes larger as the expected interest rate change is far away because the inflation responds to the cumulative consumption changes. Finally, the forward guidance under the zero lower bound is essentially an interest rate peg, which makes the solution of the system explosive.[2]

[1] Del Negro et al. [2023] find out that maintaining the federal fund rate at 25bp for 12 quarters increases quarterly real GDP by 9%, which is 30 times larger than the actual response in their data. Carlstrom et al. [2015] observed that the forward guidance can be seen as exogenous interest rate pegs, and the New Keynesian model's predictions are sensitive to the duration of the peg. As the duration increases, the reaction of the current output explodes.

[2] One of the eigenvalues of the solution matrix to be outside of the unit circle.

To resolve this puzzle, we build a simple New Keynesian model where consumers have heterogeneous awareness. The key assumption is that only a fraction of people are aware of a particular shock. In our example, there are two shocks, TFP and monetary policy. The TFP is shared by all agents, but some consumers are not aware of the monetary policy shock. This heterogeneity in the awareness of the shock makes our model depart from the rational expectation equilibrium since agents with different awareness perceive the structure of the economy and market clearings differently. We first focus on the 'temporary equilibrium' as in Woodford [2013], and Farhi and Werning [2019]. In this equilibrium, the heterogeneous awareness generates a discounting factor in the higher order expectation as in Angeletos and Lian [2018]. As the horizon of the forward guidance increases, agents are required to think over higher-order expectations, and the effectiveness of the guidance diminishes because of the discounting factor.

We also compare our model with two standard models: a homogeneous unaware model, and a homogeneous aware model. These homogeneous cases correspond to the rational expectation equilibrium New Keynesian models with one shock and two shocks, respectively. Comparative statics suggest that the effect of forward guidance is larger in the heterogeneous awareness model than in the homogeneous aware model (fully aware case). Compared to the homogeneous unaware model (fully unaware case), our model exhibits a weaker reaction of the output on the forward guidance.

We then inspect a central bank's incentive to raise the awareness of the consumers. Under the assumption that the aware consumer's interpretation of the forward guidance is aligned with the central bank's intention, the central bank may have an incentive to raise awareness. If there are not many people who are aware of the monetary policy shock, then the bank will not announce the future change at all hence the marginal increase of awareness has no effect. On the other hand, if there are enough people who are aware of the shock, then it is beneficial to make the unaware consumers aware of the monetary policy shock.

Finally, we present a reflective equilibrium, where the unaware consumers can revise their model so that they can rationalize the observed aggregates. What is missing in the temporary equilibrium is the equilibrium condition of the belief. In the standard New Keynesian model, rational expectation equilibrium requires the expected path of endogenous variables to coincide with an actual realization at every period. In the temporary equilibrium, we don't require the belief to be consistent with the actual realization. Initial beliefs are exogenous, and it is updated when there's additional information (such as an announcement from the central bank). The updated belief changes individual action and action-belief feedback continues within one's (possibly misspecified) model. This can be seen as level ∞ equilibrium in Farhi and Werning [2019] or level ∞ reflection in Woodford [2013] with a partial awareness model. After the convergence of action-belief feedback, the current actions add up to aggregates and prices clear the markets, but the aggregates need not be the same as the updated beliefs. This is the biggest difference with level-k or reflective equilibrium. Because of the correct model specification, both equilibria converge to the rational expectation equilibrium as the feedback goes to infinity. In a partial awareness model, it converges to something else. In the temporary equilibrium, we allow the discrepancy between the converging point and the realized aggregate. In the reflective equilibrium, the belief should be consistent with the observed current aggregates. In this equilibrium, the unaware consumers discover a dummy sequence, which essentially corresponds to the missing unaware shocks of the economy, and recovers the rational expectation equilibrium result.

2.2 Literature

To deal with the 'forward guidance puzzle', many resolutions have been proposed. One direction is introducing an idiosyncratic income shock and an incomplete financial market. Most notably, McKay et al. [2016] and McKay et al. [2017] showed that the two assumptions generate a discounting intertemporal Euler equation, a less forward-looking IS relation. The uninsurable income risk weakens the intertemporal substitution with a precautionary savings motive and the possibility of hitting the binding financial constraint in the future limits the agent's planning horizon. Werning [2015b] on the other hand, expounded that the incomplete market itself may not change how the consumption reacts to the future interest rate. With a vanishing liquidity assumption, the intertemporal Euler equation does not discount future real interest rates even under the incomplete market. This 'neutral benchmark' result comes from the fact that the income risk and liquidity in his model are acyclical. Acharya and Dogra [2020] confirmed Werning [2015b] with CARA utility and Normal distribution. They derived an Euler equation that discounts the future if the income risk is pro-cyclical. At the same time, they derive an explosive Euler equation with a counter-cyclical income risk.

As Werning [2015b] and Farhi and Werning [2019] pointed out, the cyclicalities of the shock and liquidity are endogenous. Hagedorn et al. [2019] found that the forward guidance puzzle could either disappear or worsen depending on the primitives of the model; the distribution of income, profits, and tax policies. If the redistribution is from high MPC households to low MPC households, then forward guidance is less effective in incomplete market models. If the distribution works in the other direction, then the incomplete market would exacerbate the puzzle.

Another strand of resolution is relaxing the strong assumption on the standard equilibrium concept in the New Keynesian model, namely 'full information rational expectation' (FIRE). Angeletos and Lian [2018] gave up the first half, full information assumption. Specifically, by removing common knowledge of the news (announcement of the central bank), they introduced a higher-order uncertainty in the aggregate action. This information friction attenuates general equilibrium effects in the Euler equation and causes the agents to react to the news as if they were myopic. Gabaix [2020] also proposed a myopic agent, but in a different mechanism. The agent does not fully understand the world, especially events that are far into the future. Using cognitive discounting, he gets a discounting Euler equation. The difference from Angeletos and Lian [2018] is that myopia comes from relaxing the second part of the equilibrium concept, the rational expectation, not the first half. García-Schmidt and Woodford [2019] and Farhi and Werning [2019] both adopted bounded rationality, relaxing the rational expectation. García-Schmidt and Woodford [2019] suggested 'reflective equilibrium': separating a temporary equilibrium (level-k reasoning) and a reflection from aggregates, which are assumed away in a rational expectation. The reaction to the change in nominal interest rate is muted if the degree of reflection recursion is low, especially at the beginning of the reaction. Similarly, Farhi and Werning [2019] also adopts level-k reasoning, and adds an incomplete market assumption. They showed that each of the assumptions, bounded rationality, and incomplete market, is not enough separately. The interaction of the two assumptions, however, generates a desired much-muted reaction of the current output.

A closely related literature that we would like to draw a line is studies on the signaling effect of monetary policy and optimal transparency. Campbell et al. [2012] empirically investigated whether the reaction to the forward guidance is aligned with the central bank's intention. They distinguish the forward guidance into two types: Delphic, a central bank's forecast of the future economic activity and an expected monetary policy reaction, and Odyssean, the bank's commitment to a nominal interest rate path. In the data, the FOMC forward guidance was successful in delivering the intention of the central bank, indicating the forward guidance was Odyssean. This justifies the usage of the forward guidance as a policy tool in the zero lower bound. On the other hand, Andrade et al. [2019]'s result is mixed. The two interpretations (Delphic and Odyssean) coexisted in the data. They further showed that the powerful reaction of forward guidance may be counteracted by the pessimistic (Delphic) agents. Broadly, our paper touches on the reaction to the future news, a question explored by Coibion and Gorodnichenko [2015], Bordalo et al. [2020], Maćkowiak and Wiederholt [2009], and Kohlhas and Walther [2021].

Baeriswyl and Cornand [2010] also pointed out that the monetary instrument takes on a dual stabilizing role. Focusing on the role of central bank announcements as a public signal, they investigated the welfare implications of the transparency of the monetary policy. Their conclusion echoes Morris and Shin [2002] and Angeletos and Pavan [2007]. Public information generally does not necessarily improve firms' coordination; rather, its effect depends on how it interacts with the policy action. Cornand and Heinemann [2008] also seeks optimal transparency, or provision of the public signal, in a very similar setting with Morris and Shin [2002]. By distinguishing the accuracy of a signal and a provision of it, they derive a conclusion that a central bank should limit the degree of publicity rather than the precision of information.

Lastly, our paper has a connection to the studies on a misspecified model. Woodford [2010a] introduced a concept, 'near rational' equilibrium. An agent may have a different assessment of the distribution of the state. The difference between the agent's assessment and the actual distribution is measured with relative entropy (KL distance), and a robust policy is defined as a policy that minimizes the maximum of the loss function given an entropy constraint. Woodford [2013] reviewed different equilibrium concepts departing from rational equilibrium with a New Keynesian model. In a similar vein, Esponda and Pouzo [2016] establishes the 'Berk-Nash' equilibrium. Each player has a subjective model, a set of probability distributions over the consequences of the action. The subjective model may be misspecified, meaning that the set may not include the objective distribution. Then the Berk-Nash equilibrium is defined as a strategy profile that is optimal and minimizes the (K-L) distance. Fudenberg et al. [2021] investigated the learning dynamics of the Berk-Nash equilibrium showing that only uniform Berk-Nash can be a long-run outcome. Molavi [2019] built a general equilibrium model with the possibility of model misspecification and proposes constrained rational expectations equilibrium, which is Berk-Nash equilibrium in a dynamic model.

2.3 Model

We consider a dynamic economy with consumers, producers, and a central bank. Time is discrete and indexed by $t = 0, 1, ...$ There is a continuum of consumers I whose measure is standardized to 1. The consumption of consumer $i \in I$ in period t is denoted by $c_{i,t} \in \mathbb{R}_+$. Her labor supply at period t is denoted by $n_{i,t} \in [0, 1]$. Consumer i's utility at period t is $U(c_{i,t}, n_{i,t}) := \frac{(c_{i,t})^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} - \frac{1}{1+\psi} n_{i,t}^{1+\psi}$ with $\gamma > 0$ being interpreted as the elasticity of intertemporal substitution and $\psi > 0$ being the inverse of Frisch elasticity as in Woodford [2010b].

There are two types of producers. One is an intermediate good producer, and the other is a final good producer. There is a continuum of intermediate good producers denoted by J, and they are normalized to measure 1. Each intermediate good producer is a monopolist. It hires labor and produces its specialized product using a CRS technology given by $y_{j,t} = \exp(z_t) \cdot n_{j,t}$, where $y_{j,t}$ is the intermediate good that is produced by firm $j \in J$ at period t, $\exp(z_t) \in \mathbb{R}_+$ is the (common) productivity, and $n_{j,t} \in [0,1]$ is the labor input hired by firm j. There is a representative final good producer who buys intermediate goods and combines them as a final good. Y_t represents the final good at period t, and the technology is the Dixit-Stiglitz aggregator $Y_t = \left(\int_0^1 (y_{j,t})^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ where $\varepsilon < 1$ is the elasticity of the substitution.

Finally, there is a central bank that sets the nominal interest rate $R_t \in \mathbb{R}_+$ for every period t via a Taylor rule which will be specified later.

2.3.1 Shocks and awareness

There are two shocks in the economy. One is a TFP shock (z_t) in the intermediate good production function, and the other is a monetary policy shock (v_t) in the central bank's monetary policy rule. The two shocks are independently drawn from normal distributions, $N(0,\sigma_z^2), N(0,\sigma_v^2)$, for every period. We assume that there's no persistence in the shock process, and the variances of the distribution are finite ($\sigma_z^2 < \infty$, $\sigma_v^2 < \infty$).

Central to our model is that we allow consumers to have a heterogeneous awareness of shocks. There are two types of consumers that differ in their awareness. The first type of consumer is fully aware of both shocks. We call this the *aware type* and denote them as I_a. The second type of consumer is only aware of the TFP shock. That is, this second type of consumer misses the monetary policy shock. For simplicity, we call this the *unaware type* and denote them as I_u. Let the measures of the two types of consumers be μ, $1-\mu$, respectively. We use $\ell \in \{a, u\}$ as the index for the awareness level.

Aware consumers realize that there is a measure of aware consumers and a measure of unaware consumers. In contrast, unaware consumers do not think about the monetary policy shocks (v_t) and thus also do not think that others think about the monetary policy shock. That is, they consider all consumers to be the same type as themselves, namely unaware consumers considering only the TPF shocks (v_t). We illustrate the simple unawareness structure in Figure 2.3.1. The model outlined so far is a special case of it where there is no $I_{a'}$, and there are only two types, I_a and I_u. This type space and

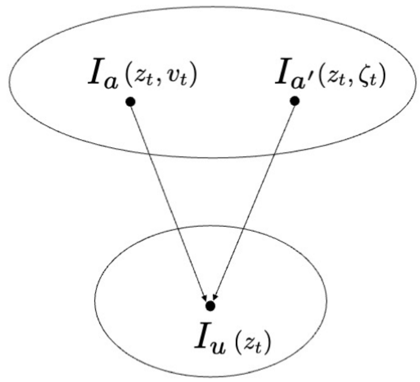

Figure 2.3.1. Unawareness Type Space

modeling of unawareness is consistent with Heifetz et al. [2006]. For simplicity, we focus on the awareness of consumers. That is, we will assume that producers and the central bank are fully aware of both shocks.

The central bank, in addition to observing past and present shocks, gets an 'early realization' of the future fundamentals/shocks (or perfect signals about them). The realization of the monetary policy shock can be interpreted for instance as internal information about upcoming changes in the management of the central bank. The TPF shock realization can be interpreted as internal research about future aggregate productivity. The fact that it is about aggregate productivity as compared to individual productivity also motivates our simplifying assumption that producers do not receive such a signal about future TPF shocks. The central bank can signal such future fundamentals via forward guidance in the form of the announcement of a future nominal interest rate, $\tilde{R}_{T|t^*}$, where T denotes the time of implementation of the interest rate and t^* is the time of announcement.[3] At the moment, we do not consider the case of the central bank directly communicating about future fundamentals/shocks to consumers. This will be considered later in Section 2.4.3. Let $z_{T|t^*}$ and $v_{T|t^*}$ be the early realization of the fundamentals. That is, $z_{T|t^*}$ is the early

[3]It is convenient to have $\tilde{R}_{T|t^*} := R_{T|t^*} + \ln\beta$ instead of the nominal interest rate itself. The advantage of this will be clear when we characterize the equilibrium in the proposition 2.4.1.

realization in t^* about the TPF shock in T (and likewise for $v_{T|t^*}$). Then the central bank announces a corresponding nominal interest rate following a Taylor rule that in our log-linearized model takes the general linear form

$$\tilde{R}_{T|t^*} := \xi_z z_{T|t^*} + \xi_v v_{T|t^*},$$

with parameters $\xi_z, \xi_v \in \mathbb{R}$.

We write the Taylor rule as a function of the two shocks rather than as a function of endogenous aggregate variables (such as output gap or inflation). This is interpreted as the composition of endogenous variables as functions of the two shocks and the "usual" Taylor rule. Our formulation is more convenient in our setting because consumers with the private signal eventually want to infer the future shocks from the reaction of the central bank.

Given the central bank's announcement in period t^*, consumers try to infer shocks realized in T. We assume that the announcement works as a private signal at t^*, shrouded by idiosyncratic noise. This can be interpreted as consumer-specific attention to the central bank policy. To differentiate the signal from the announcement, we denote by

$$\omega_{i,T|t^*} := \tilde{R}_{T|t^*} + \eta_{i,t^*}$$

consumer i's private signal about the nominal interest rate in T, where the idiosyncratic noise η_{i,t^*} is drawn from $N(0, \sigma_\eta^2)$. Since consumers in I_u are unaware of the monetary policy shock, they cannot infer anything about the monetary policy shock. Thus, they will interpret nominal interest rates differently from consumers who are aware of the monetary policy shock. An aware consumer $i \in I_a$ forms at t^* conditional beliefs (inference) about the realized values of the future fundamentals in T given the announcement according to

$$\mathbb{E}_{a,t^*}[(z_{T|t^*}, v_{T|t^*}) \mid \omega_{i,T|t^*}] = \left(\lambda_z \frac{\omega_{i,T|t^*}}{\xi_z}, \lambda_v \frac{\omega_{i,T|t^*}}{\xi_v}\right) \tag{2.3.1}$$

with $\lambda_z = \frac{\xi_z^2 \sigma_z^2}{\xi_z^2 \sigma_z^2 + \xi_v^2 \sigma_v^2 + \sigma_\eta^2}$, $\lambda_v = \frac{\xi_v^2 \sigma_v^2}{\xi_z^2 \sigma_z^2 + \xi_v^2 \sigma_v^2 + \sigma_\eta^2}$. The conditional belief contains the relative variances of the two shocks and the noise. To see this, note that under a correct common prior $(N(0, \xi_z^2 \sigma_z^2 + \xi_v^2 \sigma_v^2))$, the posterior mean of $\tilde{R}_{T|t^*}$ given the signal $\omega_{i,T|t^*}$ is

$\mathbb{E}_{a,t^*}\left[\tilde{R}_{T|t^*} \mid \omega_{i,T|t^*}\right] = \left(\frac{\xi_z^2\sigma_z^2+\xi_v^2\sigma_v^2}{\xi_z^2\sigma_z^2+\xi_v^2\sigma_v^2+\sigma_\eta^2}\right)\omega_{i,T|t^*}$. Further, note that the conditional expectation on $\xi_z z_{T|t^*}$ given $\tilde{R}_{T|t^*}$ is $\left(\frac{\xi_z^2\sigma_z^2}{\xi_z^2\sigma_z^2+\xi_v^2\sigma_v^2}\right)\tilde{R}_{T|t^*}$ since $\tilde{R}_{T|t^*}$ is a sum of two normally distributed random variables. Combining these, the conditional expectation on $\xi_z z_{T|t^*}$ is $\left(\frac{\xi_z^2\sigma_z^2}{\xi_z^2\sigma_z^2+\xi_v^2\sigma_v^2+\sigma_\eta^2}\right)\omega_{i,T|t} := \lambda_z\omega_{i,T|t^*}$. This explains the above inference rules for each shock.

An unaware consumer $i \in I_u$, on the other hand, can only infer from the announcement something about the TFP shock. Her inference rule is given by

$$\mathbb{E}_{u,t^*}[z_{T|t^*} \mid \omega_{i,T|t^*}] = \lambda_z^u \frac{\omega_{i,T|t^*}}{\xi_z} \tag{2.3.2}$$

where $\lambda_z^u = \frac{\xi_z^2\sigma_z^2}{\xi_z^2\sigma_z^2+\sigma_\eta^2}$. Recall from Figure 2.3.1 that any unaware consumer $i \in I_u$ believes that all consumers (including the aware consumers) are unaware. Thus, we must also define expectations given by equation (2.3.2) for all $i \in I_a$.

When we consider periods before the announcement in t^*, consumers form unconditional expectations. In the formal development, we avoid stating always two versions of the formulas with conditional and unconditional expectations, respectively. Instead, we only state the version with conditional expectations.

As mentioned above, we focus on the unawareness of consumers and that's why we assume that both the central bank and producers are aware of both shocks. In contrast to consumers, they observe the central bank's announcement of the future nominal interest $R_{T|t^*}$ without noise. For instance, firms may have departments specialized in market research who can perfectly observe the central bank's announcements while consumers may lack such professional support.

2.3.2 Consumers

No matter whether the consumer is aware of the monetary policy shock or not, she solves a standard consumer maximization problem. The two types of consumers only differ in how they form expectations. We use index $\ell \in \{a, u\}$ to denote their awareness level. A consumer with awareness level $\ell \in \{a, u\}$ maximizes the expected discounted sum of utilities given budget constraints conditional on her signal at t^*,

$$\max_{\{c^\ell_{i,t}, s^\ell_{i,t}, n^\ell_{i,t}\}_{t=t^*}^\infty} \sum_{t=t^*}^{\infty} \beta^{t-t^*} \mathbb{E}_{\ell,t^*} \left[U(c^\ell_{i,t}, n^\ell_{i,t}) \mid \omega_{i,T|t^*} \right] \text{ s.t.}$$

$$P_t c^\ell_{i,t} + \frac{1}{1+R_t} s^\ell_{i,t} = s^\ell_{i,t-1} + W_t n^\ell_{i,t} + D_t, \quad \forall t \in \{t^*, t^*+1, ...\}, \quad i \in [0,1]$$

where P_t is price level of the consumption good, R_t is the nominal interest rate, $s^\ell_{i,t}$ is savings, W_t is the nominal wage, and D_t is dividend, all at period t. At first glance, the superscript $\ell \in \{a, u\}$ seems redundant as either $i \in I_a$ or $i \in I_u$. However, for consumers in $i \in I_a$, we also need solutions $(c^u_{i,t})$ because unaware consumers think that every consumer, including consumers in I_a, are unaware when deciphering information from prices.

We assume that $s^\ell_{i,t}$ is in an open interval for which there no Ponzi schemes can arise. It should be clear that no matter the awareness of agents, such an interval should exist.[4] For instance, take $s^\ell_{i,t} > 0$. However, restricting to strict positive savings will not be necessary. We form the Lagrangian,

$$\mathbb{E}_{\ell,t^*} \left[\sum_{t=t^*}^{\infty} \beta^{t-t^*} U(c^\ell_{i,t}, n^\ell_{i,t}) + \sum_{t=t^*}^{\infty} \zeta_{i,t} \left(s^\ell_{i,t-1} + W_t n^\ell_{i,t} + D_t - P_t c^\ell_{i,t} - \frac{1}{1+R_t} s^\ell_{i,t} \right) \mid \omega_{i,T|t^*} \right]$$

and derive first-order conditions for $t = t^*, t^*+1, ...$ w.r.t. consumption $c_{i,t}$, savings $s_{i,t}$, and labor supply $n_{i,t}$, respectively,

$$\beta^{t-t^*} \frac{\partial U(c^\ell_{i,t}, n^\ell_{i,t})}{\partial c^\ell_{i,t}} - \zeta_{i,t} P_t = 0 \tag{2.3.3}$$

$$-\zeta_{i,t} \frac{1}{1+R_t} + \mathbb{E}_{\ell,t} \left[\zeta_{i,t+1} \mid \omega_{i,T|t^*} \right] = 0 \tag{2.3.4}$$

$$\beta^{t-t^*} \frac{\partial U(c^\ell_{i,t}, n^\ell_{i,t})}{\partial n^\ell_{i,t}} - \zeta_{i,t} W_t = 0. \tag{2.3.5}$$

Solve equation (2.3.3) for $\zeta_{i,t}$ and substitute it into equations (2.3.4), and use the partial derivative of the expected utility function to obtain the intertemporal substitution condition

$$1 = \beta \mathbb{E}_{\ell,t} \left[\frac{P_t}{P_{t+1}} \left(\frac{c^\ell_{i,t+1}}{c^\ell_{i,t}} \right)^{-\frac{1}{\gamma}} (1+R_t) \mid \omega_{i,T|t^*} \right]. \tag{2.3.6}$$

[4]For the case of heterogeneous expectations, this point must have been obvious to Angeletos and Liam (2018) as they do not explicitly state any conditions on savings.

Move the second term in equations (2.3.3) and (2.3.5) to the r.h.s. and divide (2.3.3) by (2.3.5). Use the partial derivatives of the utility function to obtain the labor supply condition

$$\left(n_{i,t}^{\ell}\right)^{\psi} = \frac{W_t}{P_t}\left(c_{i,t}^{\ell}\right)^{-\frac{1}{\gamma}}. \tag{2.3.7}$$

Define a steady state of the consumer's problem as a path with no shock and stable endogenous variables (for example, $c_{i,t}^{\ell} = c_{i,t+1}^{\ell}$). Use subscript ss for variables in the steady state. Since there are no shocks in the steady state, the process is deterministic. Awareness does not matter in the steady state. From the intertemporal substitution condition and the labor supply condition, we get $R_{ss} = -\ln\beta$ and $\psi \ln n_{i,ss}^{\ell} = \ln w_{ss} + \frac{1}{\gamma}\ln c_{i,ss}^{\ell}$, where w is the real wage ($w := \frac{W}{P}$).

We assume that the central bank announcement of the future nominal interest rate is not far away from the steady state. Hence, given normally distributed shocks and idiosyncratic noise in interpreting the central bank announcement, with a large probability consumers are not far away from their steady state no matter their awareness. Thus, we use the first-order Taylor approximation of equation (2.3.6) around its steady state to get a usual log-linearized representation of the consumption block of the New Keynesian model. That is, rewrite equation (2.3.6) for

$$1 = \mathbb{E}_{\ell,t}\left[\exp\left(\ln\beta - \pi_{t+1} - \frac{1}{\gamma}(\ln c_{i,t+1}^{\ell} - \ln c_{i,t}^{\ell}) + R_t\right) \mid \omega_{i,T|t^*}\right]$$

and use the first-order Taylor approximation around its steady state

$$1 \approx 1 + \ln\beta + \mathbb{E}_{\ell,t}\left[-\pi_{t+1} - \frac{1}{\gamma}(\ln c_{i,t+1}^{\ell} - \ln c_{i,t}^{\ell}) \mid \omega_{i,T|t^*}\right] + R_t \tag{2.3.8}$$

where $\pi_{t+1} := \ln\frac{P_{t+1}}{P_t}$.

For equation (2.3.6), simply just take log on both sides,

$$\psi \ln n_{i,t}^{\ell} = \ln w_t - \frac{1}{\gamma}\ln c_{i,t}^{\ell} \tag{2.3.9}$$

Define

$$\begin{aligned}
C_t^a &:= \int_{i\in I_a} c_{i,t}^a di + \int_{i\in I_u} c_{i,t}^u di \\
C_t^u &:= \int_{i\in I} c_{i,t}^u di \\
N_t^a &:= \int_{i\in I_a} n_{i,t}^a di + \int_{i\in I_u} n_{i,t}^u di \\
N_t^u &:= \int_{i\in I} n_{i,t}^u di.
\end{aligned}$$

Variable C_t^a is the aggregate consumption perceived by the aware consumer. It is also the actual aggregate consumption. In contrast, variable C_t^u is the aggregate consumption perceived by the unaware consumer. Analogous for N_t^a and N_t^u.

2.3.3 Firms and the central bank

The representative final good producer's profit maximization problem in period t given its packaging technology is,

$$\max_{(y_{j,t})} P_t Y_t - \int_0^1 P_{j,t} y_{j,t} dj \quad \text{s.t.}$$

$$Y_t = \left(\int_0^1 (y_{j,t})^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

and the solution to the problem gives the following factor (intermediate goods) demand functions,

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y_t. \tag{2.3.10}$$

Substituting the factor demands into the technology constraint of the maximization problem allows us to derive the aggregate price index,

$$P_t = \left(\int_0^1 (P_{j,t})^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}. \tag{2.3.11}$$

In the intermediate good production, we assume Calvo price stickiness: With probability $1-\theta$, each intermediate goods firm can reset the price of her product, and with a probability θ, the firm maintains the price that is set in the previous period. The price setting opportunities are i.i.d. across firms. The firm chooses the current price $P_{j,t}$ considering that price re-setting opportunities arrive randomly in the future. Using the aggregate price index given in equation (2.3.11), denote by P_t^* the aggregate price resulting from intermediate goods prices optimized at t by intermediate goods firms. We now have

$$\begin{aligned} P_t &= \left(\int_{j \in S(t)} (P_{j,t-1})^{1-\varepsilon} \, dj + (1-\theta)(P_t^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \\ &= \left(\theta (P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \end{aligned}$$

where $S(t)$ is the realized group of firms that are allowed to adjust their price at period t. Then, by dividing both sides by P_{t-1} and taking the log and then doing first-order Taylor approximation around its steady state (i.e., zero inflation), we obtain

$$\pi_t = (1-\theta)(P_t^* - P_{t-1}). \tag{2.3.12}$$

To get the expression for price P_t^*, consider the optimization problem of intermediate good producer j at any period $t \geq t^*$:

$$\max_{P_{j,t}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \theta^\tau \left(P_{j,t} \left(\frac{P_{t+\tau}}{P_{j,t}} \right)^{\varepsilon} Y_{t+\tau} - W_{t+\tau} n_{j,t+\tau} \right) \mid \tilde{R}_{T|t^*} \right] \text{ s.t.}$$

$$\left(\frac{P_{j,t}}{P_t} \right)^{-\varepsilon} Y_t = \exp(z_t) \; n_{j,t}.$$

The left-hand side of the constraint is the factor demand of the aggregate final goods producer, and the right-hand side is the production technology of the intermediate goods producer. Substituting the constraint into the objective function, we get

$$\max_{P_{j,t}} \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \beta^\tau \theta^\tau \left(P_{j,t} \left(\frac{P_{t+\tau}}{P_{j,t}} \right)^{\varepsilon} Y_{t+\tau} - W_{t+\tau} \left(\frac{P_{t+\tau}}{P_{j,t}} \right)^{\varepsilon} \frac{Y_{t+\tau}}{\exp(z_{t+\tau})} \right) \mid \tilde{R}_{T|t^*} \right]$$

from which we can derive the first-order condition

$$0 = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\theta)^\tau Y_{j,t+\tau} \left(1 - \varepsilon + \varepsilon \frac{W_{t+\tau}/\exp(z_{t+\tau})}{P_{j,t}} \right) \mid \tilde{R}_{T|t^*} \right]. \tag{2.3.13}$$

Since $\mathbb{E}_t[P_{j,t}] = P_{j,t}$, we get the following optimal price of the intermediate good j.

$$P_{j,t}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\theta)^\tau Y_{j,t+\tau} W_{t+\tau} / \exp(z_{t+\tau}) \mid \tilde{R}_{T|t^*} \right]}{\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\theta)^\tau Y_{j,t+\tau} \mid \tilde{R}_{T|t^*} \right]} \tag{2.3.14}$$

and the optimum price is identical to every firm that reoptimizes at period t.

We replace $P_{j,t}^*$ with P_t^*. By dividing both sides P_{t-1} and taking the first order Taylor approximation around the steady state ($P_{j,t}^* = P_{t-1}$) to obtain

$$\begin{aligned} \ln\left(\frac{P_t^*}{P_{t-1}}\right) &= \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + (1-\beta\theta)\sum_{\tau=0}^{\infty}(\beta\theta)^\tau \mathbb{E}_t\left[\ln w_{t+\tau} - z_{t+\tau} + \ln\left(\frac{P_{t+\tau}}{P_{t-1}}\right) \mid \tilde{R}_{T|t^*}\right] \\ &= (1-\beta\theta)\sum_{\tau=0}^{\infty}(\beta\theta)^\tau \mathbb{E}_t\left[\ln w_{t+\tau} - z_{t+\tau} + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + \ln\left(\frac{P_{t+\tau}}{P_{t-1}}\right) \mid \tilde{R}_{T|t^*}\right] \end{aligned}$$

where $mc^n := \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)$. Take the difference between the two equations

$$\ln P_t^* - \ln P_{t-1} = (1-\beta\theta)\sum_{\tau=0}^{\infty}(\beta\theta)^\tau \mathbb{E}_t\left[\ln w_{t+\tau} - z_{t+\tau} + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + \ln\frac{P_{t+\tau}}{P_{t-1}} \mid \tilde{R}_{T|t^*}\right]$$

$$\begin{aligned} &\beta\theta\left(\ln P_{t+1}^* - \ln P_t\right) \\ &= (1-\beta\theta)\sum_{\tau=1}^{\infty}(\beta\theta)^\tau \mathbb{E}_{t+1}\left[\ln w_{t+\tau+1} - z_{t+\tau+1} + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right) + \ln\frac{P_{t+\tau+1}}{P_{t-1}} \mid \tilde{R}_{T|t^*}\right] \end{aligned}$$

to get the following difference equation.

$$\begin{aligned} &\ln P_t^* - \ln P_{t-1} \\ &= \beta\theta\mathbb{E}_t\left[\ln P_{t+1}^* - \ln P_t \mid \tilde{R}_{T|t^*}\right] + (1-\beta\theta)\left(\ln w_t - z_t + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)\right) + \pi_t \end{aligned}$$

Combine this with the inflation-intermediate good price relation (equation (2.3.12))

$$\frac{\pi_t}{1-\theta} = \beta\theta\frac{\mathbb{E}_t[\pi_{t+1} \mid \tilde{R}_{T|t^*}]}{1-\theta} + (1-\beta\theta)\left(\ln w_t - z_t + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)\right) + \pi_t$$

we derive the inflation dynamics as follows:

$$\pi_t = \beta\mathbb{E}_t\left[\pi_{t+1} \mid \tilde{R}_{T|t^*}\right] + \frac{(1-\beta\theta)(1-\theta)}{\theta}\left(\ln w_t - z_t + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)\right) \tag{2.3.15}$$

Since the intermediate goods producers can have positive profits, they pay a dividend.

$$D_{j,t} = \left(P_{j,t} - W_t\frac{1}{\exp(z_t)}\right) y_{j,t}$$

where $D_{j,t}$ is the nominal dividend from firm j. Aggregating over all intermediate goods producers

$$D_t = \int_{j \in J} D_{j,t} dj$$

that, as we have seen in the consumers' problem, is captured by the consumers.

Finally, we close the model by specifying the central bank's reaction function (Taylor rule),

$$R_t = -\ln \beta + \phi_y \hat{X}_t + \phi_\pi \pi_t + v_t$$

or equivalently,

$$\tilde{R}_t = \phi_y \hat{X}_t + \phi_\pi \pi_t + v_t \tag{2.3.16}$$

where $\hat{X}_t$ is the log deviation of the output gap from its steady state, ϕ_y and ϕ_π are the exogenous coefficients, and v_t is the monetary policy shock. An output gap is the difference between aggregate output and 'natural' output level, $\hat{X}_t := \hat{Y}_t - \hat{Y}_t^n$. The natural output level is an output level under the full price flexibility assumption, which we will derive in the next section.

In the following, we use ˆ on variables to denote both its log deviation from the steady state or its relative deviation from the steady state, which are approximately equal to each other. E.g., $\hat{X} = \ln X_t - \ln X_{ss} \approx \frac{X_t - X_{ss}}{X_{ss}}$.

2.4 Forward Guidance

2.4.1 Temporary equilibrium

Consider a situation in which the economy is in equilibrium but no monetary policy shocks occurred yet. Agents maximize their objective functions, the central bank follows her Taylor rule, and markets clear. Since no monetary policy shocks have occurred yet, consumers behave the same no matter their awareness. Note that aware consumers anticipate that there might be some monetary policy shock in the future but unless it is announced by the central bank, the expected nominal interest rate change is zero.

Definition 2.4.1 (Temporary equilibrium). *Aggregate consumption $\{C_t^\ell\}$, aggregate output $\{Y_t\}$, labor supply $\{N^\ell\}$ and demand $\{N^d\}$, a nominal interest rate $\{R_t\}$, Dividend $\{D_t\}$, wage $\{W_t\}$, and inflation $\{\pi_t\}$ constitute a temporary equilibrium if, for every t,*

(i) Each consumer $i \in I$ optimizes leading to the intertemporal substitution condition given by equation (2.3.6) and the labor supply condition of equation (2.3.7) for $\ell = u$. Each consumer $i \in I_a$ also optimizes leading to the intertemporal substitution condition given by equation (2.3.6) and the labor supply condition of equation (2.3.7) for $\ell = a$.

(ii) The representative final goods producer optimizes leading to factor demands given by equation (2.3.10). The intermediate goods producers set optimal factor prices given by equation (2.3.14).

(iii) The nominal interest rate is set by the central bank according to the Taylor rule given by equation (2.3.16).

(iv) Unaware consumers perceive market clearing prices to solve

$$Y_t^u = C_t^u = \int_{i \in I} c_{i,t}^u di$$

in the final goods market

$$\int_{j \in J} n_{j,t} dj =: \quad N^d = N^u \quad = \int_{i \in I} n_{i,t}^u di$$

in the labor market.

Aware consumers, producers, and the central bank perceive market clearing prices to solve

$$Y_t^a = C_t^a = \int_{i \in I_u} c_{i,t}^u di + \int_{i \in I_a} c_{i,t}^a di$$

in the final goods market

$$\int_{j \in J} n_{j,t} dj =: \quad N_t^d = N_t^a \quad = \int_{i \in I_u} n_{i,t}^u di + \int_{i \in I_a} n_{i,t}^a di$$

in the labor market.

Before the announcement of the central bank, the behavior in the temporary equilibrium corresponds to the behavior in the standard rational expectations equilibrium for NK models. In particular, conditions (i), (ii), and (iii) are the building blocks of the 3-equation NK model, and (iv) is the usual market clearing condition. To see the latter, recall that as discussed above there is no difference in consumption of aware and unaware consumers before the central bank's announcement. Even though aware consumers anticipate that there will be a future central bank announcement of a monetary policy shock, the shock is mean zero ex-ante. Thus, it does not affect their behavior. There is also no difference in the labor supply of aware and unaware consumers. Thus, condition (i) is standard.

Now consider the announcement by the central bank at period t^*. Agents continue to optimize like in the baseline equilibrium but are now taking into account the announcement. Due to differences in awareness among consumers, their optimal consumption and labor supply may differ. Moreover, the perceived market clearing of unaware consumers may differ from the perceived market clearing of aware consumers. Aware consumers fully perceive actual market clearing. Unaware consumers, however, perceive market clearing as follows: Unaware consumers form beliefs about the future aggregates based on their model lacking conception of the monetary policy shocks. All information contained in the central bank announcement is attributed by unaware consumers to TFP shocks. They believe markets clear, i.e.,

$$\mathbb{E}_{u,t^*}[Y_t^u \mid \omega_{i,T|t^*}] = \mathbb{E}_{u,t^*}[C_t^u \mid \omega_{i,T|t^*}]$$
$$\mathbb{E}_{u,t^*}[N_t^d \mid \omega_{i,T|t^*}] = \mathbb{E}_{u,t^*}[N_t^u \mid \omega_{i,T|t^*}]$$

given perceived price vectors, $\mathbb{E}_{u,t^*}[(R_t, w_t, \pi_t) \mid \omega_{i,T|t^*}]$, for all $t > t^*$.

We now characterize the temporary equilibrium. We start by deriving the unawareness augmented IS curve:

Proposition 2.4.1. *The aggregate reaction of consumers forms the following unawareness augmented IS relation for each type space. For the lower space, the IS curve is*

$$\hat{Y}_t^u = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t}[\tilde{R}_{t+\tau}] + (1-\beta) \sum_{s=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t} \left[\hat{Y}_{t+\tau}^u \right]$$

and for the upper space, the IS curve is

$$\begin{aligned}\hat{Y}_t = &-\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \left(\mu \overline{\mathbb{E}}_{I_a,t}[\tilde{R}_{t+\tau} - \pi_{t+\tau+1}] + (1-\mu)\overline{\mathbb{E}}_{I_u,t}[\tilde{R}_{t+\tau} - \pi_{t+\tau+1}]\right) \\ &+ (1-\beta) \sum_{s=0}^{\infty} \beta^{\tau} \left(\mu \overline{\mathbb{E}}_{I_a,t}\left[\hat{Y}^a_{t+\tau}\right] + (1-\mu)\overline{\mathbb{E}}_{I_u,t}\left[\hat{Y}^u_{t+\tau}\right]\right)\end{aligned} \tag{2.4.1}$$

where $\overline{\mathbb{E}}_{a,t}[\cdot] := \frac{1}{\mu}\int_{i\in I_A} \mathbb{E}_{a,t}[\cdot \mid \omega_{i,T|t^*}]\, di$ *is the average expectation among the aware consumers* $i \in I_a$ *(and likewise for the unaware consumers in* I_u*).*

Next, we want to link both the aggregate demand of the consumption block of Proposition 2.4.1 and the inflation dynamics of the production block (equation (2.3.15)) with the monetary policy given by the Taylor rule (equation 2.3.16). To this end, we derive the natural rates of output and the output gaps. Start with the production side. The natural output level is defined as an output level under complete price flexibility. Recall the first-order condition of the intermediate good producer (equation (2.3.13)),

$$0 = \mathbb{E}_t \left[\sum_{\tau=0}^{\infty} (\beta\theta)^{\tau} Y_{j,t+\tau} \left(1 - \varepsilon + \varepsilon \frac{W_{t+\tau}/\exp(z_{t+\tau})}{P_{j,t}}\right) \mid \tilde{R}_{T|t^*}\right].$$

Since there is a continuum of intermediate goods producers, each of them is small. Thus, they take wages as given. Moreover, they can adjust prices each period under complete price flexibility assumed when considering the natural rate of output. Hence, there dynamic optimization problem is a sequence of one-period problems. Therefore, the above first-order condition can be written

$$0 = 1 - \varepsilon + \varepsilon \frac{W^n_t/\exp(z_t)}{P_t}$$

where $P_{j,t}$ is replaced with P^n_t since every firm will choose the same price, and n in the superscript implies the natural level. Moving $\frac{W^n_t}{\exp(z_t)P^n_t}$ to the left and side and taking the natural log gives,

$$\ln w^n_t - z_t = -\ln\left(\frac{\varepsilon}{\varepsilon - 1}\right).$$

Then, we can derive

$$\begin{aligned}-\ln\left(\frac{\varepsilon}{\varepsilon-1}\right) &= \psi \ln N^n_t + \frac{1}{\gamma}\ln Y^n_t - z_t \\ &= \psi(\ln Y^n_t - z_t) + \frac{1}{\gamma}\ln Y^n_t - z_t\end{aligned}$$

where the first equation comes from aggregate labor supply ($\psi \ln N_t^n = \ln w_t - \frac{1}{\gamma}\ln Y_t^n$) and the second equation comes from $\ln N_t^n = \ln Y_t^n - z_t$ which can be obtained from the intermediate good production technology and aggregation of the labor demand.[5] We can rearrange the above equation as

$$\ln Y_t^n = \frac{1+\psi}{\psi+\frac{1}{\gamma}} z_t - \frac{\ln\left(\frac{\varepsilon}{\varepsilon-1}\right)}{\psi+\frac{1}{\gamma}}$$

by collecting $\ln Y_t^n$. Finally, define the output gap $\hat{X}_t$ as the difference between the current output and the natural level of output. Then,

$$\begin{aligned}\hat{X}_t :=& \ln Y_t - \ln Y_t^n \\ =& \frac{1}{\psi+\frac{1}{\gamma}}\ln w_t + \frac{\psi}{\psi+\frac{1}{\gamma}} z_t - \frac{1+\psi}{\psi+\frac{1}{\gamma}} z_t + \frac{\ln\left(\frac{\varepsilon}{\varepsilon-1}\right)}{\psi+\frac{1}{\gamma}} \\ =& \frac{1}{\psi+\frac{1}{\gamma}}\ln w_t - \frac{1}{\psi+\frac{1}{\gamma}} z_t + \frac{\ln\left(\frac{\varepsilon}{\varepsilon-1}\right)}{\psi+\frac{1}{\gamma}}\end{aligned}$$

where the second equation is immediate if we combine $\ln Y_t = z_t + \ln N_t$ and $\psi \ln N_t = \ln w_t - \frac{1}{\gamma}\ln Y_t$ as $\ln Y_t = \frac{1}{\psi+\frac{1}{\gamma}}\ln w_t + \frac{\psi}{\psi+\frac{1}{\gamma}} z_t$. Multiplying $\psi + \frac{1}{\gamma}$ on both sides, we get

$$\left(\psi + \frac{1}{\gamma}\right)\hat{X}_t = \ln w_t - z_t + \ln\left(\frac{\varepsilon}{\varepsilon-1}\right)$$

and plugging this into equation (2.3.15) gives the following New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1} \mid \tilde{R}_{T|t^*}] + \kappa\left(\psi + \frac{1}{\gamma}\right)\hat{X}_t \tag{2.4.2}$$

2.4.2 Effect of Forward Guidance in Temporary Equilibrium

We would like to analyze how heterogeneous awareness of consumers affects forward guidance in temporary equilibrium. The forward guidance puzzle is that a future interest

[5]

$$\begin{aligned}\ln N_t^n :=& \ln \int_{j\in J} n_{j,t}^n dj = \ln \frac{1}{\exp(z_t)} \int_{j\in J}\left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} dj Y_t \\ =& \ln Y_t^n - z_t + \ln \int_{j\in J}\left(\frac{P_{j,t}}{\int_{j\in J}\left(P_{j,t}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} dj}\right)^{-\varepsilon} dj \\ \approx& \ln Y_t^n - z_t\end{aligned}$$

rate change has the same effect on the IS curve as a corresponding change of the current interest rate (McKay et al. [2017], Farhi and Werning [2019], Angeletos and Lian [2018]). We show that under unawareness the effect of forward guidance is weaker than the effect of a current interest rate change.

We assume that the economy is initially in a steady state. Thus, there are no shocks. In this case, temporary equilibrium coincides with rational expectations equilibrium. Since there are no shocks, differences in awareness of shocks do not matter and the reaction of consumers is the same across the two types. Then we introduce the early realization of the future shocks, $(z_{T|t^*}, v_{T|t^*})$, and the central bank's announcement, $\tilde{R}_{T|t^*}$, at period t^*. We assume that the announcement is about the nominal interest rate only at period $T > t^*$ and no other period. That is, we fix the nominal interest rate of any other periods at the steady state level and assume that all other agents in the model do not change their beliefs about it. In principle, if there is an expected change in the nominal interest rate at T, it will change the agents' action at $T-1$. Responding to this, the monetary policy should adjust R_{T-1} according to the Taylor rule. As this backward recursion goes on, all nominal interest rates at the in-between periods should adjust. We exclude this consideration by assuming that the economy is in a steady state until period t^* and will deviate from the steady state for all periods after t^*+1. The nominal interest rate, however, is fixed at the steady state level until period $T-1$. There are three reasons for this assumption: First, we do not know how to solve the model analytically without this assumption. The related literature like Angeletos and Lian [2018] or Farhi and Werning [2019] uses the same assumption. Second, as suggested by Angeletos and Lian [2018], the unmodeled zero nominal interest rate lower bound may be binding for all periods before T, constraining how the central bank could react in periods before T. Third, like Farhi and Werning [2019] we are interested in the comparative statics between two announcements of the change of the nominal interest rate at different horizons keeping nominal interest rates for any other periods constant. In some sense, we isolate an upper bound on the potential effect of forward guidance.

As we have seen in the previous chapter, aware and unaware consumers have different evaluations of the fundamentals/shocks, respectively. Hence the shift of the aggregate IS curve differs from the benchmark of rational expectations equilibrium under full awareness. In the following proposition, we show how the current output gap changes when there is a central bank announcement at t^* about the nominal interest rate at period $T > t^*$ (i.e., forward guidance).

Proposition 2.4.2. *The temporary equilibrium reaction of the current output gap, $\hat{X}_{t^*}$, on the announcement of the future nominal interest rate, $\tilde{R}_{T|t^*}$, is*

$$\hat{X}_{t^*}(\tilde{R}_{T|t^*}) = \mu\left(\Phi^a_{t^*}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] - \Phi^{u*}_{t^*}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]\right) + \Phi^u_{t^*}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]$$

where Φ^a_{t*} and Φ^{u*}_{t*} are the average output gap reactions on $\tilde{R}_{T|t}$ of types I_a and I_u, respectively. Φ^u_{t*} is the perceived average reaction of unaware consumers in the lower space. Φ^a_{t*}, Φ^{u*}_{t*}, Φ^u_{t*} are defined as follows:

$$\Phi^a_{t^*}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] - \Phi^{u*}_{t^*}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] = \begin{pmatrix}1 & -1 & 0 & 0\end{pmatrix} \cdot (M_a)^{T-t^*-1}\, b_a \tilde{R}_{T|t^*}$$

$$\Phi^u_t = \begin{pmatrix}1 & 0\end{pmatrix}(M_u)^{T-t^*-1}\, b_u \tilde{R}_{T|t}$$

$$M_a := \begin{pmatrix} \beta + ((1-\beta)\mu + \gamma\Xi\mu)(\lambda_z + \lambda_v) & 0 & \gamma\beta & 0 \\ 0 & \beta + ((1-\beta)\mu + \gamma\Xi\mu)\lambda^u_z & 0 & \gamma\beta \\ \Xi\mu & 0 & \beta & 0 \\ 0 & \Xi\mu & 0 & \beta \end{pmatrix}$$

$$M_u := \begin{pmatrix} \beta + (1-\beta+\gamma\Xi)\lambda^u_z & \gamma \\ \beta\Xi & \beta \end{pmatrix}$$

$$b_a := \begin{pmatrix} \left(1 + \frac{(1-\beta)\mu + \gamma\Xi\mu}{\beta}\right)\left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right) \\ \left(1 + \frac{(1-\beta)\mu + \gamma\Xi\mu}{\beta}\right)\left(\frac{\Lambda_{11}}{\Lambda_{21}}\frac{\beta\lambda^u_z + (1-\beta)\mu\lambda_z}{\beta + (1-\beta)\mu} + \frac{\Lambda_{12}}{\Lambda_{22}}\frac{(1-\beta)\mu\lambda_v}{\beta + (1-\beta)\mu}\right) \\ \frac{\Xi\mu}{\beta}\left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right) \\ \frac{\Xi\mu}{\beta}\left(\frac{\Lambda_{11}}{\Lambda_{21}}\frac{\beta\lambda^u_z + (1-\beta)\mu\lambda_z}{\beta + (1-\beta)\mu} + \frac{\Lambda_{12}}{\Lambda_{22}}\frac{(1-\beta)\mu\lambda_v}{\beta + (1-\beta)\mu}\right) \end{pmatrix} \tilde{R}_{T|t^*}$$

$$b_u := \begin{pmatrix} (1+\gamma\Xi)\frac{\Lambda_{11}}{\Lambda_{21}}\lambda^u_z \\ \beta\Xi\frac{\Lambda_{11}}{\Lambda_{21}}\lambda^u_z \end{pmatrix}$$

where λ_z, λ_v, *and* λ_z^u *are the relative variances defined previously, and* $\Lambda_{11} := \frac{-\gamma}{1+\gamma\phi_y}\frac{\gamma+\gamma\psi}{1+\gamma\psi}$, $\Lambda_{12} := \frac{-\gamma\beta+\frac{1-\beta}{\phi_y}(1-\mu)}{\beta+\gamma\beta\phi_y}\frac{1}{\mu}$, $\Lambda_{21} := \frac{-\gamma\phi_y}{1+\gamma\phi_y}\frac{\gamma+\gamma\psi}{1+\gamma\psi}$, $\Lambda_{22} := \frac{1-(1-\beta)\mu}{\beta+\gamma\beta\phi_y}\frac{1}{\mu}$, *and* $\Xi := \kappa\left(\psi + \frac{1}{\gamma}\right)$.

To understand the proposition, first note that the aggregate output gap reaction, $\hat{X}_{t^*}$, is a weighted average of aware type and unaware type consumers' average reactions. The weights are the measures of the type of consumers $(\mu, 1-\mu)$. Then, notice that each type of the consumer's average reaction at period t^* is calculated by a backward recursion. The transition matrices from $t+1$ to t are M_a for the aware type and M_u for the unaware type. Finally, b_a and b_u are the reactions of aggregates (i.e., output gap and inflation) at period $T-1$ which is the "beginning" point of the recursion. Therefore, $M_a b_a$ is the reaction of $T-2$, $(M_a)^2 b_a$ is the reaction of $T-3$, etc.

The proof consists of six steps. First, we build a contemporaneous reaction of the output gap to the nominal interest rate change announcement in the lower space. Second, we derive the reaction of the output gap to the announcement for a general period in the lower space using backward induction. Third, given the lower space results, we move to the upper space where the market clearing prices may differ from the unaware consumers' perceived ones in the lower space. We derive the perceived-actual reaction relations for the unaware consumers. Fourth, we derive the contemporaneous reaction of the output gap among the aware consumers. Fifth, we invoke backward induction and get the result for a general period. Lastly, in step 6, we get the aggregate output gap reaction by taking the weighted average between the reactions for the aware and unaware consumers.

To get a better idea of the Proposition 2.4.2, focus on the movement of the IS curve. Because we assumed heterogeneous unawareness among the consumers, it is enough to investigate the consumption block to get intuitions. To this end, assume that the probability of resetting the price is 0 (i.e., the fraction of firms that do not change their price is $\theta = 1$), hence the New Keynesian Philips Curve is fixed at the steady state level. Further, we also simplify the exposition by assuming that the signal is perfect ($\sigma_\eta^2 = 0$). Then, the proposition can be simplified as follows:

Corollary 2.4.1. *The reaction of the IS curve on the announcement* $R_{T|t^*}$ *with a perfect* *signal is as follows:*

$$\hat{X}_{t^*}^{IS}|_{\sigma_\eta^2=0} = \frac{\Lambda_{11}}{\Lambda_{21}}\tilde{R}_{T|t^*} + \mu\left((\beta+(1-\beta)\mu)\right)^{T-t^*-1}(1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}} - \frac{\Lambda_{11}}{\Lambda_{21}}\right)\tilde{R}_{T|t^*} \tag{2.4.3}$$

where $\hat{X}_{t^*}^{IS}$ *is the IS curve movement at period* t^* *after the announcement, and* $\lambda = \frac{\sigma_v^2}{\sigma_z^2+\sigma_v^2}$.

For a better interpretation of the result, it is instructive to separate equation (2.4.3) into three parts: the information content of the forward guidance (i.e., the announcement) for each type of consumers, the general equilibrium discounting, and the model misspecification correction.

$$\begin{aligned}\frac{\hat{X}_{t^*}^{IS}|_{\sigma_\eta^2=0}}{\tilde{R}_{T|t^*}} = & \underbrace{(1-\mu)\frac{\Lambda_{11}}{\Lambda_{21}}}_{\substack{\text{information content}\\ \text{for unaware consumers}}} + \mu\underbrace{\left(\lambda\frac{\Lambda_{11}}{\Lambda_{21}} + (1-\lambda)\frac{\Lambda_{12}}{\Lambda_{22}}\right)}_{\substack{\text{information content}\\ \text{for aware consumers}}}\left(\underbrace{(\beta+(1-\beta)\mu)}_{\text{GE discounting}}\right)^{T-t^*-1} \\ & + \underbrace{\mu\left(\frac{\Lambda_{11}}{\Lambda_{21}} - \frac{\Lambda_{11}}{\Lambda_{21}}(\beta+(1-\beta)\mu)^{T-t^*-1}\right)}_{\text{model misspecification correction}}\end{aligned} \tag{2.4.4}$$

The 'Information content of Forward Guidance' comes from the fact that the consumers cannot observe the fundamentals $(z_{T|t}, v_{T|t})$ directly and have to infer them from the announcement. The unaware consumers' interpretation of the forward guidance is the central bank's reaction to future productivity, and it is unambiguously positive $(\frac{\Lambda_{11}}{\Lambda_{21}})$. The aware consumers' interpretation depends on the distribution of the shocks and their relative variance $(\lambda, 1-\lambda)$.

If the aware consumers believe that the forward guidance is mostly the reaction to the monetary policy shock as in Angeletos and Lian [2018], which corresponds in our model to when λ is close to zero, then the information content part is close to $\frac{\Lambda_{12}}{\Lambda_{21}}$. In the language of Campbell et al. [2012], this case may correspond to 'Odyssean' forward guidance when consumers think that the announcement is a binding commitment by the central bank. On the contrary, if the aware consumers think that the guidance mostly indicates the central bank's internal knowledge of the future productivity (i.e., λ is close to one), then the coefficient is close to $\frac{\Lambda_{11}}{\Lambda_{21}}$ and the forward guidance is 'Delphic' in the language of Campbell et al. [2012].

In the remainder of the text, we assume that the information content part of the aware consumers is negative so as to emphasize the difference between aware and unaware consumers. In other words, while the unaware consumers account for the announcement only on the TFP shock, the aware consumers account for the monetary policy shock more heavily than the TPF shock. The following assumption on primitives guarantees that the information content part of aware consumers is negative.

Assumption 2.4.1.

$$\frac{1}{\phi_y} > 0 > \frac{1}{\phi_y}\lambda + \left(\frac{-\gamma\beta + \frac{(1-\beta)(1-\mu)}{\phi}}{1-(1-\beta)\mu}\right)(1-\lambda)$$

The general equilibrium discounting in equation (2.4.4) originally comes from two facts. One is the idiosyncratic noise of the signal and higher-order uncertainty as in Angeletos and Lian [2018]. The parameters λ_z and λ_v in Proposition 2.4.2 are the higher-order expectation related discounting factors. In equation (2.4.4), it is muted because of the perfect signal assumption. In addition to that, we have potentially heterogeneous awareness, which means $\mu \leq 1$. This means that μ can function as an unawareness-driven discounting factor. The idea is that an aware consumer $i \in I_a$ can correctly anticipate that only a fraction of aware consumers perceive the existence of the monetary policy shock, hence the general equilibrium effect in the future is diminished. This is our novel resolution of the 'forward guidance puzzle'. Recall that the puzzle stems from the IS reaction being independent of the time horizon (under complete information). When awareness is homogeneous, the reaction in the corollary is $\frac{\Lambda_{11}}{\Lambda_{21}}$ or $\frac{\Lambda_{11}}{\Lambda_{21}}\lambda + \frac{\Lambda_{12}}{\Lambda_{22}}(1-\lambda)$ (for all being unaware or all being aware), respectively. In these cases, the reaction does not change even if the horizon of the guidance $T - t^*$ differs. With heterogeneous awareness, on the other hand, the reaction diminishes as the horizon increases. In the extreme, when the horizon is very long, the effect of forward guidance on the output gap disappears.

Finally, the model misspecification correction, the last part of equation (2.4.4) comes from the aware consumers' actual market clearing. Note that unaware consumers disregard GE discounting if the signal is perfect. Aware consumers, on the other hand, understand that the effect of the future event is discounted with $(\beta + (1-\beta)\mu)^{T-t^*-1}$.

The model misspecification correction in equation (2.4.4) is the difference between what the unaware consumers do ($\frac{\Lambda_{11}}{\Lambda_{21}}$) and what they should do ($\frac{\Lambda_{11}}{\Lambda_{21}}(\beta + (1-\beta)\mu)^{T-t^*-1}$).

Proposition 2.4.3. *(Comparative Statics) The output gap reaction to the announcement in the heterogeneous awareness model is always less than the homogeneous* unawareness *model, and always more than the homogeneous* awareness *model. Increasing the horizon of the forward guidance, $T - t^*$, increases the reaction of the output gap. Lowering awareness, μ also increases the reaction of the output gap.*

To see this, consider first the case when all consumers are unaware (i.e., $\mu = 0$) vis-a-vis heterogeneous awareness (i.e., $\mu \in (0,1)$). As for the corollary, we continue to assume that information is complete $\sigma_\eta^2 = 0$. Then from the corollary we get

$$\frac{\hat{X}_{t^*}^{IS} - \hat{X}_{t^*}^{IS}|_{\mu=0}}{\tilde{R}_{T|t^*}} = \mu(1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}} - \frac{\Lambda_{11}}{\Lambda_{21}}\right)((\beta + (1-\beta)\mu))^{T-t^*-1}$$

For any given horizon of the forward guidance ($T - t^*$), the above difference is always negative for any μ with the Assumption 2.4.1. That is, compared to the homogeneous unaware case, heterogeneous awareness lowers the reaction of the output gap.

Now consider the second case when all consumers are aware (i.e., $\mu = 1$) vis-a-vis heterogeneous awareness (i.e., $\mu \in (0,1)$). From the corollary, we obtain:

$$\begin{aligned}
&\frac{\hat{X}_{t^*}^{IS} - \hat{X}_{t^*}^{IS}|_{\mu=1}}{\tilde{R}_{T|t^*}} \\
&= \mu\left((\beta + (1-\beta)\mu)\right)^{T-t-1}(1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}} - \frac{\Lambda_{11}}{\Lambda_{21}}\right) - (1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}} - \left.\frac{\Lambda_{11}}{\Lambda_{21}}\right|_{\mu\to 1}\right) \\
&= \left(\mu\left((\beta + (1-\beta)\mu)\right)^{T-t-1} - 1\right)(1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}} - \frac{\Lambda_{11}}{\Lambda_{21}}\right) - (1-\lambda)\left(\frac{\Lambda_{11}}{\Lambda_{21}} - \left.\frac{\Lambda_{11}}{\Lambda_{21}}\right|_{\mu\to 1}\right)
\end{aligned}$$

where $\left.\frac{\Lambda_{12}}{\Lambda_{22}}\right|_{\mu\to 1}$ is the solution of the New Keyesian Model when all consumers are aware of the monetary policy shock. The sign of the difference is also strictly positive for any $\mu < 1$. The reaction in the heterogeneous awareness case is stronger than in the homogeneous aware case. That is, compared to the homogeneous aware case, heterogeneous awareness increases the reaction of the output gap. To sum up, the reaction under heterogeneous

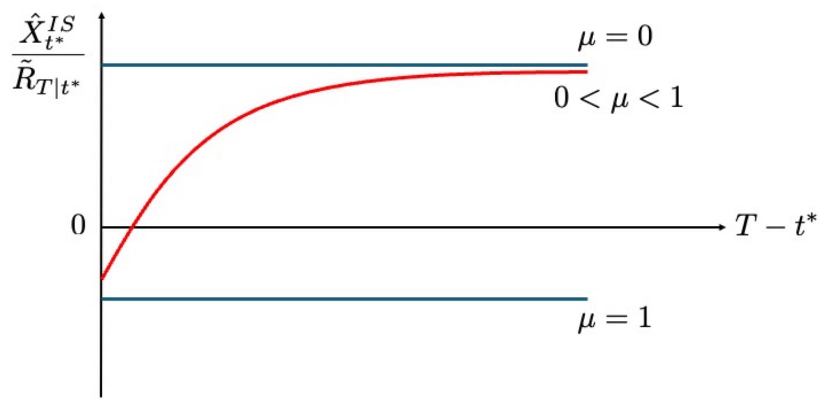

Figure 2.4.1. Output gap reaction to the announcement

awareness is between the reaction under homogeneous awareness and homogeneous unawareness.

Finally, we check how the two comparative statics change when we increase the horizon of the forward guidance. Intuitively, increasing the horizon of forward guidance diminishes the general equilibrium effect. Similarly, when more consumers are unaware, less consumers take into account the full general equilibrium effect. That's why both increasing the horizon or increasing the fraction of unaware consumers increases the reaction of the output gap to shock(s). More formally, from equation (2.4.3), we can easily confirm that the current output reaction increases (shifts upward) as the horizon $T - t^*$ increases because of the general equilibrium discounting factor. It means that the difference to the unaware case becomes smaller, and the difference to the full aware case tends to be larger. We borrow intuition from Angeletos and Lian [2018] for this observation. Increasing the horizon of forward guidance is similar to increasing the order of average expectation because as the horizon gets longer, we get more backward recursions. More backward recursion implies multiple iterations of expectation on an aggregate action, and the aware type consumer $i \in I_a$ expects that fewer consumers can understand the monetary policy shock. The result, therefore, is similar to lowering the awareness of the consumers.

2.4.3 Raising Awareness

So far, we treated the measure of unaware types as being exogenously given. However, if a central bank can communicate with the consumers, it could raise awareness of the monetary policy shock and thus change the effect of its monetary policy. Note that changing awareness is just one-directional. The central bank can raise awareness but cannot make them unaware of things that they are already aware of. While raising awareness maybe in interesting to study in a variety of macroeconomic models, let us consider it in our model.

Remember that the Taylor rule, $\tilde{R}_t = \phi_y \hat{X}_t + v_t$, is the central bank's reaction function. Once a shock realizes (i.e., z_t or v_t), then the bank sets the nominal interest rate accordingly. The early realization of the shock $z_{T|t^*}$ or $v_{T|t^*}$ also changes the future nominal interest rate $\tilde{R}_T$. Recall that forward guidance is about the central bank's announcement on the planned change of the nominal interest rate. Why does the bank want to *announce* the plan rather than just implement it in the future? In an economy that is close to the zero lower bound, the monetary policy has limited room for further action even if the output gap is negative. Because of this, the central bank may want to announce the future policy so that it can boost the current economy. In what follows, we consider such a case. To focus on the effect of awareness and simplify the transition matrix, we assume as in Corollary 2.4.1 that signals are perfect and inflation is fixed at the steady state, thus eliminating asymmetric information.

When the central bank announces its future nominal interest rate cut, $\Delta\tilde{R}_{T|t^*} < 0$, it intends to boost the economy with an expansionary monetary policy at the current period, t^*. For that to be possible, the 'Information Content of Forward Guidance' in the equation (2.4.3) should be negative,[6], which is implied for aware consumers only by Assumption 2.4.1.In this sense, the assumption implies that the aware consumers' expected contemporaneous reaction is aligned with the central bank's intention. However, because of the presence of unaware consumers, the reaction in temporary equilibrium is biased toward the TFP shock, z_t, and the overall effect on the current economy may not be

[6] $\frac{\Lambda_{11}}{\Lambda_{21}}\lambda + \frac{\Lambda_{12}}{\Lambda_{22}}(1-\lambda) < 0$

aligned with the central bank's intention. For example, if the unawareness is widespread (i.e., $\mu \to 0$), the reaction in the temporary equilibrium is unambiguously negative:

$$\Delta \hat{X}_{t^*}^{IS} = \frac{\Lambda_{11}}{\Lambda_{21}} \Delta \tilde{R}_{T|t^*} < 0$$

In such a case, it is better not to make any announcement about the future nominal interest rate change. When there is no announcement, marginally raising awareness by marginally increasing μ has no effect on the output gap. The central bank has the incentive to make an announcement only if the measure of aware consumers $\mu > \tilde{\mu}$ is above a threshold $\tilde{\mu}$ that satisfies $\frac{\Delta \hat{X}_{t^*}^{IS}}{\Delta \tilde{R}_{T|t^*}} = 0$. It is in this case that raising awareness can amplify forward guidance. Raising awareness makes more people have information content that is consistent with the central bank's intention. At the same time, as the central bank increases awareness, the general equilibrium discounting (see equilibrium (2.4.4) becomes smaller hence the positive reaction on the current output becomes larger. We summarize the above observations as follows:

Proposition 2.4.4. *If Assumption 2.4.1 is satisfied, then the aware consumers' contemporaneous reaction to the forward guidance is in line with the central bank's intention. Raising awareness can assist the effectiveness of the forward guidance if $\mu > \tilde{\mu}$, where $\tilde{\mu}$ satisfies $\frac{\Delta \hat{X}_{t^*}^{IS}}{\Delta \tilde{R}_{T|t^*}} = 0$.*

Alternatively, consider the case where Assumption 2.4.1 is violated and the information content of the aware consumer is also positive. In this case, the current output gap reaction in the temporary equilibrium is always negative regardless of μ. This is because consumers, who don't observe the shocks directly, interpret the rate cut as a central bank's response to a negative $z_{T|t^*}$. Therefore, the contemporaneous reaction is at odds with the central bank's intention. In this case, there's no room for forward guidance, hence the bank will not announce the future nominal interest rate change, and will keep the 'early realization' as internal knowledge.

We just observed that when Assumption 2.4.1 is violated, the central bank does not want to announce. Suppose now they are required (e.g. by law) to announce nevertheless. In such a case, can raising awareness mitigate the negative effect of the announcement?

Increasing awareness about the monetary policy shock (v_t) will balance the interpretation of the forward guidance between $z_{T|t^*}$ and $v_{T|t^*}$ because the aware consumers account for both shocks. Further, as more people become aware of the monetary policy shock, the Taylor rule relies more on the aware type output. At the same time, raising awareness weakens the GE discounting hence it increases aware consumers' reaction. To see this, differentiate equation (2.4.3) with respect to μ:

$$\begin{aligned}\frac{\partial \hat{X}^{IS}_{t^*}}{\partial \mu} =&(1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}}-\frac{\Lambda_{11}}{\Lambda_{21}}\right)(\beta+(1-\beta)\mu)^{T-t^*-1}\tilde{R}_{T|t^*}\\ &-\mu(1-\lambda)(1-\beta)\frac{\beta(1+\gamma\phi_y)}{\phi_y(1-(1-\beta)\mu)^2}(\beta+(1-\beta)\mu)^{T-t^*-1}\tilde{R}_{T|t^*}\\ &+\mu(1-\lambda)(1-\beta)(T-t^*-1)\left(\frac{\Lambda_{12}}{\Lambda_{22}}-\frac{\Lambda_{11}}{\Lambda_{21}}\right)(\beta+(1-\beta)\mu)^{T-t^*-2}\tilde{R}_{T|t^*}<0\end{aligned}$$

The first line of the r.h.s. of the equation represents balancing the information content of forward guidance. As the central bank increases μ, more people interpret the announcement as a combination of two shocks and fewer people interpret it as driven only by the TFP shock. The second term of the r.h.s. of the equation shows the change of the solution in the contemporaneous case. Recall that $\frac{\Lambda_{12}}{\Lambda_{22}}$ is a function of μ. This is because the monetary policy is a function of the total output gap $\hat{X}_t$ which is again a weighted average of the output gap for aware types and the output gap for unaware types. As the bank increases the awareness, the Taylor rule itself relies more on the output gap of aware types, hence the (contemporaneous) solution of the model changes. Finally, the third term of the r.h.s. of the equation comes from a weakening of the general equilibrium discounting. The direction of the derivative is negative regardless of parameters, which implies that the central bank can mitigate the negative effect of announcement by increasing the awareness of the monetary policy shock.

2.5 Self-Confirming Equilibrium

The agents' reaction functions are purely forward-looking, hence beliefs about the future variables in addition to observe current prices fix the current equilibrium allocation. However, one important question remains: Why do consumers unaware of the second shock do not realize that their model is in some sense misspecified? Upon announcement by the central bank about future interest rates, agents form expectations about future and *current* market clearing prices. However, observed current market clearing prices may differ and expected current market clearing prices. At this point, shouldn't unaware consumers realize that their model is missing something? In this section, we define an equilibrium concept that on top of temporary equilibrium requires that behavior is consistent with beliefs and beliefs are consistent with observations. In particular, we allow unaware consumers to change their model in order to make it consistent with observed current aggregates.

Definition 2.5.1 (Self-Confirming Equilibrium)**.** *The sequence of aggregates* $\{C_t, Y_t, N_t^d, N_t^\ell\}$ *and price vectors* $\{d_t, R_t, w_t, \pi_t\}$ *constitutes a self-confirming equilibrium if*

i. it is a temporary equilibrium, and

ii. it is common belief that any unaware consumer $i \in I_u$ *chooses an inference rule* $\mathbb{E}_{u,t}^{sc}\left[z_{T|t^*} \mid \omega_{i,T|t^*}\right]$ *that in every period* t *is consistent with observed current aggregates, i.e.,*

$$C_t^u = Y_t \tag{2.5.1}$$

Equation (2.5.1) implies that market clearing perceived by unaware consumers also clears the actual market ($\hat{Y}_t^u = \hat{Y}_t = \hat{Y}_t^a$) so that the unaware consumers can rationalize their observations with their self-confirming inference rule *given* their awareness level.

In the following proposition, we show that there exists a self-confirming equilibrium with a self-confirming inference rule that in some sense is a minimal departure from Bayesian inference because it is a linear transformation of it. Moreover, it allows unaware consumers to perceive a Taylor rule that adds an additional factor similar to the monetary policy shock even though they remain unaware of the monetary policy shock. It is as if they are aware that they are unaware of some factor even though they do not know what it is.

Proposition 2.5.1. *There exists a self-confirming inference rule of the unaware consumers* $i \in I_u$*,*

$$\mathbb{E}_{u,t}^{sc}\left[z_{T|t^*} \mid \omega_{i,T|t^*}\right] = \lambda_z^u \frac{\omega_{i,T|t^*}}{\xi_z} + \delta_{i,t}$$

and an associated perceived Taylor rule,

$$R_t = \phi_x \hat{X}_t + e_t$$

such that the economy is in self-confirming equilibrium. The variables $\delta_{i,t}$ and e_t are given by

$$\delta_{i,t} = \frac{1}{\phi_y}\left(\frac{\frac{1}{\gamma}+\psi}{1+\psi}\right)\left(1 - \frac{\begin{pmatrix}1 & 0\end{pmatrix}(M_a)^{T-t-1}\begin{pmatrix}1+\gamma\Xi\mu \\ \beta\Xi\end{pmatrix}\left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right)}{\begin{pmatrix}1 & 0\end{pmatrix}(M_u)^{T-t-1}\begin{pmatrix}(1+\gamma\Xi) \\ \beta\Xi\end{pmatrix}\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z^u}\right)\tilde{R}_{T|t^*} + \frac{\lambda_z^u}{\xi_z}\left(\tilde{R}_{T|t^*} - \omega_{i,T|t^*}\right)$$

$$e_t = \overline{\mathbb{E}}_{I,t}\left[e_T\right] = \left(\lambda_z^u - \frac{\begin{pmatrix}1 & 0\end{pmatrix}(M_a)^{T-t-1}\begin{pmatrix}1+\gamma\Xi\mu \\ \beta\Xi\end{pmatrix}\left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right)}{\begin{pmatrix}1 & 0\end{pmatrix}(M_u)^{T-t-1}\begin{pmatrix}(1+\gamma\Xi) \\ \beta\Xi\end{pmatrix}\frac{\Lambda_{11}}{\Lambda_{21}}}\right)\tilde{R}_{T|t^*}$$

where λ_z, λ_v, Λ_{11}, Λ_{12}, Λ_{21}, and Λ_{22} are defined in Proposition 2.4.2, and

$$\tilde{M}_a := \begin{pmatrix}\beta + (1-\beta+\gamma\Xi)(\lambda_z+\lambda_v) & \gamma\beta \\ \Xi & \beta\end{pmatrix}.$$

There are three things to comment on Proposition 2.5.1. First, the unaware consumers' modified estimation rule and perceived Taylor rule depend on the time of the announcement. That is, $\overline{\mathbb{E}}_{I,t^*}\left[e_T\right]$ and d_{i,t^*} change when T and t^* change. For example, let $R_{T|t^*}$ is the announcement and $(\delta_{i,t^*}, \overline{\mathbb{E}}_{I,t^*}\left[e_T\right])$ are the associated modification. The unaware consumer has to adjust her belief about the Taylor rule again to clear the market in the next period ($\hat{C}^u_{t^*+1} = \hat{Y}_{t^*+1}$). To be in the reflective equilibrium, the unaware consumers should update their beliefs continuously such that the modification $(\delta_{i,t}, \overline{\mathbb{E}}_{I,t}\left[e_T\right])$ satisfy the condition in the proposition for every $t \in \{t^*, ..., T\}$. Second, the 'dummy' (δ_{i,t^*}) in the estimation rule has a tight connection to the perceived Taylor rule. $\overline{\mathbb{E}}_{I,t^*}\left[e_T\right]$ is an aggregate of linear transformations of d_{i,t^*}. Further, $\overline{\mathbb{E}}_{I,t^*}\left[e_T\right]$ is a deterministic constant in the Taylor rule, which consists of the 'natural interest rate'. That is, the unaware consumers change their belief on the natural interest rate of the economy to make sense of the aggregate unless they become aware of the other shock. Third, the aware consumers' response at period t^* comes from that they foresee future market clearing. Recall that we assume the aware consumers understand the unaware consumers' problem. The aware consumer can put herself in the unaware consumer's shoes and anticipates the lower space adjustments $(\delta_{i,t}, \overline{\mathbb{E}}_{I,t}[e_T])$ for the current and every future period $t \in \{t^*, ..., T\}$. Then, when she derives her best response in the current period, she considers all future market clearing based on the $(\delta_{i,t}, \overline{\mathbb{E}}_{I,t}[e_T])_{t\in\{t^*,...,T\}}$. The current aggregate reaction of the output gap when we fix the inflation is immediate from the above proposition.

Lemma 2.5.1. *In the reflective equilibrium, the aggregate IS reaction to the announcement recovers the full awareness.*

$$\frac{\hat{X}_{t^*}^{IS}}{\tilde{R}_{T|t^*}} = \left(\lambda_z \frac{\Lambda_{11}}{\Lambda_{21}} + \lambda_v \left.\frac{\Lambda_{12}}{\Lambda_{22}}\right|_{\mu\to 1}\right) (\beta + (1-\beta)(\lambda_z + \lambda_v))^{T-t^*-1}$$

Chapter 3

Temptation and the Role of the Public Insurance

3.1 Introduction

What is the value of an unemployment insurance policy? Why do we have such a policy as social insurance rather than leave it as a private market so that each individual can choose to buy? I answer this question by suggesting a new role of unemployment insurance as a correctional mechanism for job seekers' temptation. Because of this role, public insurance is superior to private market solutions in maximizing social welfare.

I first build a two-period model that features consumers, producers, and the government. Consumers search for a job, consume, and save. Producers hire and produce goods. The government collects taxes from workers and provides unemployment insurance to consumers who fail to get a job. In the model, I consider a temptation for early consumption. Consumers understand that they have to save some amount of their working period's income for later consumption, but they tend to consume more when they actually make a decision. A government, that is free from such a temptation, wants to maximize social welfare by setting the unemployment insurance (UI). In that sense, the UI works as a correctional tool as well as insurance.

I show that the UI can work as a correctional tool in two aspects. First, it urges workers to search more. In the labor market, the tempted workers find that the working state is less attractive and search less than the social optimum. Then by providing less UI, the government can make the job seekers search harder and increase the equilibrium labor. Second, the UI lets the consumers save more. In the consumption-saving decision, tempted consumers save less. Again, by providing lower insurance for the future, the government can correct the tempted consumers to save more. Because of these two functionalities of the UI, the socially optimal UI generosity will be lower than what the consumers will buy at the market.

Following Gul and Pesendorfer [2004] and Krusell et al. [2010a], I use the representation of temptation preference that has two parameters, the short-term discounting and the resistance cost. It turns out that the resistance cost is important for the optimal UI level. If the resistance cost is finite, then the UI is strictly less than the market outcome. If the cost is infinite, then the two insurance levels are identical. Although the model itself does not provide a separate identification for the short-term discounting and cost, I suggest one possible way of the identification by changing the size of the menu. Then I go to data and run a panel regression to present evidence of the finite resistance cost. I compare households that own a home to the households that live in a rented house. Homeowners have a larger menu for consumption than renters because of the possibility of liquidation. The regression result supported the hypothesis that the sample households have finite resistance costs. The result, however, is not very strong once I control the expectation of housing price appreciation.

3.2 Literature

This chapter extensively relies on Faruk Gul and Wolfgang Pesendorfer's representation of the temptation. Gul and Pesendorfer [2001] show that the "set betweenness" axioms with other standard expected utility axioms can be represented by a utility, which identifies a commitment ranking, temptation ranking, and cost of self-control. Gul and Pesendorfer [2004] extends the previous model to an infinite horizon model, and Gul and Pesendorfer [2005] discusses that the temptation representation can be used as an approximation of the time-inconsistent preference without the discontinuity problem.

Noor [2011] extends the concept of 'temptation'. In Gul and Pesendorfer [2001], the decision maker is tempted by the items on the menu. On the other hand, Noor [2011] models agents who may be tempted by the menu itself. That is, an opportunity that leads to a tempting consumption itself can be tempting. By expanding the concept, the author derives a 'refusal' for a commitment. Without the demand for a commitment, Noor [2011] identifies a temptation as a gap between the choice and a normative preference which is a preference one would get if the decision maker take an infinite distance from the problem. Heidhues and Kőszegi [2009] modeled an agent who underestimates future temptation. Since their model is about non-separable items, the parameter that governs the temptation is resistance cost only. In line with this, Ahn et al. [2020] defines naivete in the temptation preference and provides axioms that lead to a representation theorem. Their model considers general consumption and also goes beyond two periods. I use a model of a simplified two-period version of Ahn et al. [2020].

This chapter also has a strong connection to Krusell et al. [2010a]. They adopt the temptation preference in a consumption-saving problem and ask an optimal taxation on the savings. Because of the excess consumption of the consumers with temptation, the government can increase social welfare by imposing a subsidy on the saving hence restricting the consumption level. Attanasio et al. [2024] models the demand for illiquidity with the temptation preference. This demand for a commitment leads to hand-to-mouth behavior even when liquid assets deliver higher returns than illiquid assets. Amador et al. [2006] showed that a minimum saving is a key feature for a policy to be optimal when people suffer temptation, especially if one allows a demand for flexibility as well as the self-control problem.

Similarly, Kumru and Thanopoulos [2008] analyzes the welfare effects of social security when the agents have a temptation preference using an overlapping generations model. They confirmed that social security generally decreases lifetime welfare, which is in line with previous research. However, the temptation considerably reduces the negative effect of social security. The mechanism behind this result is that the social security restricts the agents' early consumption hence reducing the cost of resisting the temptation. Tran [2016] also studies the optimal policy; subsidizing savings and providing a commitment device through social security. Compared to these, this chapter is distinct because I include labor market and search friction, and I focus on the unemployment insurance policy.

I also draw a line with the literature on the optimal unemployment benefit. Ever since Chetty [2006]'s famous sufficient statistics approach for an optimal UI, several studies including Krusell et al. [2010b], Spinnewijn [2015b], McKay and Reis [2021], Kekre [2023], investigated the link between the generosity of UI benefit and aggregate consumption. Krusell et al. [2010b] integrated search friction into the risk-averse consumer's consumption-saving decision problem. Because of the uninsurable idiosyncratic income risk, the UI would increase social welfare. Quantitatively, however, they found that the optimal UI level is far from complete insurance. Spinnewijn [2015b] builds optimal UI policy rules when job seekers are failing to correctly anticipate their job-finding rate. McKay and Reis [2021] builds a similar condition for an optimal UI with Chetty [2006], but they also consider the general equilibrium effect of job search decision, namely, 'macro stabilization'. Including this channel increases the optimal level of UI substantially. In a similar specification, Kekre [2023] found that the UI benefit has a multiplier to the contemporaneous output close to 1.

Lastly, this chapter also aims to show evidence of temptation from data. Similar works can be found in Bucciol [2012], Kovacs et al. [2021], and also Toussaert [2018]. DeJong and Ripoll [2007] found that there's small evidence of self-control problems in the data. Further, their simulation showed that the adoption of temptation utility can only marginally contribute toward tackling some of the asset price puzzles. Huang et al. [2015] on the other hand, found the presence of temptation in the microdata. In this chapter, I present indirect evidence of temptation, although one should be careful in interpreting it.

3.3 Two Period Model

3.3.1 Environment

3.3.1.1 Consumers with temptation

There are measure 1 identical consumers who are initially unemployed. Their first stage problem is maximizing the expected utility by choosing 'search intensity'. While they do it, they conceive the per effort unit probability of getting a job as fixed, ignoring the general equilibrium effect of their search effort decision. That is, consumers do not take into account the effect of their job-searching decision on the labor market tightness. After an agent decides on the search intensity, the result of the search and matching comes out. I can write the job search decision problem as follows:

$$\max_{\alpha} p(\alpha;\theta)W^e + (1-p(\alpha;\theta))W^u - \Psi(\alpha) \tag{3.3.1}$$

where the probability of getting a job, $p(\cdot)$, is a function of the search intensity ($\alpha \in [0,1]$) and the labor market tightness (θ). W^i is the value of being employed ($i=e$) or unemployed ($i=u$), and $\Psi(\alpha)$ denotes an additive search cost.

After the matching, the agent moves to a consumption-saving decision. The consumption-saving stage has two sub-periods. Period 1 represents the working age where the agent's income depends on her employment status. Period 2 is the retirement age that she lives with a fixed endowment and savings from the working age.[1] A key distinction with a standard consumption-saving problem is that the consumers may suffer temptation for early consumption. I follow Gul and Pesendorfer [2004]'s representation of temptation preference. Similar parameterizations can also be found in Krusell et al. [2010a], and Ahn

[1] An endowment is necessary because I want to allow the agent to consume an unemployment benefit without saving it for her retirement. This endowment could be seen as an exogenous transfer for the retiree.

et al. [2020].[2] The value of each state, W^i, can be described as follows. For $i \in \{e, u\}$,

$$\begin{aligned} & W^i = \max_{c_1^i, c_2^i} U(c_1^i, c_2^i) - \gamma \left(V(\tilde{c}_1^i, \tilde{c}_2^i) - V(c_1^i, c_2^i) \right), \text{ s.t.,} \\ & c_1^i + s_1^i \leq y_1^i \\ & c_2^i \leq e_2 + s_1^i, \quad s_1^i \geq \underline{s} \\ & y_1^e = w - T + \frac{\Pi}{N} \\ & y_1^u = b \end{aligned} \tag{3.3.2}$$

where $U(c_1^i, c_2^i) = u(c_1^i) + u(c_2^i)$, $V(c_1^i, c_2^i) = u(c_1^i) + \beta u(c_2^i)$, and $\{\overline{c}_1^i, \overline{c}_2^i\} = \arg\max V(c_1^i, c_2^i)$ given the same constraints.

The first part of the utility function ($U(c_1^i, c_2^i)$) is the commitment utility. This is the agent's ex-ante, 'normative' evaluation of an allocation before she experiences any temptation, or the representation of her preference when the decision is about infinitely far away future as Noor [2011]. The rest of the utility function is the utility cost of resisting a temptation which is incurred when she makes a decision. The temptation that I consider here is an 'early consumption'. β in V represents the agent's desire to spend early. γ is the relative cost of resisting the temptation or the strength of it as in Dekel et al. [2009].

c_t^i is consumption of an agent at time t whose employment state is i. s_1^i is saving at period 1, and e_2 is an endowment at period 2. T is the tax for the workers, $\frac{\Pi}{N}$ is the dividend from the firm, and b is the UI benefit. I impose standard assumptions on the flow utility $u(\ \cdot\ )$ such as monotonicity, concavity, and differentiability.[3] I also limit the UI benefit such that the working state income is weakly higher than the unemployed state income ($y_1^e \geq y_1^u$).

Finally, the budget constraint contains a borrowing limit $\underline{s}$. In a special case where the borrowing limit is 0, the unemployed agents can only consume up to their current

[2]For the ease of notation, I assumed the discounting between the two periods is 1, and the interest rate is 0. A generalization, however, should be straightforward.

[3]The consumption plan is contingent on the income level, which depends on the employment state. The income uncertainty only applies to the first period, and there's no uncertainty in the second period. Therefore, there's no motive for precautionary saving in this example. This result will change in the 'consume first, search later' example which I will discuss shortly.

income. I first operate this 'no-borrowings' assumption as a baseline specification because it streamlines my exposition. I further assume that the borrowing constraint $\underline{s} = 0$ only binds for the unemployed people. In that case, the first order condition holds with equality only for the employed whereas the unemployed become 'hand to mouth' consumers. See Krusell et al. [2011], Werning [2015a], Broer et al. [2020], or McKay and Reis [2021] for applications of a similar assumption. The conclusion and intuition in this example, however, can be extended without the assumption. I discuss what I get without the assumption in the appendix B.2.1.

Note that the temptation utility in equation (3.3.2) implies demand for a commitment. Because of the resistance cost, a singleton state-contingent consumption profile $\{c_1^e, c_2^e, c_1^u, c_2^u\}$ is preferred over any choice on a menu if the consumption profile is in the argmax correspondence. However, since the saving amount can be freely chosen at the beginning of period 1, the economy is not equipped with a proper tool for a commitment in the employed state. Interestingly, the agent does have a *commitment device across the states*. In the unemployed state, the borrowing constraint binds, so the consumption set is reduced as a singleton. By choosing the search effort level, the agent can probabilistically commit to the unemployed consumption bundle. This is the source of the distortion that the social planner tries to correct. I will come back to this issue in the next section.

Also note that I use the temptation utility (value) for the job search decision (equation (3.3.1)). Although the job search decision precedes the consumption-saving problem, the agent evaluates the actual continuation value (W) rather than the normative commitment value (U). When the agent contemplates the choice of her search intensity, she gauges what she would get in the subsequent periods and what she would 'feel' about it. In this sense, the consumer anticipates her temptation in the consumption-saving stage, and it is similar to Fudenberg and Levine [2006]'s short-run-self-perfect Nash equilibrium (SR-perfect). The self who chooses the job search decision corresponds to the long run self in Fudenberg and Levine [2006], and the consumption period 1 self corresponds to the short run self.

3.3.1.2 Sophistication and naivete

I allow the agent's ex-ante perception on the temptation (either β or γ) to be different from the actual temptation as in Heidhues and Kőszegi [2009]. A subgame perfection implies that the agent solves the period 0 problem with the perception of the future temptation. Obviously, because of the possible difference between the actual and the perceived temptation, the perceived future problem that an agent will solve may not be the true subgame. I denote the perceived value as $\hat{V}^i$. Ahn et al. [2020] has detailed behavioral foundations and axiomatizations of the representation. I write the modified problem as follows:

$$\begin{aligned}
&\max_{\alpha} p(\alpha;\theta)\hat{W}^e + (1-p(\alpha;\theta))\hat{W}^u - \Psi(\alpha), \text{ s.t.,} \\
&\hat{W}^i = \max_{c_1^i, c_2^i} U(c_1^i, c_2^i) - \hat{\gamma}\left(\hat{V}(\hat{\bar{c}}_1^i, \hat{\bar{c}}_2^i) - \hat{V}(c_1^i, c_2^i)\right) \\
&\hat{V}(c_1^i, c_2^i) = u(c_1^i) + \hat{\beta}u(c_2^i) \\
&\{\hat{\bar{c}}_1^i, \hat{\bar{c}}_2^i\} = \arg\max \hat{V}(c_1^i, c_2^i)
\end{aligned} \tag{3.3.3}$$

with similar budget constraints as in equation (3.3.2). This representation says that the agent perceives that she will experience $\hat{\beta}$ temptation with $\hat{\gamma}$ resistance cost. Therefore, if $\hat{\beta} = \beta$ and $\hat{\gamma} = \gamma$, she understands her true temptation. I call this agent fully sophisticated. If $\beta < \hat{\beta} \leq 1$ or $0 \leq \hat{\gamma} < \gamma$, then this agent fails to correctly anticipate her temptation. I call the agent (partially) naive.[4] A full naivete is the case of either $\hat{\beta} = 1$ or $\hat{\gamma} = 0$. She thinks that she will not have any temptation in the consumption-saving decision before she experiences it, but when the second stage comes, she will suffer from the temptation.

3.3.1.3 Producers and Government

On the production side, there is a representative firm. It hires people and produces a consumption good in period 1. The profit maximization problem of the firm is as follows:

$$\begin{aligned}
&\max_{v} \Pi \equiv f(N^d) - w \cdot N^d - \Xi(v), \text{ s.t.,} \\
&N^d = q(\alpha, \theta) \cdot v
\end{aligned} \tag{3.3.4}$$

[4] I only consider optimistic perceptions. This agent expects that her future temptation is less severe than the actual. Therefore, drop the situation where the consumer is overly concerned about her temptation.

where N is the labor input, v is the number of job postings by the firm, q is the probability of filling a vacancy, and $\Xi(v, \theta)$ is the hiring cost.

Also, there is a government that executes a UI policy (b) which is funded by labor income tax from workers. For ease of notation, I specify the government's budget constraint as a following generic function rather than using the law of large numbers.[5]

$$T = G(b), \quad G(b)' > 0 \tag{3.3.5}$$

3.3.2 Market clearing and wage determination

In the labor market, consumers seek a job, the firm posts positions and matching is formed by a CRS matching function $M(\alpha, v)$. Note that the matching function is augmented with a search intensity as in Pissarides [2000].

$$N^d = M \equiv \alpha^\eta v^{1-\eta} \tag{3.3.6}$$

Define $q(\alpha, v) := \frac{M}{v} = \alpha^\eta v^{-\eta}$, $\tilde{p}(\alpha, v) := \frac{M}{u} = vq(\alpha, v)$, where $q(\cdot)$ is the vacancy filling rate per posting, and $\tilde{p}(\cdot)$ is the overall job finding rate, resepctively. I further simplify the notation by defining an 'augmented market tightness' ($\theta := \frac{v}{\alpha}$) which implies

$$\frac{\tilde{p}(\alpha, v)}{\alpha} = \frac{M}{\alpha} = \theta q(\theta).$$

Therefore, the probability of getting a job per search effort is $p(\theta) = \theta q(\theta)$, and the total labor supply is $N^s = \tilde{p}(\alpha; \theta) = p(\theta) \cdot \alpha$. The firm's total hiring is $N^d = q(\theta) \cdot v$.[6] Finally, the wage is determined as a bargaining solution between the workers and firms. Let φ be the bargaining power of the worker. Then the solution w satisfies

$$(1 - \varphi) \left(\hat{W}^e - \hat{W}^u \right) = \varphi (A - w). \tag{3.3.7}$$

[5] In these two periods example, everyone is unemployed at the beginning, so $u = 1$. Then the probability of getting a job p is equal to $\frac{M}{u} = M$ where M is the number of new matches that is identical to the number of employed people (N). Therefore, the budget balance can be stated as $N \cdot T = (1 - N) \cdot b$.

[6] Labor supply comes from the consumer's problem as $N^s = p(\alpha, \theta)$ since the whole population is unemployed at the beginning. If I generalize this to the multiple periods problem, I can write the law of motion as $\dot{u}_t = p(\alpha_t, \theta_t) \cdot u_t + \lambda_t \cdot (1 - u_t)$ where λ_t is an exogenous job separation. Then, the labor supply at each period is $N_t^s = \tilde{\theta}_t q(\tilde{\theta}_t) \cdot \alpha_t (1 - N_{t-1}) + (1 - \lambda_t) N_{t-1}$. It is evident that $N_t^s = \tilde{\theta} q(\tilde{\theta}) \cdot \alpha$ once I set $N_{t-1} = 0$.

The equilibrium is defined as follows. Given the real wage w, consumers solve the utility maximization problem with a perception perfection. That is, the search decision is made with the perceived consumption-saving (equation (3.3.3)), and the actual consumption-saving decision is made with equation (3.3.2). The firm solves profit maximization (equation (3.3.4)), government's budget balance holds (equation (3.3.5)), labor market clears (equation (3.3.6) and (3.3.7)), and finally the goods market clears as below.

$$N \cdot (c_1^e + c_2^e) + (1 - N)\,(c_1^u + c_2^u) = f(N) - \Xi(v) + e_2$$

3.3.3 Characterizing an equilibrium

To get a sharper analytical result, I assume a few simplifying parametrizations; the utility function is logarithmic and the search cost is quadratic, production technology and hiring costs are linear ($\Psi(\alpha) = \frac{\psi}{2}\alpha^2$, $f(N) = AN$, $\Xi(v) = \xi v$). From the consumer problem, we get the consumption policy function which is linear to her lifetime income as follows:

$$c_1^e = \frac{1+\gamma}{1+\gamma+(1+\beta\gamma)} y^e, \quad c_2^e = \frac{(1+\beta\gamma)}{1+\gamma+(1+\beta\gamma)} y^e$$

where y^e is the lifetime income when the consumer is employed ($y^e = y_1^e + e_2$). We also get the most tempting choice $\overline{c}$ by maximizing V.

$$\overline{c}_1^e = \frac{1}{1+\beta} y^e, \quad \overline{c}_2^e = \frac{\beta}{1+\beta} y^e$$

Note that the agent consumes more at period 1 if the temptation becomes harder to resist either because of the resistance cost (γ), or the extent of the temptation (β). For any given temptation β, c^e converges to $\overline{c}^e$ as γ goes to infinity. Also c^e converges to the 'normative' ideal ($c^{e*} = \{\frac{y^e}{2}, \frac{y^e}{2}\}$) as the cost goes to zero. I call the two extremes "full temptation" and "commitment", respectively. The full temptation consumption is the most impulsive choice given β, whereas the commitment consumption is the most temperate one. The actual consumption profile is somewhere in between.

$$\underbrace{\frac{1}{1+\beta} \cdot y^e}_{\text{full temptation } (\overline{c}^e)} \geq \underbrace{\frac{1+\gamma}{1+\gamma+(1+\beta\gamma)} \cdot y^e}_{\text{actual choice } (c^e)} \geq \underbrace{\frac{y^e}{2}}_{\text{commitment } (c^{e*})}$$

Turning to the unemployed agent's problem, I initially assume that the borrowing limit ($\underline{s}$) is zero and the endowment is large enough so that she would not save the UI for the next period of consumption. Then the unemployed agent has a binding financial constraint, and she can only consume what is given at period 1.

$$c_1^u = b, \qquad c_2^u = e_2$$

Using the above results, I pin down the ex-post value of each employment status, and compactly rewrite the additional value of getting a job as follows:

$$W^e - W^u = \ln \frac{(y^e)^2 \cdot C(\beta, \gamma)}{b}$$

where $C(\beta, \gamma)$ is a collection of parameters.[7]

The agent solves the job-searching problem given the perception of her temptation. The first-order condition implies that the marginal cost of additional effort should be identical to the perceived expected marginal benefit. Likewise, the optimal job posting comes from the firm's first-order condition.

$$\begin{aligned} \Psi'(\alpha) &= p(\theta)(\hat{W}^e - \hat{W}^u) \\ w &= f'(N^d) - \frac{\Xi'(v)}{q(\theta)} \end{aligned}$$

The simplifying parametric assumptions stated earlier and the bargaining solution lead the above equations to the following equilibrium conditions.[8]

$$w = A - \frac{\xi}{q(\theta)}, \quad \text{“Labor Demand”} \tag{3.3.8}$$

$$\psi\alpha = \frac{\varphi}{1-\varphi}(A - w)p(\theta), \quad \text{“Labor Supply”} \tag{3.3.9}$$

Equating the demand and supply gives the labor market clearing. The proposition below provides comparative statics: how the equilibrium employment changes with the unemployed insurance benefit and temptation.

[7] $C := \frac{D^{1+\gamma} \cdot (1-D)^{1+\beta\gamma} \cdot (1+\beta)^\gamma}{(\frac{\beta}{1+\beta})^{\beta\gamma} \cdot e_2}$ where $D := \frac{y^e}{c_1^e}$, and $\overline{D} := \frac{\bar{c}_1^e}{y^e}$.

[8] $\Xi(v; \tilde{\theta}) = \xi \cdot q(\tilde{\theta})v$, $\Psi(\alpha; \tilde{\theta}) = \psi \cdot p(\tilde{\theta})\alpha$ and both consumers and firms perceive the effective market tightness($\tilde{\theta}$) as a parameter that shows the market condition.

Proposition 3.3.1. *The unemployment insurance benefit negatively affects the equilibrium search effort, market tightness, and employment* ($\frac{\partial \alpha^*}{\partial b} < 0$, $\frac{\partial \theta^*}{\partial b} < 0$, $\frac{\partial N^*}{\partial b} < 0$)*. The agent provides less labor as the perceived resistance cost goes up* ($\frac{\partial \alpha^*}{\partial \hat{\gamma}} < 0$) *and as the perceived temptation increases* ($\frac{\partial \alpha^*}{\partial \hat{\beta}} > 0$)*. The equilibrium labor moves along with the search effort* ($\frac{\partial N^*}{\partial \hat{\gamma}} < 0$, $\frac{\partial N^*}{\partial \hat{\beta}} > 0$)*.*

The proposition comes from the fact that the expected value of being employed decreases with the perceived resistance cost ($\hat{\gamma}$), and increases with the perceived short-term discounting factor ($\hat{\beta}$). Therefore, as the $\hat{\gamma}$ becomes larger and $\hat{\beta}$ becomes smaller, an agent has less incentive to search for a job. This result shows that the agent uses the unemployed state as a commitment device. Note that the value of being unemployed (W^u) does not depend on $\hat{\gamma}$ and $\hat{\beta}$. As the agent expects a larger temptation, the singleton consumption profile of the unemployed state becomes relatively more attractive.

3.4 Optimal Policy

The government implements the UI policy by setting the generosity of the benefit b under the budget constraint. I assume that the government is temptation-free, hence the objective of the utilitarian government is as follows:

$$\max_b \mathcal{W} := NU(c_1^e, c_2^e) + (1-N)U(c_1^u, c_2^u) - \Psi(\alpha) \tag{3.4.1}$$

The optimal policy maximizes the weighted average of utilities given the equilibrium search effort and consumption policy functions derived in the previous section. The objective function resembles the consumer's job search problem (equation (3.3.1)). The difference is that the employment status is evaluated with the commitment utility (U), rather than the continuation value (W). This is the case where people in society delegate their insurance design problem to a government, and the government chooses the insurance level on behalf of people to their best 'long-run' interests without considering the principal's temptation. What the government cannot do is force people to consume a specific consumption sequence. The government pursues the constrained optimum by respecting the perception-perfect equilibrium derived previously. To this end, I assume that social welfare is concave hence there is an internal solution to the problem.

Assumption 3.4.1. *The social welfare function $\mathcal{W}$ is concave ($\frac{\partial^2 \mathcal{W}}{(\partial b)^2} < 0$).*

The following proposition describes the optimal UI level (b^*) under the above assumption.

Proposition 3.4.1. *The optimal unemployment insurance benefit b^* satisfies the following relation.*

$$0 = \underbrace{(\Lambda + \Omega) \left.\frac{\partial N}{\partial b}\right|_{b=b^*}}_{\textit{temptation correction}} + \underbrace{(1-N)\left(-\frac{2}{y^e} + \frac{1}{b^*}\right)\bigg|_{N=N^*}}_{\textit{insurance}} - \underbrace{N \frac{2}{y^e} \frac{\partial y^e}{\partial N} \left.\frac{\partial N}{\partial b}\right|_{b=b^*}}_{\textit{incentive}} \tag{3.4.2}$$

where Λ is the resistance cost ($U - W^e$), and Ω is the perception gap ($W^e - \hat{W}^e$).

Equation (3.4.2) in the proposition is in line with Baily [1978], Chetty [2006], and Spinnewijn [2015b], except the first part, 'temptation correction'. This correctional motive can be decomposed into two parts: the resistance cost (Λ), and a misperception of the temptation (Ω).[9]

If the temptation is resistible, which means that γ is finite, then Λ is strictly positive. Under the overwhelming temptation ($\gamma = \infty$), on the other hand, the agent fully succumbs to the temptation and hence pays no resistance cost and $\Lambda = 0$. Ω is the perception wedge between the ex-ante and and ex-post value of being employed, and it is related to the sophistication of the agent's perception. If the agent is fully sophisticated, the wedge goes away ($\Omega = 0$). If the agent is naive at least partially, the gap becomes a negative value ($\Omega < 0$).

We then link the two parts of the correction with the value of UI as social insurance. First, the UI restricts excessive insurance compared to the private market. To see this, consider 'buying insurance' from a fair private market before the agent makes any decision.

[9]DellaVigna and Malmendier [2004] also has a similar decomposition of the actual bias and perception. Theirs was under quasi-hyperbolic discounting, whereas I am using the temptation utility in this chapter.

It can be stated as follows:

$$\max_{\alpha,x^e,x^u} p(\alpha)W^e + (1-p(\alpha))W^u - \Psi(\alpha), \text{ s.t.,}$$

$$c_1^i + s_1^i \leq y_1^i$$

$$c_2^i \leq e_2 + s_1^i$$

$$y_1^e = w + x^e + \frac{\Pi}{N}$$

$$y_1^u = x^u$$

$$q^e x^e + q^u x^u = 0$$

where x^e and x^u is the state contingent claims and q^e, q^u are associated prices. Fair insurance implies that the prices are identical to the probability of the states which is $q^e = p$, $q^u = 1-p$. Then the problem is isomorphic to the optimal policy problem except that the commitment utility (U) is replaced with the actual values (W^i), and the extent of insurance only varies with the 'resistance cost'.

$$0 = \Omega \left.\frac{\partial N}{\partial x^u}\right|_{x^u=x^{u*}} + (1-N)\left(-\frac{2}{y^e} + \frac{1}{x^{u*}}\right)\bigg|_{N=N^*} - N\frac{2}{y^e}\frac{\partial y^e}{\partial N}\left.\frac{\partial N}{\partial x^u}\right|_{x^u=x^{u*}}$$

In the following lemma, I show that $b^* \leq x^{u*}$, and the inequality is strict as long as the temptation is resistible ($\Lambda > 0$). The government will set the UI at a (weakly) lower level than what the agents would buy in the private market because the value of getting a job is less appreciated among the tempted agents.

Lemma 3.4.1. *The optimal UI benefit b^* is weakly smaller than the private market insurance level x^{u*}. The optimum level is strictly smaller than the market insurance if the temptation is resistible.*

Lemma 3.4.1 implies contrasting levels of the optimal public insurance for the two possible specifications of temptation. If the utility approximates present bias (overwhelming temptation), the optimal UI is equal to the private insurance level. On the other hand, if the utility represents resistible tempted choices, the optimal UI is smaller than the private market. In the present bias limit, the agent does not pay any resistance cost since she fully succumbs to the temptation. If the agent doesn't pay the mental cost, there's no reason to punish the choice with the correctional term. On the contrary, an agent with tempted choice pays a mental cost to take a balance between the most tempting choice and the most temperate one. The existence of the mental cost is the key to understanding the optimal policy difference.

Note that the UI does not work as a correction for early consumption. Although the U-maximizing government does not find the tempted choice optimal ($\{c_1^{e*}, c_2^{e*}\} \notin \arg\max U$), UI benefit and associated labor income tax cannot affect the consumption decision within the states. Rather, they are transfer mechanisms across states (from employed to unemployed), so the tool fills the gap in the value of transition ($W^e - W^u$). That is, in the job search decision, the agent uses the unemployed states as a probabilistic commitment device, and she values the search action less than the U-maximizing government.[10] The temptation correction in the optimum UI accommodates this. As a result, the UI may expand the menu of choice in some cases (for example, sophisticates with a resistible temptation) making the early consumption problem more severe.

3.5 Consume First, Search Later

We slightly change the timing of events to observe the effect of UI policy on the consumption-saving decision. The key difference from the previous example is that the agent searches for a job right after the consumption-saving decision has been made in period 1, not before it. In this way, the change of UI benefit affects the inter-temporal allocation within the state as well as the allocation across the states.

Start with the search decision. This is after the period 1 consumption and before the period 2 consumption. The job search problem is as follows:

$$\max_{\alpha} \tilde{\beta}\left(p(\alpha;\theta)u(c_2^e) + (1 - p(\alpha;\theta))u(c_2^u)\right) - \Psi(\alpha) \tag{3.5.1}$$

where $\tilde{\beta}$ is the effect discounting factor, p is the probability of getting a job, Ψ is the search cost as in the previous section.

Let the solution of job search problem α^*. Given the solution, the consumption-saving

[10] See Bryan et al. [2010] for an extensive review of various commitment devices.

problem in period 1 is as follows:

$$
\begin{aligned}
&\max_{c_1, c_2^e, c_2^u} U(c_1, \mathbb{E}_1[c_2|\alpha^*], \alpha^*) - \gamma \left(V(\tilde{c}_1, \mathbb{E}_1[\tilde{c}_2|\tilde{\alpha}^*], \tilde{\alpha}^*) - V(c_1, \mathbb{E}_1[c_2|\alpha^*], \alpha^*)\right) \text{ s.t.,} \\
&c_1 + s_1 \leq e_1 \\
&c_2^e \leq s_1 + w - T + \frac{\Pi}{N} \\
&c_2^u \leq s_1 + b \\
&\mathbb{E}_1[c_2|\alpha] = p(\alpha;\theta)u(c_2^e) + (1 - p(\alpha;\theta))u(c_2^u)
\end{aligned}
\tag{3.5.2}
$$

and similarly to the previous example, $U(\cdot)$, $V(\cdot)$ indicates the commitment and temptation utility functions ($U(c_1, c_2, \alpha) = u(c_1) - \Psi(\alpha;\theta) + u(c_2)$, $V(c_1, c_2, \alpha) = u(c_1) - \Psi(\alpha;\theta) + \beta u(c_2)$). Note that this formulation implies the 'immediate cost and delayed benefit' as in O'Donoghue and Rabin [2001b], DellaVigna and Malmendier [2004], DellaVigna and Paserman [2005b]. The rest of the model, firm's problem, government budget constraint, labor market clearing, and goods market clearing are almost identical to those in the previous example except that the production happens in period 2.

Proposition 3.5.1. *The optimal UI generosity b^* satisfies the following marginal utility ratio.*

$$
\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{b^*} = \left(\frac{1 - (1-\tilde{\beta})\frac{\partial c_1}{\partial b} + \left(\frac{1}{\tilde{\beta}} - 1\right)\Psi'(\alpha)\frac{\partial \alpha}{\partial b}}{G'(b) + (1-\tilde{\beta})\frac{\partial c_1}{\partial b} - \left(\frac{1}{\tilde{\beta}} - 1\right)\Psi'(\alpha)\frac{\partial \alpha}{\partial b}} \right) n
$$

where $n := \frac{1-N}{N}$ is the relative number of people who will benefit from the UI policy. On the contrary, private insurance in a complete financial market x^{u*} satisfies the following marginal utility ratio.

$$
\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{x^{u*}} = 1
$$

Unlike the previous example, the two insurance schemes do not coincide even without a temptation. If there's no temptation (either $\beta = 1$ or $\gamma = 0$ so $\tilde{\beta} = 1$), the agent will buy insurance so that she can buy complete insurance for the income risk. On the other hand, the government will design public insurance considering the tax burdens.

$$\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{x^{u*}} = 1$$
$$\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{b^*} = \frac{1}{G'(b)}n$$

In what follows, I assume that the consumption in period 1 increases with the UI benefit.

Assumption 3.5.1. *Equilibrium savings at period 1 decrease if the UI benefit generosity becomes larger.*

$$\frac{\partial s_1^*}{\partial b} < 0, \quad \frac{\partial c_1^*}{\partial b} > 0$$

This is a reasonable assumption because higher UI implies less demand for savings since the importance of savings as insurance for a rainy day (precautionary savings) diminishes. With this assumption, I can further argue the following.

Lemma 3.5.1. *The optimal UI generosity is lower than the market output $b^* < x^{u*}$ even without any temptation.*

Then we introduce the temptation and claim that the temptation broadens the gap between the two insurance schemes.

Lemma 3.5.2. *The optimal UI benefit for an agent with temptation is lower than the optimal UI under no temptation. Therefore, the optimal UI is less than the private insurance level.*

The implication of the lemma is obvious. An agent who suffers from temptation saves less and consumes more at the early stage. The government can motivate larger savings by offering a lower level of UI benefit.

3.6 Identifying Temptation

3.6.1 Strategy

The temptation representation does not provide a separate identification for β and γ in many cases. That is, given an observed allocation $\{c_1^e, c_2^e\}$, an econometrician can't distinguish between (β', γ') and (β'', γ'') as long as ($\tilde{\beta} := \frac{1+\beta'\gamma'}{1+\gamma'} = \frac{1+\beta''\gamma''}{1+\gamma''}$).[11] Two extreme interpretations are readily available. One is Laibson [1997]'s quasi-hyperbolic discounting agent. As Gul and Pesendorfer [2005], and Krusell et al. [2010a] pointed out, this temptation model becomes a present biased multiple selves model as $\gamma \to \infty$. In that case, the agents fully succumb to the temptation, discounting future $\tilde{\beta}\delta$ when they make a decision, but they evaluate their utility using δ discounting. In that case, the model has an interpretation of quasi-hyperbolic discounting with a short 'present' as in Harris and Laibson [2013]. Another interpretation is Gul and Pesendorfer [2004]'s agent who resists the urge to spend as much as possible ($\beta = 0$). The maximal temptation in the choice set is $c_1^e = y_1^e - \underline{s}$ with a monotonic felicity utility function $u(\cdot)$. Then the observed allocation can be rationalized with $\beta = 0$,[12] and associated temptation cost is,

$$\gamma = \frac{\delta + 1 - (1 + \tilde{\beta}\delta)\left(\frac{y_0^e - \underline{s}}{y^e}\right)}{(1 + \tilde{\beta}\delta)\left(\frac{y_0^e - \underline{s}}{y^e}\right) - 1}$$

where $\tilde{\beta}$ is again, the observed short-term discounting. The agent is tempted to spend all her available resources, but as a result of resistance, she ends up allocating some of her income (and wealth, if she has any) for future consumption as the observed consumption profile. Which story one follows doesn't matter for a positive analysis because of the lack of identification and observational equivalence between the two. As we have seen in Lemma 3.4.1, however, it is important to distinguish the resistance cost to argue the optimal UI benefit.

To this end, I propose the following strategy to identify whether the temptation is overwhelming or resistible. First, let's say that the problem is choosing a consumption level c_1 under a budget (income and asset) z_1 and leaving a subsequent problem with a budget z_2. So the problem is choosing $c_1 \in [0, z_1]$. Denote the commitment choice

[11] In this 2 periods example, the distinction between exponential and hyperbolic discounting is impossible. Therefore I don't have identification for $\tilde{\delta}$ and $\beta\delta$. In the infinite horizon example, however, those two models have different predictions of the consumption stream. I assume that I have identified δ for now to focus on the temptation and hyperbolic discounting.

[12] some papers try to estimate the temptation cost by assuming $\beta = 0$ using the illiquid asset as a commitment device. See Bucciol [2012], Kovacs et al. [2021] Also see Toussaert [2018] for an identification in an experimental setting. Here I adopt a different identification strategy.

c_1^*, the most tempted option $\overline{c}_1$, and the observed actual choice c_1. It is most likely to have $c_1^* < c_1 < \overline{c}_1 < z_1$ if the temptation is resistible, and $c_1^* < c_1 = \overline{c}_1 < z_1$ if the temptation is overwhelming. Now I introduce a nonbinding financial constraint $\overline{s}_1 > c_1$ close enough to the actual choice c_1. For example, the constraint is a minimum saving requirement which is lower than the current saving $(z_1 - c_1)$. Note that the introduction of the nonbinding constraint doesn't change the continuation value since the possible saving amount is unaltered. The introduction of the non-binding financial constraint, however, has different implications for the overwhelming and resistible temptation. For the overwhelming temptation, the choice should be preserved. For the resistible temptation, the menu that the agent can go over is shrunk and the most tempting option is not available after the saving requirement. The constraint is then relevant to her current consumption.

3.6.2 Data

To give an idea about the strategy, I present indirect evidence of a finite resistance from data. The data I shall use is Korean Public Finance Panel data. It is a yearly survey panel data ranging from 2008 to 2021, and the number of sample households is around 5,000 each year.

Interestingly, the survey contains questions on the time preference of the interviewee. The questionnaire is as follows. Let's assume that you have 20 tokens. Each token is worth 100,000 Korean won (comparable to roughly 100 dollars) and you can change it as cash tomorrow. On the other hand, if you change it to cash in a month, you will get the following amount of cash: a) 101,000, b) 101,250, c) 101,500, d) 101,750, e) 102,000. In each of the scenarios, the survey asks how many tokens the interviewee would change for cash tomorrow and how many tokens she would keep for a month. Based on the answers to the questions, I built a discrete measure of time preference. If an individual answers that she will keep 20 tokens until a month for all scenarios, then I set the beta equal to 5 indicating the lowest discounting (discounting factor being close to 1). On the other hand, if an individual answers that she would change the tokens to cash tomorrow in all scenarios, then I set the beta to zero indicating the highest discounting (discounting factor being close to 0). From 1 to 4, increasing numbers indicate the person has changed to cash out option in the higher interest rate scenario. This question is included in the recent 3-year survey waves. I averaged the answers across 3 years for each individual. I then link the individual data to household data by averaging the betas within the household members. That is, the discounting is a recent three-year average of all individuals' time preferences in the household.

The situation I am specifically examining with the data is buying a car. In the data, there is an item indicating in which year the household bought a car at what price. Using it, I calculate how much a household spent on buying a car in a specific year. I chose this exercise rather than using questions on the monthly consumption because I believe I can find temptation more easily in the one-time consumption decision than in the monthly averaged consumption data. Further, the monthly consumption contains many subitems including expenditure on the grocery, transportation, rent, and many other items that temptation might not be a strong motive, and the composition of items may be different across households. Car buying is not very frequent, one need not consider the composition or quantity, and yet there are enough options that might be tempting.

Importantly, I need to find comparable groups to identify the 'resistible' temptation. For that purpose, I distinguish households based on their ownership of their house. There are 5 options in the survey: a) own a house, b) rent a house with a deposit only, c) rent a house with a deposit and monthly rental payment, d) rent a house with a monthly rental payment only (no deposit), and e) free of charge. In the context of identifying the resistance, I focus on the comparison between the household that owns a house and the household that stays in a house with a deposit only. This is because if other things are equal, the only difference between the two types is the availability of liquidating the housing asset. Consider two households living in a similarly valued house. A household that owns it can liquidate the asset by getting a mortgage. On the other hand, a household that lives in a rented house should give up their right to the deposit during the occupancy of the house. Other types of ownership are not as comparable as the former two since they differ in monthly disposable income after the rent payment.

Because of the availability of liquidation of the housing asset, the menu that a household can entertain is larger if it owns a house. If the household experiences an overwhelming temptation, that is a $\beta\delta$ discounting agent, the availability of funding does not affect the car-buying decision after controlling the actual debt amount. If the household is resisting the temptation, then the existence of the funding opportunity extends the menu that the household can go through and it is consequential in the car-buying decision even after controlling the actual debt amount. My hypothesis is as follows. Controlling other things, the ownership of the house positively affects the price of the newly bought car. If I observe this behavior in the data, I interpret this as indirect evidence of resistible temptation (finite resistance cost γ).

3.6.3 Result

To test the idea, I run a panel regression. The result is in Table 3.7.1. The dependent variable in each of the 6 specifications is the log price of the newly bought car. Independent variables include the homeownership dummy and control variables such as the size of the family, age of the head of the household, job status for the first two individuals in the household, average monthly income, savings, credit card debt, housing assets including rental deposits, time preference (β), other financial debt excluding the credit card debt, and year fixed effects. Note that I control for the credit card debt as well as the time preference measure β I built from the other part of the survey. Credit card debt is included as an instrument for a possible present bias. See Meier and Sprenger [2010] for the link between credit card debt and present bias. β is included to control for the time preference of the household.

The difference between specifications (1), (2), (3) and (4), (5), and (6) is the inclusion of a none-buying year. For example, assume that a household bought a car in 2015 at 10,000 USD, and bought another in 2020 at 20,000 USD. The former three specifications include observations from 2015 to 2019 and use 10,000 USD as a dependent variable. From 2020 to 2022, the dependent variables are 20,000 USD. Specifications (4), (5), and (6), on the contrary, drop observations from 2016 to 2019 as well as those from 2021 to 2022. It only contains 2015 and 2020 answers. Other than that, any differences among specifications are indicated in Table 3.7.1.

What I found is that home ownership positively affects the newly bought car price across different specifications of the regression except for model (3). As Table 3.7.1 shows, the effect is statistically significant even if we control the time preference (β), possible present bias (Credit card debt), direct liquidation of the housing asset (Total debt), income and the value of other assets. This means that the households decide to buy a more expensive car just because they own their home, compared to similar households living in a similarly valued rented house. As I have argued, the result supports the hypothesis and we can think that it is evidence of resistible temptation. Since the only difference between the two households is the amount they can liquidate from the housing, hence the set they can entertain when they make a decision. Without resistance cost, the choice shouldn't be changed after controlling the debt.

One obvious limitation of the result is that there might be a channel through the expectations of the house price appreciation. If a household that owns a house expects the value of the house to rise in the future, the household may increase consumption. There are no direct questions in the survey concerning the expectations of the house price so I can't control it directly. However, assuming that the aggregated expectations on the housing markets are correlated among the households, I control the expectation channel by including the interaction term between the year fixed effect and the home ownership. That is, I allowed the year-fixed effect to be different between homeowners and renters so that it can capture possible deviations caused by the expectations on the asset appreciation. The result is mixed and reported in Table 3.7.2. The model specification is identical to previous regressions, and the only difference is that I added the interaction terms in the fixed effects. The statistical significance of the positive effect of homeownership survives even after controlling the expectation in models (1), (2), and (3). On the other hand, in models (4)-(6), the significance goes away as I include the interactions. It seems that part of the positive relationship between car price and homeownership comes from the expectation of price appreciation, but part of it also comes from the extent of availability and associated cost to resist the possible options.

3.7 Conclusion

When a job seeker suffers temptation for early consumption, the government can use unemployment insurance as a two-way correctional device. First, in the labor market, the tempted job seeker overvalues the unemployed states and hence searches less than the social optimum. Lowering UI generosity can make the tempted agent search harder. This may hurt the agent's short-term utility, but it enhances her long-term (commitment) utility. Second, in the consumption-saving decision, the tempted consumer saves too little. UI can work as a compulsive saving through labor income taxation. Both mechanisms lead to the optimal UI level being set weakly lower than the market equilibrium insurance level. The resistance cost is important to decide whether the UI is strictly less than market insurance or not. If the temptation is resistible (finite resistance cost), then UI is strictly less than the market insurance. Using Korean data, I found evidence that people have temptation utility with the (finite) resistance cost. The identification strategy uses the size of a menu that a household can entertain when it buys a car, which is distinct from the existing literature.

Table 3.7.1: Estimation Results

	(1)	(2)	(3)	(4)	(5)	(6)
Car price(-1)	0.696***	0.696***	0.733***	0.251***	0.252***	0.274***
	(41.74)	(41.82)	(48.48)	(5.17)	(5.22)	(5.99)
# of people	-0.00144		0.00104	0.0161		
	(-0.28)		(0.25)	(0.48)		
Income	0.0358***	0.0371***	0.0509***	0.126**	0.137***	0.130***
	(5.28)	(5.66)	(7.69)	(2.55)	(2.79)	(2.96)
Financial asset	0.00303**			0.0143		
	(2.06)			(1.26)		
Credit card debt	0.00873**			0.0372		0.0409
	(2.22)			(1.26)		(1.46)
β	-0.00704*	-0.00683*	-0.00358	-0.0155	-0.0180	
	(-1.84)	(-1.79)	(-1.03)	(-0.58)	(-0.69)	
Non financial asset	0.00996***	0.00957***	0.00768***	0.0307	0.0291	0.0402*
	(3.49)	(3.44)	(3.02)	(1.39)	(1.32)	(1.92)
Total debt	-0.000844		0.000379	-0.00515		
	(-0.84)		(0.40)	(-0.59)		
Homeownership	0.0515***	0.0490***	0.0156	0.243***	0.234**	0.244***
	(4.05)	(3.98)	(1.50)	(2.68)	(2.58)	(2.96)
Control dummies	Yes	Yes	No	Yes	Yes	Yes
non buying year	Yes	Yes	Yes	No	No	No
N	13773	13773	14188	789	789	886

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3.7.2: Controlling Homeownership-specific Year Fixed Effects

	(1)	(2)	(3)	(4)	(5)	(6)
Car price(-1)	0.696***	0.696***	0.731***	0.252***	0.253***	0.276***
	(41.74)	(41.83)	(47.84)	(5.05)	(5.11)	(5.95)
# of people	-0.00158		0.00255	0.0166		
	(-0.31)		(0.61)	(0.49)		
Income	0.0357***	0.0369***	0.0496***	0.127**	0.138***	0.131***
	(5.26)	(5.63)	(7.52)	(2.49)	(2.71)	(2.92)
Financial asset	0.00300**			0.0144		
	(2.04)			(1.26)		
Credit card debt	0.00879**			0.0381		0.0373
	(2.23)			(1.26)		(1.32)
β	-0.00693*	-0.00672*	-0.00343	-0.0161	-0.0185	
	(-1.81)	(-1.75)	(-0.98)	(-0.59)	(-0.69)	
Non financial asset	0.00993***	0.00953***	0.00796***	0.0314	0.0304	0.0422**
	(3.46)	(3.41)	(3.13)	(1.43)	(1.38)	(2.01)
Total debt	-0.000832		0.000259	-0.00453		
	(-0.83)		(0.27)	(-0.52)		
Homeownership	0.0499**	0.0465**	0.0376**	0.329	0.342	0.349
	(2.09)	(1.99)	(2.48)	(1.41)	(1.48)	(1.64)
N	13773	13773	14188	789	789	886

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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Appendix A

Proofs

A.1 Chapter 1

A.1.1 Proposition 1.3.1

Proof. First, note that a time-varying UI scheme cannot admit a symmetric equilibrium.[1] That is, if the UI benefit is not a constant, the equilibrium strategy $x_t(\hat{s}^t)$ is also time-varying, and there exists at least one point $s \geq t$ such that $R_s \neq R_{s+1}$. This can be shown by contradiction. Let's say $\bar{t}+1$ is a breakpoint of z (a point that UI benefit level changes, $z_{\bar{t}+1} \neq z_{\bar{t}}$) and the equilibrium is symmetric so that R^* is constant. By the definition of the symmetric equilibrium, the reservation wage should be R^* for all periods. At period t, the symmetric equilibrium suggests the same R^*, but the different z implies different R because of the tight relationship between z and R. Writing in equations, the symmetric equilibrium satisfies $\frac{u(R_{\bar{t}})}{1-\delta} = \frac{u(R^*)}{1-\delta} = u(z_{\bar{t}+1}) - k(\hat{\alpha}^*) + \frac{k'(\hat{\alpha}^*)\hat{\alpha}^*}{\hat{\beta}}$. Moving one period ahead, $\frac{u(R_{\bar{t}-1})}{1-\delta} = \frac{u(R^*)}{1-\delta} = u(z_{\bar{t}}) + k(\hat{\alpha}^*) - \frac{k'(\hat{\alpha}^*)\hat{\alpha}^*}{\hat{\beta}}$. The two equations hold at the same time only if $z_{\bar{t}} = z_{\bar{t}+1}$, which is a contradiction since $\bar{t}+1$ is the breakpoint. Therefore, if there's an equilibrium, it must be a non-symmetric one. For the later discussion, I consider one specific form of UI sequence.

Proposition 1.3.1 consists of 3 parts: existence, uniqueness, and non-increasing se-

[1] An equilibrium is symmetric if all players (selves in different time frames) in the game play the same strategy.

quence of R for decreasing z. For the existence, equation (1.3.2) suggests

$$\frac{u(\hat{R}_s)}{1-\delta} = u(z_{s+1}) + \delta u(z_{s+2}) + \delta^2 u(z_{s+3}) + ... + \xi_{s+1} + \delta\xi_{s+2} + \delta^2\xi_{s+3} + ... \tag{A.1.1}$$

where, $\xi_s = \frac{k'(\hat{\alpha}_s)\hat{\alpha}_s}{\hat{\beta}} - k(\hat{\alpha}_s)$ which is positive.[2] Denote the maximum among $\{z_t\}_{t=0}^{\infty}$ as $\overline{z}$ and the maximum among $\{\xi_t\}_{t=0}^{\infty}$ as $\overline{\xi}$ $(= \frac{k'(1)}{\hat{\beta}} - k(1))$. Then it is straightforward that $u(R_s) \leq u(\overline{z}) + \overline{\xi}$. Since $u(\cdot)$ is a strictly monotonic concave function, there exist R_s^* that satisfies equation (A.1.1) if $\exists \tilde{R} \equiv u^{-1}[u(\overline{z}) + \overline{\xi}] \in [\underline{w}, \overline{w}]$.

For the uniqueness, note that for a given belief $\hat{s}^{t+1}$, $\hat{R}_t$ and R_t are uniquely determined because $W_{w,t+1}$ is an increasing function of w_t and $W_{u,t+1}$ is constant on (independent to) w_t. Then, $\hat{a}_t$ and a_t are also unique because of equation (1.3.1) and equation (1.3.2). Therefore, the only thing that matters is how the initial reservation wage is selected.

Lastly, I can write equation (A.1.1) as follows:

$$\frac{u(\hat{R}_s) - u(\hat{R}_{s+1})}{1-\delta} = \sum_{n=0}^{\infty} \delta^n \{u(z_{s+n+1}) - u(z_{s+n+2}) + \xi_{s+n+1} - \xi_{s+n+2}\}$$

ξ_s is a decreasing function of $\hat{R}_s$, which means the right-hand side is positive. Therefore, $\{R_t\}_{t=0}^{\infty}$ shouldn't be an increasing sequence. □

A.1.2 Proposition 1.4.1

Proof. The Lagrangian of the dual problem is,

$$\begin{aligned} L &= z_t + (1-p_t)(\delta z_{t+1} + h_t) \\ &- \lambda_t \left\{ u(z_t) - k(\alpha_t) + \delta\alpha_t \int_{u^{-1}\{(1-\delta)W_{u,t+1}\}}^{\overline{w}} W_{w,t+1} dF(w) + (1-p_t)\delta W_{u,t+1} - \bar{V}_{u,t} \right\} \end{aligned}$$

where α_t is implicitly defined as

$$k'(\alpha_t) = \beta\delta \int_{u^{-1}\{(1-\delta)W_{u,t+1}\}}^{\overline{w}} (W_{w,t+1} - W_{u,t+1}) dF(w)$$

as in equation (1.3.1) and h_t collects terms that is associated with $t+2$ and further future variables ($h_t := \delta^2 z_{t+2}(1-p_{t+1}) + \delta^3 z_{t+3}(1-p_{t+1})(1-p_{t+2}) + ...$). The first order condition

[2] The transversality condition ($\lim_{T\to\infty} \delta^T \left(u(z_T) + \frac{k'(\hat{\alpha}_T)\hat{\alpha}_T}{\hat{\beta}} - k(\hat{\alpha}_T) \right) = 0$) is imposed.

for z_{t+1} is

$$\delta(1-p_t) - \frac{\partial p_t}{\partial z_{t+1}}(\delta z_{t+1} + h_t)$$
$$= \lambda_t \delta(1-p_t)\frac{\partial W_{u,t+1}}{\partial z_{t+1}} - \lambda_t \left\{ k'(\alpha_t) - \delta \int_{u^{-1}\{(1-\delta)W_{u,t+1}\}}^{\bar{w}} (W_{w,t+1} - W_{u,t+1}) dF(w) \right\} \frac{\partial \alpha_t}{\partial z_{t+1}}$$

Because of the individual optimum condition (equation (1.3.1)), we know that

$$\delta \int_{u^{-1}\{(1-\delta)W_{u,t+1}\}}^{\overline{w}} (W_{w,t+1} - W_{u,t+1}) dF(w) = \frac{1}{\beta} k'(\alpha_t)$$

Using the implicit function theorem and replacing $\frac{\partial \alpha_t}{\partial z_{t+1}}$, I can derive the below ratio between two marginal utilities.

$$\delta(1-p_t) - \frac{\partial p_t}{\partial z_{t+1}}(\delta z_{t+1} + h_t) = \left\{ \delta(1-p_t) - \delta(1-\beta)(1-F(R_t))\frac{k'(\alpha_t)}{k''(\alpha_t)} \right\} \frac{u'(z_{t+1})}{u'(z_t)}$$

The equation implies that $\frac{u'(z_{t+1})}{u'(z_t)} > 1$ because

$$\frac{\partial p_t}{\partial z_{t+1}} = (1-F(R_t))\frac{\partial a_t}{\partial z_{t+1}} - \alpha_t f(R_t)\frac{\partial R_t}{\partial z_{t+1}} < 0$$

The negative sign comes from equation (1.3.1) and the definition of the reservation wage. Since the utility function is concave, the ratio implies $z_t > z_{t+1}$. □

A.1.3 Lemma 1.4.1

Proof. First, the expenditure of the UI in a constant scheme is

$$\overline{E} = z + \delta(1-p(\alpha))z + \delta^2(1-p(\alpha))^2 z + ... = \frac{1}{1-\delta(1-p)} z$$

Next, I drop the time subscript because it is stationary, hence invariant to time. Then the continuation value is

$$(1-\delta(1-p))W_u(x) = -\exp\left(-\gamma\left(z + \frac{r}{1+r}x + \hat{s} - \hat{\alpha}\right)\right)$$
$$+ \delta p \frac{-\exp\left(-\gamma\left(w + \frac{r}{1+r}x - r\hat{s}\right)\right)}{1-\delta}$$

which proves the lemma. □

A.1.4 Proposition 1.4.2

Proof. The first-order conditions for the current period problem are as follows:

$$\begin{aligned}
&\beta\delta\exp(r\gamma s_t)p'\left(\frac{u(w)}{1-\delta}-W_u(\hat{\alpha}_{t+1},\hat{s}_{t+1},\overline{E}_{t+1},x_t=0)\right)\\
&\quad=u'((1-\delta(1-p))\overline{E}_t+s_t-\alpha_t)\\
&u'(1-\delta(1-p)\overline{E}_t+s_t-\alpha_t)\\
&\quad=-r\beta\delta\gamma\exp(r\gamma s_t)\left(p\frac{u(w)}{1-\delta}+(1-p)W_u(\hat{\alpha}_{t+1},\hat{s}_{t+1},\overline{E}_{t+1},x_t=0)\right)
\end{aligned}\tag{A.1.2}$$

Combining the two equations, I get the following relation that the equilibrium search effort satisfies.

$$p'\left(\frac{u(w)}{1-\delta}-W_{u,t+1}\right)=-r\gamma\left(p(\alpha_t)\frac{u(w)}{1-\delta}+(1-p(\alpha_t))W_{u,t+1}\right)\tag{A.1.3}$$

or equivalently,

$$\left(\frac{p'+r\gamma p(\alpha)}{p'+r\gamma p(\alpha)-r\gamma}\right)\frac{u(w)}{1-\delta}=W_{u,t+1}.$$

Note that it does not depend on the present bias (β), but the perceived one ($\hat{\beta}$) because $\mathcal{W}_{u,t+1}$ is a function of $\hat{\beta}$. Let the equilibrium search effort level α^*. The two first-order conditions (equation (A.1.2)) indicate that equilibrium liquidation (s_t^*) is a decreasing function of β for any given α_t^* because $\frac{\partial W_{u,t+1}}{\partial s_t}=\frac{\partial W_{u,t+1}}{\partial x_{t+1}}\frac{\partial x_{t+1}}{\partial s_t}<0$.

□

A.1.5 Proposition 1.4.3

Proof. Let z^* be the optimal level of UI when the planner can only choose a constant scheme. To get the best level of UI z^*, I first introduce a labor income tax as a generic function $T(z)$ that satisfies the government budget constraint. That is,

$$T(z^*)=\frac{z^*}{1-\delta(1-p(z^*))}$$

Then, I define the optimal level as follows:

$$z^*=\arg\max\mathcal{W}_u:=\frac{u(z+s-\alpha)+\frac{\delta}{1-\delta}p(z)\left(u(w-rs)-T(z)\right)}{1-\delta(1-p(z))\exp(r\gamma s)}$$

where s and α is a function of z in the equilibrium. Once I get the level z^*, then I will set the exogenous UI budget as $\overline{E}$ to be $T(z^*)$. After all, my focus is the temporal distribution of $\overline{E}$, not the level of it.

Now start with the optimal level z^*. The question is whether I can increase social welfare by increasing z at $t+1$ at the expense of z at time t while keeping $\overline{E}$ intact. From the equation (1.4.3) in the main text, I express the question as the following derivative.

$$\begin{aligned}\frac{d\mathcal{W}_{u,t}}{dz_{t+1}} =& u'_z(z_t + s_t - \alpha_t)\frac{\partial z_t}{\partial z_{t+1}} + \delta \exp(r\gamma s_t)(1 - p(\alpha_t)) \left(\frac{\partial \mathcal{W}_{u,t+1}}{\partial z_{t+1}}\right) \\ &+ \frac{\partial \mathcal{W}_{u,t}}{\partial \alpha_t}\frac{\partial \alpha_t}{\partial z_{t+1}} + \frac{\partial \mathcal{W}_{u,t}}{\partial s_t}\frac{\partial s_t}{\partial z_{t+1}}\end{aligned}$$

Because of the Proposition 1.4.2, we know that $\frac{\partial \mathcal{W}_{u,t}}{\partial \alpha_t}$ is zero with envelope theorem even if the agent has present bias. On the other hand, $\frac{\partial \mathcal{W}_{u,t}}{\partial s_t}$ is less than zero because the agent maximizes her value, and that leads to an over-liquidation of her asset. Further, because of the optimality of z^* we started from, the constant scheme from $t+1$ is optimal hence the rest of the equation $(u'_z(z_t + s_t - \alpha_t)\frac{\partial z_t}{\partial z_{t+1}} + \delta \exp(r\gamma s_t)(1-p)\frac{\partial \mathcal{W}_{u,t+1}}{\partial z_{t+1}})$ is also zero. Therefore, the key for the sign of the derivative is how the current asset liquidation moves with the future UI benefit $(\frac{\partial s_t}{\partial z_{t+1}})$. From the first order condition of the individual optimization (equation (A.1.2)), I deduce that equilibrium α is decreasing with z_{t+1} since the optimal α decision rule (equation (A.1.3)) implies that

$$\left(\frac{p' + r\gamma p(\alpha)}{p' + r\gamma p(\alpha) - r\gamma}\right)\frac{u(w)}{1-\delta} = W_{u,t+1}.$$

and the right-hand side is increasing with z_{t+1}, and the left-hand side is decreases with α for any $p(\alpha) \in [0, 1]$.

Now, I go back to the first equation of the individual optimization problem (equation (A.1.2)), and take the total derivative to get the condition for the asset liquidation behavior.

$$\frac{ds_t}{dz_{t+1}} = \frac{\beta\delta \exp(r\gamma s_t)p'(\alpha)\frac{\partial W_{u,t+1}}{\partial z_{t+1}} + u''_z\frac{\partial z_t}{\partial z_{t+1}} - \beta\delta \exp(r\gamma s_t)p''(\alpha_t)\frac{\partial \alpha_t}{\partial z_{t+1}}(\frac{u(w)}{1-\delta} - W_{u,t+1})}{\beta\delta \exp(r\gamma s_t)p'(\alpha_t)(\frac{u(w)}{1-\delta} - W_{u,t+1}) - u''_s}$$

and from the numerator of the above equation, I can get the condition in the proposition.

□

A.2 Chapter 2

A.2.1 Proposition 2.4.1

Proof. Consider the consumer problem. Similar to Angeletos and Lian [2018], the budget constraint can also be log linearized as follows: The budget constraint of consumer i at period t is

$$\frac{1}{1+R_t}s^{\ell}_{i,t} = s^{\ell}_{i,t-1} + W_t n^{\ell}_{i,t} + D_t - P_t c^{\ell}_{i,t}$$

In the next period, the budget constraint is

$$\frac{1}{1+R_{t+1}}s^{\ell}_{i,t+1} = s^{\ell}_{i,t} + W_{t+1} n^{\ell}_{i,t+1} + D_{t+1} - P_{t+1} c^{\ell}_{i,t+1}.$$

Multiplying both sides of the previous equation by $\frac{1}{1+R_t}$ and taking the difference with the period t budget constraint cancels out $\frac{1}{1+R_t}s^{\ell}_{i,t}$ and we obtain:

$$\begin{aligned} &\frac{1}{(1+R_{t+1})(1+R_t)}s^{\ell}_{i,t+1} \\ &= s^{\ell}_{i,t-1} + W_t n^{\ell}_{i,t} + D_t - P_t c^{\ell}_{i,t} + \frac{1}{1+R_t}\left(W_{t+1} n^{\ell}_{i,t+1} + D_{t+1} - P_{t+1} c^{\ell}_{i,t+1}\right). \end{aligned}$$

Iterating this process generates

$$\prod_{\tau=0}^{\infty}\frac{1}{1+R_{t+\tau}}s^{\ell}_{i,\infty} = s^{\ell}_{i,t-1} + \sum_{\tau=0}^{\infty}\left(\prod_{k=1}^{\tau}\frac{1}{1+R_{t+k}}\right)\left(W_{t+\tau} n^{\ell}_{i,t+\tau} + D_{t+\tau} - P_{t+\tau} c^{\ell}_{i,t+\tau}\right).$$

Since $s^{\ell}_{i,\infty}$ is bounded, the l.h.s. converges to 0. Hence the budget constraint can be stated as

$$\sum_{\tau=0}^{\infty}\left(\prod_{k=1}^{\tau}\frac{1}{1+R_{t+k}}\right)P_{t+\tau}c^{\ell}_{i,t+\tau} = s^{\ell}_{i,t-1} + \sum_{\tau=0}^{\infty}\left(\prod_{k=1}^{\tau}\frac{1}{1+R_{t+k}}\right)\left(W_{t+\tau} n^{\ell}_{i,t+\tau} + D_{t+\tau}\right).$$

Now, we approximate the above budget constraint around the steady state. First, take the total derivative of the above equation evaluated at the steady state. The left-hand side of it becomes

$$\sum_{\tau=0}^{\infty}\left(\frac{(P_{t+\tau}-P_{ss})c_{ss}}{(1+R_{ss})^{\tau}} + \frac{P_{ss}\left(c^{\ell}_{t+\tau}-c_{ss}\right)}{(1+R_{ss})^{\tau}} + \sum_{k=1}^{\tau}\left(\frac{P_{ss}c_{ss}}{R_{t+k}-R_{ss}}\right)\right),$$

whereas the right-hand side is

$$s^{\ell}_{i,t-1} + \sum_{\tau=0}^{\infty}\left(\frac{(W_{t+\tau}-W_{ss})n_{i,ss}}{(1+R_{ss})^{\tau}} + \frac{W_{ss}(n^{\ell}_{i,t+\tau}-n_{i,ss})}{(1+R_{ss})^{\tau}} + \frac{D_{t+\tau}-D_{ss}}{(1+R_{ss})^{\tau}} + \sum_{k=1}^{\tau}\left(\frac{W_{ss}n_{ss}+D_{ss}}{R_{t+k}-R_{ss}}\right)\right).$$

Setting both sides equal, dividing both sides by $P_{ss}c^{\ell}_{ss}$, and using $\frac{1}{1+R_{ss}} = \beta$ yields

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \hat{c}^{\ell}_{i,t+\tau} = \frac{s^{\ell}_{i,t-1}}{P_{ss}c^{\ell}_{ss}} + \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\underbrace{\frac{W_{ss}n_{i,ss}}{P_{ss}c_{ss}}}_{:=L_{ss}} (\hat{w}_{t+\tau} + \hat{n}^{\ell}_{i,t+\tau}) + \underbrace{\frac{D_{ss}}{P_{ss}c_{ss}}}_{=1-L_{ss}} \hat{d}_{i,t+\tau} \right) \tag{A.2.1}$$

where L denotes the labor share of income in the steady state, $w_t := \frac{W_t}{P_t}$, $d_t := \frac{D_t}{P_t}$, and the hat variables are the log deviations from their steady state as before.

Recall the two optimality conditions for the consumer (i.e., equations (2.3.8) and (2.3.9)),

$$1 \approx 1 + \ln \beta + \mathbb{E}_{\ell,t} \left[-\pi_{t+1} - \frac{1}{\gamma}(\ln c^{\ell}_{i,t+1} - \ln c^{\ell}_{i,t}) \mid \omega_{i,T|t^*} \right] + R_t$$

$$\psi \ln n^{\ell}_{i,t} = \ln w_t - \frac{1}{\gamma} \ln c^{\ell}_{i,t}$$

Moving $\ln c^{\ell}_{i,t}$ to the left-hand side, taking the difference from its steady state, and defining $\tilde{R}_t := R_t + \ln \beta$ gives the following equations for each $\ell \in \{a, u\}$:

$$\hat{c}^{\ell}_{i,t} = -\gamma \left(\tilde{R}_t - \mathbb{E}_{\ell,t}[\pi_{t+1} \mid \omega_{i,T|t^*}] \right) + \mathbb{E}_{\ell,t}[\hat{c}^{\ell}_{i,t+1} \mid \omega_{i,T|t^*}] \tag{A.2.2}$$

$$\hat{c}^{\ell}_{i,t} = \gamma(\hat{w}_t - \psi \hat{n}^{\ell}_{i,t}) \tag{A.2.3}$$

Using equations (A.2.2) and (A.2.3) as well as the above log linearized budget constraint given by equation (A.2.1), we can rewrite the consumer block as a dynamic beauty contest,

$$\hat{c}^{\ell}_{i,t} = \left(\frac{(1-\beta)\psi\gamma}{\psi\gamma + L_{ss}} \right) \frac{s^{\ell}_{i,t-1}}{P_{ss}c^{\ell}_{ss}} - \gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \mathbb{E}_{\ell,t}[\tilde{R}_{t+\tau} - \pi_{t+1+\tau} \mid \omega_{i,T|t^*}]$$
$$+ (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \mathbb{E}_{\ell,t} \left[\underbrace{(1+\psi)L_{ss} \frac{\gamma}{\psi\gamma + L_{ss}} \hat{w}_{t+\tau} + \psi(1-L_{ss}) \frac{\gamma}{\psi\gamma + L_{ss}} \hat{d}_{t+\tau}}_{:=\hat{m}_{i,t+\tau}} \mid \omega_{i,T|t^*} \right] \tag{A.2.4}$$

where $\hat{m}_{i,t+\tau}$ is consumer i's income deviation at period $t + \tau$. To see this, first replace $n^{\ell}_{i,t+\tau}$ for $s = 0, 1, ...$ in equation (A.2.1) using equation (A.2.3).

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \hat{c}^{\ell}_{i,t+\tau} = \frac{s^{\ell}_{i,t-1}}{P_{ss}c^{\ell}_{ss}} + \sum_{\tau=0}^{\infty} \beta^{\tau} \left(L_{ss} \left(\hat{w}_{t+\tau} + \frac{1}{\psi} \hat{w}_{t+\tau} - \frac{1}{\psi\gamma} \hat{c}^{\ell}_{i,t+\tau} \right) + (1-L_{ss}) \hat{d}_{i,t+\tau} \right)$$

Moving $\hat{c}^{\ell}_{i,t+\tau}$ to the left side,

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \left(1 + \frac{L_{ss}}{\psi\gamma}\right) \hat{c}^{\ell}_{i,t+\tau} = \frac{s^{\ell}_{i,t-1}}{P_{ss} c^{\ell}_{ss}} + \sum_{\tau=0}^{\infty} \beta^{\tau} \left(L_{ss} \left(\frac{1+\psi}{\psi}\right) \hat{w}_{t+\tau} + (1 - L_{ss}) \hat{d}_{i,t+\tau} \right)$$

or equivalently,

$$\sum_{\tau=0}^{\infty} \beta^{\tau} \hat{c}^{\ell}_{i,t+\tau} = \frac{\psi\gamma}{\psi\gamma + L_{ss}} \frac{s^{\ell}_{i,t-1}}{P_{ss} c^{\ell}_{ss}} + \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\frac{(1+\psi)\gamma L_{ss}}{\psi\gamma + L_{ss}} \hat{w}_{t+\tau} + \frac{\psi\gamma(1 - L_{ss})}{\psi\gamma + L_{ss}} \hat{d}_{i,t+\tau} \right). \tag{A.2.5}$$

Further, from equation (A.2.2), we obtain

$$\sum_{\tau=1}^{\infty} \beta^{\tau} \hat{c}^{\ell}_{i,t} = -\gamma \sum_{\tau=1}^{\infty} \beta^{\tau} \left(\tilde{R}_{t+\tau} - \mathbb{E}_{\ell,t}[\pi_{t+1+\tau} \mid \omega_{i,T|t^*}] \right) + \sum_{\tau=1}^{\infty} \beta^{\tau} \mathbb{E}_{\ell,t}[\hat{c}^{\ell}_{i,t+1+\tau} \mid \omega_{i,T|t^*}] \tag{A.2.6}$$

by multiplying with β^t and summing from t to ∞. Equation (A.2.4) follows now by multiplying equation (A.2.5) with $1 - \beta$ and adding equation (A.2.6).

As a next step, we show the income-production identity. Intermediate goods producers' surplus is distributed as a dividend,

$$d_{j,t} = \left(\frac{P_{j,t}}{P_t} - \frac{W_t}{P_t} \frac{1}{\exp(z_t)} \right) y_{j,t}$$

where $d_{j,t}$ is the real dividend from firm j. Recall that $y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} Y_t$ is the factor demand. Also, recall the price aggregation, $P_t = \left(\int_0^1 (P_{j,t})^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$. Integrating the dividend over intermediate goods producers yields the aggregate (real) dividend,

$$\begin{aligned} d_t &= \int_{j \in J} d_{j,t} dj = \left(\frac{\int_j \left(P_{j,t}^{1-\varepsilon}\right) dj}{(P_t)^{1-\varepsilon}} \right) Y_t - w_t \int_{j \in J} n_{j,t} dj \\ &= Y_t - w_t N_t \end{aligned}$$

We log-linearize this equation as follows: First, taking total derivatives evaluated at the steady state,

$$\Delta d_t = \Delta Y_t - \Delta w_t N_{ss} - w_{ss} \Delta N_t$$

Dividing both sides by $d_{ss} = Y_{ss} - w_{ss} N_{ss}$,

$$\hat{d}_t = \frac{Y_{ss}}{Y_{ss} - w_{ss} N_{ss}} \hat{Y}_t - \frac{w_{ss} N_{ss}}{Y_{ss} - w_{ss} N_{ss}} (\hat{w}_t + \hat{N}_t) \tag{A.2.7}$$

In this equation, $\frac{Y_{ss}}{Y_{ss}-w_{ss}N_{ss}} = \frac{1}{1-L_{ss}}$ is the inverse of the dividend share of income in the steady state. Thus, $\frac{w_{ss}N_{ss}}{Y_{ss}-w_{ss}N_{ss}} = \frac{L_{ss}}{1-L_{ss}}$.

Next, we aggregate the individual labor supply. Recall the labor supply condition in the consumers' problem (equation (2.3.9)).

$$\psi \ln n^{\ell}_{i,t} = \ln w_t - \frac{1}{\gamma} \ln c^{\ell}_{i,t}$$

By integrating both sides, we get

$$\begin{aligned}\psi \int_{i\in I} \ln n^{\ell}_{i,t} di &= \int_{i\in I} \ln w_t di - \frac{1}{\gamma} \int_{i\in I} \ln c^{\ell}_{i,t} di \\ \psi \ln N^u_t &= \ln w_t - \frac{1}{\gamma} \ln Y^u_t \qquad \text{(A.2.8)} \\ \psi \hat{N}^u_t &= \hat{w}_t - \frac{1}{\gamma} \hat{Y}^u_t\end{aligned}$$

at the lower space, and similarly, we get

$$\psi \hat{N}^a_t = \hat{w}_t - \frac{1}{\gamma} \hat{Y}^a_t \qquad \text{(A.2.9)}$$

at the upper space.[3] Then, plugging equation (A.2.7) into the definition of $\hat{m}_{i,t+\tau}$ of equation (A.2.4) and imposing market clearing gives,

$$\begin{aligned}&\mathbb{E}_{\ell,t}\left[\hat{m}_{i,t+\tau} \mid \omega_{i,T|t^*}\right] \\ &:= \mathbb{E}_{\ell,t}\left[\frac{(\psi+1)\gamma L_{ss}}{\psi\gamma + L_{ss}} \hat{w}_{t+\tau} + \frac{\psi\gamma(1-L_{ss})}{\psi\gamma + L_{ss}} \hat{d}_{t+\tau} \mid \omega_{i,T|t^*}\right] \\ &= \mathbb{E}_{\ell,t}\left[\frac{(\psi+1)\gamma L_{ss}}{\psi\gamma + L_{ss}} \hat{w}_{t+\tau} + \frac{\psi\gamma(1-L_{ss})}{\psi\gamma + L_{ss}} \left(\frac{1}{1-L_{ss}} \hat{Y}^{\ell}_{t+\tau} - \frac{L_{ss}}{1-L_{ss}} (\hat{w}_{t+\tau} + \hat{N}^{\ell}_{t+\tau})\right) \mid \omega_{i,T|t^*}\right] \\ &= \mathbb{E}_{\ell,t}\left[\frac{\gamma L_{ss}}{\psi\gamma + L_{ss}} \hat{w}_{t+\tau} - \frac{\psi\gamma L_{ss}}{\psi\gamma + L_{ss}} \hat{N}^{\ell}_{t+\tau} + \frac{\psi\gamma}{\psi\gamma + L_{ss}} \hat{Y}^{\ell}_{t+\tau} \mid \omega_{i,T|t^*}\right] \\ &= \mathbb{E}_{\ell,t}\left[\frac{\gamma L_{ss}}{\psi\gamma + L_{ss}} (\hat{w}_{t+\tau} - \psi \hat{N}^{\ell}_{t+\tau}) + \frac{\psi\gamma}{\psi\gamma + L_{ss}} \hat{Y}^{\ell}_{t+\tau} \mid \omega_{i,T|t^*}\right]\end{aligned}$$

Then,

$$\mathbb{E}_{\ell,t}\left[\hat{m}_{i,t+\tau} \mid \omega_{i,T|t^*}\right] = \mathbb{E}_{\ell,t}\left[\frac{L_{ss}}{\psi\gamma + L_{ss}} \hat{Y}^{\ell}_{t+\tau} + \frac{\psi\gamma}{\psi\gamma + L_{ss}} \hat{Y}^{\ell}_{t+\tau} \mid \omega_{i,T|t^*}\right] = \mathbb{E}_{\ell,t}\left[\hat{Y}^{\ell}_{t+\tau} \mid \omega_{i,T|t^*}\right]$$

[3] We interchange the natural log and integral using the approximation result.

$$\int_i \ln x_i di \approx \int_i 1 + x_i di = 1 + \int_i x_i di = 1 + X_i \approx \ln X_i = \ln \int x_i di$$

because of the aggregate labor supply ($\hat{w}_t - \psi \hat{N}^\ell_t = \frac{1}{\gamma}\hat{Y}^\ell_t$) in the equation (A.2.8) and equation (A.2.9).

So far we derived the individual reactions to shocks. Next, we aggregate individual reactions to the aggregate reaction of the economy. We begin by considering the economy in the lowest space from an unaware consumer's point of view. For such a consumer, every consumer is unaware.

First, take the average of the individual beauty contest (equation (A.2.4)) In the lower state space. We get,

$$\hat{C}^u_t = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t}[\tilde{R}_{t+\tau} - \pi_{t+1+\tau}] + (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t}\left[\hat{Y}^u_{t+\tau}\right] \tag{A.2.10}$$

where $\overline{\mathbb{E}}_{I,t}[\cdot] := \int_{i \in I} \mathbb{E}_{u,t}[\cdot \mid \omega_{i,T|t^*}]di$ denotes the average expectation of the consumers. As Angeletos and Lian [2018], we use the fact that the aggregate saving $\int_I s^u_{i,t-1}di$ is zero in the aggregation.

Moving to the upper space, the aware type consumer $i \in I_a$, taking the average among the aware type consumers gives

$$\frac{1}{\mu}\int_{i \in I_a} \hat{c}^a_{i,t}di = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I_a,t}[\tilde{R}_{t+\tau} - \pi_{t+1+\tau}] + (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I_a,t}\left[\hat{Y}^a_{t+\tau}\right] \tag{A.2.11}$$

where $\overline{\mathbb{E}}_{I_a,t}[\cdot] := \frac{1}{\mu}\int_{i \in I_a} \mathbb{E}_{u,t}[\cdot]di$ is the average expectation of the aware type consumers. Because the aggregate savings in the lower space is zero, the aggregate savings in the upper space also becomes zero ($\int_{I_a} s^a_{i,t-1}di = 0$).

Finally, recall that the aggregate reaction from all consumers is $\hat{C}^a_t$ because the aware type understands the market structure correctly, or equivalently, it is the weighted average of $\hat{C}^u_t$ and $\frac{1}{\mu}\int_{i \in I_a} \hat{c}^a_{i,t}di$ as follows:

$$\begin{aligned}
\hat{Y}_t =& \int_{i \in I_a} \hat{c}^a_{i,t}di + (1-\mu)\hat{C}^u_t \\
=& -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \left(\mu \overline{\mathbb{E}}_{I_a,t}\left[\tilde{R}_{t+\tau} - \pi_{t+1+\tau}\right] + (1-\mu)\overline{\mathbb{E}}_{I_u,t}\left[\tilde{R}_{t+\tau} - \pi_{t+1+\tau}\right]\right) \\
&+ (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\mu \overline{\mathbb{E}}_{I_a,t}\left[\hat{Y}^a_{t+\tau}\right] + (1-\mu)\overline{\mathbb{E}}_{I_u,t}\left[\hat{Y}^u_{t+\tau}\right]\right)
\end{aligned}$$

as in the proposition. □

A.2.2 Proposition 2.4.2

Proof. **Step 1**. We begin by considering the contemporaneous effect in the lower space. We first derive the output gap using the augmented IS relation from the Proposition 2.4.1. Recall from Proposition 2.4.1 that the unaware consumer's perceived IS curve is

$$\hat{Y}_t^u = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t} \left[r_{t+\tau} + \ln \beta \right] + (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t} \left[\hat{Y}_{t+\tau}^u \right]$$

where $r_t := R_t - \pi_{t+1}$ is the real interest rate, and this relation also holds at the natural level of output,

$$\hat{Y}_t^n = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t} \left[r_{t+\tau}^n + \ln \beta \right] + (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t} \left[\hat{Y}_{t+\tau}^n \right] \tag{A.2.12}$$

By taking the difference between the two equations, we obtain

$$\hat{X}_t^u = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t} \left[r_{t+\tau} - r_{t+\tau}^n \right] + (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t} \left[\hat{X}_{t+\tau}^u \right] \tag{A.2.13}$$

where $\hat{X}_t^u$ is the output gap in the lower space. To get an expression for r_t^n, multiply the next period's counterpart of equation (A.2.12) by β. Then we take the difference with equation (A.2.12) to obtain

$$\hat{Y}_t^n - \hat{Y}_{t+1}^n = -\gamma \left(r_t^n + \ln \beta \right).$$

Recall that $\hat{Y}_t^n := \ln Y_t^n - \ln Y_{ss}^n = \frac{1+\psi}{\psi + \frac{1}{\gamma}} z_t$. Therefore,

$$\begin{aligned} \frac{1+\psi}{\psi + \frac{1}{\gamma}} \left(z_t - z_{t+1} \right) &= -\gamma \left(r_t^n + \ln \beta \right) \\ r_t^n &= -\ln \beta + \frac{1}{\gamma} \left(\frac{1+\psi}{\psi + \frac{1}{\gamma}} \right) \left(z_{t+1} - z_t \right). \end{aligned}$$

Plugging r_t^n into equation (A.2.13),

$$\hat{X}_t^u = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t} \left[r_{t+\tau} + \ln \beta - \frac{1}{\gamma} \left(\frac{1+\psi}{\psi + \frac{1}{\gamma}} \right) \left(z_{t+1} - z_t \right) \right] + (1-\beta) \sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t} \left[\hat{X}_{t+\tau}^u \right] \tag{A.2.14}$$

where $\hat{X}_t^u$ indicates the output gap perceived by the unaware type consumers.

Now consider the contemporaneous effect in the lower space when an announcement on the nominal interest rate $\tilde{R}_{T|T}$ is made at period T. Given the announcement $\tilde{R}_{T|T}$, the aggregate reaction in the lower space is derived from equation (A.2.14) is as follows:

$$\begin{aligned}
\hat{X}_T^u &= -\gamma\beta\overline{\mathbb{E}}_{I,T}\left[r_{T|T} - r_{T|T}^n\right] + (1-\beta)\hat{X}_T^u \\
&= -\gamma\beta\overline{\mathbb{E}}_{I,T}\left[\tilde{R}_{T|T} - \pi_{T+1}\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}[z_{T|T}] + (1-\beta)\hat{X}_T^u \\
&= -\gamma\overline{\mathbb{E}}_{I,T}\left[\tilde{R}_{T|T}\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}[z_{T|T}] \qquad \text{(A.2.15)}
\end{aligned}$$

To understand this equation, recall that the shock occurs only at period T and the output gap is zero thereafter.

Note that we drop the forward-looking terms beyond T in the first line by assuming that the economy is initially in the steady state, and especially at the natural output level. In the lower space, the Taylor rule is $\tilde{R}_T = \phi_y\hat{X}_T$ since there is no monetary policy shock. Hence the announcement follows this Taylor rule as well.

$$\begin{aligned}
\hat{X}_T^u &= -\gamma\phi_y\overline{\mathbb{E}}_{I,T}\left[\hat{X}_T^u\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}[z_{T|T}] \\
&= \frac{1}{1+\gamma\phi_y}\left(-\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}[z_{T|T}]
\end{aligned} \qquad \text{(A.2.16)}$$

Therefore, the Taylor rule perceived by unaware types as a function of the TFP shocks is

$$\begin{aligned}
\overline{\mathbb{E}}_{I,T}\left[\tilde{R}_{T|T}\right] &= \overline{\mathbb{E}}_{I,T}\left[\phi_y\hat{X}_T^u\right] \\
&= \underbrace{\frac{\phi_y}{1+\gamma\phi_y}\left(-\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)}_{:=\xi_z}\overline{\mathbb{E}}_{I,T}[z_{T|T}]
\end{aligned}$$

Recall the estimate of the shock (equation (2.3.2)). Aggregate over all consumers (in the lower space), we obtain

$$\begin{aligned}
\overline{\mathbb{E}}_{I,T}[z_{T|T}] &= \int_{i\in I}\frac{\lambda_z^u}{\xi_z}\omega_{i,T|T}di \\
&= \int_{i\in I}\frac{1+\gamma\phi_y}{\phi_y}\left(-\frac{\frac{1}{\gamma}+\psi}{1+\psi}\right)\lambda_z^u\omega_{i,T|T}di
\end{aligned}$$

Plugging this expression into equation (A.2.16) gives

$$\hat{X}_T^u = \frac{1}{\phi_y}\int_{i\in I}\lambda_z^u \omega_{i,T|T}di = \frac{1}{\phi_y}\lambda_z^u \tilde{R}_{T|T} = \frac{1}{\phi_y}\overline{\mathbb{E}}_{I,T}\left[\tilde{R}_{T|T}\right]$$

where the second equation follows from the law of large numbers, $\int_{i\in I_u}\eta_{i,T}di = 0$, and the last equation makes use of the Taylor rule.

To study forward guidance that goes beyond the contemporaneous effect, we now consider that the announcement time t^* differs from the time of the realization of the shock and nominal interest rate change. We show the effect of forward guidance by induction. To this end, we introduce some notation. Denote by Φ_t^u the reaction of the output gap in period t given the signals of unaware consumers in the lower space at t^*. Similarly, we use Ω_t^u for the reaction of inflation at period t to the signal at period t^* perceived by unaware consumers in the lower space.

$$\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}_t^u\right] = \Phi_t^u\,\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \tag{A.2.17}$$

$$\overline{\mathbb{E}}_{I,t^*}\left[\pi_t\right] = \kappa\left(\psi + \frac{1}{\gamma}\right)\hat{X}_t^u + \Omega_t^u\,\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \tag{A.2.18}$$

Using this notation, the contemporaneous reaction on the output gap we derived earlier can be stated as

$$\Phi_T^u = \frac{1}{\phi_y}. \tag{A.2.19}$$

Moreover, from the Phillips curve, equation (2.4.2), we observe that the contemporaneous reaction on inflation must be $\Omega_T^u = 0$.

Step 2. To get a reaction at period t, we will use a mathematical induction using period $T-2$ as a base case. As a preliminary work, we derive the $T-2$ output gap and introduce some notation to simplify our exposition. Assume now that the announcement on the nominal interest rate at period T is made one period ahead at $T-1$. The perceived output gap comes from equation (A.2.14) as follows:

$$\begin{aligned}\hat{X}_{T-1}^u = &-\gamma\beta^2\overline{\mathbb{E}}_{I,T-1}\left[r_{T|T-1} - r_{T|T-1}^n\right] - \gamma\beta\overline{\mathbb{E}}_{I,T-1}\left[r_{T-1|T-1} - r_{T-1|T-1}^n\right]\\ &+ (1-\beta)\hat{X}_{T-1}^u + (1-\beta)\beta\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}_T^u\right]\end{aligned}$$

Moving $(1-\beta)\hat{X}^u_{T-1}$ to the left side and dividing β gives

$$\hat{X}^u_{T-1} = -\gamma\beta\overline{\mathbb{E}}_{I,T-1}\left[R_{T|T-1} - r^n_{T|T-1}\right] - \gamma\overline{\mathbb{E}}_{I,T-1}\left[-\pi_T\right] + (1-\beta)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right]$$

because $\pi_{T+1} = 0$, $\tilde{R}_{T-1} = 0$, and $z_{T-1} = 0$. Using the Phillips curve, equation (2.4.2), we get

$$\begin{aligned}\hat{X}^u_{T-1} &= -\gamma\beta\overline{\mathbb{E}}_{I,T-1}\left[R_{T|T-1} - r^n_{T|T-1}\right] + \gamma\kappa\left(\psi + \frac{1}{\gamma}\right)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] + (1-\beta)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] \\ &= -\gamma\beta\overline{\mathbb{E}}_{I,T-1}\left[R_{T|T-1} - r^n_{T|T-1}\right] + (1-\beta+\gamma\Xi)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] \qquad \text{(A.2.20)}\end{aligned}$$

where $\Xi := \kappa\left(\psi + \frac{1}{\gamma}\right)$. We now from the analysis of the contemporaneous effect, equation (A.2.15), that at period T we have

$$\hat{X}^u_T = -\gamma\overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r^n_{T|T}\right]. \qquad \text{(A.2.21)}$$

Considering now forward guidance at $T-1$ and taking expectations at $T-1$, we obtain

$$\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] = -\gamma\overline{\mathbb{E}}_{I,T-1}\left[R_{T|T-1} - r^n_{T|T-1}\right].$$

Therefore, we can restate equation (A.2.20) using Φ^u_t as

$$\begin{aligned}\hat{X}^u_{T-1} &= \beta\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] + (1-\beta+\gamma\Xi)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] \\ &= (1+\gamma\Xi)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] \qquad \text{(A.2.22)} \\ &= (1+\gamma\Xi)\Phi^u_T\,\overline{\mathbb{E}}_{I,T-1}\left[\tilde{R}_{T|T-1}\right]\end{aligned}$$

For inflation, recall the Phillips curve, equation (2.4.2), which we can write

$$\begin{aligned}\pi_{T-1} &= \beta\overline{\mathbb{E}}_{I,T-1}\left[\pi_T\right] + \Xi\hat{X}^u_{T-1} \\ &= \beta\overline{\mathbb{E}}_{I,T-1}\left[\beta\pi_{T+1} + \Xi X_T\right] + \Xi\hat{X}^u_{T-1} \\ &= \beta\Xi\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] + \Xi\hat{X}^u_{T-1} \qquad \text{(A.2.23)} \\ &= \beta\Xi\Phi^u_T\,\overline{\mathbb{E}}_{I,T-1}\left[\tilde{R}_{T|T-1}\right] + \Xi\hat{X}^u_{T-1}. \qquad \text{(A.2.24)}\end{aligned}$$

where the second equation follows from the next period Phillips curve, the third equation follows from the fact that steady inflation at $T+1$ is zero, and the last equation follows from the equation (A.2.17). Therefore, the two coefficients are

$$\Phi^u_{T-1} = (1+\gamma\Xi)\Phi^u_T, \quad \Omega^u_{T-1} = \beta\Xi\Phi^u_T$$

respectively.

For periods $t \leq T-2$, we show that the following relation is satisfied:

$$\begin{pmatrix} \Phi^u_t \\ \Omega^u_t \end{pmatrix} = M_u \cdot \begin{pmatrix} \Phi^u_{t+1} \\ \Omega^u_{t+1} \end{pmatrix}$$

where

$$M_u := \begin{pmatrix} \beta + (1-\beta+\gamma\Xi)\lambda^u_z & \gamma \\ \beta\Xi & \beta \end{pmatrix}$$

The proof uses mathematical induction, using the $T-2$ reaction as a base case. That is, we first show that the claim holds at period $T-2$, and then we show that the claim also holds for a general t^* with an assumption that the claim holds for every $\tau \in \{t^*+1, ..., T-2\}$.

From the perceived IS curve (equation (A.2.14)),

$$\begin{aligned}
\hat{X}^u_{T-2} = &-\gamma\beta^3\overline{\mathbb{E}}_{I,T-2}\left[r_{T|T-2} - r^n_{T|T-2}\right] - \gamma\beta^2\overline{\mathbb{E}}_{I,T-2}\left[r_{T-1|T-2} - r^n_{T-1|T-2}\right] \\
&- \gamma\beta\overline{\mathbb{E}}_{I,T-2}\left[r_{T-2|t} - r^n_{T-2|t}\right] + (1-\beta)\hat{X}^u_{T-2} \\
&+ (1-\beta)\beta\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_{T-1}\right] + (1-\beta)\beta^2\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right]
\end{aligned}$$

Moving $(1-\beta)\hat{X}^u_{T-2}$ to the l.h.s. and dividing both sides by β yields

$$\begin{aligned}
\hat{X}^u_{T-2} = &-\gamma\beta^2\overline{\mathbb{E}}_{I,T-2}\left[R_{T|T-2} - r^n_{T|T-2}\right] - \gamma\beta\overline{\mathbb{E}}_{I,T-2}\left[-\pi_T\right] - \gamma\overline{\mathbb{E}}_{I,T-2}\left[-\pi_{T-1}\right] \\
&+ (1-\beta)\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_{T-1}\right] + (1-\beta)\beta\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right]
\end{aligned}$$

using $\pi_{T+1} = 0$, $z_{T-1} = z_{T-2} = 0$, and $\tilde{R}_{T-1} = \tilde{R}_{T-2} = 0$. From the equations for inflation in periods T (equation (A.2.18)) and $T-1$ (equation (A.2.23)), we get

$$\begin{aligned}
\hat{X}^u_{T-2} = &-\gamma\beta^2\overline{\mathbb{E}}_{I,T-2}\left[R_{T|T-2} - r^n_{T|T-2}\right] + \gamma\beta\Xi\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] + \gamma\Xi\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_{T-1} + \beta\hat{X}^u_T\right] \\
&+ (1-\beta)\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_{T-1}\right] + (1-\beta)\beta\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right]
\end{aligned}$$

Further, using the equation for the output gap in period T, equation (A.2.21), we replace the first interest rates in the above equation, and using equation (A.2.22), we replace the $T-1$ output gap with the T output gap as follows:

$$\hat{X}^u_{T-2} = \beta^2 \overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] + \gamma\beta\Xi\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] + \gamma\Xi\overline{\mathbb{E}}_{I,T-2}\left[(1+\gamma\Xi)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right] + \beta\hat{X}^u_T\right] \tag{A.2.25}$$

$$\begin{aligned}
&+ (1-\beta)\overline{\mathbb{E}}_{I,T-2}\left[(1+\gamma\Xi)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}^u_T\right]\right] + (1-\beta)\beta\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] \\
&= \beta^2\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] + \gamma\beta\Xi\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] + \gamma\Xi(1+\gamma\Xi)\overline{\mathbb{E}}^2_{u,T-2}\left[\hat{X}^u_T\right] + \gamma\beta\Xi\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right] \\
&+ (1-\beta)(1+\gamma\Xi)\overline{\mathbb{E}}^2_{I,T-2}\left[\hat{X}^u_T\right] + (1-\beta)\beta\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^u_T\right]
\end{aligned} \tag{A.2.26}$$

where $\overline{\mathbb{E}}^2_{I,T-2}[\cdot] := \frac{1}{\mu}\int_{i\in I}\mathbb{E}_{I,T-2}\left[\overline{\mathbb{E}}_{I,T-2}[\cdot] \mid \omega_{i,T|T-2}\right] di$ is the average second order expectation. We can see that the claim holds for the first row of M_u since the equation (A.2.26) can be rewritten using Φ and Ω as follows:

$$\begin{aligned}
\Phi^u_{T-2} &= \left(\beta^2 + \gamma\beta\Xi + \gamma\Xi(1+\gamma\Xi)\lambda^u_z + (1-\beta)(1+\gamma\Xi)\lambda^u_z + (1-\beta)\beta\right)\Phi^u_T + \gamma\beta\Xi\Phi^u_T \\
&= (1+\gamma\Xi)(\beta + (1-\beta+\gamma\Xi)\lambda^u_z)\Phi^u_T + \gamma\beta\Xi\Phi^u_T \\
&= (1+\gamma\Xi)(\beta + (1-\beta+\gamma\Xi)\lambda^u_z)\Phi^u_T + \gamma\Omega^u_{T-1} \\
&= (\beta + (1-\beta+\gamma\Xi)\lambda^u_z)\Phi^u_{T-1} + \gamma\Omega^u_{T-1}
\end{aligned}$$

The inflation at period $T-2$ is,

$$\begin{aligned}
\pi_{T-2} &= \beta\overline{\mathbb{E}}_{I,T-2}\left[\pi_{T-1}\right] + \Xi\hat{X}^u_{T-2} \\
&= \beta\overline{\mathbb{E}}_{I,T-2}\left[\Xi\hat{X}^u_{T-1} + \Omega_{T-1}R_{T|T-2}\right] + \Xi\hat{X}^u_{T-2} \\
&= \beta\left(\Xi\Phi^u_{T-1} + \Omega_{T-1}\right)\overline{\mathbb{E}}_{I,T-2}\left[R_{T|T-2}\right] + \Xi\hat{X}^u_{T-2}
\end{aligned}$$

where the second and the third lines come from the definition of Φ and Ω (equations (A.2.17) and (A.2.18)). This can be equivalently stated

$$\Omega_{T-2} = \beta\Xi\Phi^u_{T-1} + \beta\Omega_{T-1}$$

which proves the second row of M_u.

Now assume as an induction hypothesis that the claim holds for every $\tau \in \{t^*+1, t^*+2, ..., T-2\}$. We would like to show that the claim also holds for t^*. The period t^* reaction at the lower space is

$$\hat{X}^u_{t^*} = -\gamma \sum_{\tau=0}^{T-t^*} \beta^\tau \overline{\mathbb{E}}_{I,t^*}\left[r_{t^*+\tau|t^*} - r^n_{t^*+\tau|t^*}\right] + (1-\beta)\sum_{\tau=1}^{T-t^*} \beta^{\tau-1}\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_{t^*+\tau}\right].$$

From the assumption that the claim holds for every $\tau \in \{t^*+1, t^*+2, ..., T-2\}$, we rewrite above equation in terms of Φ and Ω. First, because $\pi_{T+1} = 0$ and $R_{t^*+\tau|t^*} - r^n_{t^*+\tau|t^*} = 0$ for all $\tau \geq 0$ except $\tau = T - t^*$, the above equation is equivalent to

$$\begin{aligned}\hat{X}^u_{t^*} =& -\gamma\beta^{T-t^*}\overline{\mathbb{E}}_{I,t^*}\left[R_{T|t^*} - r^n_{T|t^*}\right] + \gamma\sum_{\tau=0}^{T-t^*-1}\beta^\tau\overline{\mathbb{E}}_{I,t^*}\left[\pi_{t^*+\tau+1|t^*}\right] \\ &+ (1-\beta)\sum_{\tau=1}^{T-t^*}\beta^{\tau-1}\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_{t^*+\tau}\right].\end{aligned}$$

Using the induction hypothesis, we replace the inflation and get

$$\begin{aligned}\hat{X}^u_{t^*} =& -\gamma\beta^{T-t^*}\overline{\mathbb{E}}_{I,t^*}\left[R_{T|t^*} - r^n_{T|t^*}\right] + \gamma\Xi\sum_{\tau=0}^{T-t^*-1}\beta^\tau\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_{t^*+\tau+1}\right] \\ &+ \gamma\sum_{\tau=0}^{T-t^*-1}\beta^\tau\Omega_{t^*+\tau+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] + (1-\beta)\sum_{\tau=1}^{T-t^*}\beta^{\tau-1}\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_{t+\tau}\right].\end{aligned}$$

Using the result for period T (equation (A.2.21)) and collecting $\hat{X}^u_{t+\tau+1}$ gives

$$\begin{aligned}\hat{X}^u_{t^*} =& \beta^{T-t^*}\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_T\right] + \sum_{\tau=0}^{T-t^*-1}\beta^\tau(1-\beta+\gamma\Xi)\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_{t^*+\tau+1}\right] \\ &+ \gamma\sum_{\tau=0}^{T-t^*-1}\beta^\tau\Omega_{t^*+\tau+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right].\end{aligned}$$

Again, we use the induction hypothesis to replace $\hat{X}^u_{t^*+\tau+1}$,

$$\begin{aligned}\hat{X}^u_{t^*} =& \beta^{T-t^*}\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_T\right] + \sum_{\tau=0}^{T-t^*-1}\beta^\tau(1-\beta+\gamma\Xi)\overline{\mathbb{E}}_{I,t^*}\left[\Phi^u_{t^*+\tau+1}\overline{\mathbb{E}}_{I,t^*+\tau+1}\left[\tilde{R}_{T|t^*}\right]\right] \\ &+ \gamma\sum_{\tau=0}^{T-t^*-1}\beta^\tau\Omega_{t^*+\tau+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \\ =& \beta^{T-t^*}\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_T\right] + \sum_{\tau=0}^{T-t^*-1}\beta^\tau(1-\beta+\gamma\Xi)\lambda^u_z\Phi^u_{t^*+\tau+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \\ &+ \gamma\sum_{\tau=0}^{T-t^*-1}\beta^\tau\Omega_{t^*+\tau+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]\end{aligned}$$

Moving the above equation one period forward ($t^* + 1$), multiplying by β, and taking the difference with the equation for period t^*,

$$\Phi^u_{t^*} = \beta\Phi^u_{t^*+1} + (1 - \beta + \gamma\Xi)\lambda^u_z\Phi^u_{t^*+1} + \gamma\Omega^u_{t^*+1},$$

which proves the first row of the M_u in the claim. The second row is straightforward from the New Keynesian Phillips Curve and the definition of Ω and Φ.

Step 3. We now move to the upper space. First, note that the unaware consumers' reaction in the lower space (step 2) is perceived reaction that may be different when realized market clearing is taken into account. To see this, recall the IS relation in the lower space (equation (A.2.13)):

$$\hat{X}^u_t = -\gamma\sum_{\tau=0}^{\infty}\beta^{\tau+1}\overline{\mathbb{E}}_{I,t}\left[r_{t+\tau} + \ln\beta - \frac{1}{\gamma}\left(\frac{1+\psi}{\psi+\frac{1}{\gamma}}\right)(z_{t+1} - z_t)\right] + (1-\beta)\sum_{\tau=0}^{\infty}\beta^{\tau}\overline{\mathbb{E}}_{I,t}\left[\hat{X}^u_{t+\tau}\right].$$

$\hat{X}^u_{t+\tau}$ at the right-hand side comes from the market clearing as perceived by unaware consumers. For example, the unaware consumer may expect

$$\hat{X}^u_T = -\gamma\beta\overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r^n_{T|T}\right] + (1-\beta)\hat{X}^u_T$$

to hold at period T, but the realized output gap is

$$\hat{X}^u_T = -\gamma\beta\overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r^n_{T|T}\right] + (1-\beta)\hat{X}^a_T$$

because the unaware consumers observe the actual market clearing price at period T. To deal with this perceived-realized reaction difference of the unaware consumers, we introduce the following notation. While we keep $\hat{X}^u_t$ for the perceived reaction of the unaware consumers at period t, we denote $\hat{X}^{u*}_t$ as the realized reaction of the unaware consumers when they observe the current market clearing price at period t. In line with this new notation, we also introduce Φ^{u*}_t to denote the realized reaction of the output gap in period t given the signals:

$$\overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^{u*}_t\right] = \Phi^{u*}_t\ \overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \quad \text{(A.2.27)}$$

Then, we can write the realized IS relation for the unaware consumers as follows:

$$\hat{X}_t^{u*} = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I,t} \left[r_{t+\tau} + \ln\beta - \frac{1}{\gamma} \left(\frac{1+\psi}{\psi + \frac{1}{\gamma}} \right) (z_{t+1} - z_t) \right] \\ + (1-\beta)\hat{X}_t^a + (1-\beta) \sum_{\tau=1}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I,t} \left[\hat{X}_{t+\tau}^u \right]. \quad \text{(A.2.28)}$$

That is, unaware consumers react $\hat{X}_t^{u*}$ given the belief $(\hat{X}_\tau^u)_{\tau \geq t+1}$, and the current market clearing $(\hat{X}_t^a)$. By taking the difference between equation (A.2.28) and equation (A.2.13), we get the following relation which will be handy later:

$$\begin{aligned} \hat{X}_t^{u*} - \hat{X}_t^u &= (1-\beta)\left(\hat{X}_t^a - \hat{X}_t^u\right) \\ &= (1-\beta)\left(\mu \frac{\hat{X}_t^a - (1-\mu)\hat{X}_t^{u*}}{\mu} + (1-\mu)\hat{X}_t^{u*} - \hat{X}_t^u\right) \\ &= (1-\beta)\mu \left(\frac{\hat{X}_t^a - \hat{X}_t^{u*}}{\mu}\right) + (1-\beta)\left(\hat{X}_t^{u*} - \hat{X}_t^u\right) \\ &= \frac{(1-\beta)\mu}{\beta}\left(\frac{\hat{X}_t^a - \hat{X}_t^{u*}}{\mu}\right) \end{aligned}$$

and equivalently,

$$\hat{X}_t^a - \hat{X}_t^u = \frac{\mu}{\beta}\left(\frac{\hat{X}_t^a - \hat{X}_t^{u*}}{\mu}\right). \quad \text{(A.2.29)}$$

Step 4. Consider now the problem of aware consumers. Converting output $(\hat{Y}_t^a)$ to the output gap $(\hat{X}_t^a)$ is analogous to Step 1 except that we now use equation (A.2.11). We obtain:

$$\frac{1}{\mu}\int_{i\in I_a} \ln c_{i,t}^a di - \ln Y_t^n = \frac{\ln C_t^a - (1-\mu)\ln C_t^{u*}}{\mu} - \ln Y_t^n = \frac{\hat{X}_t^a - (1-\mu)\hat{X}_t^{u*}}{\mu} \\ = -\gamma \sum_{\tau=0}^{\infty} \beta^{\tau+1} \overline{\mathbb{E}}_{I_a,t} \left[r_{t+\tau} + \ln\beta - \frac{1}{\gamma}\left(\frac{1+\psi}{\psi+\frac{1}{\gamma}}\right)(z_{t+1} - z_t) \right] + (1-\beta)\sum_{\tau=0}^{\infty} \beta^{\tau} \overline{\mathbb{E}}_{I_a,t}\left[\hat{X}_{t+\tau}^a\right] \quad \text{(A.2.30)}$$

where $\hat{X}_t^a$ is the average output gap perceived by aware consumers, which is also the realized one. Note that $\frac{\hat{X}_t^a - (1-\mu)\hat{X}_t^{u*}}{\mu}$ is the contribution to the output gap of aware consumers only.

Next, we derive the contemporaneous reaction of the aware consumers. We can write the average reaction among $i \in I_a$ as follows:

$$\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu} = -\gamma\beta\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right] - \beta\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$
$$+ (1-\beta)\left((1-\mu)\hat{X}^{u*}_T + \mu\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_t}{\mu}\right)$$

because, $z_{t+1} = 0$ and $\pi_{T+1} = 0$. Then, since the aware consumer anticipates the lower space Taylor rule ($\tilde{R}_{T|T} = \phi_y \hat{X}^{u*}_T$), we can replace $\hat{X}^{u*}_T$ using the Taylor rule:

$$\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu} = \left(-\gamma\beta + \frac{(1-\beta)(1-\mu)}{\phi_y}\right)\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right] - \beta\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$
$$+ (1-\beta)\mu\left(\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_t}{\mu}\right)$$

Collecting the average output gap of aware consumers, $\frac{\hat{X}^a_T-(1-\mu)\hat{X}^{u*}_T}{\mu}$, gives,

$$(1-(1-\beta)\mu)\left(\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu}\right) = \left(-\gamma\beta + \frac{(1-\beta)(1-\mu)}{\phi_y}\right)\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right]$$
$$- \beta\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]. \tag{A.2.31}$$

Note that the Taylor rule in the upper space ($\tilde{R}_T = \phi_y \hat{X}^a_T + v_T$) can be written as

$$\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right] = (1-\mu)\phi_y\hat{X}^{u*}_T + \mu\left(\phi_y\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu}\right) + \overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right]$$
$$= (1-\mu)\tilde{R}_{T|T} + \mu\left(\phi_y\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu}\right) + \overline{\mathbb{E}}_{I_a,t}\left[v_{T|T}\right]$$
$$= \phi_y\left(\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu}\right) + \frac{\overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right]}{\mu}. \tag{A.2.32}$$

Substituting the last equation into equation (A.2.31), we get

$$(1-(1-\beta)\mu)\left(\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu}\right)$$
$$= \left(\frac{-\gamma\beta\phi_y(1-\beta)(1-\mu)}{\phi_y}\right)\left(\phi_y\left(\frac{\hat{X}^a_T - (1-\mu)\hat{X}^{u*}_T}{\mu}\right) + \frac{\overline{\mathbb{E}}_{I_a,t}\left[v_{T|T}\right]}{\mu}\right)$$
$$- \beta\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$

We collect the term $\frac{\hat{X}_T^a-(1-\mu)\hat{X}_T^{u*}}{\mu}$ once again and obtain

$$\frac{\hat{X}_T^a-(1-\mu)\hat{X}_T^{u*}}{\mu} = \underbrace{\frac{\beta}{\gamma\beta\phi_y+\beta}\left(-\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)}_{:=\Lambda_{11}} \overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right] + \underbrace{\frac{-\gamma\beta\phi_y+(1-\beta)(1-\mu)}{\gamma\beta\phi_y+\beta}\frac{1}{\phi_y\mu}}_{:=\Lambda_{12}} \overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right],$$

We substitute the last equation into the equation (A.2.32) to obtain the actual Taylor rule, which is now represented as a function of the two shocks:

$$\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right] = \underbrace{\phi_y\Lambda_{11}}_{:=\xi_z=\Lambda_{21}} \overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right] + \underbrace{\frac{1-(1-\beta)\mu}{\gamma\beta\phi_y+\beta}\frac{1}{\mu}}_{:=\xi_v=\Lambda_{22}} \overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right]$$

Replacing the shocks with the inference by aware consumers, i.e., equation (2.3.1), gives us the contemporaneous reaction (output gap) of the aware consumers:

$$\begin{aligned}\frac{\hat{X}_T^a-(1-\mu)\hat{X}_T^{u*}}{\mu} &= \frac{1}{\phi_y}\left(\lambda_z+\lambda_v\left(\frac{-\gamma\beta\phi_y+(1-\beta)(1-\mu)}{1-(1-\beta)\mu}\right)\right)\tilde{R}_{T|T} \\ &= \frac{1}{\phi_y}\left(\lambda_z+\lambda_v\left(\frac{-\gamma\beta\phi_y+(1-\beta)(1-\mu)}{1-(1-\beta)\mu}\right)\right)\left(\frac{1}{\lambda_z+\lambda_v}\right)\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right]\end{aligned}$$

We define Φ^a and Ω^a similar to their analogues in the lower space:

$$\overline{\mathbb{E}}_{I_a,t^*}\left[\frac{\hat{X}_t^a-(1-\mu)\hat{X}_t^{u*}}{\mu}\right] = \Phi_t^a\, \overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] \tag{A.2.33}$$

$$\overline{\mathbb{E}}_{I_a,t^*}\left[\pi_t\right] = \Xi\hat{X}_t^a + \Omega_t^a\, \overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] \tag{A.2.34}$$

Therefore, we get

$$\Phi_T^a = \frac{1}{\phi_y}\left(\frac{\lambda_z}{\lambda_z+\lambda_v}+\left(\frac{\lambda_v}{\lambda_z+\lambda_v}\right)\left(\frac{-\gamma\beta\phi_y+(1-\beta)(1-\mu)}{1-(1-\beta)\mu}\right)\right) \tag{A.2.35}$$

and $\Omega_T^a=0$.

Step 5. Recall from Step 2 that the base case for the inductive argument concerns period $T-2$. Before we can prove the base case, we need to state the reaction for $T-1$. In order to invoke backward recursion, we first write the IS curve in the upper space as a recursive formula. To this end, we first take the difference to the contemporaneous reactions. Recall that the unaware consumers' (realized) reaction at the upper space is

$$\hat{X}_T^{u*} = -\gamma\beta\overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r_{T|T}^n\right] + (1-\beta)\hat{X}_T^a$$

and the aware consumers' reaction is

$$\frac{\hat{X}_T^a - (1-\mu)\hat{X}_T^{u*}}{\mu} = -\gamma\beta\overline{\mathbb{E}}_{I_a,T}\left[R_{T|T} - r_{T|T}^n\right] + (1-\beta)\hat{X}_T^a.$$

Therefore, the difference between the contemporaneous reactions is

$$\frac{\hat{X}_T^a - \hat{X}_T^{u*}}{\mu} = -\gamma\beta\left(\overline{\mathbb{E}}_{I_a,T}\left[R_{T|T} - r_{T|T}^n\right] - \overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r_{T|T}^n\right]\right) \tag{A.2.36}$$

Using equations (A.2.35) and (A.2.19), we can write an expression for the difference in perceived output gaps:

$$\begin{aligned}
&\frac{\hat{X}_T^a - (1-\mu)\hat{X}_T^{u*} - \mu\hat{X}_T^u}{\mu} \\
&= \frac{1}{\phi_y}\left(\lambda_z + \lambda_v\left(\frac{-\gamma\beta\phi_y + (1-\beta)(1-\mu)}{1-(1-\beta)\mu}\right)\right)\tilde{R}_{T|T} - \frac{1}{\phi_y}\lambda_z^u\tilde{R}_{T|T} \\
&= \Phi_T^a\ \overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right] - \Phi_T^u\ \overline{\mathbb{E}}_{I,T}\left[\tilde{R}_{T|T}\right] \\
&= -\gamma\beta\left(\overline{\mathbb{E}}_{I_a,T}\left[R_{T|T} - r_{T|T}^n\right] - \overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r_{T|T}^n\right]\right) + (1-\beta)\left(\hat{X}_T^a - \hat{X}_T^u\right)
\end{aligned}$$

where the last line comes from the perceived IS relation. What we want to get is the difference in the realized output gaps. To this end, we use the perceived-realized relation in the previous step (equation (A.2.29)) to rewrite the last line of the above equation as follows:

$$\begin{aligned}
&\frac{\hat{X}_T^a - (1-\mu)\hat{X}_T^{u*} - \mu\hat{X}_T^u}{\mu} \\
&= -\gamma\beta\left(\overline{\mathbb{E}}_{I_a,T}\left[R_{T|T} - r_{T|T}^n\right] - \overline{\mathbb{E}}_{I,T}\left[R_{T|T} - r_{T|T}^n\right]\right) + (1-\beta)\frac{\mu}{\beta}\left(\frac{\hat{X}_T^a - \hat{X}_T^{u*}}{\mu}\right) \\
&= \left(1 + \frac{(1-\beta)\mu}{\beta}\right)\left(\frac{\hat{X}_T^a - \hat{X}_T^{u*}}{\mu}\right)
\end{aligned}$$

Therefore, the actual contemporaneous reaction is

$$\frac{\hat{X}_T^a - \hat{X}_T^{u*}}{\mu} = \left(\frac{\beta}{\beta + (1-\beta)\mu}\right)\left(\Phi_T^a\ \overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T}\right] - \Phi_T^u\ \overline{\mathbb{E}}_{I,T}\left[\tilde{R}_{T|T}\right]\right)$$

Since $\hat{X}^a_t$ in equation (A.2.30) contains $\hat{X}^{u*}_t$, taking the difference with the unaware consumers' reaction allows us to obtain a relation between $\frac{\hat{X}^a_t - \hat{X}^{u*}_t}{\mu}$ and $\frac{\hat{X}^a_{t+1} - \hat{X}^{u*}_{t+1}}{\mu}$. At $T-1$, the difference is

$$\begin{aligned}&\left(\frac{\hat{X}^a_{T-1} - \hat{X}^{u*}_{T-1}}{\mu}\right) \\ &= -\gamma\beta^2 \left(\overline{\mathbb{E}}_{I_a,T-1}\left[R_{T|T-1} - r^n_{T|T-1}\right] - \overline{\mathbb{E}}_{I,T-1}\left[R_{T|T-1} - r^n_{T|T-1}\right]\right) \\ &\quad + \gamma\beta\left(\overline{\mathbb{E}}_{I_a,T-1}\left[\pi_T\right] - \overline{\mathbb{E}}_{I,T-1}\left[\pi_T\right]\right) + (1-\beta)\beta\overline{\mathbb{E}}_{I_a,T-1}\left[\hat{X}^a_T - \hat{X}^u_T\right].\end{aligned} \tag{A.2.37}$$

Using equation (A.2.36) and (A.2.29), we write the right-hand side as

$$\beta\overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right] + \gamma\beta\left(\overline{\mathbb{E}}_{I_a,T-1}\left[\pi_T\right] - \overline{\mathbb{E}}_{I,T-1}\left[\pi_T\right]\right) + (1-\beta)\mu\overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right].$$

Further, the difference of the inflation in the above equation is

$$\begin{aligned}\overline{\mathbb{E}}_{I_a,T-1}\left[\pi_T\right] - \overline{\mathbb{E}}_{I,T-1}\left[\pi_T\right] =& \overline{\mathbb{E}}_{I_a,T-1}\left[\Xi\hat{X}^a_T\right] - \overline{\mathbb{E}}_{I,T-1}\left[\Xi\hat{X}^u_T\right] \\ =& \Xi\frac{\mu}{\beta}\overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right]\end{aligned}$$

if we use the notation in equation (A.2.34). Combining these two observations, equation (A.2.37) becomes

$$\begin{aligned}\frac{\hat{X}^a_{T-1} - \hat{X}^{u*}_{T-1}}{\mu} &= \beta\overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right] + (\gamma\Xi\mu + (1-\beta)\mu)\overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right] \\ &= \left(\beta + (1-\beta)\mu + \gamma\Xi\mu\right)\overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right].\end{aligned} \tag{A.2.38}$$

Therefore, we can get the expression for Φ^a_{T-1} as follows:

$$\begin{aligned}&\Phi^a_{T-1}\ \overline{\mathbb{E}}_{I_a,T-1}\left[\tilde{R}_{T|T-1}\right] - \Phi^{u*}_{T-1}\ \overline{\mathbb{E}}_{I,T-1}\left[\tilde{R}_{T|T-1}\right] \\ &= \left(\beta + (1-\beta)\mu + \gamma\Xi\mu\right)\left(\Phi^a_T\ \overline{\mathbb{E}}_{I_a,T-1}\left[\tilde{R}_{T|T-1}\right] - \Phi^{u*}_T\ \overline{\mathbb{E}}_{I,T-1}\left[\tilde{R}_{T|T-1}\right]\right)\end{aligned}$$

For inflation, recall the New Keynesian Phillips Curve (equation (2.4.2)),

$$
\begin{aligned}
\overline{\mathbb{E}}_{I_a,T-1}\left[\pi_{T-1}\right] - \overline{\mathbb{E}}_{I,T-1}\left[\pi_{T-1}\right] &= \Xi \hat{X}^a_{T-1} + \beta \overline{\mathbb{E}}_{I_a,T-1}\left[\pi_T\right] - \Xi \hat{X}^a_{T-1} - \beta \overline{\mathbb{E}}_{I,T-1}\left[\pi_T\right] \\
&= \beta \overline{\mathbb{E}}_{I_a,T-1}\left[\Xi \hat{X}^a_T + \beta \overline{\mathbb{E}}_{I,T}\left[\pi_{T+1}\right]\right] - \beta \overline{\mathbb{E}}_{I,T-1}\left[\Xi \hat{X}^u_T + \beta \overline{\mathbb{E}}_{I_a,T}\left[\pi_{T+1}\right]\right] \\
&= \beta \Xi \overline{\mathbb{E}}_{I_a,T-1}\left[\hat{X}^a_T - \hat{X}^u_T\right] \\
&= \Xi \mu \overline{\mathbb{E}}_{I_a,T-1}\left[\frac{\hat{X}^a_T - \hat{X}^{u*}_T}{\mu}\right] \\
&= \Xi \mu \left(\Phi^a_T \ \overline{\mathbb{E}}_{I_a,T-1}\left[\tilde{R}_{T|T-1}\right] - \Phi^{u*}_T \ \overline{\mathbb{E}}_{I,T-1}\left[\tilde{R}_{T|T-1}\right]\right)
\end{aligned}
$$

which implies $\Omega^a_{T-1} - \Omega^{u*}_{T-1} = \Xi\mu\left(\Phi^a_T - \Phi^{u*}_T\right)$. The second line comes from the next period (T) New Keynesian Phillips Curve, the third line is from $\pi_{T+1} = 0$, the fourth line uses equation (A.2.29), and the last line uses the definition of Φ^a (equation (A.2.33)).

Next we claim that $\Phi^a_t - \Phi^{u*}_t$ and $\Omega^a_t - \Omega^{u*}_t$ follow the recursive description below for any period $t \leq T-2$

$$
\begin{pmatrix} \Phi^a_t \\ \Phi^{u*}_t \\ \Omega^a_t \\ \Omega^{u*}_t \end{pmatrix} = M_a \begin{pmatrix} \Phi^a_{t+1} \\ \Phi^{u*}_{t+1} \\ \Omega^a_{t+1} \\ \Omega^{u*}_{t+1} \end{pmatrix}
$$

where the transition matrix M_a is defined as

$$
M_a := \begin{pmatrix} \beta + ((1-\beta)\mu + \gamma\Xi\mu)(\lambda_z + \lambda_v) & 0 & \gamma\beta & 0 \\ 0 & \beta + ((1-\beta)\mu + \gamma\Xi\mu)\lambda^u_z & 0 & \gamma\beta \\ \Xi\mu & 0 & \beta & 0 \\ 0 & \Xi\mu & 0 & \beta \end{pmatrix}
$$

We prove the claim by induction starting with proving the base case, i.e., the reaction for $T-2$. Recall that the reaction in $T-2$ can be written as follows using equation (A.2.30):

$$\begin{aligned}
&\frac{\hat{X}^{a}_{T-2}-\hat{X}^{u*}_{T-2}}{\mu}\\
&=-\gamma\beta^{3}\left(\overline{\mathbb{E}}_{I_a,T-2}\left[R_{T|t^*}-r^{n}_{T|t^*}\right]-\overline{\mathbb{E}}_{I,T-2}\left[R_{T|t^*}-r^{n}_{T|t^*}\right]\right)+\gamma\beta^{2}\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_T\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_T\right]\right)\\
&\quad+\gamma\beta\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_{T-1}\right]\right)+(1-\beta)\beta\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T}\right]}{\mu}\right)\\
&\quad+(1-\beta)\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T-1}\right]}{\mu}\right) \qquad \text{(A.2.39)}
\end{aligned}$$

We replace the first term in the right-hand side (the difference of the interest rates) using period T result (equation (A.2.36)):

$$\begin{aligned}
&\beta^{2}\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T}\right]}{\mu}\right)+\gamma\beta^{2}\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_T\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_T\right]\right)\\
&+\gamma\beta\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_{T-1}\right]\right)+(1-\beta)\beta\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T}\right]}{\mu}\right)\\
&+(1-\beta)\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T-1}\right]}{\mu}\right)
\end{aligned}$$

The above expression features inflation differences. Using the New Keynesian Phillips Curve (equation (2.4.2)), we can replace both inflation differences with corresponding output gaps:

$$\begin{aligned}
&\gamma\beta^{2}\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_T\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_T\right]\right)=\gamma\beta\Xi\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T}\right]}{\mu}\right)\\
&\gamma\beta\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_{T-1}\right]\right)\\
&\quad=\gamma\Xi\mu\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^{a}_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T-1}\right]}{\mu}+\gamma\beta^{2}\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_T\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_T\right]\right)
\end{aligned}$$

Applying these observations to the right-hand side of equation (A.2.39) yields

$$\beta(\beta+\gamma\Xi\mu+(1-\beta)\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_T^{u*}\right]}{\mu}\right)$$
$$+((1-\beta)\mu+\gamma\Xi\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_{T-1}^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_{T-1}^{u*}\right]}{\mu}\right)$$
$$+\gamma\beta\Xi\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_T^{u*}\right]}{\mu}\right)$$

Using the result for period $T-1$, equation (A.2.38), we can write the above as

$$\beta(\beta+(1-\beta)\mu+\gamma\Xi\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_T^{u*}\right]}{\mu}\right)$$
$$+((1-\beta)\mu+\gamma\Xi\mu)\times$$
$$\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[(\beta+(1-\beta)\mu+\gamma\Xi\mu)\overline{\mathbb{E}}_{I_a,T-1}\left[\hat{X}_T^a\right]\right]-\overline{\mathbb{E}}_{I,T-2}\left[(\beta+(1-\beta)\mu+\gamma\Xi\mu)\overline{\mathbb{E}}_{I,T-1}\left[\hat{X}_T^{u*}\right]\right]}{\mu}\right)$$
$$+\gamma\beta\Xi\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_T^{u*}\right]}{\mu}\right)$$

Using the higher-order (average) expectations for each of the aware and unaware consumers, we rewrite the above as

$$\frac{\hat{X}_{T-2}^a-\hat{X}_{T-2}^{u*}}{\mu}$$
$$=\beta(\beta+(1-\beta)\mu+\gamma\Xi\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_T^{u*}\right]}{\mu}\right)$$
$$+((1-\beta)\mu+\gamma\Xi\mu)(\beta+(1-\beta)\mu+\gamma\Xi\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[(\lambda_z+\lambda_v)\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\lambda_z^u\hat{X}_T^{u*}\right]}{\mu}\right)$$
$$+\gamma\beta\Xi\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}_T^a\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}_T^{u*}\right]}{\mu}\right)$$

and again using the result for period $T-1$ of equation (A.2.38) we obtain

$$\begin{aligned}
&\frac{\hat{X}^a_{T-2}-\hat{X}^{u*}_{T-2}}{\mu}\\
&=\beta\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^a_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T-1}\right]}{\mu}\right)\\
&\quad+((1-\beta)\mu+\gamma\Xi\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[(\lambda_z+\lambda_v)\hat{X}^a_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\lambda^u_z\hat{X}^{u*}_{T-1}\right]}{\mu}\right)\\
&\quad+\gamma\beta\Xi\mu\left(\frac{\overline{\mathbb{E}}_{I_a,T-2}\left[\hat{X}^a_{T}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\hat{X}^{u*}_{T}\right]}{\mu}\right).
\end{aligned}$$

Now we are ready to restate the above equation using Φ and Ω as follows. This proves that our claim holds at period $T-2$ for the first and second rows of M_a:

$$\begin{aligned}
&\Phi^a_{T-2}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]-\Phi^{u*}_{T-2}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\\
&=\left(\beta+((1-\beta)\mu+\gamma\Xi\mu)(\lambda_z+\lambda_v)\right)\Phi^a_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]\\
&\quad-\left(\beta+((1-\beta)\mu+\gamma\Xi\mu)\lambda^u_z\right)\Phi^{u*}_{T-1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\\
&\quad+\gamma\beta\left(\Omega^a_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]-\Omega^{u*}_{T-1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\right)
\end{aligned}$$

For the inflation,

$$\begin{aligned}
&\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_{T-2}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\pi_{T-2}\right]\\
&=\beta\overline{\mathbb{E}}_{I_a,T-2}\left[\pi_{T-1}\right]-\beta\overline{\mathbb{E}}_{I,T-2}\left[\pi_{T-1}\right]\\
&=\beta\left(\overline{\mathbb{E}}_{I_a,T-2}\left[\Xi\hat{X}^a_{T-1}\right]-\overline{\mathbb{E}}_{I,T-2}\left[\Xi\hat{X}^u_{T-1}\right]+\Omega^a_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]-\Omega^{u*}_{T-1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\right)\\
&=\Xi\mu\overline{\mathbb{E}}_{I_a,T-2}\left[\frac{\hat{X}^a_{T-1}-\hat{X}^{u*}_{T-1}}{\mu}\right]+\beta\left(\Omega^a_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]-\Omega^{u*}_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]\right)\\
&=\Xi\mu\left(\Phi^a_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]-\Phi^{u*}_{T-1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\right)\\
&\quad+\beta\left(\Omega^a_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]-\Omega^{u*}_{T-1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right]\right)
\end{aligned}$$

which implies $\Omega^a_{T-2}-\Omega^{u*}_{T-2}=\Xi\mu\left(\Phi^a_{T-1}-\Phi^{u*}_{T-1}\right)+\beta\left(\Omega^a_{T-1}-\Omega^{u*}_{T-1}\right)$, and this proves our claim for the third and fourth rows of M_a.

Assume as the induction hypothesis that for any $\tau \in \{t^*+1, t^*+2, ..., T-2\}$, the above claim on the transition matrix holds. We like to show that the claim holds for t^*. Again, the difference of the period t^* reactions can be written as follows:

$$\frac{\hat{X}^a_{t^*} - \hat{X}^{u*}_{t^*}}{\mu} = -\gamma \sum_{\tau=0}^{T-t^*} \beta^{\tau+1} \left(\overline{\mathbb{E}}_{I_a,t^*}\left[r_{t^*+\tau|t^*} - r^n_{t^*+\tau|t^*} \right] - \overline{\mathbb{E}}_{I,t^*}\left[r_{t^*+\tau|t^*} - r^n_{t^*+\tau|t^*} \right] \right)$$
$$+ (1-\beta)\mu \sum_{\tau=0}^{T-t^*} \beta^{\tau} \left(\frac{\overline{\mathbb{E}}_{I_a,t^*}\left[\hat{X}^a_{t^*+\tau} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^{u*}_{t^*+\tau} \right]}{\mu} \right) \tag{A.2.40}$$

Since $\pi_{T+1} = 0$, $\tilde{R}_{t^*+\tau} = 0$, and $z_{t^*+\tau} = 0$ for any $\tau < T - t^*$, the first term in the right hand side is

$$\gamma \sum_{\tau=0}^{T-t^*} \beta^{\tau+1} \left(\overline{\mathbb{E}}_{I_a,t^*}\left[\pi_{t^*+\tau+1} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\pi_{t^*+\tau+1} \right] \right).$$

Considering above equation one step forward at t^*+1, multiplying it with β, and taking difference to equation (A.2.40) gives,

$$\left(\frac{\hat{X}^a_{t^*} - \hat{X}^{u*}_{t^*}}{\mu} \right) - \beta \left(\frac{\overline{\mathbb{E}}_{I_a,t^*}\left[\hat{X}^a_{t^*+1} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^{u*}_{t^*+1} \right]}{\mu} \right)$$
$$= \gamma\beta \left(\overline{\mathbb{E}}_{I_a,t^*}\left[\pi_{t^*+1} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\pi_{t^*+1} \right] \right) + (1-\beta)\mu \left(\frac{\overline{\mathbb{E}}_{I_a,t^*}\left[\hat{X}^a_{t^*+1} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^{u*}_{t^*+1} \right]}{\mu} \right)$$

By replacing π with $\hat{X}$ and $\tilde{R}$ using equation (A.2.34), we obtain the following expression for the right-hand side of the above equation.

$$= \gamma\beta \left(\overline{\mathbb{E}}_{I_a,t^*}\left[\Xi \hat{X}^a_{t^*+1} \right] + \Omega^a_{t^*+1} \overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\Xi \hat{X}^u_{t^*+1} \right] - \Omega^{u*}_{t^*+1} \overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*} \right] \right)$$
$$+ (1-\beta)\mu \left(\frac{\overline{\mathbb{E}}_{I_a,t^*}\left[\hat{X}^a_{t^*+1} \right] - \overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^{u*}_{t^*+1} \right]}{\mu} \right)$$

Then we rearrange the expression as

$$
\begin{aligned}
&= (\gamma\Xi\mu + (1-\beta)\mu)\left(\frac{\overline{\mathbb{E}}_{I_a,t^*}\left[\hat{X}^a_{t^*+1}\right] - \overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^{u*}_{t^*+1}\right]}{\mu}\right) \\
&\quad + \gamma\beta\left(\Omega^a_{t^*+1}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] - \Omega^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]\right) \\
&= (\gamma\Xi\mu + (1-\beta)\mu)\left(\overline{\mathbb{E}}_{I_a,t^*}\left[\frac{\hat{X}^a_{t^*+1} - (1-\mu)\hat{X}^{u*}_{t^*+1}}{\mu}\right] - \overline{\mathbb{E}}_{I,t^*}\left[\hat{X}^u_{t^*+1}\right]\right) \\
&\quad + \gamma\beta\left(\Omega^a_{t^*+1}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] - \Omega^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]\right) \\
&= (\gamma\Xi\mu + (1-\beta)\mu)\left(\overline{\mathbb{E}}_{I_a,t^*}\left[\Phi^a_{t^*+1}\overline{\mathbb{E}}_{I_a,t^*+1}\left[\tilde{R}_{T|t^*}\right]\right] - \overline{\mathbb{E}}_{I,t^*}\left[\Phi^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,t^*+1}\left[\tilde{R}_{T|t^*}\right]\right]\right) \\
&\quad + \gamma\beta\left(\Omega^a_{t^*+1}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] - \Omega^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]\right)
\end{aligned}
$$

where the last line uses the definition of Φ^a (equation (A.2.33)). Therefore, we can rewrite equation (A.2.40) using Φ and Ω as follows:

$$
\begin{aligned}
&\Phi^a_{t^*}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right] - \Phi^{u*}_{t^*}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right] \\
&= \beta\left(\Phi^a_{t^*+1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right] - \Phi^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\right) \\
&\quad + ((1-\beta)\mu + \gamma\Xi\mu)\left((\lambda_z+\lambda_v)\Phi^a_{t^*+1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right] - \lambda^u_z\Phi^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\right) \\
&\quad + \gamma\beta\left(\Omega^a_{t^*+1}\overline{\mathbb{E}}_{I_a,T-2}\left[\tilde{R}_{T|T-2}\right] - \Omega^{u*}_{t^*+1}\overline{\mathbb{E}}_{I,T-2}\left[\tilde{R}_{T|T-2}\right]\right),
\end{aligned}
$$

and this concludes the claim.

Step 6. From equation (A.2.29), we know that the overall reaction of the current output gap $\hat{X}^a_{t^*}$ is as follows:

$$
\hat{X}^a_{t^*} = \frac{\mu}{\beta}\left(\frac{\hat{X}^a_{t^*} - \hat{X}^{u*}_{t^*}}{\mu}\right) + \hat{X}^u_{t^*}.
$$

Also, from Step 4, we get

$$
\frac{\hat{X}^a_{t^*} - \hat{X}^{u*}_{t^*}}{\mu} = \begin{pmatrix}1 & -1 & 0 & 0\end{pmatrix}(M_a)^{T-t^*-1}\cdot\begin{pmatrix}\Phi^a_{T-1}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] \\ \Phi^{u*}_{T-1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \\ \Omega^a_{T-1}\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] \\ \Omega^{u^*}_{T-1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right]\end{pmatrix}
$$

From Step 2, we get

$$\hat{X}^u_{t^*} = \begin{pmatrix} 1 & 0 \end{pmatrix} (M_u)^{T-t^*-1} \cdot \begin{pmatrix} \Phi^u_{T-1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \\ \Omega^u_{T-1}\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] \end{pmatrix}$$

Recall that the $T-1$ results in the lower space and upper space are

$$\begin{aligned}
\Phi^a_{T-1} &= (\beta + (1-\beta)\mu + \gamma\Xi\mu)\,\Phi^a_T \\
\Phi^{u*}_{T-1} &= (\beta + (1-\beta)\mu + \gamma\Xi\mu)\,\Phi^{u*}_T \\
\Omega^a_{T-1} &= \Xi\mu\Phi^a_T \\
\Omega^{u*}_{T-1} &= \Xi\mu\Phi^{u*}_T \\
\Phi^u_{T-1} &= (1+\gamma\Xi)\Phi^u_T \\
\Omega^u_{T-1} &= \beta\Xi\Phi^u_T
\end{aligned}$$

and the period T results are

$$\begin{aligned}
\Phi^a_T\overline{\mathbb{E}}_{I_a,t^*}\left[\tilde{R}_{T|t^*}\right] =& \frac{1}{\phi_y}\left(\lambda_z + \lambda_v\left(\frac{-\gamma\beta\phi_y + (1-\beta)(1-\mu)}{1-(1-\beta)\mu}\right)\right)\tilde{R}_{T|t^*} \\
=& \left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right)\tilde{R}_{T|t^*}
\end{aligned}$$

and

$$\Phi^u_T\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] = \frac{1}{\phi_y}\lambda^u_z\tilde{R}_{T|t^*} = \frac{\Lambda_{11}}{\Lambda_{21}}\lambda^u_z\tilde{R}_{T|t^*}$$

$$\Phi^{u*}_T\overline{\mathbb{E}}_{I,t^*}\left[\tilde{R}_{T|t^*}\right] = \frac{(1-\beta)\mu}{\beta+(1-\beta)\mu}\Phi^a_T + \frac{\beta}{\beta+(1-\beta)\mu}\Phi^u_T$$

Substitution allows us now to derive $\hat{X}^a_{t^*}$ as a function of $\tilde{R}_{T|t^*}$ as follows:

$$\hat{X}^a_{t^*}(\tilde{R}_{T|t^*}) =$$

$$\frac{\mu}{\beta}\begin{pmatrix} 1 & -1 & 0 & 0 \end{pmatrix}(M_a)^{T-t^*-1} \cdot \begin{pmatrix} (\beta + (1-\beta)\mu + \gamma\Xi\mu)\left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right) \\ (\beta + (1-\beta)\mu + \gamma\Xi\mu)\left(\frac{\Lambda_{11}}{\Lambda_{21}}\frac{\beta\lambda^u_z + (1-\beta)\mu\lambda_z}{\beta+(1-\beta)\mu} + \frac{\Lambda_{12}}{\Lambda_{22}}\frac{(1-\beta)\mu\lambda_v}{\beta+(1-\beta)\mu}\right) \\ \Xi\mu\left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right) \\ \Xi\mu\left(\frac{\Lambda_{11}}{\Lambda_{21}}\frac{\beta\lambda^u_z + (1-\beta)\mu\lambda_z}{\beta+(1-\beta)\mu} + \frac{\Lambda_{12}}{\Lambda_{22}}\frac{(1-\beta)\mu\lambda_v}{\beta+(1-\beta)\mu}\right) \end{pmatrix}\tilde{R}_{T|t^*}$$

$$+ \begin{pmatrix} 1 & 0 \end{pmatrix}(M_u)^{T-t^*-1} \cdot \begin{pmatrix} (1+\gamma\Xi)\frac{\Lambda_{11}}{\Lambda_{21}}\lambda^u_z \\ \beta\Xi\frac{\Lambda_{11}}{\Lambda_{21}}\lambda^u_z \end{pmatrix}\tilde{R}_{T|t^*}$$

This proves the proposition. □

A.2.3 Corollary 2.4.1

Proof. By setting $\theta = 1$, we can see $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta} = 0$ hence $\Xi = \kappa(\psi + \frac{1}{\gamma}) = 0$. Then, the transition matrices in the Proposition 2.4.2 are reduced as follows:

$$M_a = \begin{pmatrix} \beta + ((1-\beta)\mu)(\lambda_z + \lambda_v) & 0 & \gamma\beta & 0 \\ 0 & \beta + ((1-\beta)\mu)\lambda_z^u & 0 & \gamma\beta \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix}$$

$$M_u = \begin{pmatrix} \beta + (1-\beta)\lambda_z^u & \gamma \\ 0 & \beta \end{pmatrix}$$

Note that the transition matrices are upper triangular hence $M_a^{T-t^*-1}$ and $M_u^{T-t^*-1}$ are also upper triangular. Further, the $T-1$ results are condensed as

$$\begin{pmatrix} \Phi^a_{T-1} \\ \Phi^{u*}_{T-1} \\ \Omega^a_{T-1} \\ \Omega^{u*}_{T-1} \end{pmatrix} = (\beta + (1-\beta)\mu) \begin{pmatrix} \left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right) \\ \left(\frac{\Lambda_{11}}{\Lambda_{21}}\frac{\beta\lambda_z^u + (1-\beta)\mu\lambda_z}{\beta + (1-\beta)\mu} + \frac{\Lambda_{12}}{\Lambda_{22}}\frac{(1-\beta)\mu\lambda_v}{\beta + (1-\beta)\mu}\right) \\ 0 \\ 0 \end{pmatrix} \tilde{R}_{T|t^*}$$

$$\begin{pmatrix} \Phi^u_{T-1} \\ \Omega^u_{T-1} \end{pmatrix} = \begin{pmatrix} \frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z^u \\ 0 \end{pmatrix} \tilde{R}_{T|t^*}$$

Therefore, the overall reaction when we fix the inflation at 0 is,

$$\begin{aligned} \hat{X}^{IS}_{t^*} =& (\beta + (1-\beta)\lambda_z^u)^{T-t^*-1} \frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z^u \tilde{R}_{T|t^*} \\ &+ \frac{\mu}{\beta}\left((\beta + (1-\beta)\mu)(\lambda_z + \lambda_v)\right)^{T-t^*-1} (\beta + (1-\beta)\mu) \left(\frac{\Lambda_{11}}{\Lambda_{21}}\lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}}\lambda_v\right) \tilde{R}_{T|t^*} \\ &- \frac{\mu}{\beta}\left((\beta + (1-\beta)\mu)(\lambda_z^u)\right)^{T-t^*-1} \left(\frac{\Lambda_{11}}{\Lambda_{21}}(\beta\lambda_z^u + (1-\beta)\mu\lambda_z) + \frac{\Lambda_{12}}{\Lambda_{22}}(1-\beta)\mu\lambda_v\right) \tilde{R}_{T|t^*} \end{aligned}$$

Under complete information, i.e., $\sigma_\eta^2 = 0$, we have:

$$\hat{X}^{IS}_{t^*}|_{\sigma_\eta^2=0} = \frac{\Lambda_{11}}{\Lambda_{21}}\tilde{R}_{T|t^*} + \frac{\mu}{\beta}\left((\beta + (1-\beta)\mu)\right)^{T-t-1}\beta(1-\lambda)\left(\frac{\Lambda_{12}}{\Lambda_{22}} - \frac{\Lambda_{11}}{\Lambda_{21}}\right)\tilde{R}_{T|t^*}$$

□

A.2.4 Proposition 2.5.1

Proof. Recall the perceived market clearing in the baseline equilibrium.

$$Y_t = C_t^u = \int_{i\in I} c_{i,t}^u di$$

With the modified inference rule, it should satisfy,

$$\begin{aligned} C_t^u = Y_t = C_t^a &= (1-\mu)C_t^u + \int_{i\in I_a} c_{i,t}di \\ &= \frac{1}{\mu}\int_{i\in I_a} c_{i,t}di \end{aligned}$$

which implies $\hat{X}_t^u = \frac{\hat{X}_t^a - (1-\mu)\hat{X}_t^u}{\mu}$. Therefore, we will find $\mathbb{E}_{u,t}^{sc}[z_{T|t^*} \mid \omega_{i,T|t^*}]$ such that the above equation is satified. Denote the inference rule $\mathbb{E}_{u,t}^{sc}[z_{T|t^*} \mid \omega_{i,T|t^*}] = \lambda_z^u \frac{\omega_{i,T|t^*}}{\xi_z} + \delta_{i,t}$ and the unaware type of consumers' Taylor rule as $\tilde{R}_t = \phi\hat{X}_t^u + e_t$ where $\delta_{i,t}$ and e_t to be determined. Recall the best response at period T at the lower space, equation (A.2.15).

$$\begin{aligned} \hat{X}_T^u &= -\gamma\overline{\mathbb{E}}_{I,T}^{sc}\left[\tilde{R}_{T|t^*} - \pi_{T+1}\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}^{sc}\left[z_{T|T}\right] \\ &= -\gamma\overline{\mathbb{E}}_{I,T}^{sc}\left[\phi_y\hat{X}_T^u + e_T\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}^{sc}\left[z_{T|T}\right] \\ &= -\frac{\gamma}{1+\gamma\phi_y}\overline{\mathbb{E}}_{I,T}^{sc}\left[e_T\right] - \frac{1}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I,T}^{sc}\left[z_{T|T}\right] \end{aligned}$$

We rewrite the perceived Taylor rule as a function of shocks using the above result.

$$\overline{\mathbb{E}}_{I,T}^{sc}\left[\tilde{R}_{T|t^*}\right] = \frac{1}{1+\gamma\phi_y}\overline{\mathbb{E}}_{I,T}^{sc}\left[e_T\right] \underbrace{-\frac{\phi_y}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)}_{=\xi_z}\overline{\mathbb{E}}_{I,T}^{sc}\left[z_{T|T}\right]$$

Given this rule, the unaware type consumer's inference is

$$\begin{aligned} \overline{\mathbb{E}}_{I,T}^{sc}\left[z_{T|t^*}\right] &= \frac{\lambda_z^u\tilde{R}_{T|T} - \frac{1}{1+\gamma\phi_y}\overline{\mathbb{E}}_{I,T}^{sc}\left[e_T\right]}{\xi_z} \\ &= \frac{\lambda_z^u}{\xi_z}\tilde{R}_{T|T} + \frac{1}{\phi_y}\left(\frac{\frac{1}{\gamma}+\psi}{1+\psi}\right)\overline{\mathbb{E}}_{I,T}^{sc}\left[e_T\right] \end{aligned}$$

From the definition of the modified inference rule, $\mathbb{E}_{u,t}^{sc}\left[z_{T|t^*}\right] = \lambda_z^u\frac{\omega_{i,T|t^*}}{\xi_z} + \delta_{i,t}$, we get

$$\overline{\delta}_{i,T} := \frac{1}{1-\mu}\int_{i\in I_u}\delta_{i,T}di = \frac{1}{\phi_y}\left(\frac{\frac{1}{\gamma}+\psi}{1+\psi}\right)\overline{\mathbb{E}}_{I,T}^{sc}\left[e_T\right]$$

and

$$\overline{\mathbb{E}}^{sc}_{I,T}\left[\hat{X}^u_T\right] = -\frac{\gamma}{1+\gamma\phi_y}\overline{\mathbb{E}}^{sc}_{I,T}\left[e_T\right] - \frac{1}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}^{sc}_{I,T}\left[z_{T|T}\right]$$
$$= \frac{1}{\phi_y}\lambda^u_z\tilde{R}_{T|T} - \frac{1}{\phi_y}\overline{\mathbb{E}}^{sc}_{I,T}\left[e_T\right].$$

Once we have the above contemporaneous reaction, we can derive the reaction at period t,

$$\hat{X}^u_t = \begin{pmatrix}1 & 0\end{pmatrix}(M_u)^{T-t-1}\begin{pmatrix}(1+\gamma\Xi) \\ \beta\Xi\end{pmatrix}\frac{\Lambda_{11}}{\Lambda_{21}}\left(\lambda^u_z\tilde{R}_{T|t^*} - \overline{\mathbb{E}}^{sc}_{I,t}\left[e_T\right]\right) \tag{A.2.41}$$

using Proposition 2.4.2. Moving to the aware type of consumer, recall that $\hat{X}^a_T = \hat{X}^u_T$ with $\overline{\mathbb{E}}^{sc}_{I,T}\left[e_T\right]$ in the self-confirming equilibrium.

$$\hat{X}^a_T = -\gamma\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|T} - \pi_{T+1}\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$
$$= -\gamma\overline{\mathbb{E}}_{I_a,T}\left[\phi_y\hat{X}^a_T + v_{T|T}\right] - \left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$
$$= \frac{-\gamma}{1+\gamma\phi_y}\overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right] - \frac{1}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$

Then, the Taylor rule is,[4]

$$\overline{\mathbb{E}}_{I_a,T}\left[\tilde{R}_{T|t^*}\right] = \underbrace{\frac{1}{1+\gamma\phi_y}}_{:=\xi'_v}\overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right] \underbrace{-\frac{\phi}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)}_{=\xi_z}\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$

and the output reaction is

$$\hat{X}^a_T = \frac{-\gamma}{1+\gamma\phi_y}\overline{\mathbb{E}}_{I_a,T}\left[v_{T|T}\right] - \frac{1}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\overline{\mathbb{E}}_{I_a,T}\left[z_{T|T}\right]$$
$$= \frac{-\gamma}{1+\gamma\phi_y}\frac{\tilde{R}_{T|T}}{\xi'_v}\lambda_v - \frac{1}{1+\gamma\phi_y}\left(\frac{1+\psi}{\frac{1}{\gamma}+\psi}\right)\frac{\tilde{R}_{T|T}}{\xi_z}\lambda_z$$
$$= \frac{1}{\phi_y}\lambda_z R_{T|T} - \gamma\lambda_v\tilde{R}_{T|T}$$
$$= \left(\lambda_z\frac{\Lambda_{11}}{\Lambda_{21}} + \lambda_v\left.\frac{\Lambda_{12}}{\Lambda_{22}}\right|_{\mu\to 1}\right)\tilde{R}_{T|T}$$

[4]Note that $\xi'_v = \lim_{\mu\to 1}\xi_v$

where the second equation comes from the estimation rule (equation (2.3.1)), and the third equation comes from the definition of ξ_v' and ξ_z, and the last equation comes from the definition of Λs. Further, in the reflective equilibrium, $\hat{X}_t^a = \hat{X}_t^u$ for all $t \in \{t^*, ..., T\}$. Therefore, $\Phi_t^a = \Phi_t^{u^*} = \Phi_t^u$, $\Omega_t^a = \Omega_t^{u^*} = \Omega_t^u$ the transition matrix M_a becomes simply

$$\tilde{M}_a := \begin{pmatrix} \beta + (1 - \beta + \gamma\Xi)(\lambda_z + \lambda_v) & \gamma\beta \\ \Xi & \beta \end{pmatrix}.$$

Therefore, the aware consumer's reaction at period t^* is

$$\hat{X}_t^a = \begin{pmatrix} 1 & 0 \end{pmatrix} (M_a)^{T-t-1} \begin{pmatrix} 1 + \gamma\Xi\mu \\ \beta\Xi \end{pmatrix} \left(\frac{\Lambda_{11}}{\Lambda_{21}} \lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}} \lambda_v \right) \tilde{R}_{T|t^*}$$

Finally, $\hat{X}_t^a$ should be identical to $\hat{X}_t^u$. We equate the above result to equation (A.2.41) to get

$$\overline{\mathbb{E}}_{I,t}\left[e_T\right] = \left(\lambda_z^u - \frac{\begin{pmatrix} 1 & 0 \end{pmatrix} (M_a)^{T-t-1} \begin{pmatrix} 1 + \gamma\Xi\mu \\ \beta\Xi \end{pmatrix} \left(\frac{\Lambda_{11}}{\Lambda_{21}} \lambda_z + \frac{\Lambda_{12}}{\Lambda_{22}} \lambda_v \right)}{\begin{pmatrix} 1 & 0 \end{pmatrix} (M_u)^{T-t-1} \begin{pmatrix} (1 + \gamma\Xi) \\ \beta\Xi \end{pmatrix} \frac{\Lambda_{11}}{\Lambda_{21}}} \right) \tilde{R}_{T|t^*}$$

□

A.3 Chapter 3

A.3.1 Proposition 3.3.1

Proof. From the labor demand (equation (3.3.8)) and the definition of $q(\theta)$, we get

$$\xi\theta^{\eta} = A - w \tag{A.3.1}$$

which implies a decreasing labor demand function in a w-θ plane. Next, from the labor supply (equation (3.3.9)) we get,

$$\psi\alpha = \theta^{1-\eta} \frac{\varphi}{1 - \varphi} (A - w) \tag{A.3.2}$$

Combining the two equations gives

$$\psi\alpha = \frac{\varphi}{1-\varphi}\xi\theta \tag{A.3.3}$$

Recall the first-order condition of the search effort decision.

$$\psi\alpha = \theta^{1-\eta}\left(2\ln y^e + \ln\hat{C} - \ln b\right)$$

Using equation (A.3.3), we can replace the left-hand side.

$$\frac{\varphi}{1-\varphi}\xi\theta = \theta^{1-\eta}\left(2\ln(A - \xi\theta^\eta - G + e_2) + \ln\hat{C} - \ln b\right)$$

Therefore, the equilibrium market tightness satisfies

$$(\theta^*)^\eta = \frac{1-\varphi}{\varphi}\frac{1}{\xi}\left(2\ln(A - \xi\,(\theta^*)^\eta - G + e_2) + \ln\hat{C} - \ln b\right).$$

By implicit function theorem,

$$\frac{\partial\theta^*}{\partial b} = -\frac{2\frac{1}{y^e}G'(b) + \frac{1}{b}}{\eta\frac{\varphi}{1-\varphi}\xi(\theta^*)^{\eta-1} + \frac{2}{y^e}\xi\eta(\theta^*)^{1-\eta}} < 0$$

Then from equation (A.3.3), $\frac{\partial\alpha^*}{\partial b} < 0$ follows. Finally, since $N = \theta q(\theta)\alpha = \theta^{1-\eta}\alpha$, it is obvious that $\frac{\partial N^*}{\partial b} < 0$.

To see the effect of perceived temptation, first note that $C_1'(\hat{\beta},\hat{\gamma}) > 0$, $C_2'(\hat{\beta},\hat{\gamma}) < 0$ because

$$\begin{aligned}
\frac{\partial \ln C(\hat{\gamma})}{\partial\hat{\gamma}} &= \ln\hat{D} + \hat{\beta}\ln(1-\hat{D}) + \underbrace{\frac{1+\hat{\gamma}}{\hat{D}}\hat{D}' - \frac{(1+\hat{\beta}\hat{\gamma})}{1-\hat{D}}\hat{D}'}_{=0} + \ln(1+\hat{\beta}) - \hat{\beta}\ln\frac{\hat{\beta}}{1+\hat{\beta}} \\
&= (1+\hat{\beta})\ln\left(\frac{1+\hat{\beta}}{1+\tilde{\beta}}\right) + \hat{\beta}\ln\left(\frac{\tilde{\beta}}{\hat{\beta}}\right) < 0, \quad \left(\because \frac{\partial(x+\hat{\beta})\ln\left(\frac{x+\hat{\beta}}{x+\tilde{\beta}}\right)}{\partial x} < 0 \text{ for } \hat{\beta} < \tilde{\beta}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \ln C(\hat{\beta})}{\partial\hat{\beta}} =& \underbrace{\frac{1+\hat{\gamma}}{\hat{D}}\hat{D}' - \frac{(1+\hat{\beta}\hat{\gamma})}{1-\hat{D}}\hat{D}'}_{=0} + \hat{\gamma}\ln(1-\hat{D}) \\
&+ \hat{\gamma}\frac{1}{1+\hat{\beta}} - \hat{\gamma}\ln\frac{\hat{\beta}}{1+\hat{\beta}} - \hat{\beta}\hat{\gamma}\frac{1+\hat{\beta}}{\hat{\beta}}\left(\frac{1}{1+\hat{\beta}} - \frac{\hat{\beta}}{(1+\hat{\beta})^2}\right) \\
=& \hat{\gamma}\ln\frac{\tilde{\beta}}{1+\tilde{\beta}} - \hat{\gamma}\ln\frac{\hat{\beta}}{1+\hat{\beta}} > 0
\end{aligned}$$

where $\tilde{\beta}$ is the effective discount rate $\frac{1+\hat{\beta}\hat{\gamma}}{1+\hat{\gamma}}$. Then, again from the implicit function theorem,

$$\frac{\partial\theta^*}{\partial\hat{\gamma}} = \frac{\frac{1}{\hat{C}}\hat{C}'(\hat{\gamma})}{\eta\frac{\varphi}{1-\varphi}\xi(\theta^*)^{\eta-1} + \frac{2}{y^e}\xi\eta(\theta^*)^{1-\eta}} < 0$$

$$\frac{\partial\theta^*}{\partial\hat{\beta}} = \frac{\frac{1}{\hat{C}}\hat{C}'(\hat{\beta})}{\eta\frac{\varphi}{1-\varphi}\xi(\theta^*)^{\eta-1} + \frac{2}{y^e}\xi\eta(\theta^*)^{1-\eta}} > 0$$

□

A.3.2 Proposition 3.4.1

Proof. The first-order condition of the optimum policy problem (equation (3.4.1)) indicates that the optimal b^* should satisfy the following identity.

$$\frac{\partial\mathcal{W}}{\partial b} = \frac{\partial N}{\partial b}\left((U(c_1^e, c_2^e) - U(c_1^u, c_2^u)) - \frac{\partial\Psi}{\partial N}\right) + N\frac{\partial U(c_1^e, c_2^e)}{\partial b} + (1-N)\frac{\partial U(c_1^u, c_2^u)}{\partial b} = 0$$

Using the first-order condition of the job search decision ($\frac{\partial\Psi}{\partial N} = \hat{W}^e - \hat{W}^u$) and the value of the unemployed state does not depend on the temptation ($\hat{W}^u = W^u = U(c_1^u, c_2^u)$), the right-hand side becomes

$$\begin{aligned}&\frac{\partial N}{\partial b}\left(U(c_1^e, c_2^e) - \hat{W}^e\right) + N\frac{\partial U(c_1^e, c_2^e)}{\partial b} + (1-N)\frac{\partial U(c_1^u, c_2^u)}{\partial b}\\ &= \frac{\partial N}{\partial b}\left(U(c_1^e, c_2^e) - W^e + W^e - \hat{W}^e\right) + N\frac{\partial U(c_1^e, c_2^e)}{\partial b} + (1-N)\frac{\partial U(c_1^u, c_2^u)}{\partial b}\end{aligned}$$

We denote $\Lambda := U(c_1^e, c_2^e) - W^e$ as the resistance cost in the equilibrium, and $\Omega := W^e - \hat{W}^e$ as the perception gap. The partial derivatives of the utilities are

$$\frac{\partial U(c_1^e, c_2^e)}{\partial b} = -\frac{2}{y^e}G'(b)$$

$$\frac{\partial U(c_1^e, c_2^e)}{\partial b} = \frac{1}{b}$$

Replacing this in the original equation, we get

$$\frac{\partial\mathcal{W}}{\partial b} = \frac{\partial N}{\partial b}(\Lambda + \Omega) - N\frac{2}{y^e}G'(b) + (1-N)\frac{1}{b} = 0$$

Further, recall that the budget constraint $G(b) = \frac{1-N}{N}b$ using the law of large numbers. Therefore,

$$G'(b) = \frac{1-N}{N} - \frac{1}{N^2}\frac{\partial N}{\partial b}b$$

Now we plugin this into the original equation to get

$$\begin{aligned}\frac{\partial \mathcal{W}}{\partial b} &= \frac{\partial N}{\partial b}(\Lambda+\Omega) - N\frac{2}{y^e}\left(\frac{1-N}{N} - \frac{1}{N^2}\frac{\partial N}{\partial b}b\right) + (1-N)\frac{1}{b} \\ &= \frac{\partial N}{\partial b}(\Lambda+\Omega) + (1-N)\left(\frac{1}{b} - \frac{2}{y^e}\right) + \frac{1}{N}\frac{\partial N}{\partial b}b\end{aligned}$$

This proves our proposition. □

A.3.3 Proposition3.4.1

Proof. First, notice that y^e and N do not depend on γ. N or θ are functions of the perceived cost $\hat{\gamma}$, not the actual one since consumers choose search effort based on the perception on future temptation. Therefore, y^e also does not depend on $\hat{\gamma}$ since

$$\begin{aligned}y^e :=& w + \frac{\Pi}{N} - G(b) + e_2 \\ =& A - \xi\theta^\eta - G(b) + e_2\end{aligned}$$

Further, the cross partial derivative $\frac{\partial^2 N^*}{\partial b \partial \gamma}$ in the equation (3.4.2) is also zero since $\frac{\partial \theta^*}{\partial b}$ is a function of $\hat{\gamma}$, not γ. Therefore, all labor market decisions are related only to the perceived parameter and independent to the actual one.

Finally, the resistance cost and the perception gap are as follows:

$$\begin{aligned}\Lambda(\beta,\gamma) &:= U - W^e = \gamma \ln\left(\frac{1+\tilde{\beta}}{1+\beta}\right) + \beta\gamma\ln\left(\frac{\beta(1+\tilde{\beta})}{\tilde{\beta}(1+\beta)}\right) \geq 0 \\ \Omega(\beta,\hat{\beta},\gamma,\hat{\gamma}) &:= W^e - \hat{W}^e \ln\left(\frac{C(\beta,\gamma)}{C(\hat{\beta},\hat{\gamma})}\right) \leq 0\end{aligned}$$

where $\tilde{\beta}$ is the perceived effective discount rate ($\frac{1+\hat{\beta}\hat{\gamma}}{1+\hat{\gamma}}$). Then, the only difference of the first order condition that the private insurance to be optimal is the absence of $\Lambda \geq 0$, which implies that the cross partial with respect to $\hat{\gamma}$ of the social welfare maximizing problem is negative.

$$\frac{(\partial \mathcal{W})^2}{\partial b \partial \hat{\gamma}} = \frac{\partial N}{\partial b}\Lambda' < 0$$

Also, because of the concavity assumption (Assumption 3.4.1), the second derivative is negative. Therefore, the implicit function theorem implies

$$\frac{\partial b^*}{\partial \hat{\gamma}} = -\frac{\frac{\partial^2 \mathcal{W}}{\partial b \partial \hat{\gamma}}}{\frac{\partial^2 \mathcal{W}}{(\partial b)^2}} < 0$$

which means that b^* is smaller than x^{u*}. □

A.3.4 Proposition 3.5.1

Proof. The first order condition of equation (3.5.1) is,

$$\tilde{\beta} p(\theta) \left(u(c_2^e) - u(c_2^u)\right) = \Psi'(\alpha). \tag{A.3.4}$$

Given that, I go to the first-period consumption-saving decision. The first order conditions are,

$$\begin{aligned} u'(c_1) &= \lambda_1 \\ \tilde{\beta} \frac{\partial p}{\partial c_2^e} \left(u(c_2^e) - u(c_2^u)\right) + \tilde{\beta} p u'(c_2^e) - \frac{\partial \Psi}{\partial c_2^e} &= \lambda_2 \\ \tilde{\beta} \frac{\partial p}{\partial c_2^u} \left(u(c_2^e) - u(c_2^u)\right) + \tilde{\beta}(1-p) u'(c_2^u) - \frac{\partial \Psi}{\partial c_2^u} &= \lambda_3 \\ \lambda_1 - \lambda_2 - \lambda_3 &= 0 \end{aligned}$$

where λs are associated Lagrange multipliers. Combining the results,

$$u'(c_1) = \tilde{\beta} \left(p u'(c_2^e) + (1-p) u'(c_2^u)\right) \tag{A.3.5}$$

which confirms the usual envelope theorem. The social welfare (U-maximization) is,

$$\max_b u(c_1) + N u(c_2^e) + (1-N) u(c_2^u) - \Psi(\alpha)$$

and, the first-order condition for the social optimum is as follows:

$$0 = u'(c_1) \frac{\partial c_1}{\partial b} + \left(\frac{1}{\tilde{\beta}} - 1\right) \Psi'(\alpha) \frac{\partial \alpha}{\partial b} + N u'(c_2^e) \frac{\partial c_2^e}{\partial b} + (1-N) u'(c_2^u) \frac{\partial c_2^u}{\partial b} \tag{A.3.6}$$

The (constrained) optimum allocation is described by the Euler equation (equation (A.3.5)), three budget constraints in the problem (equation (3.5.2)), labor market equilibrium (equation(A.3.4)), and the optimal policy rule (equation(A.3.6)). Combining all these equations, we get

$$N u'(c_2^e) \left(G'(b) + (1-\tilde{\beta}) \frac{\partial c_1}{\partial b}\right) = \left(\frac{1}{\tilde{\beta}} - 1\right) \Psi'(\alpha) \frac{\partial \alpha}{\partial b} + (1-N) u'(c_2^u) \left(1 - (1-\tilde{\beta}) \frac{\partial c_1}{\partial b}\right).$$

Now compare this with W-maximizing private insurance. The model including the job search problem is identical. The only difference is the consumer's consumption saving problem at period 1.

$$\begin{aligned}
&\max_{c_1, c_2^e, c_2^u} U(c_1, \mathbb{E}_1[c_2|\alpha^*], \alpha^*) - \gamma \left(V(\tilde{c}_1, \mathbb{E}_1[\tilde{c}_2|\tilde{\alpha}^*], \tilde{\alpha}^*) - V(c_1, \mathbb{E}_1[c_2|\alpha^*], \alpha^*)\right) \\
\text{s.t. } & c_1 + q^e x^e + q^u x^u \leq e_1 \\
& c_2^e \leq x^e + w + \frac{\Pi}{N} \\
& c_2^u \leq x^u \\
& \mathbb{E}_1[c_2|\alpha] = p(\alpha;\theta)u(c_2^e) + (1-p(\alpha;\theta))u(c_2^u) \\
& q^e x^e + q^u x^u \leq p x^e + (1-p)x^u, \quad \forall x^e, x^u
\end{aligned}$$

where q^e, q^u is the price of the contingent claims, x^e, x^u is the Arrow-Debreau securities. The last condition in the problem is the no-arbitrage condition. The individual optimization results in the following.

$$\begin{aligned}
q^e u'(c_1) &= \tilde{\beta} p u'(c_2^e) \\
q^u u'(c_1) &= \tilde{\beta}(1-p)u'(c_2^u) \\
\Rightarrow u'(c_1) &= \tilde{\beta} u'(c_2^e) = \tilde{\beta} u'(c_2^u)
\end{aligned}$$

□

A.3.5 Proposition 3.5.1

Proof. Note that we can write G' from the law of large numbers as

$$G'(b) = \frac{1-N}{N} - \frac{1}{N^2}\frac{\partial N}{\partial b} b$$

and the marginal utility ratio is

$$\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{b^*} = \frac{\frac{1-N}{N}}{\frac{1-N}{N} - \frac{1}{N^2}\frac{\partial N}{\partial b} b} = \frac{1}{1 - \frac{b}{N(1-N)}\frac{\partial N}{\partial b}}.$$

Finally, the equilibrium employment is a decreasing function of b because the first-order condition of the search effort decision indicates

$$\tilde{\beta}\theta^{1-\eta}\left(u(c_2^e)-u(c_2^u)\right)=\frac{\varphi}{1-\varphi}\xi\theta$$
$$\tilde{\beta}\theta^{1-\eta}\left(\ln(s_1+A-\xi\theta^{\eta}-G)-\ln(s_1+b)\right)=\frac{\varphi}{1-\varphi}\xi\theta.$$

Hence, the equilibrium market tightness satisfies

$$\frac{\varphi}{1-\varphi}\frac{\xi}{\tilde{\beta}}(\theta^*)^{\eta}=\left(\ln(s_1+A-\xi(\theta^*)^{\eta}-G)-\ln(s_1+b)\right)$$

and the implicit function theorem gives,

$$\frac{\partial\theta^*}{\partial b}=\frac{\frac{1}{c_2^e}\left(\frac{\partial s_1}{\partial b}-G'(b)\right)-\frac{1}{c_2^u}}{\frac{\varphi}{1-\varphi}\frac{\xi}{\tilde{\beta}}\eta(\theta^*)^{\eta-1}+\frac{1}{c_2^e}\xi\eta(\theta^*)^{\eta-1}}<0$$

because of the Assumption 3.5.1. Therefore, we get $\frac{\partial N^*}{\partial b}<0$, and the marginal utility ratio $\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{b^*}$ is less than 1. Since $u(\cdot)$ is a concave function, we can conclude that $c_2^e(b^*)>c_2^e(x^{u*})$ and $c_2^u(b^*)<c_2^u(x^{u*})$. □

A.3.6 Lemma 3.5.2

Proof. It is straightforward from the Proposition 3.5.1 by noting that $\frac{\partial c_1^*}{\partial b}>0$, $\Psi(\alpha)\frac{\partial\alpha^*}{\partial b}<0$ hence

$$\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{b^*}=\left(\frac{1-(1-\tilde{\beta})\frac{\partial c_1}{\partial b}+\left(\frac{1}{\tilde{\beta}}-1\right)\Psi'(\alpha)\frac{\partial\alpha}{\partial b}}{G'(b)+(1-\tilde{\beta})\frac{\partial c_1}{\partial b}-\left(\frac{1}{\tilde{\beta}}-1\right)\Psi'(\alpha)\frac{\partial\alpha}{\partial b}}\right)n<\frac{1}{G'(b)}n<1=\left.\frac{u'(c_2^e)}{u'(c_2^u)}\right|_{x^{u*}}$$

□

Appendix B

Alternative specification

B.1 Chapter 1

B.1.1 Alternative welfare criterion sympathetic planner

For the sympathetic planner, the constraint is

$$\bar{V}_{u,t} \leq u(z_t) - k(\alpha_t) + \alpha_t \beta\delta[\int_{\underline{w}}^{R_t} W_{u,t+1} dF(w) + \int_{R_t}^{\overline{w}} \frac{u(w)}{1-\delta} dF(w)] + (1-\alpha_t)\beta\delta W_{u,t+1}$$

with the individual decision rules in the main text (equation 1.3.1 and 1.3.2). The sympathetic planner problem's first-order condition is as follows.

$$\frac{u'(z_{t+1})}{u'(z_t)} = \frac{1}{\beta}\left[\frac{\delta(1-p_t) - \overbrace{\frac{\partial p_t}{\partial z_{t+1}} E_{t+1}}^{\text{UI inefficiency}}}{\delta(1-p_t)}\right]$$

$$= \frac{1}{\beta}\left[1 - \frac{\overbrace{\{1-F(R_t)\}\frac{\partial \alpha_t}{\partial R_t}\frac{\partial R_t}{\partial z_{t+1}} E_{t+1}}^{\text{search effort ineff.}} - \overbrace{\alpha_t f(R_t) \frac{\partial R_t}{\partial z_{t+1}} E_{t+1}}^{\text{reservation wage ineff.}}}{\delta(1-p_t)}\right]$$

Similarly to the paternalistic planner's case, this first-order condition indicates a decreasing UI sequence. The perception about the liquidation is s^* when $\beta = 1$ because the agent is fully naive. Therefore $s^*|_{\beta<1} < \hat{s} = s^*|_{\beta=1}$.

It is worthwhile to note a few observations. First, because of the present bias that the planner has, the UI scheme is also time inconsistent. A plan, once perceived as an optimum, is not going to be the best choice when the future arrives. If we interpret the planner as a government, then the populism of the government makes the plan to be revised at every period. Second, as the paternalistic planner, the slope of optimum UI is not monotonic with respect to the present bias, because there are two opposite forces in action. The direct effect of present bias makes the plan steeper, whereas the indirect effect through the job-finding rate and search effort makes the plan flatter. In the limit case, with a sufficient present bias, the UI scheme becomes steeper than the exponential agents.

$$\lim_{\beta \to 0} \frac{u'(z_{t+1})}{u'(z_t)} = \infty > \left. \frac{u'(z_{t+1})}{u'(z_t)} \right|_{\beta=1} > \lim_{\beta \to \infty} \frac{u'(z_{t+1})}{u'(z_t)}$$

B.2 Chapter 3

B.2.1 No financial friction

If there's no binding borrowing constraint for the unemployed agent, she can also smooth consumption. The value of transition (from an unemployed to an employed agent) is independent of γ and β.

$$V^e - V^u = (1 + \delta)(\ln y^e - \ln y^u) = \psi N$$

which means that the equilibrium wage and employment don't change as well. This implies that the previous results are exactly coming from the fact that the unemployed are essentially temptation-free. The utility cost of resisting temptation only applies to the employed, hence as the cost increases, the unemployed states become more attractive. The necessity of the binding financial constraint assumption, however, can be dropped if I adopt a more general CRRA utility.

B.2.2 CRRA utility function

I can derive qualitatively the same conclusion without the binding financial constraint if I use a more general CRRA utility rather than logarithmic. Assume felicity utility function is $\frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$. I will have the following full temptation and actual consumption profiles.

$$\bar{c}_0 = \frac{y^e}{1+\delta\beta^\sigma}, \quad \bar{c}_1 = \frac{\beta^\sigma y^e}{1+\delta\beta^\sigma}$$

$$c_0^e = \frac{y^e}{1+\delta\left(\frac{1+\beta\gamma}{1+\gamma}\right)^\sigma}, \quad c_1^e = \frac{\left(\frac{1+\beta\gamma}{1+\gamma}\right)^\sigma y^e}{1+\delta\left(\frac{1+\beta\gamma}{1+\gamma}\right)^\sigma}$$

Then, the value (without the financial constraint) is,

$$\begin{aligned} V^i =& (1+\gamma)\frac{\left(\frac{y^i}{1+\delta\left(\frac{1+\beta\gamma}{1+\gamma}\right)^\sigma}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \\ &+ \delta(1+\beta\gamma)\frac{\left(\frac{\left(\frac{1+\beta\gamma}{1+\gamma}\right)^\sigma y^i}{1+\delta\left(\frac{1+\beta\gamma}{1+\gamma}\right)^\sigma}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \gamma\left(\frac{\left(\frac{y^i}{1+\delta\beta^\sigma}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \beta\delta\frac{\left(\frac{\beta^\sigma y^i}{1+\delta\beta^\sigma}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}\right) \end{aligned}$$

Note that the value is linear to the income powered by the risk aversion parameter. That is, $V^i = \mathcal{C}(\beta,\gamma)\cdot(y^i)^{1-\frac{1}{\sigma}}$. Then the value of transition is

$$V^e - V^u = \mathcal{C}(\beta,\gamma)\cdot\left((y^e)^{1-\frac{1}{\sigma}} - (y^u)^{1-\frac{1}{\sigma}}\right)$$

Now, without the binding financial constraint, the transition value depends on the temptation parameters (β,γ), and the arguments above hold.