Title
ELECTRICAL ENGINEERING REVIEW COURSE - LECTURE XII

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ELECTRICAL ENGINEERING REVIEW COURSE

LECTURE XII
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(Notes by: R. Kilpatrick, K. Stone)

SYMBOLS AND UNITS

Units in MKS System

i = instantaneous current--amperes
e = instantaneous voltage--volts
I'= peak current--amperes
E'= peak voltage--volts
I = Root mean square current $= \frac{I'}{\sqrt{2}}$
E = Root mean square voltage $= \frac{E'}{\sqrt{2}}$
R = Resistance--Ohms
L = Inductance--Henry
C = Capacity--Farad

W = circular frequency-- $\frac{\text{rad.}}{\text{sec}}$
F = frequency-- $\frac{\text{cycles}}{\text{sec}} = \frac{W}{2\pi}$
WL = Inductive reactance--Ohms
WC = Capacitive reactance--Ohms
Z = Impedance--Ohms.
Z_{eq} = Equivalent series impedance of the parallel circuit.
Wo = Frequency at resonance.
θ = Phase angle
ϕ'= Peak magnetic field strength

ϕ = Instantaneous magnetic field strength

PARALLEL CIRCUITS

Definition: A parallel circuit is one in which the line voltage is common to all branches of the circuit.

Consider the simple circuit of Fig. 1 in which the supply voltage is sinusoidal and the circuit constants are as shown.

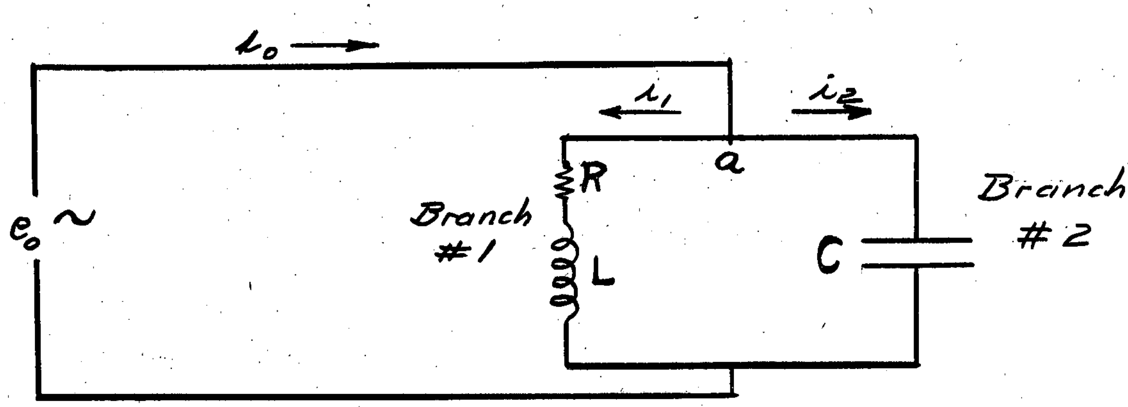


Fig. 1

NOTE: In many practical circuits the resistance in the capacitive branch is negligible and therefore, is not shown above.

DISCLAIMER

Applying Kirchoffs law at junction "a" gives:

$$I + I_2 = I_o \tag{1}$$

This is a vector equation since the branch currents are obviously out of phase with each other.

Two further equations can be written by noting that the voltage drop across each branch is the line voltage

$$L \frac{di_1}{dt} + i_1 R = e_o \tag{2}$$

$$\frac{1}{C} \int i_2 dt = e_o \tag{3}$$

To solve these equation use the complex representations

$$\begin{aligned} i_o &= I' e^{jwt} \\ i_1 &= I' e^{jwt} \\ i_2 &= I' e^{jwt} \\ e_o &= E' e^{jwt} \end{aligned}$$

Now assume the circuit has reached steady state. This will eliminate constants of integration from solutions of above equations.

Equation (2) becomes:

$$LjwI_1' e^{jwt} + RI_1' e^{jwt} = E' e^{jwt} \qquad \text{OR}$$

$$I_1' = \frac{E'}{R + jwL} \tag{4}$$

Equation (3) becomes:

$$\frac{1}{C} \; \frac{I'_2 e^{jwt}}{jw} = E' e^{jwt} \qquad \text{OR}$$

$$\frac{I'_2}{jwC} = E' \tag{5}$$

Equation (4) and (5) can be changed to the more usable form involving R.M.'s values simply by multiplying by $1/\sqrt{2}$

Then

$$I_1 = \frac{E}{R + jwL} \tag{6}$$

$$I_2 = \frac{E}{\frac{1}{jwC}} \tag{7}$$

and from (1)

$$I_o = E \left[\frac{1}{E + jwL} + \frac{1}{\frac{1}{jwC}} \right] \tag{8}$$

Define $Z_{eq} = \frac{E}{I_o}$ (8,a)

Then

$$\frac{1}{Z_{eq}} = \left[\frac{1}{R + jwL} + jwC\right] \quad (9)$$

$$\frac{1}{Z_{eq}} = jwC + \left(\frac{1}{R + jwL}\right)\left(\frac{R - jwL}{R - jwL}\right)$$

$$\frac{1}{Z_{eq}} = jwC + \frac{R - jwL}{R^2 + w^2L^2}$$

$$\frac{1}{Z_{eq}} = \frac{R}{R^2 + w^2L^2} + j\left[wC - \frac{wL}{R^2 + w^2L^2}\right] \quad (10)$$

Let

$$R_{eq} = \frac{R}{R^2 + w^2L^2} \quad (11)$$

$$X_{eq} = \left[wC - \frac{wL}{R^2 + w^2L^2}\right] \quad (12)$$

Substituting in (10)

$$\frac{1}{Z_{eq}} = R_{eq} + jX_{eq}$$

$$Z_{eq} = \left(\frac{1}{R_{eq} + jX_{eq}}\right)\left(\frac{R_{eq} - jX_{eq}}{R_{eq} - jX_{eq}}\right) = \frac{R_{eq} - jX_{eq}}{R_{eq}^2 + jX_{eq}^2}$$

$$Z_{eq} = \frac{R_{eq}}{R_{eq}^2 + X_{eq}^2} - j\frac{X_{eq}}{R_{eq}^2 + X_{eq}^2} \quad (13)$$

The magnitude of Z_{eq} is found by taking the square root of the sum of the squares of the real parts of the complex impedance given by equation (13).

$$\left|Z_{eq}\right| = \sqrt{\frac{R_{eq}^2}{(R_{eq}^2 + X_{eq}^2)^2} + \frac{X_{eq}^2}{(R_{eq}^2 + X_{eq}^2)^2}}$$

OR $\left|Z_{eq}\right| = \frac{1}{\sqrt{R_{eq}^2 + X_{eq}^2}}$

Substituting for R and X

$$\left|Z_{eq}\right| = \frac{1}{\sqrt{\frac{R}{R^2 + w^2L^2}^2 + wC - \frac{wL}{R^2 + w^2L^2}^2}} \quad (14)$$

From equation (13) the line phase angle (Θ_o) is found.

$$\Theta_o = \tan^{-1}\frac{\text{Reactive Component}}{\text{Reistive Component}} = \tan^{-1}\frac{X}{R} \qquad \text{OR}$$

$$\theta = \tan^{-1}\left[\frac{wC - \dfrac{wL}{R^2 + w^2L^2}}{\dfrac{R}{R^2 + w^2L^2}}\right] \tag{16}$$

REWONANCE IN PARALLEL CIRCUIT

The parallel circuit is similar to the series circuit in that it has a resonant frequency (w_o), but at this frequency the current is at minimum in the parallel circuit. At resonance, Z_{eq} is therefore a maximum.

Consider the case where $R \ll wL$

Then $R^2 <<< w^2L^2$

And equation (14) can be rewritten as:

$$Z_{eq} = \frac{1}{\sqrt{\dfrac{R^2}{w^4L^4} + \left[wC - \dfrac{1}{wL}\right]^2}} \tag{17}$$

It is seen that $\left|Z_{eq}\right|$ is maximum when $wC - 1/wL = 0$

Solving for the frequency which fits this condition gives the resonant frequency.

$$w_o = \frac{1}{\sqrt{LC}} \tag{18}$$

At this frequency

$$\left|Z_{eq}\right|_o \cong \frac{1}{\sqrt{\dfrac{R^2}{w_o{}^4L^4}}} = \frac{w_o{}^2L^2}{R}$$

$$\left|Z_{eq}\right|_o = w_oL\left[\frac{w_oL}{R}\right]$$

and since

$$\frac{w_oL}{R} = Q$$

$$\left|Z_{eq}\right|_o = w_oLQ \tag{19}$$

Note that at resonance, the capacitive reactance $(1/wC)$ and the inductive reactance (wL) of the two branch circuits are equal and the parallel circuit looks like a pure resistance to the line current. With R small relative to the reactances, the branch currents are approximately 180° out of phase and circulate around the parallel circuit without entering the line.

The variation of I_o with change in frequency is shown in Fig. 2.

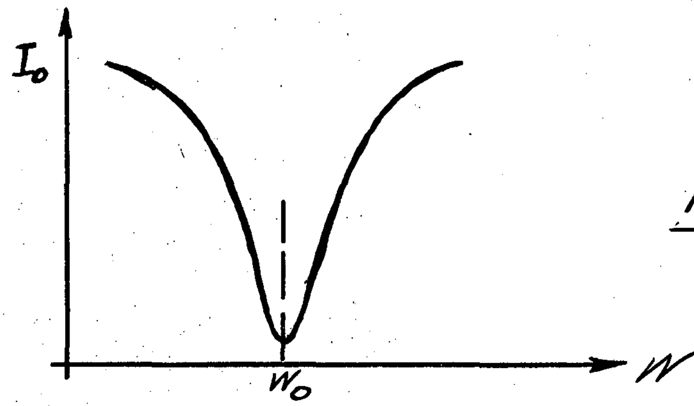


Fig. 2.

Variation of $|Z_{eq}|$ with W is shown in Fig. 3.

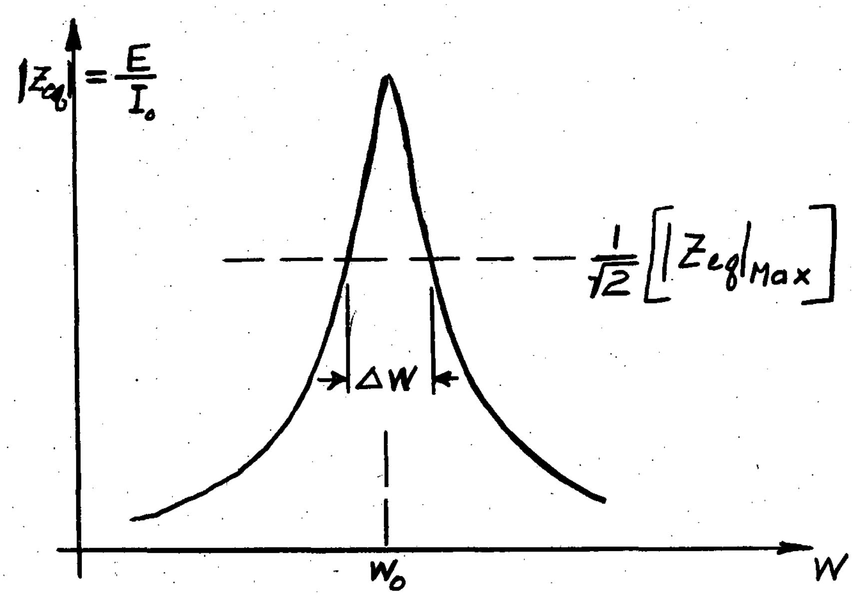


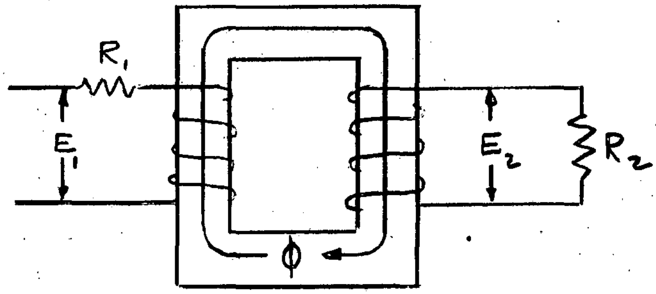
Fig. 3

Q of the circuit can be determined experimentally by driving with constant voltage, measuring line voltage and line current while the frequency is varied through known values near the resonant point. Then from a plot of $|Z_{eq}|$ (=E/I) versus the frequency, the phase shift (Δw) indicated in Fig. 3 is found and inserted in the following formula:

$$Q = \frac{w_o}{2\Delta w} \tag{20}$$

TRANSFORMERS

<u>Definition</u>: A transformer is a net of coupled circuits used to transfer electrical energy from one alternating current ciruit to another without change in frequency. This transference is usually, but not always, accompanied by a change in voltage.



R_1 is resistance of primary and R_2 is the combined secondary and load. Load is assumed to be pure resistance. Note: Subscript 1 refers to primary and subscript 2 refers to secondary. Assume that the transformer is a perfect coupling, i.e., all flux links both circuits also neglect affects of distributed capacitive. Considering voltage drop across primary:

$$i_1R_1 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} = e_1 \qquad (21)$$

$$i_2R_2 + L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} = 0 \qquad (22)$$

Double subscripts: L_{21} means the inductance of the secondary due to the change in current in primary, etc.

In all practicle transformers the resistance is very small with respect to the inductive reactance.

Symbolically $R_1 \ll wL_{11}$
Primary amp. turns $N_1\phi$
Second amp. Turns $N_2\phi$

Ignoring losses in primary circuit, i_1R_1 terms

$$\frac{N_1}{C}\frac{d\phi}{dt} = e_1 = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} \qquad (23)$$

$$\frac{N_2}{C}\frac{d\phi}{dt} + i_2R_2 = 0 = i_2R_2 + L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} \qquad (24)$$

Let

$$\phi = \phi' e^{jwt} \qquad (25)$$

$$e_1 = E' e^{jwt} \qquad (26)$$

Differentiating (25) and substituting for $\frac{d\phi}{dt}$ in (23)

$$jw\frac{N_1}{C}\phi' = e_1$$

$$\phi' = \frac{e_1}{jw\frac{N_1}{C}} \qquad (27)$$

Following the same procedure on (25) and (26)

$$\frac{N_2}{C} jw\phi' + i_2R_2 = 0$$

Combining (27) and (28)

$$\frac{N_2}{N_1}e_1 = - i_2R_2 = - e_2$$

or

$$\frac{E_1}{E_2} = - \frac{N_1}{N_2}$$

Note: Ratio of in-put and out-put voltage is equal to ratio of primary and secondary terms. This ideal condition is approached in transformer with low flux leakage. To find equivalent series circuit equations (21) and (22) give:

$$L_{12} = \sqrt{L_{11}L_{22}}$$ Mutual inductance of two perfectly coupled circuits.

or

$$L_{12} = k\sqrt{L_{11}L_{12}}$$ where k is loss in coupling, $k<1$

$$e_1\left[R_2 + jwL_2\right] = jwL_1R_2i_1 \tag{29}$$

by definition

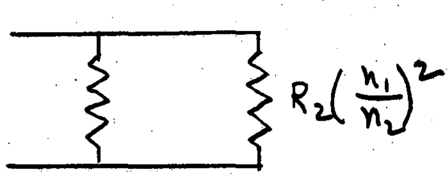
$$e_1(\tfrac{1}{Z}) = i_1 \tag{30}$$

and substituting (30) in (29) we get

$$\frac{1}{Z} = \frac{1}{jwL_1} + \frac{1}{R^2 \frac{L_1}{L_2}}$$

Z = Primary impedance of resistance loaded transformers.

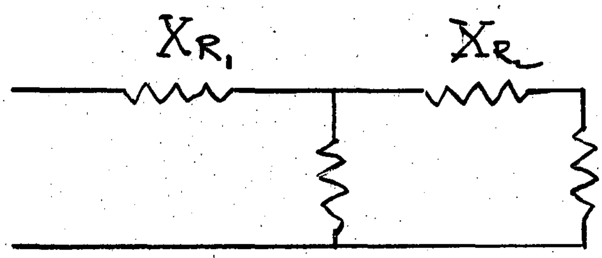
Equivalent circuit



For a good transformer wL_1 is low so input line sees resistance load times

$$\left(\frac{N_1}{N_2}\right)^2$$

For transformer with leakage



X_R = Leakage reactance.

Find XR_1 on open circuit test.

Find XR_2 on short circuit test.