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Authors
Coccia, C
McCartney, J

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Geotechnical instructional centrifuge modeling of stress distribution

C.J.R. Coccia and J.S. McCartney
University of Colorado Boulder, Boulder, Colorado, USA

ABSTRACT: This paper focuses on a new instructional module for the physical modeling of stress distributions within a layer of sand developed for use with an instructional centrifuge. This study is designed to help undergraduate geotechnical students to develop a deeper understanding of the assumptions behind the analytical solutions for the stress distribution under a square footing. A new centrifuge container was designed with the ability to measure the change in stress at a point within a sand layer due to the application of a static applied load at the surface of the layer. An experimental procedure was developed allowing 3 to 4 tests to be run during a typical 2 hour-long laboratory session. A template for an excel-based laboratory report is presented that can be used by undergraduate students during class to prepare the soil specimen and compare the experimental results with existing analytical solutions. The report also includes observational questions requiring the students to consider the influences of boundary effects and arching in centrifuge modeling.

1 INTRODUCTION

Knowledge of the change in stress with depth in a soil layer due to an applied footing stress is necessary to predict footing settlement, which is a basic component of an undergraduate geotechnical education. Undergraduate students are often given analytical solutions, based on the concepts of elasticity theory, that indicate that changes in stress within a soil layer will decay with distance away from the location of an applied load or footing stress (Boussinesq 1885, Newmark 1935). However, students often find it challenging to understand the assumptions behind these solutions as they are usually not derived from first principles in lectures. Further, students often do not have the intuition that soil not directly underneath the footing may also experience a change in stress. Accordingly, a physical modeling module for estimating the change in stress at a point in a soil layer during application of an applied stress was developed for an instructional geotechnical centrifuge. Such a module allows students to observe the distribution of stress changes within a soil layer within a typical 2-hour laboratory session, leading to experimental data that can be analyzed and compared with the analytical solutions discussed during lecture. This intuition-building educational approach has been found to deepen the learning experience for students (Wartman 2006). Along these lines, instructional centrifuge modules have been developed for bearing capacity, slope stability, and lateral earth pressure problems (Craig 1989; Dewoolkar et al. 2003). Beyond capturing the basic physics of the geotechnical problem, the features of the educational models developed in these studies include simplicity in instrumentation and soil layer preparation, straightforward analysis, and easy cleanup.

2 BACKGROUND

2.1 *Instructional Centrifuge Testing*

It is expensive and time consuming to construct a full-scale soil layer, a surficial footing, and a loading system for the sake of instructional demonstrations. Although bench-scale soil models can be prepared readily for these purposes, inconsistencies are often encountered when comparing measurements from small-scale model footings to the behavior of full-scale prototype footings in the field. This is because the behavior of a soil layer is closely tied with the stress state induced by its self-weight. Centrifuge modeling alleviates these concerns, and permits testing of small-scale models by using centripetal acceleration to increase the self-weight of the soil layer. Several studies have developed scaling relationships to extrapolate behavior from centrifuge-scale models to full-scale models. The concept of geometric similitude indicates that the stresses in a model-scale soil layer will be the same as those in a prototype-scale soil layer that is N_g times larger, where N_g is equal to the centrifuge acceleration divided by the acceleration due to earth's gravity (i.e. the g-level).

3 APPROACH

3.1 *Experimental Setup*

A new physical modeling module for instructional purposes was designed to demonstrate the distribution of stress changes within a soil layer due to application of a surficial footing stress, as shown in Figure 1(a). The complete setup includes the module (a soil container and an integrated mechanical loading system) and the instructional centrifuge. A photo of the soil container and mechanical loading system is shown in Figure 1(b).

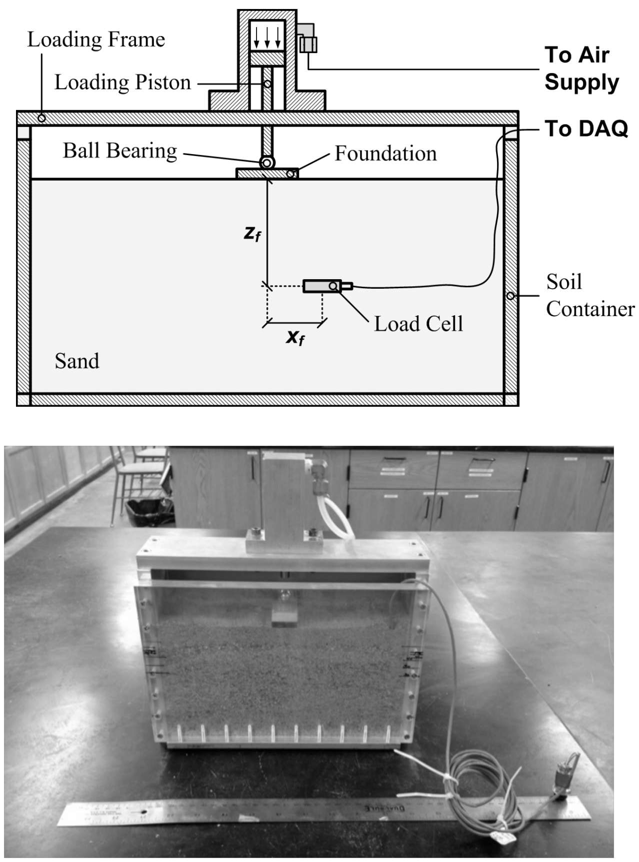
a)
b)


Figure 1. Experimental setup: a) Schematic; b) Photo

3.1.1 *Soil Container*

A soil container, originally developed by Abaidalla (2011), with internal dimensions of 75 mm-deep by 255 mm-wide by 178 mm-high was selected for this instructional module. The container is constructed from an 8 mm-thick aluminum base with two 13 mm-thick aluminum plates to support the left and right sides of the soil layer, and an 8 mm-thick plate to support the rear. The front wall of the container is a 19 mm-thick Acrylic face that permits visual observation during testing.

3.1.2 *Mechanical Loading System*

Surficial stresses are applied to the soil layer through a 6 mm-thick, 50 mm-square aluminum loading plate (i.e. the "footing"). A square footing was selected over a strip "type" footing in order to minimize the potential for frictional interaction between the footing and the soil container during testing. Loads are applied to the footing using a pneumatic piston, which functions by applying air pressure to a reservoir. The air pressure acts upon a low-friction 25.4 mm-diameter Teflon disk, shown in Figure 1(a), which converts the air pressure into a mechanical load applied to the footing. A ball bearing is used to ensure that no moments are transferred to the footing. The force from the loading piston is controlled manually using an air pressure regulator, shown in Figure 2(a). Changes in stress within the soil layer are measured using a 400 N Futek miniature load cell, shown in Figure 2(b), which can be placed at any location within the soil layer.

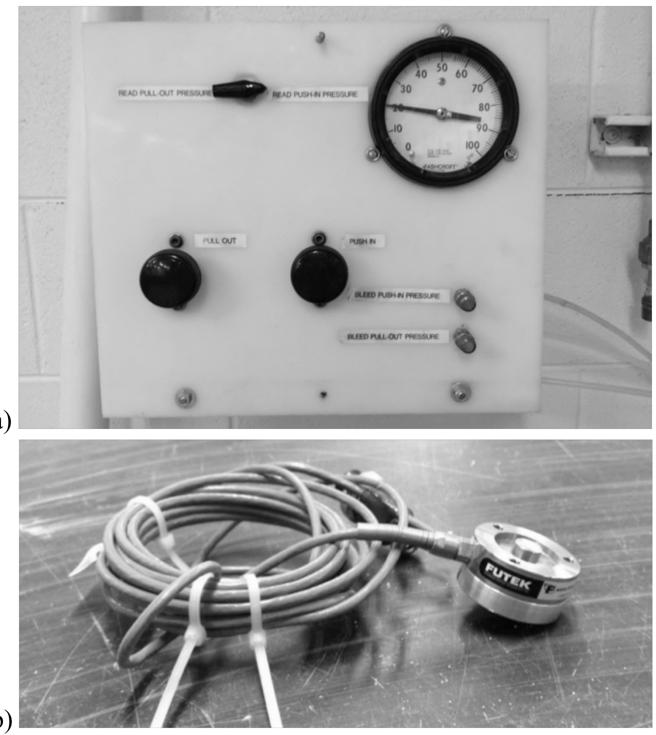
a)
b)


Figure 2. Mechanical loading system: a) Pressure control panel; b) Futek miniature load cell

3.1.3 *Instructional Centrifuge*

An instructional centrifuge located at the University of Colorado Boulder was utilized for demonstration of the new module. A photo of the instructional centrifuge is shown in Figure 3. The centrifuge is a swinging bucket type centrifuge that achieves a full swing radius of 565 mm from the center of the arm to the testing basket platform under full extension. The typical height of the specimen in this module is approximately 140 mm, giving a height:radius ratio of 0.25. Geotechnical centrifuge setups used for research usually should have a height:radius ratio less than 0.10 so that the g-level does not vary significantly within the model; however, the higher ratio of 0.25 is acceptable for instructional modules.

The testing basket is designed to carry a 7 kg payload up to an acceleration of 250 g's, corresponding to a 1.9 g-ton capacity. The centrifuge motor is controlled in a closed-loop speed control scheme by a

Pacific Scientific PC-834 brushless servo drive. The PC-834 drive allows direct control of both speed and torque of the drive train. This precise control allows centrifuge test specimens to be accelerated smoothly to the desired g-level. Once the centrifuge has achieved the required speed, the drive is capable of regulating the g-level with an accuracy of ±0.05 g.

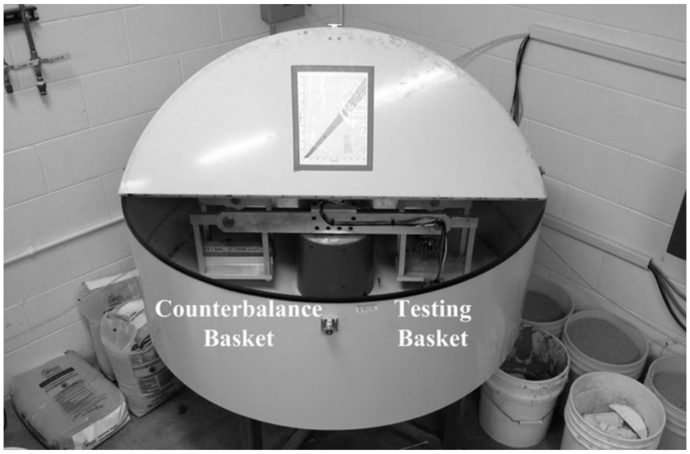


Figure 3. Instructional centrifuge

The centrifuge arm includes a payload basket and a counterbalance basket on opposite sides of the arm, respectively. Two air supply lines are included at the center of the centrifuge arm to allow application of different pressures to the selected soil box for testing. In addition, the centrifuge is equipped with one load cell input to monitor changes in load for various applications, and displacements can be observed through a digital imaging software connected to a digital camera mounted to the testing basket.

3.1.4 *Materials*

The instructional centrifuge tests were performed on a layer of a dry poorly graded medium to fine sand (SP). A dry sand was chosen for demonstration as it permits the use of air pluviation to achieve a uniform target density in a short period of time. Further, pluviation introduces students to a compaction method not typically utilized in most introductory geotechnical lab courses. The grain size distribution of the sand used in the experiments is shown in Figure 4. The sand has coefficients of curvature and uniformity of 2.8 and 5.2, respectively, with a fines content of 0.9 %. The sand was placed at a target dry density of 1600 kg/m^3. The relevant geotechnical properties of the sand are summarized in Table 1.

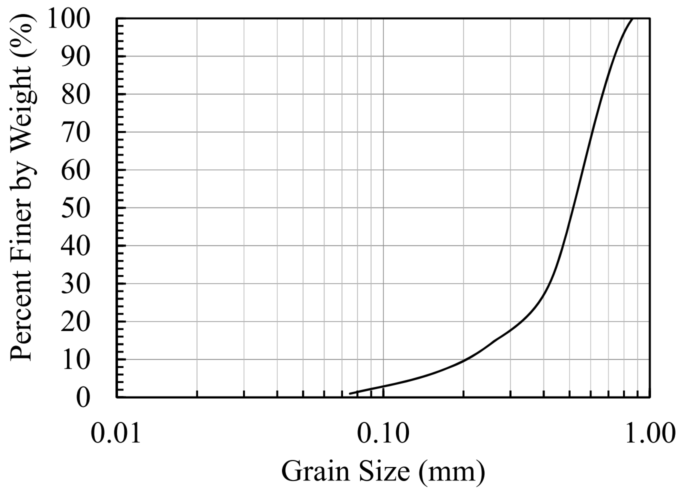


Figure 4. Grain size distribution for the poorly graded sand

Table 1. Summary of the geotechnical properties of sand

Parameter	Value
D_{10}	0.11 mm
D_{30}	0.42 mm
D_{60}	0.57 mm
% Passing No. 200 sieve	0.9 %
Coefficient of uniformity (C_u)	5.2
Coefficient of curvature (C_c)	2.8

3.1.5 *Procedures*

Prior to preparation of the sand layer, an MS Excel spreadsheet was provided to the students (and displayed on a monitor next to the centrifuge) that allows the students to input their measured parameters. Provision of this spreadsheet helps facilitate proper set up of the instructional centrifuge, and permits rapid analysis of the results and comparison between experimental data to existing analytical solutions during the laboratory session. A key portion of the spreadsheet is shown in Figure 5.

First, the initial mass of the container with no loading frame was measured and recorded into the spreadsheet (Box 1 – Figure 5). Following measurement of the mass of the soil box, the sand was placed into the centrifuge container through pluviation to a predetermined height corresponding to the assigned load cell depth. A drop height of 3.5 m and a funnel opening of 15 mm were used to achieve the target dry density. Prior to pluviation, all holes were covered with tape to avoid damage to the threads.

Sand Layer Density (model scale):

Mass Container (no frame)	**3711**	g	Box 1
Mass Container and Sand (no frame)	**8171**	g	Box 5
Mass Container, Sand, Frame	**9429**	g	Box 8
Mass of Load Cell	124	g	
Mass of Sand	4337	g	
Actual Depth of Sand	**142**	mm	Box 4
Density	1597	kg/m^3	

Loading System Details:

D_{piston} (model scale)	25	mm	
A_{piston} (model scale)	507	mm^2	
$B_{footing}$ (model scale)	**50**	mm	Box 6
$L_{footing}$ (model scale)	**50**	mm	Box 7
$A_{footing}$ (prototype scale)	1.00	m^2	
$A_{loadcell}$ (prototype scale)	0.13	m^2	

Load Cell Location and g-level Details:

Height of Load Cell (from base)	**100**	mm	Box 2
z_f (prototype scale)	0.84	m	
x_f (model scale)	**0**	mm	Box 3
x_f (prototype scale)	**0**	m	
Baseline Load (model scale)	**20**	N	Box 10
Baseline Load (prototype scale)	**8**	kN	
g-level	**20**		Box 9

Figure 5. Sample input file for centrifuge box preparation

After pluviation, the dry sand was leveled and the load cell was placed into the sand layer at a specified distance off-center as shown in Figure 6. The actual height and horizontal distance from center of the load cell was then measured and recorded into Boxes 2 and 3, respectively (Figure 5).

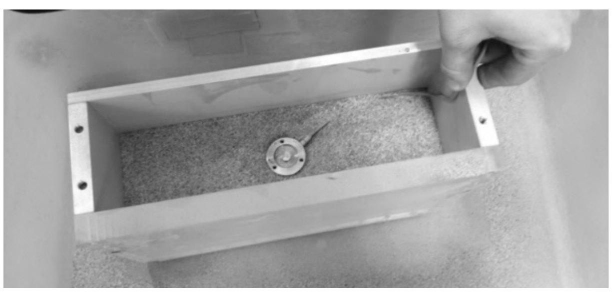


Figure 6. Photo of the load cell placement

After the load cell was placed and the location recorded, compaction continued to a target height of 140 mm. Following pluviation, the actual depth of sand achieved as well as the total mass of the container plus dry sand and load cell was measured and recorded into Boxes 4 and 5, respectively (Figure 5). The final mass and depth of sand achieved is used to determine the attained density of the sand layer as well as the center of gravity of the soil box used for preparing the instructional centrifuge for spinning.

The width (B) and depth (L) of the square footing used in the demonstrations were measured and recorded for Boxes 6 and 7, respectively (Figure 5). The top of the sand layer was then leveled and the square footing plus ball bearing was placed atop the center of the layer (Figure 7).

Figure 7. Top of sand layer with square footing

The loading frame was assembled onto the soil container while carefully bringing the loading piston into contact with the ball bearing. The mass of the complete testing apparatus was measured and recorded for Box 8 (Figure 5). Following final assembly of the testing apparatus, the container was placed onto the testing platform within the instructional centrifuge as shown in Figure 8, and all electronics and plumbing connections were connected.

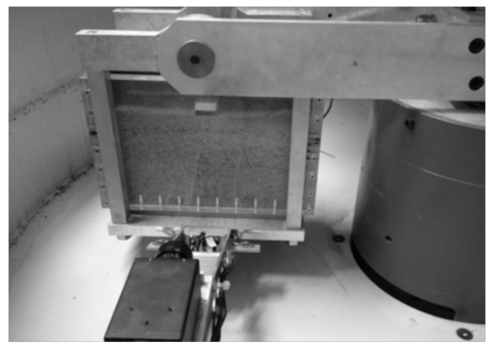


Figure 8. Container in the instructional centrifuge

Every specimen is slightly different, so careful balancing is critical. A center of gravity calculator is included in the spreadsheet given to the students (Figure 9) along with a counterbalance chart, which are required to define the appropriate counterbalance mass. These calculations help familiarize students with safe operation of a geotechnical centrifuge.

Center of Gravity Calculator:

Mass of Container and Frame	4914	g
COG Container and Frame	122	mm
Mass of Container, Frame, Sand	**9429**	g
Mass of Sand and Cell	4515	g
Actual Depth of Sand Layer	**142**	mm
COG Sand	84	mm
COG of Setup	**103.8**	**mm**

Figure 9. Spreadsheet for calculation of center of gravity

After the data acquisition system, camera, and lights were tested, the centrifuge lid was closed and locked. Next, the centrifuge was spun to a target g-level of 20 g (Box 9 – Figure 5). After equilibration, the initial load measured via the embedded load cell before application of the footing stress was recorded into Box 10 (Figure 5). This serves as the baseline measurement for the determination of the change in stress at the location of the load cell.

Seven stress increments were applied to the footing during testing, as shown in Figure 10. For each stress increment, the measured load cell reading was recorded into the provided spreadsheet once readings achieved equilibrium (Figure 11). The sand layer was observed to behave in an elastic manner for this range of stresses, and the influence factor (defined as the change in vertical stress measured by the load cell divided by the applied footing stress) from the different loading stages was averaged. The force measured by the load cell was converted to stress by dividing the recorded load by the area of the "load button" on the load cell (Figure 5).

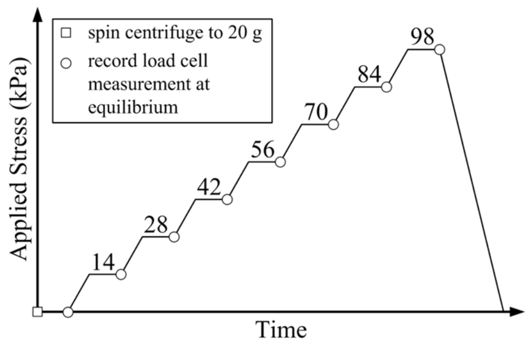


Figure 10. Loading and measurement scheme for testing

$\sigma_{applied}$ (kPa)	$F_{loadcell}$ (kN)	$\Delta F_{loadcell}$ (kN)	$\Delta\sigma$ (kPa)	$I_{\sigma,measured}$
14	**8.8**	0.8	6	0.46
28	**9.8**	1.8	14	0.51
42	**10.8**	2.8	22	0.53
56	**11.6**	3.6	29	0.51
70	**12.4**	4.4	35	0.50
84	**13.4**	5.4	43	0.51
98	**14.6**	6.6	53	0.54
			Average =	**0.51**

Figure 11. Spreadsheet for load cell readings in prototype scale

4 TYPICAL RESULTS

Results from 6 tests obtained during three 2-hour laboratory sessions in which the load cell was placed at various locations within the sand layer are shown in Table 2. The location of the load cell is specified in terms of distance away from the center of the square footing (x_f,z_f) normalized by the footing width (B). The overall influence factor for each test was determined by averaging the results for each applied stress increment from 14 to 98 kPa.

As expected, the measured influence factor decreases with increasing vertical and horizontal distance away from the center of the footing, which is consistent with the predictions from the analytical solution. The spreadsheet permitted the students to quickly compare the experimental data with the solution of Newmark (1935) in terms of the theoretical stress bulbs underneath the footing [Figure 12(a)] and in terms of the influence factors under the center of the footing [Figure 12(b)].

Table 2. Influence factors at various locations in the sand layer

Test	x_f/B	z_f/B	$I_{\sigma,measured}$
1	0.0	0.44	0.589
2	0.0	0.84	0.509
3	0.0	1.42	0.214
4	0.0	1.56	0.157
5	0.9	0.48	0.013
6	0.9	1.40	0.084

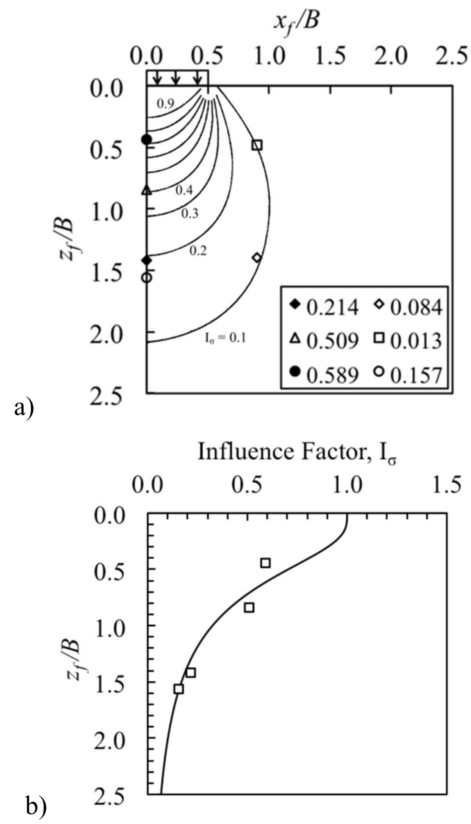


Figure 12. Comparison of measured results with Newmark's solutions for square footings (Newmark 1935): a) Two dimensional stress bulbs; b) Influence factors directly beneath footing

A good fit was observed between the experimental results and Newmark's solution, with increasing error with closer proximity to the footing. Following completion of the experimental portion of the laboratory module, the students were asked to hypothesize on the reasons for the differences between the measured and analytical influence factors. The students mentioned the likelihood that the presence of the sensor may affect the results. Students also identified potential issues such as settlement or shifting of the load cell during placement and spin-up, causing inaccuracies of the dimensions utilized in analysis (x_f, z_f).

After the students have had time to brainstorm, the teaching assistant discussed the results from previous experimental studies that quantified the reasons for the errors in the measurements. First, the fact that the analytical solutions were derived for a linear elastic and isotropic infinite half space is discussed, conditions likely not present during loading of a footing on a sand layer. Experimental errors associated with several topics are discussed, including: placement of the load cell (Garnier et al. 1999), the size, shape, and stiffness of the load cell (Askegaard 1995), "arching effects" (Dewoolkar et al. 2007; Ellis and Aslam 2009a, 2009b), "silo effects" due to friction at the soil/container interface (Garnier 2001), and potential non-uniformities in the g-level with height in the specimen. Garnier et al. (1999) observed load cell measurements to be impacted by the method of placement when measuring vertical stresses in a granular soil layer. Specifically, cells embedded at higher pressures tend to overestimate vertical stress while cells placed at low to zero pressure will underestimate the vertical stress. In addition to placement technique, the size, shape, and stiffness of the load cell may also cause error in the registered measurements (Askegaard 1995). During placement, a zone of disturbance, or "influence", may develop around the load cell as a function of placement technique and size/shape of the load cell. This zone is typically interpreted as an ellipsoidal shape surrounding the embedded object and may exhibit a stiff or soft behavior relative to the undisturbed soil media, shown in Figure 13.

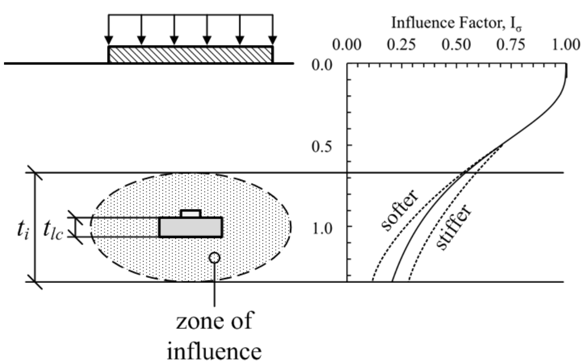


Figure 13. Impact of load cell on stress measurements

Several centrifuge studies have established that the stiffness of the zone of influence will create an "arching" effect affecting the stress measured by the load cell (Dewoolkar et al. 2007; Ellis and Aslam 2009a, 2009b). Softer zones (i.e. load cell having a lower stiffness than that of the surrounding soil) will cause a decrease in vertical stress (i.e. decreased influence factor) compared to that expected in a homogenous soil layer, while stiffer zones will cause an increase in vertical stress (Figure 13). This variability may be reduced by slowly twisting a load cell of minimal thickness (t_{lc} approaches 0) into the soil layer during placement (Askegaard 1995; Garnier et al. 1999). In addition to load cell properties and placement conditions, recorded vertical loads may also be decreased due to "silo" effects resulting from soil-container interface friction (Garnier 2001).

5 CONCLUSIONS

This paper describes a new instructional centrifuge module for evaluation of stress distributions in a sand layer. Relatively simple experiments were designed to be implemented in a short time period, while still providing empirical data to build confidence and generate discussions of the assumptions behind analytical solutions for stress distribution.

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