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ABSTRACT

The velocity distribution of electrons in a very weakly ionized gas under many conditions tends to be nearly isotropic even in the presence of electric and magnetic fields. It is shown that in such cases the rate of electron heating can be represented as a spherically symmetric diffusion in velocity space without the usual restriction to steady-state conditions. In the case of a steady electric field perpendicular to a strong magnetic field, the pertinent velocity-diffusion coefficient has the simple form $D = v_d^2 \nu/3$, where $v_d = cE/B$ is the drift speed and ν is the momentum-transfer collision frequency.

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The behavior of electrons in very weakly ionized gases in the presence of electric and magnetic fields has been investigated theoretically by many authors.[1-3] In most studies the emphasis is on conditions in a stationary state, in which case a number of simplifying assumptions are easily justified. It is the purpose of this note to show that some of these simplifications are applicable without being restricted to the steady state, so that the evolution in time of the electron velocity distribution can be described with fair accuracy long before the stationary state is reached. Our reasoning has certain features in common with that presented by Holstein,[4] who treated the case of high-frequency electric fields in the absence of magnetic fields.

In this paper we restrict ourselves to static electric and magnetic fields that are everywhere orthogonal. Furthermore, although some of our basic arguments are quite general, we pay particular attention to the case of a strong magnetic field. By this we mean conditions such that the electron gyrofrequency $\underset{\sim}{\omega} = e\underset{\sim}{B}/mc$ is much larger than the electron-molecule collision frequency for momentum transfer. In hydrogen, for instance, the momentum-transfer collision frequency ν is always less than $2\times10^{-7} n_g$ sec^{-1}, where n_g denotes the number of gas molecules per cubic centimeter. This means that we must have $B > 10^{-14} n_g$ gauss. Note that the momentum transfer due to inelastic collisions must be included here, but the order of magnitude of ν is usually not changed by these effects. The basic motion of the electrons in the planes normal to B in such cases is then primarily cycloidal, as shown in fig. 1, provided $E < B$. The guiding centers all move with a common drift velocity $\underset{\sim}{v}_d = c\underset{\sim}{E}\times\underset{\sim}{B}/B^2$, and only the occasional collisions with the gas molecules cause an irregular walk of the guiding centers. The net effect of the collisions is an additional drift that is almost exactly in the $-\mathbf{E}$ direction if $\mathbf{E}$ is the electric field observed in the frame of reference in which the gas is at rest, the gas frame. We may regard the drift along the electric-field direction as responsible for the average energy gain by the electrons.

It is also interesting to consider this electron motion in a frame of reference that moves at the velocity $\mathbf{v}_d$ through the gas. We shall call this the drift frame. In this frame the electric field is zero so that the electrons—in the absence of collisions—move with constant energy on helical paths along the magnetic field lines. Now, however, the gas is in motion and represents a wind with velocity $-\mathbf{v}_d$. The energy gain of the electrons must then be ascribed to an interaction with this wind rather than to an electric field. The resulting drift in the $\mathbf{v}_d \times \mathbf{B}$ direction now appears as a consequence of this interaction rather than as a cause for the energy gain. Furthermore, since under our assumptions this drift velocity is much smaller than $\mathbf{v}_d$, one may conclude that the electron velocity distribution in the drift frame is more nearly isotropic than in the gas frame. This latter fact can prove useful in calculations of the electron velocity distribution whenever v_d is not small compared with the mean random speed of the electrons.[3] The only disadvantage of the description in the drift frame lies in the complicated forms of the collision terms, since scattering phenomena are always best treated in the frame in which the center of mass is at rest.

In the present discussion we restrict ourselves to situations where the mean random speed of the electrons is already large compared with v_d, so that the distribution of electron velocities can be considered as nearly isotropic in the gas frame. Numerically, this means that we must have $\bar{\epsilon} >> 3E^2/B^2$, where $\bar{\epsilon} = \frac{1}{2} m \langle v^2 \rangle$ is the mean electron energy expressed in electron volts, while E is in volts per centimeter and B is measured in gauss. In this case the mathematical techniques used in refs. 1, 2, and 4 are justified and we could simply make use of the appropriate results arrived at there.

$$\frac{\partial f_1}{\partial t} + a \frac{\partial f_0}{\partial v} - \omega \times f_1 - \left(\frac{\partial f}{\partial t}\right)_{cl} = 0 . \tag{4}$$

The collision terms appearing here have been discussed extensively in the literature cited. In particular it is to be noted that we can write $(\partial f_1/\partial t)_{cl} = -\nu(v) f_1$, where in general $\nu(v)$ is to be interpreted as the collision frequency for momentum transfer summed over all types of collisions regardless of whether they are elastic or inelastic.

If we choose a coordinate system such that $E = E\hat{y}$ and $B = B\hat{z}$, so that $v_d = (cE/B)\hat{x}$, the components of eq. (4) become

$$\frac{\partial f_{1x}}{\partial t} + \omega f_{1y} + \nu f_{1x} = 0 , \tag{5a}$$

$$\frac{\partial f_{1y}}{\partial t} - a \frac{\partial f_0}{\partial v} - \omega f_{1x} + \nu f_{1y} = 0 , \tag{5b}$$

$$\frac{\partial f_{1z}}{\partial t} + \nu f_{1z} = 0 . \tag{5c}$$

We immediately see that f_{1z} is uncoupled and simply damps to zero by collisions. The components f_{1x} and f_{1y} are related to each other by the gyrofrequency ω and are driven by the term $(a\ \partial f_0/\partial v)$. If $\omega >> \nu$ and $\langle v^2 \rangle >> v_d^2$, most electron orbits have the character shown in fig. 1. Thus we expect that, to a first approximation, f_{1x} will be relatively insensitive to collisions even if f_0 has not nearly reached the steady-state condition. This means that we expect to have

$$\left| \frac{\partial f_{1x}}{\partial t} \right| << \nu f_{1x} . \tag{6}$$

If eq. (6) is valid, great simplification results. Thus we proceed by first assuming that relation (6) holds in general and then use the result to verify under what conditions the assumption is indeed justified.

From eqs. (5a) and (5b) it now follows that

$$f_{1y} = -\frac{\nu}{\omega} f_{1x} = \frac{a\nu}{\omega^2+\nu^2} \frac{\partial f_0}{\partial v} . \tag{7}$$

We see, therefore, that the original requirement $|\underset{\sim}{f}_1| << f_0$ is always satisfied whenever we have

$$\left| \gamma v_d \frac{\partial f_0}{\partial v} \right| << f_0 , \tag{8}$$

where we have substituted $a = \omega v_d$ and introduced the dimensionless factor $\gamma = \omega/(\omega^2+\nu^2)^{1/2}$. When eq. (7) is used to eliminate $\underset{\sim}{f}_1$ from eq. (3) we obtain

$$\frac{\partial f_0}{\partial t} = \frac{v_d^2}{3v^2} \frac{\partial}{\partial v} \gamma^2 \nu v^2 \frac{\partial f_0}{\partial v} + \left(\frac{\partial f}{\partial t}\right)_{c0} . \tag{9}$$

The right-hand side of eq. (9) is of course identical with the steady-state expressions obtained in refs. 1 and 2 for static fields. Also, eq. (9) is indistinguishable from the result obtained for slowly varying distribution functions in ref. 4 if ω here is interpreted as the applied high frequency rather than the electron gyrofrequency. Evidently, the collision term denoted by $(\partial f/\partial t)_{c0}$ here can therefore be taken directly from these earlier publications because its derivation never requires the steady-state assumptions.

The first term on the right of eq. (9), which we may call the "heating" term, can in the case $\omega >> \nu$, i.e., for $\gamma \approx 1$, also be derived without expansion in spherical harmonics. This can be done by studying the distribution function in the drift frame, assuming that $\bar{\epsilon} >> \frac{1}{2} m v_d^2$ and that elastic collisions are more important than any other.[5] This result is quoted in ref. 3

where it is already pointed out that this term represents a spherical diffusion of the isotropic distribution $f_0(v,t)$ in velocity space with a diffusion coefficient $D = \frac{1}{3} v_d^2 \nu(v)$. Moreover, it is shown there as well as in ref. 1 that the effect of a finite gas temperature T_g can be included in this heating term by generalizing the velocity diffusion coefficient to read

$$D = \frac{\nu}{3} (\gamma^2 v_d^2 + 3kT_g/M) , \qquad (10)$$

where k is Boltzmann's constant and M represents the mass of the gas molecules. In the case $\omega >> \nu$ we have $\gamma \approx 1$, and $(v_d^2 + 3kT_g/M)$ is recognized as the mean square velocity of the gas molecules in the drift frame; i. e., in the frame in which the electrons are tied to the stationary magnetic field lines. It should be noted that now $\nu(v)$ is essentially the frequency of momentum transfer for elastic collisions.

All other contributions from $(\partial f/\partial t)_{c0}$ remove energy from the electrons if hyperelastic collisions (collisions of the second kind) are excluded from our considerations. The effect of elastic collisions is very small as it is due to molecular recoil only. If inelastic collisions produce only rotational and vibrational excitation of the molecules and thus also represent only relatively small energy losses, these can be symbolically combined with the elastic collisions by simply introducing a dimensionless multiplier $\lambda(v)$ which modifies the recoil losses. The empirical function $1 \leq \lambda(v) << M/2m$ must be regarded as an approximation to take the place of a more elaborate and cumbersome numerical solution such as was carried out, for instance, by Carleton and Megill.[2] In this case eq. (9) can be written in the form

$$\frac{\partial f_0}{\partial t} = \nabla_v \cdot \left(D \nabla_v f_0 + \frac{m}{M} \lambda \nu \underset{\sim}{v} f_0\right) , \qquad (11)$$

where D is given by eq. (10), and the operator ∇_v denotes differentiation with respect to v in spherical symmetry. For noble gases, of course, where $\lambda = 1$, eq. (11) is quite satisfactory as long as electronic excitation and ionization are negligible.

When $\lambda(v)$ and $\gamma(v)$ can be approximated by constants, the steady-state solution of eq. (11), as is well known, is a Maxwellian distribution independent of $\nu(v)$. The mean energy in that case is given by $\bar{\epsilon}_s \equiv \frac{1}{2} m \left\langle v^2 \right\rangle_s = (\gamma^2 M v_d^2 + 3kT_g)/2\lambda$. It is clear, then, that with rather modest values of γv_d this mean energy may become very large provided the electrons do not drift out of the uniform field region. Therefore inelastic collisions can frequently not be neglected in the stationary state, even in the case of noble gases, and eq. (11) is not adequate. Before such conditions are reached, however, we may use eq. (11) to estimate the rate of gain of mean energy by multiplying by $\frac{1}{2} mv^2$ and integrating. The result is[3]

$$\frac{\partial \bar{\epsilon}}{\partial t} = \frac{m}{M}\left[\left\langle(\gamma^2 M v_d^2 + 3kT_g)\left(\nu + \frac{v}{3}\frac{\partial \nu}{\partial v}\right)\right\rangle - \frac{3}{2} m \left\langle \lambda \nu v^2 \right\rangle\right]. \tag{12}$$

If T_g and $\partial\nu/\partial v$ are neglected, the initial value of $\partial\bar{\epsilon}/\partial t$ can be approximated by $\gamma^2 m v_d^2 \nu$. The evolution of $\bar{\epsilon}(t)$ for $\gamma = 1$ is shown qualitatively in fig. 2 with the assumption that $\epsilon(0) = 0$ and that inelastic collisions limit $\bar{\epsilon}_s$ to a value much lower than $\frac{1}{2} M v_d^2$.

We will now explore the conditions under which our assumptions (2), (6), and (8)—and hence the result (9)—are justified. We consider only cases in which $\bar{\epsilon}_s >> \epsilon_1 \equiv \frac{1}{2} m(\gamma^2 v_d^2 + 3kT_g/M)$ so that these assumptions are valid for the steady state. From eq. (12) we see that the condition $\bar{\epsilon}(t) >> \epsilon_1$ is then assured already after about ten mean collision times. As these are momentum-transfer collisions, the distribution function will be nearly spherically symmetric so that the form (2) is at least reasonable, except in the relatively unimportant region near the origin of velocity space.

The reasoning we have employed in justifying our assumptions may be criticized because we use conclusions that could be derived only by making the assumptions. However, this reasoning is permissible because we know the assumptions are justified in the steady state and we are simply investigating how close to steady state we must be.

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FIGURE CAPTIONS

Fig. 1. Electron orbit in crossed fields.

Fig. 2. Mean electron energy as a function of time.

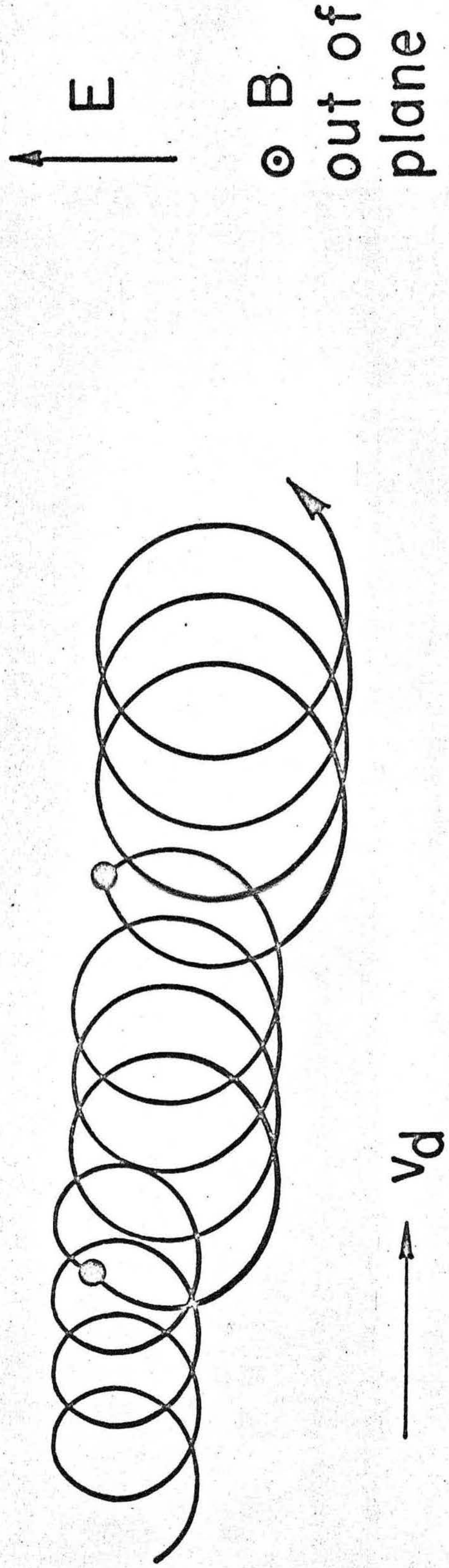

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Fig. 1.

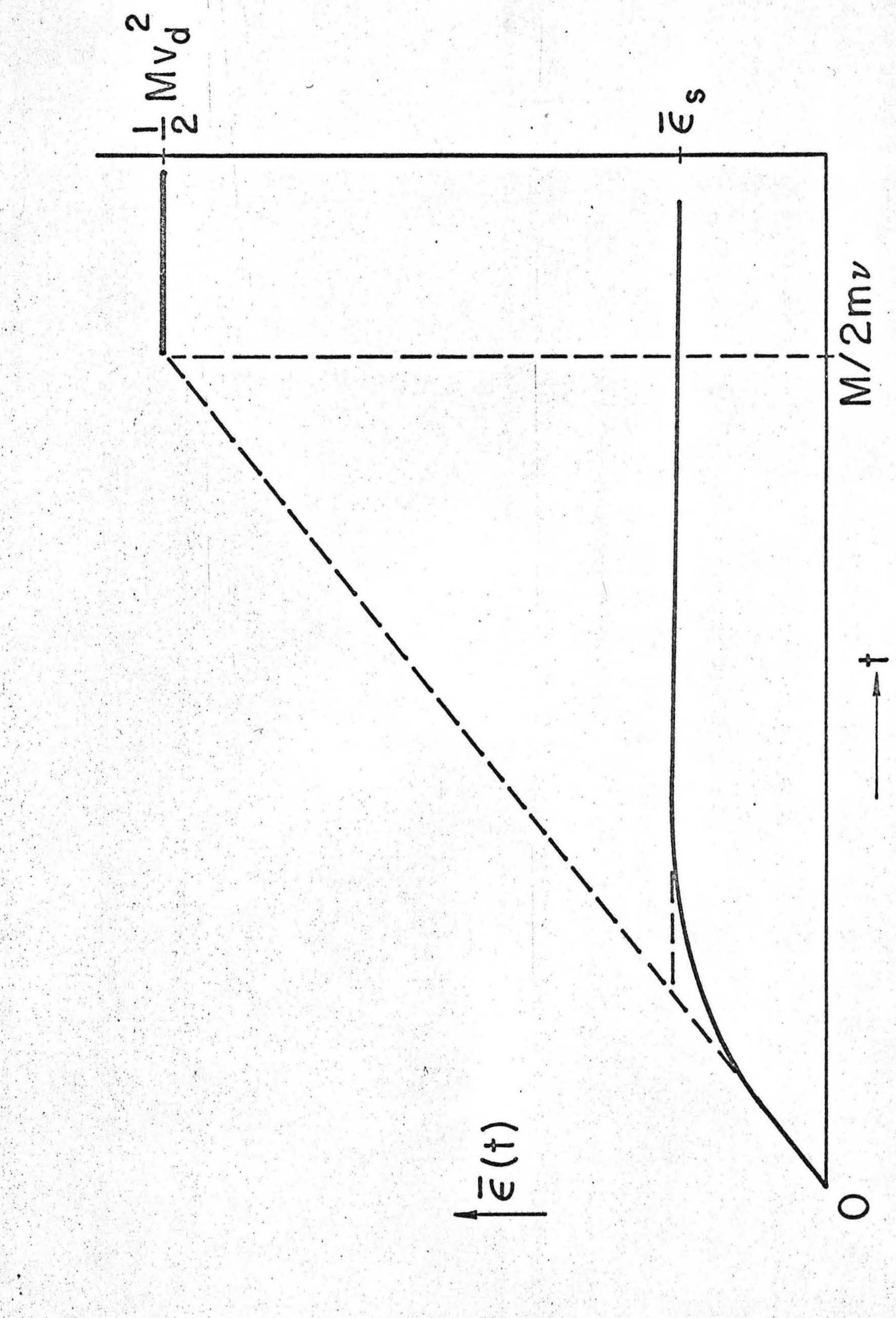

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Fig. 2.