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ESTIMATION OF HOUSEHOLD DEMAND FOR GOODS AND SERVICES IN CALIFORNIA'S DYNAMIC REVENUE ANALYSIS MODEL

by

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Estimation of Household Demand for Goods and Services in California's Dynamic Revenue Analysis Model

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I. INTRODUCTION

California Senate Bill 1837, passed in August 1994, charges the state's Department of Finance (DOF) to conduct a dynamic revenue analysis on all proposed legislation expected to have an annual fiscal impact of $10 million dollars or more.[2] Such analysis is intended to provide the Governor and Legislature with reliable information concerning likely behavioral responses of taxpayers, businesses, and other citizens to changes in state tax law and also to predict the effect of such responses on the state's finances.

Dynamic revenue analysis of the sort mandated by SB 1837 was not common practice at the time of the law's passage. In Massachusetts, the one state using such analysis, the work was being done under contract by external consultants; the results obtained were thus difficult to attribute, explain, and defend. In a concerted attempt to avoid such a scenario, California's DOF chose to conduct its analyses in-house with a model well understood by its own.

In consultation with Dr. Peter Berck and others at the Department of Agricultural and Resource Economics, University of California at Berkeley (ARE), the DOF decided that a computable general equilibrium modeling (CGE) approach would be best suited to dynamic revenue analysis. Under the supervision of Ted Gibson, Chief Economist at DOF, Dr. Bruce Smith built a CGE model of the California economy with the assistance of Dr. Peter Berck and a team at ARE (see Berck, *et. al.*, 1996). The original Dynamic Revenue Analysis Model (DRAM) consisted of 1,100 equations that not only explicitly represent the utility and profit maximizing behavior of households and firms, but also captures economy-wide ripple and feedback effects of such behavior.

DRAM continues to undergo fine-tuning. First, functions in the model are being refined as envisioned in the original report (Berck, *et. al.*, 1996). Second, economic and fiscal data are being updated. Third, parameter values taken from published work are being replaced with those estimated from appropriate data.

The purpose of this paper is to present the results of investigations into more flexible functional forms of household demand than those incorporated in the original DRAM. What follows is a brief review of consumer demand theory and a description of our procedure, including data development, for estimating coefficients of a desirable functional form. In brief, a Linear Approximate Almost Ideal Demand (LA/AIDS) is chosen to replace commonly used Cobb-Douglas demand equations. The former is estimated using current and geographically relevant Bureau of Labor Statistics (BLS) data. How these new functional forms will be implemented in DRAM and their impact on results of the model are then discussed. Conclusions follow.

II. MODELING CONSUMER DEMAND

The economic theory of consumer demand coherently explains peoples' behavior in the market. It begins with the supposition that consumers maximize their utility (u) as a function of what they consume (**x**), subject to a budget constraint that they not spend more money (y) than they have.[3] In mathematical notation, each person's objective is to:

$$\text{(1)} \qquad \max_{\mathbf{x}} \ u(\mathbf{x}), \text{ such that } \sum_{i=1}^{k} p_i x_i = y$$

Each x_i is a single element of the k-by-1 vector of available goods with corresponding price p_i.

[2] Statutes of 1994, Chapter 383.

[3] Here and following, boldfaced variables are vectors of appropriate dimension.

II.1. SOME BASIC DEFINITIONS

Understanding the above supposition and appreciating the explanatory power of the model that follows requires familiarity with the following terminology.

Utility is the economic term for (at least ordinally) measurable well-being. It is what all 'rational' people seek to maximize through their consumption of *goods.*[4]

Marshallian demand is the relationship between the quantity of a good purchased, prices, and income, or x(y,p).

Hicksian or *'compensated' demand* is the relationship between quantity of a good purchased, prices, and utility. It is the schedule of demand for a good when any price change it undergoes is accompanied by a payment to (or forfeiture by) the consumer which keeps them at the their original utility level when facing that new price. Any price increase (decrease) must necessarily then be accompanied by a compensating addition to (subtraction from) the affected consumer's income in order to trace out their Hicksian demand for that item. Whereas Marshallian demand is directly observable, Hicksian demand is a theoretical construct useful for isolating the components of demand driven by price versus wealth effects, or h(**p**,u).

An *expenditure function*, e(u,**p**), is the minimum amount of money necessary to reach a fixed level of utility when faced with a given set of prices.

A *budget share* (s_i) is the fraction of total annual expenditure spent on good i.

II.2. THEORETICAL AND INTUITIVE PROPERTIES OF DEMAND

Theory dictates three basic properties of demand. First, total expenditure *'adds up'* to total income less taxes and savings (y). Second, demand is *homogenous of degree zero*, i.e. if all prices and income were to go up by the same proportion, quantities consumed would not change. Third, the Slutsky substitution matrix is *symmetric*, a technical requirement that will not be discussed here.[5] In addition, common sense dictates that demand is not upward sloping - all else being equal, people demand no more of a good (and usually less) when its price rises. Any set of equations used to model consumer demand must meet these four criteria.

II.3. ADVANTAGES OF FLEXIBLE DEMAND SYSTEMS AND THE CHOICE OF LA/AIDS

When choosing a functional form to describe consumer behavior, 'flexibility' should be weighed against computational ease and interpretability. A flexible functional form is one which imposes few restrictions on the relationship between prices, quantities and income; the fewer restrictions, the more flexible the form. A completely flexible form would let each of the four properties discussed above be tested empirically as well as allow variable price and income elasticities. Although desirable for these reasons, flexibility comes at a price. It often entails non-linear estimation and a proliferation of parameters, some of which may have no intuitive economic interpretation.[6]

Although relatively inflexible, Cobb-Douglas utility functions are often chosen for their computational ease. They yield demand equations of the form:

[4] At the risk of circularity, but for the sake of clarity, *goods* refer to any *goods or services* which affect a person's sense of well-being.

[5] Symmetry requires that the second cross partial derivatives of the expenditure function, or identically the first cross-partials of Hicksian demand, be equal. More specifically,

$$\frac{\partial h_j(\mathbf{p},u)}{\partial p_i}=\frac{\partial^2 e(\mathbf{p},u)}{\partial p_i \partial p_j}=\frac{\partial^2 e(\mathbf{p},u)}{\partial p_j \partial p_i}=\frac{\partial h_i(\mathbf{p},u)}{\partial p_j}$$

[6] See Blanciforti, Green and King (1986) for a further discussion of these issues.

$$x_i = \lambda_i \frac{y}{p_i} \quad (2)$$

The λ_i parameters, one per good per household, are the only ones to be estimated. They are interpreted as the share of income spent on each good and due to the functional form are necessarily fixed as income rises[7]. In reality, however, it is very plausible that the relative demand for many goods changes as income changes. It is attractive to use a functional form that allows for this.

Other common demand models include the Linear Expenditure System (LES), Constant Elasticity of Substitution (CES) and Translog. While more flexible than Cobb-Douglas, LES and CES demand forms are still fairly restrictive. Translog forms are flexible, but involve difficult non-linear estimation.[8]

In a 1980 paper, Deaton and Muellbauer introduced an Almost Ideal Demand System (AIDS) with several attractive properties. First, it is flexible enough to test the four properties of demand. Second, it is rooted in the PIGLOG (price independent generalized logarithmic) class of preferences which allow perfect aggregation over consumers. And third, its functional form is consistent with known household budget data. Deaton and Muellbauer (1980) also demonstrated the ease with which a linear approximate AIDS (LA/AIDS) can be estimated.

A major refinement of DRAM, envisioned in the original research, has been the switch from Cobb-Douglas demand equations using arbitrary and/or previously published parameter estimates to a flexible LA/AIDS with parameters estimated from current regional data. As mentioned at the outset, the primary purpose of this paper is to discuss these refinements. After laying out a formal derivation of the LA/AIDS model and describing the data, we will present the results of our estimation and discuss their significance.

II.4. DERIVATION OF LA/AIDS

The AIDS model begins with an expenditure function of the form

$$\log e(u,\mathbf{p}) = \alpha_0 + \sum_k \alpha_k \ln p_k + \frac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \ln p_j + u\beta_0 \prod_k p_k^{\beta_k}, \quad (3)$$

where α, γ^*, and β are parameters.

The demand functions, stated in expenditure-share form, are derived using the fact that the derivative of an expenditure function with respect to price p_i is Hicksian (compensated) demand h_i. Differentiating (3) with respect to p_i thus yields the share equations

$$\frac{\partial \log e(u,\mathbf{p})}{\partial \log p_i} = \frac{\partial e(u,\mathbf{p}).p_i}{\partial p_i . e(u,\mathbf{p})} = \frac{p_i c_i}{e(u,\mathbf{p})} = s_i, \quad (4)$$

where s_i is the share of total expenditure e spent on good i. Since

$$\frac{\partial \log e(u,\mathbf{p})}{\partial \log p_i} = \alpha_i + \sum_j \lambda_{ij} \log p_j + \beta_0 \beta_i u \prod_k p_k^{\beta_k} \quad (5)$$

with

$$\lambda_{ij} = \tfrac{1}{2}\left(\gamma^*_{ij} + \gamma^*_{ji}\right), \quad (6)$$

substituting (5) into (4) and solving for h_i yields Hicksian demand:

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[7] Even if one sub-divides households into groups, there would be a constant share for each commodity as income changes within each household group - a strong assumption to be avoided, if possible.

[8] See Chapter 3, Berck, *et. al.*, 1996 for a more complete discussion

(7) $$c_i = \frac{e(u,\mathbf{p})}{p_i}\left(\alpha_i + \sum_j \lambda_{ij} \log p_j + \beta_o \beta_i u \prod_k p_k^{\beta_k}\right).$$

Taking advantage of the theoretical requirement that expenditure adds-up to income minus taxes and savings lets us substitute y for e in (3) yielding

(8) $$\log y = \alpha_o + \sum_k \alpha_k \log p_k + \tfrac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \log p_j + u\beta_o \prod_k p_k^{\beta_k}.$$

Inverting (8) to solve for u gives

(9) $$u = \frac{\log y}{\beta_o \prod_k p_k^{\beta_k}} - \frac{\alpha_o + \sum_k \alpha_k \log p_k + \tfrac{1}{2}\sum_k \sum_j \gamma^*_{kj} \log p_k \ln p_j}{\beta_o \prod_k p_k^{\beta_k}}.$$

Substituting (9) into (5) then (5) into (4) allows expenditure shares to be written as a function of income and all prices:

(10)

$$s_i = \alpha_i + \sum_j \lambda_{ij} \log p_j + \beta_o \beta_i \prod_k p_k^{\beta_k}\left(\frac{\log y}{\beta_o \prod_k p_k^{\beta_k}} - \frac{\alpha_o + \sum_k \alpha_k \log p_k + \tfrac{1}{2}\sum_k \sum_j \gamma^*_{ij} \log p_k \ln p_j}{\beta_o \prod_k p_k^{\beta_k}}\right)$$

This simplifies to

(11) $$s_i = \alpha_i + \sum_k \lambda_{ij} \log p_j + \beta_i \log\left(\frac{y}{P}\right)$$

where P is a price index of the form

(12) $$\log P = \alpha_o + \sum_k \alpha_k \log p_k + \tfrac{1}{2}\sum_k \sum_j \gamma^*_{ij} \log p_k \log p_j$$

For estimation purposes, it is often easier (and fairly standard) to replace (12) with

(13) $$\log P = \sum_k s_k \log p_k,$$

an index linear in log prices and weighted by s_k, the *average* expenditure shares of the households. The set of equations (11) and (13) together make up the linear approximate AIDS (LA/AIDS) model to be estimated.

Using the fact that at any given point, Hicksian demand h_i equals Marshallian (ordinary or uncompensated) demand x_i, it straightforward to derive the following equations for own-price, cross-price, and income elasticities of demand respectively in the LA/AIDS framework.

(14) $$\varepsilon_{ii} = -1 + \frac{\lambda_{ii}}{s_i} - \beta_i$$

(15) $$\varepsilon_{ij} = \frac{\lambda_{ij}}{s_i} - \frac{\beta_i S_j}{s_i}$$

(16) $$\varepsilon_{iy} = \beta_i + 1$$

III. THE DATA

To estimate a consumer demand system, one generally needs data on the price of goods (p_i), the quantity of goods consumed at that price (h_i), and disposable income (y^d). To estimate LA/AIDS in particular, we need prices (p_i), expenditure shares ($s_i=p_ih_i/y$), average expenditure shares (s_k), and total expenditure (y). With price data constructed from U.S. Bureau of Labor Statistics (BLS) consumer price indices (CPIs) and expenditure share and income data extracted from the BLS's Consumer Expenditure Survey (CES), our final data set consisted of 59 quarterly observations on prices, average expenditure shares, and mean income for households in the Western U.S. from 1980 to 1994. Additional data was retained from the 1994 CES to create a static demographic profile of California households by income decile.

III.1. PRICES

We broke total consumer demand down into the following nine aggregate categories corresponding to readily available CPIs: food & beverages, shelter, fuel & other utilities, household furnishing & operation, apparel & upkeep, transportation, medical care, entertainment, and other goods & services. Monthly (or bimonthly) region specific CPI's for these sectors are available from the BLS web site at www.bls.gov. Quarterly sector specific prices (p_i) were constructed by taking the average of available monthly prices. The overall price index (P) was created by weighting these prices by the overall mean shares (s_k) according to (13).[9]

III.2. SHARES AND INCOME

Expenditure and income data were extracted from the CES—available on CD-ROM, diskettes or tape from the BLS.[10] The CES provides continuous, comprehensive data on consumption by American households. The survey is a rotating panel design. Each quarter approximately 5,000 households are interviewed in three nationally representative panels (one per month). Households participate for five consecutive quarters, providing demographic and spending information. The survey data are catalogued in two parts, each with files arranged by calendar quarter. The *Interview* portion contains family characteristic and demographic information (FMLY files) as well as monthly expenditure records (MTAB files) with household spending identified by month of purchase and universal classification code (UCC).[11]

The quarterly FMLY files contain approximately 5,000 observations each – one per household. We sorted these observations by region, keeping only those households located in the western United States (roughly 1,000 per quarter).[12] This was done in an attempt to use only data reflecting the unique demand behavior of

[9] This overall index cannot, therefore, be constructed until the share data is compiled as outlined below.

[10] Write to Division of Consumer Expenditure Surveys, Branch of Information and Analysis, Bureau of Labor Statistics, Room 3985, 2 Massachusetts Ave. N.E., Washington, DC 20212-0001 or call 202-606-6900.

[11] The *Expenditure* portion of the CES contains microdata files with extremely detailed information taken directly from the interview. It was not used in this analysis.

[12] Households were selected by region because state identifiers were not available for all years. California identifiers were, however, available for 1994 and thus used in creating the demographic profile of CA households highlighted in Table 2 below.

California households as accurately as possible.[13] Identifying Western household in this manner allowed us to select the expenditure records of interest from the MTAB files.[14]

Due to the way MTAB data are collected and reported, some finesse is needed in regrouping expenditure records listed by *reporting* quarters according to *calendar* quarters matching the CPI data. For example, first quarter MTAB files contain data from households interviewed in January, February and March reporting expenditures made the immediately preceding Nov.-Dec., Oct.-Jan. and Dec.-March respectively; second quarter MTAB files report with a similar lag. Counting all first *calendar* quarter expenditures therefore requires pooling observations from both files. Such reaggregation is essential since the periodicity of expenditure and price data must match if any meaningful demand relationship is to be uncovered.

The quarterly MTAB files contain 460,000+ observations each - one for every expenditure reported. Match merging these with what remained of the FMLY file records (Western households only) left just under 100,000 observations per quarter. After regrouping observations by calendar quarter as explained above, expenditures classified by UCC were mapped into our nine CPI categories.[15] The next step was to sum each household's expenditures within each of these categories. The data were then reshaped so that there was one observation per household listing their own quarterly expenditure by sector. Within each observation, these expenditures were summed to yield total household quarterly expenditure, and converted into expenditure shares. Each quarterly share (s_i) and income net of taxes and savings datum was then constructed as a weighted average of their individual household counterparts, validated by virtue of having started with PIGLOG preferences. These weighted means were thus the mean values for *all* households reporting *any* economic activity that calendar quarter. Each household's contribution was weighted by the fraction of their reporting months that fell within that calendar quarter.[16]

III.3. PUTTING IT TOGETHER

As a last step before estimation, the price data and mean share and income data were matched quarter by quarter. The means of mean shares (s_k) were then computed (see Table 1 below) and used to create the quarterly grand price indices (P) according to equation (13).

Table 1: Mean Household Expenditure Shares (s_k), 1980 to 1994 Composite[17]:

Food	Shelter	Fuel	House	Apparel	Trans	Health	Enter	Other
0.241	0.257	0.139	0.049	0.053	0.093	0.052	0.040	0.076

The final data set thus consisted of 59 quarterly observations, each containing weighted mean income (y), nine weighted mean expenditure shares (s_i), nine sector specific price indices (p_i) and a corresponding overall price index (P).

[13] Western region expenditure data should match California's well due to the state's disproportionate population and economic share of the West. Using regional expenditure data also allows us to take advantage of regional CPI data.

[14] Other variables were also taken from these files for use in the above mentioned profile.

[15] Expenditures were also mapped directly into our 28 DRAM categories plus one for property taxes and another for motor vehicle related taxes. This second mapping was used in two ways: first to construct a static (1994) referential profile of consumer spending by DRAM sector according to income decile; and second, in combination with the first mapping to compute the mean allocation of our nine CPI based aggregate categories among the 28 DRAM (plus two tax) sectors - a breakdown critical for integrating the results below into DRAM. These mappings are presented in a technical appendix.

[16] For a further explanation of the mechanics of such weighting, see *1994 Interview Survey CD ROM/Public Use Tape Documentation*, January 26, 1996 (Updated 2/15/96), Section V.A.1.b. *Calendar Period versus Collection Period.*

[17] The following labels for aggregate consumption categories have been used: Food - Food & Beverage, Shelter – Shelter, Fuel - Fuel & Utilities, House - Household Furnishing & Operations, Apparel - Apparel & Its Upkeep, Trans - Transportation, Health - Medical Care, Enter – Entertainment, Other - Other Goods & Services.

As indicated above, additional demographic data for California households was extracted from the 1994 CES and analyzed according to income decile. This was done to provide DOF and other government agencies with a rough idea of the attributes of households at various income levels and tax brackets. Table 2 below presents a sample of the type of information contained in these profiles.

Table 2: Household Profile Summary

	Minimum Value	Mean Value	Maximum Value
Final Income Before Taxes	$22,000	$25,705	$29,975
Total Personal Taxes Paid	-$2,200	$2,119	$10,833
Federal Income Taxes Paid	-$2,000	$1694	$8,333
State Income Taxes Paid	-$313	$423	$3,083
Number of Children	1	2.7	11
Amount Held in Savings Account	$0	$5,305	$79,000
Amount Held in Checking Account	$0	$3,318	$100,000
Amount Held in Securities	$125	$30,646	$100,000
Amount Deposited in IRA	$0	$231	$13,000
Amount of Debt	$0	$480	$8119

Note: Values are for those reporting, which may be significantly less than the number surveyed.

IV. ESTIMATION

Our LA/AIDS consisted of nine equations (one for each sector) of form (13), each with parameters α, λ, and β. Estimation was done using the Ordinary Least Squares (OLS) version of SAS's 'syslin' procedure designed to handle interdependent systems of equations. The data satisfied the requirement that consumption plus savings and taxes by construction. Homogeneity and Slutsky symmetry were imposed via the following parameter restrictions:[18]

(17) $$\sum_{i=1}^{9} \beta_i = 0$$

(18) $$\sum_{i=1}^{9} \lambda_{ij} = 0$$

(19) $$\sum_{j=1}^{9} \lambda_{ij} = 0$$

(20) $$\lambda_{ij} = \lambda_{ji}$$

Next, price and income elasticities were calculated according to equations (14)-(16) evaluated at the overall mean shares listed in Table 1. After initial estimation yielded a counterintuitive (but not empirically uncommon[19]) positive own-price elasticity of demand for the food & beverage sector (a violation of downward sloping demand), we restricted its parameter (λ_{11}) to the value 0.240 re-estimated. This value, given the share of consumption for foods and beverages, implies a nearly zero, but negative own price elasticity of demand. The new results, presented below, then satisfied the four theoretical and intuitive properties of demand. All other own-price elasticities came out significantly negative with the exception of 'apparel & its upkeep' which was negative, but barely misses the test of statistical significance.

V. RESULTS

V.1. COEFFICIENT ESTIMATES

Table 3 below contains the estimation results. For the first nine rows, each row represents a separate share (s_i) equation and each column a price coefficient (λ_{ij}). Income coefficients (β_i) for the nine equations are given in the last row. The upper left triangle of price coefficients has been omitted because its elements are

[18] See Deaton and Muellbauer (1980).

[19] According to Deaton and Muellbauer (1980) p. 319.

identical to their lower left images due to the imposition of symmetry which cut the number of parameters to be estimated from 90 to 54.

Table 3: Coefficient Estimates

	Food	Shelter	Fuel	House	Apparel	Trans	Health	Enter	Other
Food	0.2400 (*)								
Shelter	-0.1190 (-2.965)	0.0569 (1.531)							
Fuel	-0.0846 (-3.801)	0.0201 (1.364)	-0.0204 (-1.686)						
House	0.0300 (-0.738)	-0.0240 (-1.046)	0.0379 (2.439)	-0.0378 (-0.980)					
Apparel	-0.0068 (-0.159)	-0.0377 (-1.283)	-0.0197 (-1.175)	0.0735 (2.544)	-0.0269 (-0.588)				
Trans	0.0071 (0.193)	0.0036 (0.140)	0.0535 (3.631)	-0.0325 (-1.413)	-0.0288 (-1.004)	-0.0448 (-1.275)			
Health	-0.0800 (-3.334)	0.0672 (3.563)	-0.0268 (-2.251)	-0.0459 (-1.676)	-0.0190 (-0.783)	0.0914 (4.497)	-0.0686 (-1.933)		
Enter	0.0010 (0.024)	0.0207 (1.087)	0.0278 (2.289)	-0.0454 (-1.717)	0.0680 (2.737)	0.0619 (3.297)	0.0175 (0.638)	-0.1429 (-2.437)	
Other	0.0122 (0.937)	0.0122 (0.937)	0.0122 (0.937)	0.0441 (1.572)	-0.0027 (-0.114)	-0.1115 (-4.972)	0.0641 (1.873)	-0.0086 (-0.290)	-0.0219 (-0.539)
Income	-0.004 (-0.328)	0.004 (0.328)	-0.106 (-11.693)	0.029 (3.495)	0.047 (3.252)	0.004 (0.322)	0.004 (0.476)	0.027 (4.425)	-0.004 (-0.346)

Notes: T-statistics are in parentheses under the estimated coefficients.

V.2. ESTIMATES OF ELASTICITIES

Own- and cross-price elasticities calculated according to equations (14) and (15) respectively, using the parameter estimates from Table 3 and evaluated at the mean shares in Table 1, are listed in Table 4 below.

Table 4: Calculated Own and Cross Price Elasticities:

	Food	Shelter	Fuel	House	Apparel	Trans	Health	Enter	Other
Food	-0.0003 (-0.029)	-0.4895 (-2.910)	-0.3488 (-3.756)	0.1254 (0.742)	-0.0275 (-0.155)	0.0311 (0.204)	-0.3311 (-3.322)	0.0048 (0.027)	0.0516 (0.959)
Shelter	-0.4664 (-3.004)	-0.7826 (-5.313)	0.0763 (1.314)	-0.0940 (-1.054)	-0.1476 (-1.291)	0.0127 (0.127)	0.2607 (3.543)	0.0800 (1.078)	0.0462 (0.912)
Fuel	-0.4243 (-2.633)	0.3413 (3.106)	-1.0403 (-11.847)	0.3101 (2.773)	-0.1013 (-0.842)	0.4559 (4.336)	-0.1531 (-1.778)	0.2304 (2.638)	0.1455 (1.558)
House	0.4692 (0.563)	-0.6425 (-1.349)	0.6906 (2.160)	-1.8009 (-2.288)	1.4693 (2.495)	-0.7183 (-1.541)	-0.9682 (-1.729)	-0.9510 (-1.760)	0.8554 (1.495)
Apparel	-0.3415 (-0.422)	-0.9391 (-1.658)	-0.4947 (-1.545)	1.3443 (2.464)	-1.5535 (-1.805)	-0.6257 (-1.160)	-0.4035 (-0.882)	1.2478 (2.658)	-0.1175 (-0.265)
Trans	0.0671 (0.168)	0.0286 (0.101)	0.5696 (3.533)	-0.3512 (-1.419)	-0.3119 (-1.011)	-1.4857 (-3.958)	0.9812 (4.474)	0.6639 (3.286)	-1.2016 (-4.974)
Health	-1.5564 (-3.348)	1.2726 (3.444)	-0.5260 (-2.267)	-0.8869 (-1.683)	-0.3685 (-0.793)	1.7516 (4.512)	-2.3223 (-3.400)	0.3334 (-3.135)	1.2268 (-0.359)
Enter	-0.1344 (-0.125)	0.3474 (0.723)	0.6023 (1.981)	-1.1683 (-1.766)	1.6650 (2.687)	1.4856 (3.182)	0.4029 (0.586)	-4.5980 (-3.135)	-0.2655 (-0.359)
Other	0.1726 (0.975)	0.1734 (0.976)	0.1672 (0.964)	0.5833 (1.580)	-0.0323 (-0.105)	-1.4618 (-4.991)	0.8461 (1.876)	-0.1111 (-0.284)	-1.2847 (-2.401)

Note: T-statistics are in parentheses.

Price elasticity ε_{ij} indicates the percentage by which quantity of good i demanded varies with a 1% change in price p_j. For example, reading from the table above, a 1% increase in the price of shelter leads to a 0.4664%

decrease in the demand for food. Positive (negative) elasticities indicate substitutes (compliments). All-own price elasticities are negative in keeping with the expectation of downward sloping demand.
Seven of the nine own-price elasticities and 27 of 36 cross-price elasticities are statistically significant at the 95 percent confidence level - results on par with the published works of Deaton and Muellbauer (1980)and Blanciforti, Green and King (1986).
Note that the symmetric parameter estimates from Table 3 do not translate into symmetric elasticities when equations (14) and (15) are evaluated at the non-identical mean expenditure shares listed in Table 1. More specifically, elasticities vary with expenditure shares.

V.3. CALCULATED INCOME ELASTICITIES

Income elasticities calculated according to equations (16) and again using the parameter estimates from Table 3 are presented in Table 5.

Table 5: Calculated Income Elasticities

	Food	Shelter	Fuel	House	Apparel	Trans	Health	Enter	Other
Income Elasticity	.9962	1.0038	0.8938	1.0292	1.0468	1.0037	1.0039	1.0265	0.9960

Note: T-statistics are omitted as all are significant to well beyond the 95% confidence level.

Not surprisingly, our results indicate that increased income leads to a significant increase in expenditure across all categories. The degree of uniformity, however, is noteworthy and may be due at least in part to such broad aggregates. Income elasticities less than one for food, fuel and other goods and services indicate that they are necessities, i.e. that the relative share of income spent on them decreases with wealth. Values of β_i greater than one for the other sectors indicate that they are "luxuries", i.e. their expenditure share increases with affluence.

VI. IMPLEMENTATION INTO DRAM

DRAM is a complex CGE of the California economy. In its current version, it consists of approximately 1,300 equations and data reflecting economic conditions for fiscal year 1996-97. In the sections following, information gained about the design of LA/AIDS consumer demand equations and their estimation are implemented into the current model, including tests of how the re-formulation affects typical results flowing from the model.
It is important to remember in this section that extremely simplified demand equations were implemented into the original version DRAM for two main reasons: timeliness and data availability. DRAM was built, tested and documented in just over six months (although peer review extended its publication by another three months). There was simply insufficient time to perform original estimations as part of this process. The dearth of published results of consumption estimation and the resource demands to acquire and condition the data and then estimate consumption functions, contributed to this decision.

VI.1. MODIFICATIONS TO DRAM

Changes begin with the household demand equations, which appear in the current DRAM as:

$$(21) \qquad x_{ih} = \bar{x}_{ih}\left(\frac{y_h}{\bar{y}_h}\right)^{\xi_{ih}} \prod_{j=1}^{k}\left(\frac{p_j^{\tau}}{\bar{p}_j^{\tau}}\right)^{\lambda_{ij}}$$

These may seem unlike Cobb-Douglas consumption functions presented earlier in this report:

$$(22) \qquad x_{ih} = \lambda_{ih}\frac{y_h}{p_i}$$

However, in its implementation, the DRAM consumption equations had own-price elasticities of minus 1.0, cross-price elasticities of 0.0 and income elasticities of 1.0. Thus the connection between the DRAM and Cobb-Douglas consumption functions is as follows:

$$x_{ih} = \bar{x}_{ih}\left(\frac{y_h}{\bar{y}_h}\right)^{\xi_{ih}} \prod_{j=1}^{k}\left(\frac{p_j}{\bar{p}_j}\right)^{\lambda_{ij}}$$

$$= \bar{x}_{ih}\left(\frac{y_h}{\bar{y}_h}\right)^{1} \times \left(\frac{p_1}{\bar{p}_1}\right)^{0} \times \ldots \times \left(\frac{p_i}{\bar{p}_i}\right)^{-1} \times \ldots \times \left(\frac{p_k}{\bar{p}_k}\right)^{0}$$

(23)
$$= \bar{x}_{ih}\frac{y_h}{\bar{y}_h} \times 1 \times \ldots \times \frac{\bar{p}_i}{p_i} \times 1$$

$$= \left(\frac{\bar{x}_{ih}\bar{p}_i}{\bar{y}_h}\right)\frac{y_h}{p_i}$$

$$= \lambda_{ih}\frac{y_h}{p_i} \quad \text{where } \lambda_{ih} = \frac{\bar{x}_{ih}\bar{p}_i}{\bar{y}_h}$$

Thus, due to particular choices in parameters, the existing equations are standard Cobb-Douglas consumption functions. The share equations for a LA/AIDS form with several households would be:

(24)
$$s_{ch} = \alpha_{ch} + \sum_{c \in C}\lambda_{c'c}\log p_j + \beta_c \log\left[\frac{y_h^d(1-\sigma_h)}{P_h}\right]$$

where: s_{ch} is the share of non-saved after-tax income of household 'h' spent on commodity 'c'

α_{ch} is a scale parameter for the share

$\lambda_{c'c}$ is elasticity parameter relating the change in price of commodity c' to the change in quantity demanded of good c (when c'=c, this is the parameter for own-price elasticity)

p_c is the tax-included aggregate price of commodity 'c' (and aggregate of the prices of goods and services that comprise commodity 'c')

y^d_h is after-tax income of household 'h'

σ_h is the marginal propensity to save

p_h is the price index for household 'h'

Implementing the share equations into DRAM involves introducing one new set, two new types of variables and a modification of a set of existing variables. The set is the CPI composite commodities, defined as {1, 2, ... , 9} and following the notation of DRAM, we use lower case 'c' to denote a member of the set 'C' (*i. e.* $c \in C$). Then, the variables s_{ch} are introduced as the share of household 'h' income spent on commodity 'c'. In the numerator of the expression in parentheses in the equation above is the expression for disposable income (income less taxes) less the fraction saved (σ_h). The savings fraction is shown as a parameter, meaning that the marginal propensity to save is fixed in this model. This is a narrow (or strong) assumption. However, the institutional particulars of a regional economy means that the properties induced by this assumption impact the results in insignificant ways.

Alternative assumptions about savings exist. Some regional models incorporate the assumption that regional savings always equal regional investment. Modelers who base policy analysis upon models with this assumption expose themselves to criticism from economists who demand representation of rational economic agents. DRAM does not incorporate such an assumption of irrational behavior, so savings in the model becomes a reasonably benign feature (although this would a fit topic for future research when time and other resources permit).

The modified variables are the overall price indices defined for households in the current model as arithmetic means of consumption, but now defined as harmonic means:

(25)

$$\log P_h = \sum_{c \in C} s_{ch} \log p_c^{\tau}$$

In the share and price index equations, a new product price was introduced: one inclusive of sales and excise taxes. These are new variables, calculated in a straightforward ways:

(26)

$$p_c^{\tau} = \frac{\sum_{i \in I} \phi_{ic}\, p_i \left(1 + \sum_{g \in GS} \tau_{gi}^{x}\right)}{\sum_{i \in I} \phi_{ic}\, \overline{p}_i \left(1 + \sum_{g \in GS} \tau_{gi}\right)}$$

This is in ratio terms, with all commodity prices being one in the base case (where goods prices and taxes are unchanged from original data). The coefficient ϕ_{ic} is one element in a matrix distributing the fractions of a unit of composite commodity 'c', such as 'food and beverage' to DRAM industry outputs, such as 'food' and 'retail'. This matrix was established from analysis of the CES components (these being far more disaggregated than the nine categories) and overall ratios of the two trade sectors of DRAM (wholesale and retail) to consumption commodities in existing data. The key issue is that CES surveys are completed in terms of purchases of goods, such as a loaf of bread for $1.50. Perhaps 50 cents of that purchase was 'cost of goods sold' or output of the DRAM 'food' sector. Perhaps 25 cents was value added by wholesaler and 75 cents value added by retailers. The CES data simply show the total.

A minor change in the savings function is required for each household. Savings becomes an explicit fixed share of after-tax income for each household. In the original model, the following functions were in place:

(27)

$$s_h = y_h^d - \sum_{i \in I} c_{ih} p_i \left(1 + \sum_{g \in GS} \tau_{gih}^{x}\right) \quad \forall h \in H$$

While apparently different from a fixed share of income, these take on the same computational value. As consumption of each good was a fixed share of income, as discussed above, the result of the right hand side of the equation above is to keep savings at a fixed share. In the new version, this is made more explicit:

(28)

$$s_h = \sigma_h y_h^d \quad \forall h \in H$$

The final modification is to translate the household share variables into demand for DRAM industry goods and services. Given that the ϕ matrix is not only common to all households, but also constant, a major simplification was possible. The demand from all households can be calculated by adding up the shares of each household's demand for each composite, times the share of the composite supplied by the DRAM sector, as follows:

(29)

$$c_i^h = \frac{\sum_{h \in H} \sum_{c \in C} \phi_{ic}\, s_{ch} \left[y_h^d (1 - \sigma_h) \right]}{p_i \left(1 + \sum_{g \in GS} \tau_{gi}^{x}\right)}$$

The denominator returns the calculation to real, tax-excluded quantities.

It is interesting to note the numbers of variables and equations. There were 196 household demand variables and equations in DRAM (7 households times 28 goods), plus 7 household-specific CPIs for a total of 203. In the revised model, there are 63 (7 households times 9 goods) share varaibles, plus seven CPIs, nine commodity prices and 28 consumption demand equations, for a total of 107. Considerable subtlety was added to the model with an outright reduction in its size.

However, having the cross-price elasticities non-zero adds greatly to the computational burden. Each commodity price and income enters each household's demand for each commodity. In the previous model, only one commodity price and income entered each demand equation. Since the second derivatives are exploited by the solver, we now have 100 second derivatives times 63 equations or 6,300 second derivatives for household demand. Previously we had 4 second derivatives times 196 equations for a total of 784.

Thus, despite a significant size reduction in terms of variables and equations, the overall computational complexity of the model has been increased dramatically.

VI.2. CALIBRATION

The data for testing the model are those currently in use for revenue bill analysis in the Department of Finance: those for fiscal 1996-97—both for economic and governmental fiscal data. The changes discussed above were implemented, including revisions to the main compendium of economic and fiscal data for the model: the Social Accounting Matrix. The nine rows and columns for the new composite commodities were added and then the model re-balanced with households consuming commodities which are then distributed over goods, services and sales taxes. Only two significant sets of coefficients remained: those for the scale parameter in the household consumption share equations (24) and the matrix that allocates commodities to goods, serivices and taxes (ϕ_{kc}).

As prices for composite commodities are expressed in terms of changes from a base of one, the second expression in equations (24) disappears in data at the starting point (the log of one being zero). Similarly, the third element of this simplifies, as the overall price index would become unity in the original conditions. Thus, with the initial shares and income elasticities from the estimation, plus observations of after-tax incomes, solving for the final parameter α_{ih} is reasonably straightforward:

(30) $$\alpha_i = \bar{s}_{ih} - \beta_i \log\left[\bar{y}_h\left(1 - \sigma_h\right)\right]$$

The equations for price indices, composite commodity prices and for goods and services demand require only the matrix of allocation from commodities to goods and services (ϕ_{kc}) for calibration purposes. This matrix has been appended to this report. It was derived from CES data allocating components of commodities to DRAM-type goods and services. For goods, the economy-wide share of wholesale and retail sectors was applied to goods purchases and then the entire social accounting matrix re-calibrated to the new sectoral output values.

The model was solved without tax changes to replicate current economic conditions to, at least, the fifth significant digit.

Readers are reminded that calibration is a necessary, but not sufficient condition for having a well-ordered model. Without it having being calibrated, a model cannot be treated as a reliable tool, even for replicating current economic conditions, let alone for predicting the impact of policy change. After being calibrated, a model is ready for further testing.

VI.3. SENSITIVITY ANALYSIS

As part of the Department's ongoing efforts at maintaining quality control of its highly complex dynamic revenue model, the test version with the new consumption functions was exposed to two sets of policy experiments and the results compared with the same model currently in use for revenue bill analysis (which embodies Cobb-Douglas consumption functions). The base case set of experiments mirrors tests made with the original DRAM: one billion dollar across-the-board tax reductions for each of the three largest California taxes: Bank and Corporation, Personal Income, and Sales and Use taxes. The results follow:

Table 6: Base Case Results

			Bank and Corp. Tax		Pers. Income Tax		Sales and Use Tax	
			Old Model	New Model	Old Model	New Model	Old Model	New Model
Bank & Corp. Revenue		$ Billion	4.797	4.802	5.868	5.870	5.870	5.872
Personal Inc. Tax Revenue		$ Billion	22.355	22.370	21.219	21.228	22.254	22.265
Sales & Use Tax Revenue		$ Billion	18.419	18.434	18.370	18.372	17.400	17.402
General Fund Revenues		$ Billion	46.777	46.811	46.664	46.676	46.830	46.845
Special Funds Revenues		$ Billion	13.181	13.181	13.161	13.160	13.072	13.072
Static Estimate of Rev. Loss		$ Billion	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)	(1.000)
Dynamic Estimate	General Fund Change	$ Billion	(0.866)	(0.832)	(0.979)	(0.967)	(0.813)	(0.798)
	Special Funds Change	$ Billion	0.026	0.026	0.006	0.005	(0.083)	(0.083)
	$ Dynamic Effect	$ Billion	0.160	0.194	0.028	0.038	0.104	0.119
	As percent of Static	Percent	15.994	19.359	2.763	3.826	10.410	11.929
California Personal Income		$ Billion	775.937	776.333	773.608	773.679	774.489	774.659
Gross Investment		$ Billion	81.261	81.352	80.803	80.827	80.840	80.876
Households		Millions	22.659	22.661	22.645	22.645	22.651	22.652
Wage Index		Base=100	100.176	100.195	99.867	99.881	100.027	100.032
Capital Index		Base=100	100.007	100.008	100.001	100.001	100.002	100.002
Labor Demand		Millions	13.070	13.072	13.078	13.077	13.069	13.068
Capital Demand		$ Billion	16.252	16.270	16.161	16.165	16.168	16.175
Net Job Creation		Thousands	11.979	13.746	19.906	19.190	10.527	10.176
Net Private Investment		$ Million	535.609	626.610	77.660	101.295	114.422	150.366

The results of these base case experiments reveal small changes in the overall results due to the reformulation. The order of magnitude and ordering of the revenue feedback results do not change. Bank and corporation tax reduction continues to lead, followed by sales and use tax. Personal income tax reduction nets little in the way of revenue feedback results. The nineteen percent revenue feedback (*versus* sixteen with the previous formulation) for bank and corporation tax reduction implies that 81 percent of the tax reduction is just that in the long run. The differences between sixteen and nineteen percent feedback, five or six years after a tax policy change would be lost in all of the other economic changes that would impact California tax revenues.

The investment increases are all noticeably larger, but these changes must be compared with the overall magnitude of change. Bank and corporation tax reduction implies a change of about two-thirds of one percent in investment for the old model and three-quarters of one percent with the new model. The results, as with revenue feedback continue: bank and corporation tax reduction impacts investment the most, followed by sales and use tax reduction and then personal income tax reduction.

Employment effects mirror those for investment and revenue effects and repeat the pattern from the previous model: personal income tax reduction has the most significant employment effects, followed by bank and corporation tax reduction and then by sales and use tax reduction.

It would appear that the reformulation of the model did not change the fundamental results—despite considerable new flexibility in the model. The sources of these observations are twofold. First, while the original model incorporated income elasticities of 1.0, the new elasticities do not stray far from this number (see Table 5 above). Second, the own and cross price elasticities (Table 4) retain a very important property of consumer demand theory: they are homogeneous of degree zero in income and prices. Thus, without major changes in any one composite commodity price in relation to others and incomes, the relative share consumed by each group of households remains relatively constant. That is not to say that there are no changes: the lack of perfect congruence between model results is proof of that. This does say that the overall economic changes between model formulations are relatively modest.

The changes that do exist between formulations imply a greater level of private economic reaction to changes in tax law—and much of this comes from consumers in the new model effecting at least some changes in shares of consumption in the face of income and relative price changes. This new feedback effect is from firms adjusting first to business profits tax changes and then their changes influencing consumers (through income and price changes) whose effects then encourage firms to re-adjust their quantity and price decisions (and on until an equilibrium is found). In the old model, the first order changes of firms induced a fairly rigid effect from consumers. They spent the new incomes or compensated for lost incomes by reducing their consumption in fixed shares. The new model implies a much more subtle and realistic set of rational consumption decisions by households.

Table 7: Selected Tax Experiments

			Manufacturing Investment Credit		Agricultural Investment Credit		Income Tax Bracket Widening	
			Old Model	New Model	Old Model	New Model	Old Model	New Model
Bank & Corp. Revenue		$ Billion	5.587	5.589	5.477	5.478	5.871	5.873
Personal Inc. Tax Revenue		$ Billion	22.270	22.270	22.273	22.279	21.234	21.243
Sales & Use Tax Revenue		$ Billion	18.381	18.383	18.385	18.391	18.380	18.382
General Fund Revenues		$ Billion	47.443	47.448	47.340	47.353	46.691	46.705
Special Funds Revenues		$ Billion	13.166	13.165	13.166	13.166	13.167	13.165
Static Estimate of Rev. Loss		$ Billion	(0.285)	(0.284)	(0.388)	(0.386)	(0.977)	(0.977)
Dyn-	General Fund Change	$ Billion	(0.200)	(0.195)	(0.303)	(0.289)	(0.951)	(0.938)
amic	Special Funds Change	$ Billion	0.011	0.010	0.011	0.011	0.012	0.010
Est-	$ Dynamic Effect	$ Billion	0.096	0.098	0.096	0.107	0.038	0.050
imate	As percent of Static	Percent	33.539	34.655	24.687	27.785	3.894	5.090
California Personal Income		$ Billion	774.144	774.293	774.244	774.350	773.965	774.038
Gross Investment		$ Billion	80.903	80.918	81.070	81.135	80.841	80.870
Households		Millions	22.650	22.651	22.651	22.651	22.650	22.651
Wage Index		Base=100	100.094	100.083	100.088	100.101	99.872	99.886
Capital Index		Base=100	100.002	100.002	100.003	100.003	100.002	100.002
Labor Demand		Millions	13.061	13.062	13.061	13.062	13.085	13.084
Capital Demand		$ Billion	16.181	16.184	16.214	16.227	16.168	16.174
Net Job Creation		Thousands	2.885	3.821	3.455	3.860	26.765	26.110
Net Private Investment		$ Million	177.173	192.809	344.549	409.302	115.474	144.078

Three new experiments were conducted (new in relation to the sensitivity analysis experiments for the original model). They spring from three topics raised by actual bill analyses of legislative proposals in 1997. In the first, the Manufacturers' Investment Credit (MIC) was raised by 50 percent, from six to nine percent of qualifying equipment and structures, for industries that now qualify for it. In the second, the same type of credit (at six percent) is applied to agricultural activities. In the final experiment, the income tax brackets are broadened in such a way as to deliver about one billion dollars (static estimate) of personal income tax reduction.

The first was chosen as it has demonstrated the largest feedback effects found to date: a widely-applicable investment-oriented tax credit for an industry group with high average wages (thus workers and owners who pay significant personal income, sales and other forms of taxes). The second is an investment-oriented tax credit for a highly capital-intensive industry, but a narrowly-defined one which pays little or no net tax itself (net of cash subsidies) in relation to the value of its output. The last is a personal income tax proposal that has become part of the recent public debate in California for its properties of making the dollars of tax relief more evenly distributed across incomes, as opposed to proportionate marginal rate reductions which provide more dollars of tax relief to those who are paying more of the taxes today.

Considering that the results reflect the proportions of change to the economy five or six years following tax law change, the revenue feedback percentage differences are almost trivial, as are the differences in investment and employment. The construction of household consumption functions much more congruent with core microeconomic theory and with estimated coefficients does not change the fundamental results—whether measured against large theoretical tax reductions or more specific types of legislative proposals.

VII. CONCLUSION

Construction and (especially) use of a complex analytical engine using state-of-the-art methods exposes researchers to criticism of three main types. The easiest to endure is that from those who reject new methods out-of-hand. They generally are practioners of methods that led to the demand for the new approach and often successfully resist the implementation of the new method. Fortunately, in California, the enactment of legislation mandating the use of new approaches has made this a relatively minor source.

The second comes from those who espouse results, claiming congruence with economic theory, while possessing little actual knowledge. For example, some believe that government has a 'right' to tax revenues and that any tax reduction is a 'giveaway'... often called a 'tax expenditure'. They decry the use of equilibrium models as being 'voodoo economics.' A model that predicts any real economic change in the face of tax law change, and (especially) one with the largest revenue feedback effects from business tax reduction, would attract this epithet. At the opposite extreme, politically, is another group who subscribe to the idea that 'every tax cut pays for itself.' They appear to believe that the taxation side of fiscal policy has a 'free lunch' awaiting it that the expenditure side never did. By undergoing thorough peer review by learned professionals who have earned the respect of a wide range of well-trained professionals, this second group's various criticisms can be withstood.

The third group's criticisms have been less easily withstood, but critiques have been tempered by the issue of time. DRAM was built, tested, documented and exposed to peer review in about eight months' time. It was refined, updated and put into active use sixteen months after the project was staffed. The lack of estimated parameters, restrictive representation of consumer demand and the lack of explicit connection between some kinds of public activity and private economic outcomes (such as public safety, physical infrastructure and education) exposed the model's results to the criticism of well-trained professionals keen on having available a well-rounded, reliable and fair representation of the California economy that was congruent with core economic theory. The current research answers one of the deficiencies in the original model. The answer is that the results change when better consumer demand functions with estimated parameters are used, but they do not change much.

While some ongoing deficiencies of DRAM will endure due to an almost total absence of data (trade being the most important of these and most notable for its lack of reliable data), at least one further avenue of model enhancement has begun to be explored: the connection between public provision of goods, services and factors of production and private market outcomes. That research is underway and will be reported when complete.

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