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ABSTRACT

Dynamical symmetry breaking in non-Abelian gauge theories is studied by computing an effective potential for composite operators. We obtain consistent solutions of chiral and gauge symmetry breaking which are, in some cases, compatible with a short distance behavior. The effective theory determined is in agreement with the tumbling hypothesis.

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1. INTRODUCTION

Many of the present studies in gauge theories are devoted to the gauge symmetry breaking mechanism. In order to circumvent the problems, which arise in the Higgs mechanism with fundamental scalar bosons,[1,2] different models have been proposed, such as the so called technicolor models,[2,4] which imply the existence of dynamical symmetry breaking (= composite Higgs bosons), and the supersymmetric models, where supersymmetry save us from the mixing of different scales in grand unified theories.[5]

The technicolor hypothesis brought on interesting ideas such as the tumbling hypothesis,[6] in spite of the failures to achieve a realistic model.[7] Analytical studies were done towards a better understanding of this hypothesis, but were not successful in explaining how that mechanism occurs in a realistic four-dimensional non-Abelian gauge theory.[8,9] Parallel to these works, recent Monte Carlo simulation on the scales of chiral symmetry breaking of gauge theories, gives support to the tumbling hypothesis, showing that these scales are not necessarily related to the confinement scale.[10]

This paper is an attempt to improve the knowledge on dynamical symmetry breaking (DSB) of non-Abelian gauge theories. We will calculate the effective potential for composite fermionic operators as introduced in Ref. (11), with the aim to show fermion condensation. Our results indicate that there is not necessity of confining forces to achieve condensation, which is what one can expect from a naive comparison with the similar phenomenon in solid-state physics.

It is clear that if a theory has a non-vanishing vacuum expectation value (VEV) at large momentum,[10] it can only be explained by a non-perturbative behavior. Consequently, we believe that an effective potential approach, based on the non-perturbative asymptotic behavior of two-point functions, must be a good way to show this effect.

In Section 2 we mostly deal with the effective potential calculation. Section 3 contains the analysis of a general non-Abelian gauge theory, showing the existence of fermion condensation, discussing the stability of the effective potential, and what kind of dynamics produces this phenomenon.

We discuss the problem of gauge hierarchies in dynamically broken gauge theories, and comment on the hypothesis of condensation in the maximally attractive channel (MAC).[6]

The conclusions are given in Section 4. Appendix A is devoted to a brief discussion of dynamical symmetry breaking in an Abelian case.

2. THE EFFECTIVE ACTION FOR COMPOSITE OPERATORS

The effective action to be computed (Γ), is a functional of the Green's functions G_i, and is stationary with respect to variations of G_i.

$$\frac{\delta \Gamma}{\delta G_i} = 0 \quad . \tag{1}$$

The effective potential is defined by

$$V(G_i) \int d^4x = - \Gamma(G_i) \Big|_{\text{TRANSLATION INVARIANT}} \tag{2}$$

For a non-Abelian theory it has the form[11]

$$\begin{aligned} V(S,D,G) = & -i \int \frac{d^4p}{(2\pi)^4} \operatorname{Tr} \left(\ln S_0^{-1} S - S_0^{-1} S + 1 \right) \\ & -i \int \frac{d^4p}{(2\pi)^4} \operatorname{Tr} \left(\ln G_0^{-1} G - G_0^{-1} G + 1 \right) \\ & + \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \operatorname{Tr} \left(\ln D_0^{-1} D - D_0^{-1} D + 1 \right) \\ & + V_2 (S,D,G) \quad . \end{aligned} \tag{3}$$

In Eq. (3) S,D and G are respectively the complete propagators of fermions, gauge bosons and Fadeev-Popov ghosts; S_o, D_o and G_o the respective bare propagators.

$V_2(S,D,G)$ is the sum of all two particle irreducible vacuum diagrams, as drawn in Fig. 1. It is determined by requiring that the solutions of the equations

$$\frac{\delta V}{\delta S} = \frac{\delta V}{\delta D} = \frac{\delta V}{\delta G} = 0 \tag{4}$$

give the Schwinger-Dyson equations for fermions, gauge bosons and ghosts.

The physically meaningful quantity we need to compute is the vacuum energy density given by

$$\Omega = V(S, D, G) - V(S_m, D_m, G_m) \quad , \tag{5}$$

where we are subtracting from the asymmetric potential the symmetric one, denoted by $V(S_n, D_n, G_n)$.

Due to our ignorance of the complete propagators, we use their asymptotic linearized approximation. Therefore, Ω may be trusted as far as the linearized solutions reflect the true behavior of the complete propagators.

The energy density has a graphical representation given in Fig. 2. The determination of which diagrams contribute to Ω, is summarized in Appendix B.

In the diagrams of Fig. 2, the insertions correspond to the linearized solution of the Schwinger-Dyson equation for the fermions self-energy ($\Sigma(p^2)$), and the blobs to linearized gauge boson self-energy ($\Pi(p^2)$). The double gauge boson lines correspond to the complete gauge boson propagator, which is given by (in Landau gauge)

$$D^{\mu\nu}(p) = \frac{-i\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)}{p^2 - \Pi(p^2)} \quad . \tag{6}$$

The linearized $\Sigma(p^2)$ can be determined by transforming the integral Schwinger-Dyson equation into a differential equation, with the condition $\Sigma(p^2 = 0) = \mu$.[12] One obtains the following asymptotic solutions

$$\Sigma_I(p^2) \underset{p^2 \to \infty}{\sim} \mu \left(\frac{p^2}{\mu^2}\right)^{-\delta} \tag{7a}$$

$$\Sigma_{II}(p^2) \underset{p^2 \to \infty}{\sim} \frac{\mu^3}{p^2} \left(\frac{p^2}{\mu^2}\right)^{\delta} \quad , \tag{7b}$$

where

$$\delta = \frac{3\, g^2(p^2)}{16\, \pi^2}\, c \quad ,$$

and in the case where we have, for example, two fermion representations R_ψ and R_χ, c is given by[F1]

$$c = \frac{1}{2} \left[C_2(R_\psi) + C_2(R_\chi) - C_2(R_\phi) \right] \quad . \tag{8}$$

$C_2(R_i)$ is the second Casimir operator for the representation R_i, and R_ϕ is the representation, contained in the product $R_\psi \times R_\chi$, which minimizes the effective action.

It has been argued that the solution (7b) should be the physical one[14,15] since it can be derivable from operator product expansion (OPE), and gives the known power counting behavior for the pion electromagnetic form factor ($F_\pi(q^2)$). However, this is not enough to rule out solution (7a)[16,F2]. We shall see in the next section, that only $\Sigma_I(p^2)$ leads to an effective potential with a VEV proportional to $1/g^2$, invalidating the naive OPE technique.

There are two arguments in favor of $\Sigma_I(p^2)$ as the good physical solution: 1) when inserted in the effective potential, $\Sigma_I(p^2)$ leads to the deepest minimum (as we will see in the next section), 2) in a recent renormalization group analysis, of the dynamical symmetry breaking in QCD based on the Nambu-Jona-Lasinio approach, solution (7a) was determined as the physical one.[17]

We parametrize $\Sigma_I(p^2)$ possessing the asymptotic behavior (7a) in the following form:

$$\Sigma_I(p^2) = -i\phi\left[1 + b g^2(\mu^2)\ln\frac{p^2}{\mu^2}\right]^{-3c/16\pi^2 b} , \qquad (9)$$

where

$$\phi = \sum_i \mu\, P^i \qquad (10)$$

b is the coefficient of g^3 in the expansion of the β function

$$b = \frac{1}{16\pi^2}\left[\frac{11}{3} C_2(G) - \sum_{R_i}\frac{4}{3} T(R_i)\right]$$

$T(R_i)$ is the index of the representation R_i. ϕ will act as a variational parameter to be determined from the effective action.

Notice that ϕ looks like a composite field, in the sense that the Schwinger-Dyson equation which dresses the fermion ($\Sigma(p^2)$), is related to the Bethe-Salpeter kernel ($\Phi_{BS}(p,q)$), which binds a fermion-antifermion pair together in a pseudoscalar s wave at zero momentum transfer

$$\Sigma(p^2) = \Phi_{BS}(p,q)\big|_{q\to 0} \qquad (11)$$

As we choose Eq. (7a), the vacuum polarization tensor will be given by[13]

$$\Pi(p^2) \underset{p^2\to\infty}{=} -i M_G^2\left[1 + b g^2(\mu^2)\ln\frac{p^2}{\mu^2}\right]^{-3c/8\pi^2 b} , \qquad (12)$$

where in the same way as ϕ, M_G^2 will be considered as a variational parameter. Equation (12) is the result of the interaction of the gauge bosons with the composite mesons (fermion condensates), and M_G^2 is proportional to $-\mathrm{Tr}\{[t^a,\phi][e^b,\phi]\}$, which is zero in the case of singlet condensation.

Using Eq. (9) and (12) we are able to determine the energy density Ω (all the calculations were done in Landau gauge). Computing the diagrams of Fig. 2 (see Appendix B, and Eq. (B.(7)), taking the proper vertexs at the lowest order, and using the running coupling constant, we arrive at

$$\Omega = \frac{1}{16\pi^2}\,\gamma^{-1}\left(1+\tfrac{1}{2}\gamma\right)\mathrm{Tr}\,\phi^4 - \frac{1}{16\pi^2}\mathrm{Tr}\,\mu^4\left(1+\frac{\phi}{\mu}\right)^4 \ln\left(1+\frac{\phi}{\mu}\right)^2$$
$$+\frac{7}{16\pi^2}\mathrm{Tr}\,\phi^2\mu^2 + \frac{3}{64\pi^2}\,\gamma^{-1}\mathrm{Tr}\,M_G^4\left(1-\gamma\ln\frac{M_G^2}{\mu^2}-\frac{1}{2}\gamma\right) \tag{13}$$
$$+\frac{3}{64\pi^4}\,\gamma^{-1}\frac{\mathrm{Tr}([t^a,\phi][t^b,\phi]\,M_G^2)}{(3c/8\pi^2-b)}\left(1-\gamma\ln\frac{M_G^2}{\mu^2}-\gamma\right),$$

where

$$\gamma = g^2\left(\frac{3c}{4\pi^2}-b\right) \quad . \tag{14}$$

Some of the pure gauge bosons diagrams vanish at the order we are working in the Ω calculation. The two-loop diagrams with quartic gauge coupling (Fig. 2), vanish as a result of a conspiracy between Lorentz and gauge group indices. The two-loop diagrams with triple coupling are proportional to $\sum_{cd} f_{acd} f_{bcd}(M_c^2 - M_d^2)$, which is identically zero in our approximation.[13] This result implies that the contributing diagrams for the energy density, are the same as in the Abelian case. (As a consequence, the proof of gauge invariance of the effective potential is similar to the Abelian case). This fact was noticed earlier by Cornwall in its phenomenological approach to the problem.[13] (Notice that Eq. (13) can be reduced to the Abelian result of Ref. (11), aside from some corrections of 0(1) factors).

The variation of Ω with respect to M_G^2

$$\frac{\delta\Omega}{\delta M_G^2} = 0 \quad , \tag{15}$$

entails

$$M_G^2 = -\frac{1}{2\pi^2(3c/8\pi^2-b)}\,\mathrm{Tr}\left([t^a,\phi][t^b,\phi]\right) \quad , \tag{16}$$

where we have neglected terms of order $g^2 \ln(M_G^2/\mu^2)$.

Inserting Eq. (16) into Eq. (13) we obtain Ω as a function of ϕ

$$\Omega = \frac{1}{16\pi^2}\,\gamma^{-1}\left(1+\tfrac{1}{2}\gamma\right)\mathrm{Tr}\,\phi^4 - \frac{1}{16\pi^2}\mathrm{Tr}\,\mu^4\left(1+\frac{\phi}{\mu}\right)^4\ln\left(1+\frac{\phi}{\mu}\right)^2$$
$$+\frac{7}{16\pi^2}\mathrm{Tr}\,\phi^2\mu^2 + \frac{3}{256\pi^6}\,\frac{c^2\gamma^{-1}}{(3c/8\pi^2-b)^2}\,\mathrm{Tr}\,\phi^4\left[1-\tfrac{1}{2}\gamma-\gamma\ln\frac{c\,(\phi/\mu)^2}{2\pi^2(3c/8\pi^2-b)}\right]$$
$$-\frac{3}{128\pi^6}\,\frac{c^2\gamma^{-1}}{(3c/8\pi^2-b)^2}\,\mathrm{Tr}\,\phi^4\left[1-\gamma-\gamma\ln\frac{c(\phi/\mu)^2}{2\pi^2(3c/8\pi^2-b)}\right] \quad . \tag{17}$$

$|\phi/\mu|$ must be smaller than 1, to prevent Ω from taking complex values. This also ensures that loop corrections of higher orders in powers of $g\phi$ are not relevant.[11,18] It allows us to expand Eq.(17) for small values of (ϕ/μ), which gives

$$16\pi^2\mu^{-4}\Omega = -\left[\frac{3}{16\pi^4}\frac{c^2\gamma^{-1}}{(3c/8\pi^2-b)^2} - \gamma^{-1} + \frac{11}{3} - \frac{9}{32\pi^4}\frac{c^2}{(3c/8\pi^2-b)^2}\right]\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^4$$
$$+\frac{3}{16\pi^4}\frac{c^2}{(3c/8\pi^2-b)^2}\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^4 \ln\frac{c(\phi/\mu)^2}{2\pi^2(3c/8\pi^2-b)} \qquad (18)$$
$$+\frac{1}{15}\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^6 + \frac{1}{140}\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^8 + \ldots$$

We will see in the next section that (18) has a stable solution for (ϕ/μ) different from zero.

Notice that in the first term of Eq. (18) we have two contributions of order $1/g^2$, while all the other terms are of $O(1)$. If we take into consideration only the $O(1/g^2)$ contribution, we arrive at an unstable energy density. The $O(1)$ is essential to study the effective potential, which can explain negative results obtained earlier, in the study of the stability of dynamically broken gauge theories.[19] Actually, our procedure is not free of ambiguities at $O(1)$, due to the approximation in the trial functions (Eq. (9) and Eq. (12)), as a consequence of the truncation of the Schwinger-Dyson equation, and also of the neglect of higher order loops (which can contribute to $O(1)$) in the dressed expansion of the effective action.[18]

We can determine a complete effective theory, including the kinetic term of the effective action. To start with, let us suppose that the real vacuum leads to fermion condensation, and denote the true ground state by $|\Omega>$. Taking into account the structure of the real vacuum, the fermion propagators are described by a fermion bilinear which is not translationally invariant

$$S(x,y)_{\alpha\beta} = -i\left\langle\Omega\left|T\left[\chi_\alpha(x+\tfrac{1}{2}y)\,\psi_\beta(x-\tfrac{1}{2}y)\right]\right|\Omega\right\rangle . \qquad (19)$$

The Fourier transform of Eq. (19) can be written as

$$S(p,k) = S_o(p,k) + \Sigma(p,k) , \qquad (20)$$

where $S_o(p,k)$ is the bare propagator (which is translationally invariant) given by

$$S_o(p,k) = (2\pi)^4\,\delta^4(p-k)/\not{k} , \qquad (21)$$

and $\Sigma(p,k)$ is a gap equation, which can be separated in its regular part ($\Sigma(k)$), and its singular symmetry breaking part ($\Sigma_b(p,k)$)

$$\Sigma(p,k) = (2\pi)^4\,\Sigma(k)\,\delta^4(p-k) + \Sigma_b(p,k) . \qquad (22)$$

Notice that $\Sigma_I(p^2)$ given by Eq. (7a), is nothing other than the linearized solution of $\Sigma_b(p,k)$.

If we suppose that the expectation value of the fermion bilinear has the following operator expansion[18]

$$\left\langle\Omega\left|T\left[\chi(x+\tfrac{1}{2}y)\,\psi(x-\tfrac{1}{2}y)\right]\right|\Omega\right\rangle \underset{y\sim 0}{\sim} C(y)\,\phi(x) , \qquad (23)$$

($C(y)$ is a c-number function, and $\phi(x)$ acts like a scalar field with anomalous dimension 2δ) we can write

$$\sum_b (p,k) \sim -i\,\phi(k)\left[1 + b\,g^2(\mu^2)\ln\frac{p^2}{\mu^4}\right]^{-3c/16\pi^2 b} \qquad (24)$$

As seen in Eq. (24), working in the true vacuum generates a nontrivial dependence on the momentum k for our variational parameter ϕ.

The kinetic term for our effective theory is obtained inserting $\phi(k)$ in the effective action, and expanding around $k = 0$.

The diagrams contributing to the kinetic part of the energy density in order $1/g^2$ are drawn in Fig. 3. We could say, in an intuitive way, that they are related to the vacuum polarization tensor of some composite Higgs field $\phi(x)$. The computation of these diagrams give

$$\Omega_{\text{kinetic}} = \frac{1}{2}\int d^4x\,\frac{(\partial_\mu \phi(x))^2}{4\pi^2 g^2(3c/8\pi^2 - b)} \qquad (25)$$

Comparing (25) with a common scalar field kinetic term, we can define

$$\Phi(x) = Z^{-1/2}\,\phi(x)\,, \qquad (26)$$

where Φ plays the role of the physical field, and Z is a renormalization constant

$$Z = 4\pi^2 g^2 (3c/8\pi^2 - b)\,. \qquad (27)$$

With Eqs. (26), (27) and (17) we finally arrive at an effective Lagrangian with the following form

$$\mathcal{L}_{ef} = \int d^4x\left[\frac{1}{2}(\partial_\mu \Phi)^2 + \frac{\lambda_1}{4}\Phi^4 - \frac{\lambda_2}{6}\Phi^6 - \ldots\right], \qquad (28)$$

where λ_1 and λ_2 are the effective coupling constants, and are functions of g^2, c, b and μ^2. (There is an implicit trace in the Φ polinomial.)[F3] The characteristics of this effective Lagrangian will be investigated in the next section.

3. APPLICATIONS TO DYNAMICALLY BROKEN GAUGE THEORIES

In this section we are going to analyze the effective theory obtained above, looking for the existence of fermion condensation.

For the sake of simplicity we will discuss separately the cases of chiral and gauge symmetry breaking, and, for completeness, we describe in Appendix A how this formalism works, in the Abelian case (represented by the electrodynamics of Baker, Willey and Johnson).[20]

3a. Chiral symmetry breaking

In the case where the bound-state (fermion self-energy) is a singlet, only the chiral symmetry can be broken ($M_G^2 = 0$), and the energy density in Eq. (18) is reduced to

$$16\pi^2\mu^{-4}\Omega = -\left[\frac{11}{3}-\gamma^{-1}\right]\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^4 + \frac{1}{15}\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^6 + \frac{1}{140}\mathrm{Tr}\left(\frac{\phi}{\mu}\right)^8 + \cdots \quad (29)$$

Equation (29) has a minimum for $(\phi/\mu)^2 \neq 0$ manifesting the existence of condensation. But as previously said we have the condition

$$0 < (\phi/\mu)^2_{MIN} < 1 \quad , \qquad (30)$$

neccessary for consistency of the approach.

From (28) and (30) we see that γ is bounded, and we obtain

$$0.273 \lesssim g^2(\mu^2)\left(\frac{3c}{4\pi^2} - b\right) \lesssim 0.280 \quad . \qquad (31)$$

The upper bound has no physical meaning, because in this case $(\phi/\mu)^2 \simeq 1$, and the method loses its consistency.

We so arrived at a lower bound on the gauge coupling constant, related to the scale where chiral symmetry is broken. In a realistic model we can expect a phase transition, near the critical coupling constant determined by Eq. (31). Actually, this would be a good value, if the chiral symmetry breaking phenomenon is associated to a short distance behavior. However, we expect that for some values of c and b, g^2 will not be small enough to justify our procedure, and to exclude a big contribution from low momenta.

It is possible to give a prediction for the critical coupling constants, in the case of chiral symmetry breaking of a SU(N) theory, with fermions in the fundamental representation[F4]

$$\alpha_{CRIT} \approx 1.73\left(\frac{7}{6}N - \frac{3}{N} + \frac{1}{3}N_f\right)^{-1} \quad , \qquad (32)$$

N_f is the number of flavors. Notice that the number of flavors is important in the determination of the critical scale.

We can test the above result for QCD with 6 flavors. From Eq. (32) we obtain $\alpha(\mu^2) \simeq 0.39$, which corresponds to $\mu \simeq 9.9\ \Lambda_{QCD}$ (using naively the running coupling constant). This gives a value of μ of O(1 GeV), which is 3 times bigger then the best estimate to the dynamical mass of QCD.[22]

One might think how the above scenario is modified by the low momentum effects, if the theory is described not by Eq. (7a), but by a linear combination of (7a) and (7b) (Eq. (7b) has a behavior more characteristic to the large distance physics), or, as another interesting possibility, if the theory exhibits a self-energy solution proportional to $1/p^4$ (which is expected to give the linear confining potential).[29] Our intention is to check the validity of our high momentum approach.

Let us first compute the contribution of Σ_{II} to the energy density.

$$\Omega_{\Sigma_{II}} = i \int \frac{d^4 p}{(2\pi)^4} \, \mathrm{Tr} \ln (1 - \Sigma_{II} S) \quad . \qquad (33)$$

Due to the extremely convergent character of Σ_{II}, we can take $\Sigma_{II} \sim \mu^3/p^2$ in a good approximation, and obtain

$$16\pi^2 \mu^{-4} \Omega_{\Sigma_{II}} = \frac{1}{20} \mathrm{Tr} \left(\frac{\phi}{\mu}\right)^4 + O(10^{-2}) \, \mathrm{Tr} \left(\frac{\phi}{\mu}\right)^6 + \cdots \qquad (34)$$

Notice that Σ_{II} gives a stable potential, but does not lead to condensation.

Assuming that we have a linear combination of Eq. (7a) and Eq. (7b), we arrive to

$$16\pi^2 \mu^{-4} \Omega_{(\Sigma_I + \Sigma_{II})} = -\left[\frac{11 - (3/20)A}{3} - \gamma^{-1}\right] \mathrm{Tr} \left(\frac{\phi}{\mu}\right)^4 + \left[\frac{1}{15} + O(10^{-2})B\right] \mathrm{Tr} \left(\frac{\phi}{\mu}\right)^6 + \cdots, \qquad (35)$$

where A and B are factors taking into account the linear combination of (7a) and (7b). The inclusion of Σ_{II} contributes to an increase of $g^2_{critical}$ (A needs to be of O(10) to give μ ~300 MeV for QCD).

Some comments on expression (35) are in order. We have to point out that only the irregular solution ($\Sigma_I(p^2)$) gives a VEV proportional to $1/g^2$. (Softer solutions, as the regular one, do not show this quasi-divergent aspect of Σ_I). Solutions like $\Sigma(p^2) \sim 1/p^2$

or others decreasing faster (expected in the low momentum region), at least in our approximation do not contribute to condensation, but are important in the stabilization of the effective potential. If the scale of chiral breaking is distinct from the confinement scale, this can only be due to the effect of the irregular solution. The effects of softer solutions almost disappear at high momenta. We claim that the irregular solution is the physical one, since it leads to the deepest state of minimum energy.

The effective Lagrangian for the chiral symmetry breaking case is given by Eq. (28), where

$$\lambda_1 = 4\pi^2 g^4 (3c/8\pi^2 - b)^2 (11/3 - \gamma^{-1}) \quad , \qquad (36a)$$

$$\lambda_2 = \frac{[2\pi^2 g^2 (3c/8\pi^2 - b)]^3}{5\pi^2 \mu^2} \quad . \qquad (36b)$$

From (28) we define a mass, as the second derivative of $\mathcal{L}$ with respect to ϕ, $m_\phi^2 \sim \lambda_1/\lambda_2$. The positivity of mass gives us the constraint

$$\left(\frac{3c}{8\pi^2} - b\right) > 0 \quad , \qquad (37)$$

which one can also find from equations (16) and (25). We thus recover a result of Lane,[14] which was a consequence of Mandelstam's condition of Beth-Salpeter wave function normalization[24] to solution (7a).

For a SU(N) gauge theory with fermions in the fundamental representation Eq. (37) gives a constraint on the number of flavors (N_f)

$$N_f > N + \frac{9}{2} N^{-1} \qquad (38)$$

If we apply Eq. (38) to QCD, the stability of the effective potential can only be achieved for $N_f \geq 5$. This strong result is a consequence of $\Sigma_I(p^2)$ (Eq. (7a)), and only appears in the case of a four dimensional space-time. (Condition (37) puts also a constraint in the formation of diquarks, in which case we must have $N_f > 10$.)

In Table 1 we give the result of calculations of critical couplings, and constraints in the number of flavors, for the chiral breaking of SU(2) with fermions in the representations $\underline{2}$, $\underline{3}$ and $\underline{4}$.

3b) Gauge symmetry breaking

If we form a non-singlet condensate it will break the gauge symmetry. The effective potential in this case is given by Eq. (18), and the value of $(\phi/\mu)^2$ at minimum is [F5]

$$\left(\frac{\phi}{\mu}\right)^2_{MIN} = \left\{\frac{15}{8\pi^4}\frac{c^2}{(3c/8\pi^2-b)^2}\left[\gamma^{-1} - 2 - \ln\frac{c(\phi/\mu)^2}{2\pi^2(3c/8\pi^2-b)}\right] - 10\gamma^{-1} + \frac{110}{3}\right\} . \qquad (39)$$

To see if $(\phi/\mu)^2_{min}$ given by Eq. (39) satisfies condition (30), we can study two cases: a) $3c/8\pi^2 \sim b$ and b) $3c/8\pi^2 \gg b$, and give some numerical results for specific models.

In the case where $3c/8\pi^2 \sim b$, the first term in the right hand side of Eq. (39), which is proportional to $(3c/8\pi^2-b)^{-2}$, dominates, and to maintain (30) we must have

$$\left[\frac{1}{g^2(3c/4\pi^2-b)} - 2 - \ln\frac{c(\phi/\mu)^2}{2\pi^2(3c/8\pi^2-b)}\right] \longrightarrow 0 \qquad (40)$$

as $(3c/8\pi^2 - b) \to 0$. Since the logarithm argument gets bigger, the only way to obey (40) is decreasing the value of g^2 (because $(3c/4\pi^2 - b) \not\to 0$). This scenario is consistent with small critical coupling constants. For asymptotically and anomaly free theories, we can obtain $3c/8\pi^2 \sim b$ only for groups of lower rank.

When $3c/8\pi^2 \gg b$ it is difficult to satisfy the condition given by Eq. (30), without getting out from the regime of small coupling constants. In general we obtain $3c/8\pi^2 \gg b$ for groups of higher rank. In this case we could expect more influence of the low momenta behavior.

Table 1 gives the numerical computation of the critical coupling constants, for the gauge symmetry breaking of a SU(5) theory with fermions in the representations $\underline{5}$ and $\underline{10}$, and for a SU(7) theory with fermion in the representations $\underline{\bar{7}}$, $\underline{\bar{7}}$ and $\underline{35}$.

We call the attention to the fact that the numbers in Table 1 are approximate, since there is a small range of coupling constants for which the condition (30) is not violated.

Table 1 gives also the calculation of the critical coupling constants from the naive expectation that $\alpha c \approx 1$.[6] We can see that in some cases there is a noticeable difference in the scale of symmetry breaking in those two calculations (these differences can also be seen from the results presented in Ref. (10)). This means that the aspects of gauge hierarchy in technicolor models should be reviewed,[25] mainly due to the fact that now not only the attractiveness given by the coefficient c, but also the coefficient b are involved in the calculation. For example, in one theory which tumbles twice, with attractiveness c_1 and c_2, even if c_1 is almost equal c_2, their coefficients b_1 and b_2 shall be different, if we assume a complete decoupling of the heavy particles.

Up to now we have found that the non-Abelian gauge theories admit condensation, and this phenomenon starts happening, in a short distance region, eventually receiving contribution from the low momenta physics, in accord with the theory studied. This can be expected from the naive comparison with superconductivity, where condensation is not associated to a strong confining force. For non-Abelian gauge theories, if the force between fermions reaches some given strength, which is related directly to the values of c and b, the theory goes through a phase transition to the broken phase.

The effective Lagrangian for the dynamical gauge symmetry breaking is given by

$$\mathcal{L}_{ef} = \int d^4x \left[\frac{1}{2} (\partial_\mu \Phi)^2 + \frac{\lambda_1}{4} \mathrm{Tr}\, \Phi^4 - \frac{\lambda_2}{6} \mathrm{Tr}\, \Phi^6 - \cdots \right. \qquad (41)$$

$$\left. - \frac{3}{16\pi^2} c^2 g^4 \, \mathrm{Tr}\, \Phi^4 \ln \frac{2cg^2}{\mu^2} \Phi^2 \right] ,$$

where

$$\lambda_1 = 4\pi^2 g^4 (3c/8\pi^2 - b)^2 \left[\frac{3c}{16\pi^4} \frac{c^2 \gamma^{-1}}{(3c/8\pi^2 - b)^2} - \gamma^{-1} + \frac{11}{3} - \frac{9}{32\pi^4} \frac{c^2}{(3c/8\pi^2 - b)^2} \right] , \qquad (42)$$

and λ_2 is the same as in Eq. (36b).

We assumed in the calculations that the condensation always occurs in the maximally attractive channel (MAC). If we just look at the effective Lagrangian (Eq. (41)), we see that the term responsible for the symmetry breaking is proportional to c, and computing the value of the potential at minimum, we see that it is associated to the biggest value of c, which confirms the MAC hypothesis. The only problem with this simple reasoning, is that a true minimization is only valid locally, since we are restrained to $0 < (\phi/\mu)^2 < 1$. We might have cases, in which two different channels can exist in two completely distinct regions of coupling constants, and we are not formally able to compare them. Besides that, the concept of MAC arises naturally, and, even more, in many cases the constraint given by Eq. (37) is enough to determine the MAC.

4. CONCLUSIONS

We have calculated an effective action for composite operators in a non-Abelian gauge theory, and obtained an effective Lagrangian which describes the chiral and gauge symmetry breaking, showing the existence of condensation, when the coupling constants are bigger then some value determined by the characteristics of the theory.

Most of our conclusions are based on the use of the irregular solution of the Schwinger-Dyson equation for the fermion propagator, and we have shown that this is the only physical solution since it leads to the deeper minimum.

As a consequence of the irregular solution behavior, we have a strong constraint on the fermion representations, and on the number of families, in order to obey the condition $(3c/8\pi^2 - b) > 0$. In the case of QCD with 6 flavors, this inequality tells us that we only can form bound states in the singlet channel.

Our results are in agreement with the tumbling hypothesis, with the result that for some values of c and b, the scale of symmetry breaking can really be associated with a short distance phenomenon, which is also in agreement with recent Monte Carlo simulations. We were able to determine the critical scale for chiral and gauge symmetry breaking, with a better accuracy always when the phenomenon occurs at higher momentum.

We hope to discuss in another paper some phenomenological aspects of our results. Among those, one consequence of the use of the irregular solution for the self-energy, is that this shall help to solve the problem of flavor changing neutral currents in technicolor

models,[7] allowing a raise of the extended technicolor boson masses.

Note added: After we finished this work our attention has been drawn to the paper of Mahanthappa and Randa (Phys. Lett. 121B, 156 (1983)), in which they study the dynamical breaking in a SU(2) x U(1) model. Their unsuccessful results can be explained by the violation of the constraint given by Eq. (37), and are not an indication of failure of the effective potential computation. In conformity with our results, we do not expect to break the symmetry of a realistic SU(2) x U(1) model, without the introduction of new interactions.

ACKNOWLEDGMENT

I am grateful to B. Machet for useful discussions and to Dr. M. Suzuki for critically reading the manuscript. This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under contract DE-AC03-76SF00098.

APPENDIX A

THE EFFECTIVE POTENTIAL FOR FINITE QED

It is interesting to study the chiral breaking in the Abelian case, using the effective potential for composite operators. We are going to analyze the Baker, Willey and Johnson finite QED,[20] and see if it has a stable potential and can generate fermion condensation.

The asymptotic fermionic self-energy in finite QED is given by

$$\Sigma_I(p^2) \underset{-p^2\to\infty}{\sim} \mu \left(\frac{-p^2}{\mu^2}\right)^{-\epsilon} , \qquad (A\text{-}1)$$

where

$$\epsilon = \frac{3\alpha}{4\pi} ,$$

and $\alpha = e^2/4\pi$.

The terms which contribute to the effective potential are

$$\Omega = \frac{i}{2} \mathrm{Tr}\, \Sigma_I S_0 \Sigma_I S_0 + i\, \mathrm{Tr}\, \ln(1 - \Sigma_I S_0)$$
$$+ \frac{i}{2} \mathrm{Tr}\, S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma D_0 , \qquad (A\text{-}2)$$

which correspond to the fermionic contributions in the expression (B-17) of Appendix B, and to some of the last fermionic diagrams in Fig. 2. The computation of the effective potential is straightforward, and the result is

$$16\pi^2\Omega = \frac{\phi^4}{4\epsilon} - \mu^4\left(1+\frac{\phi}{\mu}\right)^4 \ln\left(1+\frac{\phi}{\mu}\right)^2 + 7\phi^2\mu^2 \quad , \qquad (A\text{-}3)$$

where we used $\Sigma_I \sim \phi(-p^2/\mu^2)^{-\epsilon}$ and assumed the existence of a mass μ in the theory.

As we did in Section 3, we expand the logarithmic term in (A-3) (even if this is not a good procedure for many values of ϵ), which simplifies our analysis, and obtain

$$16\pi^2\mu^{-4}\Omega = -\left(\frac{25}{6} - \frac{1}{4\epsilon}\right)\left(\frac{\phi}{\mu}\right)^4 + \frac{1}{15}\left(\frac{\phi}{\mu}\right)^6 + \frac{1}{140}\left(\frac{\phi}{\mu}\right)^8 + \dots \qquad (A\text{-}4)$$

We can also calculate the kinetic term which is given by (Fig. (3)).

$$\Omega_{kin} = \frac{1}{2}\int d^4x \, \frac{(\partial_\mu\phi)^2}{8\pi^2\epsilon} \quad , \qquad (A\text{-}5)$$

with a physical effective field which can be defined as

$$\Phi^2 = \frac{1}{8\pi^2\epsilon}\,\phi^2 \quad . \qquad (A\text{-}6)$$

From (A-4) we can see that the theory is stable for small α $(e^2/4\pi)$, and that finite QED can generate condensation, but this will happen only for $\alpha \gtrsim 1/4$, where QED probably has already lost its meaning, since its coupling constant reaches these values at scales, where we cannot neglect the gravitational effects or a possible unification with other interactions.

From these results we do not expect any possibility to use a vectorial Abelian theory to generate masses through condensation in an Euclidean space-time.

APPENDIX B

DETERMINATION OF Ω

In this appendix we give a brief description of how to determine the analytical expression for Ω, which gives the result in Eq. (13).

The effective potential is given by Eq. (3) which we repeat here for convenience

$$
\begin{aligned}
V(S,D,G) = & -i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left(\ln S_o^{-1}S - S_o^{-1}S + 1\right) \\
& - i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left(\ln G_o^{-1}G - G_o^{-1}G + 1\right) \\
& + \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left(\ln D_o^{-1}D - D_o^{-1}D + 1\right) \\
& + V_2(S,D,G)\quad ,
\end{aligned}
\qquad (B\text{-}1)
$$

where S,D, G (S_o, D_o, G_o) are the complete (bare) propagators of fermions, gauge bosons and ghosts, and V_2(S,D,G) is the sum of all two particle irreducible vacuum diagrams, and can be written as

$$
\begin{aligned}
iV_2(S,D,G) = & -\frac{1}{2}\int \frac{d^4p\,d^4k}{(2\pi)^8}\,\mathrm{Tr}\,\Gamma^{\mu a}(k,p)S(p)\Gamma^{\nu b}(k,p)S(p+k)D^{ab}_{\mu\nu}(k) \\
& - \frac{1}{2}\int \frac{d^4p\,d^4k}{(2\pi)^8}\,\mathrm{Tr}\,F^{abc}_{\mu}(k,p)G^{bd}(p)F^{efd}_{\nu}(k,p)G^{fc}(p+k)D^{\mu\nu ae}(k) \\
& + \frac{1}{6}\int \frac{d^4p\,d^4k}{(2\pi)^8}\,\mathrm{Tr}\,T^{\mu\nu\sigma}_{abc}D^{cd}_{\mu\alpha}(p)T^{\alpha\beta\delta}_{def}D^{ae}_{\nu\beta}(k)D^{bf}_{\sigma\delta}(p+k) \\
& + \frac{1}{8}\int \frac{d^4p\,d^4k}{(2\pi)^8}\,\mathrm{Tr}\,Q^{abcd}_{\alpha\beta\gamma\delta}(k,p)D^{\alpha\beta ad}(k)D^{\gamma\delta cd}(p)\quad .
\end{aligned}
\qquad (B\text{-}2)
$$

In (B-2) Γ^a_μ, F^{abc}_μ, $T^{\mu\nu\sigma}_{abc}$ and $Q^{abcd}_{\alpha\beta\gamma\delta}$ are respectively the proper vertex of fermions, ghosts, triple and quartic gauge boson coupling. Eq.(B-2) is drawn in Fig. 1.

For the sake of simplicity we will write Eq. (B-1) (with (B-2)) removing all indices and integrations

$$
\begin{aligned}
V(S,D,G) = & -i\,\mathrm{Tr}\left(\ln S_o^{-1}S - S_o^{-1}S + 1\right) \\
& - i\,\mathrm{Tr}\left(\ln G_o^{-1}G - G_o^{-1}G + 1\right) + \frac{i}{2}\mathrm{Tr}\left(\ln D_o^{-1}D - D_o^{-1}D + 1\right) \\
& + \frac{i}{2}\mathrm{Tr}\left(\Gamma S\Gamma S D\right) + \frac{i}{2}\mathrm{Tr}\left(F G F G D\right) \\
& - \frac{i}{6}\mathrm{Tr}\left(TDTDD\right) - \frac{i}{8}\mathrm{Tr}\left(QDD\right)\quad .
\end{aligned}
\qquad (B\text{-}3)
$$

From (B-3), and remembering that we want to compute the vacuum energy density defined in Eq. (5):

$$
\Omega = V(S,D,G) - V(S_m, D_m, G_m)\quad , \qquad (B\text{-}4)
$$

and also making use of the definitions

$$
S_{om}^{-1} = S_m^{-1} + \Sigma_m \qquad (B\text{-}5a)
$$

$$
G_{om}^{-1} = G_m^{-1} + \Sigma_{Gm} \qquad (B\text{-}5b)
$$

$$
D_{om}^{-1} = D_m^{-1} + \Pi_m\quad , \qquad (B\text{-}5c)
$$

we arrive at

$$\Omega = -i\,\mathrm{Tr}\left(\ln S_m^{-1} S - S_m^{-1} S + 1\right) - i\,\mathrm{Tr}\left(\ln \mathcal{G}_m^{-1} \mathcal{G} - \mathcal{G}_m^{-1} \mathcal{G} + 1\right)$$
$$+ \frac{i}{2}\,\mathrm{Tr}\left(\ln D_m^{-1} D - D_m^{-1} D + 1\right) + i\,\mathrm{Tr}\,\Sigma_m (S - S_m)$$
$$+ i\,\mathrm{Tr}\,\Sigma_{\mathcal{G}m}(\mathcal{G} - \mathcal{G}_m) - \frac{i}{2}\,\mathrm{Tr}\,\Pi_m (D - D_m) \qquad \text{(B-6)}$$
$$+ \frac{i}{2}\,\mathrm{Tr}\left(\Gamma S \Gamma S D - \Gamma S_m \Gamma S_m D_m\right) + \frac{i}{2}\,\mathrm{Tr}\left(F \mathcal{G} F \mathcal{G} D - F \mathcal{G}_m F \mathcal{G}_m D_m\right)$$
$$- \frac{i}{6}\,\mathrm{Tr}\left(TDTDD - TD_m T D_m D_m\right) - \frac{i}{8}\,\mathrm{Tr}\left(QDD - QD_m D_m\right),$$

where the Green functions labeled by n are those of the normal theory, i.e., with no symmetry breaking.

We can simplify (B-6), using the definitions of the fermionic self-energy and vacuum polarization

$$\Sigma_m = -\Gamma S_m \Gamma D_m \qquad \text{(B-7a)}$$

$$\Pi_m = \Gamma S_m \Gamma S_m , \qquad \text{(B-7b)}$$

and assuming that we do not have poles in the Schwinger-Dyson equation for the Fadeev-Popov ghosts ($\Sigma_{Gn} = 0$). And also with the following approximations: $S_n = S_o$ and $D_n = D_o$ we arrive at

$$\Omega = -i\,\mathrm{Tr}\left(\ln S_o^{-1} S - S_o^{-1} S + 1\right) + \frac{i}{2}\,\mathrm{Tr}\left(\ln D_o^{-1} D - D_o^{-1} D + 1\right)$$
$$+ \frac{i}{2}\,\mathrm{Tr}\left(F \mathcal{G}_o F \mathcal{G}_o (D - D_o)\right) - \frac{i}{8}\,\mathrm{Tr}\left(QDD - QD_o D_o\right)$$
$$- \frac{i}{6}\,\mathrm{Tr}\left(TDTDD - TD_o T D_o D_o\right) + \frac{i}{2}\,\mathrm{Tr}\left(\Gamma S \Gamma S - \Gamma S_o \Gamma S_o\right)(D - D_o)$$
$$+ \frac{i}{2}\,\mathrm{Tr}\,\Gamma (S - S_o) \Gamma (S - S_o) D_o \qquad \text{(B-8)}$$

If we multiply (B-5c) by D_o, the resulting equation can be used to rewrite the fourth term of (B-8)

$$-\frac{i}{8}\,\mathrm{Tr}\left(QDD - QD_o D_o\right) = -\frac{i}{4}\,\mathrm{Tr}\,QD\Pi D_o D$$
$$+ \frac{i}{8}\,\mathrm{Tr}\,QD\Pi D_o D \Pi D_o \quad . \qquad \text{(B-9)}$$

The fifth term in (B-8) can also be rewritten in the following form

$$-\frac{i}{6}\,\mathrm{Tr}\left(TDTDD - TD_o T D_o D_o\right) = \qquad \text{(B-10)}$$
$$-\frac{i}{2}\,\mathrm{Tr}\,T(D - D_o) T (D - D_o)(D - D_o) - \frac{i}{2}\,\mathrm{Tr}\,TDT(D - D_o) D_o ,$$

Making use of Eqs. (B-9) and (B-10) we transform (B-8) into

$$\Omega = -i\,\mathrm{Tr}\left(\ln S_0^{-1}S - S_0^{-1}S + 1\right) + \frac{i}{2}\mathrm{Tr}\left(\ln D_0^{-1}D - D_0^{-1}D + 1\right)$$
$$+ \frac{i}{2}\mathrm{Tr}\left(F G_0 F G_0 (D - D_0)\right) - \frac{i}{4}\mathrm{Tr}\; Q D \Pi D_0 D$$
$$+ \frac{i}{8}\mathrm{Tr}\; Q D \Pi D_0 D \Pi D_0 + \frac{i}{2}\mathrm{Tr}\left(\Gamma S \Gamma S - \Gamma S_0 \Gamma S_0\right)(D - D_0)$$
$$+ \frac{i}{2}\mathrm{Tr}\; \Gamma(S - S_0)\Gamma(S - S_0) D_0 - \frac{i}{6}\mathrm{Tr}\, T(D - D_0) T(D - D_0)(D - D_0)$$
$$+ \frac{i}{2}\mathrm{Tr}\; T D T (D - D_0) D_0 \quad . \qquad \text{(B-11)}$$

We can expand the fermionic propagator as a function of its self-energy, which can be divided in three pieces, given by

$$\Sigma = \Sigma_m + \Sigma_s + \Sigma_I \quad , \qquad \text{(B-12)}$$

Σ_n is the self-energy part without symmetry breaking, and is given by

$$\Sigma_m = -\Gamma S \Gamma D \quad , \qquad \text{(B-13)}$$

Σ_n is drawn in Fig. 4(a). Σ_s in (B-12) is the part that has insertions, but with a symmetric result

$$\Sigma_s = \Sigma_s^{(1)} + \Sigma_s^{(2)} \quad , \qquad \text{(B-14)}$$

$\Sigma_s^{(1)}$ and $\Sigma_s^{(2)}$ are drawn in Fig. 4(b). Finally Σ_I in (B-12) is the asymmetric part of the self-energy

$$\Sigma_I = -\Gamma S \Sigma_I S \Gamma D \quad , \qquad \text{(B-15)}$$

which corresponds to Fig. 5.

Now it is possible to expand the terms $-i\,\mathrm{Tr}\,\ln S_0^{-1}S$ and $i/2\,\mathrm{Tr}\,\ln D_0^{-1}D$ contained in (B-11). The first will be expanded in powers of Σ_I (notice that Σ_s falls of faster then Σ_I, and Σ_n is swept in the subtraction $\Omega - \Omega_n$), the second is expanded in powers of Π. With these expansions, and some algebraic manipulation we get

$$\Omega = \frac{i}{2}\mathrm{Tr}\,\ln(1 - \Pi D_0) - \frac{i}{2}\mathrm{Tr}\,\frac{\Pi D_0}{(1 - \Pi D_0)}$$
$$+ \frac{i}{2}\mathrm{Tr}\left(F G_0 F G_0 D \Pi D_0\right) - \frac{i}{4}\mathrm{Tr}\; Q D \Pi D_0 D$$
$$+ \frac{i}{8}\mathrm{Tr}\; Q D \Pi D_0 D \Pi D_0 - \frac{i}{2}\mathrm{Tr}\; T D T D D_0 \Pi D_0$$
$$- \frac{i}{6}\mathrm{Tr}\; T D \Pi D_0 T D \Pi D_0 D \Pi D_0 \qquad \text{(B-16)}$$
$$+ \frac{i}{2}\mathrm{Tr}\; \Sigma_I S_0 \Sigma_I S_0 + i\,\mathrm{Tr}\,\ln(1 - \Sigma_I S_0)$$
$$+ \frac{i}{2}\mathrm{Tr}\; S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma D_0$$
$$+ \frac{i}{2}\mathrm{Tr}\left(\Gamma S \Gamma S - \Gamma S_0 \Gamma S_0\right)(D - D_0) \quad .$$

Rewriting and expanding the last term in (B-16) in powers of Σ_I we finally arrive at

$$\Omega = -\frac{i}{2}\,\mathrm{Tr}\ln\left(1-\Pi D_0\right) - \frac{i}{2}\,\mathrm{Tr}\,\frac{\Pi D_0}{(1-\Pi D_0)}$$
$$+\frac{i}{2}\,\mathrm{Tr}\left(\mathcal{F} G_0 \mathcal{F} G_0 D \Pi D_0\right) - \frac{i}{4}\,\mathrm{Tr}\;Q D \Pi D_0 D$$
$$+\frac{i}{8}\,\mathrm{Tr}\;Q D \Pi D_0 D \Pi D_0 - \frac{i}{2}\,\mathrm{Tr}\;T D T D\, D_0 \Pi D_0$$
$$-\frac{i}{6}\,\mathrm{Tr}\;T D \Pi D_0 T D \Pi D_0 D \Pi D_0 + \frac{i}{2}\,\mathrm{Tr}\;\Sigma_I S_0 \Sigma_I S_0$$
$$+\,i\,\mathrm{Tr}\ln\left(1-\Sigma_I S_0\right) + \frac{i}{2}\,\mathrm{Tr}\;S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma D_0$$
$$+\frac{i}{2}\,\mathrm{Tr}\;S_0 \Sigma_I S_0 \Gamma S_0 \Sigma_I S_0 \Gamma D \Pi D_0 \qquad \text{(B-17)}$$
$$+\,i\,\mathrm{Tr}\;S_0 \Gamma S_0 \Sigma_I S_0 \Sigma_I S_0 \Gamma D \Pi D_0\,.$$

Eq. (B-17) contains all diagrams that contribute to the effective potential, and are described in Fig. 2.

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FOOTNOTES

F1. c is given by the solution of $\sum_\alpha t^\alpha P^i \ell^\alpha = c\, P^i$ where t^α and ℓ^α are respectively the generators of the representations R_ψ and R_χ, and P^i is a projection operator which select a particular representation R_ϕ from the product $R_\psi \times R_\chi$. c needs to be positive as a necessary condition for the existence of solutions (7a) and (7b).

F2. The pion electromagnetic form factor decreases at least as a power of logarithm for $\Sigma_I(p^2)$. Unfortunately there is little experimental knowledge of $F_\pi(p^2)$ to compare the asymptotic predictions. We hope to discuss this aspect in a future paper about the phenomenological consequences of $\Sigma_I(p^2)$.

F3. The absence of a ϕ^2 term in Eq. (28) is explained in Ref. (11).

F4. Some of the results presented here are in disagreement with those of Ref. (21), where the author says that $c < 0$ for fermions in the fundamental representation.

F5. We are supposing that ϕ is in a vector representation.

Group, Representation, # familes	Constraint on # families	MAC	α_c ef. pot.	$\alpha_c \sim 1/c$
SU(2)				
$\underline{2}$, $N_f = 6$	$N_f \geq 5$	$\underline{1}$	0.61	1.33
$\underline{2}$, $N_f = 8$	$N_f \geq 5$	$\underline{1}$	0.49	1.33
$\underline{3}$, $N_f = 1$	$N_f \geq 1$	$\underline{1}$	0.18	0.50
$\underline{4}$, $N_f = 1$	$N_f = 1$	$\underline{1}$	0.02	0.27
SU(5)				
$\underline{\bar{5}} + \underline{10}$, $N_f = 2$	$N_f \geq 2$	$\underline{\bar{5}}$	0.19	0.42
$\underline{\bar{5}} + \underline{10}$, $N_f = 3$	$N_f \geq 2$	$\underline{\bar{5}}$	0.24	0.42
SU(7)				
$\underline{\bar{7}} + \underline{\bar{7}} + \underline{35}$, $N_f = 1$	$N_f \geq 1$	$\underline{7}$	0.13	0.19

Table 1. For a given gauge group, representation and number of families (N_f), we show the constraint on N_f (Eq. (37)), the MAC and the critical coupling constants calculated from the effective potential and from the hypothesis that $\alpha c \sim 1$.

FIGURE CAPTIONS

Fig. 1. Two particle irreducible diagrams contributing to $V_2(S,D,G)$.

Fig. 2. Diagrams contributing to the energy density Ω.

Fig. 3. Diagrams contributing to the kinetic term in the effective action.

Fig. 4. Diagrams contributing to the self-energy. In Figure 4(a) we have the completely symmetric contribution to Σ, and in Fig. 4(b) we have symmetric contributions resulting from a combination of asymmetric insertions.

Fig. 5. Asymmetric part of fermion self-energy.

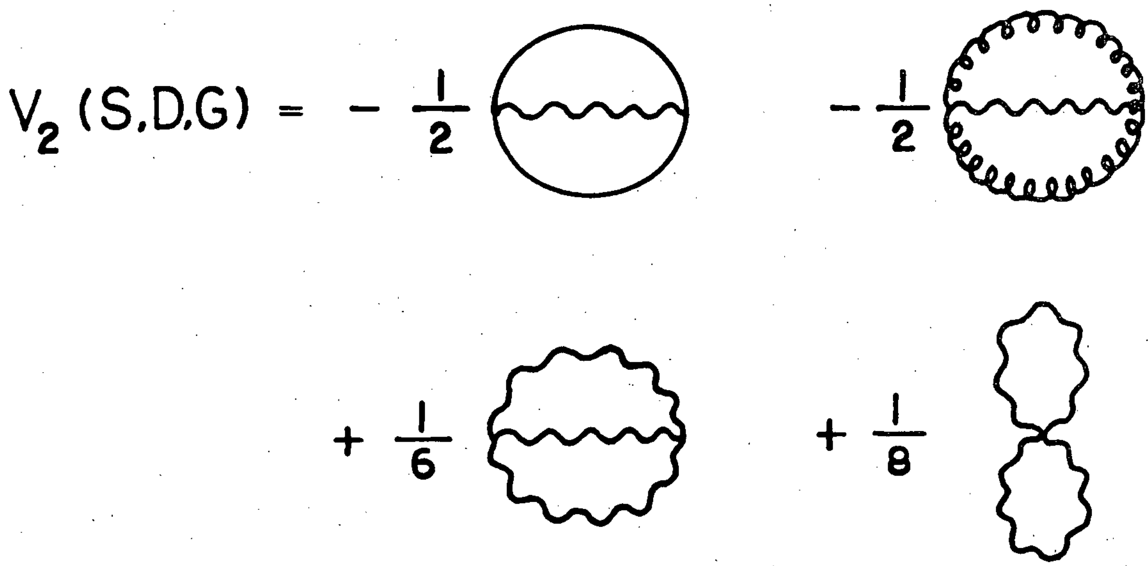


FIGURE 1

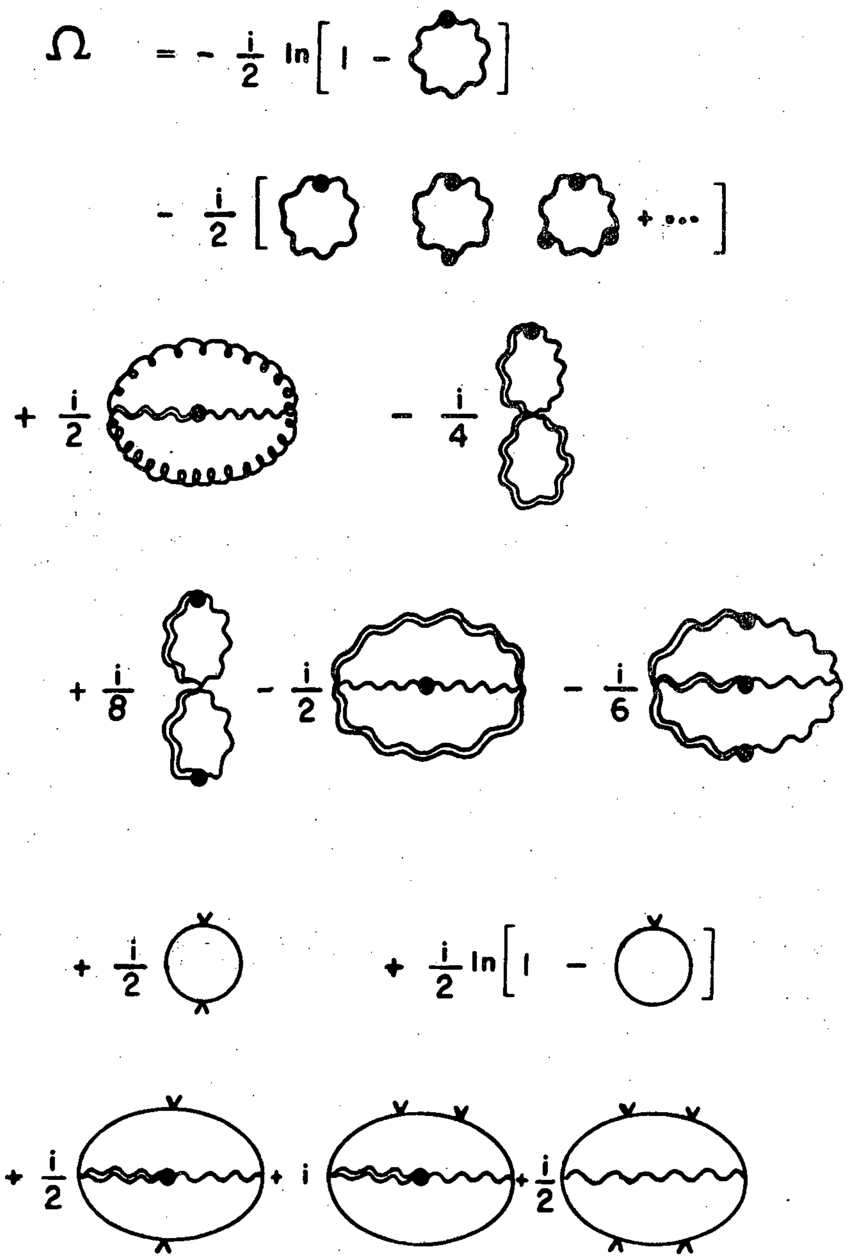


FIGURE 2

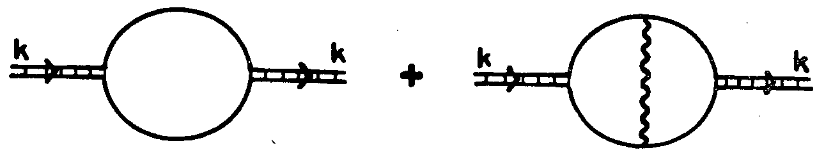


FIGURE 3

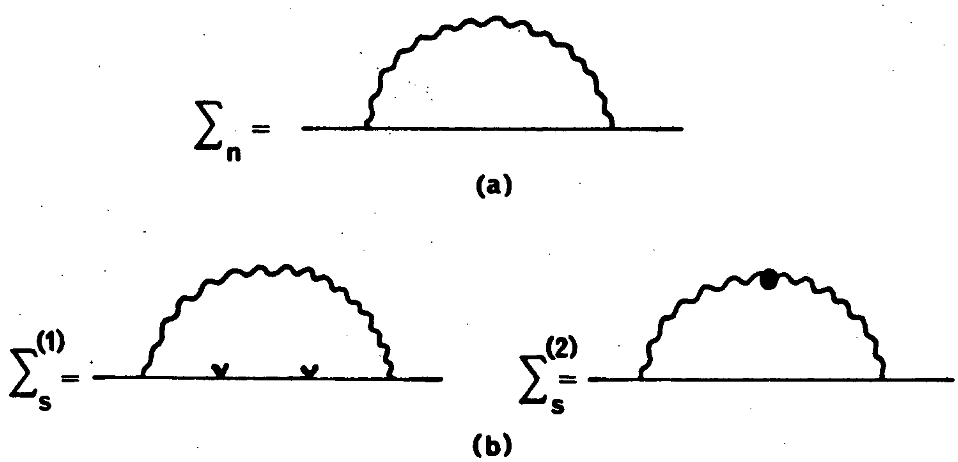


FIGURE 4

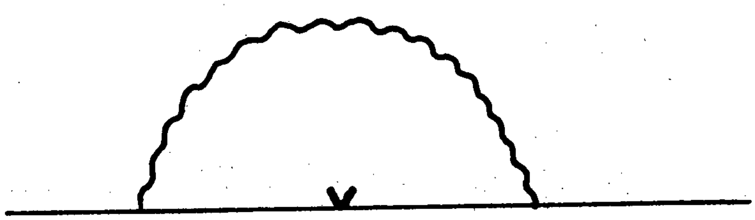
$\Sigma_1 =$ 

FIGURE 5

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