Facilitating Low-Achieving Students' Diagram Use in Algebraic Story Problems

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Abstract

Recent research indicates that when solving algebraic story problems, adding a diagram is beneficial for seventh and eighth grade students, however, sixth graders—particularly low-achieving ones—do not benefit from the diagrams. In the present study, we further investigate the diagrammatic advantage in low-achieving pre-algebra students and examine whether and how picture algebra instruction improves diagram comprehension and use in the target population. Results replicate the lack of diagrammatic advantage in this population for two types of diagrams. Picture algebra instruction on mapping information in word problems to one type of diagrams yields improvement in both diagrammatic forms, but not story problems without diagrams; a diagrammatic advantage emerges following this instruction. Though low-achieving students may fail to use diagrammatic representations to their benefit when solving word problems, instruction on the use of one specific form may be sufficient to facilitate a more general diagrammatic advantage.

Keywords: Multiple representations; Algebraic problem solving; mathematics education

Introduction

Problem representation is a critical issue in education, as the way that information is conveyed to students can have a great impact on the degree to which they learn. (e.g., Cummins, Kintsch, Reusser, & Weimer, 1988). In the domain of mathematics, use of more grounded representations rather than more abstract ones (e.g., verbal descriptions of situations as opposed to equations), has been found to be useful for presenting simple algebra problems (Koedinger, Alibali, & Nathan, 2008); this practice may be especially useful for making problems concrete when students are early in their transition to algebraic thinking and are not yet capable of the abstract thinking necessary to comprehend equations (Koedinger & Nathan, 2004).

Another way that instructors often attempt to make problems or situations more concrete is to include external representations. External representations are an important part of mathematics education (Seeger, 1998), and are intended to increase understanding of mathematical concepts by allowing children to build relations between mathematical ideas (Hiebert & Carpenter, 1992). Pictorial representations, such as diagrams, charts, graphs, and tables, are often used in math classrooms because they are thought to be useful for helping students communicate and reason about mathematical concepts (Greeno & Hall, 1997), and the National Council of Teachers of Mathematics recommends that teachers include multiple forms of representations when teaching mathematical concepts (NCTM, 2000).

Indeed, evidence abounds on the cognitive benefits of external representations, including diagrams. For example, students learn better when a diagram is added to text than if they are studying the text alone (Mayer, 1989; 2009), likely because learners are able to build two mental representations of multimedia material, a verbal representation and a visual one, and build connections between them (Mayer, 2005). Diagrams may also be beneficial because the spatial organization and grouping of related components that are characteristic of diagrams better enables users to search, recognize relevant pieces in, and draw inferences about the represented information (Larkin & Simon, 1987). Diagrams may also be beneficial because they promote users to engage in self-explanation, which is in itself beneficial for learning (Ainsworth & Loizou, 2003).

Are diagrams universally helpful?

Despite the intention of these tools to help students succeed, the use of diagrams is not always beneficial. Larkin and Simon (1987) posited that diagrammatic representations of any sort are only useful if they are constructed in a way that groups information and facilitates inference in a better way than is possible with text. Further, even a well-constructed diagram will not be useful unless the user knows the computational processes that are necessary for taking advantage of them. Ainsworth (2006) also cautions that the usefulness of diagrams is influenced by characteristics of the user such as expertise in the content domain and familiarity with the structure and components of the representation, as well as characteristics of the diagram and interactions between the two.

Consistent with these assertions, recent research on using diagrams with algebraic story problems suggests that not all students benefit from the addition of diagrams. Booth & Koedinger (2007) found that older and higher-achieving middle school students do benefit from the diagrams as intended—they solve more diagram problems correctly than problems without a diagram. However, low-achieving students do not benefit from the diagrams; they perform better on story problems that do not have accompanying diagrams. In fact, the diagrams may actually hurt their performance—they perform just as poorly on the diagram problems as they do when solving the problems as symbolic equations.

For students that do experience a diagrammatic advantage, results suggested that the benefit comes from protecting those students from making common conceptual errors in interpreting the problem (Booth & Koedinger, 2007). For example, for a problem where students are given a sale price and asked to determine the original price if the buyer purchased it at 1/5 off, having a diagram showing the pieces of the equation makes it less likely that higher-achieving students solve the problem by multiplying the original price by 5, which is a common strategy for students solving the problem in story format. Of course, this benefit can only be realized when students are able to effectively use diagrams to understand the problem.

For lower-achieving students, there appear to be two barriers to successful diagram use. One is that they are less likely than higher-achieving peers to attempt diagram problems. This is perhaps unsurprising, as low-ability students generally perceive problems to be more difficult than high or average ability students, and are more likely to shut down and not attempt the problems as a result (Ericcson & Simon, 1980). The real or perceived need to attend to more than one representation at a time causes split attention demands on working memory (Chandler & Sweller, 1991), making the problem seem overwhelming, and these students' limited diagram comprehension skills preclude the realization that the problem could be solved by simply ignoring the diagram and working from the story alone.

The other barrier to success with diagrams was the failure to glean a correct conceptual understanding of the problem from looking at the diagram. Young and low-achieving students were *more* likely to make conceptual errors in problems that included diagrams than ones with stories alone. This is likely due to either misinterpretation of the diagram itself or, more crucially, failure to accurately map the story problem to the diagram. How can we help low-achieving students to better comprehend the diagrams?

Using instruction to improve diagram use

Research from the fields of cognitive development and mathematics education suggests that effective instruction on external representations is necessary for correct student use (Sowell, 1989; Fueyo & Bushell, 1998; Uttal, Scudder, & DeLoache, 1997). Brief instruction on a particular visual representation may not suffice (Rittle-Johnson & Koedinger, 2001), but more involved representation-specific instruction could consume a significant amount of precious classroom time, and may not transfer well to other representations.

An alternative to instruction on utilizing particular types of diagrams is having students construct diagrams to represent story problems themselves. Middle school students can use self-created representations to successfully solve algebraic story problems they wouldn't ordinarily be able to solve (Koedinger & Terao, 2002), and both constructing diagrams from scratch or filling in partially completed diagrams have yielded increases in student learning in a variety of domains (Lewis & Mayer, 1987; see Van Meter & Garner, 2005 for a review). Constructing diagrams has the potential to help students learn to coordinate and integrate text with visual representations (Ainsworth, 2006; Easterday, Aleven, & Scheines, 2007).

In the present study, we directly target low-achieving pre-algebra students to determine whether guided experience constructing one type of diagram from simple story problems facilitates broader use of diagrams for more complex problems. We also aim to investigate the mechanism underlying any resulting benefit by testing students on more than one type of diagram. If improvement is due to increased familiarity with the type of diagram used during instruction, benefits should manifest as increased willingness to attempt familiar-looking problems and improved performance on those items. However, if, as intended, the instruction provides students with the necessary tools for mapping between story problems and diagrams, benefits should be more likely to transfer to the other type of diagram as well.

Methods

Participants

Participating in this study were four classrooms of non-honors Pre-Algebra students (N = 73 eighth grade students; typically age 13) from a school in which only 8.5% of students reach the required state level of math proficiency. Eighty-nine percent of students at the participating school were economically disadvantaged; the ethnic breakdown of the school was approximately 95% Hispanic, 5% African-American, and < 1% Caucasian or other. Three additional students participated in the study, but were excluded because they were not given the correct version of the posttest.

All four classrooms used the Bridge to Algebra Cognitive Tutor curriculum. The Cognitive Tutor is a computer-based intelligent tutoring system which provides on-demand, step-specific help at any point in the problem-solving process and feedback on errors (Koedinger, Anderson, Hadley, & Mark, 1997).

Procedure

Prior to beginning the first Tutor unit in the curriculum (approximately three weeks after the beginning of the school year), participants completed a written pretest on which they were asked to solve algebraic story problems in three presentation formats: story alone, story with a vertical diagram, or story with a horizontal diagram. The test included six problem situations, each representing one of two underlying algebraic equations: 1) $ax + b = c$, and 2) $x + (x + a) + (x + b) = c$. Both equation types were represented in each of the three presentation formats on every test (See Figure 1 for examples of each presentation format for the second equation). There were three counterbalanced forms of the test, such that each problem

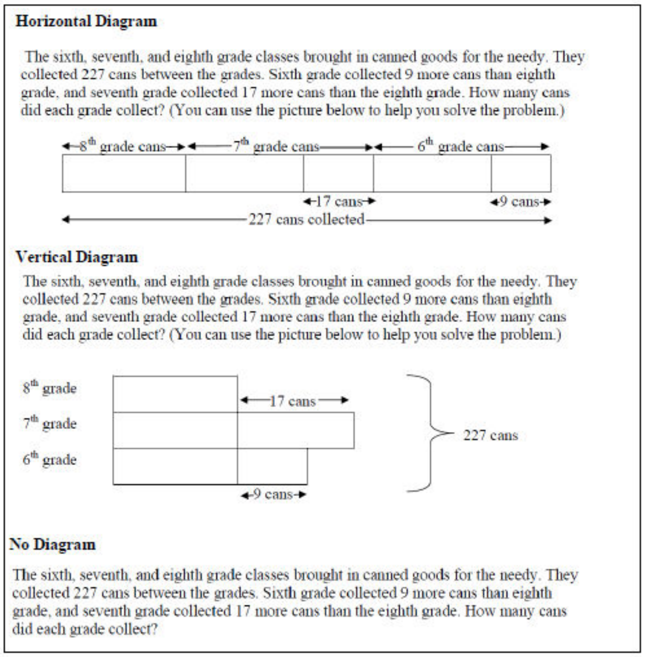

Figure 1: Sample problem in each presentation format

situation appeared in each of the three presentation formats on one of the three test versions. After completing the pretest, students began the interactive Tutor unit on Picture Algebra, in which they created, labeled, and used vertical diagrams to solve simple story problems using multiplication, addition, or subtraction The vertical training problems were simpler than those included in the test, in that they required the student to manipulate only two components compared with three or more components in test problems; thus, all test problems were transfer problems (see Figure 2 for examples of a problem in the Picture Algebra unit). Each student completed the unit at his or her own pace. As each student completed the unit, he or she was given a posttest by the classroom teacher; students were given the same version of the test that they had taken at pretest.

Results

Pretest and posttest scores for each of the three presentation types can be found in Figure 3. A 3 (presentation format: vertical diagram, horizontal diagram,

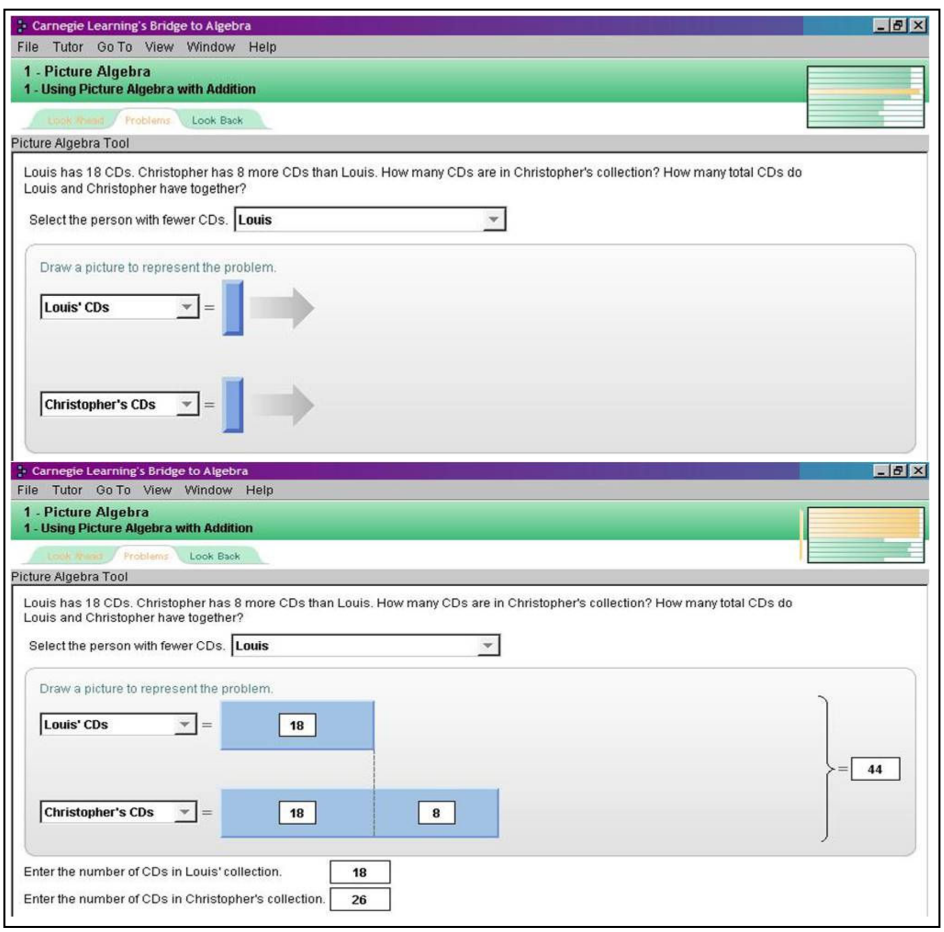

Figure 2: Screenshots from a Picture Algebra problem in the Bridge to Algebra Tutor. Students stretch the blocks out to represent the number of CDs owned by Louis and Christopher. Christopher's CDs are represented using the same sized box as for Louis and additional length to represent the 8 extra CDs.

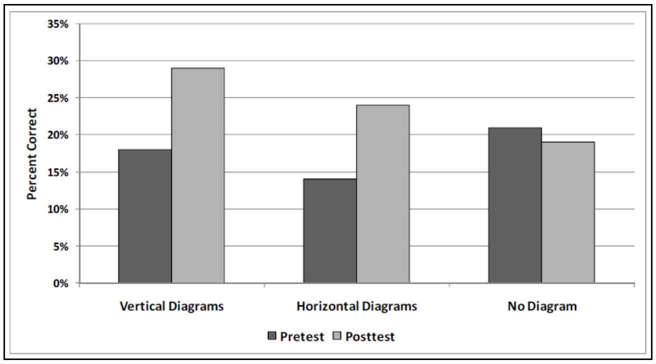

Figure 3: Percent correct at pretest and posttest for each presentation format.

no diagram) x 2 (test time: pretest vs. posttest) repeated measures ANOVA on the percent of problems answered correctly yielded a main effect of test time, $F(1, 72) = 5.10$, $p < .05$, $\eta_p^2 = .07$. There was no main effect of presentation format $F(2, 144) = 1.56$, *ns.* However, the interaction between presentation format and test time was significant, $F(2, 144) = 5.25$, $p < .01$, $\eta_p^2 = .07$. To interpret this interaction, we conducted follow-up repeated measures ANOVAs on presentation format, separately for pretest and posttest scores. No significant differences among presentation types were found at pretest $F(2, 144) = 1.91$, *ns.* In contrast, at posttest, a main effect of presentation format was found, $F(2, 144) = 3.95$, $p < .05$, $\eta_p^2 = .05$. Follow-up pairwise comparisons with Bonferroni correction indicated that students scored higher on problems with vertical diagrams than those with no diagrams ($p < .05$). No differences were found between scores on horizontal diagrams problems compared with either of the other presentation formats.

Students solved more vertical problems correctly on the posttest after receiving training on creating and using simpler versions of those diagrams ($t(72) = 2.83$, $p< .01$). Students also improved on horizontal diagrams problems after vertical diagram training ($t(72) = 2.80$, $p< .01$), but no improvement was found on problems that did not contain diagrams ($t(72) < 1$, *ns*). A repeated measures ANOVA on the amount of improvement shown yielded a main effect of presentation type, $F(2, 144) = 5.7$, $p < .05$, $\eta_p^2 = .07$. Follow-up pairwise comparisons with Bonferroni correction indicated that students improved more on problems with vertical diagrams (11%) and horizontal diagrams (10%) than those with no diagrams (both p's < .05). No difference was found between improvement on vertical and horizontal problems.

Error Analysis: The nature of the improvement

To investigate the source of this improvement in scores on the diagrams problems, we conducted a qualitative analysis of the types of errors made by students while solving each type of problem on the pretest and posttest. Four pretest and thirteen posttest problem attempts were found in which students drew diagrams to help them solve the no diagrams problems; these incidences were thus excluded from the subsequent response and error analysis.

One possible source of improvement was that exposure to the diagrams could have made students more comfortable with, and thus more likely to attempt, diagrams problems on the posttest, leading to a higher possible number of diagrams problems answered correctly. To examine this hypothesis, we coded whether students attempted to solve each problem or if they failed to respond to it. We then computed the percentage of each type of problem that was attempted at pretest and posttest. As can be seen in Figure 4, students attempted more problems of each of the three formats at posttest compared with the pretest. This suggests that a higher response rate for diagrams problems is not a viable explanation for the improvement.

A second, more plausible hypothesis was that students better understood the mapping between the diagrams and the stories as a result of instruction. Given that their experience with the Picture Algebra unit trained them to build components of diagrams to represent the information in a story problem, this hypothesis seemed plausible. This improved understanding should lead students to make fewer errors in which they demonstrate failure to make sense of the information in the problem. To test this, we coded student responses in terms of whether they were correct, contained an arithmetic error (e.g., adding 4 + 6 and getting 9), or contained a conceptual error—one that indicated a misunderstanding of the role of the numbers in the problem (e.g., for the problem pictured in Figure 1, solving the problem as if 7th graders collected 17 fewer cans than 8th graders, instead of 17 more cans). In previous work, adding diagrams to story problems was shown to prevent older, high-achieving students from making common conceptual errors when solving the problems (Booth & Koedinger, 2007), but younger and lower-achieving students did not receive this benefit. Results from the present study indicate that after instruction it appears that fewer horizontal and vertical diagrams problem attempts contained conceptual errors than did at pretest whereas no reduction was apparent for problems without diagrams (see Figure 5).

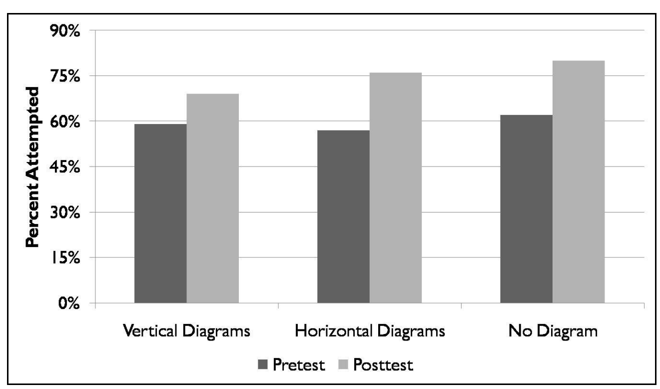

Figure 4: Percent of problems attempted at pretest and posttest for each presentation format.

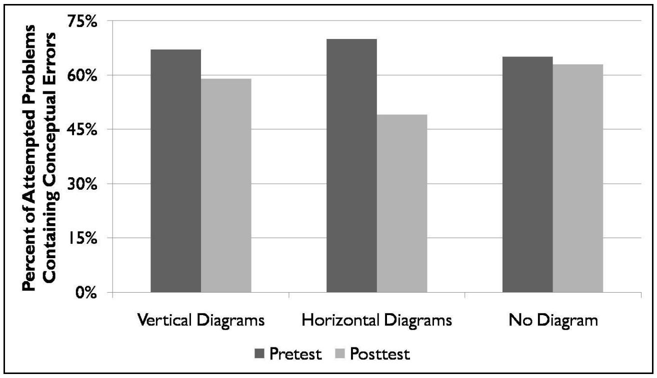

Figure 5: Percent of pretest and posttest problem attempts for each presentation format containing a conceptual error

Discussion

Results from the present study replicated the previous finding that diagrams are not inherently beneficial for solving algebraic word problems. At pretest, no diagrammatic advantage was found for low-achieving pre-algebra students. After instruction, however, the same students experienced a diagrammatic advantage, and there was evidence of transfer of instruction benefits to the non-instructed diagram format. Students made fewer conceptual errors with both types of diagrams after instruction, suggesting that increasing students' skill at mapping between story problems and supplemental diagrams can afford low-achieving students the same benefits as those enjoyed by their higher-achieving peers. Interestingly, no improvement was found for the no diagram condition, suggesting that students' general word problem solving abilities did not increase. Presumably, if students had drawn diagrams to help them solve those problems, as they did in their training, they would have had greater success.

One specific mechanism of the diagrammatic advantage is that it has been shown to increase the likelihood that students will achieve a conceptually sound understanding of a problem, and avoid common conceptually flawed solution paths (Booth & Koedinger, 2007). Consistent with this finding, results from the present study indicated that students reduced the number of conceptual errors made at posttest on transfer problems with diagrams compared to those without diagrams. The likely mechanism by which the diagrammatic advantage emerges is through increased experience coordinating information from two sources, which helps students learn to create appropriate links between the information (e.g., the components of the diagram with the corresponding components of the text). This general ability enables successful mapping between sources in new diagrammatic problems, yielding a sound representation of the overall problem, which leads to fewer critical conceptual mistakes in solution. The process of constructing the diagram facilitated this process by forcing students to make connections explicit; this is consistent with Van Meter & Garner's (2005) assertion that the benefit of diagram construction is that it necessitates integration between text and diagram. The Picture Algebra lesson provided practice opportunities with feedback for students to gain the general mapping ability, which they were then able to apply successfully to the test problems. Further research is needed to determine whether and how developing students naturally acquire this skill, whether through cognitive maturation (and perhaps the development of more formal reasoning skills), through certain types of experiences that become more prevalent as children age, or some combination thereof.

Results from this study suggest that, while low-achieving students have difficulty interpreting diagrams and using them to their benefit when solving problems, it may not be necessary for students to have specific instruction or experience with a given type of diagram in order to use it effectively. Rather, acquiring more general diagram-parsing skills that facilitate mapping between text and any sort of diagram may be more beneficial. The instruction presented in this study did not just increase comprehension of more complex vertical diagrams; it helped cultivate a broader skill which allowed them to also use complex horizontal diagrams to solve the problem. Future research should investigate the nature of broad diagram-parsing skills and determine how best to teach them to help all students benefit from diagrams and other external representations of instructional information in Algebra.

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