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A HYBRID SEMI-ANALYTICAL AND NUMERICAL METHOD FOR MODELING WELLBORE HEAT TRANSMISSION

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ABSTRACT

Fluid flow in geothermal production and injection wells can be strongly affected by heat transfer effects with the formations surrounding the wellbore. Various techniques and approximations to model wellbore heat transmission have been presented in the literature. The objective of the present work is to develop a treatment of conductive heat transfer in the formations surrounding a wellbore that is simple, yet provides good accuracy for transient effects at early time. This is accomplished by adapting the well-known semi-analytical heat transfer method of Vinsome and Westerveld (1980) to the problem of heat transfer to and from a flowing well. The Vinsome-Westerveld method treats heat exchange between a reservoir and adjacent cap and base rocks by means of a hybrid numerical-analytical method, in which temperature distributions in the conductive domain are approximated by simple trial functions, whose parameters are obtained concurrently with the numerical solution for the flow domain. This method can give a very accurate representation of conductive heat transfer even for non-monotonic temperature variations over a broad range of time scales. The only enhancement needed for applying the method to wellbore heat transmission is taking account of the cylindrical geometry around a flowing well, as opposed to the linear flow geometry in cap and base rocks. We describe the generalization of trial functions needed for cylindrical geometry, and present our implementation into the TOUGH2 reservoir simulator. The accuracy of the method is evaluated through application to non-isothermal flow through a pipe.

INTRODUCTION

The components of a hot dry rock (HDR) geothermal reservoir include injection and production wells, and a network of fractures at a depth where sufficiently high temperatures are encountered, nominally around 200 °C or more, to permit electricity generation from the produced fluids at a "reasonable" level of thermodynamic efficiency. The wells not only convey fluid to and from the deep, hot fracture system, but due to their large surface area form an important part of the total heat exchange system. For an 8" well ($r \approx 0.1$ m), surface area is 628.3 m^2 per km depth, so that for a 4 km deep production-injection system, total surface area is more than 5,000 m^2, a number that approaches heat transfer areas of major fractures.

The migration of fluids through a permeable domain coupled with heat transfer to or from adjacent rocks of low permeability is a common problem in reservoir engineering. In addition to heat transmission between a wellbore and surrounding formations (Ramey, 1962; Wu and Pruess, 1990), examples include non-isothermal fluid injection into a permeable layer sandwiched between impermeable formations (Lauwerier, 1955), and non-isothermal injection into fractured reservoirs (Pruess and Bodvarsson, 1984; Pruess and Wu, 1993). In many cases rock permeability may be negligibly small for the time scales considered, and the problem can be reduced to solving a heat conduction equation in the low-permeability domain. This can be easily accomplished by means of finite differences or other space-discretized techniques, but at considerable comput-ational expense. Analytical and semi-analytical treatments have been presented in the literature that use various simplifying assumptions. The classical treatment of Ramey (1962) provides a good approx-imation for the longer-term quasi-steady heat exchange between wellbore fluids and surrounding formations. Wu and Pruess (1990) obtained an analytical solution in Laplace space that accurately represents transient heat transfer effects in layered formations. The goal of the work presented here is to develop a simple treatment for the heat conduction problem around a pipe of cylindrical cross section that can accurately represent transient effects at early times.

THE METHOD OF VINSOME AND WESTERVELD

Vinsome and Westerveld (1980) developed a semi-analytical approach for the problem of non-isothermal fluid injection into a permeable layer that is sandwiched between impermeable base and cap rock. Their method greatly simplifies the heat conduction problem, while providing satisfactory accuracy. Vinsome and Westerveld considered that heat conduction perpendicular to the conductive boundary will be more important than parallel to the boundary. Noting that heat conduction will tend to wipe out sharp temperature differences, they suggested that the temperature profile in the conductive domain may be approximated by means of a simple trial function that contains a few adjustable parameters. More specific-ally, they proposed that the temperature profile in the cap or base rock may be represented by a low-order polynomial with an exponential tail, as follows.

$$T_{lin}(x,t) - T_i \quad = \quad \left(T_f - T_i + px + qx^2\right)\exp(-x/d) \quad (1)$$

Here, $T_{lin}(x, t)$ is the temperature at time t and distance x from the conductive boundary, T_i is the initial temperature in the conductive domain (assumed uniform in the direction perpendicular to the boundary), T_f is the time-varying temperature at the conductive boundary, p and q are time-varying fit parameters, and d is the penetration depth for heat conduction, given by

$$d \quad = \quad \sqrt{\Theta t}/2 \qquad (2)$$

where $\Theta = \lambda/\rho C$ is the thermal diffusivity, λ the thermal conductivity, ρ the density of the medium, and C the specific heat.

In the context of a finite-difference simulation of nonisothermal flow, each grid block at the conductive boundary will have an associated temperature profile in the adjacent impermeable rock given by Equation (1). The parameters p and q are different for different grid blocks and are determined concurrently with the flow simulation from the following physical constraints: (1) temperatures throughout the conductive domain must satisfy a heat conduction (diffusion) equation, and (2) cumulative heat flow across the boundary must equal the change of thermal energy in the conductive domain. Numerous test calculations have shown the Vinsome-Westerveld technique to provide excellent accuracy for conductive heat exchange, even under conditions of non-monotonic temperature variations in the fluid flow domain (Vinsome and Westerveld, 1980; Pruess and Wu, 1993).

EXTENSION TO HEAT CONDUCTION AROUND A WELL

Here we propose to adapt the Vinsome-Westerveld method for the problem of conductive heat transfer in the region surrounding a wellbore. The general concept involves representing the wellbore itself as a 1-D feature that is discretized into grid blocks in the direction of flow, while heat transfer perpendicular to the wellbore is treated by a semi-analytical technique. Depending on depth and prevailing geothermal gradients, each wellbore grid block would have a different initial temperature T_i associated with it. The main difference in comparison to the system investigated by Vinsome and Westerveld is in the geometry for heat transfer, which is linear for their base and cap rock problem, while it is cylindrical in the region surrounding the wellbore. Total rate of heat flow at distance x from the conductive boundary is given by

$$G(x) \quad = \quad -A(x)\lambda\nabla T \qquad (3)$$

where the cross-sectional area for heat transfer is $A(x) = A_0$ = const. for the linear case, while it is $A(x) = 2\pi rh$ in the case of a wellbore segment of length h. Here, $r = r_0 + x$ is the radial distance from the center of the wellbore and r_0 is the wellbore radius. For small x, the leading term in the temperature gradient derived from Eq. (1) involves an exponential tail multiplied by a constant. This will provide a leading term for conductive heat flow, Eq. (3), that in the linear case also involves an exponential tail with a constant coefficient. In the cylindrical case, however, the leading term will involve an exponential tail multiplied by radial distance r, which means that total heat flow would increase with distance from the boundary, which is unphysical. To better account for heat transfer in the radial flow geometry around a wellbore, we therefore propose to use a modified temperature trial function T_{rad} that is defined as follows,

$$T_{rad}(x,t) - T_i \quad = \quad \frac{r_0}{r}\left(T_{lin}(x,t) - T_i\right) \qquad (4)$$

where $x = r - r_0$ measures the distance from the conductive boundary. This form maintains proper limiting behavior $T_{rad} \Longrightarrow T_f$ for $r \Longrightarrow r_0$ and $T_{rad} \Longrightarrow T_i$ for $r \Longrightarrow \infty$, and will provide a leading term proportional to 1/r in ∇T, which will cancel the r-coefficient in the cross-sectional area. For modeling conductive heat exchange around a cylindrical pipe, we have incorporated Eq. (4) in our TOUGH2 code (Pruess et al., 1999). The coefficients p and q in Eq. (4) are determined concurrently with the flow simulation as in the linear case by requiring that (a) temperatures at the conductive boundary satisfy a heat conduction equation,

$$\frac{\partial T_f}{\partial t} = \Theta \frac{\partial^2 T_{rad}}{\partial x^2}\bigg|_{x=0} + \Theta \frac{\partial T_{rad}}{\partial x}\frac{\partial \ln A}{\partial x}\bigg|_{x=0} \quad (5)$$

and (b) the rate of change of thermal energy in the conductive domain is equal to the rate of conductive heat loss at the boundary,

$$\frac{d}{dt}\int_{r_0}^{\infty} \rho C T_{rad}\, dV = -2\pi r_0 h \lambda \frac{\partial T_{rad}}{\partial r}\bigg|_{r=r_0} \quad (6)$$

EXAMPLE

As a demonstration and test of the method, we consider non-isothermal flow along a 1 km long pipe of r = 0.1 m radius. To better focus on the heat transfer problem, we leave out initial temperature and pressure gradients, i.e., we assume that the pipe is horizontal. The impermeable medium surrounding the pipe is assumed at a uniform initial temperature of T = 200 °C, and the following typical values are chosen for thermal parameters: conductivity λ = 2.1 W/m °C, density ρ = 2650 kg/m^3, and specific heat C = 1000 J/kg °C. These parameters result in a thermal diffusivity of $\Theta = \lambda/\rho C = 0.80 \times 10^{-6}$ m^2/s. The pipe is discretized into 50 segments of 20 m length. Fluid at a temperature of 30 °C is injected at one end at a constant rate of 5 kg/s; at the other end of the pipe pressures are maintained constant at the initial value of 100 bar. For comparison, the same problem is run with a fully numerical approach, using a radial grid of 48 blocks around each of the pipe segments, for a total of 2,450 blocks. Radial discretization is very fine near the pipe, starting with Δr = 4 mm, becomes coarser at increasing radial distance, and extends to a large outer radius of r = 100 m so the system will be infinite-acting for the time periods considered. The numerical solution is considered highly accurate and serves as a benchmark against which the accuracy of the semi-analytical method can be ascertained. All simulations were done with the TOUGH2 code (Pruess et al., 1999). Results are presented in Figs. 1 and 2 as temperature profiles along the pipe at different times and as time-varying temperatures at specific locations.

The semi-analytical temperature profiles are seen to be very accurate at earlier times, but accuracy is deteriorating over time. Temperatures at late time are systematically underpredicted by the semi-analytical solution. The same pattern is evident in the temperature breakthrough curves. Near the pipe inlet, where breakthrough is rapid, the semi-analytical and numerical solutions virtually coincide at all times. Near the center and towards the outlet of the pipe, temperatures agree closely to 10^5 s. Subsequently the semi-analytical temperatures trend lower than the numerical solution. From these comparisons it appears that at later time the semi-analytical solution underestimates the conductive heat supply to the cooling pipe.

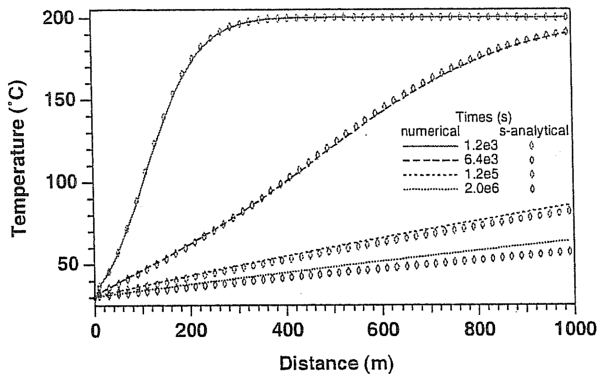

Figure 1. Temperature profiles along the pipe at different times.

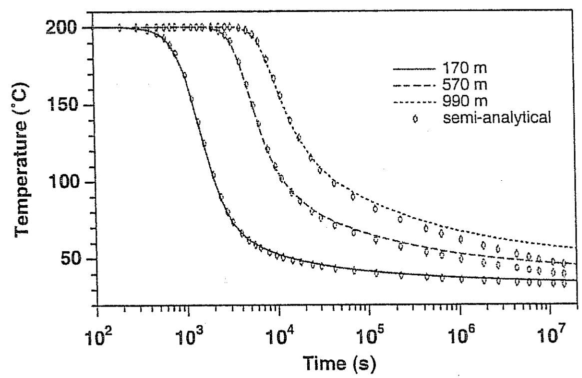

Figure 2. Thermal breakthrough curves at different distance from the pipe inlet.

In order to better pinpoint the limitations in the semi-analytical solution, we examine the temperature profiles in the conductive domain. Fig. 3 shows temperature profiles in the conductive domain attached to the first pipe segment (inlet). The numerical solutions are shown as data points at the radial distances corresponding to the nodal points in the finite difference solution; the semi-analytical profiles were calculated from Eqs. (4) and (1), using the values for the parameters p, q, and d obtained in the simulation.

From Fig. 3 it can be seen that the semi-analytical solution tends to underpredict temperatures near the pipe while overpredicting them at greater distance, with discrepancies increasing as time goes on. The semi-analytical solution systematically underpredicts temperature gradients, hence conductive heat supply, near the pipe, which explains why the semi-analytical temperatures trended too low in Figs. 1 and 2. In effect, the semi-analytical solution is taking too much heat out of the region near the pipe, and not enough from larger distance. We conclude that the trial function proposed in Eqs. (4, 1) is adequate for early times, but for larger time scales does not allow the temperature perturbation at the conductive boundary to adequately penetrate into the interior of the conductive domain, and does not permit an adequate heat supply from larger distances.

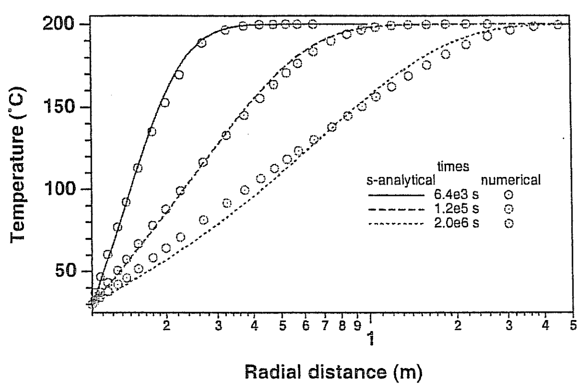

Figure 3. Simulated temperature profiles in the conductive domain surrounding the pipe inlet.

A BETTER TRIAL FUNCTION

How can the trial function be improved to deliver a more accurate description of the conductive profile for longer time? The analysis presented above suggests that a larger depth of penetration (parameter d in Eq. 1) may help to better tap into heat reserves at greater radial distance. However, calculations with a larger penetration depth, such as $d = \sqrt{\Theta t}$, yielded even lower temperatures at small radial distance, while shifting the region of overpredicted temperatures to larger distance.

Fig. 4 shows numerically simulated conductive profiles at the pipe inlet at different times. It is seen that temperatures near the conductive boundary fall on a straight line in a semi-log plot, suggesting that the conduction solution for "small" r should have the form (Wu and Pruess, 2000)

$$T_{rad} = T_f + c \cdot \ln(r/r_0) \qquad (7)$$

In other words, the polynomial in Eq. (1) should not be written in terms of $x = (r - r_0)$, but in terms of $(\ln r - \ln r_0)$. The physical interpretation is straightforward: differentiating Eq. (7) with respect to r delivers a temperature gradient and heat flux proportional to 1/r, which in turn produces a constant rate of heat flow when multiplied by the flow area $A = 2\pi rh$. Heat flow rate would be expected to be essentially constant in the quasi-steady heat transfer regime at "small" r, out to a distance of approximately $\sqrt{\Theta t}$ beyond the boundary (pipe surface at r_0 = 0.10 m). For the four times shown in Fig. 4, this amounts to distances of r = 0.17, 0.41, 1.36, and 4.10 m (measured from the center of the pipe at r = 0). Comparison with Fig. 4 shows that the conductive profiles at the different times indeed match straight line behavior out to those distances.

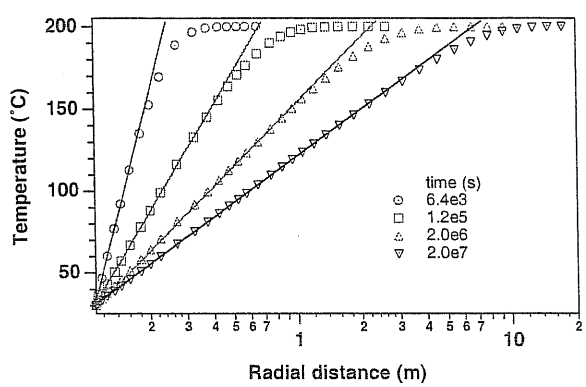

Figure 4. Numerically simulated conductive temperature profiles at different times. The straight lines are drawn to guide the eye.

Implementation of a trial function with a logarithmic expansion in terms of $(\ln r - \ln r_0)$ is currently underway. One difficulty is that for a logarithmic temperature profile it is no longer possible to evaluate the integral in the energy balance, Eq. 6, in closed analytical form.

CONCLUDING REMARKS

This paper has presented an adaptation of the semi-analytical technique of Vinsome and Westerveld (1980) to the problem of conductive heat transfer around a cylindrical pipe, such as an injection or production well. A slight generalization of the temperature trial function to account for the radial geometry, as opposed to the linear geometry considered by Vinsome and Westerveld, is sufficient to obtain an accurate treatment of conductive heat transfer for time periods of order one week. At later time the accuracy of the method deteriorates, which could be traced to the inability of the trial function to represent the quasi-steady heat transfer regime around the well that over time extends to increasing distance. Our analysis provided guidance for selecting a better trial function, computational implementation of which is currently underway.

A semi-analytical approximation for conductive heat transfer around a cylindrical pipe can find applications not only for wellbore flow, but also for non-isothermal flow in fractured media. Indeed, it is well established experimentally and theoretically that flow in heterogeneous fractures is not an area-filling phenomenon, but instead tends to proceed primarily along localized preferential pathways, or "channels" (Tsang and Tsang, 1987; Tsang and Neretnieks, 1998). The model of a cylindrical pipe may provide a better representation of heat transfer to these pathways than the idealization of sheet flow in a homogeneous fracture.

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