LBNL-40689
UC-414

ERNEST ORLANDO LAWRENCE
BERKELEY NATIONAL LABORATORY

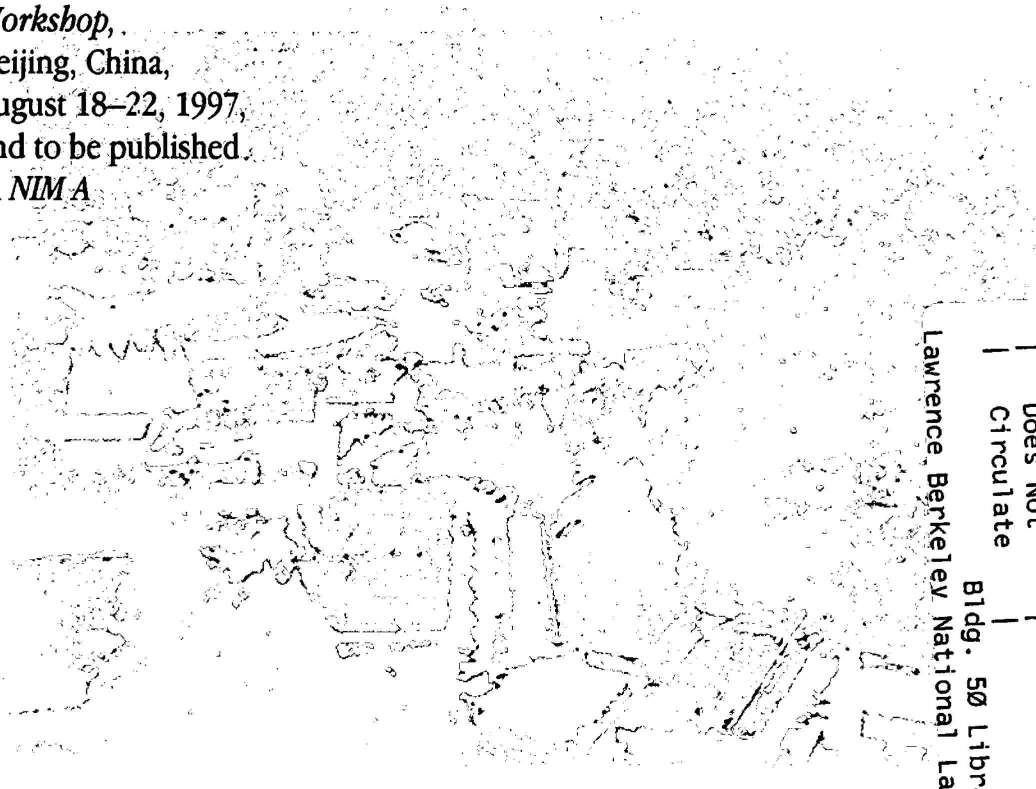
Undulator Interruption in High-Gain Free Electron Lasers

Kwang-Je Kim

Accelerator and Fusion Research Division

October 1997

Presented at the *International FEL Conference 1997 and 4th FEL Users' Workshop*, Beijing, China, August 18–22, 1997, and to be published in *NIM A*



LBNL-40689
CBP Note-237
UC-414

Undulator Interruption in High-Gain Free Electron Lasers*

Kwang-Je Kim

Ernest Orlando Lawrence Berkeley National Laboratory
One Cyclotron Road, MS 71-259
Berkeley, CA 94720 USA

October 1997

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC 03-76SF00098.

Undulator Interruption in High-Gain Free Electron Lasers*

Kwang-Je Kim

Ernest Orlando Lawrence Berkeley National Laboratory
One Cyclotron Road, M.S. 71-259
Berkeley, CA 94720 USA

Fax No. 510-486-6485

October 1997

Abstract

The effect of interrupting an undulator on the performance of high-gain free-electron lasers (FELs) is evaluated by analyzing 1-D Maxwell-Vlasov equations. It is found that the effect is small for a reasonable length of the interruptions for FEL parameters envisaged for short wavelength self-amplified spontaneous emission (SASE). Since the interruptions provide valuable space for quadrupoles and diagnostics, and at the same time permit a greater flexibility in mechanical design, the result of this paper is encouraging for construction of long undulator magnets required for SASE.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC 03-76SF00098.

1. Introduction

In a previous paper we reported an analysis of how much the gain could be reduced when a long undulator for a high-gain free-electron laser is interrupted with short drift sections [1]. The interruptions, if they could be introduced without significant loss in gain, would greatly simplify the mechanical construction and also provide valuable space for installing focusing quadrupoles and diagnostic elements for the long undulators considered for x-ray SASE projects [2],[3]. Three effects were considered; the diffraction loss of the optical mode, the free-space slippage, and the loss of the phase coherence due to the electron beam energy spread and emittance. Of these, the first effect was found to be small, and the second can be corrected by adjusting the length of the interrupted sections. It is the purpose of this paper to correct an algebraic error in the calculation of the third effect in Ref. [1]. The correct calculation reveals that the effect is quite small for short gaps.

The error in Ref. [1] was in Eq. (12), in which the factor $1/\left(1+\sigma_{x'}^2 kd\right)^2$ should be replaced by $1/\left(1+\left(\sigma_{x'}^2 kd\right)^2\right)$. With this replacement, gain reduction for the SLAC project becomes 0.5% per interruption rather than the 17% reported in that paper.

The analysis of Ref. [1] was based on qualitative argument. Here we will carry out a more precise analysis based on the coupled Vlasov-Maxwell equation, following closely the method used in Ref. [4]. The gain reduction is then found to be 3.4% per interruption. The result is encouraging for construction of long undulator magnets for SASE.

2. Derivation

The coupled Maxwell-Vlasov equations for FELs are [4]

$$\left(\frac{\partial}{\partial z}-2ik_u\eta\nu\right)F(\nu,\eta,z)=\kappa_1 A(\nu;z)\frac{\partial}{\partial\eta}V(\eta), \tag{1}$$

$$\left(\frac{\partial}{\partial z} - i\Delta\nu k_u\right)A(\nu;z) = \kappa_2\int F(\nu,\eta;z)d\eta \ . \tag{2}$$

Here z is the distance along the undulator; $k_u = 2\pi/\lambda_u$, λ_u = undulator period length; η = relative energy deviation; $\nu = \omega/\omega_0$; ω= the radiation frequency; ω_0= resonance frequency; $F(\nu,\eta,z)$ is essentially the electron beam bunching parameter; $A(\nu;z)$ is the slowly varying amplitude of the radiation field; $V(\eta)$ is the distribution function of the initial electrons' energy spread; and $\Delta\nu = \nu - 1$ = detuning parameter. The constants κ_1 and κ_2 are related such that

$$\kappa_1\kappa_2 = k_u{}^2\rho^3 \ , \tag{3}$$

where ρ is the FEL scaling parameter [5].

Equations (1) and (2) can be solved by the Laplace transform technique to express $F(\nu,\eta;z)$ and $A(\nu;z)$ for $z > z_1$, given the initial functions $F(\nu,\eta;z_1)$ and $A(\nu;z_1)$, as explained in Ref. [4]. The solution is especially simple when

$$k_u x_I(z - z_1) >> 1. \tag{4}$$

Here x_I is the imaginary part of x, where x is the solution, with the largest imaginary part, of the dispersion relation

$$x + \frac{\Delta\nu}{2} + \rho^3\int d\eta\frac{dV/d\eta}{x+\eta\nu} = 0 \ . \tag{5}$$

In this exponential gain regime, the solution is

$$A(\nu;z) = \frac{e^{-2ik_u x(z-z_1)}\left[A(\nu;z_1) - \frac{\kappa_2}{2ik_u}\int d\eta\frac{F(\nu,\eta;z_1)}{(x+\eta\nu)}\right]}{1-\rho^3\int d\eta\frac{dV/d\eta}{(x+\eta\nu)^2}} , \tag{6}$$

$$F(\nu,\eta;z) = -\frac{\kappa_1}{2ik_u}\frac{(dV/d\eta)A(\nu;z)}{x+\eta\nu} \ . \tag{7}$$

Equations (6) and (7) are valid for z sufficiently far away from $z_1(< z)$ so that the gain is dominated by the fastest growing single mode, i.e., when the inequality (4) is valid. Now suppose that the FEL was started from $z_0 << z_1$, so that the field at z_I is also dominated by the fastest growing mode. Then the bunching parameter F and the field amplitude will be related by an equation identical to Eq. (7) except that z is replaced by z_1. Using this, Eq. (6) becomes

$$A(\nu;z) = e^{-2ik_u x(z-z_1)} A(\nu;z_1) . \qquad (8)$$

This is as it should be in the exponential gain regime.

To study the effect of an interruption, we consider the case where the undulator is interrupted by a drift section between z_1 and $z_1 + d$. We assume that z_1 is in the exponential gain regime so that Eq. (7) is valid with $z = z_1$. In passing the drift section, we have

$$A(\nu;z_1 + d) = A(\nu;z_1)e^{i\Delta\nu k_u d} , \qquad (9)$$

$$F(\nu,\eta;z_1 + d) = F(\nu,\eta;z_1)e^{i\Delta\theta} . \qquad (10)$$

The field at $z >> z_1 + d$ is obtained by replacing $A(\nu;z_1)$ and $F(\nu,\eta;z_1)$ by $A(\nu;z_1 + d)$ and $F(\nu,\eta;z_1 + d)$, respectively, in Eq. (6). The phase shift $\Delta\theta$ is given by

$$\Delta\theta = k(d - ct(\eta)) = kd\left(-\frac{1+K_D^2}{2\gamma_0^2} + \frac{1+K_D^2}{\gamma_0^2}\eta\right). \qquad (11)$$

Here c is the light speed, and $t(\eta)$ is the time for an electron with relative energy spread η to travel the interruption, and

$$K_D = \frac{e}{mc}\int_0^d B(z')dz' , \qquad (12)$$

where B is the magnetic field in the drift section, e = electron charge, m = electron mass, and γ_0 is the reference electron energy in units of mc^2.

The first term in Eq. (11) does not affect the FEL gain if it is chosen to be an integral multiple of 2π. The second term can be written as

$$\Delta\theta = D\frac{\eta}{\rho} . \qquad (13)$$

Here

$$D = kd\rho \frac{1+K_D^2}{\gamma_0^2} \tag{14}$$

is the dimensionless dispersion parameter [6],[7].

Using these results, we obtain for the field amplitude $z >> z_1 + d$:

$$A(\nu;z) = e^{-2ik_u x(z-z_1)} R. \tag{15}$$

The factor R gives the modification of the exponential gain due to the drift section, and is given by

$$R = \frac{1 - \int d\xi \frac{d\overline{V}/d\xi \; e^{iD\xi}}{(\mu+\xi)^2}}{1 - \int d\xi \frac{d\overline{V}/d\xi}{(\mu+\xi)^2}}. \tag{16}$$

In the above we have assumed for simplicity that $\Delta\nu = 0$, and introduced the scaled variables $\mu = x/\rho$, $\xi = \eta/\rho$, and $\overline{V}(\xi)$ is the distribution in ξ with normalization $\int \overline{V}(\xi) d\xi = 1$.

Equation (16) is the main result of this paper [8], which needs to be numerically evaluated for a general distribution function $\overline{V}(\xi)$. However, the result is simple for a rectangular distribution

$$\begin{aligned} \overline{V}(\xi) &= 1/\Delta \quad \text{for } |\xi| < \Delta/2 \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{17}$$

In this case

$$R = \frac{1 - 2\mu^3 \cos(D\Delta/2) - iD\mu\left(1 - 2\mu(\Delta/2)^2\right)(\sin(D\Delta/2))/(D\Delta/2)}{1 - 2\mu^3}. \tag{18}$$

For vanishing energy spread $\Delta = 0$, we obtain $R = 1 - iD\mu/3$, suggesting that the gain may be increased significantly for a large D [6]. However, the improvement is difficult for a finite energy spread, as observed previously [7].

3. Numerical Examples

Figure 1 shows the correction to gain $|R|$ as a function of D for the case $\Delta \equiv \Delta\gamma/(\rho\gamma_0) = 0.01$, where $\Delta\gamma$ is the full width of the energy spread. It is seen that by choosing $D \approx 150$, the gain can be enhanced by a factor of more than 30. The periodic variation of $|R|$ is an artificial feature of the rectangular distribution.

However, the value of Δ corresponding to the electron beam parameters for short wavelength SASE projects are much larger, being of order unity. The effect of interruption is then to reduce the gain. There are some ambiguities in the correct value of Δ to employ, since our analysis here is 1-D, and the effective energy spread due to emittance is important. A reasonable estimate of Δ can be obtained by requiring that the growth rate of the 1-D model used here is the same as that obtained by simulation, taking into account the Gaussian energy spread and the electron beam energy spread.

For the LCLS, the growth rate is 40% of the ideal 1-D gain. From Fig. 2, this corresponds to $\Delta = 1.23$. Figure 3 gives a plot of $|R|$ as a function of D for $\Delta = 1.23$. It is seen that $|R| \approx 1 - D$ for $0 \le D < 1$. The value of D for the LCLS, using $K_D = 0$, $d = 25\text{cm}$, $\gamma = 30{,}000$, turns out to be $D = 0.017$. Thus the reduction is about 1.7% in amplitude and 3.4% in intensity per interruption, much smaller than the 13% estimated previously with an erroneous formula.

4. Acknowledgments

The author thanks N. Vinokurov for pointing out the algebraic error in Ref. [1].

References

[1] K.-J. Kim, M. Xie and C. Pellegrini, Nucl. Instrum. Methods A375 (1996) 314.
[2] M. Cornacchia, et al., Proceedings of 18th International Free Electron Laser Conference, Rome, Italy, 1996.
[3] J. Rossbach, et al., to be published in the Proceedings of 19th International Free Electron Laser Conference, Beijing, China, 1997.
[4] K.-J. Kim, Nucl. Instrum. Methods A250 (1980), 396.
[5] R. Bonifacio, C. Pellegrini and N. Narducci, Opt. Commun. 50 (1984) 373.
[6] J. Gallardo and C. Pellegrini, Nucl. Instrum. Methods A296 (1990) 448.
[7] R. Bonifacio, R. Corsini and P. Pierini, Phys. Rev. A45 (1992) 4091
[8] A more general type of undulator interruption was considered by N.A. Vinokurov, Nucl. Instrum. Methods A375 (1996) 264.

Figure Captions

Fig. 1. The gain correction factor $|R|$ as a function of D for $\Delta = 0.01$.
Fig. 2. Imaginary part of the solution of the dispersion relation for a rectangular energy distribution (proportional to the growth rate) as a function of the scaled energy spread Δ. For $\Delta = 1.23$, the growth rate is reduced 40% from the ideal 1-D growth rate.
Fig. 3. The gain correction factor $|R|$ as a function of D for $\Delta = 1.23$.

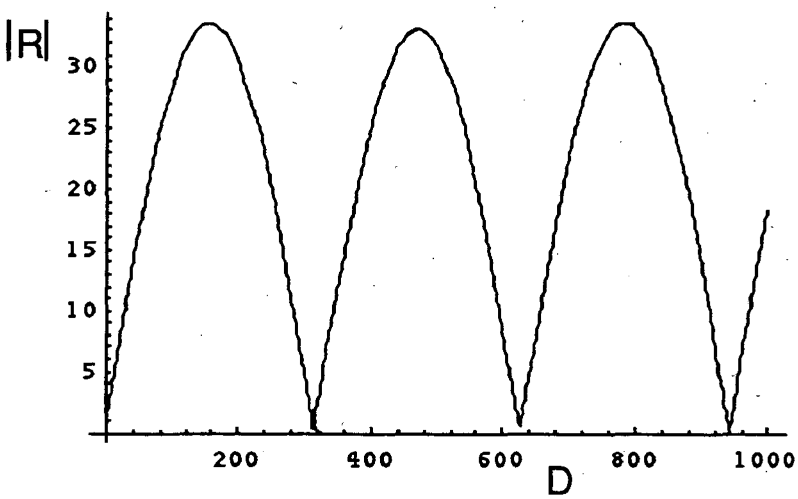


Figure 1

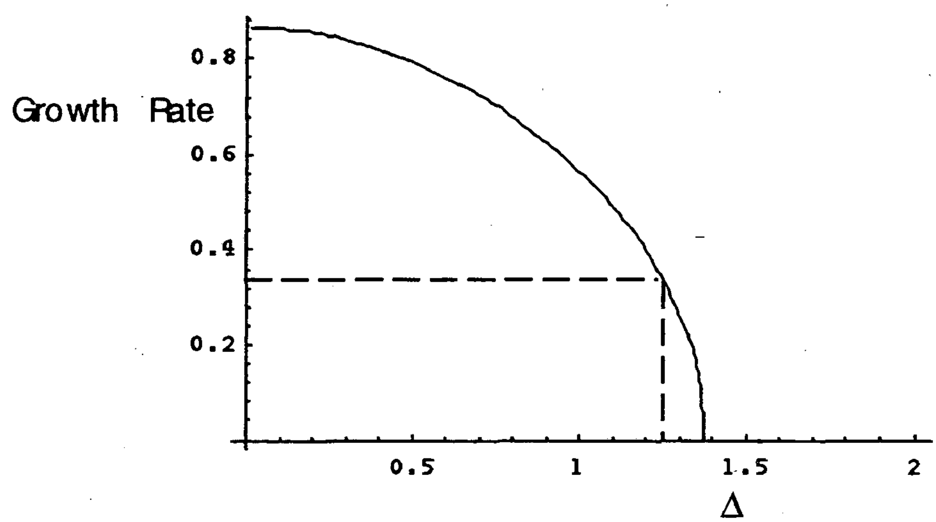


Figure 2

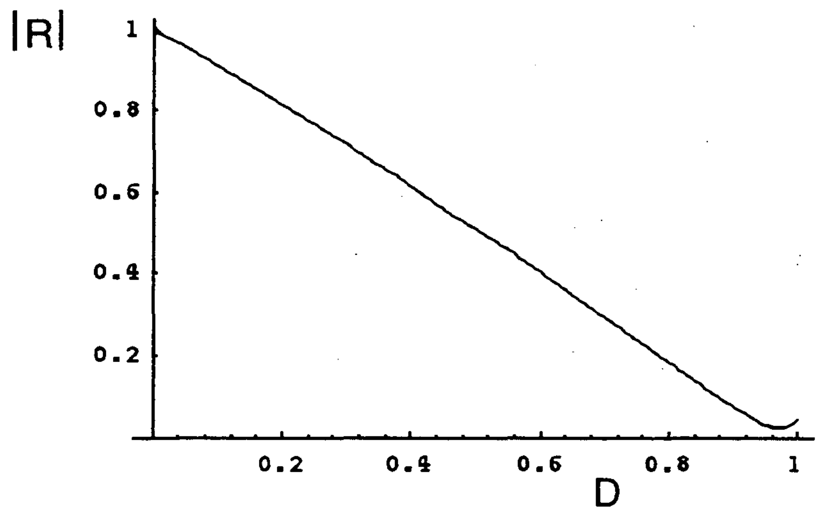


Figure 3

ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY
ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720

Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098