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Stochastically estimated covariance matrices for independent and cumulative fission yields in the ENDF/B-VIII.0 and JEFF-3.3 evaluations

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ABSTRACT

A Monte-Carlo method for the generation of correlation and covariance matrices for independent and cumulative fission yields has been developed. The method uses a constrained Monte-Carlo resampling structure in order to vary evaluated fission yield libraries in a way that meets basic conservation principles. This results in the generation of correlation/covariance matrices with limited model bias and uncertainty; the matrices are primarily reflective of the evaluated fission yield uncertainties and correlations that arise from the evaluation process. This method has been applied to generate correlation and covariance matrices for all of the fissioning systems of the ENDF/B-VIII.0 and JEFF-3.3 evaluations, marking the first time such matrices have been generated for all of these systems. These covariance matrices have been published online for immediate public use. These correlation and covariance matrices can be used to improve uncertainty estimation in calculations of reactor antineutrino emission rates, decay heat problems, and nuclear forensics.

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Contents

1. Motivation

In fission, a nucleus undergoes a deformation that leads to the scission of the nucleus into at least two fragments. These fragments have high excitation energy and undergo prompt neutron and photon emission. When the prompt neutron emission has ceased, the fragments are referred to as "products". The probability that a particular fission product will be produced directly from a fission event is called an "independent yield". The probability that a particular fission product will exist at some point in time after fission, either due to direct production from fission or due to production from the decay of a parent fission product, is called a "cumulative yield".

The measurement and evaluation of independent and cumulative fission yields is the result of decades of exceptional research by scientists from across the globe. Continued research in this area is needed to meet the ever-advancing needs of users. Neither the fission yield evaluation (based on Refs. [1,2]) in the ENDF/B-VIII.0 (Evaluated Nuclear Data File) evaluated library [3] nor the fission yield evaluation (based on Refs. [4,5]) in the JEFF-3.3 (Joint Evaluated File for Fission and Fusion) evaluated library [6] contain an estimation of correlation/covariance between fission product yields. These covariance and correlation matrices for independent and cumulative fission product yields have been identified as a pressing nuclear data need [7,8]. These matrices are needed for applications in reactor antineutrino rate calculations [9–12], decay heat calculations [13], and any other calculations that incorporate fission yield data such as nuclear forensics.

Nuclear data libraries that are used in applications, such as those listed above, are produced by scientists with specialized skills in a process called "evaluation". The evaluation process brings together experimental measurements of nuclear properties, nuclear physics modeling, and the expertise of the evaluator to produce these nuclear data libraries. Therefore, there are three keys sources where correlation arises from in evaluated nuclear data libraries: **physics, experiment,** and **evaluation**.

Each of these three sources introduces a unique set of error and correlation and all three of these sources are present in all evaluated nuclear data libraries to some degree. In an ideal case – if experimental capabilities and measurements were perfect and errorless and the evaluation was conducted flawlessly with exact modeling capabilities – the correlation between the values in a nuclear data library would be purely physical (i.e., those that arise from the underlying physics of the measured property). However, this ideal case does not occur in reality and consideration of correlations arising from experimental and evaluation sources is required in order provide users of nuclear data libraries with realistic uncertainties and covariance matrices.

Ideally, fission yield covariances would be generated with the evaluation in order to maximize consistency. In the interim, methods for estimating fission product yield covariance matrices have been proposed [13–17]. These methods rely on an underlying model of the fission process to determine correlations between fission products, and therefore give an estimation of the physical correlations that are discussed above. For example, the works of Rochman et al. [14] and Leray et al. [15] use the GEF code [18,19] to generate their matrices. These methods require that the model of fission is reliably accurate and that model parameters exist for a compound system of interest. Often parameters for these models have been determined for only a small number of well-known compound systems [15,16], limiting their scope to be less than that of the compound systems currently listed in the ENDF/B-VIII.0 and JEFF-3.3 evaluations. These model-based methods are an important component of determining fission yield covariance matrices; they estimate the physical component of correlations in evaluated nuclear data libraries. Nevertheless, these only provide a part of the correlation information that users need and must be complemented with estimations of the experimental and evaluation components.

The fact that these model-based methods do not take experimental correlations fully into consideration has been previously noted in literature [20]. It should also be noted that the evaluation process and its potential to introduce additional error and correlation into evaluated fission yield libraries has been previously observed through inconsistencies between evaluated fission yields and fission neutron multiplicity distributions [21]. This observed inconsistency, and the evaluation correlation introduced by it, is captured by the new method presented in this publication, as will be detailed in Section 2.3.

In order to address the topics discussed above, the method presented was formulated. It seeks to limit model dependence and focuses primarily on correlations that arise from the ENDF/B-VIII.0 and JEFF-3.3 evaluations and the experimental data that underlies them. In addition to this, this publication also seeks to ensure open access to the correlation and covariance matrices resulting from this method. While the alternative methods listed above for covariance/correlation matrix generation exist, the results of these methods have not been made publicly available. To address this issue, the matrices that result from the method presented have been made immediately available to the nuclear science community at nucleardata.berkeley.edu/FYCoM. In the interest of reproducible science, a workflow for the calculation of these matrices has been preserved in Ref. [22].

2. Method

2.1. Independent yields

Independent fission yield libraries should obey a number of conserved relationships. The following Monte-Carlo resampling method is structured to conserve the five conditions given below. These conditions are modified from similar conditions proposed by Fiorito et al. [13].

In a fission event, at least two fission products must be produced. Therefore, binary fission yields should sum to two:

$$\sum_i Y_i = 2 \tag{1}$$

where Y_i is the independent yield of nuclide i.

The total charge must be conserved, therefore the total charge of the compound system, Z_{CN}, should be recovered:

$$\sum_i Y_i Z_i = Z_{CN} \tag{2}$$

where Z_i is the atomic number of nuclide i.

The total baryon number must be conserved, therefore the total baryon number of the compound system, A_{CN}, less the average number of fission neutrons emitted, $\bar{\nu}$, should be recovered:

$$\sum_i Y_i A_i = A_{CN} - \bar{\nu} \tag{3}$$

where A_i is the mass number of nuclide i.

Assuming charged particle emission from fission fragments is negligible, the net yield to products with a particular atomic number, Z, should be equal to the net yield to products with the complementary atomic number, $Z_{CN} - Z$:

$$\sum_i Y(Z, A_i) = \sum_j Y(Z_{CN} - Z, A_j) \tag{4}$$

For a given fission yield library there should exist some midpoint mass number, A_{mid}, such that yields on either side of this midpoint should sum to one:

$$\sum_{A_i > A_{mid}} Y(A_i) = \sum_{A_i \le A_{mid}} Y(A_i) = 1 \tag{5}$$

Eq. (5) determines which nuclei are heavy ($A > A_{mid}$) and which are light ($A \le A_{mid}$) in the resampling method. It also states the midpoint of the fission product distribution is constant. This is expected even in the extreme case of symmetric fission as changing this midpoint also changes the total mass distributed to the fission fragments from the compound nucleus. In reality, Eq. (5) may only be approximately true due to mass number being an integer and not a continuous variable. However, exploiting this condition allows the Monte-Carlo resampling method to be structured such that the conditions in Eqs. (1)–(3), and (4) are also conserved. In this method, A_{mid} is selected by finding the A in each fission yield library that best reproduces Eq. (5).

The following steps give the method that is used to produce resampled fission yield libraries that meet the conditions given in Eqs. (1)–(5):

1. Select a random number, X, between 0 and 1. If X is less than 0.5, the yields on the 'light' side ($A \le A_{mid}$) will be resampled. Otherwise, yields on the 'heavy' side ($A > A_{mid}$) will be resampled.
2. For each A chain on the selected side, randomly select a fission product yield to be resampled. The probability that a given product is selected the should be set such that high-yield, low-uncertainty products are preferentially selected. Resample that yield about a normal distribution with a centroid equal to its evaluated yield and width equal to its evaluated yield uncertainty.
3. Scale the other fission product yields in the A chain by the same percent change realized for the product yield in Step 2.
4. Normalize the yields on the selected side such that their sum equals 1.
5. Generate fission yields on the complementary side using the fission neutron multiplicity distribution, $P(\nu, A)$:

$$Y_{frac}(Z_{CN} - Z, A_{CN} - A - \nu) = P(\nu, A) Y(Z, A) \tag{6}$$

$$\begin{aligned} Y(Z, A) &= \sum_A \sum_\nu Y_{frac}(Z_{CN} - Z, A_{CN} - A - \nu) \\ Y(Z, A) &= \sum_A \sum_\nu \Big[P(\nu, A_{CN} - A - \nu) \\ &\quad \times Y(Z_{CN} - Z, A_{CN} - A - \nu) \Big] \end{aligned} \tag{7}$$

6. Repeat Steps 1–5 N times. Select N such that statistical noise is minimized.
7. Calculate the resulting correlation and covariance matrices from the N trials.

Conservation of Eq. (5) is a given of the method. The conservation of Eq. (1) and (4) can be proven analytically. Eqs. (2) and (3) are numerically verified to be conserved to within 0.01% for the ENDF/B-VIII.0 evaluation and to within 0.05% for the JEFF-3.3 evaluation. In principle, one can combine Steps 2 and 3 and simply resample each fission product yield about its evaluated yield and yield uncertainty. However, the ENDF/B-VIII.0 evaluation assumed a Gaussian distribution of yield in Z for each A chain [1]. Therefore, Step 3 is justified as it introduces the positive correlation between product yields within a given A chain that the ENDF/B-VIII.0 evaluation process would have introduced. Step 5 relies on the accuracy of the $P(\nu, A)$ data used and Section 2.3 will address how $P(\nu, A)$ data is obtained for all of the compound systems in the evaluations. For Step 6, this study used $N = 10000$ to produce the presented matrices. The Mersenne Twister pseudo-random number generator with a seed of 0 was used for each matrix generated.

As will be detailed in Section 2.3, the fission yield evaluations did not take into consideration the consistency of fission neutron multiplicity distributions with independent fission yields. Because of this, the covariance matrix that is directly obtained from Step 7 gives variances in the yields that are larger than their corresponding evaluated variances. The correlation and covariance matrices obtained from Step 7 will be called "primary" matrices throughout this publication as they are the matrices that are obtained directly from the method presented.

The covariance matrices that result from this process exhibit variances in the independent yields that are larger than those in the evaluations. In order to address this, a pair of "normalized" correlation and covariance matrices are calculated. The fission yield variances in the normalized covariance matrix are equal to those in the evaluation. The normalized covariance matrix is calculated as the product of the primary correlation matrix and the evaluated fission yield uncertainties. In order to conserve total yield, the sum of the normalized covariance matrix must be zero (as it is in the primary covariance matrix). For all of the compound systems considered, the sum of the covariance matrix obtained by simply taking the product of the primary correlation matrix and the evaluated fission yield uncertainties was greater than zero. To enforce that the sum of the normalized covariance matrix is zero, the negative correlations in the primary correlation matrix were scaled slightly. This scaling was less than 2% in all cases. Both the primary and normalized correlation and covariance matrices are presented to the user at nucleardata.berkeley.edu/FYCoM.

2.2. Cumulative yields

Correlation and covariance matrices can be generated for cumulative yields using the covariance matrices generated for the independent yields. In order to do this, the transformation of independent yields into a given cumulative yield must be known. Evaluations make specific adjustments to these transformations. For example, the ENDF/B-VIII.0 evaluation obtained cumulative yields from independent yields by taking a weighted average of two different methods [1]. Replicating these adjustments would enhance the consistency of this method with the evaluations, however, a full tabulation of these specific adjustments is not readily available. Instead, this method transformed the independent yields to cumulative yields directly using evaluated decay data. The cumulative yields were obtained by calculating the probability that an independent product will follow a decay path leading to a cumulative product using Eq. (8):

$$Y_C(Z,A) = \sum_i \left[Y_I(Z_i,A_i) \prod_{k=1}^{i} \beta_{k\rightarrow k+1} \right] \tag{8}$$

where $Y_C(Z,A)$ is the cumulative yield being calculated, $Y_I(Z_i,A_i)$ are the independent yields that contribute to the cumulative yield, and $\prod_{k=1}^{i} \beta_{k\rightarrow k+1}$ represents the probability that product (Z_i,A_i) follows a decay path to product (Z,A) where each $\beta_{k\rightarrow k+1}$ is the decay branching ratio of the kth product into the $(k+1)$th product in the decay chain.

The decay chains required for Eq. (8) are generated using the Fission Induced Electromagnetic Response code (FIER) [23]. The decay chains generated by FIER include all possible decay paths for each fission product. FIER also provides a table of decay branching ratios parsed from ENDF/B-VIII.0 File 8 [3]. The independent yields are statistically resampled about a multivariate normal distribution using their evaluated values and their covariance matrices generated from the process in Section 2.1. Cumulative yields are then calculated from these resampled independent yields using Eq. (8). This is repeated N times such that statistical noise is minimized and the correlation and covariance matrices are calculated from the resulting N trials. In this study, $N = 10000$ was used to produce the presented matrices.

It was again seen that the covariance matrices that result from this process exhibit variances that are larger than those of the evaluated variances in the cumulative yields. This is because the cumulative yield covariance matrices are generated from the primary independent yield covariance matrix. Evaluated independent yields generally have larger evaluated uncertainties than cumulative yields and, as mentioned in Section 2.1, the primary independent yield covariance matrix has larger variances than the evaluation. Therefore, a normalized cumulative yield covariance matrix is also produced using the correlation matrix and the evaluated variances in the cumulative yields. This normalized covariance matrix is simply the product of the correlation matrix and the evaluated uncertainties. Both the primary and normalized cumulative yield covariance matrices are presented to the user at nucleardata.berkeley.edu/FYCoM.

2.3. Generation of consistent $P(\nu,A)$ data

Neither the ENDF/B-VIII.0 nor JEFF-3.3 evaluations enforced consistency between fission neutron multiplicity distributions and independent fission yields. Because of this there is no evaluated or experimental dataset that gives $P(\nu,A)$ values that are fully consistent with the independent yields in the evaluation, nor is there complete $P(\nu,A)$ data that covers all of the compound systems in the evaluation. In order to address this issue, a procedure was developed to obtain $P(\nu,A)$ data that has the

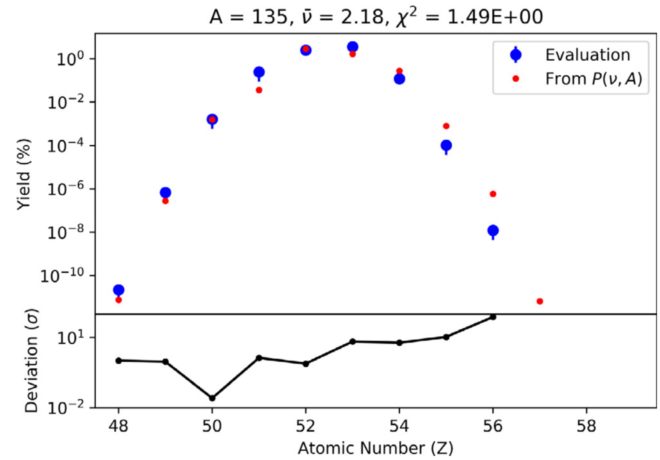


Fig. 1. Result of the minimization of χ^2 in Eq. (9) for the $A = 135$ chain of the ^{235}U fast fission ENDF/B-VIII.0 evaluation. The blue data are the evaluated yields and the red data are yields generated using Eq. (7) and $P(\nu,A)$ data that minimized Eq. (9). . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

greatest degree of consistency possible with evaluated yields. Perfect consistency would be achieved if each independent fission yield in the library could be reproduced using Eq. (7). This is the basis for the χ^2 metric in Eq. (9) which judges the consistency between evaluated independent fission yields and those generated using $P(\nu,A)$ data and Eq. (7); perfect consistency would result in $\chi^2 = 0$.

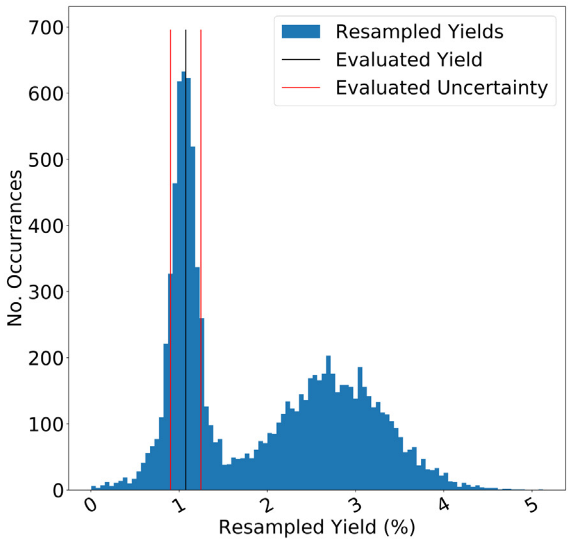
$$\chi^2 = \sum_i \frac{[Y_{eval}(Z_i,A) - Y_{gen}(Z_i,A)]^2}{Y_{eval}(Z_i,A)} \tag{9}$$

where Y_{eval} are the evaluated independent yields in a given A chain and Y_{gen} are those same yields that are generated using $P(\nu,A)$ data using Eq. (7).

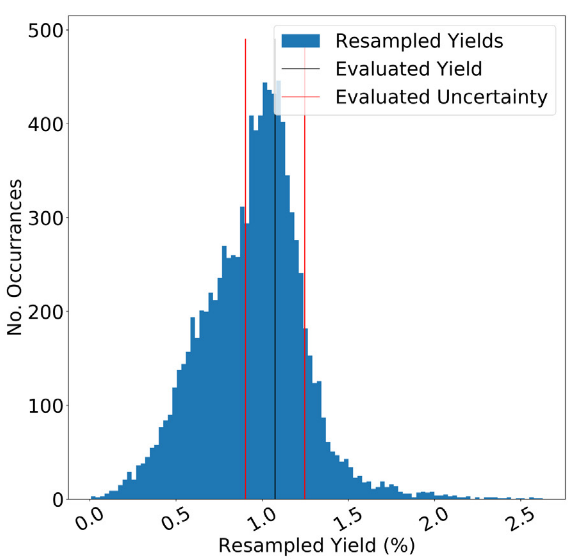
The χ^2 metric in Eq. (9) was minimized for each A chain in each of the fissioning systems in the evaluations in order to generate a set of $P(\nu,A)$ data for use in the method presented in Section 2.1. An example of this minimization technique is shown in Fig. 1 which shows the result of minimizing χ^2 in Eq. (9) to obtain $P(\nu,A)$ data for the $A = 135$ chain of the ^{235}U fast fission ENDF/B-VIII.0 evaluation.

In order to conserve mass, $P(\nu,A)$ should ideally obey the physical condition that $P(\nu,A) = P(\nu,A_{CN} - A - \nu)$. An attempt was made to introduce a term to minimize the differences between $P(\nu,A)$ and $P(\nu,A_{CN} - A - \nu)$ in Eq. (9), however, this introduction made the minimization of Eq. (9) intractable. An iterative normalization method was developed in an attempt to force $P(\nu,A) = P(\nu,A_{CN} - A - \nu)$, however, this resulted in the χ^2 metric becoming unacceptably large. Future work could include attempts to improve the minimization method and metric such that $P(\nu,A) = P(\nu,A_{CN} - A - \nu)$ is met.

It should again be noted that inconsistency between the evaluated yields and those generated using the $P(\nu,A)$ data results directly from the evaluation itself and the fact that it did not take fission neutron multiplicity data into consideration. This inconsistency has been previously noted by Jaffke et al. [21]. This $P(\nu,A)$ data is generated to mitigate the effects of this inconsistency on the method presented in Section 2.1. An example of how this generated $P(\nu,A)$ data improves this method is presented in Fig. 2. It can be seen that a simplistic choice of $P(\nu)$ creates a bimodal distribution when resampling fission yields: one peak is seen when the heavy side of the fission product distribution is chosen in Step 1 of the method and another when the light side



(a) The independent yield of ^{132}Te resampled 10000 times using this method. Here the choice of the $P(\nu)$ distribution used in the method was an A-independent distribution for ^{235}U(n,f) taken from Reference [24].



(b) The independent yield of ^{132}Te resampled 10000 times using this method. Here the $P(\nu, A)$ distributions used in the method are the generated distributions described in Sec. 2.3.

Fig. 2. Histograms of resampled yields for ^{132}Te with different choices of $P(\nu, A)$. Each histogram contains 10000 entries. The evaluated yields are shown at the black line, banded by red lines representing the evaluated uncertainty of that yield [24]. . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

is chosen. Using the generated $P(\nu, A)$ data yields much improved results.

The authors do not recommend the use of this generated $P(\nu, A)$ data for other applications. Certainly ensuring consistency between fission neutron multiplicity data and independent fission yields would add to the complexity of an evaluation and may very well be impractical. Future evaluations could attempt to address this by reporting "event" yields rather than independent yields. The "event" yields would report the probability that a given pair of fission products and number of prompt fission neutrons are produced from a given fission event.

2.4. Limitations and benchmarking

This method is able to capture fission yield correlations within a given A chain through Step 3 and correlations between complementary fission products through the use of $P(\nu, A)$ data in Step 5. However, this method does not fully capture correlations between A chains on the same side of the fission product distribution. This is because those yields are resampled independently of each other. As a result of this deficit, the correlations calculated using this method are expected to be somewhat underestimated.

Without an underlying model of fission, it is difficult to conceive how these correlations would be introduced. This method should be viewed as complementary to the model-based methods mentioned in Section 1. Where this method offers the capability to focus on correlations from the evaluation itself, model-based methods offer the ability to see physical correlations, such as those existing between A chains.

In order to assess the efficacy of the method and the effect of the limitations that are acknowledged above, a benchmarking of the method was performed with a model of mass yields. This simple model is detailed in Eq. (10) and consists of two Gaussians, one for the heavy peak and one for the light peak of the fission product distribution.

$$Y(A) = \frac{1}{\sqrt{2\pi}} e^{(A-\mu)^2/2\sigma^2} + \frac{1}{\sqrt{2\pi}} e^{(A-(A_{CN}-\mu-\bar{\nu}))^2/2\sigma^2} \tag{10}$$

where μ is the centroid of the heavy-product Gaussian, A_{CN} is the mass of the compound nucleus, $\bar{\nu}$ is the average neutron multiplicity of the fissioning system, and σ is the width of both Gaussians.

The neutron multiplicity distribution in this model was set to be a Poisson distribution with a mean of 2.0 for each mass number. This ensures the important condition discussed in Section 2.3 that $P(\nu, A) = P(\nu, A_{CN} - A - \nu)$ is met.

This model of fission has three parameters: μ, σ, and $\bar{\nu}$. Reasonable selections for the values of these three parameters are $\mu = 132 \pm 0.5$, $\sigma = 5 \pm 0.1$, and $\bar{\nu} = 2.0 \pm 0.1$. Using the model in Eq. (10), the mass yields for each A were calculated. A Monte-Carlo resampling of the covariances between the mass yields was then performed: the model parameters were varied about their uncertainties 10000 times, the mass yields were recalculated on each of these trials, and the correlations between the mass yields were assessed from these trial results. Fig. 3 shows the correlation matrix between the mass yields calculated from the model given in Eq. (10).

To benchmark the efficacy of the method presented, the yields from Eq. (10) were input to the method to see if their known correlations could be reproduced. In the first test, Step 3 was modified such that the mass yields on the selected side of the fission product distribution were varied using their respective half of the correlation matrix shown in Fig. 3. This was done because of the above-stated limitation that correlations between mass chains are underestimated. By using half of the model correlation matrix in this test, this known limitation is compensated for, thus offering a more direct comparison for benchmarking. The correlation matrix that results from this test is shown in Fig. 4 and the difference between this correlation matrix and the model correlation matrix is shown in Fig. 5. The average absolute

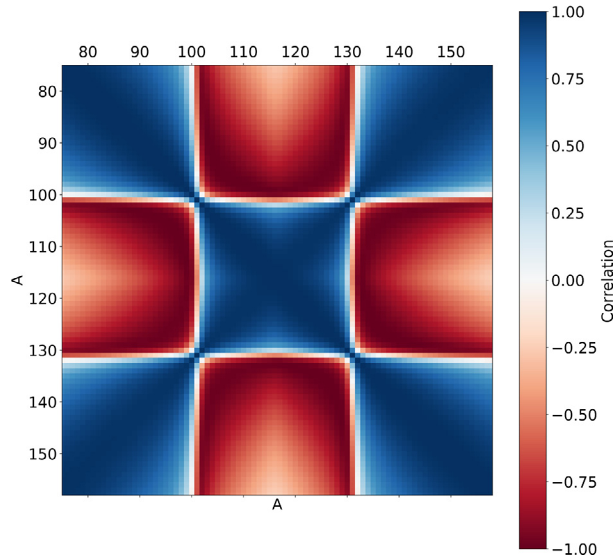


Fig. 3. Correlations between the mass yields calculated from the model given by Eq. (10).

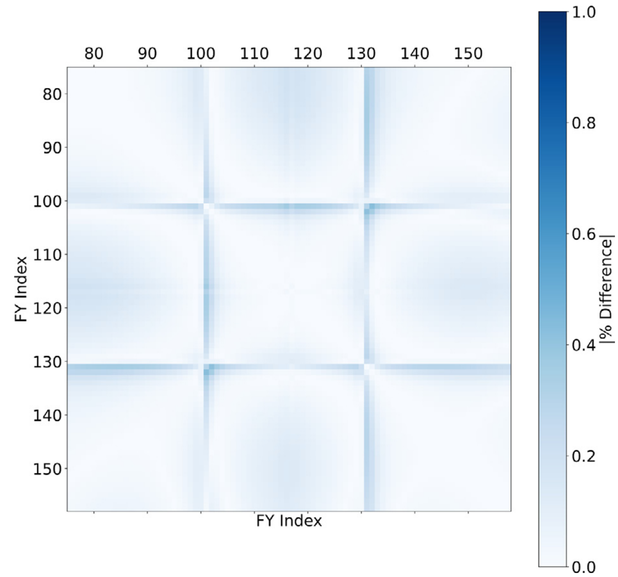


Fig. 5. Comparison between the model correlation (Fig. 3) and method correlation (Fig. 4).

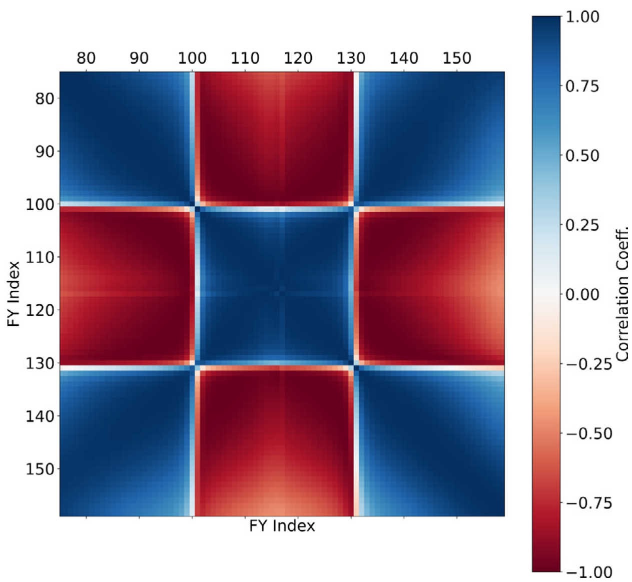


Fig. 4. Correlations between the mass yields generated from this method with resampling of half the model correlation matrix (Fig. 3).

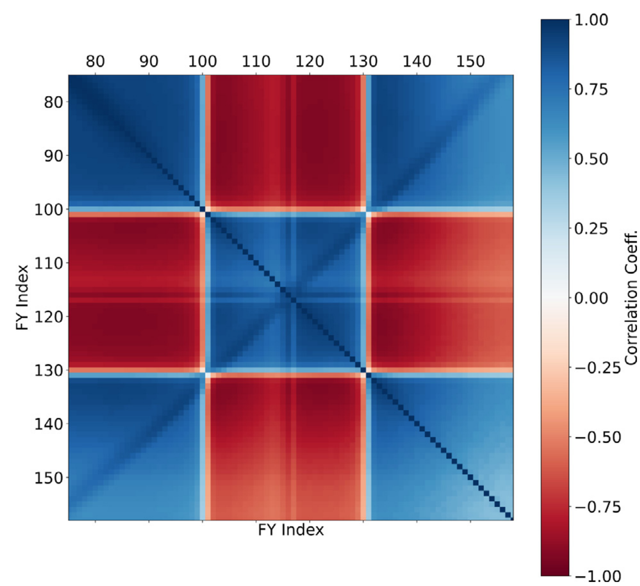


Fig. 6. Correlations between the mass yields generated from this method (without modifications to the method).

difference between the model correlations and these correlations was 9%.

In the second test, the method was not modified as in the first test; the model yields and their uncertainties were input to the method without any prior knowledge of their correlations. This reflects the same situation as when the evaluated yields are used: only their uncertainties are known, not their correlations. In this test, it is expected that the correlations will be underestimated due to the above-stated limitation that correlations between mass chains are underestimated. The correlation matrix that results from this test is shown in Fig. 6 and the difference between this correlation matrix and the model correlation matrix is shown in Fig. 7. The average absolute difference between the model correlations and these correlations was 18%. On average, the correlations from the method were 15% less than those of the model, reflecting the known limitation of the method.

This benchmarking demonstrates the overall efficacy of this method. The first test demonstrates that the method is able to reproduce known model correlations. The second test demonstrates the limitation of the method. It shows that while the known limitation is non-trivial, it is still reasonably small for a first-order estimate of fission yield correlations.

3. Results

Fig. 8 shows the independent fission yield correlation matrix that was calculated for fast fission of ^{235}U from the ENDF/B-VIII.0 evaluation. Both positive and negative correlations can be seen and indeed the diagonal is identically one. Fig. 9 shows a more illustrative subset of this data; it shows the covariance between the independent yield of ^{135}Te and those of other fission products as a function of Z and A. A number of expected trends can be

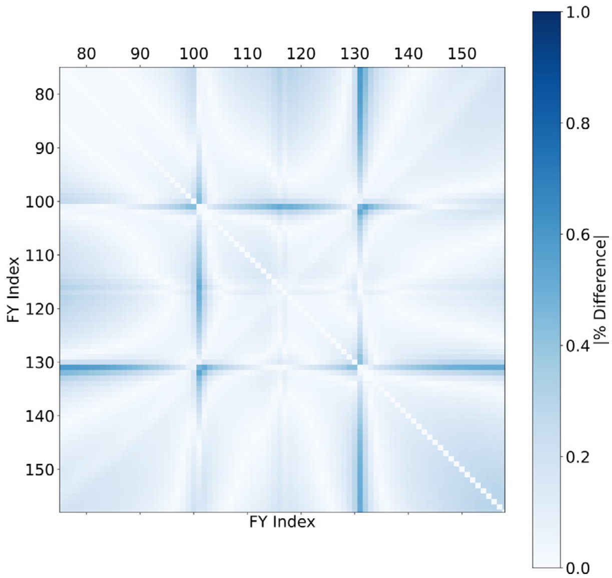


Fig. 7. Comparison between the model correlation (Fig. 3) and method correlation (Fig. 6).

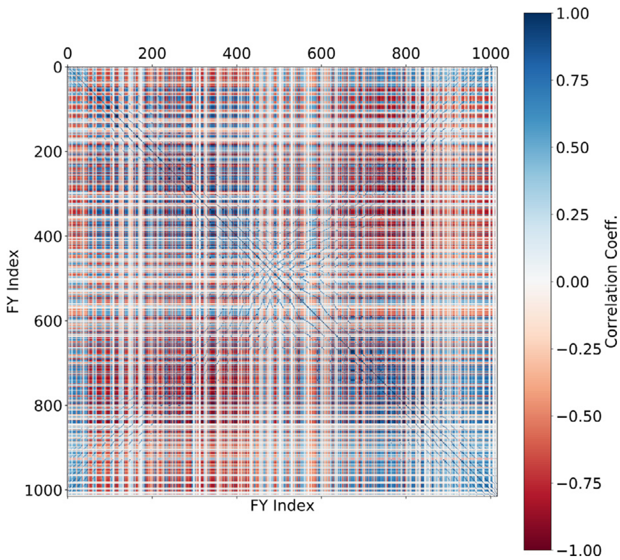


Fig. 8. The primary correlation matrix for the independent fission yields of the ^{235}U fast fission ENDF/B-VIII.0 evaluation. "FY Index" is an index assigned to each fission product and is sorted by atomic number, mass number, and isomeric number in descending order. Thus FY Index 0 has the heaviest Z and A while FY Index 1016 has the lightest Z and A.

seen in this figure. First, the yields along the $A = 135$ axis are all positively correlated; this is expected as the method varies all yields in a given A chain in tandem. Second, this positive correlation is reflected strongly along the $A = 99$ axis; this is expected as this A chain corresponds to the most probable complementary mass number for the $A = 135$ chain, $A_{CN} - A - 2$. Finally, negative covariance can be seen surrounding each voxel of strong positive covariance along each Z axis. For example, the $Z = 39$ axis features positive covariance at $A = 98$ and $A = 99$ with all other A on that axis exhibiting negative covariance. This is expected in order to conserve the normalization of $P(\nu, A)$ distributions; if the yield to one complementary product in the $P(\nu, A)$ distribution is increased the yield to other complementary products must be decreased.

Because this method does not require an underlying model of fission, it is able to be applied to any fission yield library with

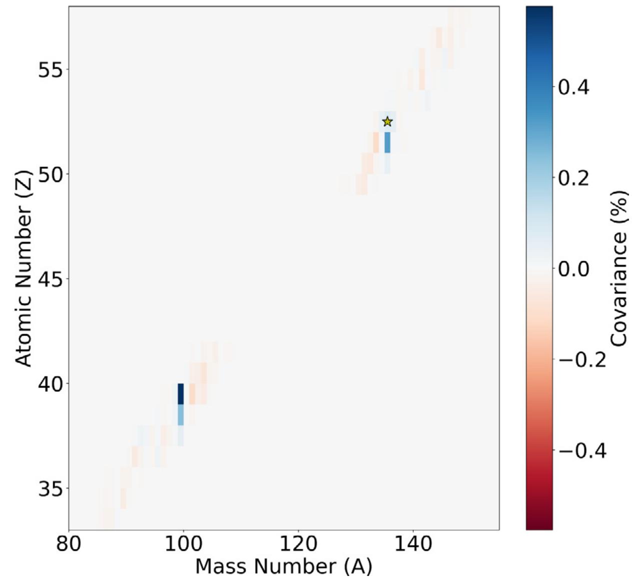


Fig. 9. A plot of the covariance between the independent yield of ^{135}Te and those of other fission products as a function of Z and A for the ^{235}U fast fission ENDF/B-VIII.0 evaluation.

uncertainties. This method was successfully applied to all of the target nuclei and energy groups in the ENDF/B-VIII.0 and JEFF-3.3 evaluations. Table A lists the systems to which this method was applied.

4. Conclusions

The method presented in Section 2 has been applied to all compound systems in the current ENDF/B-VIII.0 and JEFF-3.3 evaluations to produce independent and cumulative yield correlation and covariance matrices. This method has been benchmarked and code to generate the consistent $P(\nu, A)$ data and calculate these matrices has been preserved in Ref. [22] as an annotated reproducible workflow. In addition to this, these matrices have been published online at nucleardata.berkeley.edu/FYCoM, making them available for immediate use in applications and calculations by the nuclear science community. This marks the first time that correlation and covariance matrices have been produced for both the independent and cumulative fission product yields for all of the fissioning systems in the ENDF/B-VIII.0 and JEFF-3.3 evaluations. The presented matrices offer a first-order estimate of correlation/covariance between evaluated fission yields and serve as an interim solution until a new evaluation with a full treatment of these correlations and covariances is conducted.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table A
The target nuclei and energy groups in the ENDF/B-VIII.0 and JEFF-3.3 evaluations. This method was successfully applied to all of the systems listed in this table.

	ENDF/B-VIII.0	JEFF-3.3
Compound System	Energy Group	
^{227}Th	Thermal	
^{229}Th	Thermal	
^{232}Th	Fast DT neutrons (14 MeV)	Fast DT neutrons (14 MeV)
^{231}Pa	Fast	
^{232}U	Thermal	
^{233}U	Thermal Fast DT neutrons (14 MeV)	Thermal Fast DT neutrons (14 MeV)
^{234}U	Fast DT neutrons (14 MeV)	Fast
^{235}U	Thermal Fast DT neutrons (14 MeV)	Thermal Fast DT neutrons (14 MeV)
^{236}U	Fast DT neutrons (14 MeV)	Fast
^{237}U	Fast	
^{238}U	Fast DT neutrons (14 MeV) Spontaneous fission	Fast DT neutrons (14 MeV)
^{237}Np	Thermal Fast DT neutrons (14 MeV)	Thermal Fast
^{238}Np	Fast	Thermal Fast
^{238}Pu	Fast	Thermal Fast
^{239}Pu	Thermal Fast DD neutrons (2 MeV) DT neutrons (14 MeV)	Thermal Fast
^{240}Pu	Thermal Fast DT neutrons (14 MeV)	Fast
^{241}Pu	Thermal Fast	Thermal Fast
^{242}Pu	Thermal Fast DT neutrons (14 MeV)	Fast
^{241}Am	Thermal Fast DT neutrons (14 MeV)	Thermal Fast
^{242m}Am	Thermal	Thermal Fast
^{243}Am	Fast	Thermal Fast
^{242}Cm	Fast	Spontaneous fission
^{243}Cm	Thermal Fast	Thermal Fast
^{244}Cm	Fast Spontaneous fission	Thermal Fast Spontaneous fission
^{245}Cm	Thermal	Thermal Fast
^{246}Cm	Fast Spontaneous fission	
^{248}Cm	Fast Spontaneous fission	
^{249}Cf	Thermal	
^{250}Cf	Spontaneous fission	
^{251}Cf	Thermal	
^{252}Cf	Spontaneous fission	Spontaneous fission
^{253}Es	Spontaneous fission	
^{254}Es	Thermal	
^{254}Fm	Spontaneous fission	
^{255}Fm	Thermal	
^{256}Fm	Spontaneous fission	

Appendix. Supplementary data

In order to provide open and permanent access to the results presented in this publication, the correlation/covariance matrices and the code used to generate them are preserved in an annotated, DOI-citable Zenodo database in Ref. [22]. Zenodo is a general-purpose open-access repository developed under the European OpenAIRE program and operated by CERN [25].

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