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Authors
Kuzmiak, V
Maradudin, AA
Méndez, ER

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Surface plasmon polariton Wannier–Stark ladder

V. Kuzmiak,[1] A. A. Maradudin,[2,*] and E. R. Méndez[3]

[1]*Institute of Photonics and Electronics, Academy of Sciences of the Czech Republic, v.v.i., Chaberska 57, Praha 8 18251, Czech Republic*

[2]*Department of Physics and Astronomy, University of California, Irvine, California 92697, USA*

[3]*División de Física Aplicada, Centro de Investigación Científica y Educación Superior de Ensenada, Carretera Tijuana-Ensenada No. 3913, B.C. 22860, Mexico*

**Corresponding author: aamaradu@uci.edu*

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The propagation of a surface plasmon polariton on a planar metal surface perturbed by N equally spaced rectangular grooves, each with the same width but with varying depths, is investigated by the finite-difference time-domain method. For a linear dependence of the depth of the nth groove on n, the transmissivity of the surface plasmon polariton and of the power radiated into the vacuum above the surface, as functions of its frequency, consist of N equally spaced dips and peaks, respectively. These are the signatures of the surface plasmon polariton analog of a Wannier–Stark ladder.

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In the 1950s, Wannier [1] re-examined the problem of the motion of an electron in a periodic potential while it was being accelerated by a constant, uniform, external electric field, a problem first studied by Bloch in 1928 [2]. He found that the energy spectrum of the electron consists of equidistant discrete energy levels in the presence of the electric field, with the separation between consecutive levels proportional to the electric field strength, instead of the band structure it possesses in the absence of the electric field. These equally spaced energy levels have come to be called an electronic Wannier–Stark ladder. Wannier's prediction was confirmed in the laboratory some 20 years later [3].

Closely related with a Wannier–Stark ladder are Bloch oscillations, a periodic motion of the electrons under the influence of the external field. A Wannier–Stark ladder is the frequency domain counterpart of time-resolved Bloch oscillations [4]. These oscillations were first observed experimentally in 1992 [5,6].

In the years following this early work, experimental and theoretical searches were conducted for simpler systems, consisting of electrically neutral particles instead of electrons displaying this phenomenon. In an early effort of this kind, a Wannier–Stark ladder was observed in a system consisting of atoms moving in an accelerating optical lattice formed by two interfering laser beams [7].

More recently, an optical Wannier–Stark ladder was studied theoretically by Monsivais *et al.* [8], who investigated the transmission of transverse electromagnetic waves through a finite stratified structure whose dielectric constant at a given frequency was the sum of a periodic function of the coordinate normal to the interfaces of the structure, and a linear function of that coordinate. The transmission coefficient as a function of the angle of incidence of the electromagnetic wave displayed a Wannier–Stark ladder for some values of the parameters characterizing the structure. The first experimental observation of an optical Wannier–Stark ladder was carried out for a structure consisting of a linearly chirped Moiré grating written in the core of an optical fiber [9].

Mechanical systems have been devised that display analogs of Wannier–Stark ladders. These include stratified elastic media in which the square of the shear wave speed is a periodic function of the coordinate normal to the interfaces of the structure, supplemented by a contribution that increases linearly with that coordinate [10]; and rods with free ends whose cross sections vary in special ways along their axes, such that their torsional oscillations possess a frequency spectrum in the form of a Wannier–Stark ladder [11]. The latter frequency spectrum has been measured experimentally [11].

In this Letter, we study theoretically the analog of a Wannier–Stark ladder for a surface plasmon polariton. Specifically, we examine the propagation of a surface plasmon polariton of frequency ω across a planar metal surface perturbed by N equally spaced rectangular grooves, each of width l and varying depths $h_n (n = 0, \ldots, N-1)$, separated by planar segments of length d. The corresponding surface profile function $\zeta(x_1)$ is depicted in Fig. 1, where the propagation direction of the incident wave is indicated by an arrow on the left hand side of the structure. The region $x_2 > \zeta(x_1)$ is vacuum while the region $x_2 < \zeta(x_1)$ is a metal characterized by an isotropic, complex, frequency-dependent dielectric function $\epsilon(\omega) = \epsilon_1(\omega) + i\epsilon_2(\omega)$. We work in the frequency range in which the real part of $\epsilon(\omega)$, $\epsilon_1(\omega)$, is negative. The imaginary part of $\epsilon(\omega)$, $\epsilon_2(\omega)$ is non negative, and is assumed to be much smaller than $|\epsilon_1(\omega)|$. We will show that, for a particular choice for

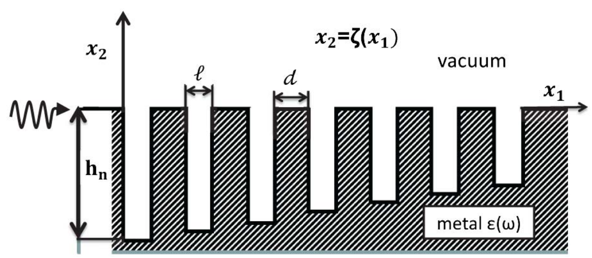

Fig. 1. Surface structure employed to obtain a surface plasmon polariton Wannier–Stark ladder.

the dependence of the depth h_n on the index n, the transmissivity of the surface plasmon polariton and the power radiated into the vacuum as functions of its frequency, display structures that have the form of a Wannier–Stark ladder.

We first consider the design of the surface presented in Fig. 1. We begin with an independent groove model [11] in which each groove supports electromagnetic resonances independently from the other grooves. To obtain analytic expressions for the frequencies of these resonances, we assume that a rectangular groove of width l and depth h is cut into the planar surface of a metal, which we assume to be silver. The width of the groove is $l = 25$ nm, and its depth is $h = 1000$ nm. The dielectric function of silver in the vicinity of the wavelength $\lambda = 616$ nm was fitted to a Drude model expression $\epsilon(\omega) = 1 - \omega_p^2/[\omega(\omega + i\gamma)]$, and the values of the plasma frequency $\omega_p = 13.07 \times 10^{15}$ rad/s and the electron scattering frequency $\gamma = 8.3607 \times 10^{13}$ rad/s [12] were obtained. The transmittance, reflectance, and power radiated into the vacuum region above the surface are calculated as functions of the frequency of a surface plasmon polariton incident normally on the groove (see Fig. 2). In the case of TM-polarization, the nonzero components H_3, E_1, E_2 of the electromagnetic field in this system are calculated within a 2D computational domain in the x_1x_2 plane, and each of the fields is represented by a 2D array. The radiated power P_2 in the x_2 direction is calculated by using the standard expression in the form $P_2 = 1/2 \int E_2 H_3^* \mathrm{d}x_1$, where the integration is carried out along an observation line parallel to the x_1 axis that is placed typically 1 μm above the metal–vacuum interface, i.e. in the near field. The results were obtained by using the OptiFDTD software [13], which employs the finite-difference time-domain (FDTD) approach [14] and produces a direct numerical solution of the time-dependent Maxwell's curl equations. Both the transmittance and the power radiated into the vacuum reveal several well-defined dips and peaks, respectively, which start to appear below the plasma frequency when the depth of the groove surpasses a threshold value.

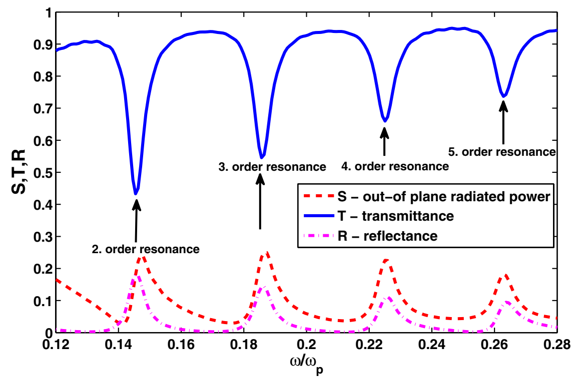

Fig. 2. Transmittance, reflectance, and power radiated into the vacuum as functions of the normalized frequency of a surface plasmon polariton incident normally on a groove of width $l = 25$ nm and depth $h = 1000$ nm cut into a planar silver surface.

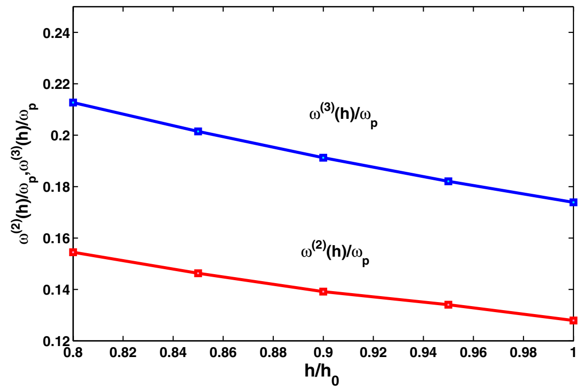

Fig. 3. Dependence of the $j = 2$ and $j = 3$ resonance frequencies on the depth h of the groove of width $l = 25$ nm cut into a planar silver surface.

By varying the depth of the groove, we have shown that the frequencies of the dips associated with both the radiated and transmitted power increase when h is decreased. Namely, in Fig. 3 we present the dependence of the frequencies associated with the second and third lowest frequency dips in transmittance for a single groove when h is varied in the range: 800 nm $< h <$ 1000 nm. If we label the frequencies of the resonances supported by the groove in the order of increasing magnitude by $\omega^{(j)} (j = 1, 2, \ldots)$ with $\omega^{(j+1)} > \omega^{(j)}$, then we can make a linear fit to the depth dependence of $\omega^{(j)}$ of the form

$$\omega^{(j)}(h) = \omega_0^{(j)} \frac{[1 + a^{(j)}(h/h_0)]}{1 + a^{(j)}}, \tag{1}$$

where $h_0 = 1000$ nm, and $a^{(j)}$ is a fitting parameter. If we now define $h_n = h_0(1 + \gamma^{(j)} n)$, then the frequency of the jth resonance in the nth groove $(n = 0, 1, \ldots)$ is given by

$$\omega_n^{(j)} = \omega_0^{(j)} \left[1 + \frac{a^{(j)}}{1 + a^{(j)}} \gamma^{(j)} n\right]. \tag{2}$$

The differences between consecutive frequencies

$$\Delta\omega_n^{(j)} = \omega_{n+1}^{(j)} - \omega_n^{(j)} = \omega_0^{(j)} \frac{a^{(j)}}{1 + a^{(j)}} \gamma^{(j)} \tag{3}$$

are then independent of n. The parameter $\gamma^{(j)}$ thus mimics the role of the electric field strength in the electronic Wannier–Stark ladder, based on the jth resonance. The values of the parameters $a^{(2)}$ and $a^{(3)}$ obtained from the results presented in Fig. 3 are found to be $a^{(2)} = -0.884$ and $a^{(3)} = -0.848$.

We now apply the preceding results to the propagation of a surface plasmon polariton of frequency ω on the structured metal surface depicted in Fig. 1. At the lowest frequencies, when ω is of the order of $\omega_0^{(2)}$, the $j = 2$ resonance in groove number 0 is excited. The modes in the remaining $N - 1$ grooves are out of resonance, so that the amplitude of the field decreases with increasing distance from the groove 0. The resulting state is, therefore, localized around this groove. When the frequency of the incident surface plasmon polariton is increased by

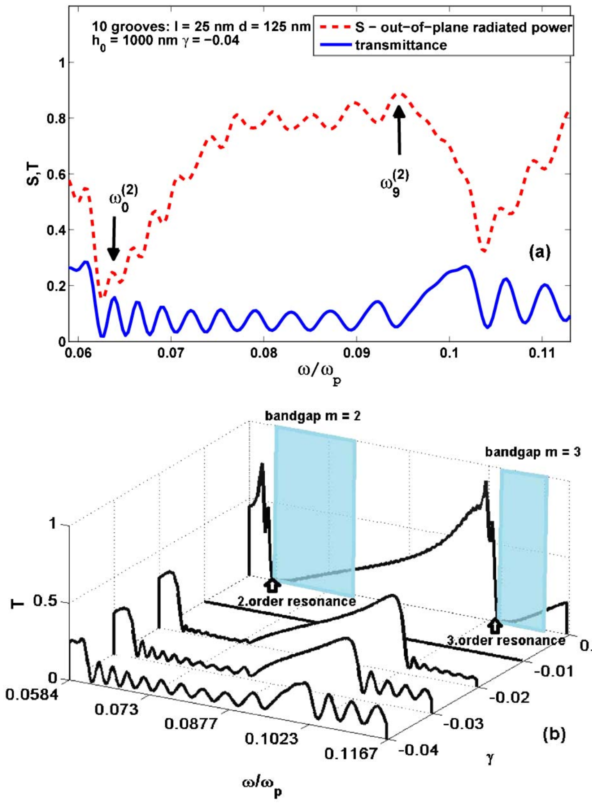

Fig. 4. (a) Transmittance and the power radiated into the vacuum as functions of the normalized frequency. (b) Transmissivity of the surface plasmon polariton as a function of its frequency for several values of the parameter γ.

$\Delta\omega_1^{(2)}$, the $j = 2$ resonance in groove 1 will now be excited, and the $j = 2$ modes in the remaining grooves will be out of resonance. The amplitude of the corresponding state is, therefore, localized around this groove. The same kind of result is obtained when the $j = 2$ resonance in groove n is excited. Thus, we have produced a finite Wannier–Stark ladder consisting of N localized states with a constant difference in frequency given by Eq. (3). These states are expected to manifest themselves as nearly equally spaced dips when the transmissivity of the surface plasmon polariton is plotted as a function of its frequency. These dips are only nearly equally spaced because the dielectric function of the metal is frequency dependent, not constant, and this can modify the resonance frequency. However, for resonance frequencies in a narrow frequency range, within which the dielectric function is a slowly varying function of frequency, the departure of the frequency differences from constancy is expected to be small.

We have calculated the power radiated into the vacuum and the transmissivity of a surface plasmon polariton propagating on a silver surface on which $N = 10$ rectangular grooves have been ruled. We have chosen to work in a frequency range that includes the frequency of the second lowest frequency resonance in each groove. The width of each groove is $l = 25$ nm and the distance between consecutive grooves is $d = 125$ nm, while the depth h_0 is $h_0 = 1000$ nm. The value of the parameter $a^{(2)}$ for this resonance frequency was found

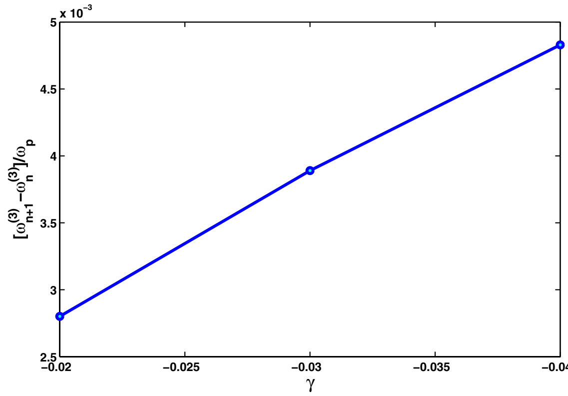

Fig. 5. Difference between the frequencies of consecutive Wannier–Stark resonances as a function of γ.

to be $a^{(2)} = -0.884$. In Fig. 4(a), we plot the dependence of the transmittance (dashed line) and the power radiated into the vacuum (solid line) as functions of the frequency of the surface plasmon polariton incident on the array of 10 grooves of decreasing depth when $\gamma = -0.04$ and where $\gamma = \gamma^{(2)}$. We observe 10 peaks in the radiated power at the frequencies $\omega_n^{(2)}$, $n = 0, \ldots, 9$ in the frequency range between the two resonances $\omega^{(2)}$ and $\omega^{(3)}$, which correspond to the dips in the transmittance at the same frequencies. To illustrate the gradual modification of the frequency dependence of the transmittance in this frequency range as the parameter $|\gamma|$ is increased, in Fig. 4(b) we plot this dependence for several values of $|\gamma|$ in the range $-0.04 \leq \gamma \leq 0$. In the case where $\gamma = 0$, the bandgaps associated with the corresponding finite periodic structure are indicated by shaded areas. The frequency of the lower and upper edges of each gap were determined from the dips and peaks in the transmittance and in the radiated power, respectively, in the manner described by López-Tejeira *et al.* [15]. This curve demonstrates the difference between the behavior of the transmittance associated with a periodic array of 10 identical grooves and that of the single groove shown in Fig. 2. We note that the transmittances for both cases resemble each other to some extent, since the interactions among the grooves in the case considered are weak.

In Fig. 5, we show the dependence of the difference between the frequencies of consecutive dips in the frequency dependence of the transmittance as a function of γ. It is seen to be very close to the linear dependence expressed by Eq. (3) for $\Delta\omega_n^{(3)}/\omega_p = (\omega_{n+1}^{(3)} - \omega_n^{(3)})/\omega_p$, for the values $n = 0, 1, 2, \ldots$. The index n now labels the dips in the transmittance, rather than the grooves, and $\Delta\omega_n^{(3)}/\omega_p$ is, for all practical purposes, found to be independent of n.

In conclusion, we have constructed an analog of a Wannier–Stark ladder for surface plasmon polaritons. We note that grooves with shapes other than rectangular also support electromagnetic surface shape resonances [16], so that the metal surfaces with surface profile functions different than the one assumed here can also be expected to display surface plasmon polariton Wannier–Stark ladders. The observation of these ladders through the use of a prism or grating coupler should be feasible.

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