Title
CONSIDERATIONS ON RADIOFREQUENCY HEATING

Authors
Garren, A.
Riddell, R.J.

Publication Date
1956-07-01

UCRL-3467

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

BERKELEY, CALIFORNIA

DISCLAIMER

UNIVERSITY OF CALIFORNIA
Radiation Laboratory

Cover Sheet
Do not remove

INDEX NO. UCRL-3467
This document contains 12 pages
This is copy 4 of 5 series A
Date July 1956

~~RESTRICTED DATA~~
~~This document contains restricted data as defined in the Atomic Energy Act of 1954. Its transmittal or the disclosure of its contents in any manner to an unauthorized person is prohibited.~~

DECLASSIFIED

Issued to Information Division

~~SECRET~~
Classification

Each person who receives this document must sign the cover sheet in the space below.

Route to	Noted by	Date	Route to	Noted by	Date
EOLawrence					

RL-220

~~SECRET~~

UCRL-3467

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

4

DECLASSIFIED

CLASSIFICATION CANCELLED
BY AUTHORITY OF THE DECLASSIFICATION
BRANCH USAEC ON 9-30-58
DATE
By B Fokeit
SIGNATURE OF THE
PERSON MAKING THE
CHANGE

CONSIDERATIONS ON RADIOFREQUENCY HEATING

A. Garren and R. J. Riddell, Jr.

July 1956

unauthorized
person is prohibited.

Printed for the U. S. Atomic Energy Commission

~~SECRET SECRET~~

CONSIDERATIONS ON RADIOFREQUENCY HEATING

by

A. Garren and R. J. Riddell, Jr.

University of California Radiation Laboratory
Berkeley, California

The possibility of accelerating ions in an axial dc magnetic field by means of a comparatively small ac component alternating at the ion gyrofrequency is discussed from a single-particle standpoint. The individual particle motions are described, and estimates made for the individual fields and consequent limitations on density.

INTRODUCTION

At the Princeton meeting the possibility was discussed of accelerating the ions in a plasma contained by an axially symmetric longitudinal magnetic field by means of an azimuthal electric field alternating at the cyclotron frequency of the ions in the static field. We outline here the conclusions we have reached since then. Such an ac electric field may be provided by an alternating current in coils concentric with those providing the dc field. As we will see, such a field would accelerate individual particles. The question is whether, in a plasma, such a field will penetrate enough to cause the acceleration. Our approach is to assume that it will penetrate, then to calculate the particle motions in that field. Knowing the behavior of the particles, one then calculates the fields induced by the resultant charge and current densities. At densities for which these induced fields are small compared with the external fields, one is inclined to believe that the acceleration will occur as predicted. At higher densities, for which the induced field so calculated is larger than the applied field, one suggests that it will not--at least to such a high degree. Thus we regard the calculations, outlined below, as determining roughly the density at which direct coupling between the external field and the ions is broken.

IONIC ORBITS

We consider the orbits of the ions in combined dc and rf axial magnetic fields, wherein the azimuthal electric field associated with the latter accelerates the ions. To simplify the analysis we specialize to the case in which both magnetic fields are uniform and purely axial. Then the nonvanishing field components may be taken to be

$$H_z = -H_0(1 + \alpha \cos \omega t), \quad E_\theta = -\frac{\alpha\omega H_0}{2c} r \sin \omega t .$$

We describe the motion in the transverse x-y plane by a complex number

$$w = x + iy ,$$

and, further, introduce

$$\omega_0 = \frac{eH_0}{Mc}, \quad \tau = \omega t , \quad \frac{\omega_0}{\omega} = 1 + \eta .$$

We assume that both α and η are much less than one. Then the equation of motion in the transverse plane is, to the first order in α and η,

$$\frac{d^2 w}{d\tau^2} - i(1 + \eta + \alpha \cos \tau) \frac{dw}{d\tau} + \frac{1}{2} i \alpha \sin \tau \, w = 0 .$$

Since this is an equation with periodic coefficients, we may apply Floquet theory, and one finds that the solutions may be written as follows, to the first order in α and η:

$$w = a + be^{i\tau} ,$$

$$\delta = \sqrt{\eta^2 - \alpha^2/4} , \quad \lambda = \alpha/2\delta , \quad \mu = \eta/\delta = \sqrt{1 + \lambda^2} , \quad x = \frac{1}{2} \delta(\tau - \tau_0) ,$$

$$a(\tau) = e^{i\mu x}\left\{(\cos x - i\mu \sin x)a(\tau_0) - i\lambda \sin x \, b(\tau_0)\right\} ,$$

$$b(\tau) = e^{i\mu x}\left\{i\lambda \sin x \, a(\tau_0) + (\cos x + i\mu \sin x) \, b(\tau_0)\right\} ,$$

whence

$$r_0^2 = |a|^2 - |b|^2 = \text{constant} .$$

The ion moves in a circle of radius $|b|$ centered at a (Fig. 1). Both a and b change very slowly compared with the motion of the particle in its circle as a result of the slow x variation, and their value at τ depends linearly on their value at τ_0, the coefficients depending only on the elapsed time $\tau - \tau_0$ (see Fig. 1).

Suppose the ion starts from rest at $\tau = \tau_0$, so that $|a(\tau_0)| = r_0$, $b(\tau_0) = 0$. Then the velocity $v(\tau) \approx ibe^{i\tau}\omega$, so that the energy is

$$E = \frac{1}{2} m\omega^2 b^2 = \frac{1}{2} m\omega^2 r_0{}^2 \lambda^2 \sin^2 \frac{\delta}{2}(\tau - \tau_0) .$$

If $|\eta| < |\alpha/2|$, δ is imaginary, and the particle's orbit radius and energy will increase indefinitely until it strikes the wall of the container and is lost. To prevent such loss it is necessary to keep $|\eta| > |\alpha/2|$, so δ is real. In this case $|b| = \lambda \sin x \cdot r_0$, and the circular orbit swells and shrinks sinusoidally, with a period of $1/\delta$ rf cycles. One gets the greatest acceleration when δ is very small, but it takes longer to reach the peak energy (see Fig. 2).

IONIC CHARGE, CURRENT, AND INDUCED FIELD

We suppose that sometime after the rf is turned on an equilibrium is established in which cold ions are continuously being formed, then picked up by the rf, undergoing a good many acceleration cycles, and eventually lost by charge exchange or scattering. We assume that the cold ions are formed randomly in space and time within the tube with density n_0. Since we know their position at time τ in terms of their starting time τ_0 and position w_0, we can find their density at position w and time τ by suitable integrations over their initial distribution and starting time τ_0. Let $n(w, \tau, \tau_0)$ be the density of particles at position w, and time τ that started from rest at τ_0. The calculation is briefly summarized by the following equations:

$$w(\tau, \tau_0) \equiv f(\tau, \tau_0) w(\tau_0) ;$$

$$f(\tau, \tau_0) = e^{i\mu x} [(\cos x - i\mu \sin x) + i\lambda \sin x \, e^{i\tau}] ; \quad x = \frac{\delta}{2}(\tau - \tau_0) ;$$

$$n(w, \tau, \tau_0) d^2 w = n(w_0, \tau_0, \tau_0) d^2 w_0 \qquad \text{(Conservation of particles)}$$

$$= n_0 d^2 w_0$$

$$= n_0 \frac{d^2 w}{|f(\tau, \tau_0)|^2} ;$$

$$n(w, \tau) = n_0 \frac{1}{T} \int_{-T}^{\tau} \frac{d\tau_0}{|f(\tau, \tau_0)|^2} \Rightarrow n(w, \tau) = \psi(|w|^2 (\mu - \lambda \cos \tau)^{-1}) .$$

Actually a further complication is caused by the fact that we want to exclude from $n(w, \tau)$ the contribution of particles that at some time will strike the wall. This is taken into account by excluding certain values of τ_0 from the above integration.

The density $n(w, \tau)$ so obtained is shown in Fig. 3, where n is plotted against $r = |w|$, for various values of τ. One sees that the positive charge distribution is fluctuating in a complicated way with the rf. The nature of the fluctuation is that of a scale transformation; specifically, n is a function only of $r^2/(\mu - \lambda \cos \tau)$. A consequence is that the total charge within any circle whose radius fluctuates according to $r^2/(\mu - \lambda \cos \tau) =$ const. is constant in time, that is, the charge "pants." The curves shown are for the case $\lambda = \alpha/2\delta = 1$, and $\omega_0 > \omega$. If $\omega_0 < \omega$ there is a phase change of 180° in the behavior of n.

In a similar way one may calculate the ionic current density, and from this the magnetic field thereby induced. This field has the form

$$H_z^{\text{ind.}} = \frac{\pi e \omega a^2 n_0}{c} h(r, \tau) .$$

Curves of h vs r for various τ's are shown in Fig. 4. This induced field has an inphase component opposed to the applied field, therefore one expects that it will stop the acceleration when it is comparable with the applied field, that is, when we have

$$\frac{\pi e \omega a^2 n_0}{c \alpha H_0} \sim 1 .$$

If $\alpha = H_{rf}/H_{dc} \sim 1\%$, $a \sim 10$ cm, this gives a limiting density of about 10^{12} particles/cm^3. This density limitation may be expressed in terms of β as

$$\beta \equiv \frac{n_0 E_{av}}{H^2/8\pi} \lesssim \frac{1}{5} \frac{H_{rf}}{H_{dc}} .$$

ELECTRONIC MOTION

Besides the limitation imposed on density by the ionic diamagnetism, we have also to consider the effects of possible charge separation and electronic currents. First of all we observe that the direct response of the electrons to the external field is comparatively unimportant. For the rf is so slow compared to the electron's cyclotron frequency that for them the adiabatic approximation is good. Thus we may conclude that the electron's transverse energy will always be proportional to the magnetic field, which only fluctuates by a few percent. Hence originally thermal electrons cannot be accelerated by the applied field. Also the electrons cannot cross the field lines, so that they cannot directly follow the ions. If some other means is not available for the electrons to follow the fluctuations in positive charge density, large charge separations and enormous electric fields (capable of stopping the ionic acceleration) will ensue.

A possible mechanism for charge neutralization consists of axial electronic currents to and from metal plates. The electrons would flow out of the plasma to the plates at radii where the positive density is decreasing, through the plates, and back from the plates into the plasma at radii where the positive charge is increasing.

We have attempted an approximate treatment of such a mechanism as follows. We assume, first, that the positive charge distribution is known to be that just described. Next, as a first approximation, we assume that the electron density is exactly the same as the ionic--there is no charge separation. Using the equation of continuity, we can easily deduce the axial electronic velocities and currents that will make this so. If we know the axial electron velocity we also know the axial electric field that will accelerate the electrons to this velocity. We can then add other field components in such a way as to satisfy Maxwell's equations.

To simplify the analysis we replaced the charge distribution shown previously by replacing that part of n that varies with r by a constant, but keeping the inner (constant) density, the boundaries of the two regions, and the total charge within them unchanged. The resultant fields are

$$H_\theta = -\frac{2\pi e n_0 \omega}{c}\,\frac{\lambda \sin\tau}{\xi^2}\,rz\begin{pmatrix} 1 \\ \dfrac{1-\gamma}{2\lambda}\end{pmatrix}, \quad \begin{array}{l} r^2 < \gamma^3\xi a^2, \\ \gamma^3\xi a^2 < r^2 < r\xi a^2, \end{array}$$

$$E_z = -\frac{m\omega^2\lambda\cos\tau}{e\xi}\,z,$$

$$E_r = \frac{\pi n_0 e\omega^2}{c^2}\;\frac{2\lambda\sin^2\tau - \xi\cos\tau}{\xi^3}\,r(\ell^2 - z^2)\begin{pmatrix} 1 \\ \dfrac{1-\gamma}{2\lambda}\end{pmatrix} \quad \begin{array}{l} r^2 < \gamma^3\xi a^2, \\ \gamma^3\xi a^2 < r^2 < e\xi a^2, \end{array}$$

$$\xi = \mu - \lambda\cos\tau .$$

By taking div $\vec{E}$ one may obtain the charge separation. Since for cases of interest this turns out to be very small compared with n itself, we believe the above fields give a correct order-of-magnitude estimate of the fields induced by charge separation and electronic currents. The electric field so obtained is comparable to the applied one at densities of the order of $10^{11}/cm^3$, so that if one is conservative one should regard this as the limiting density for penetration. However, since it appears that the phase relationships are such as to make this field aid the ionic acceleration, it may be that the ionic diamagnetism previously described is the only mechanism that prevents acceleration.

NONUNIFORM FIELDS

We have done certain calculations with axially symmetric nonuniform fields in order to get an idea of the extent to which the general nature of the results previously described might be altered for this reason.

We considered the departures from flatness of the field to be small, and then derived equations for the radius of curvature b and the center of the orbit a. Suppose that the field is derived from a vector potential:

$$A_\theta = - H_0 \sum_0^\infty \frac{1}{n} (\beta_n + \gamma_n \cos \tau) \cos nz\, I_1(nr) ,$$

where $\quad \Sigma\beta_n = 1 , \quad -\Sigma n^2 \beta_n = \beta , \quad \Sigma\gamma_n = \gamma , \quad \beta, \gamma << 1 ,$

and then define

$$u \equiv |b|^2 , \quad v \equiv |a|^2 , \quad x \equiv i(a^* b - ab^*) , \quad y \equiv a^* b + ab^* ,$$

where $w \equiv a + be^{i\tau}$ and $\dot{w} \equiv ibe^{i\tau}$. It follows that

$$x^2 + y^2 = 4\, uv .$$

Then the equations of motion are approximately

$$\dot{u} = \frac{-\gamma}{4} x + \beta z\dot{z}u , \quad \dot{v} = \frac{-\gamma}{4} x - \beta z\dot{z}v , \quad \dot{x} = - \frac{\gamma}{2}(u + v) - [\eta + \frac{\beta}{2}(z^2 - u - \frac{1}{2} v)]y ,$$

$$\dot{y} = [\eta + \frac{\beta}{2}(z^2 - u - \frac{1}{2} v)]x .$$

For motion in the median plane we have $\dot{z} = z = 0$, and by elimination one obtains

$$\ell = v - u = \text{const.} \quad E \equiv u + \frac{\ell}{3} ,$$

$$\frac{d^2E}{d\tau^2} = - \frac{9}{32} \beta^2 E^3 + \frac{9}{8} \beta\eta E^2 + (-\eta^2 - \frac{3}{4} \beta\eta E_0 + \frac{9}{32} \beta^2 E_0{}^2 - \frac{3}{16} \gamma\beta y_0 + \frac{\gamma^2}{4})E$$

$$+ (+ \frac{\gamma^2}{24} \ell + \frac{\gamma\eta}{4} y_0 + \eta^2 E_0 - \frac{3}{8} \eta\beta E_0{}^2) .$$

If we regard the energy E as a coordinate, we see that it moves in a quartic potential given by the integral of the R. H. S. Since for large enough E the potential becomes arbitrarily large and positive, E is always bounded, and there are no indefinitely runaway solutions such as we found could exist in the flat-field case. The energy in all cases will oscillate between certain limits imposed by the initial conditions. Thus the general behavior of the particles is not radically different from the flat-field case formerly described.

CAPTIONS FOR FIGURES

Fig. 1. An orbit for small x, when particle is still in phase with the rf. At $\tau = 270^\circ$ the electric field is counterclockwise, and to be accelerated the particle must be at the outward point of its orbit.

Fig. 2. A sequence of orbits for a particle during an acceleration cycle. The particle starts at rest at time $x = (1/2)\,\delta(\tau - \tau_0) = 0$ at the point on the horizontal axis labeled $x = 0^\circ$. The dots show the position of the particle in its orbit when the rf phase angle $\tau = 0$ (so that $E_\theta = 0$). The circles show the orbits of the same particle for subsequent values of x. The largest orbit is 90° out of phase with the rf (cf Fig. 1).

Fig. 3. Positive-ion density as a function of radius r from the axis. A is the tube radius. Each curve is for a different rf phase angle τ. The positive-ion density pants with the rf.

Fig. 4. Curves showing the variation of the axial magnetic field induced by ionic currents with radius and rf phase angle τ.

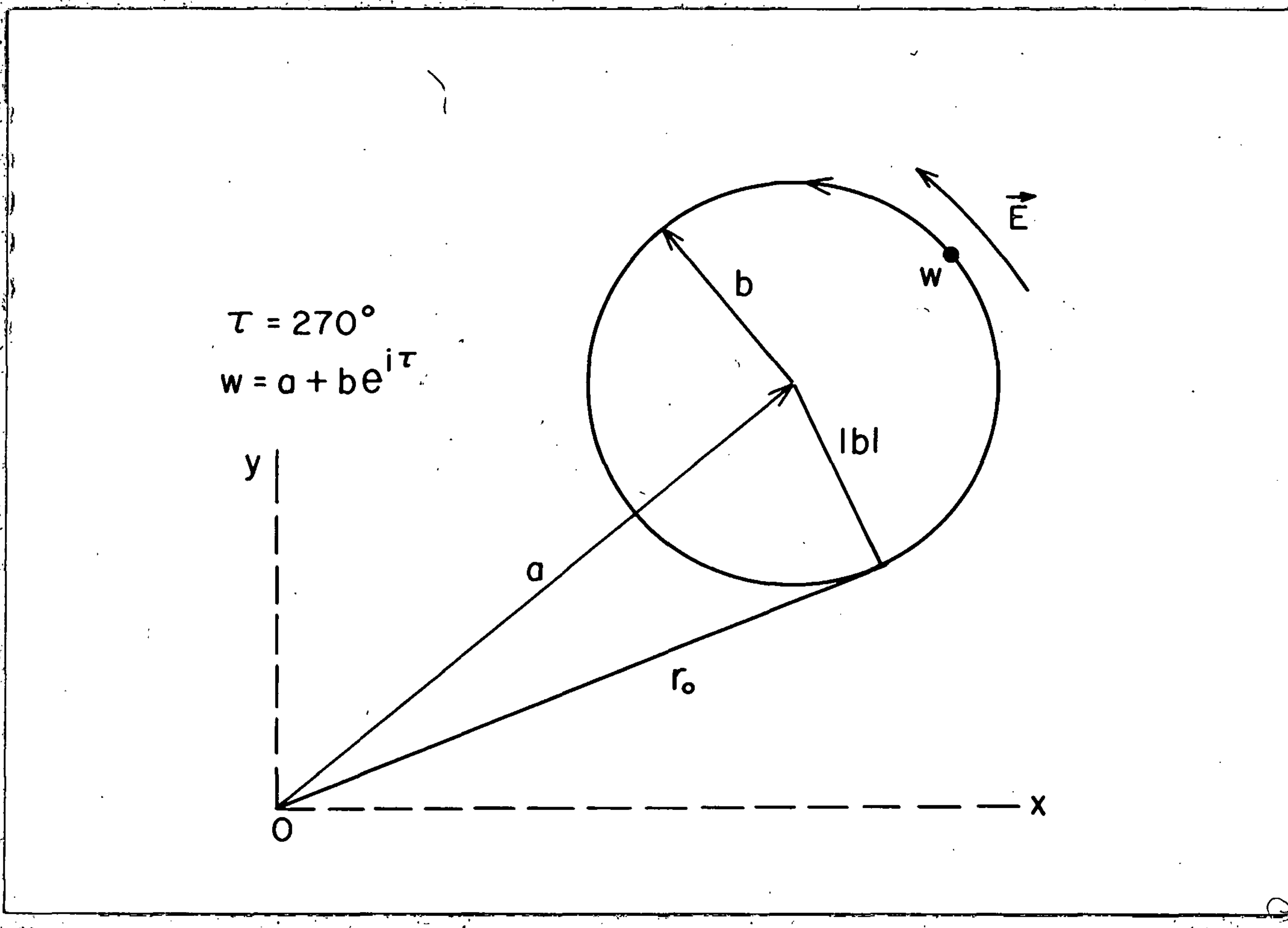


Fig. 1

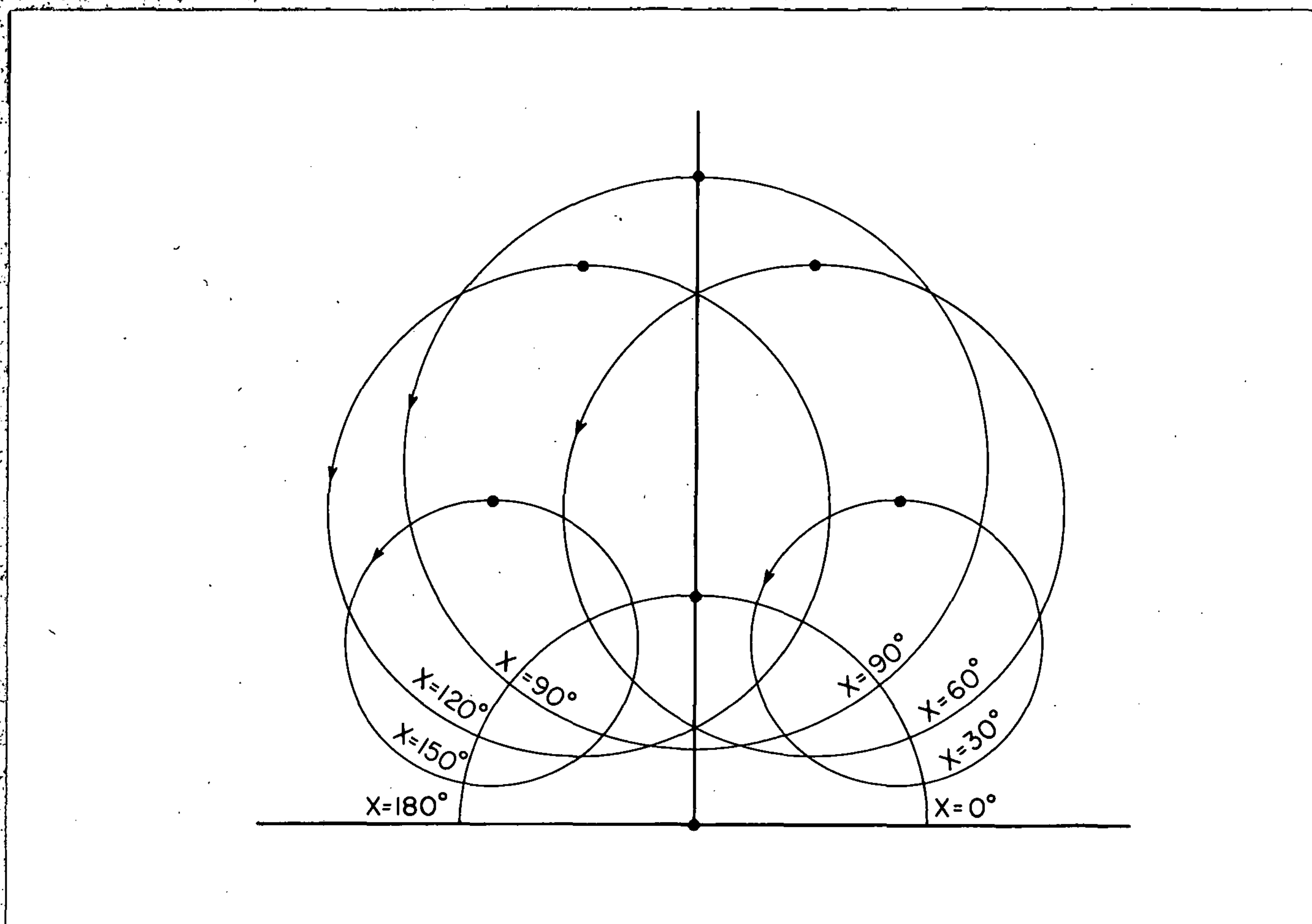


Fig. 2

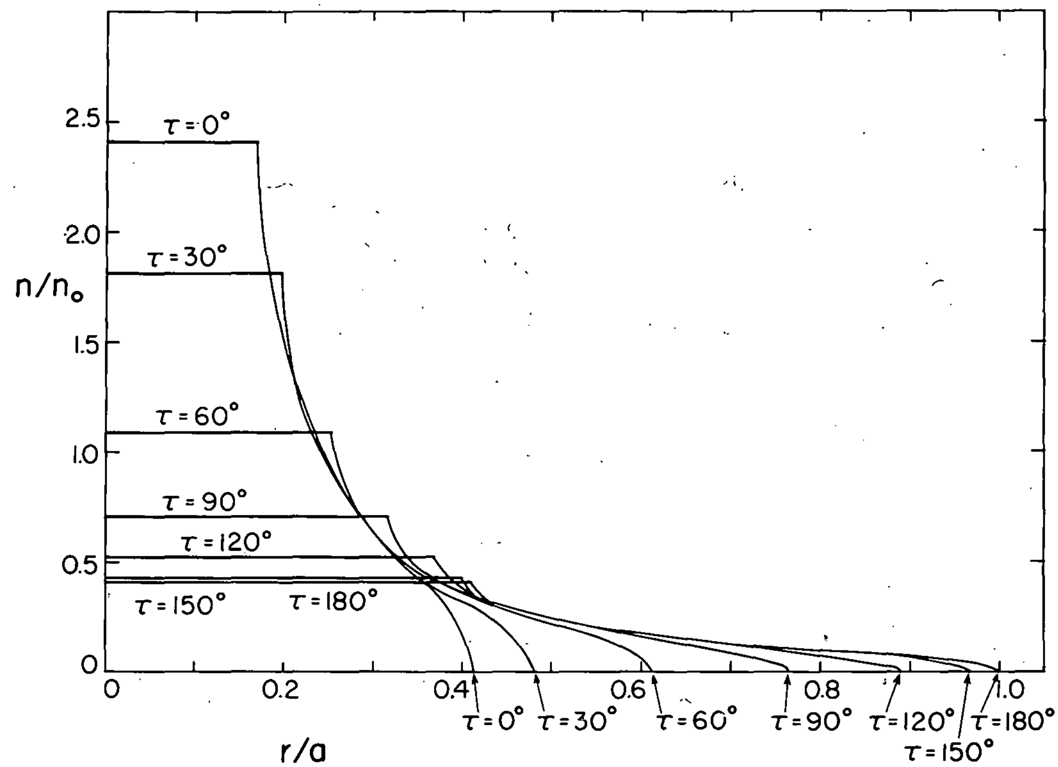
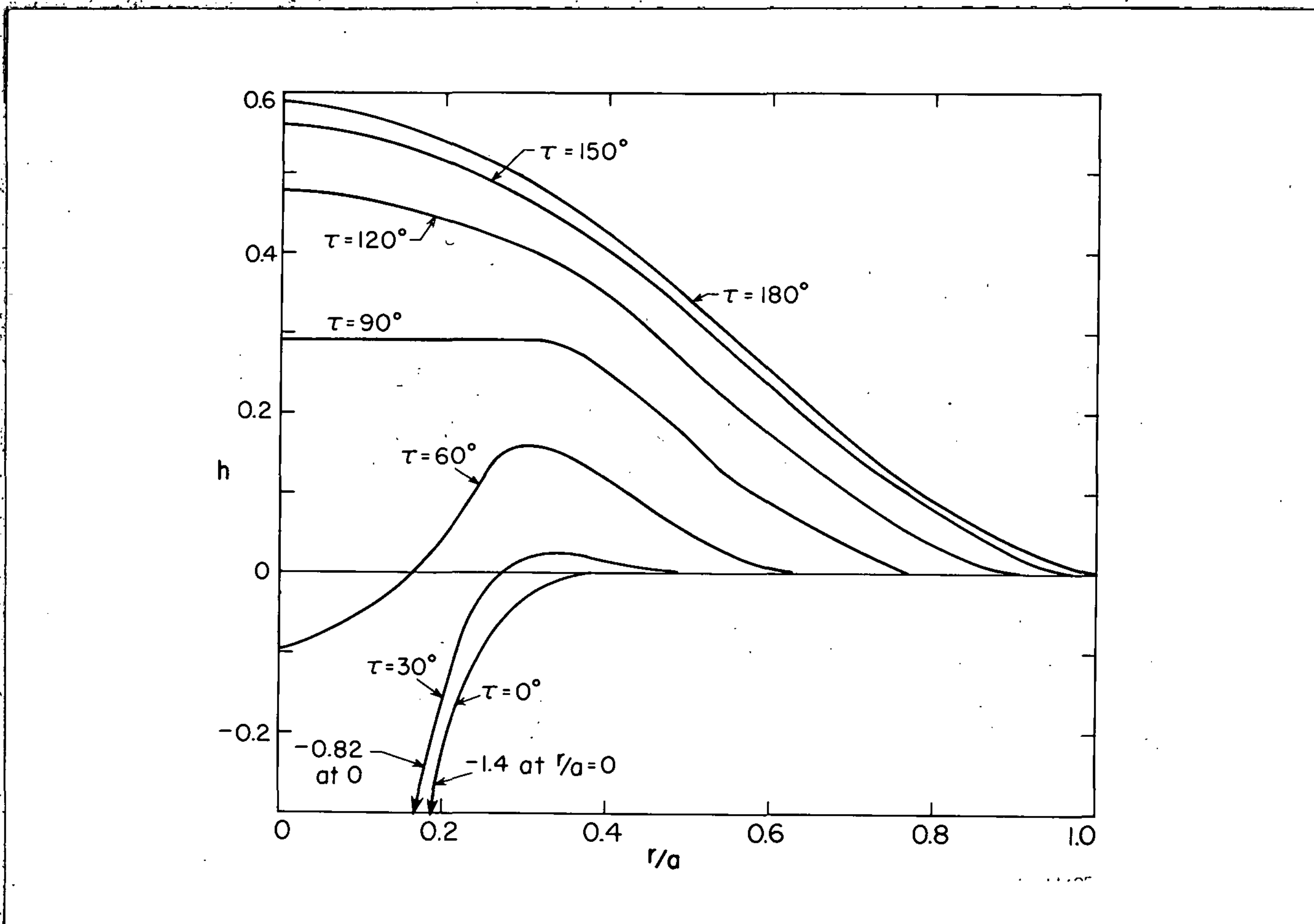

n/n$_o$
2.5
2.0
1.5
1.0
0.5
0
0
0.2
0.4
0.6
0.8
1.0
r/a
τ = 0°
τ = 30°
τ = 60°
τ = 90°
τ = 120°
τ = 150°
τ = 180°
τ = 0°
τ = 30°
τ = 60°
τ = 90°
τ = 120°
τ = 180°
τ = 150°

Fig. 3



τ = 150°
τ = 120°
τ = 180°
τ = 90°
τ = 60°
τ = 30°
τ = 0°
−0.82 at 0
−1.4 at r/a = 0
h
r/a
0.6
0.4
0.2
0
−0.2
0
0.2
0.4
0.6
0.8
1.0

Fig. 4

~~SECRET~~

DECLASSIFIED

DECL

~~SECRET~~