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ABSTRACT

A model is proposed to directly relate observed tidal action with orbital movements of the moon and the earth. The basic model consists of a unit mass acting as a free point on the rotating earth's surface and being carried into a field gradient. The field is such as to cause the free point to "fall" along the surface, gain a velocity with respect to the earth's surface, and thus produce a tidal interaction at coastal barriers. Gyroscopic and resonant oscillatory current effects are not treated.

The Tides: A Dynamic Explanation Based Upon Fundamental Physical Concepts*

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Historically speaking, the problem of relating tidal action to earth rotation and to gravitational interaction with the moon and sun has been examined by many investigators[1] including Newton. Apparently, little success has been met in explaining specific tidal action from fundamental concepts with the conventional equilibrium and resonant oscillation models[1-5]. This tract presents an approach which explains as natural consequences, certain "anomalies", i.e., substantial tidal amplitude differences in areas less than a hundred miles apart, 180° phase shift in tidal actions across thin north-south land barriers, and diurnal tides. As an illustration, we may compare the tide data[6]

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* The major portion of this study was completed late in 1960.

1. H. A. Marmer, The Tide, Appleton and Co., 1926. Chapter Two presents a brief historical development of the subject.

2. A. Defant, Ebb and Flow, University of Michigan Press, c. 1958.

3. W. H. Munk and G. J. F. McDonald, The Rotation of the Earth, Cambridge at the University Press, 1960.

4. J. Proudman, Dynamical Oceanography, John Wiley & Sons, 1953.

5. P. Schureman, Manual of Harmonic Analysis and Prediction of Tides, U.S. Dept. of Commerce, Coast and Geodetic Survey, 1941.

6. U.S. Dept. of Commerce, Coast and Geodetic Survey, Tide Tables, 1961.

for three locations around the Florida Cape on January 1, 1961. At Miami Harbor entrance, high tide appears at the time 0748, followed by a low tide at 1357; at Pavilion Key, directly west and across the peninsula from Miami, a low tide appears at 0746, followed by a high tide at 1344. The tide amplitude at Miami compares with that at Pavilion Key, but is essentially 180° out of phase. A station at Key West shows a tide somewhere in between in phase; however, the amplitude is approximately one-third as large due to the freer water movement allowed in the Keys. Other stations of the same approximate latitude show a similar correlation. However, allowance for different zenith times must be considered with wider land masses.

Without denying they exist, no consideration is taken herein for resonant oscillatory water movements.

The approach here is to consider a free point P, on the surface of the rotating earth, moving with the earth surface from an essentially neutral plane into a field gradient which is predominately either gravitational or centrifugal in the classical sense. The simplified model is diagrammed and explained in Figure 1. Figure 2a illustrates the force amplitudes on P_1 and P_2 resulting from the earth-moon interaction. Figure 2b illustrates the similar forces due to the earth-moon system's interaction with the sun. Superimposed upon the illustrated interactions would be additional variation due to earth wobble resulting from the earth and moon rotating about their common center of gravity, and to eccentricity factors. The lunar orbit is eccentric due in part to solar "tidal" forces expanding the orbit twice during each lunar rotation.

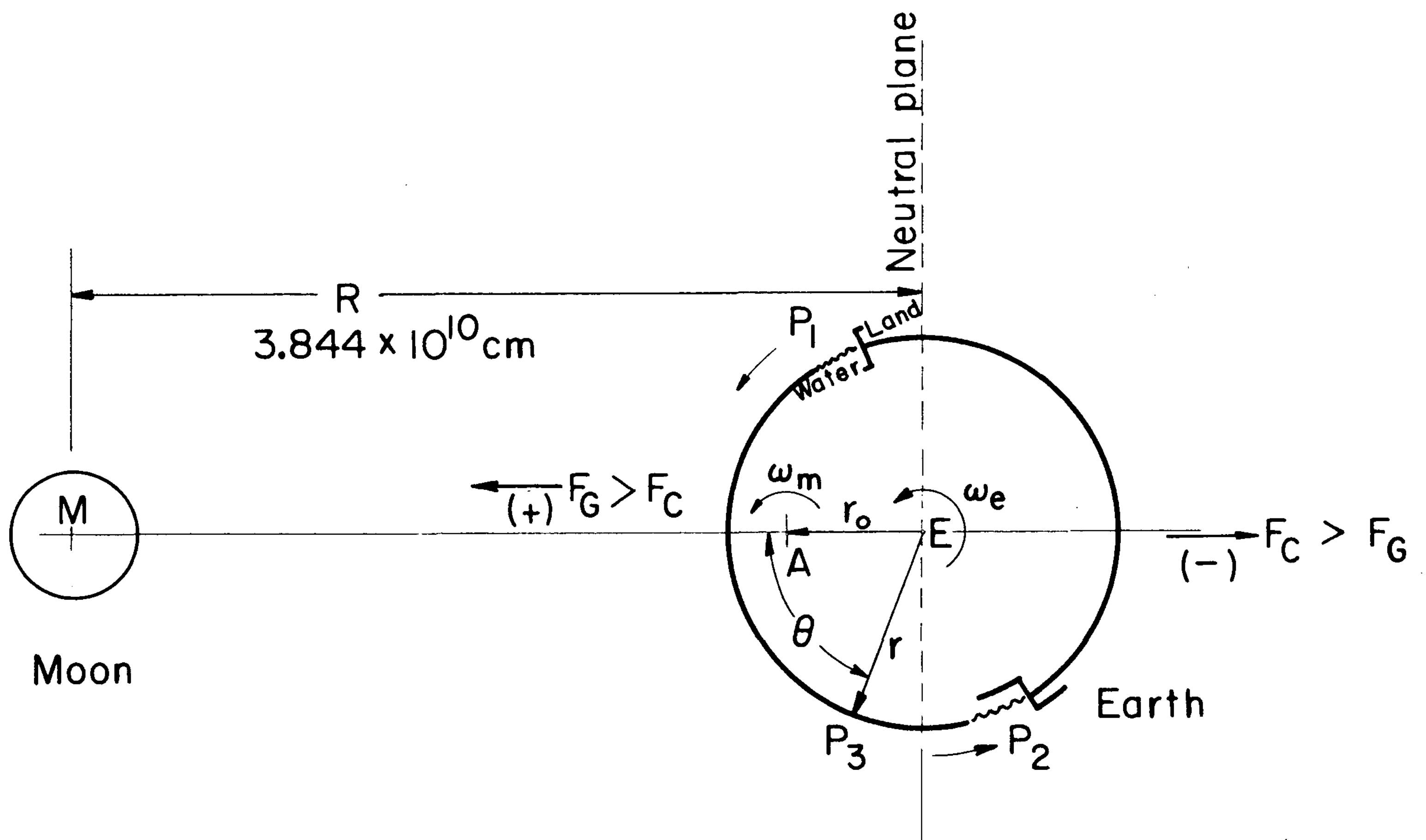

Fig. 1. The rotating earth-moon system. A plane intersecting the earth at its center E, and perpendicular to line ME, may be considered a neutral (or reference) plane with respect to the earth-moon system. F_C is the centrifugal force resulting from the earth-moon rotation ω_m around A, and F_G is the gravitational interaction of a unit mass on the earth's surface with the moon M. The centrifugal force due to rotation around A is essentially balanced at the neutral plane intersection by gravitational attraction between the moon and earth. A fluid at P_1 (east of barrier) will tend to flow away from the coastline, and a fluid at P_2 (west of barrier) will tend to flow toward the coastline, as the earth rotates from the indicated position. A low tide is displayed by P_1 each time it crosses the line through M and E. A high tide is displayed by P_2 at these same intersections.

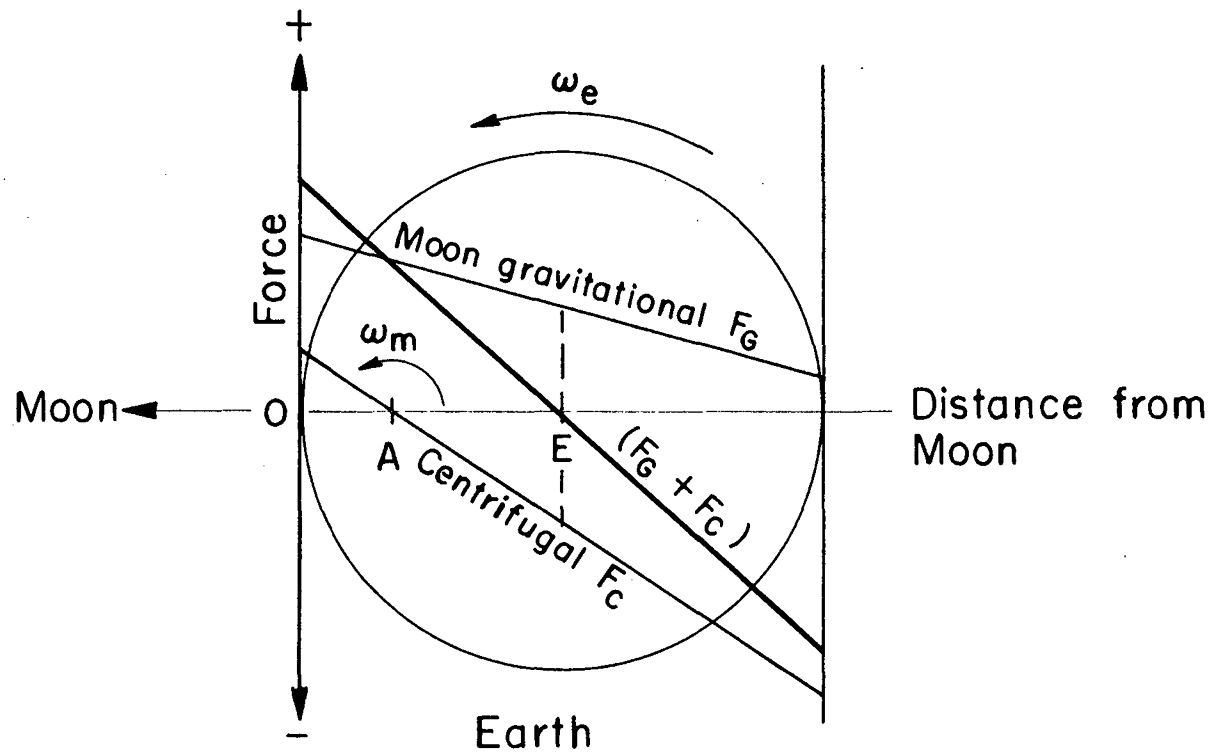

Fig. 2A. Forces on a unit mass as a result of earth-moon-system gravitation and rotation ω_m around A.

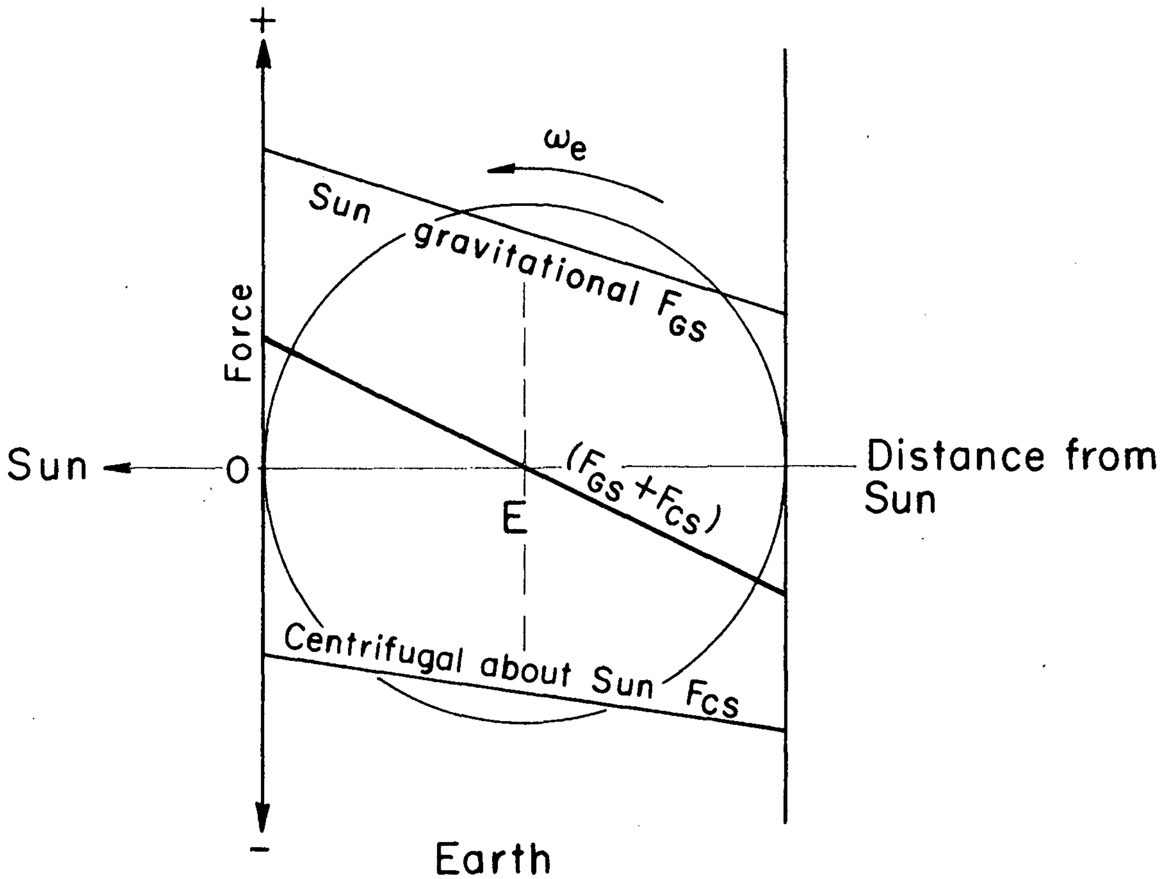

Fig. 2B. Elementary forces due to sun gravitation and rotation of the earth-moon system about the sun. Expanded force scale.

The effect is that a free point[7] on the rotating surface facing the moon sees a predominantly gravitational force directed toward the moon, and the surface away from the moon sees predominately centrifugal force directed away from the moon. As a consequence, we see the two tides each lunar day -- this is of course modified by a similar, although smaller, pair of tides each solar day due to the interaction between the sun and earth.

In Figure 1, the earth is rotating around its axis E so as to carry a fluid at P_1 away from the neutral plane intersection and into an increasing force field. We have a situation analogous to an unbalanced rotating wheel with the heavier portion being carried over the upper neutral center and starting to gain velocity. The result is that a fluid will gain a velocity faster than the earth surface as it leaves the area of the neutral plane. At P_2 we have a similar situation, but with a fluid following rather than preceding a barrier -- fluid will accumulate against the barrier, rather than flow away from it. With this viewpoint, we immediately see two consequences: (1) A thin north-south line barrier having a rising tide on one side, should have an ebbing tide on the other, and (2) a barrier (coast line) along the equator would have essentially no tide at all if the planes of the equator, the ecliptic, and the lunar orbit were all coincident.

It would appear that in an ideal sense, coastal tide actions due to the moon and sun are simple sine-like vector functions which are multiplied by Sine β, where β is the coastal direction angle (0 - 360°) in reference to the equator. The amplitude of the sine-like function is

7. We will consider a unit mass of a fluid to be a free point.

determined by the kinetic energy gained as the fluid falls in the force fields of Figure 2a and interacts with local bottom features. Considering an ideal Poisson ratio such that a radial force will produce an equivalent lateral hydraulic force toward the point of lowest potential, we may calculate a velocity amplitude due to the sine-like forces resulting from interaction with the moon in an ideal circular orbit system with the ecliptic, equatorial and lunar orbit planes coincident. The velocity functions would then be modified to allow for an inclined equatorial plane. Since the free point of unit mass is moving with the surface of a rotating earth, it sees a continually changing force. We are interested in the velocity of this unit mass, as the evergy available for producing coastal tides must be derived from the kinetic energy associated with the current (velocity) gained as the unit mass "falls" into the force field.

Referring to Figure 1, we may set up the following equations for the lunar effect parallel to the line M E.

(1) $$\frac{F_G}{m} = \frac{+G\,M_m}{(R - r\ \mathrm{Cos}\ \theta)^2}$$

Gravitational force on a unit mass at P_3 due to the moon. The gravitational constant $G = 6.670 \times 10^{-8}$.

(2) $$\frac{F_c}{m} = -(r_o - r\ \mathrm{Cos}\ \theta)\ \omega_m^2$$

Centrifugal force on a unit mass at P_3 due to rotation of the earth-moon system

Using Newton's second law, and combining (1) and (2):

(3) $$dv = \Sigma \frac{F}{m}\,dt = [\frac{G\,M_m}{(R - r\ \mathrm{Cos}\ \theta)^2} - (r_o - r\ \mathrm{Cos}\ \theta)\ \omega_m^2]\ dt$$

A companion equation for the quadrature force F* (perpendicular to the line ME) takes the form:

$$(4) \qquad dv^* = \Sigma \frac{F^*}{m} dt = [\frac{r \operatorname{Sin} \theta}{r} \quad \frac{G M_m}{(R - r \operatorname{Cos} \theta)^2} - (r \operatorname{Sin} \theta)\omega^2] dt$$

The resultant amplitude V of the velocity function would be the difference between the quadrature velocities for the rotation carrying a point P from the "neutral" plane to intersect line ME.

$$(5) \qquad V = \int_{\theta=\frac{3\pi}{2}}^{\theta=2\pi} dv - \int_{\theta=\frac{3\pi}{2}}^{\theta=2\pi} dv^*$$

Solving these equations gives a value for V of approximately 2.4 cm/sec greater than, and in the same direction as, the earth's surface movement due to axial rotation. This current is not large, but is in the same direction for each tide[8]. The result is, effectively, an alternating current component superimposed upon the continuous West to East current inherent in this model. To account for observed tidal currents, we must consider the lower water depths as more or less confined, and invoke Bernoulli's equation which energetically predicts an increased velocity in the areas of less restricted motion -- near the surface.

To extend the approach to account for the tide normally observed at a coast line in the form of changes in water level, the potential energy associated with a unit mass near the surface would be determined by halving the square of the current velocity. The energetic situation is

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8. This is opposite in direction to the current suggested by Groves and Munk, J. Marine Research, Vol. 17, pp 199-214.

is similar to that of a pendulum -- the kinetic energy (proportional to velocity squared) is essentially all converted to potential energy when the mass comes to rest at the end of each swing. Thus, with water west of a coast, we have a tide in phase with the current, and proportional to the square of the current. A coastal direction parallel to the current direction will convert little of the kinetic energy of motion into the potential energy of a rise in water level and will thus display very small tide level changes. This effect is particularly noticeable along the northern coast of South America and of Java, where the tide level variation is principally diurnal due to the obliquity of the earth's axis of rotation, and the semidiurnal currents produce little observable coastal tide.

The tide (vertical rise) to be found in open seas or areas a few hundred miles from any coastal or major sea bottom disturbance, may be estimated from energetic considerations by setting the potential energy equal to the kinetic energy associated with the current velocity:

$$(6) \quad Mg\,(\Delta h) = \frac{1}{2}\,mv^2;\ \Delta h = \frac{v^2}{2g} = 5.1\ \text{cm.}$$

Taking a value of 100 cm/sec., for a near-surface high tide current and using 980 cm/sec^2 for g, the deviation from a mean level is found to be ±5.1 cm with the phase shifted 90° from the current. An island only a few miles across would have little effect on this tide. Coastal or large undersea barriers will cause a major portion of the kinetic energy in the fluid body to be transferred up to sea level and thus may display a tide level variation ten or more times the open sea value of ±5.1 cm. Atmospheric tides due to these effects are almost completely masked by solar heating and circulation. Earth tides cannot be considered by this free-point approach as the movement of the earth surface would be quite small.

Conclusions

1. The 180° difference in the phase of tides simultaneously observed on opposite sides of north-south land masses may be explained by considering current as the prime tidal effect, with an increase in tide level due to water collecting against a barrier, and a decrease in tide level due to water flowing away from a barrier. A coastal direction other than north-south has the effect of multiplying the tide current function by sine θ, where θ is a coastal angle of 0° - 360° with respect to the equator. If the earth's axis were not tilted, there would be essentially no tide along coastlines parallel to the equator.

2. The predominantly diurnal tide observed at coastlines parallel to the equator is due to the obliquity of the earth's axis inducing a north-south current component (in phase with the earth's rotation) on the luni-solar west-to-east current. This north-south component is subject to barrier and phase effects similar to that described in 1 above. The amplitude of the north-south current is determined at the equator by multiplying the fundamental west-to-east current function by the sine of the angle between the earth's axis and lunar orbit axis.

3. Land barriers with dimensions small compared with the distance traversed by the fluid mass in one cycle (~ 15 miles) would have little effect in producing tides as in 1 and 2. A small land barrier would have dimensions of 1.5 miles or less. A large mass would have dimensions of 150 miles or more.

4. There will be a residual west-to-east current component with a superimposed alternating component. The continental land masses obviously prevent any clearly defined movement, but this eastward flow should be observable around the southern capes.

APPENDIX

A. The instantaneous earth-moon interaction force seen by a unit mass m on the equator, as in Figure 2A, may be determined by solving equation 3 for $\frac{dv}{dt}$.

Moon at zenith, Cos $\theta = +1$

$$(7) \quad \frac{dv}{dt} = \frac{F_m}{m} = \frac{GM_m}{(R - r)^2} - (r_o - r)\,\omega_m^2$$

$$\frac{F_m}{m} = \frac{6.670 \times 10^{-8} \times 7.343 \times 10^{25}}{(3.844 - .064)^2\, 10^{20}} + (6.378\text{-}4.667)\, 10^8\, (7.079)10^{-12}$$

$$\frac{F_m}{m} = 3.428 \times 10^{-3} + 1.211 \times 10^{-3} = 4.639 \times 10^{-3} \text{ dynes/gm}$$

Moon 180° from zenith, Cos $\theta = -1$

$$(8) \quad \frac{F_m'}{m} = \frac{GM_m}{(R - r)^2} - (r_o + r)\,\omega_m^2$$

$$= (3.207 - 7.819)\, 10^{-3} = -4.612 \times 10^{-3} \text{ dynes/gm}$$

The quadrature force, radially outward from E, Figure 1, and normal to the line M E, is:

Sine $\theta = 1$, Cos $\theta = 0$

$$(9) \quad \frac{F^*}{m} = \frac{rGM_m}{R^3} - r\omega^2 = 4.46 \times 10^{-3} \text{ dynes/gm}$$

This is our "zero reference" force at the neutral plane for $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$. The "effective" force for the moon at zenith would be $(4.64 - 4.46)\ 10^{-3} = 0.18 \times 10^{-3}$ dynes. Similarly, for the moon 180° from zenith $(4.61 - 4.46)\ 10^{-3} = 0.15 \times 10^{-3}$ dynes.

B. The force on a unit mass resulting from the earth-sun interaction may be determined by differentiating the algebraic sum of the gravitational force and the centrifugal force and multiplying by the earth radius r. The resultant force is essentially the same with the sun at zenith or 180° from zenith. M_s, R_s, and ω_s refer to the mass of the sun, average c to c distance between the earth and the sun, and the rotational frequency of the earth around the sun.

$$(10)\quad \frac{F_s}{m} = \pm \frac{G\ M_s}{R_s^2} \pm R_s \omega_s^2$$

$$(11)\quad \pm \frac{r\ d(\frac{F_s}{m})}{dR_s} = \frac{2\ G\ M_s r}{R_s^3} + r\omega_s^2$$

$$= \frac{2 \times 6.670 \times 1.987 \times 10^{33}}{(1.495)^3\ \ 10^{39}} + 6.378 \times 10^8 \times 3.965 \times 10^{-14}$$

$$= 5.05 \times 10^{-5} + 2.53 \times 10^{-5} = 0.076 \times 10^{-3} \text{ dynes/gm}$$