Title
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Course Notes
UCB-ITS-CN-2010-2

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COURSE NOTES
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Homework #1: Robustness of EOQ

We often encounter optimization problems of the form

$$z^* = \min\left\{ z = Ax^n + Bx^{-m} : 0 < x \le \infty; m, n > 0 \right\}$$

where z is a cost, x is a design variable, and the optimum is achieved for some x^*. Show how the actual cost achieved, z_a, depends on our design, x_a ($x_a \neq x^*$). To do this use dimensionless variables $\Delta \equiv z_a/z^*$ and $\delta \equiv x_a/x^*$, find Δ and a function of δ and plot Δ as a function of δ for different values of n and m. Discuss the sensitivity of the cost as the design deviates from the optimal and the effect of the constants n and m.

Extra credit: find a simple form for the relative errors (Δ-1) in terms of (δ-1) where $\delta \approx 1$.

Homework #2: Point-to-Point Travel

Distribution of benefits for transferring drivers to transit during a congested morning commute

A residential suburb has N = 15,000 commuters who travel to jobs in a central business district with uniformly distributed desired arrival times to the CBD from 8:00am to 9:00am. All car traffic must cross a bridge with limited capacity μ = 10,000 veh/hr to enter the CBD, but road capacity upstream and downstream can be assumed sufficient to serve demand without delay. Assume that all commuters value their time of delay at β = 15 \$/hr, and that the out-of-pocket cost of driving a car is c_d = \$2 per trip.

A. Initially, the only way to get to work is by car. Using Vickrey's model of the morning commute with earliness time valued at ½ the rate of queuing delay (e = 0.5) and lateness time unacceptable ($L \rightarrow \infty$), draw to scale a queuing diagram for a typical day. Show on it the maximum queuing delay, and the maximum earliness. (Note: free flow travel time can be ignored; explain why.) What are the total generalized cost, and the average generalized cost per commuter? The city government is considering starting a high-speed ferry service to relieve congestion and reduce the cost of the morning commute. How many commuters (n), uniformly distributed between 8:00am to 9:00am, must be diverted to the ferry to eliminate the delay caused by the bridge bottleneck?

B. Write an expression of the generalized cost per passenger in terms of the ferry ship's scheduled headway h << 1 (hrs/trip), the prorated cost per commute period for the ferry terminal infrastructure c_f (\$/period) and the variable cost per revenue ferry trip c_v (\$/trip) (Assume: fares reflect the ferry's costs; the ferry carries the n commuters identified in part A; a ferry trip excluding the waiting delay takes the same amount of time as a car trip exclusive of the bottleneck delay; headway are so short that people do not bother checking the schedule; and the number of seats on the ferry always exceeds the demand.)

C. What h^* minimizes the average ferry rider's time cost if c_f = 10,000 \$/period, c_v = 1000 \$/trip, and all the costs of ferry operation are passed on to ferry riders as fare? What is the average generalized cost to each ferry rider, and how does it compare to the average cost to each driver on the now uncongested bridge? Who is better off? How much total generalized cost would now be associated with the morning commute?

D. Could the solution of part C be an equilibrium solution if commuters were free to choose their transportation mode? If your answer is "no", suggest a toll and ferry fare policy that would achieve the solution we have assumed, and discuss the role of cross-subsidies.

E. Extra credit: Determine whether the total generalized cost could be reduced by changing n.

Homework #3: Many-to-One

Design of transit line carrying commuters along a corridor to CBD, and effect of TOD

Trips originate along a 10 mile radial corridor that runs from the edge of a city (x = 0) to its central business district (x = 10) where everyone works, shops, or just wants to be. The schedule of train arrivals at the city is given and such that every train carries the same number of people. We assume that schedules are published and passengers arrive just in time to avoid waiting. The cumulative demand for rail service to the CBD is described by $N(x)$: the number of people in each train with origins before x. Travelers spend time to access their stations by walking, and once aboard each traveler endures an in-vehicle-delay of τ = 2 min for each stop experienced along the ride. The running speed of the train not including stops is v = 40 miles per hour, and passengers access the stations by walking along the corridor at v_a = 3 miles per hour.

A. Assume that stops are evenly spaced with spacing S (such that $10/S$ is an integer), demand is evenly distributed with density D ($N(x) = D \cdot x$) and passengers walk to the nearest <u>downstream</u> station. Write an exact expression for total passenger time in terms of S and D. Assume that travelers suffer τ min of stop delay at the station they board. Find the value of S that minimizes the expression you wrote. Which value of S would you choose? What would be the total passenger time?

B. The total time for passengers using the train line can be expressed approximately by treating station spacing and demand as smooth continuous functions ($S(x)$ and $N(x)$), and integrating the time consumed in walking and stopping along the length of the corridor. Using this approach: (i) write an expression for the time consumed per unit distance at or near x in terms of $S(x)$, the cumulative demand $N(x)$ and the demand density $D(x) = dN(x)/dx$; (ii) express the total as an integral; and (iii) solve this integral for the data of part A and compare the result with the exact answer of part A.

C. The station spacing function can be chosen to minimize the total passenger time. Using the results of part B, write an expression for the optimal station spacing, $S^*(x)$.

D. Simplify the expression for $S^*(x)$ if the origins are uniformly distributed along the route, as in part A, but we do not restrict ourselves to uniform spacings. If $N(x) = 100x$, what is the total passenger time per train run into the city? What is the average travel time per passenger? Compare with part A and comment.

E. Now, before the stations are constructed, the city has decided to encourage transit oriented development along the corridor (TOD). If policies are designed so that TOD is built at every station constructed and will comprise 30% of the travel demand, 30% of trip origins will be right at the stations eliminating access time for these travelers. If station spacings are redesigned to minimize total passenger time, recognizing the effect of TOD, find the new overall passenger travel time, the average travel time per TOD passenger, and the average time per non-TOD passenger.

F. Extra: Densities are not typically uniform along the length of a radial corridor. Often the population density (and therefore the demand density) is roughly exponential so

$$D(x) = D_0 \cdot e^{\gamma x}$$

If the density at the edge of the city is $D_0 = 25$ trips per mile, and the density gradient is $\gamma = 0.25$, what is the spacing $S^*(x)$? How many stations should be built along the corridor to minimize travel time? What is the average travel time per passenger in this case? (To make the math easier, assume that trains depart $x = 0$ with D_0 / γ passengers from outside the CBD already aboard.)

For more about population density profiles see:

Clark, Colin (1951). "Urban population densities." *Journal of the Royal Statistical Society. Series A.* 114(4) pp. 490-496.

Homework #4: Improving congestion-prone modes

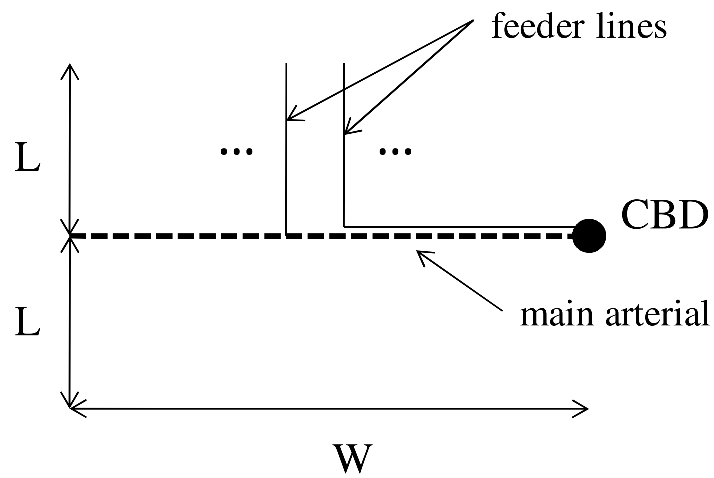
Short term effect of investing funds into congestible modes under market equilibrium.

Two lightly congested transportation modes, which jointly carry N passengers per day, are to be improved with a small short-term investment of $\$$ dollars. The modes (i = 1, 2) are always in an equilibrium so that their travel times, T_i, are equal. The travel time by each mode, i, is given by a performance function F_i that increases with the number of users, N_i, and decreases with the amount invested, $\$_i$. We are interested in determining a short-term investment strategy for the $\$$ dollars, assuming that the total number of daily users is fixed: $N_1 + N_2 = N$. Let the following constants, $\alpha_i = dF_i/dN_i$ (congestion proneness) and $\beta_i = -dF_i/d\$_i$ (improvability), be defined for the current equilibrium. Show that the all the funds should go to the mode with the smallest ratio α_i/β_i, independent of N. (Hint: since this is a question involving small changes in the arguments of the performance functions, we can assume that these functions are linear.)

Homework #5: System Hierarchy

Design of a scheduled feeder bus system for a metro station

A commuter bus system with headways, H (hrs) has a demand rate such that $N/2$ trips head into the CBD in this time, and $N/2$ trips head out. The streets form a dense square grid in the rectangular service area (dimensions $2L$ x W, see Figure).



We look for a set of bus lines with frequency H and the location of their stops. Bus lines travel perpendicularly to the main arterial with stop spacing $s << L, W$, and then along it without making any stops (see Figure). The bus line spacing is also s. Walking speed is much less than bus speed ($v_w << v_b$). The time for a stop is t_s.

A. Draw a pattern for the stops that will minimize the maximum distance walked, and show the catchment areas for each stop. Write a formula for the maximum walking time for any passenger.

B. Write a formula for the maximum travel time, T, of any outbound travelling passenger, assuming that they know the service headway and do not have to wait for the next dispatch.

C. Write a formula for the number of bus-hrs traveled (you can find bus-mi as a stepping stone) to transport the passengers.

D. If the cost per bus-hr is c_t (\$/bus-hr), write a mathematical program for s^* that will minimize that fare required, F^*, to avoid subsidies while achieving a total time service standard, T_o, for the worst case passenger. Prove that s^* is the largest spacing that would meet the time requirement.

E. Give the values of s^* and F^* that would apply to the distribution of travelers to and from downtown San Francisco (L = 3 mi; W = 7 miles; N = 1000 pax/dispatch; H = 8 min; v = 15 mph; v_w = 3mph; c_t = 30 $/hr; and t_s = 0.01 hrs) for different maximum travel time design standards, T_o. What is the minimum feasible standard, T_o?

F. Extra Credit 1: Assume that the bus lines provide express service and only make stops on a length $l << L$, $l > s$. Then repeat Part E.

G. Extra Credit 2: Repeat Part E if we allow the line spacing to be different from the stop spacing. (without the extra parameter of Part F.)

Homework #6: Demand Responsive Transit (dial-a-ride)

Design of demand responsive feeder-bus service from a transit station with scheduled service

We want to provide feeder service from a metro station with demand responsive minibuses during the evening commute (distribution only). The service area is "round" with area R km^2. The minibuses travel at a speed of v (km/hr) and take t_s (hrs) to stop and deliver a passenger. The metro operates with headways, H, and brings N customers to the station with each train. Customers have uniform and independently distributed destinations in the service region, and they board the first available minibus, so each bus carries a random mix of passengers. Once a bus has loaded $n << N$ customers (a decision variable) it departs and the next bus begins loading. (Assume for simplicity that N/n is an integer.) Each minibus chooses a route to distribute passengers by minimizing their total time (and distance traveled) including their return to the station.

A. Derive an expression for the average bus cycle time C (recognizing that they will be idle while waiting for a train, $H/2$ time units on average), and for the average time in the bus for the last person to be delivered, T (this will be our service standard for design). Also derive an expression for the fleet size required, M. Remember that the fleet must consist of an integer number of buses. (The distance for a traveling salesman problem with area R and number of stops (including the origin) n_s with an L1 metric is $L = 0.92\sqrt{(Rn_s)}$.)

B. Consider where the feeder buses are nearly fully used all of the time and the cost of running the buses is recovered from the fare box. If the hourly cost of running a bus is c_t (\$/bus-hr) = 30, use the answer from part A to develop a table linking the necessary fare, F, to various design standards, T_o, for the Downtown Berkeley BART. Use the following data: $R = 3$ mi^2; $N = 75$ pax/train; $H = 8$ min; $v = 15$ mph; and $t_s = 0.01$ hrs.

C. To gain some insight, repeat parts A and B generically, but assume $n >> 1$ ($n \approx n+1$) and $t_s \rightarrow 0$, $H \rightarrow 0$ (so they can be neglected). Write an expression relating F to the choice of T_o.

D. Repeat parts B and C if we now assume that the service region has been partitioned into w wedges served separately (w to be chosen optimally). You should imagine here that w buses with clearly labeled destination areas are simultaneously loading passengers at the station; and that when a bus is filled another one takes its place, until all passengers in the train are served. Is F still a function of the choice of T_o?

Homework #7: Bus Pairing

Design of a Control Strategy to Mitigate Bus Pairing

Consider an unscheduled bus line being served by a pair of buses $i = 1, 2$. We assume that the average speed of one bus, i, in time period t depends only on the spacing in front of the bus at the beginning of the time period, $x_i(t)$. A rough rule could be as follows: the distance advanced by bus i in time period t, $y_i(t)$, is $1 - x_i(t)/10 + \varepsilon_i(t)$, where $\varepsilon_i(t)$ is a random disturbance. Assume that the system is in equilibrium for $t < 0$, with $\varepsilon_i(t) = 0$ and $x_i(t) = 5$ for all $t<0$ and $i = 1,2$. Assume too that $\varepsilon_i(t) = 0$ for $t \geq 0$ and $i = 1,2$, except that $\varepsilon_i(t) = 1$ for $t = 0$ and $i = 1$.

A. Write the dynamic equations for the system; i.e. express the vector $\boldsymbol{x}(t+1)$ as a function of its history [$\boldsymbol{x}(t\text{-}1)$, $\boldsymbol{x}(t\text{-}1)$,…] and the history of the disturbances [$\boldsymbol{\varepsilon}(t)$, $\boldsymbol{\varepsilon}(t\text{-}1)$,…]. Express them in as simple a form as possible. You should eliminate the y's from the expression, and can also eliminate one of the x's.

B. Verify from the dynamic equations that the system is in equilibrium for $t < 0$. What happens for $t > 0$? How long until the two vehicles become a pair?

C. Assume vehicles are given information about the current position of the other one. Give a modified rule of motion that will alleviate the positive feedback problem. Note that there are many possible answers but you may only slow vehicles down, never speed them up. Justify your proposal qualitatively and demonstrate its effectiveness with our disturbance data.

D. Why would the control strategy proposed in part C be difficult to implement in the field? Qualitatively discuss another (commonly used) strategy that could be used that would alleviate some of these problems and still prevent bus pairing.

E. Assume you want to model a general urban bus line with the relation $y_i(t) = a - bx_i(t) + \varepsilon_i(t)$, where $\varepsilon_i(t)$ has mean zero and standard deviation, c. What units of measurement (dimensions) would you use for all the variables of this equation? What real-life factors would determine the values of a, b, and c? Give rough estimates for the values that would pertain to a typical Berkeley bus.

Mini Project 1

Modifying an existing local bus line to Bus Rapid Transit (BRT)

The Geary transit corridor has one of the highest transit riderships in San Francisco. Current service on Geary Boulevard has high scheduled frequencies, but is plagued by poor reliability due to conflicts from operating in mixed traffic, high dwell times at bus stops, excessive signal delays and other difficulties.
In this project you will examine the current situation on Geary Blvd. during the evening rush, as well as the benefits that can be obtained through the introduction of BRT and a hierarchy of service.

Route Characteristics

The route consists of a 1.5 mile section in the CBD and 5 miles in the residential area. During a round trip, the bus currently stops at 84 relatively evenly spaced stops along the route including terminals at both ends (thus in each direction there are 41 stops plus the terminal) and the round trip of a bus is 120 min. 50% of this time is spent cruising, 25% loading and unloading (including loss time due to accelerating and decelerating), and 25% of the trip is caused by signal delay. You can assume a fixed loss time of 10 sec per stop due to acceleration and deceleration, and that buses do not skip stops. Users take 4 times as long to board as to alight and we will assume that at each stop boarding and alighting movements occur in sequence. Users access bus stops by walking at a speed of 2mph. Muni has budgeted 40 buses for the route, giving a headway of 3 minutes.

Standards for LOS

You will be analyzing 3 types of users for this project:

1. Residential to Residential (R→R)
2. CBD to Residential (C→R)
3. Residential to CBD (R→C)

Data

The boarding and alighting demand density (PM rush) per hour per mile for service in both directions is shown in Figure 1.

Using these data, you are to perform the following tasks. Some assumptions are deliberately left for you to make, and you are expected to explain your assumptions.

i. What is the commercial and cruising speed of a bus? What is the loss time (boarding and alighting) per passenger served? What is the optimal stop spacing in the residential area for R→R passengers? What is the optimal stop spacing in the CBD and in the residential area for C→R passengers? Reconcile the two conflicting optimal values, and find the optimal stop spacings for each zone for all users. Discuss why the actual spacings on the route may differ from the optimal ones calculated.

ii. By implementing several features of BRT, service can be improved. Converting a lane to bus service (only) will allow buses to cruise 50% faster. Transit Signal Priority (TSP) will cut the time spent at signals by 50%. What is the new cycle time? With these new data what is the optimal stop spacing for each zone? Compare these values to part I and discuss the effect upgrading to BRT has on the stop spacing.

iii. Service can be improved further by introducing a hierarchy of service such that local buses are available in the residential section only and express buses run the entire length of the route. For worst case analysis you can assume R→R users will use local service, C→R users will board an express bus and transfer to a local bus at the express stop closest to their destination, R→C users will board a local bus and transfer to an express bus at the express stop closest to their origin (Express and local schedules are not coordinated.) Determine the optimal headway and the optimal stop spacing for each zone, and for each bus type. Remember you only have 40 buses to be split between local and express service. Since this is a complex optimization problem you may solve it numerically (using Excel or a similar program)

You are to submit a formal report describing your tasks and your findings. For the discussion of part iii you should quantify and discuss the costs of switching from a continuum approximation solution to a real-world solution as well as what constraints are imposed when implementing the latter solution. You may work in small groups of up to 3 people and submit a group report.

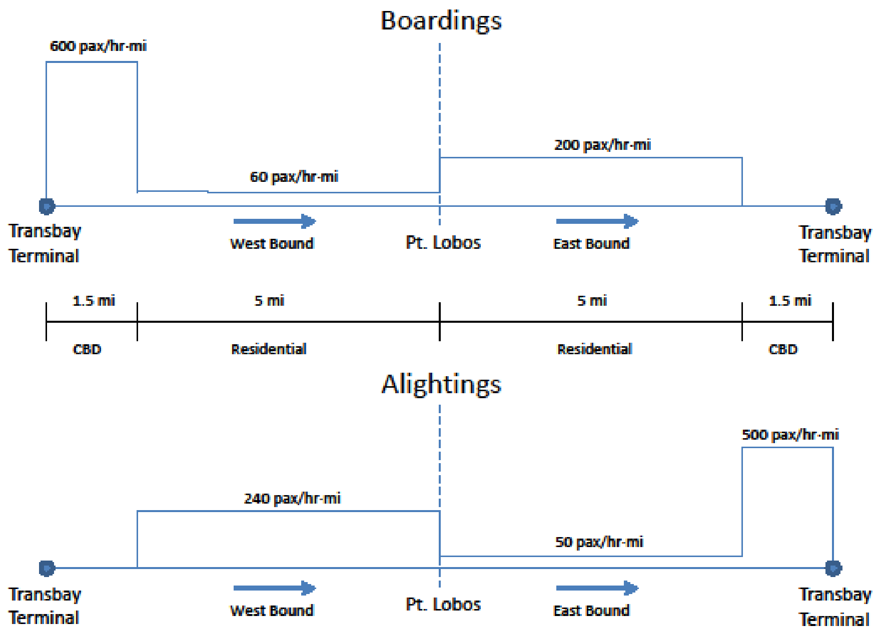


Figure 1. Boarding & Alightings vs. Route Location

Geary 38 Route

Mini Project 2

Designing a Transit Network

The Chicago Transit Authority currently serves the city of Chicago and the surrounding area with rail and bus transit. In this project you will create your own transit design for the Chicago area, combining grid bus service with a particular type of radial rail service.

Region Characteristics

The area of design is bounded by a quarter-circle of radius 15 mi northwest of the downtown area and a quarter-circle of radius 10 mi southwest of the downtown, as shown in Figure 1. Streets cover the region in an infinitely dense grid.

Vehicle Characteristics

The rail service operates independent of the street grid at a cruising speed of 30 mph and has fixed lost time per stop (including loss time due to acceleration and deceleration and passenger loading) per stop of 1 min. The bus service operates on the grid at a cruising speed of 15 mph (accounting for traffic and signal delays), has a loss time (due to acceleration and deceleration) of 10 sec per stop, plus 4 sec per boarding passenger, and negligible time per alighting passenger. Users access the system by walking at a speed of 2 mph.

Data

The transit agency values user time at \$20/hr. Bus and rail service have operating costs of \$3/bus-mile and \$10/train-mile respectively, while rail infrastructure has a cost of \$650/mile-hr. During peak hours, trips to/from the downtown region are generated uniformly at a rate of 360 pax/mi^2-hr and trips with origins and destinations outside the downtown region are generated uniformly at a rate of 60 pax/mi^2-hr.

Using these data, you are to perform the following tasks. For each part analyze a branching-radial design for the rail service as shown in Figure 2, combined with a grid system for bus service.

i. Write an expression for the hourly agency cost for this system.

ii. Define the different trip profiles for the following trip types: NW region to downtown, SW region to downtown, NW region to SW region, and SW region to NW region (e.g. a trip profile from NW region to downtown might include Walk→Bus→Rail→Walk). For each trip type and trip profile, formulate the worst case travel time in terms of the system parameters and decision variables listed below, or explain why that trip profile would not produce a reasonable worst-case trip.

iii. For the following set of standards, optimize the bus and rail system with respect to the reasonable trip profiles from part ii (i.e. minimize generalized cost subject to standards). To do this, first show how the agency cost can be optimized with respect to a single profile in part ii analytically (assuming that this particular profile is the binding constraint). Then optimize the agency cost with respect to all constraints numerically using Excel or a similar solver program. Identify the worst case passenger profile for the optimal design.

STANDARDS

- NW→Downtown – 2 hours
- SW→Downtown – 1.5 hours
- NW→ SW, SW→ NW – 3.5 hours

iv. Using a street map of Chicago (an aerial is provided to show the regions we will be analyzing), lay out the bus and rail lines using the optimal spacing values determined in part ii as a guideline. You should keep in mind that the existing infrastructure provides constraints that must be considered (for example, bus lines should only be placed on existing city streets and you should consider a minimum of 10 key locations in Chicago that should be connected to the transit service). Comment on the differences between the continuum approximation solution and the more realistic "on-ground" system with respect to the worst case passenger.

You are to submit a formal report describing the tasks and your findings. Extensive calculations should be placed in an appendix (and referred to in the report). You may work in small groups of up to 3 people and submit a group report.

Decision Variable List

<table>
<tr><td rowspan="3">Bus (grid)</td><td colspan="2">S_b</td><td>route spacing</td></tr>
<tr><td colspan="2">s_b</td><td>stop spacing</td></tr>
<tr><td colspan="2">H_b</td><td>headway</td></tr>
<tr><td rowspan="3">Branching-Radial</td><td>$S_{rad, NW}$</td><td>$S_{rad, SW}$</td><td>max route spacing</td></tr>
<tr><td>$s_{rad, NW}$</td><td>$s_{rad, SW}$</td><td>stop spacing</td></tr>
<tr><td>$H_{rad, NW}$</td><td>$H_{rad, SW}$</td><td>Headway on the periphery</td></tr>
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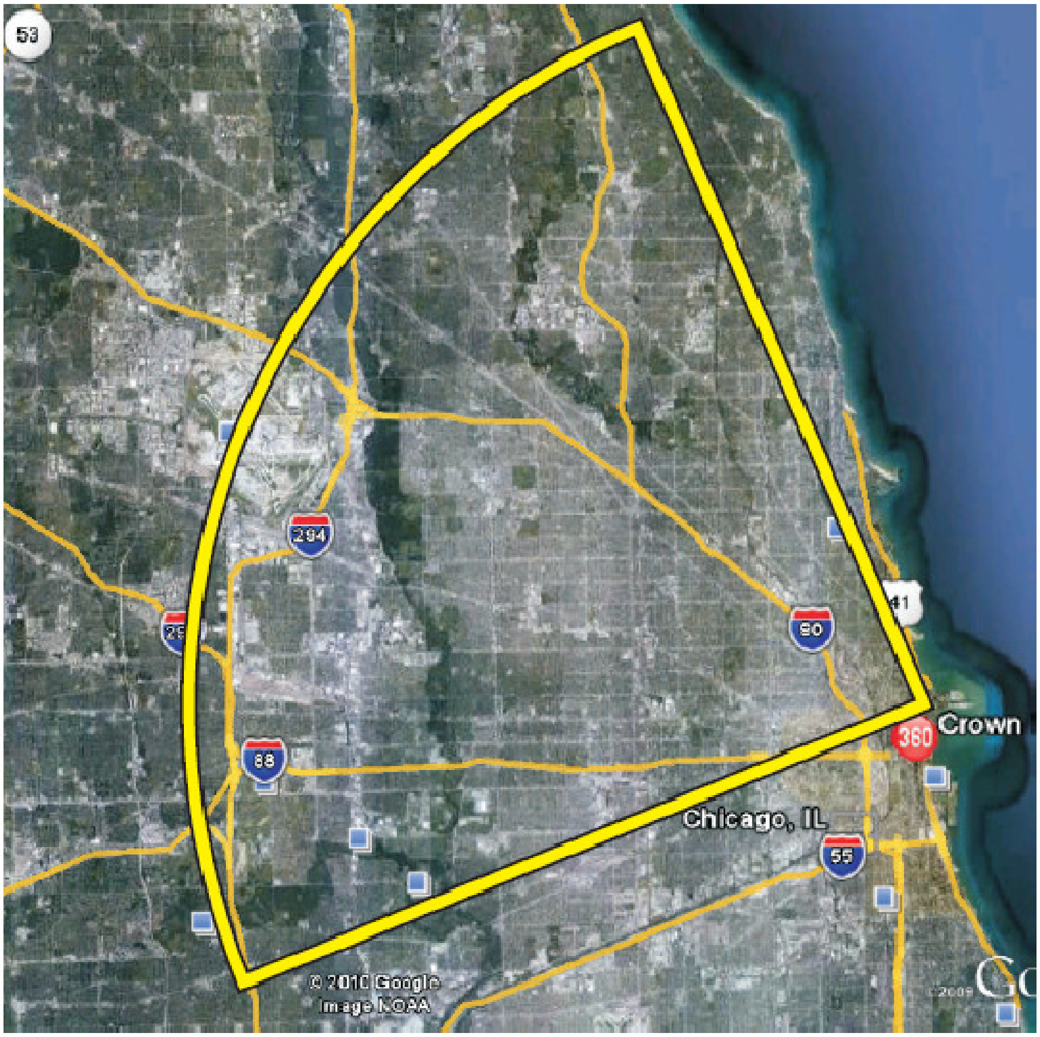
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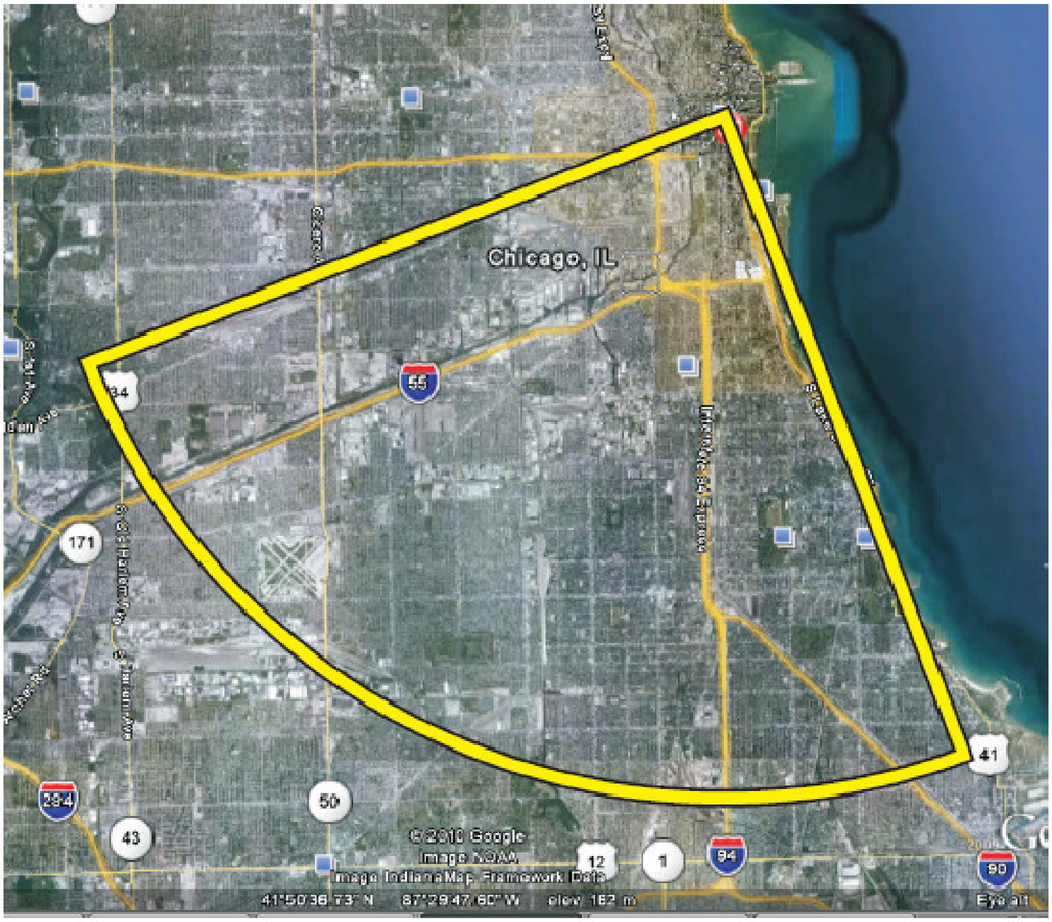

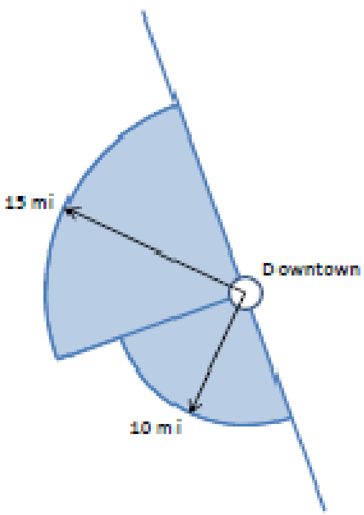

Figure 1.

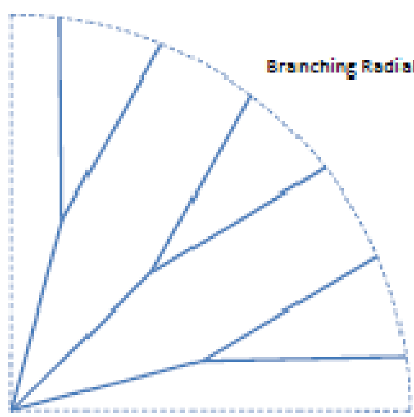

Figure 2.

Mini Project 3

Covering a schedule with buses; designing bus runs;Covering runs with drivers; designing tasks/shifts

AC Transit serves Berkeley, Oakland, and many of the surrounding areas with almost 200 routes and a fleet of over 600 buses. The area next to Berkeley's West Gate is a major transit hub served by a BART station and 15 different bus routes.

In this project you will determine the optimal way to assign buses and drivers to three of the bus routes that serve this transit hub.

Route Characteristics

The three routes examined in this mini project are the 7, which provides service between Rockridge and El Cerrito Del Norte BART stations; the 18, which serves the Shattuck corridor and downtown Oakland; and the 52L, which provides service between El Cerrito and the UC Berkeley campus. The weekday schedules for these routes are shown in Tables 2-4.

A. For each route and direction determine the total *bus-hrs in service* per day. This will be a lower bound used for comparison for the rest of the project. What are the peak service times for each route and direction? What is maximum number of bus loops over the day (the minimum number of buses needed) for each route?

B. Assuming that the routes are treated independently, determine the maximum number of buses needed to cover the route if deadheading between termini is allowed. Determine the actual number of buses needed to cover the loops for each route using the greedy algorithm; identify the runs for each bus and the total bus-hrs spent, including any dwell times not at a depot. If necessary, a bus can deadhead to the other end of its route to cover a loop. Times to deadhead between termini and to a depot are shown in Table 1. Compare the bus-hrs and number of buses with the lower bounds from Part A.

C. In order to decrease the wasted bus-hrs dwelling and the number of buses used, the three routes named above will use the same pool of buses to serve their routes. Determine the maximum number of buses needed to cover the system if deadheading between all terminii is allowed. Program and run an iterative covering algorithm to find the optimal arrangement of runs that minimizes the bus-hrs required to serve the routes. Compare the bus-hrs and number of buses with the lower bounds from Part A.
(Pseudocode and logic for the algorithm are included at the end of this problem statement)

D. Cover the runs from your solution to Part C. An asymmetric work rotation if offered such that each driver gets 4 days of 9-hour shifts and 1 day with a 4-hour shift (such that each day has a ratio of 9-hour to 4-hour shifts of 4:1). In addition, overtime is offered at double the wage rate. In order to avoid hiring more drivers than needed, you can also use 8-hour shifts. Drivers can only be assigned to a bus at the Depot or at the Berkeley BART station. Determine the number of drivers needed, the job for each driver, and the driver hours wasted (when a driver is not on a loop). Using a wage rate of $25/hr, what is the cost of covering the runs?

E. Absenteeism can be a problem for transit agencies. If we assume that each driver has a 5% chance of skipping their assigned shift, determine how many backup drivers (who can cover any shift) the city must employ to ensure that there is 99% chance that all shifts will be covered for the work schedule you solved for in Part E.

You are to submit a formal report describing your tasks and your findings. You may work in small groups of up to 3 people and submit a group report.

Table 1. Deadheading Travel Time between locations

Deadheading Time	Rockridge BART	Del Norte BART	San Pablo / Marin	Moraga / Medau	Bancroft / Telegraph	El Cerrito BART	
Rockridge BART	0:00:00	0:24:00	0:14:00	0:06:00	0:07:00	0:18:00	**Route 7**
Del Norte BART	0:24:00	0:00:00	0:07:00	0:25:00	0:18:00	0:08:00	
San Pablo/Marin	0:14:00	0:07:00	0:00:00	0:19:00	0:10:00	0:04:00	**Route 18**
Moraga/Medau	0:06:00	0:25:00	0:19:00	0:00:00	0:12:00	0:19:00	
Bancroft/Telegraph	0:07:00	0:18:00	0:10:00	0:12:00	0:00:00	0:14:00	**Route 52L**
El Cerrito BART	0:18:00	0:08:00	0:04:00	0:19:00	0:14:00	0:00:00	
Emeryville Depot	0:08:00	0:16:00	0:09:00	0:16:00	0:11:00	0:13:00	**Depot**

Route	Northbound				Southbound			
	Rockridge BART	Berkeley BART	El Cerrito Del Norte BART	Bus #	El Cerrito Del Norte BART	Berkeley BART	Rockridge BART	Bus #
7	6:22 AM	6:36 AM	7:08 AM		6:18 AM	6:52 AM	7:10 AM	
7	6:42 AM	6:56 AM	7:28 AM		6:36 AM	7:10 AM	7:28 AM	
7	7:05 AM	7:19 AM	7:51 AM		6:53 AM	7:28 AM	7:48 AM	
7	7:25 AM	7:41 AM	8:17 AM		7:14 AM	7:49 AM	8:09 AM	
7	7:45 AM	8:01 AM	8:37 AM		7:34 AM	8:09 AM	8:29 AM	
7	8:05 AM	8:21 AM	8:57 AM		7:57 AM	8:32 AM	8:52 AM	
7	8:24 AM	8:42 AM	9:19 AM		8:25 AM	9:00 AM	9:22 AM	
7	8:45 AM	9:03 AM	9:40 AM		8:55 AM	9:30 AM	9:52 AM	
7	9:05 AM	9:23 AM	10:00 AM		9:25 AM	10:00 AM	10:22 AM	
7	9:30 AM	9:48 AM	10:25 AM		9:54 AM	10:29 AM	10:51 AM	
7	10:00 AM	10:18 AM	10:55 AM		10:24 AM	10:59 AM	11:21 AM	
7	10:30 AM	10:48 AM	11:25 AM		10:54 AM	11:29 AM	11:51 AM	
7	11:00 AM	11:18 AM	11:55 AM		11:24 AM	11:59 AM	12:21 PM	
7	11:30 AM	11:48 AM	12:25 PM		11:54 AM	12:29 PM	12:51 PM	
7	12:00 PM	12:18 PM	12:55 PM		12:24 PM	12:59 PM	1:21 PM	
7	12:30 PM	12:48 PM	1:25 PM		12:54 PM	1:29 PM	1:51 PM	
7	1:00 PM	1:18 PM	1:55 PM		1:24 PM	1:59 PM	2:21 PM	
7	1:30 PM	1:48 PM	2:25 PM		1:53 PM	2:28 PM	2:50 PM	
7	1:57 PM	2:15 PM	2:52 PM		2:24 PM	2:59 PM	3:21 PM	
7	2:27 PM	2:45 PM	3:22 PM		2:50 PM	3:25 PM	3:47 PM	
7	2:56 PM	3:14 PM	3:51 PM		3:15 PM	3:50 PM	4:12 PM	
7	3:27 PM	3:45 PM	4:22 PM		3:37 PM	4:12 PM	4:34 PM	
7	3:58 PM	4:16 PM	4:53 PM		3:57 PM	4:32 PM	4:54 PM	
7	4:21 PM	4:39 PM	5:16 PM		4:19 PM	4:54 PM	5:14 PM	
7	4:46 PM	5:02 PM	5:38 PM		4:39 PM	5:14 PM	5:34 PM	
7	5:06 PM	5:22 PM	5:58 PM		4:59 PM	5:34 PM	5:54 PM	
7	5:26 PM	5:42 PM	6:18 PM		5:22 PM	5:57 PM	6:17 PM	
7	5:46 PM	6:02 PM	6:38 PM		5:44 PM	6:19 PM	6:39 PM	
7	6:06 PM	6:22 PM	6:58 PM		6:05 PM	6:40 PM	7:00 PM	
7	6:28 PM	6:44 PM	7:20 PM		6:35 PM	7:10 PM	7:30 PM	
7	6:55 PM	7:16 PM	7:52 PM		7:05 PM	7:43 PM	8:03 PM	
7	7:25 PM	7:46 PM	8:22 PM		7:35 PM	8:13 PM	8:33 PM	
7	7:55 PM	8:16 PM	8:52 PM		8:05 PM	8:43 PM	9:03 PM	

Route	Northbound				Southbound			
	Rockridge BART	Berkeley BART	El Cerrito Del Norte BART	Bus #	El Cerrito Del Norte BART	Berkeley BART	Rockridge BART	Bus #
7	6:22 AM	6:36 AM	7:08 AM		6:18 AM	6:52 AM	7:10 AM	
7	6:42 AM	6:56 AM	7:28 AM		6:36 AM	7:10 AM	7:28 AM	
7	7:05 AM	7:19 AM	7:51 AM		6:53 AM	7:28 AM	7:48 AM	
7	7:25 AM	7:41 AM	8:17 AM		7:14 AM	7:49 AM	8:09 AM	
7	7:45 AM	8:01 AM	8:37 AM		7:34 AM	8:09 AM	8:29 AM	
7	8:05 AM	8:21 AM	8:57 AM		7:57 AM	8:32 AM	8:52 AM	
7	8:24 AM	8:42 AM	9:19 AM		8:25 AM	9:00 AM	9:22 AM	
7	8:45 AM	9:03 AM	9:40 AM		8:55 AM	9:30 AM	9:52 AM	
7	9:05 AM	9:23 AM	10:00 AM		9:25 AM	10:00 AM	10:22 AM	
7	9:30 AM	9:48 AM	10:25 AM		9:54 AM	10:29 AM	10:51 AM	
7	10:00 AM	10:18 AM	10:55 AM		10:24 AM	10:59 AM	11:21 AM	
7	10:30 AM	10:48 AM	11:25 AM		10:54 AM	11:29 AM	11:51 AM	
7	11:00 AM	11:18 AM	11:55 AM		11:24 AM	11:59 AM	12:21 PM	
7	11:30 AM	11:48 AM	12:25 PM		11:54 AM	12:29 PM	12:51 PM	
7	12:00 PM	12:18 PM	12:55 PM		12:24 PM	12:59 PM	1:21 PM	
7	12:30 PM	12:48 PM	1:25 PM		12:54 PM	1:29 PM	1:51 PM	
7	1:00 PM	1:18 PM	1:55 PM		1:24 PM	1:59 PM	2:21 PM	
7	1:30 PM	1:48 PM	2:25 PM		1:53 PM	2:28 PM	2:50 PM	
7	1:57 PM	2:15 PM	2:52 PM		2:24 PM	2:59 PM	3:21 PM	
7	2:27 PM	2:45 PM	3:22 PM		2:50 PM	3:25 PM	3:47 PM	
7	2:56 PM	3:14 PM	3:51 PM		3:15 PM	3:50 PM	4:12 PM	
7	3:27 PM	3:45 PM	4:22 PM		3:37 PM	4:12 PM	4:34 PM	
7	3:58 PM	4:16 PM	4:53 PM		3:57 PM	4:32 PM	4:54 PM	
7	4:21 PM	4:39 PM	5:16 PM		4:19 PM	4:54 PM	5:14 PM	
7	4:46 PM	5:02 PM	5:38 PM		4:39 PM	5:14 PM	5:34 PM	
7	5:06 PM	5:22 PM	5:58 PM		4:59 PM	5:34 PM	5:54 PM	
7	5:26 PM	5:42 PM	6:18 PM		5:22 PM	5:57 PM	6:17 PM	
7	5:46 PM	6:02 PM	6:38 PM		5:44 PM	6:19 PM	6:39 PM	
7	6:06 PM	6:22 PM	6:58 PM		6:05 PM	6:40 PM	7:00 PM	
7	6:28 PM	6:44 PM	7:20 PM		6:35 PM	7:10 PM	7:30 PM	
7	6:55 PM	7:16 PM	7:52 PM		7:05 PM	7:43 PM	8:03 PM	
7	7:25 PM	7:46 PM	8:22 PM		7:35 PM	8:13 PM	8:33 PM	
7	7:55 PM	8:16 PM	8:52 PM		8:05 PM	8:43 PM	9:03 PM	
18	1:47 PM	2:11 PM	3:10 PM		1:38 PM	2:32 PM	2:48 PM	
18	2:02 PM	2:26 PM	3:25 PM		1:53 PM	2:47 PM	3:03 PM	
18	2:17 PM	2:41 PM	3:42 PM		2:08 PM	3:02 PM	3:18 PM	
18	2:32 PM	2:56 PM	3:57 PM		2:23 PM	3:17 PM	3:33 PM	
18	2:47 PM	3:11 PM	4:12 PM		2:38 PM	3:32 PM	3:48 PM	

18	3:02 PM	3:26 PM	4:27 PM		2:53 PM	3:47 PM	4:03 PM	
18	3:17 PM	3:41 PM	4:42 PM		3:08 PM	4:02 PM	4:18 PM	
18	3:31 PM	3:55 PM	4:56 PM		3:23 PM	4:17 PM	4:33 PM	
18	3:51 PM	4:13 PM	5:10 PM		3:39 PM	4:34 PM	4:52 PM	
18	4:07 PM	4:29 PM	5:26 PM		3:54 PM	4:49 PM	5:07 PM	
18	4:23 PM	4:45 PM	5:42 PM		4:09 PM	5:04 PM	5:22 PM	
18	4:38 PM	5:00 PM	5:57 PM		4:24 PM	5:19 PM	5:37 PM	
18	4:53 PM	5:15 PM	6:12 PM		4:39 PM	5:34 PM	5:52 PM	
18	5:09 PM	5:31 PM	6:28 PM		4:55 PM	5:49 PM	6:07 PM	
18	5:25 PM	5:47 PM	6:44 PM		5:09 PM	6:03 PM	6:21 PM	
18	5:46 PM	6:08 PM	6:59 PM		5:25 PM	6:19 PM	6:37 PM	
18	6:04 PM	6:26 PM	7:17 PM		5:41 PM	6:33 PM	6:51 PM	
18	6:23 PM	6:45 PM	7:36 PM		5:56 PM	6:48 PM	7:06 PM	
18	6:41 PM	7:03 PM	7:54 PM		6:11 PM	7:01 PM	7:19 PM	
18	7:01 PM	7:23 PM	8:13 PM		6:27 PM	7:19 PM	7:37 PM	
18	7:20 PM	7:42 PM	8:32 PM		6:43 PM	7:29 PM	7:45 PM	
18	7:40 PM	8:02 PM	8:52 PM		6:58 PM	7:43 PM	7:58 PM	
18	8:00 PM	8:22 PM	9:12 PM		7:14 PM	7:59 PM	8:14 PM	
18	8:22 PM	8:44 PM	9:32 PM		7:33 PM	8:18 PM	8:33 PM	
18	8:42 PM	9:04 PM	9:52 PM		7:53 PM	8:37 PM	8:52 PM	
18	9:04 PM	9:26 PM	10:14 PM		8:12 PM	8:56 PM	9:11 PM	
18	9:24 PM	9:46 PM	10:34 PM		8:31 PM	9:15 PM	9:30 PM	
18	9:44 PM	10:06 PM	10:54 PM		8:51 PM	9:35 PM	9:50 PM	
18	10:04 PM	10:26 PM	11:14 PM		9:11 PM	9:55 PM	10:10 PM	
18	10:24 PM	10:46 PM	11:34 PM		9:31 PM	10:15 PM	10:30 PM	
					9:51 PM	10:35 PM	10:50 PM	
					10:13 PM	10:54 PM	11:09 PM	
					10:33 PM	11:14 PM	11:29 PM	
					10:53 PM	11:34 PM	11:49 PM	

Route	Northbound				Southbound			
	Bancroft Way & Telegraph Ave.	University Ave & Shattuck Ave	El Cerrito Plaza BART	Bus #	El Cerrito Plaza BART	University Ave & Shattuck Ave	Bancroft Way & Telegraph Ave.	Bus #
52L	6:00 AM	6:12 AM	6:41 AM		5:42 AM	6:21 AM	6:32 AM	
52L	6:27 AM	6:39 AM	7:16 AM		6:37 AM	7:09 AM	7:20 AM	
52L	7:00 AM	7:12 AM	7:41 AM		6:55 AM	7:27 AM	7:38 AM	
52L	7:30 AM	7:42 AM	8:19 AM		7:03 AM	7:35 AM	7:46 AM	
52L	8:00 AM	8:12 AM	8:41 AM		7:24 AM	7:56 AM	8:07 AM	
52L	8:30 AM	8:42 AM	9:13 AM		7:38 AM	8:10 AM	8:21 AM	
52L	9:00 AM	9:12 AM	9:43 AM		7:48 AM	8:20 AM	8:31 AM	
52L	9:30 AM	9:42 AM	10:13 AM		7:58 AM	8:30 AM	8:41 AM	
52L	9:58 AM	10:10 AM	10:41 AM		8:08 AM	8:41 AM	8:52 AM	
52L	10:30 AM	10:42 AM	11:13 AM		8:18 AM	8:51 AM	9:02 AM	
52L	11:00 AM	11:12 AM	11:43 AM		8:29 AM	9:02 AM	9:13 AM	
52L	11:30 AM	11:42 AM	12:12 PM		8:40 AM	9:13 AM	9:24 AM	
52L	12:00 PM	12:12 PM	12:42 PM		8:51 AM	9:24 AM	9:35 AM	
52L	12:30 PM	12:42 PM	1:12 PM		9:02 AM	9:35 AM	9:46 AM	
52L	1:00 PM	1:12 PM	1:42 PM		9:13 AM	9:46 AM	9:57 AM	
52L	1:30 PM	1:42 PM	2:12 PM		9:25 AM	9:58 AM	10:09 AM	
52L	2:00 PM	2:12 PM	2:51 PM		9:37 AM	10:10 AM	10:21 AM	
52L	2:32 PM	2:44 PM	3:15 PM		10:08 AM	10:41 AM	10:52 AM	
52L	3:02 PM	3:14 PM	3:47 PM		10:38 AM	11:11 AM	11:22 AM	
52L	3:29 PM	3:41 PM	4:14 PM		11:08 AM	11:41 AM	11:52 AM	
52L	3:44 PM	3:56 PM	4:29 PM		11:38 AM	12:11 PM	12:22 PM	
52L	3:54 PM	4:06 PM	4:39 PM		12:08 PM	12:41 PM	12:52 PM	
52L	4:04 PM	4:17 PM	4:49 PM		12:37 PM	1:10 PM	1:21 PM	
52L	4:15 PM	4:28 PM	5:00 PM		1:07 PM	1:40 PM	1:51 PM	
52L	4:25 PM	4:38 PM	5:10 PM		1:35 PM	2:08 PM	2:19 PM	
52L	4:35 PM	4:48 PM	5:19 PM		2:05 PM	2:45 PM	2:56 PM	
52L	4:45 PM	4:58 PM	5:29 PM		2:32 PM	3:12 PM	3:23 PM	
52L	4:55 PM	5:08 PM	5:39 PM		3:04 PM	3:37 PM	3:48 PM	
52L	5:05 PM	5:18 PM	5:49 PM		3:35 PM	4:08 PM	4:19 PM	
52L	5:15 PM	5:28 PM	5:59 PM		4:05 PM	4:38 PM	4:49 PM	
52L	5:25 PM	5:38 PM	6:09 PM		4:35 PM	5:08 PM	5:19 PM	
52L	5:36 PM	5:49 PM	6:20 PM		5:06 PM	5:39 PM	5:50 PM	
52L	5:46 PM	5:59 PM	6:30 PM		5:35 PM	6:08 PM	6:19 PM	
52L	5:57 PM	6:10 PM	6:41 PM		6:05 PM	6:38 PM	6:49 PM	
52L	6:10 PM	6:23 PM	6:53 PM		6:35 PM	7:08 PM	7:19 PM	
52L	6:20 PM	6:33 PM	7:03 PM		7:01 PM	7:41 PM	7:52 PM	
52L	6:29 PM	6:42 PM	7:12 PM		7:34 PM	8:14 PM	8:25 PM	

52L	6:40 PM	6:53 PM	7:23 PM		8:01 PM	8:34 PM	8:45 PM	
52L	6:55 PM	7:08 PM	7:38 PM		8:27 PM	9:00 PM	9:11 PM	
52L	7:10 PM	7:23 PM	8:01 PM		8:56 PM	9:29 PM	9:40 PM	
52L	7:25 PM	7:38 PM	8:08 PM		9:26 PM	9:59 PM	10:10 PM	
52L	7:39 PM	7:52 PM	8:22 PM		9:56 PM	10:29 PM	10:40 PM	
52L	7:58 PM	8:11 PM	8:40 PM		10:26 PM	10:59 PM	11:10 PM	
52L	8:20 PM	8:32 PM	9:01 PM		10:58 PM	11:31 PM	11:42 PM	
52L	8:51 PM	9:03 PM	9:32 PM					
52L	9:25 PM	9:37 PM	10:06 PM					
52L	9:55 PM	10:07 PM	10:36 PM					
52L	10:27 PM	10:39 PM	11:08 PM					
52L	11:01 PM	11:13 PM	11:42 PM					

Pseudocode and Logic for Iterative Covering Algorithm

Define Variable Type Loop
    ***Start** as Time*
    ***StartLocation** as String*
    ***End** as Time*
    ***EndLocation** as String*

Define Variable Type Bus
    ***Run** as Loop Array*

***Main Program**()*

*Load the Loops into a Loop array **ScheduleData**()*

*Create a bus array **Fleet**()*

*Create a test bus array **TestFleet**()*

*Iterate through **ScheduleData(j)***

    *If there is no bus at the depot then create one in **Fleet**()*

    *Iterate through **Fleet(i)***

        *Reinitialize **TestFleet**() to **Fleet**()*

        *If current bus is feasible with **Feasible(Fleet(i)**, **ScheduleData(j)**) then*

            *Assign **ScheduleData(j)** to **TestFleet(i)***

            *Run **GreedyAlgorithm(j+1**, **TestFleet**)*

            *Save cost of result **CheckCost(TestFleet**)*

        Else

            Assign very high cost to infeasible assignment

        *Find lowest cost assignment **i***

        *Assign **ScheduleData(j)** to **Fleet(i)***

    *End **Fleet(i)** iteration*

*End **ScheduleData(j)** iteration*

```
Function GreedyAlgorithm(StartRun, TestFleet)

    Iterate through ScheduleData(j) starting at StartRun

        If there is no bus at the depot then create one in TestFleet()

        Iterate through TestFleet(i)

            If current bus is feasible with Feasible(Fleet(i), ScheduleData(j)) then

                Assign ScheduleData(j) to TestFleet(i)

                Exit Loop

        End TestFleet(i) iteration

    End ScheduleData(j) iteration

--------------------------------------------------------------
Function Feasible(Loop, Bus) as Boolean

    If Bus has never been assigned anything then Return True

    If Bus can travel from its last location to the start location of Loop before the start time of Loop then

        Return True

    Else

        Return False

--------------------------------------------------------------
Function CheckCost (TestFleet) as Integer

    Iterate through TestFleet(i)

        Increase cost by the time between the start time of TestFleet(i)'s first loop and end time of
        TestFleet(i)'s last loop

        Increase cost by time to deadhead from the depot to the start location of TestFleet(i)'s first loop

        Increase cost by time to deadhead from the start location of TestFleet(i)'s last loop to the depot

        Iterate through TestFleet(i).Run(r)

            If the dwell time between loops is longer than the time to deadhead to the depot from the
            end location and back to the start location of the next then

                Decrease the cost by the time spent at the depot

        End TestFleet(i).Run(r) iteration

    End TestFleet(i) iteration
```