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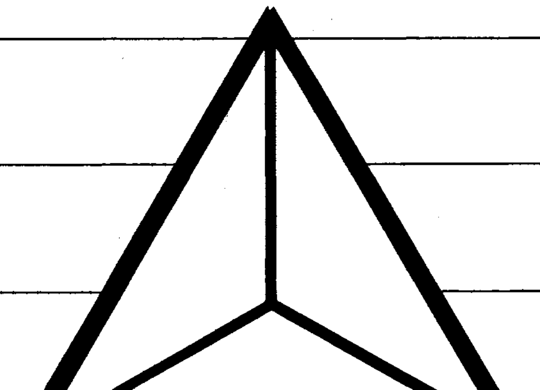
by
Carl Eckart



The Author

Carl Eckart was an eminent mathematical physicist. He was a professor at the University of Chicago and later at the University of California at San Diego. His earliest well-known achievement was in quantum mechanics where he established unifying relationships between the matrix and wave mechanical formulations. After leaving his mark on quantum mechanics he turned to classical physics and developed some of the fundamentals of irreversible thermodynamics. Subsequently his interest shifted to the physics of oceanography, in particular to acoustics and hydrodynamics. He published a number of papers on these subjects as well as a book *Hydrodynamics of Oceans and Atmospheres.* During this period he also served as Director of the Scripps Institution of Oceanography. He died in 1973 at the age of 71.

OUR MODERN IDOL: MATHEMATICAL SCIENCE

by

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Preface

At the time of the author's death this book was in the form of an unpublished manuscript. The manuscript consists of seven loose-leaf notebooks, handwritten in pencil, on lined notebook paper. Each page is carefully numbered and dated. It appears that it is a first draft, yet the initial wording is precise, with almost no after-thoughts.

The title *Our Modern Idol: Mathematical Science* belies its content. Although the author was an eminent mathematical physicist the book is not mathematical in nature; there are no mathematical equations or derivations. The few mathematical expressions are simple and the book is clearly intended for the non-mathematical reader. It does not appear to have a definitive, coherent theme; rather it is a collection of eclectic essays connected historically or sociologically. In short, this book is about people, ancient and recent, as individuals and in society.

It was the opinion of several commercial publishers to whom this manuscript was submitted that this would not be a commercially viable book without extensive editing. Their proposals required condensation of the whole and the addition and deletion of material so that the theme could be more clearly developed. Upon consultation with friends and colleagues of the author, it was decided to publish the manuscript privately as written, without substantive editing.

In this endeavor I was ably assisted by Maureen Perry and Laurie Nord who carefully read the manuscript and made necessary but minor editorial changes. In addition Ms. Perry set the entire manuscript in type using the University of California computing facilities. Their knowledgeable contributions and persistent efforts made private publication possible.

Leonard Liebermann
Professor of Physics

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Introduction

One hears it said that, for the first time, man now has the power to determine his own future, if only people will be sensible. This is a popular summary of a number of serious books, none of which are quite so incautious. Still, their authors are enthusiastic about the achievements of science and technology, and are convinced that others are coming; moreover that these achievements have improved the lives of people and will continue to do so. These achievements have been largely those of the physical and biological sciences, and those in biology have come mainly after biologists began to use the tools and methods of the physical scientists. Among these methods are the mathematical theories of physics and chemistry.

Remarkably, those sciences that are specifically devoted to the study of people, sociology, psychology and political science, have made no corresponding achievements. This requires explanation. One facile explanation is widely accepted, especially by the scientific and technological communities. This is that these behavioral sciences do not use mathematical theories. Many behavioral scientists accept this explanation, and promise to do better in the future. In fact, they have been sincerely trying to do so for at least a generation, but without striking success. It is therefore reasonable to investigate the validity of this explanation.

There is another phenomenon that casts doubt on this explanation. Despite the demonstrable improvement in living conditions, many people are still dissatisfied with them. Some of these people are, for economic and other reasons, deprived of the advantages provided by the physical and life sciences, and live under antiquated conditions. Without being unsympathetic, or denying this phenomenon, their opinions need not be examined in detail at this point. But the phenomenon does cast its shadow on the proposed explanation. Moreover, there are people who are not denied the advantages of the technological changes, but still find their lives unsatisfactory. They say that these very changes have had unsatisfactory consequences, that they have so changed the environment that the world is now a less pleasant place in which to live than in the past. They are tempted to ascribe this to the abandonment of ancient wisdom and customs. Extremists advocate destruction of the present Establishment and a return to even more antiquated ways of life than those now open to the underprivileged. Those who are less extreme, devote themselves to the study of ancient writings and attempt to shape their lives in accordance with the precepts contained in them, or rather, with their own view of the lives of these ancient writers. One cannot deny the phenomena of which these people complain, and this broadens the scope of the present investigation.

The field to be investigated becomes so huge that it seems to require new methodology, a methodology that is neither based on admiration of the present, nor on admiration of the past. It also seems likely that the investigation cannot be systematic. One will start somewhere, in hope that other starting points will be suggested as one proceeds. Thus, it will be a collection of investigations, rather than a single one. At least, that is how I have written this book.

The initial investigation should be general, yet simple enough to make progress possible. I chose to start with the question, "How does man differ from the other animals?" It is immediately found that man (called *homo sapiens* by zoologists) is the only surviving species of the genus *homo*. Today, most genera are represented by many species, or else by none. Those which have no living species became extinct in the distant past. This suggests that man may be well along the road to extinction, and may have no appreciable future. Yet the number of people, the number of members of the species *homo sapiens* that are alive today is many times the number that were alive a thousand years ago. This suggests the opposite conclusion. One has already arrived at a significant fact about the popular opinion with which this chapter opened: one should distinguish between man and people. The popular opinion is verified, if only in this minor way. The question should be, "How do people differ from animals ?"

Two anatomical differences come to mind. People have hands with opposable thumbs and can use them to make artifacts; they also have mouths that can make the sounds of speech. Birds build nests, which are artifacts. Antelopes make no artifacts, but they make a cry that puts the whole herd into flight when one of them senses the approach of a lioness. People are not absolutely unique, but differ from other animals in at least two ways that are obvious.

If one person acquires new knowledge by experience, this can be communicated to others by speech, and by them to their children. Because of speech, the acquisition of knowledge becomes a cumulative process. Other animals do not accumulate knowledge as the generations pass. This accumulation of knowledge by human beings has been enhanced by their invention of the artifacts of writing and printing. One may, with some reason, hope that this accumulated knowledge will enable man to exercise more and more control over his future. But this same process of the old teaching the young can also cause errors and false conclusions to accumulate with the passage of time. One should therefore study ancient writings, not so much in the hope of finding lost wisdom as in the hope of locating the origin of errors that have been, and still are, accepted as truths.

Thus, the consideration of these few almost obvious matters has led to the formulation of a methodology with which to continue the proposed investigation.

The Failure of Universal Education

It is very difficult, if not impossible to define knowledge, but all of us attach some meaning to the word. Today, most people will agree that the ability to read and write is knowledge, although this is an almost irreducible minimum of formal knowledge. Education is the systematic transmission of knowledge from one person to another. Most modern systems of education are complex organizations whose origin is to be found, if at all, in the distant past. It is also generally agreed that everyone should receive some education. There were times when only children of wealthy parents received formal education. It is difficult to date the beginning of the doctrine of universal education, or to find the reason for its adoption. An important event in this history occurred toward the end of the French Revolution, when reconstruction began to replace its original destructive violence. Apparently, it was thought that education for all would make all cooperate in changing society for the better. People still express disappointment when an educated person does not cooperate in this way.

The effort to increase the number of French elementary schools was made difficult by the lack of qualified teachers. On the initiative of Napoleon Bonaparte (who was not yet a dictatorial emperor), the Revolutionary Convention established the Ecole Normale to train the required corps of teachers and to certify their competence. Much the same system was adopted in the United States. Here, teachers' colleges were called normal schools until well into the twentieth century; this was long after the original Ecole Normale had ceased to exist. The normal schools of the U.S. not only continued to train and certify teachers for the increasing number of elementary schools, but they also standardized the curriculum of these schools, which in turn certified the competence of their graduates. These certificates or diplomas were often accepted by colleges and universities for the admission of applicants, without further examination of the knowledge and aptitude of the applicant. This practice has not yet been completely abandoned.

Attendance at elementary schools was made compulsory. One reason for this was certainly the hope that education would produce better citizens. It also discouraged the exploitation of child labor, which had begun in the East. But the major industry of the United States was still farming, and many parents could not afford to do without the help of their children during the busy summer; hence the long summer vacation was a compromise. Moreover, the people of the U.S. were more inclined to move about than the people of Europe. The settlement of the West was beginning and even cities were growing. The standardization of school curricula was therefore a convenience: a child could move from one school to another without requiring much special attention from the teacher, or having to make up deficiencies. Standardized education was efficient, at least superficially, under the circumstances.

Elementary education was free, paid for from public funds. Free higher education developed gradually, but was never compulsory or universal. In 1900, one public high school sufficed for the entire city of St. Louis, then the fourth largest city in the United States with a population of a half million. Since then, the number of public high schools, colleges, and universities has increased rapidly, as has the number of their students. The educational system of the U.S. has increased in size and complexity, but it still bears the marks of its origin in the standardized elementary school.

The early normal schools did not provide their graduates with a very good education; their knowledge was scarcely more than that which they were expected to pass on to their pupils. They were prepared to teach only the standard curriculum, not to improve it. There were, of course, exceptionally fine teachers among these graduates, but not enough of them to invalidate this generalization. Moreover, teachers were overworked; even in the smallest schools they could not give each student individual attention. The regimentation of pupils was inevitable, but not recognized as such, since it was "for their own good." If an exceptional pupil vaguely recognized some error in the teacher's exposition and questioned it, he was likely to receive an evasive answer and a poor grade. If a pupil with a strong personality refused to be regimented, he not only received poor grades, but was punished as well. Since the pupils were immature children, they came to believe that the purpose of going to school was, not to obtain knowledge, but to "please teacher" and go one's own way after school. Of course there were exceptions to this generalization. There were also attempts to improve the system; the introduction of kindergarten is an example. Today, there is less regimentation and more permissiveness, even in the elementary schools, and most teachers do have a better education. The curriculum is less standardized, particularly in respect to current events.

The weakest part of the curriculum was history. Pupils were required to memorize lists of dates and events. Perhaps this is essential, but history is not like a string of pearls. It is more like a tapestry, with many interwoven strands, some of them tattered and torn. Perhaps children are unable to understand this, and would be confused if history were presented as it happened. In any event, the standard history course was much like a modern movie scenario. There were the good guys (somewhat more than life size) and the bad guys. The memorization of dates and events was enlivened by bowdlerized stories of battles and wars. According to these stories, the good guys usually won; if not, the teacher became sentimental about them and bitter about the bad guys. This history confined itself mainly to events that had occurred in the United States, and was glossed over with complacency about the near perfection of contemporary American social and political institutions.

The barren ineffectiveness of our early elementary educational system can be documented by reference to old textbooks and autobiographies. The significant fact is that, while it has certainly been improved over the years, many remnants of the original educational system can be found today, even in our colleges and universities.

The Brandywine Valley and Similar Experiences

Instead of continuing to discuss the complacencies and the discontents of the United States in general terms, it will be well to consider a specific case in detail. The Brandywine Valley is an area that had long supplied water and recreation for neighboring towns. Its population was increasing and changes were occurring that justified concern. This new population consisted largely of well-educated people who might be expected to cooperate with their neighbors for the general good. An intensive educational advertising campaign was conducted, alerting them to the possible pollution of the water supply, and to the reduction of the recreational potential of the park-like Valley. After this, a questionnaire was circulated among the inhabitants of the Valley. Those responding to the questionnaire were almost unanimous that the preservation of the natural environment is very important. This had been emphasized in the educational campaign. When asked why the environment was important in their lives, the Valley residents split 33:31:30 in stressing three different aspects that had been discussed during the pre-questioning period. Some were specific, very few (2%) were negative in their response. When asked what should be done about the possible pollution problem, the split was 53:15:32 (53% said the Valley must be preserved; 32% favored letting the owner of each property in the Valley to develop that property free of legal restrictions). Those who favored control were a bare majority. Nevertheless, a plan was prepared which, its proponents thought, might be put into effect with general public approval. An informal poll of the citizens, however, convinced the leaders of the community that it would be futile to place the plan on a formal ballot with a request for authority to put it into effect.

This is an example where even specialized education failed to produce cooperation; it produced unanimity only at the trivial level of lip-service to words. The more general educational system had accustomed people to "please teacher" by making the expected verbal response. When it came to action, the more fundamental "school's out, we can do as we please" took over. Their education had been irrelevant to their daily habits of life.

Educational campaigns, such as that preceding the attempt to save the Brandywine Valley, brought new words into wide use: "environment", "ecology", "pollution". These had not been emphasized when the adult residents of the Valley were in grammar school, or even in college. The phenomena they describe had existed before, but the history courses had not mentioned them. Many thought they were new phenomena. The catchwords were taken up largely by younger people, who used them as arguments against technology and their elders who, they said, "supported the Establishment." They emphasized the "irrelevance" of the education they were receiving. All of these new words were evidence of confusion, and this confusion was shared by everyone, not only by young people. Even with the best intentions, their teachers were not prepared to help them resolve their problems.

The environmental changes brought about by the automobile furnish a specific example of the complexity of these problems, as well as those of the educational system. The exhaust from gasoline engines combines with air to form smog. Smog is a form of smoke whose constituent particles are very small, and remain suspended in the atmosphere for a long time. Smog spreads over wide areas, and is therefore more noticeable than smoke from coal, fuel oil, or even diesel oil. The particles from these settle out much more rapidly as soot in the immediate neighborhood. Moreover, the use of leaded gasoline introduces some 250,000 tons of lead into the air over the United States each year. Lead is toxic to the human body; quite small amounts of lead can cause permanent damage, and the fatal dosage is not large. Thus, concern about smog is justified, although it is possible to exaggerate its dangers. Some action is now being taken to prevent or reduce the formation of smog, but only after years of special and expensive education via articles and speeches in all the news media. This action will be even more expensive than the specialized educational campaign. It will also require many people to change their habits. All of this might have been foreseen if our ordinary educational system had provided a more realistic history of technology and society.

The general ignorance about such matters can be illustrated by a conversation between two college students, overheard at a cafeteria table. One of them was describing the beauties of a Canadian island where only horse-drawn vehicles and bicycles are permitted. Suddenly, the other exclaimed "Oh! No pollution!" He had not lived in the pre-automobile environment, and no one had provided him with a realistic description of the pollution that followed horses everywhere. Flies bred in the horse dung in the streets and spread the filth into houses and food markets. Some fifty or sixty years ago, there was a vigorous "swat the fly" campaign. Unscreened food markets and open garbage pails were prohibited by law. In retrospect, it can be seen that this would all have been ineffective if the automobile had not displaced the horse. Many streets were unpaved. In wet weather, traffic churned the horse dung into a black muck, which people's shoes tracked into homes, offices, and everywhere else they went. In hot, dry weather, the muck turned into dust, which penetrated even into screened areas. The gray and brown sparrow, the chippy, provided another ecological complication. The chippy is a scavenger and an aggressive fighter. It was introduced into North America at about the same time as the horse, and it multiplied rapidly. Nature lovers feared that it would eventually displace the more colorful native birds. When the horse was displaced by the car, the chippies were deprived of their major source of food. Their numbers diminished, and native birds returned to all except the most densely populated areas.

This does not finish the account of the changes that accompanied the automobile. Unpaved streets and highways became intolerable to motorists. Roads were paved, using revenues obtained from car licenses. This reduced both dust and mud. The income of boot-blacks was greatly reduced, and this occupation for poor boys and unskilled men has almost disappeared. The automobile increased the mobility of the individual and produced urban sprawl, of which the Brandywine Valley is an example. The favorable aspects of these social changes were recognized very early, and people urged governmental intervention to accelerate them. The bad aspects might also have been recognized as they began to appear, and measures might have been taken to prevent them before they became intolerable and almost irreversible. This imbalance in the control of social changes certainly requires study and correction, if man is to control his future.

Opponents of the Establishment often describe corporations as heartless despoilers of the natural environment. The charge of recklessness is sometimes justified, and almost always so in the case of new industries. This, however, does not relieve people and their official representatives of all responsibility. The profits of an established corporation depend strongly upon the good opinion of a small fraction of its customers or potential customers. The officers of a corporation, therefore, can respond to opinions of a marginal segment of the public. They need not wait until that opinion spreads to the majority. They can even respond to intangible and aesthetic value judgments. They fear damage to their public image as much as violating a law. Thus, they cannot escape all responsibility for the undesirable effects of their activities.

Overhead electric wires, strung on rough poles, are often cited as evidence for technology's contempt for the intangible values of the environment. They were first installed by Samuel Morse, financed by a Congressional appropriation. Morse had been an art teacher, and one of his sculptures had won an international prize. He was to become the first president of the National Academy of Design in New York. Today, Morse's overhead wires are being placed underground, or being replaced by less unsightly and isolated radio towers. (Some newspapers fulminate against overhead wires, and, unreasonably, against the obstruction of traffic during the construction and repair of underground electric utilities.) Builders have changed their plans to retain the good will of their neighbors. Power stations are relocating generating stations in response not to laws enacted by the voters and their elected representatives, but to less tangible expressions of community displeasure. Manufacturing plants are devising methods of waste disposal that neither exhaust nor contaminate the local water supply. Engineers who have never attended an art class are including aesthetic qualities among the desirable attributes of their projects. Present day designers of tract houses pay more attention to the environment and to aesthetics than did their nineteenth century predecessors in Baltimore and London. The early tracts have degenerated into twentieth century slums, and our new tracts will provide the twenty-first century with problems of urban renewal. Possibly these will be less serious than ours, but they will surely not be the same.

The history curricula of our schools contain no coherent account of these matters. Samuel F.B. Morse is presented abstractly, as the great inventor of the telegraph, which has been such a benefit to man. The negative aspects of overhead wires come to general attention largely through direct observation. The efforts of corporations to remedy them are publicized through advertising designed to protect or enhance the corporation's public image. Realists properly suspect that advertising is biased by self-interest. This is one reason why the advertising campaign in the Brandywine Valley did not obtain the cooperation of businessmen engaged in developing the Valley. It too was suspected of bias. The bias of our educational system rarely receives comment. When it does, the commenter is usually accused of antisocial radicalism.

Historians of civilization often discuss the fact that people have modified the environment to their advantage. Instead of continuing to live in caves, people built houses. Even earnest students of ecology rarely discuss the damage that primitive people have inflicted on their environment. Prehistoric men exterminated whole species of animals, not because they needed them all for food, but because they used very inefficient methods of hunting. They killed far more animals than they could eat. Large herds of animals were stampeded over the edge of cliffs, though a few carcasses would have fed the hunters and their families for days. Later, many cities were built on artificial mounds, composed of the garbage, rubbish and ruins left by previous generations. Tarsus, the birthplace of St. Paul the Apostle, is one of innumerable such cities. The dead were buried in these rubbish heaps, often under the floors of their own houses. Even the small communities of prehistory contaminated their surroundings with garbage and excrement. This unpleasant aspect of human history is not widely taught, even in universities, and this is responsible for the seeming novelty of such ideas as pollution and ecological damage.

Perhaps it is the unpleasantness of these phenomena that keeps them out of the general curriculum. There may also be older reasons; a few centuries ago, plagues were considered to be punishments decreed by God. For one reason or another, the London plagues are discussed only very briefly in history classes, and their causes mentioned only with euphemistic circumlocution. Hans

Zinnser, a specialist in typhus, wrote a book entitled *Rats, Lice, and History*. He was one of the first in recent times to call attention to some of the revolting features of life in past times. He used the word "louse" freely, and did not hesitate to say that cleanliness has always lagged behind art and the social graces. He described in some detail the environmental conditions in the time of Rousseau and Voltaire. He quoted portions of a frank description of the toilet training of an eighteenth century princess. This, however, is left in the original French and is unintelligible to those who do not recognize the French words for "spit" and "snot". He confined himself to typhus, and did not describe the symbiosis of flies, horses and food shops that he must have witnessed as a boy. If one is very familiar with a phenomenon from an early age, it is difficult to realize that it merits a written record.

T.R. Forbes is an authority on general matters of public health. His article *Life and Death in Shakespeare's London* is based on the surviving records of a single London parish, for the period 1558 to 1625. The records are reasonably explicit, and it is easy to read between the lines. The appalling number of deaths in the plague years is illuminated by the artless comments of the parish clerks. In non-plague years individual deaths could be described in some detail, and this leaves no doubt that conditions were worse during the plagues. The pollution of that particular parish today is certainly far less revolting than it was in Shakespeare's time.

Concern about the environment is not new. In 1661, the British essayist and Fellow of the Royal Society of London, John Evelyn, published a small book called *Fumifugum*. In it he speaks of the "hellish and dismal cloud of sea-coal" that had become so great as to make "the city of London resemble the suburbs of Hell." He says that travellers could smell the smoke at a distance of many miles, but he does not give the distance at which they could smell the city's open sewers and garbage heaps. He quotes Lucretius, who lived in the first century B.C., on the injurious influence of smoke. Lucretius' long hexameter poem, *On the Nature of Things*, is dismissed by one recent commentator with the remark that its subject is unpleasant. As will be seen later, this is scarcely an adequate summary. Evelyn's quotation also must be considered in the context of Lucretius' times and the rest of his work. At that time, Roman houses and temples had no chimneys and were heated by braziers. They contained altars on which the meat, fat, entrails and bones of animals were burned as sacrifices to the gods. The smoke and stench must have been painful at times. Moreover, Lucretius was a religious reformer with no taste for martyrdom. His attack on the pretensions of Roman religion and ritual were indirect and inferential. An exaggeration of the bad effects of smoke was safe propaganda.

Apparently, London's smoke was a relatively recent phenomenon in Evelyn's time. He had grown up with much worse kinds of pollution, but he does not mention them in his book. It is impossible to believe that between 1558 and 1625 the conditions described by Forbe's parish record could have vanished completely, or that they were entirely confined to that parish. This is not conjecture, but certainty. The Great London Plague occurred four years after Evelyn's publication. The raw data are: 70,000 dead out of a population of less than half a million. This underestimates the lethality of the disease. Unpopular and ineffective laws caused many plague deaths to be reported otherwise. Moreover, it is estimated that two thirds of the half million inhabitants fled the London to escape death. This epidemic began in Holborn, and some months elapsed before it reached the parish studied by Forbes. The nature of these bubonic plagues is now well understood, and it is certain that coal smoke was not a major cause; its contribution to the death toll can safely be neglected. London's Great Fire, a few years later, produced great volumes of smoke, but was not followed by a plague. Fortunately, it killed more rats and lice than people. The reconstruction which followed the fire was planned with more attention to sanitation, and the new buildings were relatively rat-proof. Plagues were less severe thereafter.

Of course these records are from the Old World. It may therefore be thought that conditions were better in the New World, and possibly this is true. However, in the mid-nineteenth century, towns in the Midwestern United States repeatedly suffered major epidemics of cholera, and no summer passed without some deaths from this disease. Epidemic cholera is always caused by a water supply that is contaminated with human excrement. Again, it may be thought that urbanization was responsible, and that conditions were necessarily better in the open country. Both Boccaccio and Newton testify to the relative safety of rural places in times of plague. But cholera epidemics occurred among the people who were building the transcontinental railroads through sparsely populated areas. Almost certainly, they contaminated the water supplies of their own camps.

In the twentieth century, epidemics of bubonic plague, typhus and cholera no longer occur in Europe and North America. This was brought about by the combined efforts of physicians, biologists, public officials, the building trades and the manufacturers of plumbing fixtures. It was the end result of work by a very few people who set the goal for themselves. They used knowledge obtained by research, by independent study. Some of that new knowledge has now been incorporated in public laws and in our educational system. In most homes, children are given better training in personal hygiene than were the royal princesses a few centuries ago. There is thus some justification for a defense of our educational system, but none for complacency about it.

Town and Country

The failure of the Brandywine Plan was a rejection of the countryside and an acceptance of urbanization. It was a great disappointment to advocates for the preservation of rural beauties and virtues. It was of more than local significance because this controversy is not only nationwide, but worldwide as well. In the United States, and probably in other countries too, the matters just discussed are not taught in a coherent, or even complete fashion. Usually, the curriculum includes only the pleasanter, more optimistic parts of history. Often it is bowdlerized to conceal current horrors. But the cities are present for all to see, and it requires no instruction to notice that they are ugly and sadly in need of improvement. It is easy to reach the superficial conclusion that a return to rural life is the remedy. It then seems to follow that this remedy would eliminate the need for technology and the Establishment. The irrelevance of education seems to be another corollary. These superficialities are very reminiscent of Jean Jacques Rousseau, who lived in the mid-eighteenth century. In today's idiom, he would be described as a drifter with a talent for writing. Among other things, he revived the ancient doctrine that Natural Man was a paragon of virtue, living happily in an unspoiled Garden of Eden. He too proceeded to the conclusion that civilization and culture are the only sources of people's misery. This doctrine may properly be called romantic anarchism. It was certainly influential in bringing about the violent French Revolution. Rousseau's autobiography does not confirm his conclusion. Less biased biographers conclude that much of Rousseau's misery and misfortune were the result of his own quarrelsome personality. Yet, his ideas are still in circulation.

Anatole France was a more astute observer of human activities. He published his conclusions in a series of gently ironic plays and novels, and received the Nobel Prize for Literature in 1921. One of his most mature and carefully considered novels was published in 1909, and was soon translated into English under the title

Penguin Island. The story begins with a near-sighted missionary, who mistakes some penguins for human beings, and baptizes them. Then the penguins slowly become human beings. Continuing, the story describes the growth of Parisian society and the nineteenth-century French Establishment. This portion of the book emphasizes the greed for wealth, the ambition for power, and the superficiality of relations between people, even within the family. It does not put much emphasis on industrialization, or on the physical ugliness of the cities. The symbolism of the penguins and the near-sighted missionary is not clear; perhaps they symbolize the irrelevant education of children and young people. The last chapter is entitled "The Future." Unexpectedly, it describes the revival of romantic anarchism among university students, who destroy the cities with atomic bombs. This is related unemotionally, the violence and death being bowdlerized into a sort of sanitary slum-clearance project. Except for some sterile areas, Earth regains its original beauty and the cycle of urbanization begins again.

It is surprising to find student alienation and violence in a book written in 1909, until one recalls that the abortive European revolutions of 1848 were led by university people. It is still more surprising to find a description of atomic bombs written at this early date. There is no doubt about this, however. The author quotes Sir William Ramsay on the highly concentrated energy released by radioactivity. Ramsay, who won a Nobel Prize in 1904, had demonstrated that radium emits helium nuclei which carry great amounts of energy.

The foresight with which Anatole France described two major developments which were to occur within the next few generations had little influence on his contemporaries. According to the *Index of Reviews* for 1910, American critics were only concerned with France's technical style and the defects of the translation. This might have been because, at that time, the United States was isolated from European affairs. But *Penguin Island* is a work of fiction, and one might at least have expected some comments on its entertainment value. The introduction to a 1932 edition completely ignores the apocalyptic ending. Those who developed the atomic bomb in the 1940's had forgotten the book; perhaps they had never read it. Their security officers were horrified when, in 1941, a science fiction writer openly described a device which resembled the atomic bomb much less closely than Anatole France's bomb. So far as I am aware, none of the authoritative accounts of the development of the atomic bomb, and none of the many analyses of student alienation and urban violence, make any mention of this early warning. Neither do any of the authoritative estimates of the cost (in dollars) of providing such early warnings when new technologies become possible because of new scientific experiments.

The sum of all the Nobel Prize money awarded to Anatole France and Sir William Ramsay is less than any of the recent estimates of the cost of such an early warning, which is now called technology assessment. Moreover, if one takes the Nobel Prize as a fixed monetary standard, there has been an inflation of about a thousand percent over the last fifty years. The cost of this early warning about undesirable side effects of new technologies and the defects of social organization is negligible. The mere existence of an early warning, however, does not influence the course of events, even after its correctness has become inescapably obvious, and the underlying causes were evident years before. It is necessary that this be remembered. One will not only inquire why the France-Ramsay warning was ignored, but also whether France's apocalyptic catastrophe is now inevitable. One can console oneself with the thought that he did not foresee the arms race or the Cold War. He underestimated the mass of an atomic bomb, thinking that it would be the size of an egg. He supposed that the poisonous mushroom cloud might rise to the height of half a mile. These quantitative errors are all underestimates. Until the ultimate catastrophe has occurred, there is still time for preventive action. But the difficulty of prevention would have been much less, had Anatole France's warning been kept in mind during the years in which the danger was growing.

The Aristocratic Fallacy

The American and French Revolutions were caused, at least in part, by dissatisfaction with the monarchical form of government and its accompanying theory of society. Universal education was one of the reforms which accompanied these revolutions. Monarchies are based on the aristocratic fallacy, which may be formulated as follows: "Society is stratified into mutually exclusive classes, and these classes can be ranked in order of their value to society." It is to be noted that one does not need to be an aristocrat to believe in this fallacy. Such widely different people as Marie Antoinette and Karl Marx have subscribed to it. "The militant proletariat," wrote Lenin, "is always conscious of its spirit of class distinctions." If the fallacy is made the basis for action, differences of opinion about the rank-ordering of the classes become, very often, deadly serious. Two arguments have been advanced to support the aristocratic fallacy. The most ancient one holds that some people are descended from gods; the doctrine of the divine right of kings grew out of this idea. The more recent argument holds that all people are descended from animals. Neither argument is convincing.

Robert Ardrey has carefully assembled the evidence for the evolutionary argument in three very readable books. It has long been known that in a barnyard flock of hens, a peck-order is established. The Number 1 hen may peck any other without fear of reprisal. The Number 2 hen may peck all but Number 1, again without reprisal; Number 3 may peck all but Number 1 and Number 2; and so on. In herds of dairy cows, a similar order is established, except that cows butt each other, rather than peck. This ordered aggression is rather harmless: no blood is drawn. It is also completely useless when adequate food is available. In nature, similar peck-orders have been observed in many animals that live in herds or flocks. In only a few cases, notably among male baboons, are lethal injuries frequent; in most cases the physical injuries are minor. Sometimes the ordering seems to serve a purpose, to be based on superior physical strength. Returning to the case of the hens: the observations are based on flocks of not more than a dozen or so; if the flock is much larger, the human observer has difficulty in recognizing the individual hens, in being sure that Number 3 is always the same hen. Unless one supposes that hens have a better ability to recognize each other, the existence of peck-order cannot be established in flocks of a hundred or more. The difficulties of recognizing individual animals become greater when they are not domesticated, but live in a state of nature. Obviously, these observations cannot be relevant to human communities that contain thousands of individuals. Then the aristocratic rank-ordering can be maintained only by resorting to distinctive dress, tattooing, or other status symbols. Such status symbols are not used by animal communities. The failure of one human being to recognize the superior status of a stranger is the time-worn plot of many humorous stories and plays.

This is not intended to deny that all people wish to have themselves and their opinions respected by others, to have their decisions receive the cooperation of others. All people also have a sense of inadequacy, of their ability to foresee the future or to evaluate the present. Under the influence of the aristocratic fallacy, this has been misnamed the "inferiority complex." But whatever names are used, the wish to lead and the wish for the security of competent leadership produce a vicious circle; the higher one climbs the ladder of status, the further one may fall. The stress of maintaining the public image appropriate to one's status becomes greater, sometimes unbearable. It can lead to silly behavior, which provides the dramatist with plots, the psychiatrist with patients, the prison with inmates, and even the morgue with the corpses of suicides. Suicide is very rare among animals; the exceptional case of the lemmings has been widely publicized, but they do not seem to have a peck-order.

The aristocratic fallacy seems to have arisen only after man had evolved as a recognizable species of animal, had acquired his present anatomical characteristics, including the ability to speak. The aristocratic fallacy can be traced from modern Europe and America back to Greece and Rome and eventually to Egypt, Mesopotamia, and India. These early societies were all strongly aristocratic, and included absolute rulers, priests, merchants, tradesmen, farmers, and slaves. In India, the caste system obscured the slave status. In Greece, Rome, and even Mesopotamia, the authority of the ruler was often limited, but class distinctions, including slavery, persisted. This kind of social organization seems to have resulted from the many invasions and conquests that plagued these lands. The conquerors would kill the leaders of the defeated peoples. The leader of the conquerors would reward his followers with the land and other goods which had belonged to their defeated predecessors. The inhabitants of an conquered area were deprived of privileges, and were usually reduced to serfdom or slavery. The best they could expect was deportation to a less desirable region.

The origin of the aristocratic fallacy therefore requires an explanation of invasions or wars. Three explanations have been advanced. Again, the most ancient is theological, the second is economic, and the third is evolutionary. It is certain that theological and economic factors contribute to invasion and war, as will be seen. The evidence for the evolutionary factor has again been carefully assembled by Ardrey in his books. In recent years, evidence has accumulated to show that many animals, including fish and birds, establish territorial claims. Perhaps ownership of home and garden is too anthropomorphic a description, but monkeys, apes, lizards, cuckoos (that build no nests), doves, sparrows, seals, wolves, hippopotami, squirrels, and at least two species of fish all establish territorial claims and repel all intruders. Intruders, and especially intruders of the same species, are vigorously repelled. But this is not warfare within the species. Often it amounts to single combat, although in some cases whole troops of monkeys or packs of wolves are involved. But it is not mortal combat. Both the invading dove and the defender may lose a few feathers, but no lethal injuries are sustained. Almost always the invader retreats, leaving the defender victorious. The territorial conflict is more like that of the peck-order. Again, there is an exception: the Siamese fighting fish fights an invader of its own species until one or the other of them is dead. Consequently, this defense of territory is not like human warfare. There is no killing of members of the same species; there is no robbery; most particularly, there is no enslavement.

Animal and insect societies show many features that can be made to correspond to human affairs. The stratification of bee and ant societies into queen, drone, and worker castes can, with enough good will, be called aristocratic. But there are anatomical differences between the classes; there is no need for status symbols. The anatomical differences do seem to be brought about artificially by differences in the early diet. Presumably, the different diets contain different growth hormones. The dietary deficiencies of the most underprivileged human being do not bring about such drastic anatomical changes (they may, however, be lethal while the insect diet is not). Some species of ants may be called agricultural, for they plant the seeds of favorite plants in easily accessible places. Some ants domesticate other insect species, feeding and "milking" them. However, with the exception of man, no species of animal or insect enslaves its own kind. Goethe's Mephistopheles remarks that man "calls it Reason, but uses it only to be more bestial than the Beasts." Ardrey is not so optimistic as this. He is convinced that all of the evils of human society originated before man's brain had evolved to the stage that people could reason. If Goethe is right, there is a possibility that people can learn to be at least no more bestial than the beasts.

Returning to the invasions that accompanied the beginning of Indo-European civilization, one notes that this kind of war is nothing other than murder and armed robbery. However, the warrior kings claimed to be demigods and to act by divine direction. This was believed by their followers, and perhaps by the kings themselves. By this magic formula, the infamous crime was tranformed into a glorious mission. Even during periods of relative stability, when there were no mass migrations, there were wars. When a general had captured a town, his soldiers would destroy it and kill many of the inhabitants. The remnant of the population would be marched off to grace the general's triumphant return. After this, they would be put to work as slaves. The deportation of the Israelites was by no means unique, but their escape from captivity before they had lost their sense of unity was most unusual. In most cases, such captive remnants lost their cultural identity and became unrecognizable to later historians. Their pedigrees were irretrievably lost.

This kind of stratification was therefore not based on the value of an individual to society. It was more like the peck-order, and to this extent, Ardrey is right. The pecked accepted their hurts as inevitable, and comforted themselves by pecking those below them in the order. But in the case of humans, the injuries inflicted were often more serious than those in the peck-order of the beasts. This stratification of society was also not static, as the aristocratic fallacy implies. People did move from the stratum in which they were born to another. But in most cases the movement was downward. Only victorious military men and their families moved upward. Even in peacetime, a merchant or his family might be sold into slavery for bankruptcy. The most appalling aspect of all this is the human misery it caused. Scarcely less appalling is the moral and intellectual deterioration of the rulers. Even if the founder of a dynasty was an able man, his descendants were often merely callous tyrants obsessed by infantile megalomania and delusions of divinity. There was no respect for human life, and certainly none for intelligence and knowledge. The builder was valued primarily for his ability to construct fortifications and royal tombs. The artist, of course, contributed to the latter. The metal worker was most often an armorer or a court jeweler. The farmers and other craftsmen were valued, not as productive people, but as taxpayers and possible conscripts for the army.

The general degradation of the non-military elements, of the skilled craftsmen and merchants, resulted in a phenomenon that may be called the sedimentation of knowledge. It is a very important phenomenon, and has had consequences that persist to the present day. To some extent, the priests were an exception to all of this. However, during an invasion, a priest might also be killed, especially if his temple contained valuables. If he survived, his fate was no different than that of others; he ended up in slavery. People could be enslaved at the whim of a king or other powerful nobleman. It is said that, on a visit to Syracuse, the Athenian philosopher Plato offended the king, who sold him into slavery. There are various versions of the story, but all agree that it happened. According to one version, an admiring acquaintance, Anniceris of Cyrene, heard of Plato's enslavement, bought him and immediately freed him. According to this version, Plato and the king, Dionysius I, had disagreed about the morality of Greek plays. It is not important whether this story is true or not. It is sufficient that those who passed it on considered it plausible, not so extraordinary as to be unbelievable.

Nothing exhibits the false value placed on a pedigree as does slavery. Enslavement cancels all pedigrees. There were always a few people who recognized this and deplored slavery. They were silenced by an exceedingly ancient proverb: "Slavery will continue until the loom weaves itself."

Solon and the Aristocracy of Wealth

We are taught that Greece, especially Athens, was the birthplace of democracy. It is true that the Greeks experimented with various forms of government. By the beginning of the seventh century B.C., Athens (capital city of the country Attica) had abolished the kingship. Sometimes a coalition of the pedigreed nobles (oligarchs) attempted to run the country without a formally designated leader. In Attica, and in some other countries, a leader (archon) was chosen to serve for one year, but this term of office was often extended. The more responsible leaders resigned before they became senile. At other times, one man would seize dictatorial power; if he assumed the title of king he was called a tyrant. Otherwise he was respected, for very often he did not act in what we would call a tyrannical manner; he usually merely acted like a king, and sometimes like a good one. The word "dictator" had not yet been coined.

At this time, Greece had just emerged from a Dark Age of illiteracy. History was not recorded, but transmitted orally in the form of myths, usually as ballads. The itinerant singers, or bards, provided such entertainment and gossip in return for meals and a night's lodging. The most famous of these was Homer, and when the ballads were written down, his name was attached. If a man chose to claim descent from one of the gods or demigods portrayed in these ballads, there was no way to dispute him. It would, after a generation or two, be generally accepted as fact.

Solon, who lived in the sixth century B.C., was an impoverished nobleman, a poet, and a trader. Among other reforms that he introduced was the abolition of the enslavement of bankrupts. More clearly than any other form of enslavement, this illustrated that people were regarded as commercial commodities. At this time, all Attica was in a serious economic depression and seemed to be on the point of losing an unpopular war. Most people, other than the wealthier nobles, were bankrupt and had mortgaged their land. It would have been foolhardy to try to enslave so many people, most of them armed. But the threat was there, and it did not improve the morale of the people. By using his abilities as a poet and orator, Solon persuaded the people to continue the war. He personally organized and led an expedition that captured the nearby island of Salamis. The loot from this large island (including slaves) lifted Attica out of the depression. This success at armed robbery gave Solon great political power, amounting to dictatorship. He tried to use his power wisely, imposing new laws intended to prevent the recurrence of such an emergency. Many of his laws remained on the books but were never enforced, despite unconcealed violations. There was only one of his laws that the Athenians never forgot, although they modified it several times. Solon established four classes of citizens, with the slaves making a fifth class. Thus, for the first time, the aristocratic fallacy was formalized and legalized. The first four classes were established on the basis of wealth. More precisely, the top four classes were based on the relative worth to society of their members, as measured by the amount of taxes paid, and not by pedigree or military might. It was, both theoretically and in actual practice, possible for a citizen from the fourth class to enter the first class without doing physical violence to anyone. Peaceful business enterprise was thus encouraged since fear of enslavement was replaced by hope of advancement. The duties and privileges of each of the four classes were spelled out to provide further economic incentives. Later generations would modify or ignore many of the provisions of Solon's laws (e.g., the laws against homosexuality and libel), but the four classes of citizens plus the class of slaves remained.

Only the slaves were left without economic incentives. Their only hope to improve their condition was to inspire affection in their owners. They might even inspire such affection that their owners would free them, at some cost to themselves. In this rather unlikely event, the freed slaves became fourth-class citizens. Even this remote possibility of obtaining citizenship was later denied by Pericles.

At first, the two upper classes were composed almost entirely of the wealthier nobles, and the two lower classes were no political threat. They were ineligible for the higher political offices.

Moreover, the Ecclesias, or Town Meeting, was the legal ruler of Attica, and later the nominal ruler of the Athenian empire. Every free adult male was privileged to attend, and the vote of a fourth-class citizen counted as much as that of a first-class citizen. Since the Town Meeting was always held at Athens, this effectively disenfranchised all citizens of Attica who lived outside of Athens. Town Meetings were time consuming, so those residents of Athens whose business pursuits did not leave them sufficient time to attend the Town Meetings were also unable to exercise their franchise. At first, the practical result was that Attica was still ruled by the wealthy nobility, despite the apparently democratic nature of the Town Meeting. Some of the nobles cultivated Solon's art of persuasive oratory, embellished with filibustering filigree. Consequently, the Town Meeting usually gave all important offices to the two or three most persuasive men, and sometimes gave dictatorial powers to a single man. Sometimes the dictator established himself by persuading the Town Meeting to exile or execute some of his rivals, and to provide him with a suitably large bodyguard.

Solon voluntarily resigned his powers. He lived to see much of his work destroyed by Peisistratus. Peisistratus was as picturesque a scoundrel as ever capered across a wide screen. He so outraged the Athenians that they ran him out of town. While he was in exile, he raised a small private army, but he did not try to storm the fortifications of Athens with these inadequate forces. Instead, he tamed an owl and got a woman to dress up like the goddess Athena. With the owl on her shoulder, the woman led Peisistratus and his army into Athens. The Athenians were so superstitious that they offered no resistance to the "goddess" and her train. Once inside, Peisistratus assumed dictatorial powers although he allowed the Town Meetings to continue. Any who opposed him, however, risked their lives. Solon, watching this ridiculous affair, remarked, "Individually you are foxes; collectively you are geese." He then ostentatiously left Athens. It is said that he nailed his weapons to the door of his empty house. The symbolism of this gesture is not clear; perhaps it was meant as a rejection of war and armed robbery.

Solon did not, and probably could not, abolish the nobility of pedigree. Since his own pedigree was one of the best, it is unlikely that he gave the matter much thought. In time he became a secondary standard of nobility. Anyone who could claim Solon as an ancestor could immediately claim descent from the gods. Attica was therefore divided into two rival factions: the aristocracy of pedigree, and the aristocracy of wealth. It was inevitable that this would result in serious internal conflict.

The Silver of Mount Laurium

About a century after Solon and Peisistratus, silver was discovered at Mount Laurium, some twenty-five miles from Athens. It was this discovery, and not Solon's laws, that staved off a recurrence of the economic difficulties which had brought him into power. The mines were owned by the government, which was still the Town Meeting. Their operation was entrusted to private individuals who retained some of the silver as a reward for their services. Other public services in Athens were not financially rewarded, which further excluded the poorer classes from major public offices. At first, even soldiers on active duty received only their food from the government. They had to furnish their own weapons and armor. Obviously, the wealthier men would have better arms, and be more likely to survive a battle. This was legal; Solon's laws concerning the duties and privileges of the classes spelled it out. This division of public service into two kinds, paid and unpaid, was informal and not emphasized. It was soon ignored completely in contemporary political discussions, and consequently, by many uncritical historians.

The actual work of mining and refining the silver was done by slaves. Some of these may have been owned by the mine operators, but other Athenians invested their money in slaves, and hired them out to the mines. Again, this emphasizes the treatment of people as commercial commodities. At one time, Nicias was a major political leader and reputedly the wealthiest man in Athens. His largest property consisted of about a thousand of these mine slaves. The condition of the mine slaves was hopeless. Their lives were miserable and short. They could not hope to inspire affection or even pity in their owner, for he never saw them. He had insured himself against their early death by setting the price of their hire at such a rate that he could afford to replace them. Naturally, they were desperate men; they had nothing to lose except their agonized lives. At the peak of operations, there were at least twenty thousand slaves at Mount Laurium. This must have required large numbers of armed overseers and guards. Picks, spades, and ladles of molten metal are almost as deadly as swords and spears. An archer at a distance cannot be an effective overseer. Slave labor imposes a large and continuing overhead unless the slaves are well treated, and these slaves were not. Although archaeologists are more interested in temples and statues than in mines, Mount Laurium has been investigated sufficiently to provide material for many gruesome horror stories; but these horror stories have not been written.

It is estimated that in nearly two centuries of operation, the mines produced silver worth several billion dollars (based on the 1945 price of silver). Its contemporary purchasing power was about a hundred times that much. Spread evenly over Attica, this would have amounted to an annual income of several thousand dollars per person (not per family). This wealth was enough to raise Attica from subsistence farming to a level of ostentatious splendor as could not fail to be recorded in history and legend. This prosperity was temporarily interrupted toward the end of the first Peloponnesian War. Spartan armies occupied Attica and besieged Athens. With Spartan help, the mine slaves revolted and killed their overseers and guards. Although defeated, Athens recovered some of its prosperity after the mines were again made productive. Finally, however, all the silver was extracted from the mines. Then the glory that was Athens faded like a tropical sunset.

Pericles and the Athenian Empire

Of course, the silver was not distributed equally among the inhabitants of Attica. Much of it was used to equip fleets and armies. These conquered neighboring countries, and the tribute exacted from them increased the wealth of the Athenians. However, tribute is not fair gain. It also involves an overhead for military force. However, the Athenian navy patrolled the Aegean and made it safe for peaceful commerce. Solon's incentives for commercial activity remained effective, and Athens became a mercantile and manufacturing center. Many of the merchants and manufacturers were plebian and not even natives of Attica. As Athens thus became more cosmopolitan, the power of the nobility diminished. Plebians became first class citizens, and the nobility could use only the weapon of social contempt for those who had earned their own wealth and come up from the lower classes. Solon's constitution had not foreseen the day when Athenian citizenship would be valuable to foreigners. An empire needs different laws than a city-state. The man who guided the growth of the empire was Pericles, a descendant of Solon, and therefore a descendant of the gods. He is usually credited with making Solon's constitution more democratic. He was virtually an emperor, but he did not assume that title. He was more subtle, and relied on his ability as an orator to keep his power. Most Athenian orators appealed to the emotions of their listeners, rather than speak about the actual issues at hand. Pericles did not. His speeches were brief and to the point. The Town Meeting was so flattered by his apparent faith in its ability to reason that it elected him over and over, giving him more and more power for thirty years.

Like his ancestor Solon, Pericles tried to use his power for the good of Athens. If Athenians were to rule an empire, citizenship must be restricted to them, and not be granted to other inhabitants of the empire. These others must be dependent on the Athenians. This is one experimental form of government tried out by the

the Town Meeting to restrict citizenship to the legitimate children of two native born Athenians. Henceforth, there would be no way to obtain Athenian citizenship, except by inheritance. Marriage of a citizen to a foreigner was made illegal; this last law served no purpose, and it is not clear why it was included. Eventually, it prevented Pericles from marrying his mistress, Aspasia.

This change in the Athenian constitution was not accomplished without compromise. Previously, only first and second class citizens were eligible for the higher public offices; third and fourth class citizens were now made eligible. Since they were not wealthy enough to donate their services, and since the workload of an imperial official is greater than a municipal functionary, small and inadequate salaries were provided for all officials, including jurors. Pericles is celebrated for these democratic innovations, but fundamentally, his constitution was even more aristocratic than Solon's. Had it endured, it would have produced a new pedigreed nobility to rule the Athenian Empire which he was engineering. It contained no provisions to avert the conflict between the old nobility and the new rich, since some of the new rich had been born in Attica. The insulting marriage law may have aggravated this conflict. Because of his own tact, Pericles was able to keep this conflict between bounds, but during his last years, Cleon the Tanner was his major political rival, and finally succeeded him.

Pericles left three legacies to Athens. One was a surplus of $75,000,000 in gold and silver (1945 prices); another was the Peloponnesian War, which was occasioned by his imperialistic foreign policy; and the third was the internal conflict between the old nobility and the new rich. At his death, all semblance of unity of purpose disappeared from Athens. It was divided four ways: by the conflict between the nobles and the merely wealthy, and by the conflict between those who favored the war and those who opposed it. When Cleon succeeded Pericles, the Athenian Empire was at its height of prosperity. Plato was born at about this time; before he was thirty, the Empire had dissolved, and Athens was bankrupt and defeated. It is estimated that nearly half of the men of Attica were killed or enslaved during the war. The internal conflicts brought on two reigns of terror that exceeded even the usual bloodiness of Athenian politics. These were led by Plato's close relatives, the nobles Critias and Charmides. Defeat escalated the internal conflicts into a civil war. Both Critias and Charmides were killed in battle. The victors were remarkably lenient. They executed only a few of their captives, and freed the rest, even guaranteeing them freedom from future reprisals for their past deeds. This was the first general amnesty in history. Socrates had been the friend and teacher of Critias and Charmides, as well as of Plato. It was not until several years later that Socrates was executed and Plato began writing his dialogues.

The Structure of Modern Western Society

In western countries, the aristocratic fallacy has been formally rejected, although its vestiges continue to have more influence than is commonly recognized. Many of these will be examined in some detail, but it is profitable to begin by considering the changes that have already occurred. It is impossible to assign a specific date to the beginning of modern times. They had certainly begun when it was recognized that every white boy born in the United States had a chance of someday becoming President. They had certainly not begun in the time of Pericles, when any boy or girl, regardless of skin color, had a chance of becoming a slave. This time span is more than two thousand years in length, but it seems futile to attempt to assign a more definite date to the beginning of modern times.

In order to arrive at a more realistic view of society, it must be recognized that there are individual differences that cannot be eradicated by the most rigid curriculum, and that various individuals will choose different educational curricula if allowed to do so. As a consequence of this, and of the speed of modern transportation and communication systems, non-geographic communities have come into being. They are composed of people with common interests, education, and abilities, regardless of their place of residence. Moreover, one person may have many interests, and thus belong to several of these communities. Since they are not mutually exclusive, the non-geographic communities do not fit into the scheme of social classes. It is very likely that such communities have existed from quite early times, but they have become more obvious recently. It may not be a coincidence that the inventor of the telegraph, Samuel Morse, belonged to the art community, to the scientific community, and to the engineering and business communities as well. This seems to contradict those who deplore the increasing specialization of our times. Specialization, however, refers primarily to the occupation whereby a person earns his living, and secondarily to the lack of some kinds of knowledge. This lack is occasioned more by the increasing complexity of our knowledge than by a narrowing of people's interests. Moreover, people may be interested in matters which do not directly affect their income. The emphasis on occupation as a measure of worth to society is a remnant of Solon's vision of society. A person's value to society is not measured by his income, or by his major occupation. There is even the possibility that it cannot be quantitatively measured at all. In any case, personal interests do not necessarily coincide with occupation, and this is the reason for preferring to speak of communities rather than occupations. The rare book community includes both collectors and dealers. The golf community consists of those who play for recreation, the instructors who teach golf skills, the professionals who play in exhibition matches, those who attend these matches as spectators, the managers of golf matches and country clubs, and the sportswriters and commentators who report on golf events. Golf is the occupation of only a few of these; most belong to other communities as well. The invidious distinction between amateur and professional is certainly a remnant of the aristocratic fallacy. Yet, it is instructive.

In discussing the aristocratic fallacy, the needs of people for status, for the respect of others, and for self-respect have been mentioned. Membership in several communities multiplies the opportunities for satisfying these needs. The increased leisure time provided by technological progress adds to these possibilities. A person whose status in the business world is mediocre may find satisfaction as an amateur golf or tennis player. Thus the distinction between professional and amateur is a useful one. It refutes Solon's doctrine that status is a matter of wealth. Wealth and property may be obvious status symbols, but they do not necessarily symbolize self-respect, and self-respect is an important component of status. Men like Van Gogh and Gauguin have given dramatic demonstrations of this, but the later inflation of the price of their paintings has obscured the significance of their work. Under the influence of Solon's fallacy, their paintings have been distorted into status symbols, whose worth is measurable in terms of money.

One may therefore consider modern society as composed of overlapping communities, with membership in any of them being largely the option of each individual. For simplicity, one may call them optional communities. The differences between these communities are qualitative; they cannot be rank-ordered. In the United States, the term "scientific community" has come to include research scientists, engineers, and some physicians, sociologists, and economists. Its boundaries are not sharply drawn, and it might be better to speak of scientific communities, for it will often be necessary to speak of components such as the medical community. The scientific communities are essentially one of the two cultures to which C.P. Snow refers in his book, *The Two Cultures and the Scientific Revolution*. The other culture may be called the "humanistic communities." The humanistic communities are also not homogeneous. There are individuals who belong to both cultures. C.P. Snow is one; others of the recent past include Hermann Weyl, D'Arcy Thompson, J. Robert Oppenheimer, and John Maynard Keynes. It is not possible to describe these two cultures without reference to communities that are not components of these two. To name only a few of the others: the business community, the advertising community, the trade unions, and sports organizations. As Snow says of his two cultures, each of these communities generally has curious and distorted images of the others, and these images are not always flattering. This is inevitable, considering their diversity of interest and knowledge. Moreover, each community has a curious and distorted image of itself which is always flattering. The university community regards the advertising community with disfavor, often accusing it of dishonesty. On the other hand, the university community regards the arts community with favor. Yet, the two communities are two sides of the same coin. Advertising publicizes the desirable features of society; Dickens and Daumier publicized its undesirable aspects. Artists must eat; many have used their talent for the production of advertisements. Writers do not refuse to publish their stories in magazines that derive much of their income from advertising. The overlapping of the various communities provides a social cohesiveness that would otherwise not exist. Conflicts escalate when some of the communities exclude members of others. This has become so apparent that the exclusion of racial communities has been made illegal, and is frowned upon by a large section of the population of the United States. Of course, membership in a racial community is not optional, but the entire concept of race is itself a remnant of the fallacy of a pedigreed aristocracy.

Intelligence Quotients and Career Counseling

Schools and colleges are beginning to recognize the relations between the various communities. Many educational institutions have career counselors, whose job is to provide students with factual information about various occupations so that the available options can be exercised intelligently, and before unnecessary obstacles develop. A specialized literature has been published, making such information more widely available. It is often attractively displayed in school libraries. The emphasis on careers is, of course, an emphasis on gainful occupations. Still the career planning literature is a serious attempt to describe our society as it is. This is not always recognized. As yet, it is an ephemeral literature, since it rarely takes the possibility of social change into account. Until there is a corresponding and adequate socio-economic theory, career planning literature will always have this ephemeral quality.

There has been one attempt to provide such a theory. The model on which it is based has some similarities to modern Western society, but these similarities are not absolute. This theory resulted from dissatisfaction with the use of grade point averages for students, and the use of intelligence quotients for the rank-ordering of people of any age. Both are vestiges of the aristocratic notion that people can be rank-ordered. The intelligence quotient, which is supposed to remain unchanged throughout a person's lifetime, is not incompatible with our modern understanding of heredity. It is not supposed that the I.Q. of the children is the same as the I.Q. of either of the parents. It is, however, supposed to be innate, fixed at birth; it is supposed to be beyond the power of people to change it. It is suprahuman, at least in the original version of the theory. It occurred to the engineer and psychologist, L.L. Thurston, that there may be many intelligence quotients (he called them factors), one for each identifiable human ability.

As an introduction to Thurston's theory, one might consider Newton's theory of color, to which it is analogous. Newton supposed that the retina of the eye has three kinds of receptors: R, G, and B, each sensitive to red, green and blue light, but not to the other two. Any color could then be represented by a point, C, in the interior of a triangle (as shown in Figure 1).

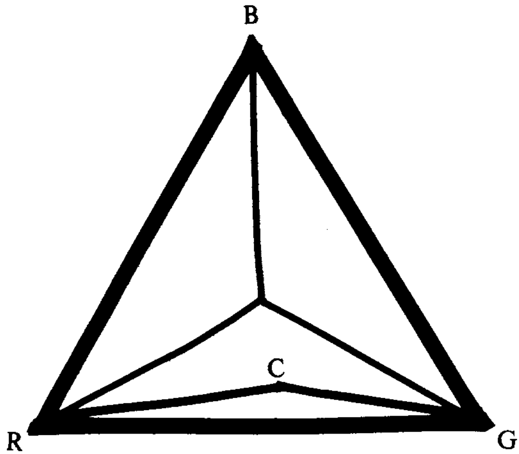

FIGURE 1
Newton's Color Triangle

Here, the lengths RC and CG are in inverse proportion to the amounts of red and green in the color C, and so forth. Since the points in the interior of the triangle are not rank-ordered, neither are the colors. Newton ascribed color blindness to the lack of one or another of these three kinds of receptors. More than a century ago, the physicist and psychologist, G.T. Fechner, coined the term "psychophysical parallelism" for the hypothesis that all sensations are related to physical phenomena in this way.

Thurston supposed that there are many abilities, and that each person has all of them, but in various measurable proportions. The proportions of these abilities would characterize the person; the proportions could be considered as points on a multidimensional sphere. Each person would be represented by such a point, and since the points are not rank-ordered, neither would people be rank-ordered. As in the case of the intelligence quotient, the abilities would be supposed to be innate, not changeable by instruction or experience. The similarity of this theory to Newton's three-color theory of vision is complete, except for one aspect. There are no anatomical features that correspond to mental abilities as the receptors of the eye correspond to color vision. Thurston went on to suppose that the successful pursuit of a career (or a curriculum) would require each of several abilities in some measurable proportion. Careers and curricula would also be represented by a point on the same multidimensional sphere as human beings. A person would therefore be well advised to choose a career whose point was not too far away from his own point. Thurston proposed to implement this theory by subjecting many people, or groups of people, to many tests that could be scored or graded numerically. Some of the tests were questionnaires, and some consisted of assigned tasks. Some of the people involved in the experiment would be selected because they were conspicuously successful in a certain career, others would be volunteers. The results of the experiment would be tabulated; the scores of each person on a line, the scores of various people on a specific test in a column. By analyzing these tables (or matrices), Thurston expected to measure the multiple abilities of each person, and the demand each test placed on his abilities. Various methods for analyzing the matrices mathematically have been proposed. Although none have been very successful, this work has continued over a period of forty years. One reviewer formulated the hypothesis that underlies this work as, "What is mathematically fundamental is psychologically fundamental." This hypothesis was not originated by Thurston; it antedates even Fechner by more than two thousand years. It has never been named, and it has never been formulated as simply as by the anonymous critic quoted above. Following Fechner, it may be called the hypothesis of psychomathematical parallelism. Thurston was not the originator of psychological tests. They had been used nearly fifty years earlier by the French physiologist and experimental psychologist, Alfred Binet. Binet did not subscribe to either of the doctrines of psychophysical or psychomathematical parallelism. However, Thurston founded the journal *Psychometrika*, and this has contributed to keeping his ideas, rather than Binet's, before the scientific public.

One must pause here and inquire how such a fundamental hypothesis entered psychology with so little debate. The full history is lengthy and must be postponed until later, but its latter nineteenth-century part is easily summarized here. Psychology had been taught and studied as a branch of philosophy. Its beginnings can be traced to Aristotle, whose doctrine that axioms are self-evident truths is a proposition about human psychology. The examples most frequently cited are mathematical, "Equals added to equals gives equals." The physical sciences had separated themselves from philosophy (then called "natural philosophy") in the seventeenth and eighteenth centuries, and had not only made much use of mathematics, but had elaborated it far beyond anything Aristotle knew. When psychology finally achieved the status of a separate science, it was only natural that it should attempt to imitate the physical sciences. Fechner's psychophysical parallelism is an imitative generalization of Newton's color triangle. Thurston's efforts to construct a similar theory of intelligence are thus understandable; it was not noticed that his theory is actually a vestige of the earlier domination of psychology by philosophy.

While Thurston's theory has not achieved any noteworthy success, it does show that the aristocratic fallacy of rank-ordering people can be avoided without abandoning the ancient doctrine of psychomathematical parallelism.

The Nature of a Plan

If we are to control our own future, it will be necessary, not only to obtain the cooperation of people, but to prepare comprehensive plans for that future. The Brandywine plan is only one of many that are now being prepared. There are community plans, city plans, regional plans, national plans and plans for the exploration of space. Doubtlessly, this hierarchy of plans will be completed with a plan for the colonization of the other planets, and the utilization of their resources.

In itself, planning is not a new activity; families often plan parties and excursions. Yet there is a consensus that this hierarchy of plans is somehow unprecedented, and would have been impossible in any earlier era. Of course, space travel is unprecedented, although it was foreseen by the soldier, writer, and physics teacher Cyrano de Bergerac in the seventeenth century. He wrote satirical novels about the inhabitants of the Moon and Sun. He foresaw some of the problems of space travel, but not always with accuracy.

The Ionian city of Miletus founded about eighty colonies on the shores of the Aegean, the Black Sea, and the Mediterranean during the long period that the city was in existence. Both Miletus and Athens were destroyed by the Persians about 490 B.C. Once the Persians were finally defeated, these cities regained their prosperity. One of the famous architects of Greece, Hippodamus, was born in Miletus about that time. He based his city plans on rectangular grids, and is sometimes credited with the invention of this type of city plan. (Actually, it was used much earlier in the Indian city of Mohenjo-Daro. Much later, it was widely used in the Midwestern United States.) Hippodamus followed this plan in restoring the Athenian port, Piraeus, and later in laying out the Egyptian town later called Alexandria. Miletus was expanding rapidly, and he laid out a suburb on this plan, thereby becoming the first designer of tract housing. He had to fill a salt marsh to do this, thereby altering the ecology of the region.

During the 2,500 years since then, the natural processes of beach erosion and deposition have changed the ecology of the Mediterranean coast far more than did Hippodamus. His changes were more rapid than the natural ones, but less widespread. The new element, at the present time, is the ability of people to make changes that are almost more widespread than the slower natural changes. This is largely because there are more people, and only secondarily because of new technologies. If there were fewer people to travel the highways, bulldozers would slice through fewer hills. The people of Crete, 1,500 years before Hippodamus, built their towns on artificially terraced mountainsides. Since there were only a few towns, connected by a few roads, most hills remained in their natural state.

It is therefore more profitable to examine the general nature of any plan, than to look for modern novelties. A plan will always have at least two parts: a goal, and a proposed course of action. If the plan involves more than one person, it will not be possible to put it into effect unless the persons involved agree that the goal is desirable and believe that the course of action is feasible, effective, and will have no undesirable side effects. All of this is no guarantee that the goal will be reached by the proposed course of action, or that there will be no undesirable consequences. The plan for a picnic may involve the transportation of a family by car over a familiar highway. When put into effect, it may fail because the highway has been blocked by a landslide. A good plan will therefore include alternative actions, the choice between them being left open until the passage of time indicates which is feasible and which is not. This is especially important when the project being planned is large. When the Egyptian king Zoser started the construction of his pyramid, its ultimate size was left undetermined. His Step Pyramid shows at least six major enlargements of the plan. It is commonly supposed that when Zoser found that he would outlive the completion of his original plan, he enlarged it, ultimately including the large temple complex that surrounds the temple itself. There may also have been other reasons for these changes in the plan.

In contemporary language, a good plan must involve feedback during its execution. This is obviously not new, but a careful analysis of feedback operations is a recent accomplishment; it is part of the present state of technology. There are many kinds of feedback. If the execution of a plan extends over a period of years, technology may change so that what was not initially feasible turns out to be feasible in the end. Recently, technology has been changing very rapidly, so that planning for periods of even a few years requires a forecast of technological change. If the execution of a plan requires many years, the group of persons involved may change. The new group may or may not be in agreement with the goal, and may or may not share the belief of the original group as to the effectiveness of the proposed actions. Undesirable side effects of these actions may have become apparent, leading the planners to change both the goal and the course of action. Planners rarely make provisions for this kind of feedback.

This analysis of an effective plan is surely incomplete; it will serve, however, for a comparison of the present planning effort with some past efforts that had some of the characteristics of an effective plan, though possibly not all. The current planning activity is characterized by a division of labor. The goal is usually established by some agency, often governmental. This agency then sponsors and supports a group of specialists, charging them with the formulation of an effective course of action. This group of planners contains not only technological professionals (in the narrow sense of the word), but, more and more frequently, economists and sociologists. When the group is finished, they present their recommendations to the sponsor in the form of a well-documented report. The documentation contains numerical data supporting their opinion that the chosen course of action is feasible. The report is written in non-technical language, so that it will be comprehensible to the sponsor and other non-specialists. The objective of the report is to persuade the sponsoring agency that the course of action selected by the planning committee is feasible. In establishing the goal, the sponsor will usually have made some effort to assure himself that the community he represents agrees that the goal is desirable. It is often thought that he has the responsibility to obtain a consensus within his community as to the course of action, as well as the authority to put the plan into effect. All of this is a recapitulation of the Brandywine experience.

Apart from the division of labor, the inclusion of sociologists and economists into the planning group is new. Even the distinction between sociology and economics is relatively recent. In the past, they were considered to be identical, so that one speaks only of the classical economists. They would have not considered themselves to be planners. They endeavored to explain the phenomena of business and industry, and published their theories in technical form, not addressed to the general public. They supposed that society and its operations were more or less static and unchangable. If they did consider the possibility of planned change, they usually thought of violent revolution, which would quickly eliminate all the undesirable features of the society of their time. In a modern planning group, the function of the economists and sociologists should be much the same as that of the engineers and physical scientists. It should be to establish the feasibility of obtaining sufficiently widespread consent to the proposed course of action.

The Revolutions of the Eighteenth Century

By the seventeenth century, the social and political organization of Western Europe was considerably different from that of Greece in the time of Solon and Pericles. These differences, however, were not the result of careful planning. Despite these differences, the influence of Grecian ideas on seventeenth-century society was greater than their influence on medieval society. The differences between classical Greece and early modern Europe were especially pronounced in England. This emphasized, by contrast, the continuing influence of Solon and Pericles.

The Ecclesias, or Town Meeting, had been replaced by a Parliament of elected representatives. The pedigreed nobility had not been abolished, but its political power had been much reduced. The House of Commons was already more powerful than the House of Lords; the Prime Minister and his Cabinet were more powerful than the King and his advisers. This trend would continue through the next three centuries. The optional communities were, in retrospect, already apparent. However, a person's freedom of choice was sharply limited by the continuing influence of the aristocratic fallacy. Only persons of some wealth had any freedom to choose, and if their choice involved religious opinions, even they were not able to exercise it. If Sir Isaac Newton had not kept his religious opinions a closely guarded secret, they would have ruined all four of his careers: in the university, in Parliament, in the stock market, and as a government official. It is now known that Newton was a private Unitarian.

Solon's stratification of society in terms of wealth remained. It was actually becoming more pronounced. It was almost impossible for a very poor person to accumulate enough wealth to rise to a higher status. That is not to say that there were no exceptions to this, but they were very few. The list would not be long and the reader would recognize most of the names. In general, poverty was inherited as well as wealth. In retrospect, it is said that this social rigidity was due to the Industrial Revolution. The Industrial Revolution was sudden only in the perspective of time; armed uprisings and bloodshed were not its primary characteristics. People living through it were rarely aware of the gradual changes in society that accompanied the changes in manufacturing technology. The means of production, machines and factories, became too expensive for a craftsman and his family to own. In fact, they were too expensive for people of moderate means; these formed corporations, pooled their wealth, and borrowed from people who had more money than they could use in their day-to-day existence. The stock market was formed, and capital became a commercial commodity, paid for with interest and capital gains. People were, in general, no longer commercial commodities, but their labor was. Labor was sold by the individual laborer, and since there was a surplus of laborers, its price was very low. Distress often made a laborer accept whatever price was offered; there was very rarely any opportunity for the poor to bargain. In continental Europe, conditions were different from those in England, but they were no better.

As Europeans emigrated to other parts of the globe, they had new experiences and, in particular, must have seen that a noble pedigree was of no use to a pioneer. This was clearly stated in the preamble to the Declaration of Independence, and when the Congress of the United States was finally organized, pedigree was given no place in either the House or the Senate. The pioneers encountered other civilizations, less industrialized, and with cultures which had developed independently of Greece and Rome. Unprepared to evaluate these other cultures, the pioneers adopted the fallacy of rank-ordering on the basis of race or nationality. In Europe, this expansion into distant lands gave rise to a number of Periclean empires. Again, this is most clearly spelled out in the bill of particulars of the Declaration of Independence.

In France, the economic and political power of the nobility had not been gradually reduced as in England. Encouraged by the success of the American Revolution, the French proceeded to change their society suddenly and forcefully. This was not the only reason for the French Revolution; the French people also had many grievances against their king. For various reasons, including famine, the Revolution occurred in the homeland, rather than in the colonies. It was a civil war, and in its aftermath Napoleon Bonaparte emerged, first as a dictator and later as the leader of the attempt to establish a French empire on the continent of Europe.

Fiction, Theory, and Social Change

In urging the improvement of society, writers of fiction are often more successful than the proponents of social theories. *Uncle Tom's Cabin* is a well-known example. It was written by Harriet Elizabeth Beecher, daughter of a prominent New England family. For several generations, its men were ministers and theologians, though by no means reactionary. Harriet and her sister Catherine were active in what was then called "female education." In 1836, she married a professor of theology, the Reverend C.E. Stowe; in 1851, she published *Uncle Tom's Cabin* in the antislavery journal *New Era*. It was soon republished and became a bestseller that mobilized wide support for the abolition of slavery in the United States. At the start of the Civil War, Harriet's brother, Henry Ward Beecher, was the pastor of a Brooklyn church that recruited and equipped a regiment of volunteers.

In England, Charles Dickens had experienced the misery of poverty. When Charles was ten, his father was imprisoned for debt, and Charles was left to earn his own living in a factory. After his father's release, poverty continued but the boy received three years of education before he again went to work. After some more years of drudgery, he became a newspaper reporter. This gave him some spare time, which he spent in the library of the British Museum. By 1833, he was contributing stories to a magazine, and in 1836 they were collected and published under the title *Sketches by Boz*. From then on, he wrote prolifically, producing many novels, most of them colored with his early knowledge of the underside of London. Like Harriet Beecher Stowe, he made no specific suggestions for reform, but attracted widespread attention to man's inhumanity to man.

Other thoughtful men were writing theoretical analyses of society, both of its structure and its dynamics. One of the most influential of these was Thomas Robert Malthus. He was a clergyman, but also a puritanical misanthrope. Possibly this was a reaction to the violence of the French Revolution and his father's enthusiasm for Rousseau. He coined the term "struggle for existence", and used it as an argument against humanitarian reforms. Paradoxically, his ideas were incorporated into the platform of the Whigs, Britain's liberal antislavery party. A generation later, when the Whigs were in power, Charles Darwin and Alfred Russell Wallace elevated the "struggle for existence" to the status of a law of nature. Darwin and Wallace were on opposite sides of the globe and arrived at the same conclusion independently; this is some indication of Malthus' great influence at the time. Even later Kipling wrote, "Life is strife, and strife means knife." Today, Malthus is best remembered for his ideas on population. As its population and industry grew, Britain was raising less food, and importing more. Should these trends continue, especially if they became worldwide, Malthus foresaw a catastrophic famine, which would escalate the struggle for existence into violence. Other economists shared his concern about the fate of British agriculture, but considered the problem to be more a matter of real estate than of people, of the interplay between factories and arable land as means of wealth. Some of these economists reached conclusions very different from those of Malthus.

In 1848, there were unsuccessful revolutions in several continental European countries. Their principal objective was to spread the reforms achieved by the American and French Revolutions. Armed rebellion as a means of social change had precedents, and was generally seen as the only effective means of changing society. In the aftermath of the uprisings, many of the people involved were forced to emigrate. One such person was Karl Marx. His principal part in the rebellions had been that of a writer of political editorials. With his associate, Friedrich Engels, Marx had also written the *Communist Manifesto*, in which the phrase, "Workers of the World, Unite", appeared for the first time. Marx went to England, where he lived in poverty. Each day, he secluded himself in the British Museum, and educated himself in economic theory and history. In 1867, he published his own theory of economics, which was based on the fact that each laborer was creating more wealth than he received from the sale of his labor. This created an unearned increment to capital, and caused it to increase cumulatively. In turn, this increased the disparity between the proletariat or working class and the capitalist class. Marx concluded that only public ownership of the means of production could prevent the disparity from becoming intolerable. This theory might have become a subject for Parliamentary debate and the enactment of measures to avert the catastrophe. However, Marx was distressed by the miseries of the poor, as Malthus had not been. He also concluded that the expropriation of capital could only be achieved through violence, and that it was his duty to hasten the revolution which he considered to be inevitable. In England, slavery had been abolished without violence, but it had not been a major economic factor. In the United States it was, and the long Civil War resulted. There was little reason to doubt the generally accepted view that major social change entailed violence. Marx did not fail to note the anti-humanitarian influence of Malthus in England. He emerged from his seclusion with his manuscript unfinished; he founded and was the first leader of the International Workingmen's League. Later, after the end of the Franco-Prussian War, the League sponsored the rebellion known as the Paris Commune. This was unsuccessful, partly because of armed intervention from outside Paris, partly from lack of widespread support, and partly from the lack of a planned course of action to be followed after public ownership of the means of production had been declared.

The Marxian theory had assigned a minor, even negligible importance to technological change. Before the end of the nineteenth century, these changes had accelerated to a pace which could not be ignored. Some thought that this would make the Marxian catastrophe more terrible. Ignatius Donnelly was of that opinion. Born in 1831, he became a political leader in Minnesota. He was first Lieutenant Governor, then a congressman, and once the Populist candidate for Vice President. In addition, he was an editor and scholar of unconventional opinions. In 1890, he published *Caesar's Column, a Story of the Twentieth Century*, under the pen name of Edmund Boisgilbert, M.D. This novel contrasted the opulent lives of the owners of the means of production with the miseries of life in the mechanized factories and mills and their accompanying slums. *Caesar's Column* foresaw a world-wide police state with its internal espionage and counter-espionage; it foresaw organized underground resistance, poison gas, and the bombing of cities from the air. Donnelly imagined armored, but lighter-than-air dirigibles, capable of a non-stop flight from New York to Uganda with a detour over Europe. Ultimately, there was open rebellion and treachery compounded upon treachery. This and the advanced military technology resulted in the destruction of civilization and most of mankind.

Donnelly was not an able novelist. He slowed the action by long philosophical conversations and tables of numbers. The book was widely read, though not always with approval. Reviewers deplored its content more than its literary shortcomings. Some said that it should never have been published. Because of the activities of Marx and his followers, any discussion of Marxian revolution was confused with advocacy of violence. In 1894, Donnelly abandoned his anonymity, republished the novel, and defended himself. It is a novelist's special function, he said, to expose what is dangerous in sentiment and pernicious in action by a vivid picture of its consequences. Like Harriet Beecher Stowe, he was devout, and believing that he read the future aright, considered it immoral to remain silent. Still, his effort did not prevent the use of poison gas and the bombing of London from dirigibles during World War I. This war was not the Marxian revolution. In the United States, at least, it was seen as an effort to extend the principles of the Declaration of Independence to Europe. In the aftermath of the war, however, the Russian Revolution was guided by Marxian terrorists.

Edward Bellamy was born and lived most of his life in Chicopee Falls, Massachussetts. He was educated in law and admitted to the bar, but he did not practice. Instead, he devoted his life to writing. He apparently accepted the inevitability of the Marxian revolution. Evidence for this seemed to be increasing toward the end of the nineteenth century. The courts denied the rights of workingmen to organize unions and bargain collectively. Police action escalated strikes into riots. It was in this turbulent atmosphere that Bellamy wrote his novel, *Looking Backward: 2000-1887.* The book passed lightly over the revolution, which was supposed to have occurred about 1887. Its calm and placid story provided a vivid picture of post-revolutionary life, as imagined by the author. Somehow, Bellamy realized, as Marx did not, that the detailed formulation of a goal was essential if social change was to be directed toward the better. Next, it would be necessary to convince a large number of people that the goal was not only desirable but attainable, and that technology would contribute to its attainment.

Bellamy foresaw the broadcasting of music and the use of credit cards. The music was broadcast by telephone, which had been invented some years earlier. It must be supposed that he did not foresee wireless telephony, but even if he had, his readers would very likely not have believed in its feasibility, and this would have been to his disadvantage. He did foresee the selection of programs by turning a knob. He did not foresee that the use of credit cards would be preceded by charge accounts. The universally acceptable card had to wait until machines had been invented to do the rapid and voluminous bookkeeping that is needed. He foresaw the formation of politically strong guilds (trade unions to us) and the intervention of government in business and industry. In both cases, his picture is more extreme than present reality. He supposed that all political power would be legally vested in the guilds, and that the election of public officials would be in accordance with this. The credit cards would not be issued as payment for either personal services or the use of capital, but would give everyone an equal share of the available goods. The means of production would be owned by the government.

Looking Backward lacked any description of the course of action that would be required to attain this goal (i.e. socialism); thus it was not a complete plan in the sense previously discussed. Bellamy seems to recognize this incompleteness, for he devoted much time to a second book, *Equality*. This book was a socio-economic treatise that took its name from the famous phrase in the Declaration of Independence. While *Looking Backward* had been an immediate bestseller, one critic described *Equality* as completely unreadable. It certainly does not hold one's attention as a novel does. In attempting to assess Bellamy's influence on the course of events, one must not ignore the size of the audience he reached, or the gradual influence of public opinion in changing society. His audience was not confined to the intellectuals. Sylvester Baxter, in a preface to the 1898 edition of *Looking Backward*, describes Bellamy's move from Massachussetts to Denver for reasons of health: "The welcome accorded him in the West, where his work was met with widespread and profound attention, was one of his last and greatest pleasures. Letters came from mining camps, from dirigibles, capable of a non-stop flight from New York to Uganda with a detour over Europe. Ultimately, there was open rebellion and treachery compounded upon treachery. This and the advanced military technology resulted in the destruction of civilization and most of mankind.

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Bellamy foresaw the broadcasting of music and the use of credit cards. The music was broadcast by telephone, which had been invented some years earlier. It must be supposed that he did not foresee wireless telephony, but even if he had, his readers would very likely not have believed in its feasibility, and this would have been to his disadvantage. He did foresee the selection of programs by turning a knob. He did not foresee that the use of credit cards would be preceded by charge accounts. The universally acceptable card had to wait until machines had been invented to do the rapid and voluminous bookkeeping that is needed. He foresaw the formation of politically strong guilds (trade unions to us) and the intervention of government in business and industry. In both cases, his picture is more extreme than present reality. He supposed that all political power would be legally vested in the guilds, and that the election of public officials would be in accordance with this. The credit cards would not be issued as payment for either personal services or the use of capital, but would give everyone an equal share of the available goods. The means of production would be owned by the government.

Looking Backward lacked any description of the course of action that would be required to attain this goal (i.e. socialism); thus it was not a complete plan in the sense previously discussed. Bellamy seems to recognize this incompleteness, for he devoted much time to a second book, *Equality*. This book was a socio-economic treatise that took its name from the famous phrase in the Declaration of Independence. While *Looking Backward* had been an immediate bestseller, one critic described *Equality* as completely unreadable. It certainly does not hold one's attention as a novel does. In attempting to assess Bellamy's influence on the course of events, one must not ignore the size of the audience he reached, or the gradual influence of public opinion in changing society. His audience was not confined to the intellectuals. Sylvester Baxter, in a preface to the 1898 edition of *Looking Backward*, describes Bellamy's move from Massachussetts to Denver for reasons of health: "The welcome accorded him in the West, where his work was met with widespread and profound attention, was one of his last and greatest pleasures. Letters came from mining camps, from farms and villages, the writers all longing to do something for him to show their love."

Those mentioned above were not the only nineteenth century novelists to deal with such themes. Donnelly's publisher listed "Serious Works for Students of Social, Economic, and Political Problems", which included four works of "Fiction: Social, Economic, and Reformative" in addition to Donnelly's. Marx's friend and collaborator, Friedrich Engels, wrote a polemic against such novels, entitling it "Socialism, Scientific and Utopian." While it is certain that no one of these novelists had a decisive influence on the United States, collectively they prepared the public for the reforms that came in the twentieth century. There was widespread interest in social problems, and this may have averted the Marxian catastrophe. These reforms began long before the economic stagnation of the 1930's, and even that disaster was not accompanied by civil war. These reforms have not affected the basic concept that labor and capital are commercial commodities. They have included anti-trust laws, and the recognition of the rights of trade unionists to organize and go on strike. The graduated income and capital gains taxes, on the one hand, and social security pensions and insurance on the other, tend to counteract the aggravation of economic differences by the unearned increment of capital. The trade unions have not, as Bellamy anticipated, been incorporated into the political system, but they have acquired political power. Intervention in public affairs by the trade unions is not frowned upon to the same extent as is the intervention by corporations that employ large numbers of people. All of this makes the present day United States very different than it was in the nineteenth century, although it is still far from perfect.

Recent Planning by the Scientific Communities

It is certain that the course of human events can be influenced, but that course has been erratic. It remains to examine the suggestion that modern science can make this course more direct. This is not an easy task. One may begin by examining some of the evidence for this in the United States. In other countries there is evidence of much the same kind, but comparisons tend to be more confusing than illuminating.

During the Civil War, the National Academy of Science was created by an Act of Congress, and signed into law by President Lincoln. It was to be, in contemporary terms, a self-governing, non-profit corporation. The Act also imposed duties: "The Academy shall, whenever called upon by any department of the Government, investigate, examine, experiment, and report upon any subject of science or art, the actual expenses...to be paid from (Government) appropriations...but the Academy shall receive no compensation whatever for any services to the Government of the United States." There have since been amendments to the Act, but they have not altered the spirit of the original, and both Government and Academy have acted in that spirit for more than a century. The official actions leading to the formation of the Academy occupied only a few days. Their background need not be considered here, but can be found in the later chapters of Coulson's biography of Joseph Henry.

At first, the Academy seems to have been called to perform its obligation to the Federal Government only on rare occasions. The Government had already established its own permanent scientific agencies; among these were the Coast and Geodetic Survey, the Geological Survey, and the Hydrographic Office. Later, the Weather Bureau and the Pure Food and Drug Administration were added. The Academy elected its members for life, with past achievement as the criterion for election to the Academy. Many

Government scientists were elected. During World War I, the Academy mobilized all the scientists in the country. For this purpose, it organized the National Research Council. Appointment to the various components of the Council was temporary, and based on ability to contribute to the work in progress. At the end of the war, the Council remained active. At this time, it is still active, although the Academy is engaged in reorganization.

During the 1920s, the Academy initiated a major program for improving scientific education and research in the United States, especially in the colleges and universities. For this purpose, it devised the system of postdoctoral fellowships. The Government was not equipped to support the project, and its cost was borne by private individuals and foundations. The success of the project induced some of the latter to assume responsibility for it, and to extend it to fields outside the sciences. Consequently, when refugees from European universities came to the United States, they found the colleges and universities staffed with people they respected and with whom they could cooperate. As a consequence, the United States became preeminent in science and technology. After 1950, the Federal Government contributed to the support of the postdoctoral fellowships.

During World War II, the Academy and Council remained rather aloof from the war effort. The scientific and technological parts of the war effort were managed by the National Defense Research Council and the Manhattan Project. Government agencies were empowered to support university research directly on a nonprofit basis, the university administrations being responsible only for fiscal supervision of staff expenditures. All of this is reminiscent of the Academy's charter. At the end of the war, the Manhattan Project was converted into a new Government agency, and the National Defense Research Council was disbanded. Many of its projects were taken over by the Academy and its permanent National Research Council. Most of these were nonmilitary; examples include the Committee on Meteorology in the Academy, and the Highway Research Board in the Council. Some military problems also received attention from the Academy, as was inevitable.

The United States emerged from World War II with undisputed preeminence in science and technology. The maintenance of this status became a national goal. This goal was not set by any formal action, unless one considers the establishment of the President's Scientific Advisory Council (PSAC) as such an action. The PSAC was headed by a scientist rather than a Cabinet member, but it certainly had a very considerable influence within the Federal Government. During the period from 1950 to 1968, this goal was endorsed, more or less explicitly, by four Presidents, and President Nixon did not depart widely from it.

Even before the establishment of the PSAC, there had been major and complex changes in the relation between the Government and the science communities. These changes may be simplified into economic terms. Before World War II, the universities and the research activities of their faculties and students had been supported almost entirely by private philanthropy. Very early, some of them had been endowed with Federal land grants, but these had ceased to be a major factor in the economics of education and research. Now Federal financial support increased and became the major economic factor. From 1956 to 1966, the Federal expenditures for scientific research increased at an average rate of 20% per year, compounded annually; which is an average rate of $400,000,000 per year. Some of these Federal expenditures were made directly to researchers by Government scientific agencies, and were thus not channeled through the universities. For practical purposes, one may suppose that 60% of Federal funds were channeled through the universities, and another 20% through industrial research and development laboratories. The large expenditures of the Department of Defense have not been included in these estimates; in any year they were about twice the amount that the Government awarded for nonmilitary research. In general, defense expenditures for research were not channeled through the universities, but they did create occupational opportunities for scientists, and thus influenced the enrollment of students in scientific curricula. This large financial support of science and the universities was unprecedented in the history of any nation, and was accompanied by equally unprecedented phenomena of other kinds.

The Academy appointed a Committee on Science and Public Policy (COSPP) to cooperate actively with PSAC. COSPP followed the precedent set by the National Research Council, and appointed panels to investigate and prepare reports on different aspects of this policy. Financial support for this work was provided by various

Governmental agencies, in accordance with the Academy's charter. Other branches of the Academy, and other professional organizations, such as the Social Science Research Council, also followed this procedure for providing advice to the Government. It will be convenient to call these the academic reports. The PSAC and other Government agencies prepared additional reports, often using the same system of panels drawn from the scientific communities. The following is necessarily a very inadequate digest or critique of these many reports, but some common characteristics are apparent.

The scientific community has never been a single organization. It consists of numerous smaller communities that may be conveniently be called disciplines: mathematics, chemistry, physics, biology, etc. In the course of this work, one or more reports on each discipline were written and published. The panels that prepared these reports were composed largely of members of the disciplines concerned. Members of other disciplines were also included, and these made a valuable contribution. Except for their liaisons with COSPP and the PSAC, as well as other Government agencies, the panels worked independently. The end products could only be sets of plans for the disciplines, and not unified plans for the scientific community. However, all recommended courses of action that would contribute to the achievement of the goal of American preeminence in science. This gave them a unity that is not superficially obvious. The reports were addressed to different agencies, and published in different formats.

A typical report consisted of four parts, the first containing the panels' recommendations. Strictly speaking, this first part was the report submitted to the Government; the other parts were supporting material designed to establish the feasibility of the recommended actions. The cost of these actions was obviously of interest to the agency sponsoring the study. The cost was usually not estimated. Instead, it was assumed that the funds available to each discipline in future years would continue to increase at the same rate as in the immediate past. Since the number of people involved in the discipline had also been increasing at the same rate, it was assumed that this growth would continue. Since the scientific disciplines are optional communities, this was an assumption about the choices people would make. These choices had undoubtably been influenced in the past by the funds available for salaries and fellowships in the sciences. The other university communities were not so liberally supported; this made the sciences more attractive as occupations. The assumption was, therefore, that financial renumeration was the only factor influencing people's preferences.

Two other parts of the reports, the forecasts and the essays, were sometimes combined. The forecasts concerned the probable achievements of the discipline in the reasonably near future. Since these were made by people working in the discipline, it may be assumed that they were accurate. It has been seen that even non-scientists can have some success in forecasting scientific and technological achievements. The essays were addressed to a wider audience and were sometimes published and circulated separately from other parts of the report. All of them were written for an audience that is wider than the discipline, but still a part of the scientific-technological communities. It is impossible to say that these essays were widely read in the building-trades or business communities. However, they are still being read on university campuses and elsewhere. They are very useful to teachers, both as source material and as collateral reading. Many have been read by congressmen concerned about the place of science in our society. Invariably, they are permanently valuable additions to our literature, and will continue to be influential for many years. This influence will be on the scientific-technological communities, and those closely allied with them.

Either implicitly or explicitly, these reports support the view that man, aided by science, can now control his own future. In the same way, they support the hypothesis of Psychomathematical Parallelism. The most ambitious of these volumes of essays is entitled *Biology and the Future of Man*.

By 1966, the unrealistic assumptions of the academic reports had become apparent to responsible Government officials. In particular, it was clear to them that the yearly increase in Federal funds for scientific research could not increase indefinitely. When the Academy was informally asked to advise on this, its reponse was essentially to state that this rate of growth should continue, at least for several more years. Congress and the Bureau of the Budget ignored this unofficial advice and took abrupt action. The lack of an increase in Federal funds for research in 1967 was the first indication of changes in the Government's attitude toward science to reach most members of the scientific communities, those who had not been involved in the broader aspects of the planning effort. Some of them reacted with anger, all of them with great concern. Some found it very difficult to meet the commitments they had made for the future support of their staff of colleagues and assistants. As is usual in such cases, the hardship was greatest for the marginal members of the communities. These were young people who had not yet established themselves as scientists, or were studying to prepare themselves for a scientific career.

Most Government agencies did not completely reject the recommendations of the academic reports. They did ask for advice on the order in which various projects should be supported. Reluctantly, COSPP transmitted this request to some of its panels. These found it impossible to reach agreement. The reluctance came from the fear that such disagreements would be disastrous for the unity of the scientific communities; this fear does not seem to have been justified. For other reasons, but with equal reluctance, the Government agencies assumed the responsibility for setting priorities. This was done neither abruptly nor thoughtlessly. Higher authority imposed restrictions on the purposes for which a given agency might spend its funds. But the people in the agency inevitably had to make the decisions themselves. The scientists had to "shop around" for financial support. Earlier, the senior members of the scientific community had welcomed the system of multiple sources of support. It was thought that this prevented any possibility that the Government would dictate scientific goals. Now the multiplicity of small and restricted sources has reduced the efficiency, and increased the difficulty of operating scientific laboratories with Government funds. Scientists are not accustomed to restricting the range of their activities, and perhaps should not be. Work in one field may unexpectedly reveal the way to solve a problem in some quite different field.

The Changing Public Images of Science and Technology

The unprecedented financial support for science and technology was closely related to the public image of science. This gave political sanction to the governmental actions just described. A brief review of the changes in that public image is therefore in order; it would be better to use the plural, for prior to World War II, science and technology were sharply distinguished in the United States, and probably elsewhere as well. In the early part of the twentieth century, the average American considered himself to be "practical," and admired Thomas Edison for his practicality. The industrialization of the United States was a matter of national pride, and was thought to be the result of this practicality. The scientist, on the other hand, was caricatured as an impractical recluse, slightly mad because of his interest in esoteric matters. Occasionally, in the fiction of the day, this madness was malevolent, but generally it was supposed to be harmless. In the period between the two World Wars, chemistry and the medical sciences received wide publicity for the benefits they bestowed on both humanity and industry. Astronomers and mathematicians were still the epitome of impracticality. Only the few people who had gone to college were personally acquainted with a scientist; "Doctor" meant either "physician" or "minister." Despite the publicity given to Albert Einstein's mysterious achievements, most people did not know that he was a physicist; to them, "physics" meant "laxatives." Immediately after World War II, the public images of science and technology fused into one: both were considered practical and beneficial. During the period 1900 to 1967, the number of Ph.D.'s awarded each year had increased by a factor of nearly one hundred. The number of people who were personally acquainted with a scientist or engineer increased by a much larger factor. The general public became aware that these specialists were friendly, well-intentioned people.

Of course, there was the atomic bomb. Some people thought the scientists ought never to have invented it. They were answered, truthfully, that the men who had built the bomb had tried (before its existence had been publicized) to prevent its use. There was a sincere doubt that the small scientific community had the political power to prevent the misuse of the achievements of science. There was even the doubt that refusal to work on weapons would be an effective deterrent to their development. Leonardo da Vinci wrote that he knew how to build submarines, but he refused to record his knowledge for fear that it would be used to commit murder on the sea floor. Despite his restraint, submarines were eventually invented. Scientists were also criticized in a rather thoughtless and ill-informed way. This can be illustrated by an anecdote concerning a general conversation shortly after television had become competitive with the movies. There was general agreement about the low literary and artistic standards of the TV programs and the flatulence of the commercials. I concurred, and this so startled a professor of romance languages that she turned to me and exclaimed "But you're a scientist!" It soon developed that she considered TV to be a great scientific achievement, and she assumed that I would therefore defend it in all its aspects. Moreover, she held "Science" responsible for the quality of the programs. This discussion remained friendly, but became futile. I then tried to find out what she knew about TV; essentially she knew only that if one pushed a button and turned a dial, the tube lighted up, and (usually) one saw and heard the actors. During the course of this conversation, she had used the word "magic."

Over the years I have followed up on this clue, and my conclusion is that one perception of the scientific community is that of a group of magicians. They provide the public with an assortment of Aladdin's lamps, and teach it the proper way to rub them in order to evoke the desired djinn. This view is not confined to the literary-artistic community; it is shared by many businessmen, politicians, school principals, etc. Of course, if questioned directly, they would deny it. After the fairy tale period of kindergarten, people rapidly become aware that it is embarrassing to admit a belief in magic. However, even today the astrological community is larger than the astronomical. It is not irrelevant to remark that Kepler earned his living as a court astrologer. Quite recently, some scientists and engineers have encouraged the view that science is magic. In a book addressed to adults, a distinguished professor at Columbia

University entitled the second chapter "Modern Magic" and opened it with the words "Aladdin's lamp"; the third chapter was entitled "The Magic Carpet."

In the late 1940's and the 1950's, these djinns were most often considered friendly, but this view began to change, as has already been noted in connection with the Brandywine plan. However, the unrealistic public estimate of the power of science, combined with equally unrealistic criticisms, produced a reaction in the scientific communities. They denied social responsibility for the uses to which their discoveries were put, and also denied the ability to foresee undesirable effects of these discoveries. The achievements of Ignatius Donnelly and Anatole France were forgotten. But the undesirable aspects of industrialization, urbanization, and war are real. In the early 1960's, Rousseau's romantic anarchism was revived among university students. The educational system had failed to provide anyone, public officials and scientists included, with the historical background needed to deal with these problems. The students, therefore, declared education and even government to be irrelevant. Their alienation escalated into violence.

This escalation from quiet alienation to violent confrontations with the police was certainly hastened by the Vietnam War and Selective Service. This system of conscription had been introduced during World War II. Exemption from military service was granted to all men who had civilian skills that were both essential to the war effort, and could not be quickly acquired by those exempted for other reasons. Almost all scientists were thus exempted from involuntary active military service. In the 1960's, this exemption was extended to all university and college students, but only for the period that they were in school. After graduation, the exemption was withdrawn. The inequity of this system soon became apparent. The underprivileged, including many blacks, were not prepared for college, and could not have afforded it in any case. Since the war in Vietnam was unpopular, much of the student dissidence focused on the draft, racial discrimination, and the defects of the educational system. The recognition that not all technological change was beneficial was more widespread among the general public than was romantic anarchism and violence. Disapproval of the latter merely led to confusion. Public officials were necessarily more sensitive to these matters than were the scientific and technological communities.

The Planning Process in the Federal Government

The annually increasing amount of federal funding of scientific research was not unplanned. If this planning process has a name, it is the Budgetary and Appropriations Procedure. The agencies that spend federal money are in the executive branch, which is headed by the president and his cabinet. Each year, each agency makes a budget request. The requests are reviewed and revised by the Bureau of the Budget, under the supervision of the president and in conference with the agency staffs. After this, they are assembled and the president sends them to the House of Representatives. Here, the budget requests are referred to permanent committees, whose memberships change only slowly; their members can therefore familiarize themselves with the problems which are their responsibility. Each committee holds public hearings at which many people are invited to testify. Anyone interested in a particular budgetary item may ask for an invitation, or submit a written statement which is incorporated in the proceedings for the committee. These committees prepare bills recommending their revised versions of the budget requests. These bills are debated and amended on the floors of both the House and the Senate. After passage and approval by the president, the appropriated funds are made available to the agencies for expenditure. This entire process requires about twenty-four months. The actual expenditure of the funds, in which the agency has some discretionary authority, requires at least another year. The process is therefore one of continuous planning for the future. It has been developed over many years; the Bureau of the Budget is a comparatively recent addition.

During the 1960s, most of the budgetary items for science research and education were referred to the Committee on Science and Astronautics and its Subcommittee on Science, Research, and Development. These committees not only followed the customary procedures, but went further. They established a Panel on Science and Technology, which met for several days each year with the

Committee. The membership of the Panel varied from year to year, but it always included guests from foreign countries. At these meetings, there was informal discussion and papers were prepared. The themes varied from meeting to meeting. In 1968, the ninth meeting had the theme "Applied Science and the World Economy"; the tenth meeting had as its theme "Science, Technology, and the Cities." In 1963, this Committee also became the first Congressional Committee to formally request the advice of the National Academy of Science. As a result of their activities, the Committee members became familiar with a number of theories about the relation between science and society.

By 1966, it had become apparent that none of these theories were adequate guides to the problems being actively discussed by the general public. Some of these problems were slums and urbanization, pollution of the environment by industry, and the waste of our natural resources. On the positive side, the Subcommittee had identified an area of study that seemed to have been neglected: Technology Assessment. Representative Daddario, Chairman of the Subcommittee defined it as "improved information and analytical inputs to the legislative process so that our management decisions can ensure the realization of full benefits from our knowledge, and minimize the unwanted, unintended, and unanticipated consequences of applied science." He emphasized that the Committee was not seeking to establish a regulatory agency (like the Federal Communications Commission) to control technology or its uses. This was the theme of the eighth meeting of the Panel on Science and Technology in 1967.

The Invention of the Steam Engine

It has been seen that for the last twenty years science and technology have received unprecedented amounts of money from the U.S. Government. Moreover, this was planned, and is therefore related to the idea that science can enable man to control his future. This raises several questions. To what extent were the planners influenced by (possibly erroneous) views inherited from the past? Have the relations between science, technology, and society always been as they are today? The second question is more easily answered than the first. In one word, the answer is "No." But a complete answer would require an examination of the entire history of society and its changing relation to science and technology. This would involve one in either complex detail or abstract and dubious generalizations. It is therefore advisable to select a few episodes for detailed analysis. The invention of the steam engine is a suitable beginning.

After Galileo and Newton had developed the science of dynamics, some members of the scientific community were more interested in the properties of gases than in the motion of the planets (and similar matters). Among these were Boyle and Huygens; a comprehensive account even of the use of steam, would require the enumeration of many others. The invention of the steam engine is sometimes described as an Horatio Alger success story. In a sense this is true; certainly the scientific community was peripheral to the principal actions. These were practical rather than intellectual, and were mostly carried out by men who had little formal education. The history of the use of steam for practical purposes is complicated. The following account is simplified by emphasis on the cylinder-piston-condenser engine: the condenser was sometimes replaced by (less efficient) means for wasting the steam into the atmosphere.

Denis Papin was born in France, completed medical studies and, a few years later, entered Huygen's laboratory in Paris. The work in this laboratory centered on the production of vacua. Papin made improvements on vacuum pumps, and published on this topic with Huygens as joint author. In 1675, he moved to London, working in Boyle's laboratory and continuing his improvement of vacuum pumps; these were mainly cylinder and piston, but Papin is also credited with the invention of a condensing pump. In 1679, he demonstrated a pressure cooker (an autoclave, complete with safety valve) to the Royal Society of London and was elected a member the following year. Shortly thereafter, he went to Venice and stayed until 1684. In 1687, he moved to Marburg, as professor, and then to the university in Cassel. During this stay in Germany, he engaged in correspondence with Huygens and Leibnitz. He published frequently, in the French language. In 1705, Leibnitz sent him a sketch of Savery's apparatus for ejecting water from mines. This did not involve the use of cylinder and piston, but did involve the condenser principle. Papin had earlier published a design for a cylinder and piston engine; it would have been inoperable since no condenser was provided. He now (1705) published, again in French, a design that provided means for manually venting the spent steam into the atmosphere. Papin did not return to London until 1707.

Thomas Savery was a military engineer, but engaged also in mining operations. In 1702, he published *Miner's Friend.* Although his engineering applications of steam are of interest in themselves, it will be enough here to emphasize his familiarity with steam condensers, devices for reconverting steam to water by cooling it.

Thomas Newcomen was a blacksmith in Dartmouth, England; he was also actively involved in the construction of mining equipment, as is shown by his correspondence. Some of this was with the prominent scientist, Robert Hooke, and concerned Papin's work. Since Hooke died in 1702, Papin's work of 1705 is not mentioned in that correspondence. There seems to be no evidence that Newcomen and Savery knew of it in that year. Yet in the same year, they and a man named Cawley constructed a cylinder and piston steam engine. It differed from Papin's mainly in that the spent steam was condensed inside the piston cylinder, by dousing the cylinder with cold water. It also differed in external form.

Little seems to be known about Cawley, not even whether he was a grazier or a glazier; he may have spelled his name Cowley. According to all accounts, Newcomen was the leader of this successful project, and he was elected a member of the Royal Society. This engine attracted much attention, but was not greatly improved for some years. Papin could certainly have done so immediately, had he made the effort to understand the condensing principle.

James Watt was the son of a hardware dealer. Going to London, James apprenticed himself to a "philosophical-instrument maker." Since the invention of the clock by Huygens, and the telescope by Galileo, the construction of experimental apparatus had increased, and this trade had become profitable, with university scientists as the principal customers. Apprenticeship had the force of a legal contract. It provided a source of cheap unskilled labor, and in return the apprentice received an initiation into the trade secrets. On completing the contract, he became a journeyman and a source of skilled labor. A few journeymen became masters; this required both literacy and some capital. Poor health made Watt leave his apprenticeship without becoming a journeyman. Both were handicaps to his early career, even though he seems to have mastered the trade in one year. Since he did not complete his apprenticeship, he was excluded from the Hammermen's Guild and the commercial practice of his trade. In 1757, he was employed by the University of Glasgow, and there made friends of some of the faculty. He must have profited especially from conversations with Joseph Black, who had discovered latent heat (i.e. the heat required to convert boiling water to steam). The University had a demonstration model of Newcomen's steam engine, and Watt repaired it in 1764. This is further evidence of the passivity of the academic community in the matter. Watt thus became aware of the inefficiency of Newcomen's engine, and must immediately have seen possibilities for improving it. He was led to perform his own experiments with steam, and to separate the condenser from the piston cylinder. He was very clear about the reason for doing this. To be efficient, the cylinder and piston must be as hot as possible and the condenser as cold as possible. These conflicting requirements can only be met by separating the condenser from the hot components of the engine.

Watt left the University, presumably in order devote himself full time to the development of the steam engine. After some years of disappointment, the partnership of Bolton (or Boulton) and Watt was formed. The partnership was a commercial success, Boulton providing the capital and managing the business affairs. In the course of business, Watt invented the throttle valve, the centrifugal governor, the double-acting cylinder and piston, and the linkage for converting reciprocating to rotary motion. The reciprocating steam engine was thus given a quite definitive form, and became an industrial necessity. None of Watt's inventions required more than a rudimentary knowledge of mathematics and physics. They did require intelligence and mechanical ability. Two men, however could not do all the work of the firm; they must have had skilled assistants. The shaping of large metal parts to the required precision must have required the invention of machine tools. Did Watt invent these too, or were some of his workmen also inventors? Certainly many of them must have understood Watt's ideas before they were converted into metal parts and mechanisms. It seems that these matters were not recorded in detail. Neither do the encyclopedias record Boulton's biography or activities.

The magnitude of this enterprise required financial resources not available to a university professor. This was certainly a factor contributing to the passivity of the scientific community. There were other reasons. Among these was the scientific community's preoccupation with more sophisticated mathematical theories and new experimental discoveries. The aristocratic fallacy also added a sense of elitism. Personal involvement in commercial and manufacturing enterprises was of a lower order than detached impersonal speculation about them. This did not preclude some interest in practical affairs, as evidenced by the election of Newcomen to the Royal Society, and his correspondence with Hooke.

Many must have noticed that even with its separate condenser, Watt's engine was not completely efficient, but these observations were not recorded in the scientific literature until the nineteenth century. This may have been due to the primitiveness of current ideas about heat and energy. The ancient Greek opinion that heat was an indestructible gas (phlogiston) had not yet been challenged. Energy was an abstraction, derived from Newton's mathematical theory. At first, only two kinds of energy were recognized, kinetic energy and potential energy. Boyle and Hooke added elastic energy as a third. Moreover heat did not fit readily into Newton's theories.

It was not until 1824 that Sadi Carnot, still working within the limits of the phlogiston theory, gave a mathematical formulation of Watt's ideas about the importance of cold, as well as heat, to the efficiency of heat engines. This bit of mathematics laid the foundation for the science of thermodynamics, but only after effective steam engines had been in use for more than fifty years. Even then, Carnot's work attracted little attention until, in the middle of the nineteenth century, heat was recognized as a form of energy. The theory of thermodynamics was not completed until the twentieth century, when the reciprocating steam engine was all but obsolete.

This one example typifies the original relation between industry (or technology) and science. The industrial development preceded the scientific. It provided the material facts about which scientists thought. Perhaps the one exception to this rule was in astronomy, but even in this case, one may identify astrology and oceanic navigation as the stimulating industries. Scientists did perform experiments and make observations that were more crucial than the industrial operations. On the whole, science took more from industry and commerce than it returned.

The Beginning of the Scientific Revolution

The scientific revolution was as complex as was the industrial revolution. Neither can be described in one sentence, nor in purely economic or sociological terms. One of the phenomena associated with the scientific revolution has already been described: the fusion of the scientific and technological communities which occurred during and after World War II. This could not have been foreseen in the years following the invention of the steam engine, and was not foreseen in the early nineteenth century. The nineteenth century did produce other phenomena that made the fusion of the two communities possible, if not inevitable. At the beginning of the nineteenth century there was no electrical industry. At its end, this industry was economically viable. The chemical industries changed in a similar, scarcely less dramatic fashion. By 1930, they bore little resemblance to their predecessors in 1830. The medical arts, especially public health activities, underwent a similar change. Cameras and phonographs became commercial goods. These developments were accompanied, or perhaps caused, by a change in the self-image of the scientific community.

Michael Faraday's father was a blacksmith. Self-educated, and with almost no knowledge of higher mathematics, he became Sir Humphrey Davy's laboratory assistant. Uniquely in the history of science, even to the present day, he succeeded Davy in the prestigious Professorship of Chemistry at the Royal Institution of London. Religious tests had just been abolished in England; otherwise this would have been impossible, as Faraday was a devout member of the Sandemanian sect of dissenters. He became a famous and popular lecturer. His Christmas Lectures were attended by children and royalty alike. At one of these, he demonstrated the heating effect of an electric current. Afterward, it is said, Queen Victoria asked him, "...But what good is it?"; to which he replied, "Madame, what good is a newborn babe?"

This anecdote contrasts strangely with one told about Euclid, who taught in Alexandria about 300 B.C. It is said that a student asked what it would profit him to study geometry. Instead of answering, Euclid told a slave to give the man a copper and then show him to the door. Of course, the authenticity of this is doubtful; but it tells much about the recent biographer who wrote that he wished he could believe it, and about those who were not shocked by the wish. The contrasting attitudes exemplified by these anecdotes is the outstanding characteristic of the scientific revolution. The change came gradually, and some of Euclid's elitism remains evident today, even in our elementary schools.

Not only was there no electrical industry in 1800, but most electrical and magnetic phenomena had not yet been discovered. Compact histories of these discoveries and the ideas generated by them are readily available. Is is both impossible and unnecessary to condense them further here. The significant fact is that almost all of these discoveries were made in universities; the reasons for this require consideration.

These discoveries were made in the course of experiments performed by university professors who were usually assisted only by students and instrument makers. The experiments were "laboratory scale": they were so inexpensive that the universities could meet their costs out of current income. Occasional gifts from private individuals and philanthropic organizations were helpful, and sometimes the professors had the means to support their own work. The enterprise was given unity by correspondence and publication, as well as by some travel and changes of residence. This manner of paying for the cost of the enterprise was unavoidable. Queen Victoria spoke for almost the entire population. Even if a few far-sighted businessmen shared Faraday's vision, they could not risk much capital with any reasonable expectation of a profitable return. Such capital as they did contribute was written off as philanthropy. Philanthropy was traditional and required no economic justification; social approval was sufficient return. The monetary costs of generating ideas from the basic discoveries was negligible; it amounted to a small fraction of a professor's salary. The essential ingredient was undisturbed time for concentrated thought, as has already been noted in the case of Archimedes, and the university campus was the only favorable milieu for this. Lectures to students and discussions with colleagues were important; they made the thinker put his ideas in order and submit them to intelligent criticism. Albert Einstein called this interaction "resonance." Correspondence and publication were also a part of resonance as well as being ways for communicating ideas to a larger audience.

No professor received additional compensation for this work; many professors did not take part in it. The doctrine of "publish or perish" had not yet been established. The work was done "for its own sake." This is not to say that it was without compensation. Part of this was quite personal and private, the euphoria that accompanies a new discovery. Another part was the approval and admiration of one's colleagues. In the laissez faire academic community, competition for these returns is quite as keen as the competition of businessmen for monetary rewards. Erwin Schrödinger has noted that these returns, euphoria and admiration, are similar to those derived from skillful athletic competition. The custom of awarding trophies for athletic success has been translated into medals for scientific success. This perpetuated the elitism that distinguishes between amateur and professional athletics. It also reinforced the distinction between the university professor and the engineer.

Not only the elite feel the need for non-monetary compensation. It is felt by all people, and it is this that distinguishes human effort from an inanimate commercial good. The invention of the steam engine, and later the invention of electrical distribution systems increased the number of people whose need for non-monetary reward is being (more or less) satisfied. The standard of living has risen. But fundamentally, human effort is still considered a commercial commodity, to be purchased in the same market as food, shelter, and automobiles.

Perhaps the electrical industry has done more to raise the standard of living than any other industry. It was non-existent when Marx wrote *Capital*, and he did not foresee it. It is important to keep this in mind when reading that book. Prior to the industrial revolution, people, draft animals, water, and wind were the primary sources of energy. In most places, wind is fickle. Windmills and sailing ships must shut down in calm weather. Water power is geographically localized, and, even with dams, mills must be located on the banks of streams. The steam engine freed the factories from this geographical constraint. Small steam engines are very inefficient; but not much energy is needed to drive small machines. The mechanical distribution of the energy of a large steam engine required a complicated system of shafts, pulleys, belts, and gears. This system was quite inefficient and in constant need of repair. Power machines could only be used in large factories, or on large construction projects. People and draft animals were still the primary sources of small amounts of energy. The development of electrical distribution systems changed all this. Energy could be delivered anywhere, in large and small amounts. Electric sewing machines, washing machines, portable power saws, and other woodworking tools; all these and many more became possible. Not only did the loom weave itself, the broom swept itself. By the mid-twentieth century, it was no longer necessary to use people as primary sources of energy. It was not only unnecessary, it was economically unprofitable. People were released for activities which utilized their more specifically human abilities and rewarded them with non-monetary returns. Draft animals were displaced by oil and gasoline engines, but initially the social importance of this was less evident.

The Optical Industry in the Nineteenth Century

Since the electrical industry originated during the nineteenth century, and is otherwise unique and important, one might next consider its history. Its history has so many ramifications that this would be unwieldy and not very instructive. The essentials would be obscured by a mass of detail. Other smaller industries exhibit the same general feature, the changed relation between science, technology, education, and society. Of these, the optical industry is an ideal textbook example. The histories of other industries are often different in more than detail, but show the same general trends.

By 1850, many optical devices were in use: spectacles, magnifiers, telescopes, and even compound microscopes. These were made by individual craftsmen, spectacle makers, and philosophical-instrument makers. Each master craftsman had a few assistants, apprentices, and journeymen. The use of spectacles was increasing and provided the major income.

Ernst Abbe was born into a poor family, but his father had a steady job in a nearby factory, and the family seems to have lived rent-free in a haunted house. The family was spared the periods of extreme misery that afflicted many of their neighbors when business was bad and many factory workers were laid off. After the unsuccessful revolution of 1848, Abbe's father hid some of the refugees in the haunted house. The police were conducting random searches, and the eight-year-old Ernst did duty as a lookout. He obtained his early education through a form of apprenticeship, modified by the factory's need for clerical help. This contract was broken, but not without high words on both sides. Somehow, the boy retained the good will of one of his employers, and finished high school on a scholarship provided by Herr von Eichel, whose generosity is deserving of record. By tutoring wealthier boys, Abbe even managed to save a small sum. This, together with his father's savings, enabled him to attend the University of Jena, though not without hardship. Two prizewinning essays brought him money and hometown fame. Some of its citizens combined to provide him with a stipend that alleviated the hardships of the first year at Jena.

After having obtained all that he could from the meager faculty at Jena, Abbe moved to Göttingen in 1859. Again, he was saved from financial disaster by a prizewinning essay. Göttingen was then, as always, an active center for mathematical and physical research, and Abbe came into contact with some of the best minds in his chosen fields. In 1861, he obtained his doctorate. His dissertation was an epistemological analysis of the experimental evidence for the equivalence of heat and mechanical work. Both Riemann and Weber were on his examining board. After two disappointing years in Frankfurt, Abbe returned to Jena in 1863. Beginning as an instructor in mathematics and physics, he became full professor in 1878, remaining active in that capacity for twenty years.

Carl Zeiss was born in 1816, the son of a tradesman, in Weimar. This is the town in which Goethe had spent many years, working simultaneously as prime minister, poet, and scientist. As prime minister he had established the position of *Hofmechaniker* (Mechanic to the Court). One of Goethe's scientific interests was visual phenomena, so that the first man who held this job developed skill in making optical instruments. Carl Zeiss learned the trade of instrument maker from this man's son. In 1846, Zeiss opened a shop in Jena, whose main business (at first) was repairing apparatus for the University. This was quite insufficient to support the shop, so Zeiss also made spectacles and magnifiers. He and Abbe became acquainted during the latter's student days. Abbe is said to have occasionally worked in the shop. Almost coincident with Abbe's departure, was the arrival of the biologist Häckel in Jena. Häckel and his colleagues were much interested in the discoveries which were then being made about the cell structure of living things. For this, they needed compound microscopes, and Zeiss began making them. The construction of these microscopes was a trade secret. Each instrument maker obtained one from each of his competitors and tried to steal their secrets. The image produced by a microscope can be distorted by ten or twenty different kinds of defects. There was therefore little chance of making a good custom built microscope in this way. Glassmaking was also a trade secret, and different batches of glass, even when they came from the same supplier, were not the same. Moreover, the precision of current machine shop methods left much to be desired. Hence, it was impossible to make an exact copy of a good microscope. The instrument makers had a demand for goods they could not produce.

Zeiss attempted to educate himself in the laws of optics, and to design microscopes by calculation. This was a difficult problem, not only for him, but for the mathematician to whom he first went for assistance. Finally, Abbe returned to Jena, and Zeiss asked for his help. Abbe's studies had not only interested him in such a project, but also made him capable of bringing it to a successful conclusion. He attacked it both theoretically and experimentally. At the beginning, he could scarcely have foreseen that it would be his life's work. He would not only make many discoveries and inventions, but he would have to construct a new optical theory of image formation. Abbe's theory was based on the wave properties of light, whereas previous theories had been based on the geometrical properties of rays of light. This new theory, as well as the microscopes that he and Zeiss produced, made it necessary for biologists to reconsider everything they thought they had seen with the older instruments. At first they were not grateful, but ultimately the demand for Zeiss' microscopes increased. The business grew and prospered.

Zeiss and his workmen invented new methods and machines for shaping metal and glass to very precise shapes and sizes. A demand for such things developed, as did demands for specialized optical instruments. The business grew at an ever increasing rate. Abbe was able to undertake experiments that the University could never have paid for. His scientific work benefitted accordingly. The business was able to employ other men with university educations, and the project was no longer a simple partnership. Both the monetary and scientific returns increased as new minds and hands were put to work.

One major obstacle to the manufacture of good microscopes was the poor quality of glass. Each new batch had to be tested, using methods and instruments invented for the purpose. Only the best glass could be used, and the rest was thrown away. Goethe had experienced the same difficulties, and had endeavored to improve matters. He had even tried, unsuccessfully, to establish a glass works in Jena. Others had also tried and failed to get the glass makers to improve their product. Otto Schott was the son of a manufacturer of household glassware. He attended a university, and obtained a doctorate with a dissertation on "Mistakes in the Manufacture of Window Glass." After some practical experience elsewhere, he returned to his father's glass works and began laboratory experiments with new kinds of glass. After some correspondence with Abbe, he came to Jena. Zeiss and Abbe provided capital for the construction of a laboratory in which somewhat more ambitious experiments could be performed. Later, the State of Prussia contributed a little to support this work. The firm of Schott and Company (the Company being Abbe, Zeiss, and Zeiss' son) was formed and a larger laboratory was built. Abbe's increasing part in business affairs led him to refuse to accept pay from the University. Still, he continued to perform his duties as professor and director of the Astronomical Observatory in addition to his scientific work with Zeiss and Schott. The commercial production of optical glasses began somewhat later.

Much of the success of these enterprises must be ascribed to the high company morale that Zeiss and Abbe created. They pioneered not only the scientific and technological aspects of the optical industry, but also the field of employee welfare. Abbe wrote papers on the economics and sociology of this. His early experiences with poverty and social unrest certainly stimulated his interest in the matter, and his experience as a wealthy industrialist made his ideas practicable. They provided the firm's employees with the non-monetary rewards that people need. The employees were given paid vacations, an eight hour workday, good working conditions, a guarantee against arbitrary dismissal, and a share of the profits. Child labor was permitted only under humane conditions. Employees could get unsecured loans, to be repaid by small deductions from their salaries. Various cooperative organizations run by the employees were encouraged by the management, and were probably subsidized with working capital. Contrary to the predictions of less farsighted economists and sociologists, Abbe and Zeiss became very wealthy men, and must have received non-monetary satisfactions as well. Unlike the misanthropic theories of Malthus, Abbe's actions were philanthropic, in the most all-inclusive use of the word.

Abbe held stern views about the unearned increment to capital, and especially about its inheritance. An early will testifies to the sincerity of this belief. After the death of Carl Zeiss, Abbe and Roderick Zeiss transferred the ownership of the entire business to a non-profit corporation. Its trustees were charged with operating the business, and with maintaining the integrity not only of its assets, but also of its organization. Furthermore, they were charged with concern for the welfare of the employees and for the public interest. The latter included the city of Jena and its University, which had already been greatly affected by the optical industry and its increasing number of employees. All funds remaining after the discharge of these duties were to be used to support research in the natural and mathematical sciences.

Not many of the new industrial corporations had all of the characteristics of the firm Zeiss-Jena, but these were the characteristics of the social changes which accompanied the scientific revolution. They were at least as great as those that the revolutions of 1848 hoped to bring about, and had much the same objectives. Slowly, and with various modifications, they spread outward from Jena, and were incorporated into the society of Western Europe and the United States. This must of course be modified by saying that there are many relics of Malthusian misanthropy in modern society

The Emergence of the Engineering Communities

With the development of the new industries and the emergence of less misanthropic social theories, the nongeographic communities became more easily recognized, increased in number, and people could more easily choose among them. The engineering or technological communities were among these nongeographic communities. It is not known where the terms "architect" and "engineer" were first used, but there were architects and engineers in very early times. There is every indication that the engineers evolved out of the military, and the architects out of the civil administration. Both were involved in larger construction projects than the building of a home. Military requirements dictated both the construction and destruction of fortifications, including city walls and gates. The design and construction of fortifications was not much different from that of bridges, dikes, aqueducts, canals, roads, harbors, docks, and jetties. The distinction between the architect who designed and constructed palaces, temples, and granaries, and the military engineer would not be sharply defined. Some individuals would be members of both communities. Vestiges of this arrangement remain in the present day United States. The Army Corps of Engineers not only has authority to approve or disapprove bridges over rivers, but it also has responsibility for flood control. Its Beach Erosion Board not only studies ways to keep harbors from silting up, but it has also become increasingly concerned with the recreational aspects of beaches and lakes.

The destruction of fortifications required the design and construction of engines of war: battering rams, Trojan horses, catapults, and ultimately cannons. Rumford made the first measurement of the mechanical equivalent of heat while supervising the Bavarian arsenal; in that capacity he also invented canned food. A successful siege may also require the construction of a tunnel. It has already been noted that Thomas Savery was a military engineer who made notable contributions to civilian mining operations. Alexander the

Great captured the island city of Tyre by having a causeway built out from the mainland. The Romans took Masada by building a ramp from the plain to the top of the fortified cliff.

The differentiation of the engineering communities into military and civil is well documented. The father of Gaspard Mongé was a peddler and scissors sharpener. At the age of twenty-two, Gaspard became professor of mathematics at the French officers' school in Méziéres. There, he had already devised his method of descriptive geometry, and used it in designing fortifications. It was promptly classified as a military secret. During the French Revolution, Mongé performed many services for the Republic. He was Minister of the Navy for a time, and a young artillery officer, Napoleon Bonaparte, visited him on some small business. Later this casual encounter ripened into a warm and enduring friendship. Mongé had a talent for such friendship.

With Mongé's help, Napoleon gathered a brilliant circle of scientists and scholars about him. Earlier, in 1794, Napoleon had persuaded the Convention to expand the French elementary school system. To provide the teachers, the Ecole Normale had been established. This was now reorganized and renamed the Ecole Polytechnique. Mongé was essentially in charge of this institution, which was now given the additional task of educating scientists and engineers. At first the curriculum seems to have made no distinction between civil and military engineering; the choice of a career was made later in life. Mathematics had a central place in the curriculum. All of the members of Mongé's group were essentially members of the Polytechnique's faculty and it was soon a leading center for science. After Napoleon's downfall, Mongé was dismissed by Louis XVIII. Another of the original group, Pierre Laplace, was more politically adroit and succeeded Mongé as head of the school.

The Ecole Polytechnique continued to provide an example for the developing educational systems of both Europe and the United States; the course of events on the two continents differed only in detail. In the United States the first engineering curriculum was established at West Point in 1802. In 1823, Norwich College, in Vermont, established a curriculum in civil engineering. In the following year, Reusellaer Polytechnic Institute (apparently patterned after the French Ecole Polytechnique) was established. The British

Institute of Civil Engineers was established in 1818, the American Society of Civil Engineers not until 1852. During the latter half of the nineteenth century, the development of the new industries resulted in further differentiation of the engineering community. Curricula in architectural, mechanical, electrical, and ultimately chemical and sanitary engineering were established.

These educational developments were somewhat different in Europe and in the United States. In the latter, the engineering schools were administratively distinct, even when they were located on a university campus. Members of the science departments often participated in the teaching of engineering students, however. Until well into the twentieth century, four years was considered adequate for an engineering education, and no research was conducted in the average engineering school. The objective of the engineering curriculum was to provide enough knowledge of the relevant sciences to enable the graduate to plan and supervise the construction of factories and new machines. Many of the older industries had never employed a college graduate, except perhaps as an administrator. The engineering graduated disseminated the new knowledge in these older industries as well as in the new industries. When the medical schools were incorporated into the universities, their relation to the general campus was much the same as that of the engineering departments. While members of the science departments taught in the engineering and medical departments, they were personally interested in those aspects of their science which were as yet well removed from utilitarian objectives. The distinction between the scientific and engineering communities became greater (rather than less) as time went by. Naturally, however, there were individuals like Abbe and Schott who belonged to both communities.

Thus, the scientific revolution caused great changes in the educational system as well as in industry. These changes occurred not only in the colleges and universities, but in the preparatory schools as well. In the United States, public high schools were established by the end of the nineteenth century. They not only replaced apprenticeship in training craftsmen, but also functioned as preparatory schools. Consequently, the high schools included algebra, geometry, and the sciences among their course offerings. While the elementary schools continued their single standard curriculum, the high schools and universities had numerous curricula. Their students could exercise free choice in entering one or another. Not all high school graduates went on to college or university, however. The high schools thus served a dual purpose: preparation for higher education, and preparation for those crafts and businesses that needed more than the standard elementary education of the grammar schools. The old distinction between occupation and avocation was continued, but supervised training in sports and fine arts was introduced at all levels.

The Instrument Makers and Technicians

The handicrafts developed in Paleolithic times, and presumably resulted in the first division of labor within social entities larger than the family. With the passage of time cities grew larger, and many of the crafts organized into guilds. This occurred in very early times. The primary purpose of the guild was the preservation of a monopoly through the protection of trade secrets, as well as through collective bargaining and political influence. A second purpose of the guilds was educational. The apprentice-journeyman-master system preserved the security of trade secrets while transmitting them to a new generation. Naturally, there were few written records of this technological knowledge.

A number of factors rendered the guilds obsolete. One was the advent of power tools, machinery, and the assembly line. Another was the patent. The intent of this legal device was to make it profitable to disclose trade secrets, especially new inventions, by guaranteeing the inventor a royalty for a period of years. Then there was the spread of free, or at least inexpensive, education in public high schools and trade schools. Finally, there was the advent of the engineer, who often surprised the guild by knowing its most cherished secrets -- and more. In the Midwestern U.S., the manufacture of glass and ceramics continued to be family enterprises until well into the twentieth century. The trade secrets had been brought from Europe by earlier generations, but sooner or later, changing conditions produced problems that the secrets could not overcome. Reluctantly, chemical engineers were called in to help. The proprieters were always astonished to find that their secrets were common knowledge among the engineers, as were the solutions to their problems. All this left the guild with only two functions: collective bargaining, and political influence. After a period of confusion and distress, the guild evolved into the modern trade union.

But there was one handicraft that appeared very late in history, and whose evolution differed from this outline in specific ways. This was the trade of the instrument maker. Abbe's most mathematically perfect design for a microscope would have remained a fiction if there had been no Zeiss. It was first necessary for Zeiss to understand the design; then it was necessary for him to devise a feasible method of building the microscope. This required a knowledge of methods for fabricating metal and glass parts, plus an intelligence that could improve on the methods used on the past. New tools (both hand tools and power tools) had to be devised and constructed. At this point, the evolutionary tree of the instrument maker has two branches. The first branch is that taken by Zeiss. A small, specialized industry is developed, with its factory located in a university town. It may, however, be very large -- the computer industry is a recent example. The instrument maker becomes a businessman and employer unless there is someone to relieve him of that responsibility, as Boulton relieved Watt. In that case, the instrument maker becomes a specialized engineer.

Not all instrument makers elect this option to join a new community. Many prefer to stay in the laboratory. It may be either a university laboratory or an industrial laboratory. In any case, it is headed by a scientist who evolves goals that have never been achieved in the past, and may have no clear utilitarian purpose. In the exact sciences, these goals are often formulated in mathematical terms, and it is the job of the instrument maker to understand them and use his skills to convert them into material objects. These objects may or may not have a utility outside of the laboratory; they may never be duplicated, much less mass-produced. The experiment may fail or be inconclusive; the scientist modifies the goal, and the feedback cycle of the planning process is completed.

If the laboratory is biological or psychological, the scientist may formulate his goals in logical rather than mathematical terms. However, there is a current tendency to replace logic and common sense with mathematics. The construction of new and unique instruments is secondary to the use of purchased instruments (e.g. microscopes) of proven value. The experiments depend more on the processing of biological materials, or the recording of responses to psychological stimuli. Even in these laboratories, there are people who have a function very similar to that of the instrument maker. They are usually called technicians. It is their job to understand the goals formulated by the scientist, and then to use their special skill and knowledge (either of instruments or processes) to achieve them. Again, the experiment may fail, the goal be modified, and the feedback cycle repeated many times.

The instrument makers and the technicians are as essential to the scientific or technological enterprise as are the scientists and engineers. They earn their living through the exercise of manual skills, and a special kind of knowledge that has come to be known as "know-how." Above all, they use their skills and knowledge with an intelligence that is not to be deprecated, even though it is rarely recorded in learned journals. They are the evolutionary successors to the old craftsmen, and they obtain the same nonmonetary satisfaction from their work as the craftsmen did when they finished tables, carpets, and jars.

The handicrafts cannot be obliterated by the assembly line, the mass-produced spare part, or the automated rolling mill. If people cannot earn their living at handicrafts, they will take them up as recreational hobbies, or invent games of skill (e.g. tennis or golf) to replace them. The scientific laboratory is a refuge for the individualistic craftsmen and craftswomen who will not conform to the regimentation of the trade unions, but must still earn their living. This fact receives no publicity, but it is essential to the understanding of the scientific revolution.

It is impossible to conclude this inadequate account of the community of instrument makers and technicians without calling attention to one of the most unjust vestiges of the aristocratic fallacy. These people never share in Nobel Prizes. There is of course an exception to this. The communities of Nobel Prize laureates and instrument makers are not mutually exclusive. Many able scientists are also able instrument makers and technicians. Some, like Abbe, have also been businessmen and sociologists.

The Scientific Revolution in the Twentieth Century United States

During the 1930's, Nazi activities made central Europe not only an unpleasant place for intellectuals, but a dangerous one. There was an exodus of thinkers; England and the United Stated benefitted from this immigration. It strengthened their universities and their industries, even though these exiles were unhappy and hoped to return to their native countries. By 1940, when World War II had escalated to a full-scale conflict, the exiles in the United States found themselves permanently allied with their new country. Their teaching had much enlarged the number of scientists in the country.

In 1942, most university professors in the sciences left their posts to engage in the war effort. Only a small proportion of the professors of science and technology entered the armed forces, however. Instead, they worked on government projects. The Manhattan Project was merely the largest of many, and it has received the greatest publicity. The work on many of these projects was conducted on sites other than university campuses. The Federal Government found it expedient to ask universities to administer and manage these projects, and entered into contracts with them for this purpose. The Government provided funds, and the universities disbursed them for the purposes specified in the contracts. Collectively, these projects provided an impressive demonstration of the ability of scientists to create new technologies, and the scientific revolution entered upon its second phase. The technologies that cluster about the A-bomb are best known. However, shipbuilding, and the electronic and aircraft industries were also revolutionized. Substitutes for unavailable materials were developed; nylon is an example. New pharmaceuticals, jeeps, bulldozers, and many other things were developed and later found their places in the civilian economy. This money also provided the university people with funds that enabled them to experiment on a scale that had previously been possible only in industry. They were provided with assistants and technicians. Some of these later became university students and professional scientists and engineers. For the first time, professors had personal secretaries, and were relieved of the necessity of typing their own letters and manuscripts. They had telephones on their desks, not down the hall in the department office.

Scientists, especially the younger ones, became fully aware of their ability to create new technologies. The old distinction between the aristocratic pure scientist and the plebian engineer began to be questioned, though some traces of this remain today. It is mentioned because there is a division of labor in the scientific enterprise. However, such terms as basic and applied research, development, and engineering are more often used without pejorative connotations. This was an unforeseen consequence of the war projects. Not only scientists and engineers, but also business and government officials, became aware of the scientific revolution. This was to have a great economic influence during the post-war years.

As the war drew to a close, it became apparent that the projects had so disrupted the universities that it would be very difficult to restore them to their pre-war condition. In fact, this disruption was of the magnitude of a national disaster. The U.S. Navy had a long tradition of providing assistance to civilians in times of disaster. It was therefore suggested that the Navy assist the universities in their recovery. The wartime system of contracts provided a feasible method; it would merely be necessary to alter their wording, not their management and administration. This was the early thought concerning the Office of Naval Research. It was not foreseen that it would continue to exist for more than twenty-five years.

At the same time, farsighted people discussed the feasibility of Federal support of the scientific community through the establishment of a civilian agency. This was not the first time that such a suggestion had been made, or the first time that it had been rejected by the scientific community. Dedication to laissez faire caused its leaders to fear that such an agency would use financial support as a means of gaining dictatorial power. This rejection certainly influenced the planning for the Office of Naval Research, although I do not know the details. The younger scientists had become personally acquainted with Navy and Army officers, and (quite correctly) did not discern any dictatorial ambitions in them.

Reciprocally, these officers had acquired an appreciation of the abilities of the scientists and engineers. This must have influenced the planning for the Office of Naval Research. Another influence was the existence within the Navy of an organization capable of channeling federal support to the universities. This combination of mutual trust and expediency contributed to the establishment of the Office of Naval Research as a stable agency of the Federal Government.

It also led to the modification of the wartime contracts in a significant way. Legally, the contracting parties were the Navy and a university. However, the contracts designated one faculty member as Principal Investigator; no expenditures could be made without his authorization. Moreover, it was the Principal Investigator, not the university administration, who negotiated the contract. It was he who "sold" the Office of Naval Research on the character of the work to be supported by the contract. The description of the work was in very general terms; it was recognized that the results of research could not be foreseen, and that the original plan might be modified during the term of the contract. The Principal Investigator was given very considerable discretion in such matters.

More and more frequently, the contracts also specified that the Principal Investigator should be provided with paid assistants. These might be full-time technicians, or part-time graduate students whose education was thus subsidized. As time went on, other Federal agencies, civilian as well as military, established their own Offices of Research, and the universities' budgets for research grew from a minor part of their income to a major part. This caused a profound change in the universities. Two centuries earlier, the primary function of an American college or university had been the education of ministers, teachers, and lawyers; the sons of a few wealthy families attended with no intention of following one of these professions. Most colleges were denominational, and supported by a church. By the twentieth century, this had changed considerably. It has been seen that scientific and engineering curricula had been added, and an increasing number of students, uncommitted to any profession, came for a general education. However, science, engineering, and medicine did not dominate the campus. In many of the Land Grant Colleges, agriculture was prominent. Under this new system of Federal support, the scientific and engineering faculties became a wealthy elite. Previously, the department chairman had been an important officer, sometimes dictatorial. Now he became the servant of his faculty. To some extent, the university president suffered the same change in status. University regulations and policies were modified to meet the needs of the federally supported projects. In some cases, the overhead fees collected by the university for the administration of research projects were indirectly used to provide more support for the rest of the campus.

This could not have happened without equally major changes in the agencies of the Federal Government. It has been seen that during the nineteenth century, a number of technical and scientific agencies had been established within the various Departments. The armed forces necessarily had their engineering branches, and these had established research laboratories prior to World War II. The civilian Departments had established numerous similar organizations: the Weather Bureau, the Geological Survey, the Coast and Geodetic Survey, the Smithsonian Institution, the Naval Astronomical Observatory, the Bureau of Standards, the Public Health Service, the National Administration for Civil Aviation; this is only a partial list of federal activities in the field of science and technology. Like the military, all of these had benefited, during World War II, by close working relations with university people. They also became aware of a need to keep abreast of scientific advances and new technologies. Eminence in science and technology became a Departmental goal, even before preeminence became a national goal. The Research and Development budget of the Federal Government increased for this reason.

All of these agencies recognized that eminence requires continuing contact with the research going on in universities. The support of that research by contracts proved to be effective, and many civilian agencies followed the precedent established by the Office of Naval Research. This has resulted in the formation of a typical-laissez-faire organization, a nightmare for the General Accounting Office and the Bureau of the Budget. But it is just what the scientific enterprise wants. If one agency cannot fund a particular research project, its proponents can seek support from other agencies. Dictatorship is impossible. This is not to say that the scientific enterprise needs unlimited amounts of money, and complete autonomy in the expenditure of those funds. If man is to determine his own future, the scientific and university communities must both recognize the preferences and needs of other communities. The future of mankind cannot be planned by one or two communities.

The present crisis in the scientific, tecnological, and university communities results, in large part, from general social disapproval of some of their recent activities. If these had remained academic, as in the late eighteenth and early nineteenth centuries, this disapproval might be ineffective or unjustified. But by becoming active agents for social change, and by accepting financial support for their activities, these communities have also accepted responsibility for respecting the wishes of other communities. If man's future is to be planned and controlled, only the whole of mankind can set the goal.

Our Numerical Society

One aspect of the scientific revolution is often emphasized. This is the important part played by mathematical theories of physical phenomena. It has been seen that other factors were also influential, but this one has received more recognition than the others. This would be sufficient reason to examine the foundations of mathematics, but there are other equally cogent reasons. The success of mathematics in the physical sciences has led to its use in the social and biological sciences.

Sociologists have been conspicuously unsuccessful in this. They are urging the establishment of an agency to supplement the Bureau of the Census in collecting more numbers for them to use, hoping that this will bring success. Economists join them in this; they have also made mathematical models of the financial system. Perhaps it is too early to judge their success, but no one has yet made a fortune by using their models. This statement requires a minor modification: the services of professional economists are in demand and are quite well paid. This is the only new industry that has yet resulted from the application of mathematics to the social sciences, a marked contrast to the physical sciences.

Insofar as biology is biochemistry and biophysics, mathematical theories have been very useful. They have been taken over, with very little change, from physics and chemistry. The instruments of the physical sciences have also been adapted for biology; the microscope is only one example. This has been less successful in psychology. Sensation and perception are not easily measured. The anatomy of the inner ear stubbornly refuses to conform to the laws of mathematical acoustics. Many people are bothered by a continuous whistling in one ear; this is called an illusion and is not considered to need an explanation. There is no completely satisfactory theory of the retina; the luminous figures seen by sufferers from migraine, and the floating lights seen by users of digitalis are also dismissed as illusions. The list of such failures of the theory of psychomathematical parallelism could be multiplied almost indefinitely, but has never been compiled by the advocates of that theory.

It has been possible to develop mathematical theories of nerve conduction, but not of the brain. Again, its anatomy and functioning do not conform to the mathematical theory of the electronic computer. There was, at one time, the hope that study of the computer would provide an understanding of the brain. John von Neumann, a leader in computer design, concluded that this was a false hope. The general public, however, continues to refer to computers as "electronic brains." It is sometimes said that the computer put men on the moon. This is not only false, but it is unjust to the astronauts, to all the men and women at Cape Kennedy and Houston, and also to the people who build computers. For while people can build computers, computers cannot yet build people.

The preceding criticisms do not exhaust the reasons for examining the foundations of mathematics in great detail. Not only the scientific community, but our entire society is deeply committed to numbers, whether as mathematics or as numerology. Society is highly arithmetized and is becoming more so. This is not because of the General Accounting Office and the Internal Revenue Service. It is not because of the Social Security and Selective Service Administrations. There is dissatisfaction with this trend. When the names of telephone exchanges were replaced by numbers, there was audible complaint. Zip codes met with passive resistance for some years, but are now accepted with passive resignation. The use of computers by university registrars at first led to student protests, but now students have become resigned to "being just an IBM card." There is anxiety lest data banks result in the invasion of privacy, and ultimately to a police state. This anxiety has not resulted in organized opposition to the social scientists who are urging the establishment of such data banks.

This dissatisfaction with numbers is relatively superficial. There is a much deeper faith in the magic of numbers. Proud parents make their children show off before guests by counting. The child's face is always eloquent of mixed feelings: fear of making a mistake, a wish to please, and bewilderment. "Why are the Big Folks so anxious about counting?" This is very different from the unselfconscious way the child learned to speak. If the child tries to resolve his bewilderment, he gets a lame answer: "Well-ah-you see when you go to school-well-we want you to get good grades." Still fearful of making a mistake, the child does not ask "Mummy, what are grades?"

Most of us are unaware of our deep-seated faith in numbers. It has been instilled by our parents, and reinforced by our entire educational system, from the elementary schools to the universities. We rarely think about it, or acknowledge it explicitly. Many people consider arithmetic to be just one of the unpleasant facts of life; others are fascinated by it. Few have a critical knowledge of the history of mathematics in its relation to society. Even the historians of mathematics are not inclined to question the "facts" that they learned while they were young and impressionable.

The Anthropology of the Whole Numbers

Many differing views as to the nature of mathematics have been expressed, without any apparent progress toward unanimity. These views range from "God is a mathematician" to "Mathematics is a meaningless game." An ancient philosopher, living in a polytheistic society wrote, "Numbers exist, even if nothing else does." A late Roman writer said, "Take from all things their numbers, and all shall perish." The nineteenth-century mathematician, Leopold Kronecker wrote, "God made the whole numbers, all the rest of mathematics is the work of man." This introduced a new idea: mathematics is an invention, an artifact, not a part of nature which has an existence independent of people. In other words, it is a tool, and Kronecker thought that the whole numbers were the "substance" of which the tool is made.

I shall here present the view that numbers, even whole numbers, are words, parts of speech, and that mathematics is their grammar. Numbers were therefore invented by people in the same sense that language, both written and spoken, was invented. Grammar is also an invention. Words and numbers have no existence separate from the people who use them. Knowledge of mathematics is transmitted from one generation to another, and it changes in the same slow way that language changes. Continuity is provided by the process of oral or written transmission. This is a quite recent view, but there are indications that it is gaining acceptance. Not surprisingly, computer scientists are especially receptive to this view, for the leaders in this field are actively engaged in the invention of new languages. More conservative linguists and philologists call these new languages "artificial" to distinguish them from the "natural" languages on which their attention is focused. There are more than a thousand natural languages, not including dialects, and less than a hundred artificial languages. Here, too, attention will be focused on the natural languages, unless specific mention is made of the others.

I shall also present the view that idealizations are fictions. Such fictions include very large numbers, very small numbers, lines without thickness, and some other mathematical concepts. I must take sole responsibility for this, without claiming originality. Those few readers who are familiar with the works of H. Vaihinger and J. Bentham will recognize both the similarity and difference between their views and mine, but I will not weary the reader with repeated attempts to distinguish my opinions from theirs. The discussion in the earlier parts of this book should make it clear that I do not hold fiction in contempt as a mere recreation, but I consider it an important component of literature, often providing a motive for social and technological change. Plato's Dialogues have exerted a great influence on the evolution of our western society, and they contain a large fictional component. His *Republic* is utopian fiction. Plans are fictions until they are made to become true through the actions of people.

It is customary to distinguish between words and thoughts or ideas. However, thinking is merely inaudible speech. It is impossible to think clearly about anything for which one's language contains no words. Writing is also an inaudible soliloquy. While speech and thought are temporary, writing has greater permanence; it permits the writer and others to review the ideas which have been written down, and to subject them to reflective analysis. When one says "thinking" rather than "soliloquizing," one invites confusion; thinking is commonly considered to be absolute, whereas soliloquizing obviously depends on a language and a vocabulary. It is true that some people are said to be able to think better (or more clearly) than others, and this difference is supposed to be independent of their language. Emphasis on vocabulary makes it clear that a child whose vocabulary is small will not "think" in the same way as a well-educated adult. This does not mean that a child's soliloquies are of no importance. In the same way, the vocabulary of ancient languages was surely different from that of modern; the ancient Egyptian or Chinese languages cannot have had a word for "automobile." It is a serious error to suppose that primitive people thought as we do; it is an equally serious error to suppose that they were therefore mentally inferior to us. This fallacy has persisted to the present time, and has been used to justify many actions which are unjustifiable. Neither should it be supposed that, because ancient writings are difficult to understand, they are more profound than less difficult modern writings. However, there is no need to be pedantic in avoiding words like "thinking," "thoughts", and "ideas" when confusion is likely to arise. Also, it must be remembered that the primary use for speech is communication. Most words have been coined in the course of casual conversation, as a need (real or fancied) arose. Perhaps the most fundamental difference between natural and artifical languages is that the former have been invented in this casual way, and the latter have been invented deliberately, and for a definite purpose. It follows that the vocabulary of the artificial languages will be smaller, and their grammars more systematic than those of the natural languages.

It is difficult to imagine a time when no one had counted a herd of animals, or a catch of fish. All natural languages contain words for use in counting, even though they make no provision for very large numbers. Anthropologists have found some isolated tribes that do not count beyond three. Perhaps all peoples passed through this stage. The Egyptians and ancient Chinese often repeated a written character three times simply to express "many." The Old Babylonian word for "three" was *esh*, which was also the suffix that converted a single noun to plural. This is remarkably like our "chair" and "chairs," but it is unlikely that an etymological connection can be established: since there are so many languages, some allowances must be made for coincidences. One South Sea language that is meager in number words has two very different words for one boat and for ten boats; for one coconut and ten coconuts. Moreover, there is no systematic similarity between the pairs of words.

It is impossible to make a generalization which would apply to all languages. To avoid losing the way and becoming involved in irrelevancies, it will be well to begin with the English words for the cardinal (whole) numbers. Our words "one," "two," "three," "eleven," "twelve" are very old. They were derived from older Anglo-Saxon words, and etymologists can trace them to even older languages. Finally, the etymologists are led to construct (or reconstruct) a basic Indo-European language from which Greek, Latin, and Sanskrit were derived, though other languages (Sumerian, Egyptian, Old Babylonian, Hebrew, Arabic, etc.) were not. These other languages had other basic or proto-languages. Our "dozen" was derived from Cornish and Breton words, and these perhaps from the Latin *duodecim.* "Hundred" and "thousand" are also so are "score" and "gross" though their meanings may have changed over the centuries. "Quire" and "ream" were coined more recently to meet the needs of the paper trade; "million" and "billion" were not coined until business and government had expanded into large, complex organizations. This happened much more recently in England, rather than in Egypt or Rome. This and similar delays contribute to the widespread belief that these early people were wiser than we. Actually, the difference is only one of chronology.

It is to be noted that these number-words show no systematic relation to one another; this is also the case in many other languages. When it comes to large numbers, we say "fourteen," "fifteen," "sixteen." Germans say, "four-ten," "five-ten," "six-ten," and so did the Romans. This shows signs of a deliberately invented system, similar to that of an artificial language. Continuing, we say, "twenty, thirty, forty, eighty, ninety"; the Germans do something similar; again there are signs of artifice. This is no less true for French, though there are some irregularities (eighty is "four score," ninety is "four score ten"). A few centuries ago, the phrase "three score and ten" was common in England, and we still understand it easily. We say, "four hundred and eighty-three," the Germans say, "four hundred three and eighty."

These examples from a few languages are typical of many other languages, living and dead. The conclusion seems certain: the smaller number-words were invented spontaneously, in response to growing needs, while the larger number-words were later invented according to a deliberate system. The system varied from language to language, and sometimes it was changed as the language changed. Number-words are used in counting. Originally, this was probably their only use. It should be noted that counting is a soliloquy about things that are being handled or looked at. It is a thoughtful activity. When a child is made to show off, he is not counting. It is mere recitation of a memorized sequence of words, a jingle which does not rhyme. This memorized succession of words will be used later, or possibly has already been used, as a soliloquy in counting apples, eggs, cows, men, and a great many other kinds of things. If one simply picks apples from a tree and puts them in a basket, this is not counting either. Only if one accompanies the action with the soliloquy, "one, two, three," is one counting. With experience, one can count a small number of objects at a glance (without the soliloquy), but only if they appear simultaneously. Otherwise, one can easily lose count of a large number of things, as anyone knows who has tried to count a show of hands at a large meeting. Even with smaller numbers, one can lose count if the things to be counted (objects, days, or transactions) appear at irregular intervals, or are separated by distractions; to avoid this, the counter must make a memorandum of each as it occurs. This is a very important conclusion: counting must be considered as the union of words and other human activities. The words are formed by the tongue and the lips, or if written, by the hand; usually they must be accompanied by other motions of the hands or the eyes.

Proponents of theories which deny that numbers are linguistic inventions frequently cite the ability of certain animals to count, although they are unable to speak. In the case of birds, an examination of the facts indicates that they have only the ability to distinguish between one egg in the nest, and more than one. One example designed to demonstrate a greater ability can be explained if the bird used as subject had the ability to distinguish between one man and another. To many animals, people are not interchangable. Every dog fawns on some people, and growls at others. It is said that horses can be taught to count. At the Louisiana Purchase Exposition, one of the sideshows claimed to prove this. Numerals from one to ten were painted on boards and arranged before the horse. At the spoken command of its trainer, the horse would take the appropriate board between its teeth and display it to the audience. This is not counting. A more accurate description would be that the horse interpreted the spoken word as the name of the board carrying the numeral. It was not demonstrated that the horse could distinguish between three cubes of sugar and four. And, even if a horse were trained to count, it would be the result of training and not an innate ability. The ability would not be inherited by its offspring. Nor do human beings inherit the ability to count; each generation must be taught by its elders. On the other hand, horses cannot teach their offspring to count as they cannot speak. The hypothesis that mathematics is a linguistic artifact, invented by people, may therefore be accepted, at least provisionally. Further evidence favoring it will appear later in this investigation.

Counting, Memoranda, and Arithmetic

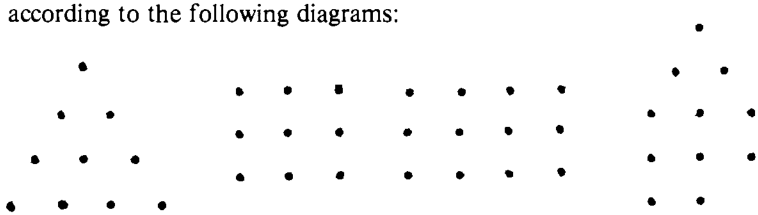
Even minor distractions can make one lose count and be unable to finish. One is rarely unable to finish a sentence unless there is a sudden and dramatic distraction. This is in itself evidence that counting is more than simple speech. When, for whatever reason, counting became important to primitive people, it brought with it a need for aids to memory, for memoranda. Counting on the fingers is such an aid, but it becomes tiresome if the counting process is long. There is ample evidence for a wide variety of other kinds of memoranda.

One may conjecture that tally marks scratched in the dust were among the first to be used. They are still used (although written on paper) by score-keepers at tennis matches. Modern dictionaries list several definitions for the word "score," one of which is to "gash in lines." Correspondingly, it can be translated into other languages in a variety of ways; two ways to translate it into German are "reckoning" and "number of points made." Now reckoning and the corresponding German word are both derived from the name for a rake that makes parallel marks in the dirt. "Point" is again a word with many meanings, and there is evidence that early tally marks were simply dots or holes imprinted into the earth with a stick.

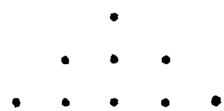
Menniger describes many other ways of tallying and there is abundant anthropological and archaeological evidence for their use by primitive people. A few of them are: cutting notches in a stick, driving nails into a post, placing pegs in holes previously bored into wood, and moving pebbles from one heap to another. Many of these methods survived into quite recent times, even to the present day. Notched sticks were used by the British Exchequer in the thirteenth and fourteenth centuries when few people were able to read and write. Our personal checks and banknotes developed from the complex uses made for these Exchequer sticks. The reader will know of many less obscure examples.

Experience with counting must have led to the use of sentences like, "Two and three make five, Two times three make six." This experience may have been gained by counting actual articles of barter or trade. The experience may have also been gained by the accidental or deliberate grouping of tally marks: II III or .. ... (two and three make five), or III and III or ... and ... (two times three make six).

Much simple arithmetic can become soliloquies about such arrangements, as well as some not-so-simple arithmetic that we now call algebra. The Greek mathematicians of the Pythagorean cult classified numbers as triangular, square, oblong, or pentagonal according to the following diagrams:



The squares have survived into modern algebra as n^2. The slightly different triangular arrangement:



in modern notation is

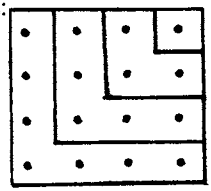
$$1+3+5=9=3^2$$

and generalizes to, "The sum of the first n odd numbers is n^2." The Pythagorean triangular number shown in the first diagram shows that:

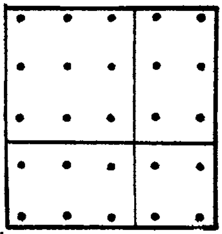
$$1+2+3+4=10=(4)5/2$$

which generalizes into: "The sum of the first n whole numbers is $n(n+1)/2$." The square numbers can also be subdivided by lines drawn in the dust.

A favorite with the Pythagoreans was:



which also exhibits the sum of the first four odd numbers as 4^2.
Another subdivision of a square is:



which generalizes into our binomial

$$(n+m)^2 = n^2 + 2nm + m^2$$

The Pythagoreans were certainly not primitive people, but there is at least some evidence that special cases of these theorems were known in much earlier times.

Games may also have provided such experiences. Cubical dice, marked with dots like ours, have been found in Egyptian tombs dating back four millenia. Knucklebones is a game played with the ankle bones of sheep, suitably marked with scratches or dots; presumably this game antedated the cubical die. In Greek and Roman times, boys were encouraged to play knucklebones in order to improve their proficiency in arithmetic. Dominoes, marked with dots in the same way as ours, are also very ancient, even though they were not introduced into England until quite recently. In Mesopotamia, games like backgammon, or some other complicated form of checkers, were played before 3000 B.C. All such games involve both actions and words, even if some of the following are silent soliloquies.

Again, it is important to remember that each generation must be taught both the games and the arithmetic by its elders. Unfortunately, they are also taught superstitions, such as a belief in luck and lucky and unlucky numbers, at the same time. Silliness as well as wisdom can be handed on from one generation to the next, and thus survives for many centuries.

Seals, Numerals, and Writing

There is much evidence that, at some early time, simple person-to-person barter was replaced by barter that involved transportation. Transportation usually involves one or more middlemen. Without venturing a precise date for this development, it certainly antedated the invention of coins by thousands of years, a fact that has many implications. The nature of the evidence can be illustrated by amber. Amber almost never occurs in nature, except in the areas adjacent to the Baltic Sea. Yet, amber jewelry has been found in all the countries north of the Mediterranean, and as far south as Egypt. This jewelry has been found in archaeological strata that are very ancient and primitive. By analyzing these amber finds, it has been possible to map fairly well-defined trade routes. Probably no one trader traveled the full length of any of these routes; the route into Egypt must have involved ships. One is thus led to the idea of very early transportation of goods, partly on foot and by relays. Heavier merchandise would have required pack animals, and perhaps caravans traveling in company for protection.

For one reason or another (one can imagine many), this kind of trade separated goods from the immediate vigilance of their owners. An element of trust, perhaps what we now call credit, entered business affairs. The people of that time are unlikely to have had words for these ideas; at least etymologists have not found evidence for them. Still, some method for identifying the owner of the goods being transported was needed. The method which was adopted was the seal-stone, a stone belonging to the owner and carefully guarded. It was carved in a distinctive fashion. This was impressed on moist clay or warm wax, which then hardened and retained the impression. Narrow-necked jars of oil or wine, stoppered with blobs of clay and sealed in this way, have been found in many places. The seal-stones were usually elaborate works of art in order to make forgery difficult. Such seal-stones (or signets) have been found in great numbers at widely separated archaeological excavations, and at some that surely antedate the invention of writing. Specimens are on exhibit in almost all of our museums. Seal-stones are still in use today in modified forms. The dies which mint our coins, the engraved plates that print our bonds, paper money and postage stamps, the punches with which notaries emboss documents to attest the deed of signature, all of these are decendants of the earliest seal-stones Perhaps all printing can be included in this genealogy.

When the commodities being transported were packaged in jars, bags, or bales, there was also a need to identify the contents, and also their number. At first, this was accomplished by drawing a crude picture (pictograph) of the commodity or its source (a grapevine for wine, a tree for olive oil) on the same material as the seal. This was done with a pointed stylus; tally marks could be added. These are the earliest stages of writing; archaeologists have found relics of all stages of its development, from the pictograph and the tally mark to proper scripts.

There is evidence that, at least in Egypt, the Near East, and the Eastern Mediterranean lands, systems of numerals replaced the tally marks while other forms of writing were in the seal-pictograph stage. This may be one reason for the sharp distinction between numerals and other kinds of script that persists to this day. Other, perhaps more cogent, reasons will be encountered later. In the present context, these early numeral systems are of more interest than the writing, but the two cannot be completely separated. Their significance for us can be more easily understood if prefaced by a brief account of the early civilizations of the Aegean islands and the Greek peninsula. The reader will understand that this is condensed and simplified by the omission of nonessential details; items that are still controversial will be identified as such.

Shortly after the Egyptian and Mesopotamian civilizations had gotten under way, the sparse Stone Age inhabitants of the Aegean islands and the Greek mainland were replaced or absorbed by a more numerous people. We have come to call them Minoans. It is not known whence they came, or even what kind of language they spoke. They seem to have been a peaceable people; at least their cities, even their palaces, were unfortified. The most southerly of the Aegean islands is Crete, and there the city of Knossos became the capital of the Minoan dominions. The lands of the Minoans were fertile and produced grain, wool, and hides for export. This, together with geography, required ships.

To maintain a balance of trade, the Minoans imported metals and other raw materials and became manufacturers. Maritime shipping is subject to piracy, and the Minoans developed a powerful fleet of warships to protect their maritime fleet. One may conjecture that the warships were intended to reinforce the element of trust so essential to business.

Such overseas business also needs trading posts or colonies. At least they have come to be called colonies, although they seem not to have been colonies in the later sense of Periclean imperialism. They were scattered along the shores of the Black Sea, the Aegean, and the Mediterranean. One of these colonies was on the eastern tip of Asia Minor, and it later became known as Miletus. It was near the large island of Samos, and has already been mentioned as the home of Hermondas, the architect and city planner.

Another Minoan outpost was the island Thera, also known as Santorini. Thera is about seventy-five miles north of Knossos, and seems to have been of more religious than commercial importance. Thera was then a very large extinct volcanic cone. There may have been hot springs and other minor volcanism. Such places have always had a religious fascination for early people. About 1550 B.C. (this and the following dates are approximate), the volcano reawakened and violently exploded. The habitations on Thera were destroyed in this blast; they have not yet been fully excavated because of the huge accumulation of pumice and ash at the site.

Somewhat before this time, the Greek mainland was invaded from the north by people now called Mycenaeans. They spoke a form of the Greek language, but had no script; they brought with them horses, two-wheeled chariots, and more elaborate weapons and armor than those of the Minoans. The latter were therefore conquered or absorbed. The details are still not clear, but the Greek language prevailed. About 1500 B.C., the Myceneans conquered Knossos and took control of Crete and the Minoan commerce. The eruptions of Thera became more violent, ejecting lava and pumice. This left a hot cavity into which seawater ultimately poured. This caused a violent explosion. The explosion of Krakatoa in the nineteenth century was of the same sort. However, the explosion of Thera was much more violent. By analogy to Krakatoa and similar events, one may conclude that great tsunamis (seismic sea waves) destroyed all exposed beach installations in the southern Aegean, including the northern shore of Crete. The fall of ash may have destroyed all vegetation on Crete, and rendered the island infertile for a generation or more. The palaces at Knossos and the other Cretan cities were all destroyed about the same time. It is still being debated whether this destruction was caused by the Thera explosion, by a rebellion against the Mycenaeans, or by a combination of the two. In any case, the mainland city of Mycenae, sheltered high on the side of a gorge at the head of a long, narrow gulf, replaced Knossos as the center of Aegean commerce. The Minoan language appears to have died out, while the Mycenaean Greek survived. The history of the Greek world does not end at this point, but this much provides the background for a first discussion of numerals and writing. It should be added that, while the general history of Egypt was very different than that of Greece, the development of numerals and writing was identical for both areas, and they developed a very similar system of numerals.

It has been seen that language, numerals, and writing were developed in that chronological order. One would like to know the number-words used by these people, but since their languages are no longer spoken by anyone, only guesses are possible. The guesses are inferences from the numeral systems they developed. For exploratory purposes, it is easier to present the inferences first, and then the evidence. They had words for one, two,...ten, hundred, thousand; later the Mycenaeans and Egyptians coined a single word for ten thousand, and still later the Egyptians continued with single words for hundred thousand and million. For 15, they said "ten five"; for 30, they said "three ten"; for 300, "three hundred" just as we do. For 2739, they said "two thousand seven hundred three ten nine." Of course, since they spoke natural languages, there may have been departures from this artificially systematic speech. Such deviation from strict rules of grammar are idioms, and it is in the nature of the evidence that no inferences can be made about idioms. The numeral system does not reflect the idiom any more than "12" reflects "twelve." Like our numeral system, theirs was deliberately systematic and artificial.

Speaking systematically and not necessarily chronologically, the first step was the invention of graphic characters for the basic number-words one, two, ten, hundred, etc. These basic characters will be called "digits"; the digits were used to spell the numerals. All of these numerals can be described simultaneously if one uses o (ones), t (tens), h (hundreds), and T (thousands) as shown in the

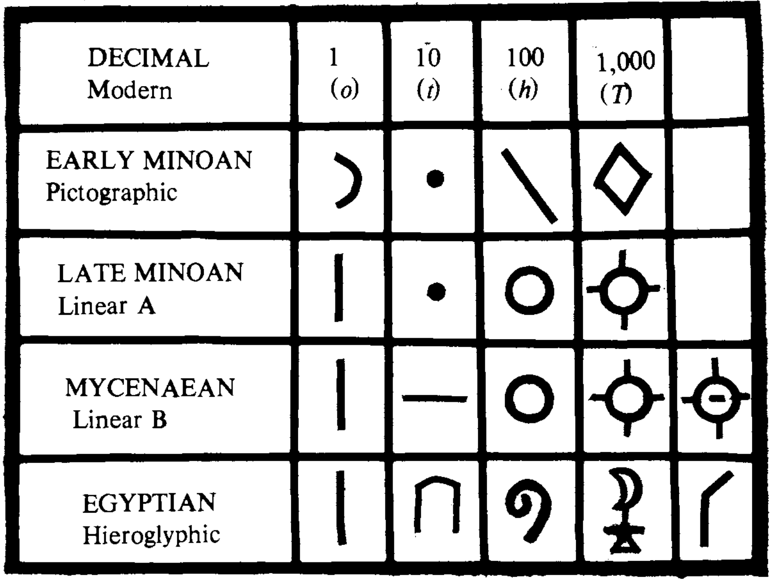


FIGURE 2
Comparison of Digits

upper row of Figure 2. Thus twelve was t-o-o, except that the Egyptians wrote o-o-t. This seems backwards to us, but not to them; this right-to-left habit of the Egyptians can be ignored in the following. The numeral 2739 was then written:

T	h	h	h	t	o	o	o
T	h	h		t	o	o	o
	h	h		t	o	o	o

It is seen that the digits appear in groups small enough to be counted at a glance; no digit ever appeared more than nine times in any numeral. The appearance of these numerals reminds us of a domino and one might call them domino-numerals. It is preferable to speak of the primary numeral system, for no other numeral system is quite so simple and easy to learn. The Egyptian hieroglyphs were difficult to draw, but they were only used for monumental purposes. For everyday matters, there were two other Egyptian scripts, the hieratic and demotic, which could be written cursively, and which correspond to our handwriting.

Before proceeding to a discussion of the way arithmetic was done with primary numerals, it will be well to describe the purposes for which the primary numerals were used. This will be both of general interest, and of significance for the foundations of mathematics. Both the Minoan and Mycenaean numerals and scripts were drawn on slabs of moist clay with a pointed stylus. The slabs were then simply allowed to dry in the sun. This does not produce a very durable record, for moisture will quickly reduce the dried slabs to unrecognizable lumps of clay. This is a sufficient reason why so few clay tablets have been preserved. Only about two thousand Mycenaean tablets, and fewer Minoan tablets are in museums. Most of these were found in houses or palaces which had been destroyed by fire; the heat turned the clay into durable brick. This scarcity increases the difficulty of decipherment. The Minoan script (Linear A) has not yet been deciphered, and the Mycenaean (Linear B) only partially. This was a difficult task, not achieved until 1952, when the British architect and linguist, Michael Ventris, deciphered it. In the later stages of his work, he was assisted by John Chadwick, a specialist on ancient Greek dialects. It is now known that the Mycenaeans used their script and numerals only to keep business records. The vocabulary of the tablets is therefore small and specialized, which increased the difficulty of decipherment. It may never be possible to reconstruct the full Mycenaean language, or its grammar and idioms. The Mycenaean tablets are not dated. They refer to this year, last year, and next year; they often give the name of a month. It is known that the bookkeepers filed the tablets in simple wicker baskets, labeled with blobs of inscribed clay. It is thought that the tablets were discarded at the end of each year. At no site has a full year's set of accounts been found. Archaeologists are thus in a peculiar situation: from the latest of a set of tablets they can find the month in which a building was burned, but not the year. It has been conjectured that before discarding the tablets, a summary of the year's accounts would have been transcribed onto papyrus or parchment, but no such summaries have been found. Very likely, the Minoan tablets are also business accounts, for they contain many numerals. The other written words have not been deciphered.

It must be emphasized that the Minoans and Mycenaeans never used their scripts for monumental inscriptions. Perhaps this was also the case in earliest Egypt. While there is abundant evidence that the hieratic script and numerals were used for business accounting, the script was elaborated, and its vocabulary increased, so that it could also be used for monumental and literary purposes. The failure of the Minoans and Mycenaeans to do the same may be explained by the relatively brief duration of their civilizations. Both were destroyed by invaders, and it is useless to speculate about what might otherwise have occurred.

One will next inquire into the way in which the primary numerals were added, multiplied, or divided. It was formerly thought that an abacus was employed, and this opinion is still widely quoted. The difficulty is that archaeologists have not found a single abacus that antedates 800 B.C., and very few from the time of Imperial Rome. This is in strong contrast to the large number of very ancient seal-stones found in this same geographic area. There are explanations for this contrast, but it is certain that the abacus hypothesis is not simple. One will first ask whether there was a simpler explanation.

By way of introduction, consider:

```
 2739
 1325
 ----
(1 1)
 4064
```

This is a memorandum of the soliloquy "five and nine make fourteen; four and carry one; one and two and three make six;..." Of course, an experienced calculator will omit "four and carry one" and remember the "one", carry it in his head. He will also omit the carries from the memorandum, and write it more simply as:

```
2739
1325
----
4064
```

There are Minoan, Mycenaean, and Egyptian records of the same sort. The foregoing addition would appear as:

T	h	h	h	t	o	o	o
T	h	h		t	o	o	o
	h	h		t	o	o	o

T	h	t	o	o
	h	t	o	o
	h		o	

Total:

T	T		t	t	o	o
T			t	t	o	
T			t	t	o	

It is certain that the calculator carried t, and most likely carried it in his memory. One also observes that the primary numeral system had no symbol for zero, and needed none. Assuming that these early calculators were familiar with the carry principle, addition in the primary numeral system would be no more difficult than in our own. Moreover, the carry principle follows very directly from the hypothesis concerning the spoken number-words.

The primary method of addition is thus similar to ours, but there is a fundamental difference. Before our children can add large numbers, they must have memorized the entire addition table, from $1+0=1$ to $9+9=18$. For young children, this is a difficult task, and requires much time and drill. With the primary numerals, it was only necessary to be able to count. The addition of two numerals required only the ability to count to twenty. The addition of long columns required the ability to count to larger numbers, and perhaps memoranda. Moist clay is an ideal medium for temporary memoranda.

The Minoan and Mycenaean tablets give no information about the multiplication of large numbers. Surviving Egyptian texts on advanced arithmetic give examples (but no explanation) of their method of multiplication. It is one that was used in Classical Greece, Rome, and until quite recently in southeastern Europe. The Egyptian multiplication of 127 by 105 is shown below, in modern notation:

1	1	127
0	2	254
0	4	508
1	8	1016
0	16	2032
1	32	4064
1	64	8128
	105	13335

The third column contains the successive doubles of 127; the second, the successive doubles of 1; the first contains only ones and zeroes. One ignores those lines with zeroes and adds, in the second and third columns, the remaining numbers. Those in the second column yield $64+32+8+1=105$; while those in the third column yield $105\times 127=13{,}335$. It is seen that everything hinges on the correct determination of the first column. The Egyptians and Greeks probably did this by trial and error. However, those readers familiar with modern computer technology will recognnize the first column, when written as 1101001, is the binary numeral for 105, and will know a systematic way of obtaining it. But, if they remember their early instruction in division by two, they will realize that even this modern method based on trial and error. Again, it is important to note the soliloquy that is part of this multiplication algorithm, and compare it with our own. Ours requires the memorization of the entire multiplication table, whereas the Egyptian requires only the ability to add and a bit of ingenuity. The increase in school-hours is considerable when our method is taught.

Division could be carried out by the inverse process: the ones and zeroes were determined from the third column rather than the second. The Egyptians sometimes used this method; the division of 19 by 8 was as follows:

1	1/8	1
1	1/4	2
0	1/2	4
0	1	8
1	2	16
	$2+\frac{1}{4}+\frac{1}{8}$	19

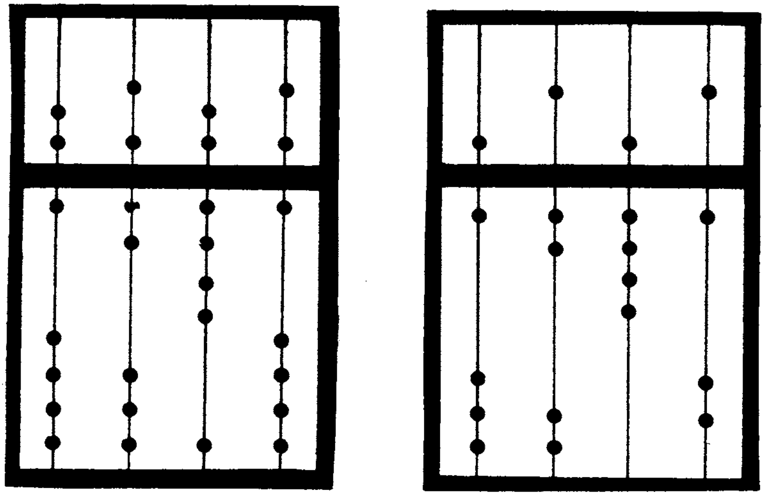
We would write the result 19/8 but the Egyptians were satisfied with the result 19 as it stands. In this case, the first column, written as 10.011, is the binary numeral for $2\frac{3}{8}$ but this is only because $8=2^3$. The Eqyptians did not say "divide 19 by 8"; instead they said "add 8 until you get 19." The Egyptians seem to have used this algorithm only for division by small numbers. For larger numbers, they used a method that required more memorization, and will be discussed later.

It is therefore possible that no abacus was used with the primary numerals. It is also surprising to find the beginnings of the binary numeral system in ancient Egypt.

The Abacus: History and Conjecture

The abacus is of interest as an early calculating machine, as well as for the archaeological problem it poses. In England, it remained in use until at least the seventeenth century. In much of Eastern Europe, it was in use even later. It is still used in the Orient and some countries, such as Finland, in Europe. A skilled operator of an abacus can compete in speed with a modern adding machine. The reader will have some familiarity with the Chinese and Japanese abaci. They are wooden frames holding rods on which rings (counters) are free to slide. The English language is no longer well adapted to a detailed description of the use of the abacus, so that some general familiarity will be presupposed in the following discussion.

The Chinese and Japanese abaci are very similar in appearance. Except for material and structural detail, the Japanese abacus is essentially identical with those few early Roman abaci that survived because they were made of bronze. Perhaps surprisingly, both Chinese and Japanese abaci can be used with Roman numerals as well as with our modern decimal numbers. The Roman digits M, V, X ,L, C, D, and I, happen also to be letters of our alphabet, and the rules for spelling numerals will also be familiar to the reader; except that, in earlier times, IV was written IIII, etc. The diagrams of Figure 3 show the two kinds of abaci, both set to show the



Chinese Japanese

FIGURE 3
Abaci

number MDCCXXXXVI. In passing, it may be remarked that today we are familiar only with that form of the Roman numerals which is suitable for monumental inscriptions and printing. A cursive form, suitable for rapid handwriting, was also developed.

At first glance, the Chinese and Japanese abaci seem to differ only on a minor way: the number of counters per rod. But this is not minor. The Chinese abacus requires only that the user be able to count to five; whenever beads are moved down on a lower rod, one bead must be moved up on the upper rod; that is, when both beads on the upper rod are raised, they are immediately to be moved down, and one bead on the next lower rod moved up. The verbal description of its use is cumbersome, but when a workable abacus is available for illustration, the rules are quickly memorized by the pupil. It will be noted that a modified count-and-carry system is involved. The Roman numerals also consist of digits in small groups, countable at a glance. The system differs only from the primary numeral system in that some are five times others, some are twice others.

The use of the Japanese abacus requires more than the ability to count-and-carry. It requires that the user have memorized an abbreviated addition table: $1+4=5$, $2+4=5+1$, ... ,$4+4=3+5$, $5+5=10$. The memorization of this short table is less of a feat than memorizing our addition table, but still requires some instruction and drill. One abacus, surviving from Imperial Roman times, is of the Japanese type; another is a modified Japanese type made of bronze. There are no other surviving examples.

Most Roman abaci are said to have been grooved wooden boards or table tops; the counters were pebbles (*calculi*), or sometimes ivory discs placed loosely in the grooves. A few later European tables of this sort have been preserved in museums, more for their artistic than for their historic value. The one surviving abacus from Classical Greece is a heavy stone slab, with narrow incised straight lines. The counters have not survived. It was found on the island of Salamis, the island that Solon's expedition had ravaged. A drawing on a Greek vase (that commemorated the repulse of Xerxes' invasion of Greece, but found in Italy) shows an abacus in use. It was apparently a wooden table; only Greek numerals are visible on it.

There is a great advantage to abaci that do not have a fixed number of counters per line or groove. By changing the number of counters deployed on the various lines, the abacus can be quickly adapted to calculations in any system of coinage (such as pounds, shillings, and pence), weights and measures (gallons, quarts, and pints) or fractions (1/2, 1/4, 1/8). No mechanical reconstruction is needed for this conversion. The bronze Roman abacus had the coinage system built into it, as well as the Roman numerals. In a general sense, every system of coinage, weights, or measures is a numeral system.

Other late European abaci were cloths into which straight lines or checkered squares were woven; the counters were discs of base metal, sometimes minted like medals or tokens. The cloths have long since disappeared into rag bins, and the counters have either been melted down or become collector's items. It is therefore entirely possible that most abaci were made of perishable material, and if there were durable parts, it would be impossible for archaeologists to now determine their use.

This explanation of the rarity of Greek and Roman abaci loses some of its cogency from the fact that the Roman numerals can be, and were, added without the use of an abacus. The count-and-carry method was used instead; the memorandum of 1778+834=2612, is as follows:

	M	D		C	C	L		X	X	V		I	I	I
		D	C	C	C		X	X	X		I	I	I	I
M	M	D			C				X				I	I

Such calculations were facilitated by ruling vertical lines on the paper before beginning the calculation. Our modern ruled ledgers and account books are developments of this practice. It is almost certain that an experienced bookkeeper using cursive Roman numerals would be no slower than one using modern decimal numerals, but the race has not been held.

Before leaving this topic, it should be said that the Romans later extended their numeral system. By the time of Julius Caesar, digits for 5000, 10,000, 50,000, and up to 5,000,000 had been added. These were not letters of the alphabet, and were shaped in a more or less deliberate and systematic manner. At least one bronze Roman abacus was large enough to deal with numerals as great as 9,999,999. It seems not to be known just when these greater digits went out of use.

The Egyptian Mathematical Texts

The classical Greeks started a tradition of very advanced mathematics and astronomy that the Egyptians had a knowledge of, even in very ancient times. This traditional belief is still widely held, although modern scholarship has found no evidence for it; in fact, it is scarcely an exaggeration to say that it has been disproven.

Herodotus and Aristotle state that this knowledge was confined to the Egyptian priests, since these were the only people with the leisure to study such matters. However, this cannot have been true until very late in Egyptian history. Even the demi-god pharaohs had secular as well as religious duties, and there were no priests with exclusively religious functions. Instead, there was the class of scribes, literate and usually capable of making these mathematical calculations. They were most often public officials, with little leisure, and did not do mathematics for its own sake. For that matter, Egyptian artists had little opportunity to pursue art for its own sake. The scribes' interest in mathematics must have had a utilitarian motive. The social status of the scribes was puzzling even to the Egyptian artists, who indicated class by the size of the figure. This must have been even more puzzling to a Greek visitor. A scribe might be prime minister, or he might earn a precarious living sitting in the market place, doing accounts and writing letters for the farmers. In later times, some craftsmen were also literate; at least master masons could sign their names and use the Egyptian system of numerals and weights and measures. Egypt made its own papyrus. Since scribes were usually public officials, they would be adequately, though not lavishly, supplied with writing materials. This may have made it quite unnecessary for them to use an abacus.

The Egyptian society was the most perverted of all the ancient societies. Its kings and nobles lived in sparsely furnished mud-brick houses, while monstrous stone structures were being built to preserve their corpses. It is difficult to understand this funerary character of Egypt, but, taking it for granted, the ascetic character of the homes and palaces can be explained. The land of Egypt had no forests, and no metal ores. Wood was imported from the forests of Lebanon, which required transportation either by sea or over difficult terrain. Large timbers were therefore very expensive. Copper was obtained from the island of Cyprus; large deposits of tin were even more distant. Consequently, wood (other than palm trunks, which are too weak for building) was a rarity, and was used only lavishly in the tombs, and for the boats and sledges that transported large stones for the tombs. The quarries were a state monopoly and the use of stone was restricted. Until very late in Egyptian history, iron and bronze were more rare than gold. Copper was the only metal used for tools. Wood was not used freely until quite late (XVIII Dynasty). The ordinary Nile mud is not suitable for making durable kiln-fired brick. The Nile valley does contain ceramic clays, but these were needed for pottery, either for domestic use or for export to keep a favorable balance of trade.

It is customary to begin a discussion of Egyptian mathematics with a discussion of the pyramids. Certainly, these are impressive space-filling objects. The people who built them must have known much about practical geometry. The bureaucrats who administered the projects must have known much about business mathematics and accounting. The Egyptians did not usually record these matters. The aristocratic fallacy kept all early societies from doing so. Not much is recorded even about the architects and engineers. There has been much speculation about the geometry of the Pyramids. The resulting literature on the subject is confusing and unhelpful. Willy Ley has written an admirably brief and critical summary.

Our definite knowledge of Egyptian arithmetic is derived from surviving mathematical texts. These fall into two very distinct groups. The most numerous group of texts dates from the Ptolemaic period. Ptolemy I had been a general in the army of Alexander the Great. When Alexander died in 323 B.C., his ephemeral empire disintegrated, and Ptolemy gained control of Egypt and made himself Pharaoh. Egypt had already been strongly influenced by the Greeks, and this late group of mathematical texts needs no further discussion.

The early group of mathematical texts is very small, and they originated during the XII Dynasty (1900 to 1780 B.C.), about the time that the Minoan civilization was reaching its climax. This remarkable concentration of texts in one out of twenty-five dynasties deserves attention. There is also a question about the audience to which these texts were addressed. There are only seven of these mathematical texts. One consists of some tattered papyrus scraps found at Kahun. Kahun was neither a temple nor a city, but the construction camp for the pyramid of Sesostris II, one of the later pharaohs of the XII Dynasty. A second text is a letter from one scribe to another, known as Papyrus Anastasi I. The recipient is the Chief Scribe of the Army, but was engaged in civil construction projects. The writer must have been a very high official indeed, for he taunts the recipient about his lack of mathematical knowledge and the frequency with which he comes for help in solving his problems. The letter concludes with a challenge to solve a problem that arises when planning to build a ramp, and also contains a further set of examples that could be studied by his friend.

A third mathematical text is in the British Museum. It is not written on papyrus but on leather. The fourth is written on two wooden tablets, now in the Cairo museum. Both wood and leather would stand up in the field much better than the fragile papyrus. The use of valuable wood, however, indicates the importance given to the preservation of the text. It is known that Egyptian architects were in the habit of sketching plans and numerals on scraps of smoothed stone, but there seems to be no other case in which large boards were used as writing material until much later than the XXII Dynasty.

The most famous of these mathematical texts is known as the Rhind Papyrus. It is a later copy of a XII Dynasty original. It was bought in two parts from two dealers, so that its discovery site is not known exactly. It is thought to be have been near the important city of Thebes, possibly in the large nearby necropolis, the area of tombs and temples. Since it is a copy of an original written a century or more earlier, one may guess that the original may have been kept in a supposedly safe place, and that the copies were expendable. The sixth text is the Moscow Papyrus, and the seventh is the Berlin Papyrus. It seems that the last two contain much of the same kind of mathematics as the others, and nothing more significant has been reported about them.

It therefore seems that these texts were intended for scribes in charge of construction projects, and they were intended for use at construction sites, not in the peace and quiet of the temples. The texts were handbooks, not research publications. They may also have been used for instruction, but the taunting scribe tells that this had not gone very far. Responsible positions were being filled with people whose knowledge of mathematics was deficient. Their subordinates were also deficient in this knowledge; the calculations could not be delegated to others.

There are thus reasons for thinking that these texts were handbooks, or reference works. If so, this should be borne out by their subject matter. This can be described as being intermediate between our advanced arithmetic and our elementary algebra and geometry. The Egyptians had not phrases corresponding to "Let x be the unknown number," or "Let n be any whole number." Consequently, the texts are not concerned with generalities, but are collections of numerical examples and exercises. If the user encountred a problem that was not analogous to any worked-out example, he would have to use ingenuity in combining the examples provided him. Neither could the texts be used by anyone who had not received oral instruction in their use. This is just like our handbooks of engineering, physics, and chemistry. These also contain few explanatory notes, and no proofs that the formulae are correct. In the field, one is not concerned with proofs, only with getting the right answer. It seems that the Egyptians never wrote out the reasoning by which the solutions had been obtained. Instruction was very likely by rote; the reasoning had been forgotten. However, this is conjecture. Little is known about the educational system that produced the scribes; it may have been a form of apprenticeship. Some murals and funerary models show many more scribes than would seem necessary for the task being performed (counting and recording sacks of grain); some of those present might have been apprentices.

The examples contained in the mathematical texts are of many kinds. Some are merely concerned with the equitable distribution of loaves of bread among different numbers of men. Others require a change of units: how much wheat is needed to make a certain number of loaves? How many "bushels" of wheat will a granary of a given size and shape contain? What is the area of a triangle? This particular example made difficulties for the translators; the result is correct only if a word that usually meant "dock" or "quay" also meant "altitude" (of the triangle). The texts contain no definitions. It also seems that Egyptian geometry developed quite independently of arithmetic. Other examples calculate the number of bricks needed to build structures of a given size and shape, such as a ramp with retaining walls and paving. Some of the granaries were cylindrical pits, lined with brick. The calculation of the number of bricks needed to build such granaries required the use of the number π. The approximation used is quite good. The high point of the examples is the calculation of the volume of an unfinished pyramid.

A few of the examples are quite strange. While one problem is formulated in terms of loaves and men, it seems very unrealistic. The most careful scholarly study implies that the properties of square and triangular numbers were known a thousand years before Pythagoras was born. This study has not yielded any suggestion as to the method of solution, other than trial and error. If it was systematic (the text gives no indication that it was), the XII Dynasty scribes must have known how to solve some special quadratic equations. These unrealistic examples do suggest leisure for studying mathematics for its own sake, but this is quite rare.

We shall now turn our attention to the problem of the why the texts are all concentrated in the XII Dynasty. This dynasty is unique in many other respects. Most writers consider that the Middle Kingdom consisted of the XI and the XII Dynasties, but there is a recent tendency to include only the XII Dynasty in the Middle Kingdom period. The Middle Kingdom was preceded by a dark age -- The First Intermediate Period -- and followed by the Hyksos invasion, or Second Intermediate Period. The XII Dynasty was a period of economic, social, and physical recovery from the preceeding dark age, a renaissance. It can properly be viewed only against the dark background of the First Intermediate period.

By the end of the VI Dynasty, so much of the national product had been squandered on pyramid building that the Old Kingdom was impoverished. Even the nobles were dissatisfied. The last pharaoh of the VI Dynasty is said to have reigned for ninety four years. He must in any case have been senile at the end, and this may have contributed to the disintegration of the government. There followed more than a century and a half of civil wars; there may also have been rebellions of slaves and artisans; conditions often approached savagery. Many of the older tombs were vandalized, not merely robbed secretly. The quality of all craftsmanship seriously degenerated. Literacy did not entirely disappear, and a kind of Homeric escapist literature developed. It exaggerated past glories, and bowdlerized bloody crimes into heroic deeds. This has led some historians to describe that dynasty as the "golden age of Egypt"; perhaps they were ignorant of the misery and chaos. This gloomy period in the history of Egypt coincided with the increasing prosperity of the Minoans and a rapid development of Cretan civilization.

The pharaohs of the XI Dynasty restored some semblance of order, and revised the ancient funerary doctrine. People could now hope for immortality even if they could not afford to have a pyramid or temple-tomb built for themselves. The morale of the middle classes, the successful traders and artisans, must have been much improved. However, the work of economic and physical reconstruction and expansion did not begin until the XII Dynasty. The Fayum oasis and its lake were enlarged by great irrigation projects until it became a great market garden, capable of supporting an increasingly large number of people. The need for planning and constructing novel kinds of projects can scarcely be doubted. This need would extend to reference works of precisely the kind that have been found.

While this provides a motive for the writing of the mathematical texts, it is not a complete explanation. The surviving texts did not write themselves. What was the earlier history of Egyptian mathematics? When and where did it originate? How was it passed on to later generations? No records of the Hyksos or Second Intermediate Period have survived, but it seems not to have been so violently anarchic as the First Intermediate Period. Even so, why are there no mathematical texts from the New Kingdom that followed? There is good evidence that mathematical knowledge survived the Hyksos invasion.

The Mesopotamian Numeral Systems

Simultaneously with the Egyptian civilization, another great but very different civilization developed in Mesopotamia (modern Iraq). It had a very long and complicated military and political history. Carleton S. Coon and Geoffrey Bibby have attempted to provide an integrated account of this history, but for the last two centuries, the rate of archaeological discoveries has been accelerating, and has always exceeded the rate at which scholars can interpret and organize this new knowledge. C.W. Ceram has given an account of this archaeological aspect of the problem. The labor of interpretation is especially great for those discoveries that are relevant to the history of Mesopotamian mathematics, even though this history is closely related to the economic and social history of that civilization. Fairly recently, Wooley, Albright, and Bibby have found much information about Mesopotamian foreign trade, and have produced a chronology that is consistent with that of Egypt, Crete, and other countries. For the present purposes, the military history of the region is almost irrelevant. The economic and commercial history is more important. Like Egypt, Mesopotamia was almost without forests or mines. Yet it had an advanced technology from very early times. This necessitated foreign trade. Clay, however, was plentiful and houses, palaces, and even some fortifications were constructed of durable kiln-fired brick. The foothills of the Taurus mountains (in modern Turkey) were forested, and were relatively close to the Euphrates River. After a short overland haul, the logs were rafted to the southern cities. Copper was obtained from Cyprus, thus bringing Mesopotamia into contact (and conflict) with Egypt. There was also trade with India, and possibly China. Much of this trade was by donkey caravan; the climate was much moister than it is now, and camels were not needed. The inhabitants of the country east of Mesopotamia (modern Iran) were not friendly to the Mesopotamians. Hence, the early trade with India was by ship through the Persian Gulf. Manufacturers of such goods as rugs, cloth, and metalwork were needed to keep a favorable balance of trade. All of this required commercial organization, including bookkeeping, contracts, and later insurance. Thus, rather elaborate arithmetic developed very early and was disseminated along the trade routes. Military conquests were sometimes also effective in this dissemination of knowledge, but seem to have been over-rated by many historians. This network of communication was slow, but by no means negligable. It often makes it difficult to locate the origin of early cultural elements, either geographically or chronologically.

In very ancient times, say before 4000 B.C., both Mesopotamia and the Arabian Peninsula seem to have been inhabited by people who spoke various Semitic languages. Arabia was then a grassland, and the area at the head of the Persian Gulf was a swampy delta formed by the Tigris and Euphrates Rivers. Sometime about 3500 B.C., a people that spoke an entirely different language, Sumerian, settled on hillocks at the head of the Persian Gulf. They had a technology that was much more advanced than that of the original inhabitants. It included metallurgy, and four-wheeled carts drawn by oxen or donkeys. It is not known where the Sumerians came from, or how they came to have such an advanced culture. One of the Sumerian cities was Ur; the King James Bible calls it "Ur of the Chaldees" for a reason that is not very easily explained. The migration of the Sumerians appears to have been relatively peaceful. It is believed that the citizens of Ur and its surroundings remained bilingual, speaking Sumerian and a Semitic dialect, for more than two thousand years. There was intermarriage, and the Semitic element was increased by continuous immigration from northern Arabia. It is becoming customary to call these immigrants Amorites. Abraham, his father Terah, and their families were Amorites, and spent some time, perhaps a generation, at Ur. The Amorites were nomadic, and found the operation of caravans congenial; those of them who adopted a sedentary life became metalworkers or farmers.

The development of seal-stones, numerals, and writing paralleled that which has already been described. It was more or less simultaneous with the corresponding developments in Egypt, but it seems to have occurred independently of Egypt. The Egyptians seem never to have made much use of clay as a writing material, but the Sumerians, like the Minoans, used it almost exclusively. The numerals were imprinted in the clay with a stylus. The stylus, both ends of which were circular, had one end larger than the other. If held slantwise, it made a D-shaped impression; held vertically, a circular depression resulted. The digits of this system are shown in

1 10 60 600 3600

FIGURE 4
Sumarian Digits

Figure 4. For simplicity, the letters D and O, with accents, will be used in the following explanation:

$$D=1,\ D'=60,\ D''=60^2=3600,\ O=10,\ O'=600.$$

There has been much inconclusive speculation about the prominence given the number 60 in this system, but no convincing answer has been found. The number twelve was written $O-D-D$; no D could appear more than nine times in a numeral, and no O more than five times. 1972 was written as:

$$\begin{array}{ccccccccc} O' & O' & D' & D' & O & O & O & D & D \\ & O' & & & O & O & & & \end{array}$$

as the reader may verify. It seems not to be known how these numerals were spoken. Since there were few digits, arranged in easily countable sets, one may guess that the spoken number-phrases had a grammatical structure not much different from ours. It was a modification of the primary numeral system, but in the opposite direction from the Roman. There is no evidence that the abacus was ever used by the Sumerians; anyone who could count to ten could easily be taught the count-and-carry method for addition. The early methods for multiplication and division seem to have been unknown.

Later, when words for greater numbers were needed, the round stylus was abandoned, and the wedge-shaped (cuneiform) was adopted. The greater variety of marks made possible the addition of many written characters. To understand the problem, it must be known that Amorite city-states were developing to the north of the Sumerians. One of these was Akkad, and the cuneiform script was used to write both Sumerian and Akkadian, just as one alphabet suffices for the Western European languages today. But the difficulty was much greater: the Semitic, or Akkadian, and the Sumerian languages differed as much as, say, Chinese and Italian. To complicate matters further, a third people, the Elamites who spoke an early form of Persian, also settled in Mesopotamia. Ultimately, at least five languages were written in cuneiform script. This is notably different from the Egyptian case. In Egypt, there were three scripts, but essentially only one language; at least only one written language for the whole country. For the present, this is of minor importance, though there will be occasion to recall it.

It is becoming customary to call the cuneiform script "Akkadian," for it came into use at about the time when the Semitic city of Akkad displaced Ur as the dominant power in southern Mesopotamia. The Akkadian numerals were based on the older Sumerian numerals, though they had a quite different character. Most stages of the change-over are represented by archaeological finds. The Akkadian numerals are more usually called sexagesimal; our sixty-minute hour and 360-degree circle are evidence that the influence of the sexagesimal system is still present.

Our decimal system has ten digits, so one would expect that the sexagesimal system had sixty digits. Actually, it had only two. For typographical convenience, they will be designated by Y and C, though this does injustice to the beauty and flexibility of the cuneiform script. The value of the digits depended on their position in the numeral, just as "2" represents different values in 23, 234, and 342. Thus, Y and C might have any of the values

$$Y=60^n,\ C=10\times 60^n$$

where n might be 0, 1, 2, etc. The number 1972 was then written:

C	*Y*	*Y*	*C*	*C*	*Y*	*Y*
C			*C*	*C*		
C			*C*			

That is,

$$1972 = 3\times 600 + 2\times 60 + 5\times 10 + 2\times 1.$$

But this expression could also mean 1972×60, 1972×3600, etc. There was no decimal point, and the system was therefore ambiguous. This seems not to have troubled the Mesopotamians any more than we are troubled by the ambiguity of words such as "bear," "still," "will," "bluff," etc. The context must have made the meaning clear. However, the lack of a digit for zero created troublesome cases: 9068 was written somewhat like

Y	*C*		*Y*	*Y*	*Y*
Y	*C*	*Y*	*Y*	*Y*	*Y*
	C		*Y*	*Y*	

to distinguish it from 159, which was written

Y	*C*	*Y*	*Y*	*Y*
Y	*C*	*Y*	*Y*	*Y*
	C	*Y*	*Y*	*Y*

The two contain the same number of *Y*'s and *C*'s. It should be noticed that the *Y*'s appear in groups of not more than nine, and the *C*'s in groups of not more than five. Again, there is the principle of repeating a digit in groups that are small enough to be counted at a glance. Addition could therefore have been performed by the same count-and-carry method used by the Egyptians and Romans.

That is it could be, once the ambiguity of the numerals was resolved; for $CY+C$ might mean $11+10=21$, or $660+10=670$, or $11+600=611$, or any one of the other possibilities. Historians of mathematics are interested in more advanced matters, and it is usually said that the ambiguity was resolved by the context. But there must have been some system that made the interpretation unique. Leonard Wooley found that the clerks in the temple at Ur were meticulous in demanding requisitions and receipts for even the smallest items that they issued from the stores. It cannot be supposed that their supervisors, who were concerned with larger amounts, would be content with any possible ambiguity in their records. The lawyers who drew up contracts for the delivery of goods, and for risk insurance (similar to Lloyd's of London), would be even less tolerant of any ambiguity.

Perhaps it is unnecessary to say that archaeologists have found no evidence that the Mesopotamians used an abacus. While it is easy to imagine one for the purpose, it would not have solved the problem of ambiguity. Whatever method was used, it must have been simple, otherwise the traders could not have spread the sexagesimal system as widely as they did. The count-and-carry method is simple; moist clay is ideal for temporary memoranda, for it can be used over and over again. It is also, in contrast to ink-on-papyrus, ideal for corrections and erasures; evidence of this has been found in surviving cuneiform tablets.

What has been said about the early method of addition applies equally to subtraction, multiplication, and division. There is more information about it in later times. In order to understand the evidence, some historical background is needed. As the Sumerian-Akkadian civilization spread northward, the numeral system did not change for several thousand years. The Sumerian language also spread northward without much change, but its use was gradually restricted to religious and legal matters. The Akkadian Semitic, however, divided into two major languages: Babylonian in the south, and Assyrian in the north. These languages both changed with time, and are classified into Old, Middle, and New. The Old period is dated from about 2000 to 1500 B.C. Education in reading and writing seems to have been bilingual in both north and south. There were bilingual dictionaries for use in the elementary schools; one was even illustrated. At Ur, Wooley has found evidence that literacy was widespread in the commercial or middle class; moreover, instruction may have been available to both boys and girls. This literacy resulted in a large number of clay tablets inscribed in cuneiform script; about a half million have found their way into our museums. Mesopotamian clay was suitable for making brick, whereas Egyptian mud was not. Egypt did have fine clay, but it was reserved for pottery. Some Mesopotamian tablets were kiln-fired to the durability of brick, but many were simply dried in an oven. These last are fragile, often found broken, and in our moist climate they deteriorate rapidly unless their curators fire them. That they survived at all was probably due to the dryness of the earth and rubble in which they were buried.

Otto Neugebauer is a mathematician who became interested in the history of Mesopotamian mathematics, and learned to decipher the cuneiform writings. As of 1945, he and his colleagues had selected about 400 of the 50,000 tablets as being mathematically oriented; neither religious, legal, nor commercial. Since then, this number has increased somewhat. Like the Egyptian mathematical texts, these texts are separated into two distinct groups. The later group was produced during the Selucid Dynasty, although a few may be a bit older. Like Ptolemy I, Seleucus I was a Greek general in the army of Alexander the Great. After the death of Alexander, Seleucus gained control of northern Mesopotamia and Asia Minor, and also of Syria and northern Palestine. These tablets are thus contemporary with the Ptolemaic papyri. Still more remarkable, the older texts are almost all written in Old Babylonian. They are dated between 1900 and 1600 B.C., and are therefore contemporary with the XII Dynasty Egyptian texts. This can scarcely be an accident. The question is, which were the originals? And, in any case, how was this knowledge communicated from one country to the other? It is impossible to answer either without considering the evidence in detail.

The Old Babylonian group of texts is again clearly divided into mathematical tables and problem texts. The problem texts make the similarity with the Egyptian texts even more striking. Most of the problems are very similar in the two countries, although the Babylonians were more careful using more words in stating them. Neither Egypt or Babylon had devised anything corresponding to our algebra, and the Babylonian problem texts may again be classified as between advanced arithmetic and elementary algebra. There are no demonstrations of correctness in the texts. The Babylonian texts do contain one type of problem that the Egyptian texts do not. This type of problem concerns the calculation of the number of days a given number of men would need to dig a canal of given dimensions. During the period in which these texts were written, the Mesopotamian civilization was expanding northward, and the construction of new canals and irrigation ditches was characteristic. The level of these texts is rather more advanced than the Egyptian. It is certain that the Babylonians knew how to solve some kinds of quadratic and even cubic equations, although like the Egyptians, they had no notation for expressing general rules.

Because of a false but widespread tradition, it is well to say at once that none of these older texts or tables have an astrological or astronomical character. There are old astrological texts, but they are few and not numerical. Systematic records of eclipses and of the planet Venus were not kept until much later times, and even then were scarcely numerical astronomical observations.

The table texts are more numerous than the problem texts and of quite a variety. There are tables of weights and measures, including the standard sizes of bricks. There are multiplication tables, tables of reciprocals, of squares (or, by courtesy, of square roots). There are tables for the solution of some kinds of equations, already mentioned above. It is significant that not one of these is an addition table; this supports the conjecture that addition was performed by the count-and-carry algorithm, and was supposed to be known to the reader. Perhaps it was taught in the elementary schools.

The Babylonian multiplication tables were all variants of the following:

7×1	7
7×2	14
7×3	21
7×19	133
7×20	140
7×30	210
7×40	280
7×50	350

The first number in the last column is the principal number, in this case 7, in general it is c. The last number is usually $50\times c$ but sometimes c^2. The format suggests that the table was prepared by successive additions of the principal number c until $20\times c$ was reached; then $10\times c$ was added successively until $50\times c$ was reached. There was no need for $60\times c$, because of the ambiguity of the sexagesimal system. This closely resembles the method of teaching simple multiplication in the U.S. elementary schools; it was known as "counting by 7's," or more generally, "counting by c's"; except that our counting by c's stopped at $10\times c$, or c^2. and

c was usually not greater than twelve. In playing games like hide-and-seek, the child who was "It" might count by fives or tens to a specified upper number (100 or 200) before uncovering his eyes and beginning to seek.

There is ample evidence that these Old Babylonian multiplication tables were often school exercises. Very often, one side of the tablet would be written by an experienced person, the teacher, and often consisted of several such tables. The other side would be written by the pupil, who supposedly had memorized the teacher's version while the clay was still moist. Often this side contained mistakes, and the tables were repeated; they were copybook exercises. The teacher's version sometimes also had a mistake, showing that the teacher wrote out the table over and over, for each individual pupil. Moreover, these tables had rarely (or never) been kiln-fired when found; they were not intended to be permanent records.

At least one of these tablets gives the impression of having been intended as a temporary memorandum. One side was blank, and the other had space for much more than a single multiplication table. Of course, it may also have been an unfinished copybook. There are other examples for which the interpretation of an unfinished copybook does not seem tenable.

Much of what has been said also applies to the tables of reciprocals, and the detailed discussion of them can be postponed until more general matters concerning fractions have been investigated.

All of this tells us something about the Babylonian educational system of that time. The schools in which these matters were taught were not elementary schools, teaching reading, writing, and simple arithmetic. Perhaps one could call them trade schools, whose graduates would become engineers and architects, planners, the administrators of projects too large for craftsmen and artisans to manage. The students were required to memorize a multiplication table that was larger than that which our children memorize. It was an expensive education. If it was borrowed from the Egyptian system, it would seem likely that they would have used simpler multiplication by doubling. This is not certain, but it may indicate that the Babylonians were the originators, the Egyptians the borrowers, of this advanced arithmetic. While the Egyptian engineers took their texts into the field with them, the Babylonian engineers were required to memorize the material.

Turning now to the spoken Akkadian numerals, or rather to the spelled-out number-words: Neugebauer and Sachs published a list of words gleaned from their study of the mathematical texts. Since there is no exact equivalence between the cuneiform characters and our modern letters, scholars use many diacritical marks to supplement our letters; these have been omitted here. The list of spelled-out number words is surprising, not only because of omissions, but also because of inclusions. Of the cardinal numbers, the following were included.:

one:	isten;
two:	sina, sittan;
four:	erbitu;
sixty:	su-si;
hundred:	me'atu, me;
six hundred:	gis'us;
thousand:	lim.

It is unfortunate that words for intermediate numerals like 73, were not found; they might have shown the same sort of construction as our word seventy-three. It is not surprising to find sixty in this list, for it is one of the meanings of *Y*, and six hundred is one possibility for *C*. But the words for one hundred and one thousand correspond to

Y C C C
C

and

C Y Y Y C C C
Y Y Y C

respectively. They are clearly foreign, probably included because of Egyptian contracts. These numbers appear frequently in the numerical work so that they must have been important to the Old Babylonians despite the emphasis their number system placed on sixty. They are further evidence of a more than casual relation between the Babylonian and Egyptian mathematical texts.

The Land of the Alphabets

The Egyptian and Babylonian mathematical texts are sufficiently similar to suggest that their writers were in communication, though not in direct personal contact. None of the texts can be dated exactly, but there is general agreement that the Babylonian texts are later than the Egyptian. Neugebauer believes that none of his texts were written earlier than 1800 B.C. Possibly the Kahun fragments were contemporary with the oldest Babylonian tablets. The Kahun text was in use during the reign of Sesostris II, in the middle of the XII Dynasty, about 1800 B.C. Consequently, there is the possibility that some overlap of the periods during which the Babylonian and Egyptian texts were written. Leonard Wooley found one set of mathematical tablets at Ur, and was able to show that they were in existence in the year 1674 B.C. This is definitely later than the XII Dynasty of Egypt. All this suggests that the Babylonians may have borrowed their mathematics from the Egyptians. The frequent appearance of the number 100 in sexagesimal numerals indicates that there was some interchange of mathematical techniques between the two cultures. Some of the Babylonian texts are definitely more advanced than any of the Egyptian texts, which may be taken to indicate that the Babylonians elaborated on what they borrowed and were not simple copyists. It is remarkable that the knowledge should have been transmitted in this direction. Babylonian, not Egyptian, was the international language of commerce and diplomacy. Moreover, the Mesopotamians were much less hampered by tradition and more innovative than the Egyptians. However, the chronology seems conclusive.

Granting this hypothesis, one will inquire where the translation and personal contact necessary for the transmission of mathematical ideas actually took place. One may surmise that it was in the neighborhood of Cyprus, the source of copper for both civilizations. But more than simple contact is needed; one also needs to find people competent to translate Egyptian into Babylonian (and vice versa) and to convert weights and measures from one system to the other.

Both conditions existed at the ancient seaport of Ugarit. Ugarit was the mainland city nearest to Cyprus. It was excavated in 1929, by Claude F.A. Schaeffer. The site is also called Ras Shamra, about ten kilometers north of Latakia in northernmost Syria. It is separated from Cyprus by less than a dawn-to-dusk sail, and copper from Cyprus was there transferred to caravans bound for Mesopotamia and elsewhere. The later Greeks called this entire coastal strip Phoenicia; the Canaanites of the English Bible were the northern Phoenicians, inhabitants of modern Syria. The government of Phoenicia fluctuated between three types: domination by Egypt, domination by Mesopotamia, and intermittent periods of organization into small independent kingdoms. Beginning with the VI Dynasty, the Egyptians stopped importing wood and resin from the forests of Lebanon, and probably copper from Cyprus as well. During the XII Dynasty of Egypt, Ugarit was the capital of an independent kingdom, but the king and his courtiers were very Egyptianized. One might imagine that there were refugees, including craftsmen, from Egypt in Ugarit, and that many of the port's inhabitants could speak both Egyptian and Babylonian. During the XII Dynasty, the kings of Ugarit received presents from the pharaohs, including Sesostris II. It has been suggested that there may have been marriage ties between the royal families of Egypt and Ugarit. Egypt was again importing copper and Asian goods. All of the conditions necessary for the communication of ideas between Egypt and Mesopotamia were met at Ugarit. Not much more can be said; before the end of the XII Dynasty, Egypt had conquered not only Ugarit, but all of Phoenicia. Egyptian armies invaded Mesopotamia, and got as far as the Euphrates. This expansion was short-lived. Egypt itself was then invaded and dominated by the Hyksos, who may have been Asian, but they are unlikely to have been Babylonians.

The later history of Ugarit is of interest for other reasons. The period of its greatest prosperity coincided with that of the Mycenean domination of the Aegean trade. At the end of the Mycenean period, about 1200 B.C., the city was suddenly deserted, and was never reoccupied. It was soon buried in sand. There has been much speculation as to why it was deserted. It was a time of restless migration throughout the Eastern Mediterranean region.

For whatever reason, the city was preserved practically intact. The excavators found large palaces and temples, both associated with archives or libraries. The secular archives were separate from the religious records; both consisted mainly of clay tablets. Most significantly, a school for scribes was discovered; the students' unfinished exercises were found on the benches. The curriculum could easily be inferred. The students were taught to write many languages: Egyptian, Akkadian, Sumerian, the local Cypriote and Ugartic, and probably Mycenean. They were taught many scripts: Egyptian and Hittite hieroglyphics, Mesopotamian cuneiform, Mycenean Linear B, and Cypriote. The inhabitants of Cyprus had developed their own script, which has not yet been deciphered. It has similarities to the Minoan and Mycenean scripts. Perhaps not every student learned all of this, but this was the curriculum. The archaeologists say nothing about evidence for instruction in arithmetic and accounting, but it is difficult to believe that none of the students at this school would enter the city's principal industry: the transfer of goods. Ugarit was certainly cosmopolitan and polyglot; it must have been so since very ancient times, and so must have been the other coastal cities of Phoenicia and Asia Minor.

The script used for the local Ugartic (or Canaanite) language was unique and quite different from any older script. Those scripts that developed directly from pictographs are syllabic. Each character represented a syllable, the accented syllable of an earlier pictograph. There were many characters, but very few characters per word. The modern Chinese script is of this kind. The Ugartic script was quite different. There were only twenty-nine characters, each much simpler than the average Akkadian character. There were often many such characters per word. In short, Ugartic was an alphabetical script, using the first alphabet of which there is any record. It was imprinted on moist clay with a wedge-shaped stylus. One may surmise that it was invented deliberately for the Amorites who inhabited the hinterland, and whose produce was exported through Ugarit.

The relative advantages of alphabetic and syllabic scripts may be debated. We are appalled by the large number of characters that Chinese children must memorize. The Chinese are equally apalled by the need to memorize so many spellings. Our alphabet does not permit phonetic spelling. To make phonetic spelling possible, an alphabet of more than twenty-six letters would be needed. Electronic devices demonstrate this even more conclusively. Especially revealing is the difference between a child's and an adult's pronunciation. However, very likely, syllabic scripts are not fully phonetic either. Whether or not the invention of the alphabet was progress, it was a change, and a change that continues to influence us today.

About a hundred miles south of Ugarit was the equally prominent city of Byblos. In the 1920's, a landslide uncovered the tomb of King Ahiram, who might have been an ancestor of Hiram, the Caananite ally of Solomon. In the tomb were found many tablets inscribed in a second alphabetic script. Its characters were inscribed with a pointed stylus, just as were the Minoan and Mycenean scripts. The development of most other alphabets can be traced to this Ahiram alphabet: they include the Hebrew, Arabic, Persian, Ethiopian, Greek, Roman, and therefore our own alphabet.

While Phoenicia and all of Palestine has long been considered subsidiary to Mesopotamia and Egypt, it is rapidly becoming certain that it was an independent contributor, not only to religious thought, but also to other intellectual aspects of civilization.

The Greek Alphabet and Numerals

This investigation need concern itself only with the transmission of the Phoenician alphabet to Greece. At the time Ugarit was abandoned, Greece was invaded for a second time. These invaders are known to us as Dorians and came from the north. They spoke various Greek dialects and had iron weapons. They were excellent horsemen, though horseshoes had not yet been invented. The invasion was not sudden, and many Mycenaean cities fortified themselves. Ultimately, the Mycenaean cities were captured by the Dorians, with the single exception of Athens. Athens became overcrowded with refugees and some of these emigrated to Asia Minor. There they captured the coastal towns and some of the nearby islands. Among these were Miletus and Samos. They renamed the area Ionia, and ultimately the Ionian League consisted of twelve quarrelsome cities. While the rulers of these cities spoke a Greek dialect, they must have intermarried with non-Greeks. Their cities, like those of Canaan, were ports of transshipment for the Mesopotamian caravans, and there is evidence that they also had cosmopolitan, polyglot populations.

Having conquered the earlier inhabitants of the Greek mainland and nearby islands, the Dorians became robber barons ashore and pirates at sea. The expedition of the Argonauts under Jason was a piratical raid into the Black Sea; it cannot have been the only one. Robbery and piracy do not require accounts and inventories. Greece entered into a dark, illiterate age; the Mycenaean script was forgotten. There is not even any evidence that it was used in Ionia; there the merchants presumably used Babylonian for their records. Eventually, the Greek pirates drove Phoenician ships out of the Black and Aegean Seas, leaving them however, the rest of the Mediterranean.

There are nine or ten Greek legends that mention Cadmus the Phoenician. If it be supposed that each of them hides a kernel of historic fact, Cadmus can only be the personification of various groups of Phoenicians. Some of them seem to have joined the Argonauts. Only two of these legends are of interest here. One tells that Ares, god of war, gave Cadmus a magic clod of earth. Theras, a Dorian, joined Cadmus and they sailed south. When Cadmus cast the clod into the sea, it became the island of Thera. The other legend is that Cadmus taught the Greeks their alphabet.

Archaeological investigations show that after the vulcanism of the explosion subsided, Thera was again inhabited. It cannot have been of much mercantile importance. It does enclose a sheltered body of water, but there is little beach. The water is enclosed by almost vertical cliffs that rise to about a thousand feet. The water is too deep for even modern ships to anchor. There are no large piers or warehouses. The principal town is located at the edge of the thousand foot cliff. In the tenth century B.C., it can have been important only as a religious center, as it was in Minoan times. Remarkably, in view of the legend, the oldest Greek inscriptions in an alphabet (as distinct from the syllabic Mycenaean script) have been found on Thera. Archaeologists have debated their date, but they now seem agreed that it is about 850 B.C. The Greek language contains many more vowels and fewer consonants than any Semitic language, so that many of the Canaanite consonants are converted into vowels. All of this becomes understandable if one supposes an original mixed population on Thera, as a result of intermarriage. Only the Greek language survived, or at least it became official. The Phoenicians, with their knowledge of writing and alphabet-making contributed the official script. The other Greeks of that period were illiterate, and the Theran alphabet was later adopted by them as they became literate.

The newly acquired alphabet seems to have been used principally to record the Greek legends and songs. These had previously been transmitted orally, often by itinerant bards of ballad singers, which were later personified as Homer. Since these myths have a strongly religious character, this supports the conjecture that Thera was a religious center. This alphabet was not accompanied by a system of numerals; none would be needed for mythology. What domestic trade there was in Greece at that time, seems to have been simple barter. Taxes were collected in kind, and rather haphazardly. If there was any foreign trade, it was probably in the hands of foreigners who spoke Babylonian or Egyptian, and used the corresponding numerals for their accounts. All of Greece lapsed into the economic apathy that preceeded Solon's reforms.

The classical Greeks did invent two numeral systems, nearly simultaneously, at the beginning of the fifth century B.C. These have become known as the Attic and Herodianic numerals. The earliest inscription bearing Attic numerals was found near Athens, and has been dated 454 B.C. The earliest Herodianic inscription was found in Halicarnassus, on the coast of Asia Minor, at about the same time. By then, Greece was becoming the dominant maritime power, and Greek the dominant language throughout the Mediterranean world. These dates are sufficient to show that the Greek numeral systems are much more recent than the others discussed above; probably our own decimal system is even more recent. Very likely, the Greek systems had been invented a few generations earlier, and were coming into widespread use at the time of the inscriptions.

It will be useful to fit the date 454 B.C. into the pattern of events and people that are more familiar to us. Solon had died a century earlier. Ten years before, Pericles had established himself firmly as the ruler of Athens, and had begun to make Athens the imperial power that dominated the Aegean. Socrates was a lad of fifteen, very likely playing knucklebones in the streets. It is said that Socrates' father was a sculptor, and that the boy wanted to be one too. Finding himself without talent, he learned to be a stonemason. If so, he may have known the man who carved those first Attic numerals, but they would not likely have talked about it. Plato, who made Socrates famous to all suceeding generations, would not be born until twenty-seven years later.

Halicarnassus is not mentioned in most history books, and thus requires explanation. Asklepios, the god of healing, was a Doric deity, and his cult prospered on mainland Greece until Sparta declared sickness a crime against the state. The Asklepian priests then moved their center first to Halicarnassus, and then to the island of Cos, some twenty-five miles offshore. Here, Hippocrates, the "Father of Medicine," was a boy of four or five when the first Herodianic numerals were carved. He was said to have been the nineteenth lineal descendent of the god; in modern terms, it is likely that he became the nineteenth leader of the cult. It is only reasonable to suppose that, after so long a time, the art of healing had become encumbered with many useless traditions. At any rate, Hippocrates resigned his leadership while still vigorous, and moved to Abdera, where he became a friend of Democritus.

Abdera was an Ionian colony in Thrace, founded much earlier to exploit the gold mines in the country just north of the Aegean. Democritus had inherited a large fortune and spent it on travel and foreign study. In a surviving book, he says that he had traveled in more countries and climates, and visited more thinkers, than any of his contemporaries. Our curiosity is disappointed, for he does not give his itinerary, nor name the people he visited; but it is generally agreed that he was the most learned man of the time. He is sometimes known as the Laughing Philosopher, for the foibles of his fellow men amused him, apparently without arousing any impulse to reform them. He seems to have had no students, yet his influence was widespread and long-lasting. Although it is not true, he is often credited with inventing the atomic theory. He does seem to have synthesized the knowledge acquired during his travels. He did give the atomic theory a distinctly modern form. It is not surprising that the combination of Hippocrates and Democritus should have an enduring influence on the intellectual development of society, even though neither seems to have been aggressive in attempting to control it.

This is the environment in which the two Greek numeral systems evolved, and they will now be considered more specifically. In its original and simplest form, the Attic numeral system is identical with the later Roman numeral system, except that capital letters were used, and some of them were written in a peculiar way to indicate that they were digits rather than letters. This resemblance is no accident. When the Romans adapted the Greek alphabet to the Latin language, they also adopted the Attic numerals. Since the use of Roman numerals has already been considered, little need be said about the Attic numeral system. The most important thing is that there is almost no evidence about the way they were actually read at the time. The meager evidence for the use of the abacus has already been discussed.

By 500 B.C., the Greek alphabet had twenty-four letters, but earlier it had twenty-seven or more. The details of this scheme are unessential here; its extreme complication, in contrast to the simplicity of the Attic and all earlier numeral systems, is the important fact. Remarkably, none of the mathematicians of classical Greece mentions the earlier systems, although Democritus must have known of them, and did write about geometry. The cuneiform script is mentioned only by one writer of the period: this is Herodotus, who lived after Athens had already begun to decline.

In Attica, the Attic numeral system continued in use until 95 B.C., when the Herodianic system was legally adopted. By then, the Romans had already adapted the Attic numeral system to their needs. Both systems were originally used outside of Attica, but by 200 B.C., the Herodianic had completely displaced it. There is a clear reason for the Roman adoption of the Attic numeral system. No one has ventured to speculate why the Herodianic numerals displaced the Attic in most of the Greek-speaking world, but they did.

Human Language and Inhuman Logic

A digression on the general characteristics of all languages will be needed as a background for the following. It used to be considered that most of these characteristics were too obvious for words. The result was that they were ignored, and this caused much confusion, especially in philosophical discussions. It is only recently that this has been realized. It was first recognized, less than fifty years ago, by people who were trying to reconcile and synthesize the many philosophical doctrines. Surprisingly, it has already become a technological necessity in the computer industry. To an electronic computer, nothing is too obvious for words. It is this moronic stupidity of the machines that has caused such ridiculous incidents as the delivery of a truckload of the same issue of a magazine to a single subscriber. Grammatically, "two plus three" is not the same as "five." "Two plus three" is a number phrase. In any sentence containing "five," one may replace "five" by "two plus three" and the sentence remains grammatically correct. Moreover, if the sentence was true, the substitution leads to a true sentence; and if it was false, the substitution leaves it false. However, "five" can also be replaced by the number phrase "four plus six" without making it grammatically incorrect; but this substitution may well change the sentence from true to false, or conversely.

It is therefore necessary to distinguish between correctness and truth. In a well-written novel, the sentences will be correct, but not true. They will also not be false; they will be fictional. It is possible to write science fiction deliberately; it is also possible to do so unwittingly. It is this second possibility that leads to confusion.

In technical terms, grammatically correct or incorrect are the possible syntactic values of a sentence. Whether a sentence is true, fictional, or false can be determined (though only partially) from its meaning; its meaning must also somehow be compared with reality before its semantic value can be determined. True, fictional, and false are the semantic values of a sentence. In retrospect, it can be seen that history abounds with cases in which correctness has been confused with truth. The history of mathematics is no exception.

It seems that a clear distinction between grammar and meaning, between syntax and semantics, was first made in India, when Sanskrit was already becoming a dead language used only by scholars. At that time, these scholars wrote a grammar of Sanskrit that made no reference to the meaning of its words. In a way, the distinction between semantic and syntax is the same as that between a dictionary and a grammar textbook. This achievement of the Sanskrit scholars did not become known to Europeans until about 1800 A.D., and did not begin to influence logicians and mathematicians until the first half of this century. The distinction is now an essential one in computer science and technology. In retrospect, it is possible to see that the problems involving this distinction were considered by earlier Europeans, including the classical Greeks. For lack of the words, these discussions are obscure and ambiguous. The classical Greeks wrote neither dictionaries nor grammar textbooks. In another sense, logic and mathematics have not yet experienced the full impact of the distinction.

Logic and mathematics are concerned with those correct substitutions that do not alter the semantic value of a sentence. Semantics is not so easily reduced to rules as is syntax. For example,

Bucephalus galloped

is a true sentence, "Bucephalus " being the name of a famous horse. But

Pegasus galloped.

is fictional, "Pegasus" being the name of a mythical winged horse.

Pegasus flew

is fictional, but

Bucephalus flew.

is false, unless interpreted metaphorically. Whether the sentence is to be interpreted literally or metaphorically can be determined only by examining its context. It cannot be determined by examining the sentence by itself.

When one notes that the writing, publication, and broadcasting of fiction is a major activity in our society, it seems strange that few professional logicians have taken it into account. The usual explanation is that logicians are not concerned with sentences, but with the propositions that sentences express. There are many reasons for doubting that there is a difference. Not every language has different words for "sentence" and "proposition." In modern German, *Satz* means both. It is doubtful whether Classical Greek had words for either. The Greeks of the time of Plato and Aristotle compiled no standard dictionaries; many different dialects were in use, and there was such linguistic confusion that it provides many problems for modern scholars. In Latin, *propositio* meant only a tentative plan or a proposal for action until late Roman times; its use to distinguish propositions from sentences did not become widespread among scholars until after the sack of Rome by the barbarians and the beginning of the Dark Ages in Europe. It is no exaggeration to say that the supposed distinction between a sentence and a proposition is a legacy from the Dark Ages. In later times, there have been a bewildering number of explanations of the difference between sentences and propositions. These all seem to be based on untenable theories of either language or psychology. Their only common element is Bertrand Russell's definition: a proposition is something that is either true or false. Formally, this is an adequate definition, and independent of all theories of language or psychology. But it does not demonstrate that propositions are true, any more than the definition "centaurs are half man, half horse" demonstrates that there really are centaurs.

To understand more clearly how theories of language and psychology enter into this matter, one must consider the "meaning of meaning," to use a fashionable phrase. For this, one can do no better than elaborate on Aristotle's discussion. Suppose three people are together; *G* speaks only Greek, *P* only Persian, while *B* speaks both. First *B* says something in Greek; *G* and *P* hear the same sounds, but their perception of them is different. This difference is called meaning. Then *B* says something in Persian; now it is *G*'s face that remains blank, while *P*'s shows animation. Almost everyone has experienced this phenomenon and there is no doubt about its reality. It requires a theory of language and psychology to go further and demonstrate that there is a difference between a sentence and a proposition, and that the latter can only be true or false. When written sentences come into consideration, it is seen that the assumption of a difference implies that a language is at least as unchanging as handwriting. This is an assumption that people, especially young people, are inclined to make. Yet it is false. William Taft's face would have remained blank, or become bewildered, had someone told him "We tuned in on the broadcast of President Wilson's inaguration." This sentence would not only have been false; it would have been meaningless.

The changes that occur in all languages are important, not only because they provide evidence for distinctions without differences, but also for differences without distinctions. A simple but important example from an unfamiliar language will show this. The Old Babylonian word *auatu* is translated into English as *Name, word,* or *thing,* depending on the context. It seems that the Babylonians had only one word for our three, and that these three have quite different meanings for us. All three are nouns, and a correct English sentence will remain correct if one word is substituted for another, but its semantic value will very likely be altered. This is exactly the same as the interchange of *Bucephalus* and *Pegasus*. These two examples are not unrelated. Today, we do not believe that winged horses exist, or ever did exist; the ancient Greeks did believe in them. There is a strong tendency to believe that, if a thing has been named, if there is a word or phrase for it, it exists. The ancients can be excused because it was almost impossible for them to search the Earth sufficiently to be sure that there are no winged horses. Today, we believe that there were dinosaurs long ago; most of us have seen the fossil bones of dinosaurs. We have also seen reconstructions of their external shape; the reconstuctions are conjectural. They have been made by careful students of comparative anatomy, who are familiar with the relations between bone, muscle, and skin in contemporary animals. But if someone writes about a blue and red dinosaur, there is no way to compare this with reality. Blue and red dinosaurs are in the same catagory as winged horses: fictional. If "blue and red bird" in a correct sentence is replaced by "blue and red dinosaur," the sentence will remain correct; but even if the first sentence is true, the second will be fictional.

All of this is simple enough, but it raises an important question. Are there distinctions for which there are no distinct words in the English language? If so, we must not be too proud when comparing English with Old Babylonian. The question has already been answered in the affirmative. Spoken English has no easy way to distinguish between a word and the name of that word. Consequently, it has not even occurred to most people that words ought to have names as well as pronunciations, spellings, and meanings. It is only very recently that grammarians, philosophers, and logicians have recognized this distinction and traced some famous and inconclusive arguments to the failure to notice it. When we speak or write about apples, we do not produce pieces of the fruit; when we eat apples we do not eat words. When we speak or write about words, we do use words. What was simple and straightforward has suddenly become complicated and confusing. Grammarians have invented a way of writing the names of words: apple is the name of the fruit, and "apple" is the name of the word. This is a simple way to avoid the confusion: "apple" is a noun while apple is a food. But no one has yet invented a simple way of saying "apple" differently than apple; in speech the same sound does for both, and the distinction depends on the context. Both "apple" and "orchid" are nouns, but an apple and an orchid are very different indeed. This is one reason for the difficulty in teaching young children the rules of grammar.

It has been maintained that questions and instructions do not express propositions, and therefore have no semantic value, and are not a part of logic. It is possible to support this view with examples. Questions like "Can smog be eliminated?" and instructions like "Speak French in thirty days," are often used as titles for books and lecture courses. It would be quite possible to replace these two with "Methods for the elimination of smog," and "A thirty day course of instruction that will enable you to speak French." Neither of these is a sentence: they are phrases used as names for the books or courses of instruction. It is also possible to use sentences like "Spring came early that year" as titles of novels. It might be possible to use these examples for the development of a theory about the difference between a sentence and a proposition, but it would not be any of the traditional theories. Numerals are used to name the pages in books, but it would not be possible to use this example as the basis for a fundamental theory of arithmetic.

It can also be maintained that questions and instructions are superfluous, at least in theory. Thus, the question "Where is my necktie?" is equivalent to the sentence "I desire to know the present location of my necktie." The instruction "Put on your necktie!" is equivalent to "I want you to put on your necktie." The question "Are you going to put on your necktie?" is ambiguous. If the word "are" is not stressed, it is "I desire to know whether you intend to put on your necktie," while if "are" is stressed, it may be the threat "If you do not put on your necktie at once, I shall spank you." However, emphasis can alter the meaning of declaratory sentences also, so that this ambiguity is not peculiar to questions.

"How shall I put on my necktie?" is "I desire instruction in the way to knot my necktie." Since this is a request instruction in a procedure, a manipulation, it may be best to answer it by a demonstration, accompanied by words that direct attention to the salient actions. On the other hand , "Why should I put on my necktie?" is "What satisfaction (or pleasure, or benefit) will accrue to me (or anyone) if I put on my necktie?" It is generally agreed that the natural sciences attempt only to answer questions beginning with "How," not those beginning with "Why." Despite the general agreement, it has not been subjected to careful analysis. One might expect that there would be equally general agreement that the social sciences should attempt to answer questions beginning with "Why," but this is not the case.

Further ramifications appear when one considers the instruction "You must put on your necktie." This is "You will inevitably put on your necktie" or "You lack the ability (or freedom) to avoid putting on your necktie." Depending on the intonation, it may be a threat of punishment if you do not put on your necktie; the traditional association of the birch cane with the schoolmaster or instructor contributes to this sinister connotation of the word "must." There have been almost endless and inconclusive philosophical discussions about the word "must"; it is a word to beware of. An innocuous example is the adage "What goes up must come down"; The successful launching of space-craft has demonstrated that this is false. Less innocuous is its association with the tyranny of social conventions and laws that infringe upon civil liberty. Ultimately, the debate over "must" becomes the theological debate over free will.

Sometimes the failure to analyze questions can lead to serious ambiguity. Consider "What is the capital of Wyoming?" If the senior Senator from Florida asks it of his secretary, it means "I wish to know the city in which the government of the state of Wyoming has its headquarters." If the secretary then answers "I don't know; I will look it up," the Senator will be content. If pupils are asked the same question by a teacher and make the same reply, they will be reprimanded and be given low grades. The teacher means "I wish to know if you remember the name of the capital of Wyoming." This has serious consequences. It wastes the time of teachers and students with useless memory drill, for the pupils (unless they live in Wyoming) will still soon forget the name of the capital, and be none the worse for it. The time could be used to better advantage in teaching the pupils the proper use of maps and other reference materials; even better in teaching them to look for ambiguities and analyze the meaning (not the grammar) of a suspected sentence.

These examples show that questions and instructions can be converted into declarative sentences, although the conversion is sometimes ambiguous. When the conversion has been made, questions and instructions are seen to be very personal, very human. Perhaps it would be better to say that they refer to interpersonal transactions, not to inanimate objects or machines. It is also seen that truth and falsehood are closely related to honesty and dishonesty. The latter are ethical and human; the former are depersonalized and logical, and seem to have nothing to do with people. Some sentences, such as "Broadcasting stations emit electromagnetic waves," do have little reference to people, and communicate knowledge about impersonal matters. Others are deceptively impersonal. The sentence "The Indus River has flooded the Punjab" deflects ones attention from the human disaster of flooded fields and homes to the inanimate river and its valley.

By excluding questions and instructions from logic, it has been possible to make it seem impersonal, inhuman. An impersonal discussion of electromagnetic waves is not only possible, but advantageous. An impersonal discussion of broadcasting stations may be irrelevant to the matter under consideration. Since our educators have ignored such matters, some students denounce our whole educational system as irrelevant and drop out.

Archimedes, Buddha, and the Uncountable

Archimedes lived from 287 to 212 B.C., during the Hellenistic period. He was the son of an astronomer and a close friend (possibly a relative) of the kings of Syracuse, the Sicilian city in which he spent most of his life. The legends concerning him are contradictory. According to some, he was a solitary, absent-minded eccentric, who only communicated with mathematicians in other cities by letter. Some of his letters have survived, at least in translation. Certainly, those of his books and letters that have survived could not have been written without long periods of undisturbed concentration. Another legend relates that, during a siege of Syracuse, the Roman fleet was destroyed by devices invented and constructed by Archimedes. No single man, much less a recluse, could have performed such a feat. Cicero tells that he saw a planetarium that Archimedes had constructed, which was so accurate that it could be used to predict eclipses. Such accuracy is incredible, but it is known that one of Archimedes' books (lost to us) did describe a mechanical model of the heavens. The invention of various other devices is ascribed to him but he seems to have left no written account of them. Two books on floating objects have survived, and suggest not only abstract thought about the subject, but a firsthand acquaintance with real floating objects and their behavior.

All of this suggests that Archimedes was a man of varied interests, on easy personal terms with both the rulers and the artisans of Syracuse; that he understood the devices used by artisans sufficiently well to make suggestions for their improvement. In Archimedes' time, Syracuse was a small town, and it would be possible for him to know everyone who lived there. It is likely that many of the artisans were illiterate, so that his suggestions had to be made verbally, by means of diagrams probably drawn in the dust. This, and the narrowly intellectual interests of his distant friends, would account for the absence of written descriptions of these inventions. Long personal acquaintance with the artisans would make the large defense project more credible. The later destruction of Syracuse by another Roman invasion would account for the failure of his devices to survive in identifiable form. Clearly, his Roman contemporaries considered him to be a sorcerer. The defeated commander of the first Roman fleet, in trying to excuse himself, said nothing to dispel the notion that Archimedes had superhuman powers. Archimedes does seem to have been an unusual person. Nothing is known about his religious beliefs, but his writings show that he did not consider himself to have been favored by any revelation of divine knowledge.

Eleven of his books and letters have survived in translation, but they make reference to others. They were usually of such a length that they fitted conveniently on one or two rolls of papyrus. For the present purpose, only one of the surviving books is of interest, for it introduces not only a new system of numerals, but also a new view of numbers. This view may also have arisen in India, as will be seen. It has dominated much, if not all, of later mathematics. Archimedes wrote two books or letters about very large numbers. One was addressed to a fellow mathematician and is usually called "The Principles." It has not survived. The other is addressed to a young man whom he had tutored, and who later became Gelon II, King of Syracuse. It is apparent that Gelon and Archimedes continued their conversations, and that the letter is an amplification of one of these. It is usually called "The Sand Reckoner," and falls naturally into three parts:

1. Astronomy and the formulation of a problem
2. The new numeral system
3. Solution of the problem

Heath's translation of the first part will be given verbatim. It is of double interest, for it contains an authoritative, though incomplete, account of astronomy, as it was some centuries before the time of the astronomer, Ptolemy. Historically, it is very significant that the geocentric and heliocentric descriptions of the solar system were both considered at thie early date. It is apparent that Aristarchus of Samos originated the heliocentric hypothesis. Since he was younger than Aristotle, the latter considered only the geocentric description; this would become of importance much later, during the

Reformation in Europe. The remainder of the text will be given in a summary form, and in modern notation. Archimedes writes:

> There are some, king Gelon, who think that the number of the sand is infinite in multitude; and I mean by the sand not only that which exists about Syracuse and the rest of Sicily but also that which is found in every region whether inhabited or uninhabited. Again there are some who, without regarding it as infinite, yet think that no number has been named which is great enough to exceed its multitude. And it is clear that they who hold this view, if they imagined a mass made up of sand in other respects as large as the mass of the earth, including in it all the seas and the hollows of the earth filled up to a height equal to that of the highest of the mountains, would be many times further still from recognizing that any number could be expressed which exceeded the multitude of the sand so taken. But I will try to show you by means of geometrical proofs, which you will be able to follow, that, of the numbers named by me and given in the work which I sent to Zeuxippus, some exceed not only the number of the mass of sand equal in magnitude to the earth filled up in the way described, but also of a mass equal in magnitude to the universe. Now you are aware that "universe" is the name given by most astronomers to the sphere whose centre is the centre of the earth and whose radius is equal to the straight line between the centre of the sun and the centre of the earth. This is the common account, as you have heard from astronomers. But Aristarchus of Samos brought out a book consisting of some hypotheses, in which the premises lead to the result that the universe is many times greater than that now so called. His hypotheses are that the fixed stars and the sun remain unmoved, that the earth revolves around the sun in the circumference of a circle, the sun lying in the middle of the orbit, and that the sphere of fixed stars, situated about the same center as the sun, is so great that the circle in which he supposes the earth to revolve bears such a proportion to the distance of the fixed stars as the center of the sphere bears to its surface. Now it is easy to see that this is impossible; for, since the center of the sphere has no magnitude, we cannot conceive it to bear any ratio whatever to the surface of the sphere. We must however take Aristarchus to mean this: since we conceive the earth to be, as it were, the center of the universe, the ratio which the earth bears to what we describe as the "universe" is the same as the ratio which the sphere containing the circle in which he supposes the earth to revolve bears to the sphere of fixed stars. For he adapts the proofs of his results to a hypothesis of this kind, and in particular he appears to suppose the magnitude of

the sphere in which he represents the earth as moving to be equal to what we call the "universe."

I say then that, even if a sphere were made up of the sand, as great as Aristarchus supposes the sphere of the fixed stars to be, I shall prove that, of the numbers named in the *Principles*, some exceed in multitude the number of the sand which is equal in magnitude to the sphere referred to, provided that the following assumptions be made.

1. *The perimeter of the earth is about 3,000,000 stadia and not greater.*

It is true that some have tried, as you are of course aware, to prove that the said perimeter is about 300,000 stadia. But I go further and, putting the magnitude of the earth at ten times the size that my predecessors thought it, I suppose its perimeter to be about 3,000,000 stadia and not greater.

2. *The diameter of the earth is greater than the diameter of the moon, and the diameter of the sun is greater than the diameter of the earth.*

In this assumption I follow most of the earlier astronomers.

3. *The diameter of the sun is about 30 times the diameter of the moon and not greater.*

It is true that, of the earlier astronomers, Eudoxus declared it to be about nine times as great, and Pheidias, my father twelve times, while Aristarchus tried to prove that the diameter of the sun is greater than 18 times but less than 20 times the diameter of the moon. But I go even further than Aristarchus, in order that the truth of my proposition may be established beyond dispute, and I suppose the diameter of the sun to be about 30 times that of the moon and not greater.

4. *The diameter of the sun is greater than the side of the chiliagon inscribed in the greatest circle in the (sphere of the) universe.*

I make this assumption because Aristarchus discovered that the sun appeared to be about 1/720 part of the circle of the zodiac, and I myself tried, by a method which I will now describe, to find experimentally the angle subtended by the sun and having its vertex at the eye.

This means that the diameter of the sun is greater than 1/1000 the diameter of the earth's orbit, and that the diameter of the sphere of the fixed stars is less than 10,000 times the diameter of the earth's orbit. Somewhat farther along, Archimedes makes a fifth assumption:

> 5. *Suppose a quantity of sand taken, not greater than a poppy-seed, and suppose it contains not more than 10,000 grains. Next suppose the diameter of the poppy-seed to be not less than 1/40 of a finger-breadth.*

Archimedes wrote in his local Greek dialect, and neither his manuscript nor his dialect have been preserved. It is obvious that the translation is faulty and that "sand" is not the right word for the stuff he describes; we would call it "dust." The diameter of each grain is less than one thousandth of an inch, and a single grain would be scarcely perceptible to the unaided eye. However, the phrases to be remembered are : "No number has yet been named" and "Of the numbers named by me and sent to Zeuxippus."

Archimedes constructed his number system with the Herodianic numerals as the starting point. It will be recalled that, while these were complicated, they only made it possible to write numbers less than H=100,000,000. Let h be any one of these; he called them numbers of the first order:

First Order:
$1,2,3,\ldots,h,\ldots,H$

The second order consisted of the next H numbers:

Second Order:
$H+1,\ H+2,\ldots,H+h,\ldots,H^2$

Continuing, he reached the Hth order:

Hth Order:
$H^{H-1}+1,\ H^{H-1}+2,\ldots,H^{H-1}+h,\ldots,H^H$

All of the numbers named thus far, Archimedes called numbers of the first Period. Let $P=H^H$; it is a very large number: a one followed by eight hundred million zeroes. Next came the numbers of the second Period and first Order, etc.:

> Second Period first Order:
> $P+1, P+2,...,HP$
>
> Second Period second Order:
> $HP+1, HP+2,...,H^2P$
>
> Second Period Hth Order:
> $H^{H-1}P+1,...,H^H P=P^2$

He continued in this manner until he reached the Hth Period Hth Order, whose largest number is P^H. Here he stopped, but he had named all numbers by Period, Order, and Herodianic numeral up to P^H. The numeral P^H is, in our decimal system, a one followed eighty thousand million million zeroes.

The solution of the problem (or rather problems) proposed by Archimedes is now possible. He interpreted them simply as the calculation of the volume of two spheres, the "universe" (or the "sphere of fixed stars") being one, and the dust grain being the other; then the ratio of the two volumes is to be calculated. There were two difficulties in Archimedes' way that are not in ours. The first was the difficulty of calculating with the Herodianic numerals, the second was that the numbers required in the calculation exceeded those named in the Herodianic system. This second obstacle was overcome by his invention of the new system of numerals, and the first had previously been mastered both by Archimedes and Gelon. Only the results need be given here: he shows that the number of dust grains required to fill the "universe" is less than $10^{51}=1{,}000H^6$ and for the "sphere of fixed stars", less than $10^{63}=10{,}000{,}000H^7$ His numeral system was more than adequate for the problems proposed.

Before discussing this further, one may consider a story from the life of Buddha, which shows that similar matters were being discussed in India. Buddha lived about three centuries before Archimedes; it is not known when the legend was first written down, but it can scarcely have been during Buddha's lifetime. Before renouncing the ways of the world, Buddha was the handsome Prince Gautama. As a young man, he was one of the many suitors for the hand of a beautiful princess. Her father agreed to bestow her on the man who most successfully passed a long series of tests. Needless to say, Gautama passed them all perfectly. One of the tests was a quiz by the mathematician Arjuna. The first question was "Can you name the numbers by hundreds?" Gautama replied by writing:

> A hundred *koti* make one *ayuta*;
> A hundred *ayuta* make one *niguta*;
> A hundred *niguta* make one *karikari*;

and so on for twenty more lines. Authorities differ as to the number *koti*, but it is thought to have been 10^7. In this way, Gautama therefore reached the number *tallakshana*, which is equal to 10^{53}. But he did not stop there: he recited eight more stanzas of the same length, thereby reaching 10^{421}, which is large, but still much less than Archimedes' largest number. This is the same method that we use:

> Ten tens make one hundred;
> Ten hundreds make one thousand;
> A thousand thousands make one million;
> A thousand millions make one billion;
> A thousand billions make one trillion

and so forth. We proceed somewhat more systematically than Buddha, less so than Archimedes.

But Arjuna's quiz was not yet ended. "How many primary atoms make one mile?" Buddha recited:

> Seven atoms make a very small particle;
> Seven very small particles make a small particle;
> Seven small particles make a large particle;

and so on, until he found a number that we would write 108,470,495,616,000. Arjuna accepts this as correct; we would be inclined to doubt whether the question was properly formulated, and whether the calculation was correct. These doubts arise because we think of a mile as a unit of length. It is known that in early India and classical Greece, the same word was used for the unit of area and the unit of length. In Mesopotamia, the same word was used to name a unit of area and a unit of volume. It is possible that Arjuna was thinking of a cubic mile. There are other versions of this story, some of which may have been modified to eliminate this doubt.

The obvious similarities between Archimedes' letter to Gelon and Buddha's recitation lead one to inquire about possible communication between India and Sicily. One immediately thinks of travelers that may have carried the story along the caravan and shipping routes. However, there is no evidence for it. Archaeologists have found no city, like Ugarit, that could have served as an intermediary. It is known that Alexander took Greek philosophers with him as advisors, and that they reached the Indus River. However, they stayed there only a very short time. Obvious linguistic and military barriers make it unlikely that the Greek philosophers could have established communication with their Indian counterparts. If there was communication between India and Sicily, it would seem to have been very indirect, as indirect as that which produced the similarities of the modern Indo-European languages.

If there is doubt about the influence of the Indian mathematicians on Archimedes, there is none about the latter's influence on more recent writers. The Dutch warehouse clerk, lens maker, and microscopist, Anton van Leeuwenhoek, observed spermatazoa for the first time, and made a calculation of their number. Extracts from his letter to Dr. Graaf follow:

> Sir:
> Since my last of the 21st February, viewing the Melt of a live Codfish, I found the *Succus* thereof, which ran from it, full of exceedingly small live Animals incessantly moving to and fro: these Trials I thrice repeated with the same success, till I was weary with seeing them. I have also viewed the Melt of Pikes or Jacks, and therein also found an incredible number of small Animals. And I judge that there were at least ten thousand of these Creatures in the bigness of a small Sand. These were smaller than those I observed in Beasts, but their Tails were longer and thinner. I viewed also the Testicles of a Dog taken out of its second skin. Viewing the matter taken presently after excision, I discovered a vast number of small Creatures. After three hours cutting the Vessel at (another place) I found multitudes of Animals there also, but most dead. The *semes* of

> a Cock about a year old, I found exceedingly full of these Animals, at least 50,000 in the bigness of a Sand.
>
> How vast and almost incredible the number of these Creatures are, you may somewhat the better conceive by the Calculation that I have hereunto annexed, depending fundamentally on accurate observation. In the quantity of the juice of the Melt of a male Codfish, of the bigness of a small Sand, there are contained more than 10,000 small living Creatures with long Tails. And considering how many such quantities (viz. of the bigness of a Sand) might be contained in the whole Melt, I was of the opinion that the Melt of one single Codfish contained more living Animals than there were living Men at one time upon the face of the Earth. That which induced me to be of this belief, was this following Calculation.
>
> I conceive that 100 Sands in length will make an inch, therefore in a cubic inch there will be a Million of such Sands. The Melt of a Codfish must therefore contain one hundred and fifty thousand Millions.
>
> I will now reckon the number of Men upon the face of the Earth at once by Guess. There are in a great Circle of the Earth 5400 Dutch Miles, thence I collect there must be 9,276,218 square Dutch Miles for the earthly superfices. 'Tis said two thirds of the superfices of the Earth are Water, and one third is Land; a third therefore of the last number is 3,092,072. I suppose a third of this is uninhabitable, and the other two thirds only inhabited, which contain 2,061,382 square Miles.
>
> I further suppose Holland and West-Friesland to be 22 Miles long and seven Miles broad, which makes 154 square Miles: the habitable part of the World therefore exceeds Holland and West-Friesland 13,385 times.
>
> According to the Computation of N.N., the number of People in Holland and West-Friesland may be about a Million. And if all the rest of the habitable World be a populous as these (which is very unlikely), there would be 13,385 Millions of Men at once on the face of the whole Earth; but in the Melt of a Codfish, there are one hundred fifty thousand Millions of Animals; the number of these therefore will exceed the number of Men more than ten times.

Leeuwenhoek made several other calculations of this kind. This is selected as an example because there are so many similarities between this and Archimedes' calculation that they cannot all be ascribed to coincidence: the repeated reference to a grain of sand, the reference to the circumference rather than the diameter of the Earth; the geographic element, the general scheme of the calculation, the use of estimates and the method of inequalities. It is not necessary to suppose that Leeuwenhoek had a copy of the "Sand Reckoner" before him as he calculated. It is sufficient if he had read it at some earlier time, or if a teacher had explained it to him.

All of this requires further comments. They can best be introduced by another bit of arithmetic. A cubic yard of coarse beach sand contains about a billion particles. Supposing that a person can count the grains at a rate of slightly more than one per second, then three people in working in eight hour shifts can count about a hundred thousand grains per day. Allowing for holidays, one hundred people could complete the task of counting all the particles in a year. But each grain of coarse sand would be much larger than one of Archimedes' dust particles. A cubic yard of his dust would contain about 100,000 billion particles. Disregarding the difficulty of seeing a single dust particle, it would therefore require ten million people to complete the count in one year; this is about the present population of New York City. To say that even one cubic yard of dust can be counted exactly (that is, not one particle more, not one less) is therefore fictional, if not false. Very large numbers are therefore fictions at best.

It is not known whether or not Archimedes believed that there were winged horses. But he tells us that he was "naming numbers." He seems to have believed that even very large numbers have an existence independent of people who can count. There are alternatives of course: the translation may be wrong, or he may not have given the matter a second thought. Most nouns, like "apple" denote things that can be seen, touched, and tasted; in short things that exist. "Number" is a noun; therefore, numbers exist. This is typical of much ancient reasoning, not only Greek. However, examples of this type of reasoning recur very frequently in Greek writings, especially in the writings of Plato.

The matter is worth more than a second thought, and should be examined from still other points of view. Every ten years, the U.S. Bureau of the Census attempts to count all the people living in the United States. Formerly, this was done by employing large numbers of census-takers, who visited each home and inquired about the residents. But people are dying and being born every minute of the day; even if there were no erroneous answers, the numbers reported would depend on the time the census-taker visited the various homes. The number reported by the Bureau of the Census is thus inexact; it is not an accurate count, in the sense of "not one more, not one less." The larger the number of things to be counted, the more likely it becomes that there has been an error. This is often recognized by estimating the probable error and it has become customary to replace the supposedly uncertain digits with zeroes. Leeuwenhoek sometimes did this, sometimes not.

The example of the census raises another question: What is the exact moment of birth or death? This is not a philosophical quibble; it has already become an important problem in medical law, and the solutions proposed change with advances in medical science. Buddha's teachers were in a better position; they believed that the primary atoms were indestructable, as did many others. They could therefore maintain that the number of atoms in a piece of matter, though large, was at any instant certain, even though unknown to any human being. The whole numbers were therefore inherent in nature, and not invented by people. The belief in the indestructability of the primary atoms (or fundamental particles) persisted well into the present century, and with it, the belief that numbers exist, independent of people who can count. Physicists no longer believe that the fundamental particles are indestructable. Some are always disappearing (disintegrating spontaneously), while new ones are appearing spontaneously (creating themselves with equal spontaneity). These process cannot yet be explained in time and space, perhaps they never will be. Thus, the census of large numbers of fundamental particles encounters the same problem as the census of human beings. The implication is that the whole numbers are not a part of nature, but this implication is never emphasized. The matter is of sufficient importance to be investigated further in later chapters.

For the present, there is still another viewpoint from which large numbers can be considered: it may be called the technological viewpoint, although it separates into two different technologies and both become involved with human psychology. First, consider one of Archimedes' largest numbers, a number that in our numeral system would require eighty thousand million million decimal digits, most of them not zeroes. One may imagine that it has been printed and therefore the numeral would have acquired at least a material existence. It would occupy about a billion volumes, each the size of Webster's Unabridged Dictionary. It would require a thousand buildings, each the size of the Library of Congress, to provide shelf space for these volumes. Who would read even one of them? Consider a much smaller numeral, one that would require only a single volume the size of the Bible. Many people have read the entire Bible, so that it would certainly be possible for this numeral to be read; but would the reader be able to understand it? It is certain that it would not be understood at all. The reader of this present book has presumably read Buddha's large numeral, a few pages back. Is 108,470,383,526,000 a repetition of that number? It is certain that people cannot comprehend very large numbers. Leeuwenhoek expressed amazement at those he came upon. Menniger remarks that very large numbers are superhuman. One begins to understand why Kronecker, who was more skeptical than most nineteenth century mathematicians, said that God created the whole numbers. But, remembering the Old Babylonian word *auatu*, one can also understand why Archimedes may not have given the matter a second thought.

There is another technological way of looking at large numbers. It is possible to construct mechanical or electronic devices that can count much more rapidly, and are much less likely to lose count than people. Paradoxically, these devices are most effective in counting insubstantial events. The old-fashioned grandfather's clock will count hundreds of thousands of swings of its pendulum with a single winding. An ordinary electric clock counts more than five million oscillations of an electric current every day. The older definition of a second was related to the time required for the Earth to make one rotation. Then astronomers found that the Earth's rotation is not quite constant. The variations are slight, but can be measured. The present definition of a second is 9,162,631,770 oscillations of the light emitted by cesium atoms. Clocks do not accompany their counting with a soliloquy; they are not human. They are also not superhuman, since they are constructed by people. Neither do they announce their count in incomprehensibly large numerals. Chimes were formerly more common than now. A child that has learned to count must still be able to tell time from the position of the hands of a clock; this is a different kind of counting. The fact that clocks are under the domination of human beings, are subhuman, is emphasized by the modifier in "present definition of the second"; about once a generation, an international congress changes the definition in order to take advantage of new inventions and technologies. One sees that people can sometimes even make incomprehensible fictions come true. But it should not be forgotten that most people have no need to know the time all that accurately. Few people have any reason to encumber their memories with the niceties of the current definition of the second.

Much of this has already been summarized. Archimedes, Buddha, and their predecessors introduced fictionally large numbers into mathematics. This is rarely recognized. All numbers are usually considered to have an existence quite independent of people who can count and can make mechanisms that can count. This belief has been held by scientists and philosophers who, as will be seen, formulated it explicitly. It has been implicitly accepted by most people, even by those who do not give much thought to such matters, but have absorbed it from our educational system. There are two forms of this belief, and their proponents have indulged in lengthy debates. The first may be called the Doctrine of Natural Mathematics, and its simplest form has just been discussed. According to it, numbers exist in all of nature, both animate and inanimate. It is closely related to the atomic theory and to the theory of space. The other form of this belief asserts that numbers, ideal straight lines, and mathematics in general, exist in the soul. Since souls are immortal, and in all people, this also ascribes an existence to numbers which transcends any single person or group of people. This may be identified with the Doctrine of Psychomathematical Parallelism. The two doctrines are not necessarily incompatible, but the attempt to reconcile them has resulted in much ingenious and inconclusive argument. The view that numbers are parts of speech, grammatical inventions, eliminates the need for such debates. Yet, these debates are historical fact, and cannot be ignored in the present investigation. Since they have included geometry as well as arithmetic, the consideration of this controversial part of the history of mathematics will be postponed. Meanwhile, it will be convenient to write as if the outcome will be favorable to the linguistic view of mathematics.

It remains to mention one historical fact. Archimedes stopped his numeral system at the third Period, but it is clear that he could have extended his system and named even larger whole numbers. This was explicitly recognized by later writers, who concluded that there is no largest whole number. This is known as Archimedes' Axiom; it has played a somewhat dramatic part in the recent history of mathematics.

How the Decimal Numbers Came to Europe

Thus far, our decimal numerals have been taken for granted, but they also have a history. We call them Arabic; the Arabs call them Persian, and the Persians call them Hindu. They were invented in India, probably before the time of Archimedes, certainly before 200 B.C. They were widely used by Persian and Arabian scholars well before 1000 A.D., during the Dark Age of Europe. These scholars had also developed algebra, much as we now know it. Decimal arithmetic and algebra were taught in the Moorish universities of Spain and North Africa. A few French scholars studied at the Spanish universities and brought this knowledge to their colleagues, but it had little impact. More significally, the Moors of North Africa established business schools and taught this arithmetic to young people. One of these was Leonardo Fibonacci, son of an Italian merchant who, with his family, spent some years in North Africa. The son also became a merchant and settled in Pisa. In 1202 A.D., he wrote an Italian version of the arithmetic he had learned from the Moors. It is not clear why he was so enthusiastic; perhaps only because the decimal numerals were different from the Roman numerals then used in Italy. It is also possible that he had not learned to keep books in Roman numerals, and therefore could not judge the relative advantages of the two systems. His manuscript was circulated and copied. His enthusiasm was contagious, and others wrote their own versions. Although the Italian Renaissance of learning and prosperity is usually dated from the fourteenth through the sixteenth centuries, the early work of Fibonacci is properly included in the Renaissance literature. It gave impetus to the mathematical and scientific studies which culminated in the work of Galileo and Newton. Fibonacci himself did original mathematical work, on the theory of population growth, but it was not of good quality.

Despite the enthusiasm of Fibonacci's followers, the new numerals met with resistance. The sober-minded recognized their disadvantages, and the advantages of the Roman numerals, which have already been mentioned in connection with the abacus. In some European countries, the decimal system was declared illegal because "0" could be changed to "6" or "9" by a single stroke. This objection could, of course, have been eliminated by changing the shape of the digits. It was, however, a simpler argument than the more fundamental one: the use of decimal numerals was more troublesome, and required more education, than the use of the Roman numerals.

Still, the enthusiasm for the novelty spread. After the introduction of moveable type in the fifteenth century, textbooks of arithmetic were bestsellers and gave needed support to the new industry. This is reminiscent of the unexpected popularity of Newman's four volumes, *The World of Mathematics*, in recent times. It is difficult to explain this wide interest in mathematics, and especially in mathematical innovations, but it is a social phenomenon that cannot be ignored. In view of the ineffective protests against the arithmetization of people's lives, it takes on a paradoxical aspect.

The author and publisher of the first of these mathematical bestsellers are not known by name. It is dated at Treviso (near Venice), December 10, 1478. The opening paragraphs, as translated by D.E. Smith follow.

> Here beginneth a Practice, very helpful to all who have to do with that commercial art commonly known as the abacus.
>
> I have often been asked by certain youths in whom I have much interest, and who look forward to mercantile pursuits, to put into writing the fundamental principles of arithmetic, commonly called the abacus. Therefore, being impelled by my affection for them, and by the value of the subject, I have to the best of my ability undertaken to satisfy them in some slight degree, to the end that their laudable desires may bear useful fruit. Therefore in the name of God I take for my subject this work in algorism, and proceed as follows:
>
> All things which have existed since the beginning of time have owed their origin to their number. Furthermore, such as now exist are subject to its laws, and therefore in all domains of knowledge is this Practica necessary....

To the professional mathematician this introduction seems confused (e.g. the equation of the material abacus with the intellectual discipline of arithmetic). To others, it may seem naive. But the frank intermingling of the practical and the religious, of creation and number, stirs the imagination. Is it also a clue to the sociological problem just mentioned? Whoever wrote it was more alert to the feelings of people than were the later scholars. The enthusiasm for the decimal numerals did not reach England until the time of Shakespeare, during the reign of Elizabeth I. Then the physician, Robert Recorde, wrote several books on arithmetic, one of which, *The Ground of Arts* went through eighteen editions, and was still popular at the time of Newton's birth (1642). It opened with an introduction entitled "The Declaration of the Profit of Arithmeticke" and treated a wide range of topics, from the use of the abacus with the new numerals to the extraction of square roots. All of this is indicative of an increase in the number of people who could read and write their native language. The invention of paper, which is cheaper than parchment or papyrus, may have contributed to this. Printing is also cheaper than handwritten manuscripts; it was introduced into England before the reign of Elizabeth's grandfather, Henry VII, about a year before the Treviso Arithmetic was written. The acceptance of the decimal system in England may also have been connected with inflation and the larger sums of money involved in commercial transactions. But this does not explain the introduction of square roots into the general curriculum of elementary schools, where it remains to this day.

This section should not be concluded without noting a recent trend away from the decimal system. Very few, if any, large electronic computers are built to operate with the decimal system. They use systems with fewer than ten digits, usually with two or eight. These are analogous to the decimal system; they do contain a digit for zero. But they all have an upper bound, usually ten or a hundred billion. This bound is usually raised by the device of multiplication by powers of two or eight, but this results in a numeral system which is still bounded and has gaps: not all numbers less than the bound can be used. It cannot be expected that any but professional users (programmers) of such machines will submit to the tedium of learning the machine's numerical system. Therefore, the machine is provided with an auxiliary device that translates the result of its calculation into the conventional decimal numerals before they are automatically typed onto the paper. This abandonment of the decimal system was deliberate; some fifteen years ago, the pros and cons of the matter were debated at length. The use of the octal (base eight) system is a compromise between the binary and the decimal. The upper bound is imposed by the nature of all artifacts, whether they be abaci, books, or electrical devices. The choice of the upper bound is to some extent free, but any increase adds to the cost of the machine.

It is not easy to foresee whether the decimal system will also be abandoned by our elementary schools. The pros and cons have not even been debated. Certainly, any proposal to abandon the decimal system will meet with as much resistance as did its original introduction. Certainly the fact that we have ten fingers will be used by opponents of such a proposal. Electronic computers have no fingers, and automated tools can be given as many fingers as needed. More importantly, it will be necessary to consider the effect of such a change on spoken language; new number-words would be needed. This might lead to even more drastic changes. Electronic computers operate with words and sentences as well as with numbers. Their rules of spelling and grammar are much simpler than those of English, but also much stricter. Errors in spelling are not tolerated, nor are metaphorical sentences. It will be well to let this sleeping dog lie.

Fractions, Equity, and Higher Education

In addition to words for whole numbers, most languages have words or phrases for "three-fifths" and "two-thirds"; these are the proper fractions. We now include the improper fractions like "eleven-thirds." The classical Greeks called them *lepta* or *logoi*, interchangeably. The Romans called them "ratios." In German, they are "broken numbers." Modern mathematicians follow the Romans and call them rational numbers. It must be emphasized that this is the original meaning of "rational"; it is now more commonly used in the sense of a sound mind. This linguistic change is closely related to the doctrine of psychomathematical parallelism. The word was originally derived from the Latin *ratus*, which meant to reckon, to keep an account. The evolution of the later meaning will be more understandable after the origin of the doctrine of psychomathematical parallelism has been found.

Terms like "two-thirds" must have been coined for use in discussing quite ordinary actions. Six melons can be shared equitably among three people by giving each two melons, provided the melons are of the same size. Five melons can be shared equitably among three people only by cutting the melons into pieces. Our children still find fractions more palatable when they are introduced while an imaginary pie is being cut.

There are no records of the origin of fractions; one can only conjecture about it, using this simple example as a guide. It involves the notion of equity, but this is abstract, and unlikely to have been recognized in any but a quarrelsome way by very early people. Quarrels are not conducive to calm soliloquies, even after they are over. The other notion is the five melons and the three people, two sets of things to be counted. Trade by barter and bargaining is known to have preceded more elaborate commerce. Bargaining is a sort of shrewd quarrel. "I will give you two coconuts for those five melons." "No, you must give me four coconuts." This might be followed by a period of aimless gossip, during which each of the traders does some reckoning, imagining what advantages would accrue from the possession of the other's goods, and the deal might be closed with a compromise: "Oh well, I'll give you three coconuts." "Done!" In some such way, two numbers, and the idea of balanced advantages, equity, must have entered into conversation, and required new words to be coined. Some of these new words are: "fraction," "numerator," and "denominator."

But oil and wine cannot be counted, and must be carried in jars, consumed in cups. Kernels of wheat can be counted, but in quantity they must be carried in sacks or jars and stored in even larger containers. Counting a useful quantity of wheat is time-consuming. Trade in such commodities would involve a new element: the purchaser would have to estimate the number of times his cup could be filled from the seller's jar. Bad bargains, discontent, and quarrels would inevitably occur. Sellers might deliberately cheat, using containers whose size was misleading. Other commodities would lend themselves more readily to weighing, to balancing in the pans of a pair of scales. Such speculations about the origin of fractions, weights, and measures can only be checked by evidence from much later times.

Even these later times are now ancient, and the societies of those times were much different than ours. Today, we make a sharp distinction between religious rituals and secular routines. This was not always so. Agricultural routines were inseparable from the rituals of the goddess of agriculture. If the spring was unreasonably cold, both the planting of corn and its accompanying religious rituals would be postponed. The appropriate days for such rituals and routines were not determined by counting days or watching the heavens. The budding of trees, the blossoming or fading of wildflowers, the ripening of fruit were the phenomena to be observed, and such observation was much more effective than any commercial calendar. In republican Rome, an annual popular election sanctified two respected and successful farmers, who then became the Aediles of the year. They had charge, not only of the Temple of Ceres, but also of announcing the day for each of her rituals, that is, the day for each important farming operation. In Latin, the word for announcement is "calend"· hence our "calendar." The Aediles also supervised the food markets, and served as magistrates to settle complaints and impose fines for cheating. It was an admirable arrangement, and served well so long as Rome was a small town, adequately supplied by nearby farms.

In pre-dynastic Egypt, before 3000 B.C., Memphis was a small village with a temple and a market place. The priests of this temple also supervised the market. We have no good word for such functionaries as the Aediles, and so "priest" must do, even though this word has given rise to a false conception of ancient history. The priests of ancient Memphis not only knew the balance (our symbol for justice) but had an established system of weights and measures. Undoubtedly, they standardized privately owned jars, cups, and weights. These would be certified by imprinting the symbol of the temple and its god, and perhaps some numerals as well. The distinction between standardization and sanctification cannot have been clear. The authority of the god would be invoked to punish cheating. The statue of a later Egyptian surveyor ("rope stretcher") has survived; his coil of measuring line is fastened by a clasp that bears the insignia of a goddess. Very likely, he had vowed to perform his duties faithfully, and the clasp was a constant reminder as well as a badge of authority.

The actual process of standardization would be a sacred trust, rather than a legal or scientific trust as it is now. One can conjecture, with fair assurance, as to the process of standardization. A large jar would be filled (or emptied) by a counted number of small cups. A set of equal weights would be prepared, equality being determined by balancing each against another; perhaps filing or grinding would be needed to bring about the balance. Thus, fractions with a unit numerator would be recognized; we call them reciprocals. Both the Egyptian and Babylonian mathematical texts give reciprocals an importance that can be taken as evidence of their antiquity. Just as very large numbers are fictions, so are their reciprocals.

However, containers and weights of intermediate sizes would also be needed. Their standardization would require two countings, and so more general fractions would be introduced. These standardized containers would be given names, and could readily be recognized. This would correspond to our gallon, quart, and pint. But since different containers would be used for wine and wheat, there would also be dry measures: our bushel and peck. These objects, the containers and weights, were thus substantial, material embodiments of fractions. Moreover, they and their proper uses would be sacred. Here one recognizes another reason for the association of divinity with mathematics, and of divinity with equity, justice. Several writers, notably A. Seidenberg, have emphasized this phenomenon. They seek the origin of counting and mathematics solely in religious ritual, ignoring the utilitarian, secular nature of much of early religion. This view of the matter is not inconsistant with that outlined above; it differs only in emphasis.

The Egyptian and Old Babylonian mathematical texts contain no explicit reference to the abstract idea of equity or justice; in fact, they are not abstract in any sense. They do, however, exhibit the solution of the problem of sharing a certain number of loaves of bread equitably among a given number of people. They solve the even more complicated problem of issuing a certain number of loaves equitably to the leaders of groups of people, each group consisting of a different number of people. Problems of the inheritance of land are also discussed, again without abstract reference to ethics. Yet this is a matter of concern to society. It places brothers into competition with each other, and injustice in the legacy of a father can cause dissention among his descendants. Moreover, a given piece of land can be made more productive if it is farmed as a whole, rather than as a number of small independent plots. Of course, it cannot be maintained that this is the only cause of internal dissention and inefficient production, but it is not a negligible one. It was one of the matters that Solon considered in framing his constitution. The Greeks were noted more for shrewdness than for honesty and justice. Homer warned against trusting them even when they came bearing gifts. Solon, the lawgiver, had the same experience as the Trojans. It should therefore not occasion surprise to find that they obscured the ethical component of fractions. Plato even objected to the use of fractions at all, as is shown by the following extracts from his *Republic*.

> For you are doubtless aware that experts in this study, if anyone attempts to cut up the one in argument, laugh at him and refuse to allow it, but if you mince it up, they multiply, always on guard lest the one should appear not as one but as a multiplicity of parts.

This does not exhibit his ethical attitude toward mathematics, and is obscure besides. It must be read in the context of another passage from the *Republic*:

> We should induce those who are to share the highest functions of state to enter upon that study of mathematics and take hold of it, not as amateurs, but to follow it until they attain to the contemplation of number by pure thought, not for the purpose of buying and selling, as if they were preparing to be merchants or hucksters, but for the uses of war and facilitating the conversion of the soul itself from the world of generation to essence and truth.

Merchants and hucksters used fractions and were vulgar, consequently fractions should not be used by the elite. But the use of mathematics in war was as noble as its use for freeing the soul from the transitory phenomena of the world. Plato himself fought in at least three campaigns and was decorated for bravery. During the whole of his long life, Athens was almost constantly at war. He could not free himself from the ancient notion that victory proved a general to be a hero, a demigod under the protection of greater gods. This is the theme of Homer's epic poems, and these were the textbooks of the Athenian nobility. This blind spot of Plato's is evident in much of his writing, and especially in his many ineffectual attempts to understand virtue. For the present, it is sufficient to note that he stripped mathematics of its ethical development, but not of its divinity.

Plato's views about fractions and mathematics in general became official with his contemporaries, and later with us. In American and British schools, the phrase "vulgar fraction" was used to distinguish 3/5 from its decimal equivalent 0.6. This custom had not disappeared in the early years of the twentieth century and the phrase can still be found in today's dictionaries. Bertrand Russell relates that as a child he calculated that he was growing four and one-seventh inches a year. He told this to his maternal grandmother, and she told him that one must not mention any fractions but halves and quarters. Yet she was certainly not overly conservative for a Victorian: she advocated votes for women years before the suffragettes were organized, and founded Girton College for Women in Oxford.

Incidentally, since the *Republic* is a work of fiction, this is a detailed illustration of the importance of fiction in shaping society. It is also a significant remnant of the aristocratic fallacy; it fails to take the actual structure of our society into account.

Plato's inability to write clearly about war and virtue can be documented with many passages from his writings. Its origin must be sought in the time and place where he lived, and it must not be supposed that he was amoral by the standards of his contemporaries. One more quotation from his writings will suffice for the present. It is the opening passage from his dialogue *Menon*; it should be prefaced by the remark that Plato's dialogues are not always reports of actual conversations. Plato put his own thoughts into the mouth of a more or less fictional character, Socrates. *Menon* opens as follows.

> MENON: Can you tell me Socrates — can virtue be taught? Or if not, does it come by practice? Or does it come neither by practice nor by teaching, but do people get it by nature, or in some other way?
>
> SOCRATES: My dear Menon, the Thessalians have always had a good name in our nation — they were always admired as good horsemen and men with full purses. Now it seems to me, we must add brains to the list. Your friend Aristoppos is a very good example, and his townsmen from Larissa. Gorgias is the man who set it all going. As soon as he got there, all the Aleuadai were at his feet — your own bosom friend Aristoppus was one — not to mention the rest of Thessaly. Here's a custom he taught you, at least — to answer generously and without fear if anyone asked you a question; quite natural, of course, when one knows the answer. Just what he did himself; he was a willing victim of the civilised world of Hellas — any Hellene might ask him anything he liked, and every mortal soul got his answer!
>
> But here, my dear Menon, it is just the opposite. There is a regular famine of brains here, and your part of the world seems to hold a monopoly on that article. At least, if you want to ask anyone here what you are asking me, all you will get is a laugh and — "My good man, you must think I am inspired! Virtue? Can it be taught? Or how does it come? Do I know that? So far from knowing whether it can be taught or can't be taught, I don't know even the least little thing about virtue, I don't even know what virtue is!"

I'm in the same fix myself, Menon. I am as poor of the article as the rest of us, and I have to blame myself that I don't know the least little thing about virtue, and when I don't know what a thing is, how can I know its quality? Take Menon, for example: if someone doesn't know in the least who Menon is, how can he know whether Menon is handsome or rich or even a gentleman, or perhaps just the opposite? Do you think he can?

MENON: Not I. But look here, Socrates, don't *you* really know what virtue is? Are we to give that report of you in Larissa?

SOCRATES: Just so, my friend, and more — I never met anyone who did, so far as I know.

MENON: What! Did you not meet Gorgias when he was here?

SOCRATES: Oh, yes.

MENON: Didn't you think *he* knew?

SOCRATES: I have a rather poor memory, Menon, so I can't say at the moment whether I did think so. But perhaps he did know, or perhaps you know what he said; kindly remind me, then, what he did say. You say it yourself, if you like; for I suppose you think as he thought.

MENON: Oh, yes.

SOCRATES: Then let us leave him out of it, since he is not here; tell me yourself, in heaven's name, Menon, what do you say virtue is? Tell me, and don't grudge it; it will be the luckiest lie I ever told if it turns out that you know and Gorgias knew, and I went and said I never met anyone who did know.

Socrates' crude and irrelevant response to Menon's modest and serious question is disconcerting to the modern reader. It is a gratuitous attack on a man named Gorgias. One must first ask who Gorgias was, and what he had done to gain Plato's (or Socrates') ill will. Gorgias was a native of Leontini, a city near Syracuse, but then independent of that city's rulers. In his younger days, Gorgias took an active part in his city's affairs, and was repeatedly elected to high offices. In the year of Plato's birth, 428 B.C., he was sent to Athens as an ambassador from Leontini. Years later, when Plato was a young man and Athens was deeply involved in its disastrous war with Sparta, Gorgias attended the Olympic Festival. This was always the occasion for a sacred truce, and all of Greece was represented. Gorgias entered the oratorical contest and was awarded the prize. In his oration he urged that Athens and Sparta make peace, and predicted disaster for all Greece if they did not. Since Plato and his family were in the pro-war faction at Athens, this would have been sufficient reason for Plato's enmity.

Gorgias lived to be a very old man, and on retiring from active participation in public affairs, became a sophist. The sophists were, more or less, the successors to the Homeric bards. Many of them were itinerant, giving lectures and instruction for a price. Sometimes this price was a day's room and board. Sometimes they settled down as tutors to the sons of some wealthy family, or even opened a school where they charged tuition. Some of them, very likely, were charlatans, but others, like Gorgias, were responsible, honest teachers. Gorgias' well-deserved fame enabled him to charge high fees for his services. This was another point against him in Plato's view: Gorgias was reducing knowledge to the level of a commercial commodity. Paradoxically, many winners in Olympic athletic contests retired to become coaches and charged fees; Plato was a famous wrestler, and was most likely paid for coaching in that art. If so, he considered that a different matter; perhaps he was paid with presents rather than with money. Plato's family, leaders of the nobility and the pro-war party, traced all of Athen's misfortunes to Pericles' establishment of salaries for public service. Money and commerce were vulgar, and associated with their opponents.

Furthermore, Gorgias and many other sophists taught rhetoric. Then, as now, rhetoric included both grammar and the art of persuading people to cooperate in achieving a common goal. It is better to settle political differences with words rather than swords. Gorgias, had he been an Athenian citizen, would have been in the anti-war party, and so would many of his colleagues. Thus Plato could not accept Gorgias without disowning his own family and the glory of war.

Toward the end of his long life (some say he lived to be a hundred), Gorgias retired to Thessaly and calmly spent his savings enjoying life. He may have chosen Thessaly not only for its climate, but because it was peaceful compared to Athens or even

Sicily. In Thessaly, he was welcomed by the Aleuadai, the nobility, of whom Menon was one. Socrates' diatribe was therefore a personal affront to Menon. In eighteenth-century France or England, it would have provoked a duel. This is one reason for believing that the dialogue is fictional. Menon was a proud man, a general in the cavalry; according to Xenophon, who knew him well, he was even haughty and overbearing. He would most likely have terminated the conversation before Socrates had finished the first paragraph. Against the hypothesis that Plato's dialogues are mostly fiction, one may cite the fact that many of their characters are, like Menon, demonstrably real people, whose lives are known to historians. But this is also true of the Athenian dramas of that time, and especially of Aristophanes' comedies. It must be remembered that Solon's law against libel was not being enforced, though it had never been repealed. When Aristophanes lampooned Socrates in a most savage manner, it is said that the latter could only defend himself by standing up in the theater, smiling and applauding. There was thus sufficient reason for Plato's animosity toward Gorgias and the Thessalians, and no reason why he should not express it in writing.

The Decimal Point and Long Division

The Akkadian numeral system was ambiguous for two reasons: the lack of a zero, and the lack of a decimal point. The decimal system, as originally introduced into Europe, was not ambiguous because it had the digit "zero," But it had no decimal point; fractions were treated in the vulgar form of numerator and denominator. This seems to have displeased some people. In 1585, Simon Stevin, a Dutch public official, engineer, and mathematician, invented the decimal fractions. His book taught how all "Computations met in Business may be performed by Whole Numbers alone without Fractions." This is reminiscent of Plato's rejection of fractions as vulgar, but the dedication is not consistant with this. It reads: "To Astrologers, Surveyors, Measurers of Tapestry, Gaugers, Stereometers in General, Mint-Masters, and to all Merchants, Simon Stevin sends Greeting." Stevin did not actually use the decimal point, but that is immaterial. More importantly, he overlooked something that even the Old Babylonians might have noticed, had they invented a systematic method of long division rather than relying so much on reciprocals.

One has $3/5=0.6$, but $2/3=0.66666...$, where the dots indicate an endless succession of 6's. It is impossible to speak or write an endless succession, but the more 6's one does write, the closer the approximation to $2/3$. It did make it possible to add fractions without reducing them to a common denominator: $3/5+2/3=1.26666...$ One can invent a notation that avoids the endless succession: $2/3=0.(6)$ the parentheses indicating that the approximation will be better, the more 6's one writes. Another example is $105/37=2.(837)$ and the approximation will be better, the more times one repeats the three digits, 837, without changing their order. In the same way, one finds that $\frac{1}{11}=0.(09)$ and $\frac{1}{101}=0.(0099)$.

It is seen that the vulgar fractions are of two kinds. Fractions like 3/5, whose denominator goes into some power of 10 without a remainder, are equal to decimal fractions that terminate. Fractions whose denominators do not go into any power of ten without a remainder are equal to repeating or periodic decimals. The system of decimal fractions obscures the fact that a fraction involves two whole numbers. Instead it brings with it the fiction of an endless succession, infinity.

Consider our method of division with the following example: 201848/511=395 remainder 3. Let n stand for one of the digits 0 through 9. Then the complete form of the soliloquy begins: "If $nx511$ is less than 2018, and $(n+1)x511$ is greater than 2018, then the first digit in the quotient is n." A decision is therefore involved. It can be made by trying the integers 0 through 9 in succession until the one satisfying the condition is found. Egyptian multiplication involved similar decisions.

These are real decisions, but of a special kind. They are very different from the decision confronting a manufacturer, whose business seems to be outgrowing his plant. If he borrows money to expand his plant, it will be some years before he can repay the loan. In that time, business may decrease and he may be left with a debt that he cannot repay. If he does not expand his plant, and his business continues to grow, he will have a constantly increasing backlog of unfilled orders; there will be delays, and he will lose customers and the profit he could make by supplying them.

Nevertheless, it has become customary to use the same word "decision" for both kinds, even though the soliloquies involved are very different. This is a consequence of the fallacy that the soliloquy of arithmetic is childish, not worth considering. To avoid it, let the decisions involved in division be called "no-risk decisions." For no risk is involved, other than that of making a mistake. The proper decision can always be found; it is determined by knowledge already present. This is not the case in the manufacturer's decision. He must make it before those events have occurred which will show it to be right or wrong.

It is possible to build mechanisms that will do long division, including all of the necessary no-risk decisions. These need not be expensive electronic computers; the much cheaper calculator will do. Moreover, it will do long division (as well as addition, subtraction, and multiplication) much faster than most people, and make fewer mistakes.

The hope that it may be possible to convert the risky decisions into no-risk decisions is a very ancient one. It has led to the various systems of omens, auguries, horoscopes, and more recently to mathematical models of economics and sociology. With the advent of electronic computers, it was hoped that these mechanisms would make this ancient wish come true. This hope has not been realized. One corporation that manufactures computers advertises explicitly that its machines do not have this capability. It is better to advertise this fact, than to have its public image damaged by allowing its customers to discover this at their own expense. Computers do not soliloquize it in the same way as businessmen, statesmen, or politicians.

Computers can be made to analyze public opinion polls, sometimes with some success. However, there is a modern fable about the two ways to determine the President's waist measurement: the one is to hold a public opinion poll, and the other is to inquire of his tailor.

Epidemics, Square Roots, and Elementary Education

Having arrived at the notion of a decimal number whose digits repeat in endless periods, it is said to be easy to imagine numbers with an endless but non-repetitious succession of digits to the right of the decimal point. Such numbers are called "irrational"; it might have been better to call them "aperiodic" or "non-repititious"; they are fictions, of course. And without methods for calculating at least the early members of the disorderly succession of digits, they would be completely useless fictions. Since about the seventeenth century, much of the work of professional mathematicians has been the invention of such methods. What motivated all of this work? A complete answer would require a detailed and technical history of mathematics, extending back in time far before Stevin, and even requiring conjectures as to events of which there are no traditions, much less records.

There are reasons (which will be discussed later) for believing that this history began with the following geometic problem: Given a square whose side has the length a, find another whose area is twice that of the first. In other words, find b of this doubled square, so that $b^2=2a^2$ Seidenberg and others have shown that this problem was most likely proposed by ancient theologians rather than mathematicians. Doubling the area of an altar was supposed to be a public health measure, causing the gods to end an epidemic. In modern notation, $b/a=\sqrt{2}$, so that the problem reduces to the calculation of $\sqrt{2}$. To this point the historical evidence is quite convincing; but at some completely unknown time and place, someone found that a and b cannot both be whole numbers: b/a is not a vulgar fraction. Naturally, it is also not known how this discovery was made. Initially it was formulated by saying that the sides a and b are incommensurable, for the general notion of irrational numbers was still far in the future. After this gap in the history, we can assign names and dates, provided that Plato's dialogue *Theaetetus* is factual history. F.M. Cornford, who has carefully studied this dialogue, believes it to be pure fiction. Nevertheless, the following discussion will proceed as if it were factual. The Pythagorean distinction between square and oblong numbers has been described. According to Plato, the geometer, Theodorus of Cyrene, in the year 399 B.C., was able to demonstrate that the square roots of the oblong numbers 3,5,...,15,17 are irrational. He taught this to the young Athenian, Theaetetus, who matured into a very able geometer. Recent studies by Eva Sachs suggest that Theaetetus then worked out a complete geometric treatment of incommensurable lengths. Since Plato fails to mention 2 in this list of oblong numbers, it is thought that the irrationality of $\sqrt{2}$ was well known in 399 B.C., and that Theodorus used this fact in his demonstration.

Lacking information about the ancient method of showing that $\sqrt{2}$ is not a rational fraction, the modern algebraic method will be presented; it consists of showing that the assumption that $\sqrt{2}$ is rational leads to a contradiction. Suppose that n and d are whole numbers, and that $\sqrt{2}=n/d$, or, which is the same, that $2d^2=n^2$; d is not 1, otherwise 2 would be a square number, not oblong. It may also be supposed that n and d are not both even, otherwise a factor of 2 could be canceled, reducing the fraction n/d to lower terms. The second form of the equation then shows that n must be even, d odd. Therefore let $n=2m$, m being a whole number, so that the equation becomes $d^2=2m^2$ after canceling a factor of 2 on both sides. But then, d is shown to be even as well as odd, a contradiction. The supposition that $\sqrt{2}$ is rational is therefore false. This method can be used to show the non-rationality of any oblong number, provided that one has learned to cancel common factor from numerator and denominator.

Having learned that $\sqrt{2}$ is not a rational number, we may guess that it is irrational. To demonstrate this, it is sufficient to find a method that enables one to calculate as many of its early digits as one has patience. This cannot lead to a repeating decimal, otherwise $\sqrt{2}$ would be a rational number. The simplest method for this purpose is based on the observation that, if a is any rational number such that $a^2>2$, and if $b=2/a$, then $b^2<2$. One may say that a and b bracket $\sqrt{2}$ and that $a-b$ is a measure of the closeness of the bracket. Consequently, the average of a and b, or $(a+b)/2=a'$ will be closer to $\sqrt{2}$ than it is to a; a' will also be greater than $\sqrt{2}$ so that it and $b'=2/a'$ will bracket $\sqrt{2}$ more closely than a and b.

Bracketing contains material of historical interest. The Indian mathematicians, who wrote in Sanskrit sometime during the first millenium B.C., knew that 577/408 was an approximation to $\sqrt{2}$. It is disappointing to find that they do not say how they found this, or whether they knew that $\sqrt{2}$ was irrational. An apparently much earlier cuneiform text suggests to D. Neugebauer that the Old Babylonians attempted to approximate the fraction 577/408 as a sexagesimal numeral, but did not quite succeed. The later cuneiform texts of the Seleucid period show that the approximation 17/12 was well known; but again, they contain no explanation of the way in which it was calculated. One regrets the taciturnity of the writers, but most early mathematicians were more interested in the result than in the method by which it was obtained. This is almost the exact opposite of the attitude of modern mathematicians, who therefore tend to disparage their ancient predecessors.

Even if this method of extracting square roots was known and used in early times, it gives no clue as to the reason for treating $\sqrt{2}$ differently from other square roots. The method can easily be modified to obtain any other roots. For $\sqrt{3}$, one need only replace $b=2/a$ by $b=3/a$. Again, bracketing is of interest because Archimedes published 1351/780 as an approximation to $\sqrt{3}$, but he also published the approximation $\sqrt{3}=265/153$. It can be shown that this second fraction cannot have been obtained using this method, no matter what initial value of a is used.

There are several methods of calculating square roots, of which this is the simplest, not only to use but to explain as well. In the United States, elementary schools teach their pupils a different method, one that is much more difficult to understand and use. They do not teach the history of the problem, especially not its origin in a superstition. The difference between rational and irrational numbers is not explained, only the difference between vulgar and decimal fractions. This educational policy is not easily understood. No adult (other than elementary school teachers) remembers this method of finding a square root. Most adults never have a need to find a square root, and those engineers and scientists who do have this need use logarithms. This instruction does not provide the pupils with any useful knowledge, nor does it give them an example of good reasoning. It is another example of the burden which the dead hand of the past places on our education, a burden that could easily be lifted by an understanding of its history.

The Perception of Space

When we ask "How far is it?," we expect an answer like "three blocks" or "ten miles." It is true that city blocks can be counted, but originally there were no cities. We ask "How large is this room?" and receive the reply "23 by 31 feet"; after some arithmetic we remark "That's 713 square feet; it will need about 80 square yards of carpet." But originally there were no rooms and no carpet; caves are not rectangular. In our society, space and arithmetic have become so intimately fused that it is difficult to separate them. It cannot always have been so. There must have been a time when the question "How far did you go?" could receive only some answer like "To the other side of the forest." It cannot be hoped that the people of those times will have left records for archaeologists to find. The earliest Mesopotamian and Egyptian mathematical texts are quite late, and show that their authors already had made calculations like the one described above. They already knew that the area of a rectangle was the product of the lengths of its sides, and the area of a triangle was the product of base times altitude divided by two. In the Punjab, the cities whose ruins are now called Moenjo-daro and Harrapa were large, and built of kiln-fired brick about 2000 B.C. Their houses were arranged in rectangular blocks, seperated by streets as wide and straight as ours. Their inhabitants may therefore also have associated numbers and arithmetic with space. Incidentally, these cities conformed to standards of sanitation that have only recently been equalled. Such cities of geometric design, and constructed of good bricks, were preceded by others of less regular form, and numerical geometry must have developed slowly. The perception of space is not intrinsically numerical, is not a matter of counting.

One opportunity for obtaining information about primitive ideas of space has been missed. Quite recently, there were still societies whose people did not build cities, and did not build houses of brick or cut stone, or even of sawn wood. But the Europeans who first encountered these people were more interested in inculcating them with European ways of thinking, than in finding out how others thought. Even modern anthropologists are not properly educated to understand primitive ideas about space. There is only one possibility of understanding the evolution of modern geometric thinking, and that is to reconstruct it imaginatively, after understanding the psychology and physiology of space perception, and using whatever evidence archaeology can provide. It is reasonable to suppose that human physiology and psychology have not changed greatly during the last ten thousand years.

One may begin with the physiology of space perception, although our knowledge of it is far from complete. The eyes are the sense organs that furnish much of our information about space-filling objects, but touch and bodily motion also contribute essentially. Sensations arising in the muscles that move the eyeballs certainly contribute. The physics of the eyes is well understood. The transparent parts of the eye, (its cornea, lens, and fluids) cast a two-dimensional image onto the retina. This occurs according to the same physical laws by which the lens of a camera produces a photographic image, the same laws that Abbe worked so long to elucidate. Even a photo is a distorted image, a perspective view, but the image on the retina is even more distorted. The image of a straight line is a curve, not a straight line. Moreover, the eyes of most people are not identical, and even if they were, the curves on the two retinas would not be the same. Few people have eyes that are identical to each other: their retinal images are often of different sizes. The image produced by the most perfect eye has worse errors than Zeiss' first microscope. We are not aware of all this, nor even that the images are upside-down. Somehow, the two curved images of a straight line are interpreted as a single straight line, and this interpretation is made without awareness. There is no soliloquy as in the case of counting. While physiologists and anatomists can give no detailed explanation of this unconscious interpretation, it is certain that it occurs partly in the optic nerves which connect the retinas to the brain, and partly in that part of the brain where these nerves terminate. After the interpretation has been made, it is transmitted to another part of the brain, probably the cortex. It is this interpreted image of which we are aware, and about which we can speak and soliloquize.

During birth, the eyelids of a human infant are tightly closed, sometimes even overlapping. Despite this, the eyes (as well as other parts of the body) are often injured. Normally, after a few days, the infant's eyes, though small, are as perfect as they ever will be. At this early stage, the infant can use its eyes to some extent. They will follow a large white ball, moved slowly against a contrasting background, but if the ball is bounced, or if the background does not contrast greatly, the child's eyes do not follow the ball. Despite this, the infant does not have the "ability to see." This is learned before the ability to speak, and seems to be connected with motions of the eyes, arms, hands, legs, and feet. Very likely, memories of simultaneous visual, tactile, and muscular sensations contribute to this learning process, but they are not memories that the child will be able to recall in later years. Despite all the study that has been devoted to these problems, and despite the technology of making spectacles for correcting "vision defects," this learning process is not well understood and is usually ignored. People who have had an eye injury or operation are conscious of having to "learn to see again"; even the adjustment to the use of bifocal spectacles provides one with some notion of the problems that a baby must solve.

They are not much different than learning to walk, although the infant's behavior while learning to see attracts less attention from adults. In later years, the child will not be able to recall memories of any of these experiences, whether of learning to walk, speak, or see. These unrecallable, unverbalizable memories of very early events in a child's life do, however, influence its later life and abilities. Other aspects of what we call sight are learned much later, after the child has learned to speak. A child must have picture books explained to him. Some people can remember their childish bewilderment when first shown a photograph of an unfamiliar object, and their frustration while trying to communicate their bewilderment to adults. In 1915, Roy Chapman Andrews visited an isolated tribe living on the steep and narrow watershed between the upper Yangtze and Mekong Rivers. They were unable to recognize themselves on photographs until after instruction. Animals seem quite unable to understand pictures. A dog does not react to a picture of itself, or even to its image when a large mirror is placed before it. Some aspects of vision are thus uniquely human, and depend on the location of the eyes in the front, rather than the sides, of the head.

There are also unrecallable, unverbalizable aspects to learning to walk. We say "I'll walk to the store." We do not say "I will periodically contract and elongate my right ---- and ---- muscles, synchronously elongating my left ---- and ---- muscles, thus moving my ---- and ---- bones so that my body will be propelled forward until it reaches the store." The blanks in this sentence stand for Latin names that anatomists have conferred on parts of the human body. Only an orthopedic surgeon could fill the blanks in properly, and he would probably wish to amplify the sentence into several pages. All of this is recorded somewhere in our brain and body, but not so that we can recall and speak about it. Moreover, if we repeatedly walk to the store over the same route, we will not make exactly the same motions each time. This has been demonstrated by slow-motion movies. This is also true of animals. The human ability to speak and soliliquize is of no help in walking, but the variability in the motions is significant. It is their variability of behavior that distinguishes people and animals from machines.

Knowing all this, one should not be surprised to find that the Greek philosophers, who were apparently the first to think about the matter, sometimes reached the conclusion that vision and other sensory perceptions are all deceptive, illusory. Even today, writers find it difficult to give a precise definition of a visual illusion as distinct from a true perception. Many books have been written on the subject. The scientific study of the psychology of space perception, of that interpretive process that does involve speech and soliloquy, was initiated by George Berkeley. He was remarkable for the variety of his achievements and is remembered as a mathematician, psychologist, educator, philosopher, humanitarian, and clergyman. His criticism of Newton's calculus has been accepted as valid by mathematicians. As an educator he is remembered at Yale and Harvard; Berkley, California, honors him by misspelling his name. As a clergyman, he became the Bishop of Cloyne, in Ireland. He spent some time with the Indians of Rhode Island, and seems to have learned enough from them so that he abandoned an ambitious missionary project. His philosophy and psychology are closely related; this is illustrated by two of his epigrams: "To be is to be perceived"; "The corporeal is the sensory." The fundamental importance he assigned to the perceiver, the observer, caused his philosophy to be rejected by both idealists and materialists. Remarkably, it has strongly influenced modern theories of atomic physics. He published *An Essay Towards A New Theory of Vision* in 1709, and elaborated it in several later books. His *Alciphron, orThe Minute Philosopher* (1732) is a series of dialogues in which he presents his considered views on nature and religion. The following extract formulates the problem of space perception as he considered it.

> *Euphranor*: Tell me, Alciphron, can you discern the doors, windows, and battlements of that castle?
>
> *Alciphron*: I cannot. At this distance it seems only a small round tower.
>
> *Euphranor*: But I, who have been at it, know that it is no small round tower, but a large square building with battlements and turrets, which it seems you do not see.

The first two sentences refer to the geometric problem of perspective, with which we are all familiar. It is said that Anaxagoras and Democritus solved this problem of graphic representation about 450 B.C. The painters of murals in the buried city of Pompeii understood it. The paintings of the Italian Renaissance, and more recently, photographs, have made it commonplace. It is to be noted that the laws of perspective are actually rules for the production of two-dimensional artifacts that give some impression of three-dimensionality. Historically, it is a rather late invention. Berkeley's important addition to this mathematical theory is contained in the third sentence.

The first time Euphranor walked through the woods and up the hill to the castle, he did not know that it was large and square. He gained this knowledge by walking around it, and perhaps by entering it. Our knowledge of a space-filling object does not consist of a single perspective. It is the memory of an orderly succession of perspectives, seen as we moved about it, or moved it about in our hands. Euphanor's words "large square building" are a concise and abbreviated description of such a memory. Such phrases are not much different from the phrase "a dozen apples"; this also describes a memory of counting apples, of a succession of visual perspectives and other bodily sensations.

A slightly different situation is encountered when one sees a ladle in a bowl of clear liquid. Its handle is apparently bent sharply at the liquid's surface. By "apparently" it is meant that the succession of perspectives, as one walks around the bowl, is not the same succession that would occur if the handle were really bent. Moreover, in order to straighten the apparently bent handle, one need only lift it out of the liquid; if the handle were really bent, the problem would be more difficult. Here are perspectives of another kind, memories of muscular rather than visual sensations. This theory, that shape is an orderly successions of perspectives, was used by Edgar Allan Poe to infer that a conjurer's trick was accomplished with mirrors. It was used by many painters and draftsmen, from earliest times to Picasso. The Egyptians habitually showed the face and feet in profile, with a front view of the torso. Some very early and some very recent painters show both right and left profiles of the head. A papyrus from the XVIII Dynasty (about 1600 to 1300 B.C.) shows that an Egyptian architect drew the front and side views of a shrine to guide his workmen; he anticipated Mongé's descriptive geometry, which has already been mentioned. This is nothing else than the representation of an object by three or more views. And these views are stylized perspective drawings. This method is still in use today. For use in designing complicated shapes, like automobile bodies, it has been computerized. The computer is given three or four views of the proposed design. From these, it generates a "Walt Disney" movie, which simulates the succession of views that would be seen by a person walking around it or looking down from a balcony. In this way, the designer can judge the aesthetic qualities of his proposed object. If the object has become familiar, or has a familiar shape, a single perspective view will evoke a memory of the whole succession. The shape is recognized at a glance. In fact, a simple word may evoke the memory. A single photograph is even more successful; Dr. Stanley Milgram has recently conducted an extensive program establishing these psychological facts beyond doubt.

Space-words, like number-words, serve to summarize thoughtful action and succession. And, as cannot be overemphasized, thinking is a soliloquy in which words play an important, but too often overlooked, part. Euphranor's space-phrase "large square building" does not designate a single visual sensation, a single perspective. It summarizes his memory of a thoughtful walk, to and around the castle and back home. He will have seen and soliloquized about the windows, doors, battlements, stonework, etc. He will have experienced the pleasure of walking; if he did not go alone, he will have spoken with others, the soliliquy would be replaced by conversation. In Alciphron, the space-phrase "square building" will evoke other, but similar memories; possibly he went to school in a small square building. This difference is not relevant to the communication he receives from Euphanor: it is the common element in their memories that is communicated.

If knowledge of a space-filling object is so closely connected with language, it must depend on the language one speaks. We are not aware of this, because all modern European languages have a common heritage from Greek and Latin. It is only when we come into contact with people who do not have this heritage, that we become aware of it. And even then, our awareness is not one of understanding but of bewilderment. Of course, this also applies to other kinds of knowledge. This confusion has been experienced by many Europeans who have been engaged in teaching non-Europeans. Berkeley must have been aware of it from his experience with the Indians of Rhode Island. Silvia Townsend Warner has illustrated this mutual bewilderment by an amusing story of the well-intentioned efforts of an Englishman trying to teach geometry to a friend who had lived all his life on an isolated island in the South Seas.

It is also important to notice that Euphranor did not open his conversation with a question about the natural landscape, but with one about an artifact -- the castle. Artifacts and languages change with time. Shakespeare allows us to hear not only the conversations, but the soliloquies of his characters. None of them speak of cameras or airplanes. Most words are coined during conversations, and one does not usually speak about objects that do not exist and whose construction is not even planned. One may write fiction about them, or tell tall tales about them, but even the exercises of the imagination are usually kept within bounds imposed by the credulity of the audience. If Shakespeare had conceived a spy story that involved a camera and an airplane, it could not have been presented to an Elizabethan audience. He would have had to convert the plane and camera into a talking eagle. As has been noted, the phrase "tune in on a broadcast" was meaningless sixty or seventy years ago.

Two cavemen could not have carried on a conversation like that between Alciphron and Euphanor; they had never seen a building. It must be concluded that space-words, like number-words, were coined as society and technology became more complex. We can only speculate about the first time the most common words of our languages were used, and it may be that they evolved slowly and gradually. The invention of new artifacts was not as sudden then as now. There are some limitations on the exercise of the imagination. The most important is our knowledge of the way in which relatively new words like *bus* and *phone* were coined in connection with hitherto unknown artifacts. Some ancient artifacts have survived, and sometimes their chronology can be fixed with more or less certainty. It can scarcely be doubted that their invention was paralleled by the invention of new words, not only for the artifacts themselves, but for their components and methods of construction. These new words would not be coined according to deliberate systematic rules, as are those of recent artificial languages, invented for computers, or for other scientific purposes. But the vocabulary of even the natural languages grew according to the same principle: no word is coined until it is needed.

The Rules of the Cord

The primeval landscape has few examples of straight objects; reeds and the stalks of some grasses are about the only ones. Puddles in sheltered locations have flat surfaces. These would not immediately be conversation pieces, leading to the coining of words like "straight" and "flat." It is possible that they were not even seen as much different from the leaves and branches of trees, or the surface of a stone. We provide our infants with blocks that have flat surfaces and straight edges; we raise them in nurseries with flat floors and walls that intersect in straight corners. Ancient infants had none of these, and presumably heard no words to direct their attention to straightness and flatness. Writers like James Fenimore Cooper, who knew the American Indians, do not tell us of their ability to draw straight lines. Instead, we read of their ability to track an animal by interpreting phenomena (bruised leaves, displaced pebbles) that the European eye does not see. Presumably, their languages had words for these phenomena. It is known that the Eskimos have many words for snow that we can translate only as phrases: "soft snow falling through the air"; "freshly fallen fine hard snow"; "snow with a hard crust"; and so on.

Hunters would find that spears with straight hafts are more effective than those with crooked hafts. The invention of the bow and arrow would place more value on straightness, and the taut cord of the bow would provide a standard of straightness. The loom would also require taut parallel strings, and produce more or less rectangular mats and pieces of cloth. The necessary words would evolve simultaneously with the inventions, and most likely, the ability to see such shapes as well. There is ample evidence for the importance of the taut cord in the construction of more elaborate geometrical objects, as well as in weaving and similar technologies.

A taut cord can serve as a standard of length. By marking the position of both ends and repeating the process, lengths equal to whole numbers of units can be measured. By folding the cord, fractions of a unit are obtained. The subdivisions can be marked either with knots or with spots of dye. Throughout the long history of Egypt, inscriptions and murals indicate that land was surveyed by "rope stretchers," but there are few details. Subdivisions of the rope are rarely indicated. Straight wooden rods with clearly marked subdivisions were used for associating numbers with smaller lengths. This raises a question: how were the straight rods constructed? The branches of trees are not straight; the rods as well as the cords were artifacts. When the Egyptians wished, they could work to remarkable precision. The base of the Great Pyramid was a square to better than one part in a thousand, but many later pyramids are far from square, or even rectangular. Egyptian masons (and presumably woodworkers as well) are also known to have used cords. Apart from a simple cord, they used an ingenious device that has come to be known as "boning rods." This consisted of three rods of equal length (about six inches), two of which were provided with diagonal holes through which a cord was passed. When an irregular block of stone was to be given a flat face, three holes, *a*, *b*, and *c*, were drilled, as shown by *A*, whose bottoms determined the final plane. A fourth hole was then started between *b* and *c*, and deepened until the stretched cord just touched the top of the third boning rod as shown by *B*. Its bottom, and those of *a*, *b*, and *c* would then be in the same plane. A fifth hole, with its bottom in the desired plane, could then be drilled, and so on. When a sufficient number of such holes had been drilled, the intervening material was broken away, leaving a more nearly flat surface. The final dressing was done by hand grinding and polishing until the whole length of a stretched string would touch the surface at all its points, no matter in what direction it was laid on the surface.

Once a plane surface had been fashioned, the cords had other uses. Coated with clay or ochre, stretched between two points on the surface and then plucked, a good straight line would be marked, which could be used to guide further work. According to V.G. Childe, lines drawn in this way have been preserved on a bitumen floor at Erech in Mesopotamia, dated about 3500 B.C. Looped around a fixed peg, the free end of the cord would describe a circle. There is abundant evidence that geometry developed in this way. Although the potter's wheel also produced circular artifacts, it did not produce planes of straight lines.

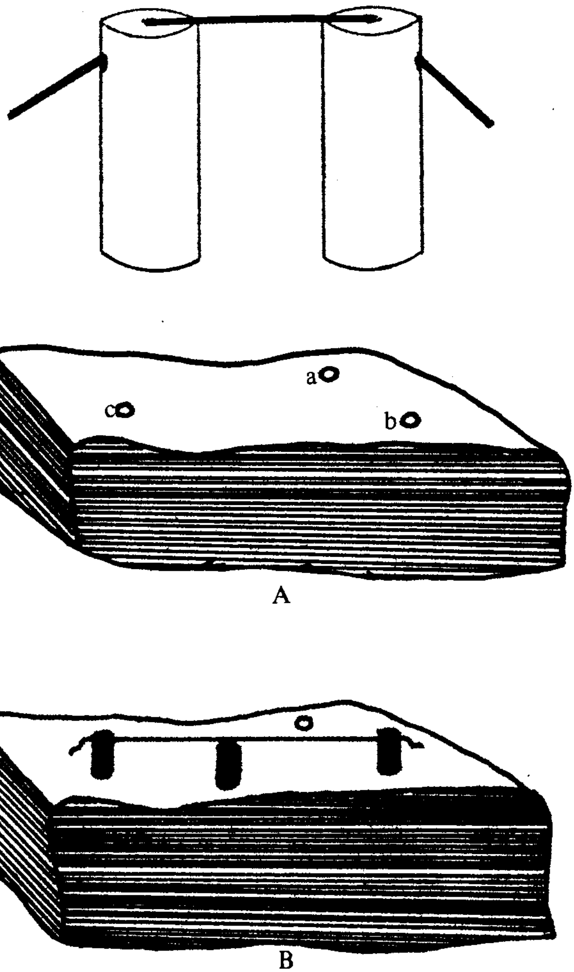


FIGURE 5
Boning Rods

Egyptian methods of stone-cutting were laborious, time-consuming, and required much skill. This was partly because of the innate conservatism of the society, and partly because all metals had to be imported. Copper was most plentiful but it is soft. It is known that soft metals, when used with abrasives, can be used to drill the hardest stone. Much of the rest of the mason's work was done with stone and wood implements. Consequently, everything was sacrificed to reduce the amount of stonecutting to a minimum. This accounts for the large, often huge, and odd-shaped blocks that they used. Their architectural styles are much admired but their craftsmanship deserves at least as much admiration and study. The unfinished blocks were transported from the quarry to the building site. There the masons selected them so as to minimize the stonecutting, making rectangular faces only when the architectural design required it. The masons thus had discretionary authority that contrasts strongly with the static rules which governed most Egyptian activities, including other forms of art. It seems to have prevented the recording of a systematic geometry. Yet, when occasion demanded, quite complicated geometric problems were solved. The accurate fit of the slanting faces shows that the masons knew how to construct parallel planes; a minimum of mortar was used. The exposed faces of the walls were not finished until all the blocks were in place, and then they were plastered if they were to have painted murals. It is known that murals and bas-reliefs were transferred from sketches to the walls by the still-used used method of coordinate squares. It is also known that on at least one occasion, an architect specified a curve by giving the numerical Cartesian coordinates of some of its points. This survived because the sketch, with its numerical coordinates, was drawn on the flat face of a stone chip. As was mentioned earlier, a tattered papyrus from the XVIII Dynasty shows front and side elevations of a shrine, superimposed on an accurately drawn coordinate grid.

This account of the geometric knowledge of the Egyptian craftsmen is based entirely on the book written by Clarke and Engelbach, which acknowledges earlier work by Flinders Petrie. In the jargon of the 1960s laboratories: this research was bootlegged. Our society is liberal in its support of the excavation of royal tombs and the restoration of temples. These activities provide golden trinkets for display in museums, and art photos that can be reproduced in four colors by historians of architecture. Colorless but careful studies of Egyptian technology do not lend themselves to such exploitation; the worn-out tools of masons and slaves are stored in the museum cellars, awaiting someone to study them.

There seem to be no studies of the masonry of other societies, but extenuating circumstances can be found. The Egyptians were very conservative; changes in method were not great, and occurred only slowly. In other societies, changes were more rapid and sometimes abrupt. This increases the difficulty of studies like those of Clarke and Englebach. It must also be said that archaeologists have made similar and much more extensive studies of pottery, and used the results in many ingenious and informative ways. One is disappointed, but not too surprised, to find that the possibilities of masonry (stone or brick) have not been exploited in the same way as have those of pottery. Petrie, Clarke, and Englebach have thus provided us with good evidence that the Egyptian craftsmen possessed quite advanced geometrical knowledge, and there must have been words to express it. Unfortunately, they were not recorded in any systematic form. The mathematical texts of the XII Dynasty are of little help in this respect. They contain only the memoranda of specific calculations, and technical terms are not clearly defined. It seems that their writers tacitly supposed that the reader would understand them without words. These must have been in current use, or else were explained orally. Many were probably like our word, "pig." Shakespeare would never have expected this to come to mean a chunk of cast iron. This leads many to follow Plato and say that the mathematics of the Egyptians was not a science. Yet, long before Plato, they knew how to calculate the areas of triangles and rectangles, the volume of granaries of various shapes, including cylindrical. For this last, they used the value

$$\pi = (16/9)^2 \doteq 3.16$$

which is not a bad approximation. The volume of the frustum of a pyramid was important for Egyptian planning, and they made the calculation correctly. The derivation of the general formula is not given; if the texts are handbooks, used for reference, this would not be expected. Yet, if and when they were used for teaching, all the necessary explanatory language must have evolved. The absence of later texts leads one to suppose that the knowledge was transmitted orally, and in words that could be easily understood and memorized. In the present unsatisfactory state of Egyptology, it is useless to speculate about this in any detail.

There are several generalities that do emerge, however. Neither the texts, nor the material evidence indicate a close connection between Egyptian theology on the one hand, and this arithmetic and geometry on the other. If there had been, the masons would not have been allowed to exercise so much discretion. It is true that there are murals showing pharaohs "stretching rope" to lay out the foundations of a temple. The accompanying inscription indicates that, while doing this, the pharaoh kept a goddess informed of his actions, assuring her of his strict adherence to procedural rules. But it is very likely that these were ceremonies similar to our modern cornerstone layings. The pharaoh probably had little real knowledge or skill, and the actual work was most likely done by craftsmen. Today the masons, who make sure that the cornerstone has the proper shape and is properly laid, do not appear in the news photos, nor do they participate in the ceremonies.

There is evidence that the cord was also used by the Mesopotamian builders. At Ur, three bas-reliefs were found on the walls of the Ziggurat (temple-pyramid), one showing the king receiving the coiled rope and measuring rod from the god, symbolizing a divine command to build the temple. A second shows the king before the god, carrying trowel and mortar-basket. A third is fragmentary but more realistic: it shows the bricklayers climbing ladders and doing the actual work. This kind of connection between religion, architecture, and mathematics is very tenuous.

The early use of elementary Mongé geometry is striking. For one thing, this indicates its fundamental character. For another, it shows that written records cannot be trusted to indicate the full extent of a society's knowledge. The phrase, "a society's knowledge" must not be taken to mean that every person in that society had the same knowledge but only that those whose business it was to know, had that knowledge. The aristocratic fallacy produced a sedimentation of knowledge into the "lower classes"; writing materials were valued so highly that little technological knowledge was recorded. Presumably most of the lower classes were illiterate. Yet the master masons were able to sign their names and use the Egyptian systems of numerals, weights, and measures, as is shown by graffiti found in their stone quarries. This fact was also not recorded on papyrus, or formally inscribed on temple walls.

The Cartesian coordinates invite more extensive speculation. The sketch of a mural superimposed on a coordinate grid suggests tile construction. While Egypt has no extensive deposits of clay suitable for the mass production of ceramic bricks, it was famous for its glazed pottery. This industry began in Neolithic times, but tile was never used extensively in architecture. It may be a coincidence, but Petrie found evidence of square tile, glazed and embossed, during the same XVIII Dynasty that preceded the evidence of the Cartesian coordinates. By the XX Dynasty, it is certain that tile was in limited use for wall covering. Brick and tile are countable objects, and, as was suggested earlier, this may have led to the association of numbers with areas and lengths. Carefully molded, kiln-fired brick and tile were used in Mesopotamia and India much earlier than in Egypt. There is also evidence that numerical calculations were more integrally related to Mesopotamian geometry than to Egyptian, but this is an inference from a very few texts. It has already been noted that the Mesopotamians used the same name for units of area and volume. This may have been because of the early use of bricks and tile of standard sizes, and rectangular shapes. The numerical measure of areas and volumes would then be found by counting the same objects: the bricks. This is a hypothesis, but it accounts for some of the differences between Mespotamian and Egyptian geometry.

While the geometrical literature from India is of a much later date than that of Egypt and Mesopotamia, it is more detailed. It also shows that a much closer connection had been established between theology, bricks, and geometry. Three authors, Baudhayana, Apastamba, and Katyayana, each wrote books entitled *Sulva Sutra*. The title is usually translated "The Rules of the Cord," and they give additional evidence that the cord and tile masonry were fundamental in the evolution of geometry. The complete texts of two of these three books have been translated into European languages; both Heath and Seidenberg give extracts and summaries, as well as references to other discussions of this material. There is general agreement that the *Sulva Sutras* were given their present form in the third and fourth centuries B.C., but there is also evidence that these were not the first editions. Moreover, the Egyptian evidence strongly suggests that they are based on knowledge acquired by the Indian bricklayers of a very much earlier period and presumably transmitted orally from generation to generation. There is also internal evidence that the *Sulva Sutras* are based on geometrical knowledge that must have taken centuries to accumulate — centuries that included the illiterate dark ages of both India and Greece. There is also the possibility that this knowledge evolved elsewhere (say in Mesopotamia) and was an importation, but if so, there is no evidence for the source or the date of the importation.

The *Sulva Sutras* are concerned mainly with the construction of brick and tile altars that conform to certain theological specifications. However, houses and temples in India were constructed of brick and tile before the end of its dark age. The theology also arose much earlier. The books were apparently written about the time India was developing its script, complete with the numerals that later evolved into our "Arabic" system. These writings therefore record only the end of a long history. The fact that the cord is still important at this late stage is quite significant in itself. For that matter, tape measures are still used in modern Europe and America, and carpenters used the chalk line until the advent of machine tools for producing straight edges. The Indian theology required the construction of altars of various shapes and sizes. The shapes included squares, circles, combinations of rectangles and squares, and stylized falcons and hawks. Depending upon circumstances, these were to have various sizes, but the shapes were always to be similar. The cord was used to lay out these shapes on flat level ground by driving pegs. The altars were then constructed of tile having specified shapes and sizes. The authors of the *Sulva Sutras* were mediating between the theologians and the master builders and bricklayers.

Before proceeding to other matters, one linguistic peculiarity deserves mention. The diameter of the cord does not seem to have been specified. Either it had been standardized by long custom, or it was assumed that the reader's common sense would keep him from using a hawser. The wording is such that, with our Greek heritage, it is easy to reach the conclusion that it was an ideal cord of zero diameter. It is more likely that the failure to specify its diameter was a linguistic device to simplify the wording of an otherwise cumbersome sentence. It was not expected that microscopic precision would be achieved; the gods were not all that superhuman, or at least did not expect superhuman efforts by their worshippers. Frequently there are instructions to mark the cord at some point, or to fasten its end to a peg. No details of these procedures are described; this again suggests reference to long established procedures, rather than to idealizations. Otherwise, why were the words "cord" and "peg" used instead of "straight line" and "point?" There is a psychological difficulty to be overcome when making this fictional abstraction, from a peg (or the imprint left by a peg in the ground) to the insubstantial point. This is recognized by Sylvia Townsend Warner in the story mentioned above. The early history of these fictions is very obscure. They appear suddenly in the writings of classical Greece, but with little explanation and with no indication of their earlier history. It seems that they arose only once in the history of mankind, and diffused slowly from a single source. Until quite recently, there were parts of the Earth that they had not reached; possibly there still are.

A single example will suffice to show how the cord and pegs were used in India. Let it be required to lay out a square whose side has a given length. A cord of this length is cut, as indicated by the line AB in Figure 6-1; its midpoint, and the midpoints of its halves are marked — indicated by C, D, and E. It is then stretched straight upon the flat ground, and pegs (indicated by circles) are driven to mark the current positions of A, B, C, D, and E. The ends, A and B, are then fastened to the second and fourth pegs, as shown in Figure 6-2; the cord is grasped at C and stretched; a sixth peg is driven to mark the current location of C. The end, A, is then fastened to the third peg, as shown in Figure 6-3; it is stretched to pass over the sixth peg, and a seventh peg is driven to mark the new location of C. This description of the procedures becomes tedious, and the reader will presumably find Figure 6-4 self-explanatory. The eighth peg, again at C, is one corner of the required square; the other three corners are pegged in the same way. It is easy for us to imagine simpler ways for accomplishing this construction, but that is because many people have worked on it, and have isolated the essentials and removed the nonessentials.

Numerous other peg-and-cord constructions are described in the *Sulva Sutras*, some of them showing the same (to us) excessive complication. But it is more interesting to describe their authors' knowledge then their failings. They knew that a rectangle whose sides were a and b units long, had an area of ab square units; that

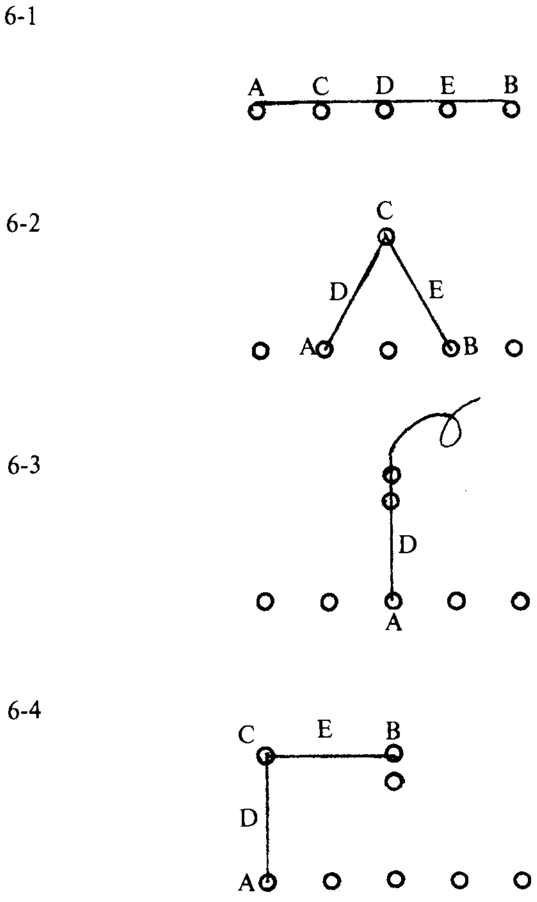


FIGURE 6
Creating a Square

the diagonal of the rectangle divided it into equal triangles; and much more. Since every straight line was associated with its numerical length, every cord-and-peg construction acquired a corresponding arithmetic or algebraic interpretation. For example, construction

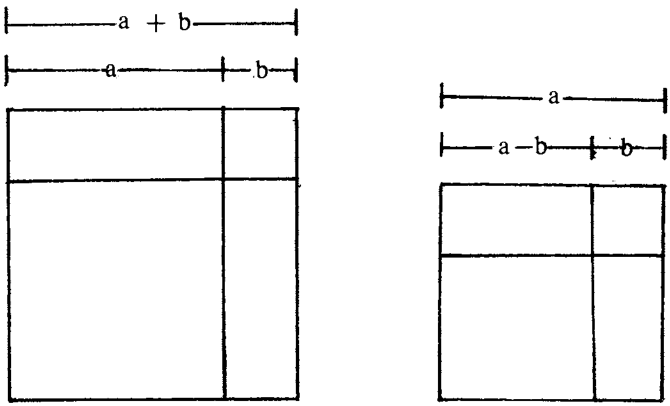


FIGURE 7

corresponding to the diagrams of Figure 7 are described, the lengths *a* and *b* being prescribed. The diagrams correspond to the algebraic formulas

$$(a+b)^2=a^2+2ab+b^2$$

$$(a-b)^2=a^2-2ab+b^2$$

These equations were not expressed in words, and the lengths were not assigned letters, as we do in algebra. Even today we say that one diagram is worth a thousand words. It has been noted above that similar arithmetic propositions could also be obtained from the study of dot-diagrams representing whole numbers. But the length of the cord need not have been a whole number of units. This is therefore an important step from the arithmetic of whole numbers toward algebra as we know it today. Again, earlier Greek mathematicians had taken this same step.

The relation of theology to Indian geometry is important, not only for its own sake, but because Plato's theology bears a somewhat different relation to Greek geometry. Only the specifications of the theologians were recorded in the *Sulva Sutras*. The theology itself was recorded in a series of associated works: the *Kalpa Sutras*, and the *Brahmanas*. It has already been noted that, in order to stop an epidemic, it was considered necessary to

1) Construct a square altar whose area was twice that of

another.

For reasons that remain obscure, under other conditions it was necessary

2) to construct a square altar whose area was equal to the sum of the areas of the other two altars, and

3) to construct a circular altar whose area was equal to that of a square altar (or conversely).

These do not exhaust the assignments that are solved in the *Sulva Sutras*, but these are fundamental. The first two were solved by the propositions:

1) The diagonal of a square produces the double of that which one of its sides produces.

2) The diagonal of a rectangle produces the sum of what the longer and shorter sides separately produce.

The context makes it clear that a line produces a square, and that areas are to be added. The second proposition is essentially what we know as the Pythagorean Theorem. With the passage of time, it has become more and more important for the study of geometry. We consider the first proposition as a special case of the second. But this separation of the two propositions also occurs in Babylonian mathematical texts, and in the writings of Plato. It seems to indicate that the second proposition was discovered before the first, and that tradition preserved the separation. This may be the reason why Theodorus of Cyrene treated $\sqrt{2}$ differently than $\sqrt{3}$, $\sqrt{5}$, etc. It has already been noted that the authors of the *Sulva Sutras* knew the very accurate approximation $577/408$ for $\sqrt{2}$. The less accurate Old Babylonian approximation, $17/12$, was also found only on a tablet devoted to the calculation of the diagonal of a square. But there is no evidence that either of the authors of the *Sulva Sutras* or the authors of the Old Babylonian texts knew that $\sqrt{2}$ is not a rational number. The origin of this knowledge remains unknown despite the new discoveries of twentieth century scholars.

The historical relations between the Indian and Greek geometers is unknown. It is known that Alexander the Great had Greek philosophers among his advisors, and that they reached the Indus River. It has been suggested that the Indian geometers were indebted to this brief contract with Greeks. This seems unlikely, for there were obvious linguistic and military barriers that would have prevented communication between scholars.

Plato and Eucleides of Megara

There is a general tradition that Plato had a great interest in mathematics and sponsored, or even required, its study. Yet writers on the history of mathematics treat Plato's own writings on mathematics very briefly, and only in paraphrase. This is justified because his contributions to technical mathematics are, to say the least, meager. Paraphrasing, however, involves a danger: the danger that the historian will use Plato's prestige to justify his own opinions, or those current in his own time. This has happened often. If one is investigating the foundations of mathematics, and its influence on present day society, Plato's writings must be considered as historical documents, not as textbooks of mathematics. The writer's opinions should be separated from the document itself, leaving the reader free to form an independent judgement.

Plato was an Athenian nobleman and is considered to have been a disciple of Socrates. Socrates was executed for reasons that remain somewhat obscure despite Plato's account of the circumstances. After the execution, Plato and other friends of Socrates left Athens and stayed for a time with Eucleides of Megara. Plato was then twenty-eight, Eucleides fifty-seven. Eucleides had often traveled the twenty miles from Megara to Athens in order to visit Socrates, and did so at the time of the execution (399 B.C.). It is now impossible to distinguish the influences of the two men on Plato, or to say how much they may have influenced each other. Plato was also influenced by the Pythagoreans. In particular, he adopted their custom of ascribing his own (and other people's) ideas to the Master; in Plato's case, the Master was Socrates, as has been noted above. Socrates seems never to have put his own opinions in writing.

So far as is known, all of Plato's writings have survived. This survival, however, is not the same as the survival of a clay tablet bearing cuneiform writing; these tablets are holographs: they bear the writing of the author. All that is needed to understand what an author meant is a knowledge of the language in which he wrote. In the case of the cuneiform texts, this knowledge is still incomplete, but it is increasing. Even if Plato's holographs had survived, this same problem would exist, for he wrote in a language that is similar to, but not identical with the Greek spoken in modern Athens, or the classical version taught in our universities. Only copies, edited versions, and translations have come down to the present time. During the period in which he wrote, the various Greek dialects were in a state of rapid change, but were scarcely fused into a single language. It is therefore more accurate to say that various interpretations of Plato's writings have survived; one need only compare three or four English versions of the same Platonic dialogue to convince oneself of this. The continuing controversies among scholars of Plato are additional evidence of this.

The order in which the dialogues were written can be determined only from their content, and there are differences of opinion. It is generally agreed that, *Phaedo* and *Menon*, are among the earlier, while *Phaedrus* and the *Republic* are later. *Timaeus* and *Critias* may be the latest. Only one, *Theaetatus*, seems to be datable on internal evidence, but even in this case, there are differences of opinion concerning the reliability of the date.

Returning to Eucleides, he seems to have originated the Megaran system of philosophy. Its central tenet was that experience and sensory perception are not essential to knowledge. What we call learning (by instruction or experience) is really the recollection of things the soul has known all along. One's soul knows everything that can be known and it is only a matter of calling it to mind. At first, Plato seems to have accepted this doctrine without modification, and his later modifications are not complete repudiations of the original. His original version (or Socrates') is given in both *Phaedo* and *Menon*. Moreover, *Menon* contains a more extensive discussion of geometry than any of the other twenty-six dialogues. It is almost the only one for which modern translators consider it necessary to supply diagrams and editorial comment in order to make it easier for the reader to understand the text. Perhaps it was meant to be acted, but there seem to have been no stage directions. Not even all of *Menon* is devoted to geometry: it is generally said that its principal topic is Virtue, and this aspect of the dialogue has already been mentioned above. The geometric portion is quoted in full below. The text is that of W.H.D. Rouse's translation; the present writer has substituted his own diagrams and editorial comments for Rouse's. The geometry is introduced almost parenthetically as an illustration of Plato's (or Socrates' or Eucleides') theory of knowledge. Menon has just objected to something Socrates has said, and the conversation then continues as follows.

> SOCRATES: I understand what you wish to say, Menon. You look on this as a piece of chop-logic, don't you see, as if a man cannot try to find either what he knows or what he does not know. Of course he would never try to find what he knows, because he knows it, and in that case he needs no trying to find; or what he does not know, because he does not know what he will try to find.
>
> MENON: Then you don't think that is a good argument, Socrates?
>
> SOCRATES: Not I.
>
> MENON: Can you tell me why?
>
> SOCRATES: Oh yes. I have heard wise men and women on the subject of things divine.
>
> MENON: And what did they say?
>
> SOCRATES: True things and fine things, to my thinking.
>
> MENON: What things, and who were the speakers?
>
> SOCRATES: The speakers were some priests and priestesses who have paid careful attention to the things of their ministry, so as to be able to give a reasoned explanation of them; also inspired poets have something to say, Pindar and many others. What they say I will tell you; pray consider, if they seem to you to be speaking truth. They say that the soul of man is immortal, and sometimes it comes to an end, which they call death, and sometimes it is born again, but it is never destroyed; therefore we must live our lives as much as we can in holiness: for from whomsoever
>
> Persephone shall accept payment for ancient wrong,
> She gives up again their souls to the upper sun in the ninth

year;
From these grow lordly kings, and men of power and might,
And those who are chief in wisdom; these for time to come
Are known among men as for holy heroes. Then, since the soul is immortal and often born, having seen what is on earth and what is in the house of Hades, and everything, there is nothing it has not learnt; so there is no wonder it can remember about virtue and other things, because it knew about these before. For since all nature is akin, and the soul has learnt everything, there is nothing to hinder a man, remembering one thing only, which men call learning, from himself finding out all else, if he is brave and does not weary in seeking; for seeking and learning is all rememberance.

MENON: Yes, Socrates. But what do you mean by saying that we do not learn, but what we call learning is remembering? Can you teach me how this is?

SOCRATES: You are a young rogue, as I said a moment ago, Menon, and now you ask me if I can teach you, when I tell you there is no such thing as teaching, only remembering. I see you want to show me up at once as contradicting myself.

MENON: I swear that isn't true, my dear Socrates; I never thought of that, it was just habit. But if you know of any way to show me how this can be as you say, show away!

SOCRATES: That is not easy, but still I want to do my best for your sake. Here, just call up one of your own men from all this crowd of servants, any one you like, and I'll prove my case in him.

MENON: All right. *(to a boy)* Come here.

SOCRATES: Is he Greek, can he speak our language?

MENON: Rather! Born in my house.

SOCRATES: Now, kindly attend and see whether he seems to be learning from me, or remembering.

MENON: All right, I will attend.

SOCRATES: Now my boy, tell me: Do you know that a four-cornered space is like this?

(Socrates begins drawing Diagram 1)

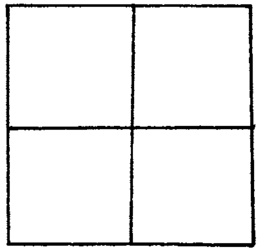


DIAGRAM 1

BOY: I do.

SOCRATES: Is this a four-cornered space having all these lines equal, all four?

BOY: Surely.

SOCRATES: Such a space might be larger or smaller?

BOY: Oh yes.

SOCRATES: Then if this side is two feet long and this two, how many feet would the whole be? Or look at it this way: if it were two feet this way, and only one the other, would not the space be once two feet?

BOY: Yes.

SOCRATES: But as it is two feet this way also, isn't it twice two feet?

BOY: Yes, so it is.

SOCRATES: So the space is twice two feet?

BOY: Yes.

SOCRATES: Then how many are twice two feet? Count and tell me.

BOY: Four, Socrates.

SOCRATES: Well, could there be another such space, twice as big, but of the same shape, with all the lines equal like this one?

BOY: Yes.

SOCRATES: How many feet will be in that, then?

BOY: Eight.

SOCRATES: *(aside to* MENON): You see Menon, that I am not teaching this boy anything: I ask him everything; and now he thinks he knows what the line is from which the eight-foot space is to be made. Don't you agree?

MENON: Yes, I agree.

SOCRATES: Does he know then?

MENON: Not at all.

SOCRATES: He *thinks* he knows, from the double size which is wanted?

MENON: Yes.

SOCRATES: Well, observe him as he remembers bit by bit, as he ought to remember.

Now boy, answer me. You say the double space is made from the double line. You know what I mean; not long this way and short this way, it must be equal every way like this, but double this eight feet. Just look and see if you think it will be made from the double line.

BOY: Yes, I do.

SOCRATES: Then this line is double this, if we add as much to it on this side.

(Socrates elaborates Diagram 1 into Diagram 2)

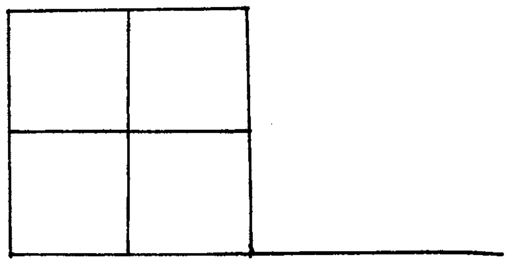


DIAGRAM 2

BOY: Of course!

SOCRATES: Then if we put four like this you say we shall get the eight-foot space.

BOY: Yes.

SOCRATES: Then let us draw these four equal lines. Is that the space which you say will be eight feet?

(Socrates now elaborates Diagram 2 into Diagram 3)

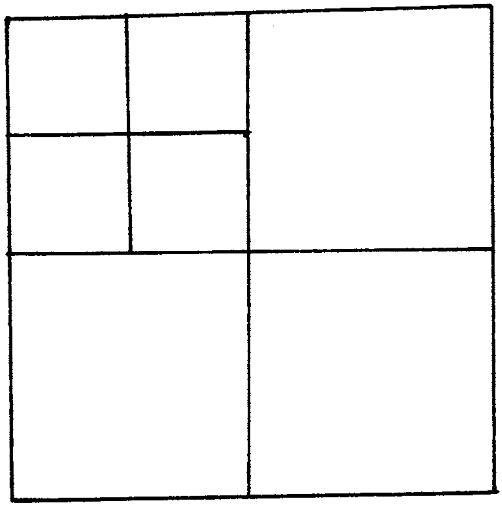


DIAGRAM 3

BOY: Of course.

SOCRATES: Can't you see in it these four spaces here, each of them equal to the one we began with, the four-foot space?

BOY: Yes.

SOCRATES: Well, how big is the new one? Is it not four times the old one?

BOY: Surely it is!

SOCRATES: Is four times the old one, double?

BOY: Why no, upon my word!

SOCRATES: How big, then?

BOY: Four times as big!

SOCRATES: Then, my boy, from a double line we get a space four times as big, not double.

BOY: That's true.

SOCRATES: Four times four is sixteen, isn't it?

BOY: Yes.

SOCRATES: Good. The eight-foot space will be double this, and half this.

BOY: Yes.

SOCRATES: Then its line must be longer than this, and shorter than this. What do you think?

BOY: That's what I think.

SOCRATES: That's right, just answer what you think. Tell me also: was this line not two feet, and this four?

BOY: Yes.

SOCRATES: Then the line of the eight-foot space must be longer than this line of two feet, and shorter than the line of four feet.

BOY: Yes, it must.

SOCRATES: Try to tell me, then, how long you say it must be.

BOY: Three feet.

(Diagram 3 is now modified into Diagram 4)

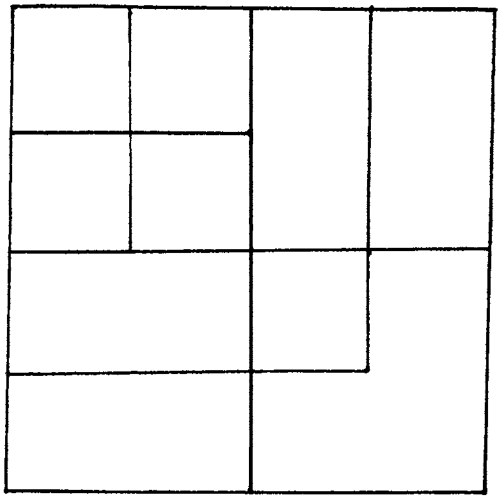


DIAGRAM 4

SOCRATES: Three feet, very well: If we take half this bit and add it on, that makes three feet, doesn't it? For here we have two, and here one, the added bit; and on the other side, in the same way, here are two, here one; and that makes the space you say.

BOY: Yes.

SOCRATES: Then if the space is three feet this way and three feet that way, the whole space will be three times three feet?

BOY: It looks like it.

SOCRATES: How much is three times three feet?

BOY: Nine.

SOCRATES: How many feet was the double to be?

BOY: Eight.

SOCRATES: So we have not got the eight-foot space from the three-foot line after all.

BOY: No, we haven't.

SOCRATES: Then how long ought the line to be? Try to tell us exactly, or don't want to give it in numbers, show it if you can.

BOY: Indeed, Socrates, on my word I don't know.

SOCRATES: Now Menon, do you notice how this boy is getting on in his remembering? At first he did not know what line made the eight-foot space, and he does not know yet; but he thought he knew then, and boldly answered as if he did know, and did not think there was any doubt; now he thinks there is a doubt, and as he does not know, so he does not think he does know.

MENON: Quite true.

SOCRATES: Then he is better off as regards the matter he did not know?

MENON: Yes, I think so too.

SOCRATES: So now we have put him into a difficulty, and like the stingray we have made him numb, have we done him any harm?

MENON: I don't think so.

SOCRATES: At least we have brought him a step onwards, as it seems, to find out how he stands. For now he would go on contentedly seeking, since he does not know; but then he could easily have thought he would be talking well about the double space, even before any number of people, again and again, saying how it must have a line of double length.

MENON: It seems so.

SOCRATES: Then do you think he would have tried to find out or to learn what he thought he knew, not knowing, until he tumbled into a difficulty by thinking he did not know, and longed to know?

MENON: I do not think he would, Socrates.

SOCRATES: So he gained by being numbed?

MENON: I think so.

SOCRATES: Just notice now after this difficulty he will find out by seeking along with me, while I do nothing but ask questions and give no instruction. Look out if you find me teaching and explaining to him, instead of asking for his opinions.

Now boy, answer me. Is not this our four-foot space? Do you understand? (Here Socrates erases Diagram 4 and begins drawing Diagram 5).

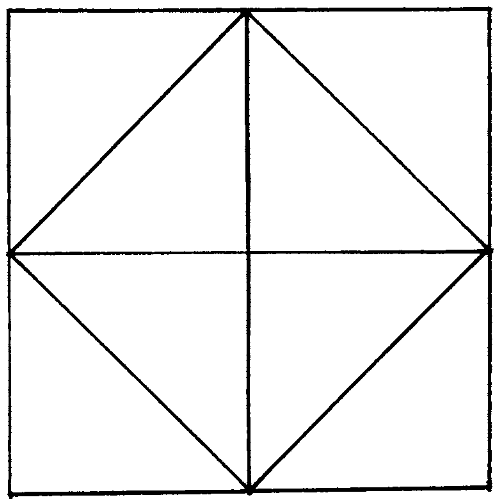


DIAGRAM 5

BOY: I do.

SOCRATES: Shall we add another equal to it?

BOY: Yes.

SOCRATES: And a third equal to either of them?

BOY: Yes.

SOCRATES: Now shall we not also fill in this space in the corner?

BOY: Certainly.

SOCRATES: Won't these be four equal spaces?

BOY: Yes.

SOCRATES: Very well. How many times the small one is this whole space?

BOY: Four times.

SOCRATES: But we wanted a double space, don't you remember?

BOY: Oh yes, I remember.

SOCRATES: Then here are lines running from corner to corner, cutting each of these spaces in two parts.

BOY: Yes.

SOCRATES: Are not these four lines equal, and don't they contain this space within them?

BOY: Yes, that is right.

SOCRATES: Just consider: how big is the space?

BOY: I don't understand.

SOCRATES: Does not each of these lines cut each of the spaces, four spaces, in half? Is that right?

BOY: Yes.

SOCRATES: How many spaces as big as that are in this middle space?

BOY: Four.

SOCRATES: How many in this one?

BOY: Two.

SOCRATES: How many times two is four?

BOY: Twice.

SOCRATES: Then how many feet big is this middle space?

BOY: Eight feet.

SOCRATES: Made from what line?

BOY: This one.

SOCRATES: From the line drawn corner to corner of the four-foot space?

BOY: Yes.

SOCRATES: The professors call this a diagonal: so if this is a diagonal, the double space would be made from the diagonal, as you say, Menon's boy!

BOY: Certainly, Socrates.

SOCRATES: Now then, Menon, what do you think? Was there one single opinion which the boy did not give as his own?

MENON: No, they were all his own opinions.

SOCRATES: Yet he did not know, as we agreed shortly before.

MENON: Quite true, indeed.

SOCRATES: Were these opinions in him, or not?

MENON: They were.

SOCRATES: Then in one who does not know, about things he does not know, there are true opinions about the things which he does not know?

MENON: So it appears.

SOCRATES: And now these opinions have been stirred up in him as in a dream; and if someone will keep asking him these same questions often and in various forms, you can be sure that in the end he will know about them as accurately as anybody.

MENON: It seems so.

SOCRATES: And no one having taught him, only asked questions, yet he will know, having got the knowledge out of himself?

MENON: Yes.

SOCRATES: But to get knowledge out of yourself is to remember, isn't it?

MENON: Certainly it is.

SOCRATES: Well then: This knowledge which he now has, he either got it sometime, or he had it always?

MENON: Yes.

SOCRATES: Then if he had it always, he was also always one who knew; but if he got it sometime, he could not have got it in this present life. Or has someone taught him geometry? For he will do just these same things in geometry, and so with all other sciences. Then there is anyone who has taught him everything? You are sure to know that, I suppose, especially since he was born and brought up in your house.

MENON: Well, I indeed know that no one has ever taught him.

SOCRATES: Has he all these opinions, or not?

MENON: He has, Socrates, it must be so.

SOCRATES: Then if he did not get them in this life, is it not clear now that he had them and had learnt at some other time?

MENON: So it seems.

SOCRATES: Is that not the time when he was not a man?

MENON: Yes.

SOCRATES: Then if both in the time when he is a man and when he isn't there are to be true opinions in him, which are awakened by questioning and become knowledge, will not his soul have understood them for all time? For it is clear that through all time he either is or is not a man.

MENON: That's clear.

> SOCRATES: Then if the truth of things is always in our soul, the soul must be immortal; so that what you do not know now by any chance, that is, what you do not remember, you must boldly try and find out and remember?
>
> MENON: You seem to argue well, Socrates. I don't know how you do it.
>
> SOCRATES: Yes, I think that I argue well, Menon.

There are many things that need to be said about this extract, including many that seem not to have been put in writing before. To begin with, it is much longer than most earlier mathematical texts that have survived. The format of a dialogue enables us to understand the motivation of the speakers (or writer) in much more detail than we understand the motivation of the Babylonian mathematicians. It begins and ends with theology. The *Sulva Sutras* also link geometry and religion, but here the connection is much closer. The *Sulva Sutras* are concerned only with the implementation of theological specifications. The theology expounded by Socrates seems almost deliberately devised to explain Eucleides' theory of knowledge. Menon's reception of it suggests that it was novel, by no means commonly accepted. Or, it may have been the other way: in Plato's time, many small religious cults had been formed, which believed in reincarnation. It is generally considered that they were the result of contact with the Orient. Eucleides may have derived his theory of knowledge from one of these cults. In the quotation, the geometrical episode is described, not as an implementation of the theology, but as a demonstration of its truth. The *Sulva Sutras* tacitly assume the truth of their theology. Neither Socrates nor his theology gives any reason for the selection of the specific geometric example chosen. Plato also expounds this theology (without reference to geometry) in another of his Dialogues, *Phaedo*. There Socrates uses it to comfort those of his friends who were present at his execution. Plato was absent, but Eucleides was present. Plato also expounds a much more specific form of the same theology, again without geometry, in his later dialogue *Phaedrus*:

> Hear now the ordinance of Necesssity. Whatsoever soul has followed in the train of a god, and discerned something of truth, shall be kept from sorrow until a new revolution shall

> begin, but if she can do this always, she shall remain always free from hurt. But when she is not able so to follow, and sees none of it, but meeting with some mischance comes to be burdened with a load of forgetfulness and wrongdoing, and because of that burden sheds her wings and falls to the earth, then thus runs the law. In her first birth she shall not be planted in any brute beast, but the soul that has seen the most of being shall enter into the human babe that shall grow into a seeker after wisdom or beauty, a follower of the Muses and a lover; the next, having seen less, shall dwell in a king that abides by law, or a warrior and ruler; the third in a statesman, a man of business, or a trader; the fourth in an athlete, or physical trainer, or physician; the fifth shall have the life of a prophet or a Mystery priest; to the sixth that of a poet or other imitative artist shall be fittingly given; the seventh shall live in an artisan or farmer; the eighth Sophist or demagogue; the ninth in a tyrant.

Here Plato gives us his aristocratic ordering of the social classes, and the sophists are ranked even lower than artisans or farmers. The passage is dogmatic: he has given up the attempt to demonstrate its truth. But in *Menon*, the boy is made the subject of a psychological experiment intended to demenstrate the validity of the theology: "I'll prove my case in him." The geometry and the diagrams are the apparatus of this experiment. One must therefore consider if the experiment does prove the case, and will immediately conclude that it does not. Almost all of Socrates' questions are loaded. They are really instructions, instructions to watch him drawing diagrams, to perceive, to experience them as they are being drawn. Sometimes he even forgets the "chop-logic" of putting them in the form of questions: "Just look and see if you think it will be made from the double line." But this is not very important. When Socrates' questions are reworded as statements or instructions, it is seen that they convey knowledge or opinions from Socrates to the boy. Very few elicit knowledge or opinions from the boy that Socrates did not provide him, either via his loaded questions or his diagrams. There is nothing to suggest that the boy is remembering anything that his soul has known all along. In real life, Menon would have noticed this, and would not have hesitated to comment on it. In fact, he had been asked to do just this: "Kindly attend and see whether he seems to be learning from me, or remembering." There is every evidence that the boy is learning, in the ordinary sense of the word, both by instruction and by guided experience.

The experiment has not proven Socrates' case. This is the more remarkable, for it is most unlikely that we are reading the report of an actual experiment. Socrates, Menon, the boy, the stick with which Socrates draws the diagrams, all these are figments of Plato's imagination. Not only the actions of the experimenter, Socrates, but the responses of Menon and his slave, all are part of Plato's fantasy. It is an imaginary experiment, the prototype of all the thought experiments (*Gedanken Experimente*) that have been discussed by modern theoretical physicists. It proves nothing about the real world except Plato and his fantasies. The manuscript of *Menon* must have been circulated among Plato's immediate friends; perhaps it was read aloud to them. They would not have failed to point out that the questions were loaded, and also that the experiment described is contrary to the spirit of Greek geometry, as it was then being developed. In any event, Plato never again ventured to write so explicitly about geometry or mathematics.

Since Plato abandoned the attempt to use psychomathematical experiments to establish Eucleides' theory of knowledge and learning, one must ask whether he also abandoned the theory itself. His other dialogues show no evidence that he did, although it is nowhere again stated as explicitly as in *Menon* and *Phaedrus.* His ideas about knowledge and mathematics changed as he grew older, but he did not explicitly repudiate *Menon.* The dialogue *Theaetatus* and others show the development of his modified theory of knowledge.

In addition to the dialogues, some letters of Plato's have survived. Their authenticity has been disputed, largely because they are stylistically different from most of the dialogues. However, this argument is not convincing. The dialogues are not stylistically uniform; there are variations in style, even within a single dialogue. The character of Socrates appears in most of them, but Plato's portrayal of his personality is neither uniform, nor does it show what modern literary critics call "character development." According to the Encyclopedia Britannica, Letter VII is very likely authentic. It contains a clear summary of what one gleans from the dialogues concerning Plato's considered theory of knowledge.

> Such writers can in my opinion have no real acquaintance with

the subject. I certainly have composed no work in regard to it, nor shall I ever do so in future, for there is no way of putting it in words like other studies. Acquaintance with it must come rather after a long period of attendence on instruction in the subject itself and of close companionship, when, suddenly, like a blaze kindled by a leaping spark, it is generated in the soul and at once becomes self-sustaining.

Besides, this at any rate I know, that if there were to be a treatise or a lecture on this subject, I could do it best. I am also sure for that matter that I should be very sorry to see such a treatise poorly written. If I thought it possible to deal adequately with the subject in a treatise or a lecture to the general public, what finer achievement would there have been in my life than to write a work of great benefit to mankind and to bring the nature of things to light for all men? I do not, however, think the attempt to tell mankind of these matters a good thing, except in the case of some few who are capable of discovering the truth for themselves with a little guidance. In the case of the rest to do so would excite in some an unjustified contempt in a thoroughly offensive fashion, in others certain lofty and vain hopes, as if they had acquired some awesome lore.

It has occurred to me to speak on the subject at greater length, for possibly the matter I am discussing would be clearer if I were to do so. There is a true doctrine, which I have often stated before, that stands in the way of the man would dare to write even the least thing on such matters, and which it seems I am now called upon to repeat.

For everything that exists there are three classes of objects through knowledge about it must come; the knowledge itself is a fourth, and we must put as a fifth entity the actual object of knowledge which is the true reality. We have then, first, a name, second, a description, third, an image, and fourth, a knowledge of the object. Take a particular case if you want to understand the meaning of what I have just said; then apply the theory to every object in the same way. There is something for instance called a circle, the name of which is the very word I just now uttered. In the second place there is a description of it which is composed of nouns and verbal expressions. For example the description of that is named round and circumference and circle would run as follows: the thing that has everywhere equal distances between its extremities and its center. In the third place there is the class of object which is drawn and erased and turned on the lathe and destroyed, processes which do not affect the real circle to which these other circles are all related, because it is different from

> them. In the fourth place there are knowledge and understanding and correct opinion concerning them, all of which we must set down as one thing more that is found not in sounds nor in shapes of bodies, but in minds, whereby it evidently differs in its nature from the real circle and from the aforementioned three. Of all these four, understanding approaches nearest in affinity and likeness to the fifth entity, while the others are more remote from it.

In the dialogue *Timaeus*, Plato gives an even more concise summary of his views, saying that truth is an eternal now, unchangeable, forever inexpressible. This inexpressibility of truth, the impossibility of reducing it to words, appears to have been his mature and final conclusion. This doctrine is variously called Platonic idealism and Socratic agnosticism. In the earlier *Menon*, one finds recollection of truth described as easy, possible for everyone, even for an uneducated slave boy. Here recollection is replaced by something like revelation, which is so difficult to obtain that very few do so, and only after a long period of instruction and meditation. But geometry is again called upon to illustrate revealed reality. It is a more sophisticated geometry. The real circle is not any of the transient round objects, or even the understanding of such material objects. The true reality of the circle is eternal, not transient, not to be confused with the understanding of round objects. This is an elaboration of *Menon*, but something of the original is left. The inaccessibility of true knowledge to all but an elite few seems to have been challenged by Aristotle. It will be seen that he elaborated *Menon* in a different way, of which Plato cannot have approved.

Plato and the Academy

If this were simply a history of mathematics, there would be very little more to be said about Plato. However, popular history tells us that Plato founded a college called the Academy. This belief is one reason for his profound effect on modern educational systems in Europe, the United States, and elsewhere. Nothing could be less factual. H.T. Cherniss has assembled the evidence against this legend; part of it is very simple. It is certain that the Academy was an actual entity and that Plato was its founder and first leader, but no early writers have described it in detail. In recent centuries, many writers have described it in many different ways. Each description is that of the writer's own Alma Mater, seen through a nostalgic haze and embellished with improvements. There are also stories about the way in which the Academy was founded. The most common one (although Cherniss considers this also to be legendary) asserts that when Plato was about forty, he and his friends incorporated, under the laws of Athens, as a religious cult devoted to the worship of the Muses. This is usually supposed to have been a technicality. Certainly there is nothing in Plato's writings to indicate any interest in Terpsichore, the Muse of the Dance. In his *Republic*, he proposes to exclude all poets and musicians from his utopia, with the exception of those who submit to a strict censorship. His argument with Dionysus of Syracuse, which according to legend resulted in his enslavement, is said to have been about this issue. Yet, it has been seen that Plato's writings have an undeniably theological element, which is not unlike that of many other Greek cults. One may suppose that the story contains a grain of truth. Under Athenian law, such an incorporated cult would be empowered to accept gifts and endowments, and be exempt from taxation. To a considerable extent, incorporation would protect its members from persecution; such cults were not required to disclose their beliefs or rituals. As long as they did not make a public nuisance of themselves by proselytizing the unwilling or by creating disturbances during public ceremonies, they were exempt from public inspection. They were free to teach their doctrines to any who wished such instruction. Plato and his friends purchased property, presumably including a building, adjacent to a public park in the suburbs of Athens. The tutelary god of this park was Academus -- hence the name "Academy." The park was used by the public as an athletic field, for practicing racing and wrestling. In Greece, wrestling was a brutal sport. There were only two rules: not to gouge out an opponent's eye, and not to kill him; violation of either rule disqualified one for the prize. A kick in the groin was considered a clever trick, not subject to reproof. In his youth, Plato had been a famous wrestler, and had won at least one major contest.

There are various legends as to the source of the Academy's original endowment, but the cult seems to have existed almost continuously for a thousand years, and seems rarely to have lacked money. It was essentially an organization of wealthy people. In 529 A.D., the Emperor Justinian abolished all such pagan cults in the Roman Empire, and confiscated their assets. It is said that the Academy was one of these outlawed cults.

The leader of the Academy was elected by its members. When Plato died, he was succeeded by his nephew, Speusippus. His election has surprised many historians, for Plato's most famous younger contemporary was Aristotle, and it has been supposed that he was therefore the favorite disciple. This does not follow, for there are many points on which the writings of Plato and Aristotle differ. These diferences may have caused their personal friendship to be less than ardent. Speusippus was succeeded by Xenocrates, a historian. His history of mathematics failed to survive, but it is referenced in other writings. The other of Plato's successors whose writings are relevant here, is Proclus Diaidochus. He lived from about 412 to 485 A.D., some eight hundred years after Plato. He was convinced that he had achieved direct communication with the Greek gods. He apparently offended the Athenians by attempting to convert Christians to his own beliefs, and had to leave Athens to avoid persecution. Returning as a reformed character, he seems not to have been molested further. He wrote a commentary on the geometry of Euclid of Alexandria which contains many historical footnotes, and even a historical introduction. The commentary itself contains much that is mystical, so that it is often supposed that the strictly factual history was added by some later editor. Be that as it may, it is customary to reference it as if Proclus was the author of this history. It is one of our few sources of information about earlier Greek mathematicians. Although he wrote about matters that occurred centuries earlier, his comments show a proper skepticism of legend. He must also have had historical sources that have not survived to the present. In any event, Proclus is often the only source of biographical and historical information available to us, and will frequently be quoted below.

The Dialogue *Timaeus* and Greek Cosmology

Before leaving Plato and his dialogues, it will be well to consider two others in more detail. These are *Timaeus* and *Critias*. The former has had a great influence on later European writers. From internal evidence, including style and content, their authenticity might well be doubted, but they have always been accepted as genuine. It seems that they were intended as the first two members of a trilogy, but *Critias* was never finished and the third dialogue was never even started. They contain two, rather different versions of the story of the Lost Continent of Atlantis. This one item has given rise to a great mass of speculative literature, and has contributed its name to the Atlantic Ocean. This is sufficient to establish the great influence of these two dialogues, but it need not detain one here. Concerning the difference between the two versions of the Atlantis story, Aristotle remarks drily "He who created it also destroyed it." Other early writers were similarly skeptical, but this has not deterred people from speculating about it, and even today some are still searching for its remnants.

Timaeus of Locri is the chief narrator of the dialogue, which is largely devoted to cosmology. It was formerly thought that he was a historical person and one of Plato's teachers. It now seems certain that he is a purely fictional character, invented by Plato for his own purposes. The quotation of a few passages from *Timaeus*, as translated by B. Jowett, will suffice to explain its great influence.

> When the Father and Creator saw the image that he had made of the eternal gods moving and living, he was delighted, and in his joy determined to make his work still more like the pattern; and as the pattern was an eternal creature, he sought to make the universe the same as far as might be. Time, then, was created with the heaven, in order that being produced together they might be dissolved together, if ever there was to be any dissolution of them; and was framed after the pattern of the eternal nature, that it might, as far as possible, resemble it, for

that pattern exists throughout all ages, and the created heaven has been, and is, and will be in all time. Such was the mind and thought of God in the creation of time. And in order to accomplish this creation, he made the sun and moon and five other stars, which are called the planets, to distinguish and preserve the numbers of time, and when God made the bodies of these several stars he gave them orbits in the circle of the other. There were seven orbits, as the stars were seven; first, there was the moon in the orbit nearest the earth, and then the sun in the next nearest orbit beyond the earth, and the morning star and the star sacred to Hermes, which revolve in their orbits as swiftly as the sun, but with an opposite principle of motion, which is the reason why the sun and Hermes and Lucifer meet or overtake, and are met or overtaken by each other. To ennumerate the places which He assigned to the other stars, and the reasons of them, if they were all to be counted, though a secondary matter, would give more trouble than the primary ones. These things at some future time, when we are at leisure, may have the consideration which they deserve, but not at present. Until the creation of time, all things had been made in the likeness of that which was their pattern, but in so far as the universe did not as yet include within itself all animals, there was a difference. This defect the Creator supplied by fashioning them after the nature of the pattern. And as the mind perceives ideas or species of a certain nature and number in the ideal animal, He thought that this created world ought to have them of a like nature and number. There are four such; one of them is the heavenly race of the gods; another, the race of birds moving in the air; the third, the watery species; and the fourth, the pedestrian and land animals. Of the divine, he made the greater part out of fire, that they might be the brightest and fairest to the sight, and he made them after the likeness of the universe in the form of a circle, and gave them to know and follow the best, distributing them over the whole circumference of the heaven, which was to be a true cosmos or glory spangled with them. And he bestowed on each of them two motions; first, the motion in the same, because they ever continue thinking about the same things, and also a forward motion in that they are controlled by the revolution of the same and the like; but the other five motions are wanting in them and thus each of them was the best possible. And for this reason also the fixed stars were created, being divine and eternal animals, ever-abiding and revolving after the same manner and on the same spot; and the other stars which revolve and also wander, as has already been described, were created after their likeness. The earth, which is our nurse, compacted (or *circling*) around the

> pole which is extended through the universe, He made to be the guardian and artificer of night and day, first and eldest of gods that are in the interior of heaven. Vain would be the labor of telling about all the figures of them moving as in a dance, and their meetings with one another, and the return of their orbits on themselves, and their approximations, and to say which of them in their conjunctions meet, and which of them are in opposition, and how they get behind and before one another, and at what times they are severally eclipsed to our sight and again reappear, sending terrors and intimations of things about to happen to those who can calculate them -- to attempt to tell of all this without looking at the models of them would be labor in vain. Let what we have said about the nature of the created and visible gods be deemed sufficient and have an end.

Much of this is reminiscent of the Book of Genesis and of Judaic monotheism. As in the cases of Archimedes and the Buddha, it is not known how this similarity came about. In any event, much of *Timaeus* was incorporated into medieval Christian theology. In commenting on this, Jowett has the following to say:

> The influence which the *Timaeus* has exercised upon posterity is partly due to a misunderstanding. In the supposed depths of this dialogue, the Neo-Platonists found hidden meanings and connections with the Jewish and Christian Scriptures, and out of them they elicited doctrines quite at variance with the spirit of Plato. Believing that he was inspired by the Holy Ghost, or had received his wisdom from Moses, they seemed to find in his writings the Christian Trinity, the Word, the Church, the creation of the world in a Jewish sense, as they really found the personality of God or Mind, and the immortality of the soul. All religions and philosophies met and mingled in the schools of Alexandria, and the Neo-Platonists had a method of interpretation that would elicit any meaning out of any words. They were really incapable of distinguishing between the opinions of one philosopher and another, or between the serious thoughts of Plato and his passing fancies. They were absorbed in his theology, and under the dominion of his name, while that which was truly great and truly characteristic of him, his effort to realize and connect abstractions, was not understood by them at all. And yet the genius of Plato and Greek philosophy reacted upon the East, and a Greek element of thought and language overlaid the deeper and more pervading spirit of Orientalism.

> There is no danger of the modern commentators on the *Timaeus* falling into the absurdities of the Neo-Platonists. In the present day we are well aware that an ancient philosopher is to be interpreted by himself, and by the contemporary history of thought. We know that mysticism is not criticism. The fancies of the Neo-Platonists are only interesting to us because they exhibit a phase of the human mind which prevailed widely in the first centuries of the Christian era, and is not wholly extinct in our own day.

Someone seems to have been more successful than Proclus in persuading medieval Christians that Plato was a divinely inspired writer. In turn, this gave the impetus to those later studies of the Greek classics that have shaped the entire education of Western Europe. Their influence was not confined either to science or to religion. The German Platonist, Erich Frank, remarks that one idiosyncracy of Plato's, one phantasm, influenced the development of our music for millenia. Possibly this particular influence is now ended. The misunderstanding of which Jowett speaks still dominates most of us today; worse yet, the misunderstanding has received no publicity outside scholarly circles. Even Jowett's optimism concerning modern commentators has not always been justified, as the continuing search for the Lost Continent of Atlantis illustrates.

Of course, not all of Plato, or even all of the *Timaeus* was incorporated into Christian theology. An example that will be investigated later is the astrological passage: "sending terrors and intimations of things about to happen to those who can calculate them." There are other cosmological and theological passages in the dialogues. Again some of them have been incorporated into Christian theology, some not. A striking example is provided by the closing passages from the *Phaedo*, which purports to tell of Socrates' last hours before execution. As background, one should know that sometime before Plato, someone whose name is not recorded evolved the idea that the Earth was round and revolved about an axis. W.H.D. Rouse's translation of the relevant passages of *Phaedo* follows; Socrates is the narrator.

> "I believe, then," said he, "that first, if it is round and in the middle of the heavens, it needs nothing to keep it from falling, neither air nor any other such necessity, but the uniformity of the heavens, themselves alike all through, is enough to keep it there, and the equilibrium of the earth itself; for a thing in

equilibrium and placed in the middle of something which is everywhere alike will not incline in any direction, but will remain steady and in like condition. First I believe that," he said.

"Quite right too," said Simmias.

"Next, I believe it is very large indeed, and we live in a little bit of it between the Pillars of Heracles and the river Phasis, like ants or frogs in a marsh, lodging round the sea, and that many other people live in many other such regions. For there are everywhere about the earth many hollows of all sorts in shape and size, into which have collected water and mist and air; but the earth itself is pure and lies in the pure heavens where the stars are, which is called ether by most of those who are accustomed to explain such things; of which all this is a sediment, which is always collecting into hollows of the earth. We then, who lodge in its hollows, know nothing about it, and think we are living upon the earth; as if one living deep on the bottom of the sea should think he was at the top, and, seeing through the water sun and stars, should think the sea was heaven, but from sluggishness and weakness should never come to the surface and never get out and peep up out of the sea into this place, or observe how much more pure and beautiful it is than his own place, and should never have heard from anyone who saw it. This very thing has happened to us; for we live in a hollow of the earth and think we live on the surface, and call the air heaven, thinking that the stars move through that and that is heaven; but the fact is the same, from weakness and sluggishness we cannot get through to the surface of the air, since if a man could come to the top of it, and get wings and fly up, he could peep over and look, just as fishes here peep up out of the sea and look round at what is here, so he could look at what is there, and if his nature allowed him to endure the sight, he could learn and know that that is the true heaven and the true light and the true earth. For this earth and the stones and all the place here are corrupted and corroded, as things in the sea are by the brine so that nothing worth mention grows in the sea, and there is nothing perfect there, one might say, but caves and sand and infinite mud and slime wherever there is any earth, things worth nothing at all as compared with the beauties we have; but again those above as compared with ours would seem to be much superior. But if I must tell you a story, Simmias, it is worth hearing what things really are like on the earth under the heavens."

"Indeed, Socrates," said Simmias, "we should be glad to hear this story."

"It is said then, my comrade," he went on, "that first of all the earth itself looks from above, if you could see it, like those twelve-patch leathern balls, variegated, with strips of colour of which the colours here, such as are used by painters, are a sort of specimens; but there the whole earth is made of such as these, and much brighter and purer than these; one is sea purple wonderfully beautiful, one is like gold, the white is whiter than chalk or snow, and the earth is made of these and other colours, more in number and more beautiful than any we have seen. For indeed the very hollows full of water and mist present a colour of their own as they shine in the variety of other colours, so that the one whole looks like a continuous coloured pattern. Such is the earth, and all that grows in it is in accord, trees and flowers and fruits; and again mountains and rocks in like manner have their smoothness and transparency and colours more beautiful, and the precious stones which are so much valued here are just chips of the those, sard and jaspers and emerald and so forth,...

"This, then, is the nature of the whole earth and all that is about it; but there are many regions in it and hollows of it all round, some deeper and spreading wider than the one we live in, some deeper but having their gap smaller than ours, some again shallower in depth than ours and wider; but these are all connected together by tunnels in many places narrower or wider, and they have many passages where floods of water run through from one to another as into a mixing-bowl, and huge rivers ever flowing underground both of hot waters and cold, where also are masses of fire and great rivers of fire, and many rivers of liquid mud, some clearer, some muddier, like the rivers of mud which run in Sicily before the lava, and the lava itself. And each of these regions is filled with this, according as the overflow comes in each case. All these things are moved up and down by a sort of seesaw which there is in the earth, and the nature of this seesaw movement is this. One of the chasms in the earth is largest of all, and, besides, it has a tunnel which goes right through the earth, the same which Homer speaks of when he says,

Far, far away, where is the lowest pit
Beneath the earth,

and which elsewhere he and many other poets have called Tartaros. For into this chasm all the rivers flow together, and from this again they flow out, and they are each like the earth through which they flow. The cause which makes all streams run out from there and run in is that this fluid has no bottom or foundation to rest on. So it seesaws and swells up and down, and the air and wind about it do the same; for they follow with it, both when the rivers move towards that side of the earth, and when they move towards this side, and just as the breath always goes in and out when men breathe, so there, too, the wind is lifted up and down with the liquid and makes terrible tempests both coming in and going out. Therefore whenever the water goes back into the place which called 'down,' it rushes in along those rivers and fills them up like water pumped in; but when, again, it leaves that part and moves this way, it fills up our region once more, and when the rivers are filled they flow through the channels and through the earth, and, coming each to those places where their several paths lead, they make seas and lakes and rivers and fountains. After that they sink into the earth again, some passing round larger regions and more numerous, some round fewer and smaller, and plunge again into Tartaros, some far below their source, some but little, but all below the place where they came out. Some flow in opposite where they tumbled out, some in the same place; and there are others which go right round the earth in a circle, curling about it like serpents once or many times, and then fall and discharge as low down as possible. It is possible from each side to go down as far as the centre, but no farther, for beyond that the opposite part is uphill from both sides.

"All these rivers are large, and they are of many kinds; but among these many are four in especial. The greatest of these, and the outermost, running right round, is that called Ocean; opposite this and flowing in the contrary direction is Acheron, the River of Pain, which flows through a number of desert places, and also flowing under the earth comes to the Acherusian Lake, to which come the souls of most of the dead, and when they have remained there certain ordained times, some longer and some shorter, they are sent out again to birth in living creatures. The third of these rivers issues forth in the middle, and near its issue it falls into a large region blazing with much fire, and makes a lake larger than our sea, boiling with water and mud; from there it moves round turbid and muddy, and rolls winding about the earth as far as another place at the extreme end of the Acherusian Lake, without mingling with the water; when it has rolled many times round it falls into a lower depth than Tartaros. This is what they call Pyriphlegethon, the River of Burning Fire, and its lava streams blow up bits of it wherever they are found on the earth. Opposite this again the fourth river discharges at first into a region terrible and wild, it is said, all having the colour of dark blue; this they call the Stygian, the River of Hate, and the lake which the river makes they call Styx. But the river, falling into this and receiving terrible powers in the water, plunges beneath the earth and, rolling round, moves contrary to Pyriphlegethon and meets it in the Acherusian Lake on the opposite side. The water of this, too, mixes with none, but this also goes round and falls into Tartaros opposite to Pyriphegethon. The name of this, as the poets say, is Cocytos, River of the Wailing.

"Such is the nature of the world. So when the dead come to the place whither the spirt conveys each, first the judges divide them into those who have lived well and piously, and those who have not. And those who are thought to have been between the two travel to the Acheron, then embark in the vessels which are said to be there for them, and in these come to the lake, and there they dwell, being purified from their wrongdoings; and after punishment for any wrong they have done they are released, and receive rewards for their good deeds each according to his merit. But those who are thought to be incurable because of the greatness of their sins, those who have done many great acts of sacrilege or many unrighteous and lawless murders or other such crimes, these the proper fate throws into Tartaros whence they never come out. Those who are thought to have committed crimes curable although great, if they have done some violence to father or mother, say, from anger, and have lived the rest of their lives in repentance, or if they have become manslaughterers in some other such way, these must of necessity be cast Tartaros; but when they have been cast in and been there a year the wave throws them out, the manslaughterers by way of Cocytos, the patricides and matricides by way of Pyriphlegethon; and when they have been carried down to the Acherusian Lake, there they shriek and call to those whom slew or treated violently, and, calling on them, they beg and beseech them to accept them and let them go out into the lake; if they win consent, they come out and cease from their sufferings; if not, they are carried back into Tartaros and from there into the rivers again, and they never cease from this treatment until they win the consent of those whom they wronged; for this was the sentence passed on them by the judges. But those who are thought to have lived in especial holiness, they are those who are set free and released from these places here in the earth as from a prison house, and come up into the pure dwelling place and are settled upon earth. Of these same, again, those who have purified themselves enough by philosophy live without bodies altogether forever after, and come into dwellings even more beautiful than the others, which is not easy to describe nor is there time enough at this present. But for the reasons which we have given, Simmias, we must do everything so as to have our share of wisdom and virtue in life; for the prize is noble and hope great.

> "No sensible man would think it proper to rely on things of this kind being just as I have described; but that, since the soul is clearly immortal, this or something like this at any rate is what happens in regard to our souls and their habitations — that this is so seems to me proper and worthy of the risk of believing; for the risk is noble.

It seems impossible to read this without being reminded of posthumous judgement, punishment and reward, purgatory, eternal damnation, fiery hell and shrieking souls, and of heaven and the eternal bliss of souls not encumbered by bodies. But Socratic agnosticism reasserts itself at the end, in a form that modern theologians call Pascal's Wager, naming it after the seventeenth-century ascetic mathematician and theologian who revived it. In 1923, the German Platonist, Erich Frank, summarized this cosmology in a diagram that contains details gleaned from other Dialogues. It shows the inhabited world, including the seas and oceans, as the flat bottom of a shallow crater. The otherwise spherical Earth is surrounded by the concentric circle of the fixed stars, with the planets in between. Most noteworthy, however, is that exterior to the sphere of fixed stars is the region of ideas. It is said, though whose authority I do not know, that Plato's vocabulary included no words for the distinction we now make between matter and concept. Whether this was a general characteristic of the Attic dialect, or one of Plato's idiosyncracies is also unknown to me. At any rate, Plato held that ideas have an existence which is independent of people who can speak, soliloquize, or write. This doctrine is still widespread today. Platonic idealism maintains that these ideas are more real than people and material objects. This, like astrology, reincarnation, and Socratic agnosticism, was not incorporated into Christianity.

Plato was not the only classical Greek to write about cosmology. Much of Greek cosmology was, like Plato's, fanciful speculation, but there was also a substratum of systematic observational astronomy. The discussion of cosmology will be interrupted at this point, and resumed later at a more appropriate place.

Pythagoras and the Pythagorean Theorem

Pythagoras is a legendary figure, although there is no doubt that he was a real person and contributed to the advancement of both physics and mathematics. Apparently he left no writings of his own, and few of the writings of his immediate followers have survived. Two discoveries have been attributed to him. The first is the mathematical law of musical strings: a single string can be made to sound its octave, fifth, and fourth by stopping it at points that divide its length into one half, two thirds, and three quarters. There is no evidence that casts doubt on his discovery of this law.

The other discovery is the theorem about rectangles or right triangles that has already been encountered in the *Sulva Sutras*. They were written long after Pythagoras' time and cannot be cited as evidence against his priority. Concerning this, Proclus remarks:

> If we listen to those who wish to recount ancient history, we may find some referring to the Pythagorean theorem, saying that he sacrificed an ox in honor of his discovery.

This has a very skeptical tone. As will be seen, there is evidence that Pythagoras opposed the killing of animals, which should encourage even more caution. Nevertheless, this passage (and the writings of others less cautious than Proclus) has caused Pythagoras' name to be attached to the theorem. It is now certain that Proclus' skepticism was entirely justified, and that one must treat Pythagoras and the Pythagorean Theorem as two distinct historical problems.

Pythagoras was born about 580 B.C. on the Ionian island Samos. Everything else about the first fifty years of his life is pure conjecture. However, Samos is separated from Miletus by less than twenty five miles of water, and Miletus was then the center of Greek science, as well as being the terminus of Mesopotamian caravans. Samos itself had a cosmopolitan culture. It is said that the young Pythagoras traveled widely. This has a certain plausibility. It is inherently likely that young men of Ionia would join caravans or board ships, thus becoming familiar with foreign lands before settling down to the family business of trading with them.

Miletus was also the center of the first, or Ionian, school of Greek philosophy. It was started by Thales of Miletus, about a generation before Pythagoras was born. Thales' major doctrine claimed that the sun, moon, and planets were all composed of the same kind of matter as the earth. While his disciples continued to hold this doctrine, it did not gain wide acceptance until Galileo had improved the telescope and used it for astronomical observation in the seventeenth century A.D. Almost certainly, Pythagoras must have known of Thales as well as much about Mesopotamia and Egypt. His followers rejected Thales' views, so it is likely that Pythagoras was not convinced by them. Some say that the Pythagoreans held that everything, including the Earth, was made of whole numbers. Others say that they only claimed that the whole numbers and their arithmetic were the great Principle of Order that pervaded the universe. Aristotle and many other later writers have ridiculed them for these views; Pythagoras' own views are even less certain. Yet, this second view re-emerged in the writings of Kronecker and other nineteenth century mathematicians; this has not stopped the ridicule of "the so-called Pythagoreans," as Aristotle dubbed them.

It seems certain that, at about the age of fifty, Pythagoras was the leader of a politically powerful religious cult in Crotona. Crotona was an Achaean (Greek) colony in the heel of the Italian boot. By then, it was the intellectual and commercial center of that part of the Mediterranean world. The doctrines of this cult, as reconstructed by scholars, included a belief in the transmigration of souls between human beings and animals. Consequently, its members were vegetarians and refused to kill animals. According to Aristotle, they also refused to eat beans. They held that the greatest good in life was "conversing with the divine." This was a silent soliloquy accompanied by the fantasy of a divine audience. It differed from a prayer in that the soliloquy was not a request for divine intervention in the affairs of the world, not a wish or hope for a miracle, merely a wish for knowledge of the divine laws. This is a definite intellectual advance beyond the primitive prayer for help from the gods in reaching commercial and military goals. Pythagoras must be included among the intellectual innovators of ancient Greece.

His political power in Crotona was ended by a violent rebellion, and it is thought that he fled to Sicily, dying about 500 B.C. Within a few generations there was a schism among the Pythagoreans, one faction calling itself the Akousmatikoi, the other the Mathematikoi. At that time, *mathemata* meant teachings in general, or doctrines. However, the Akousmatikoi devoted themselves more and more to the theory of music, while the Mathematikoi emphasized the theory of arithmetic and geometry. By the time of Plato, *mathemata* was just beginning to take on its present connotation. Some scholars think that Plato sometimes used the word with its original meaning, and sometimes with its modern meaning. This hypothesis explains a controversy that has arisen among other scholars. This controversy centers about what are now called "non-mathematical numbers," "inaudible numbers," or "idea-numbers." These terms are of modern origin, devised to discuss obscure passages in the writings of Plato and Aristotle. The hypothesis also explains the emphasis that non-mathematicians have placed on Plato's "love of mathematics."

At about the same time as the schism, Pythagoreanism was proscribed and persecuted everywhere in Italy and Sicily, and some of its adherents found refuge in Ionia and Athens. There, they influenced Plato and others, especially the mathematicians. Some of the latter seem to have been converted to Pythagoreanism. Those Greek cults that emphasized reincarnation and transmigration of souls may have been stimulated by the Pythagoreans, or they may have been more directly influenced by Oriental theology. This is another historical problem that remains to be solved. As has been seen, it would be of great interest to know more about early communication between Greece and Asia. It seems that, at some time before Plato, some of the intellectual descendents of Pythagoras were able to return to Italy.

Turning now to the Pythagorean Theorem, one will first inquire how it might have been discovered by early geometers, using peg-and-cord methods. Since there is no written record, one may begin by analogy to Socrates' discussion. Figure 8 is a generalization of his last diagram. The sides of the inner square have the length c; of the outer $a+b$. The four right triangles have the sides

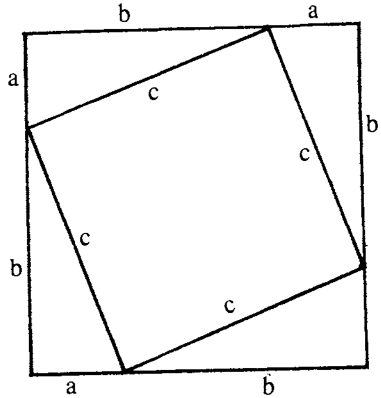


FIGURE 8

a and b, and the diagonal (or hypotenuse), c. It has been seen that the calculation of the areas of rectangles and triangles was known in quite ancient times. Inspection of Figure 8 shows that

$$(a+b)^2=c^2+4(ab/2)$$

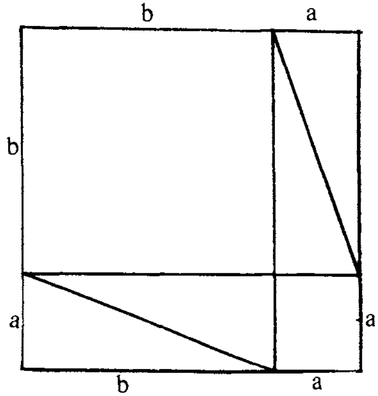


FIGURE 9

Then, rearranging the triangles, one obtains Figure 9. This shows that

$$(a+b)^2=a^2+2ab+b^2$$

Combining these two equations, one obtains

$$a^2+b^2=c^2$$

Present-day high schools still teach their students to demonstrate this theorem, but in a much more complicated way. If one inquires why this simpler way is not used, the usual answer is "the students should learn to think logically and exactly." There is nothing illogical or inexact about the above, and any high-school student could understand and remember it very quickly. Heath implies that the only objection to this simpler demonstration is that it is not in the anti-empirical spirit of Greek mathematics (or, for that matter, of modern mathematics). But, neither is the mathematical passage from *Menon* in this spirit. These more complicated demonstrations are all variants of one recorded by Euclid of Alexandria; the German philosopher, Schopenhauer, said that this is "a proof walking on stilts, a deceitful proof." Inverting his metaphor: it is a shallow complexity masquerading as a profundity. As though someone had advocated this view to him, Proclus comments:

> But for my part, while I admire those who first observed the truth of this [Pythagorean] theorem, I marvel more at the writer of the *Elements* [Euclid] not only because he made it fast by a most lucid demonstration, but because he compelled assent to a still more general theorem by the irrefutable arguments of science.

Few high school students would agree that Euclid's demonstration is lucid; most adults have forgotten it.

The equation, $a^2+b^2=c^2$ has many solutions for which a, b, and c are integers, whole numbers. If any unit of length is chosen, a triangle whose sides are a, b and c units in length will be a right triangle. These are the integral right triangles. Let n be a fourth whole number; then $A=a/n$, $B=b/n$, $C=c/n$ will be rational numbers, and $A^2+B^2=C^2$. A, B, and C then lead to the rational right triangles. Before the invention of irrational numbers, it would be supposed that all right triangles were rational. Naturally, no writer who was ignorant of irrational numbers would say this explicitly; he would have no words with which to say it. Conversely, if a writer discusses rational right triangles without saying that there are others, we may suppose that he did not know of irrational numbers. This hypothesis will be implicit in the following.

It seems certain that integral right triangles were known before 1500 B.C. It also seems certain that irrational right triangles were recognized about 400 B.C. The evidence for this last is in Plato's dialogue, *Theaetetus*. Unfortunately, there is a gap in the historical record that lasts from about 1500 B.C. until about 500 B.C. Consequently, nothing is known about the way in which the idea of irrational right triangles evolved. This is another problem in the history of mathematics whose solution would be most welcome. It is very difficult even to speculate about the way in which the idea might have evolved. Alfred North Whitehead has explained how a modern mathematician would develop the idea that a point has no parts. It is certain that early Greek philosophers would not have appreciated the fine points of Whitehead's explanation. Zeno of Elea was a contemporary of Pythagoras, and held much the same views as Eucleides. His writings show that he would not have understood Whitehead's reasoning, much less have accepted it as valid.

Surviving editions of Euclid's work on geometry show that he knew a way to calculate all integral right triangles. As background, one must know that either a, b or both must be even integers; the algebraic demonstration of this will be omitted. If a is even, Euclid's formulae are

$$a=2pq$$

$$b=p^2-q^2$$

$$c=p^2+q^2$$

where p and q are any two whole numbers, subject only to the condition that p be greater q. If b and not a is even, the formula can obviously be revised by interchanging the right sides of the equations for a and b. The "Pythagoras" formula is obtained from $p=n+1$, $q=n$ (with a and b interchanged), while the "Plato" formula is $p=2n$, $q=1$.

The writers of the *Sulva Sutras* knew some integral right triangles; they are listed in the following table.

Sulva Sutras

a	b	c
3	4	5
5	12	13
7	24	25
8	15	17
12	35	37

The first three were known by Pythagoras, the last two were known by Plato. The 3-4-5 right triangle appears in all three tables; it has a special history. There has long been a tradition that the early Egyptians knew it and used it to construct right angles. When Petrie discovered the Kahun Fragments, this was confirmed, for they were found to contain the following equations

$$1^2+(3/4)^2=(5/4)^2$$

$$8^2+6^2=10^2$$

$$2^2+(3/2)^2=(5/2)^2$$

$$16^2+12^2=20^2$$

any one of which is equivalent to

$$3^2+4^2=5^2$$

Quite properly, skeptics have raised the question: Did the writer of the papyrus know the relation between this arithmetic and right triangles? There is, as yet, no certain answer. It is certain that the XII Dynasty texts display no interest in abstract arithmetic, only in arithmetic that was useful for some practical purpose. The question thus reduces to: Is there any use for this arithmetic other than the construction of right triangles?

At about this same time, the writings of the Chinese mathematician, Chou Pei, came to the attention of Europeans. He lived about 1100 B.C., and demonstrated that the 3-4-5 triangle is a right triangle. It seems that he did not know of any other integral right triangles, or of the general Pythagorean Theorem.

This brings one to the discovery, by Neugebauer and Sachs, of an Old Babylonian table of integral right triangles. It is in the Plimpton Library of Columbia University, and was originally catalogued as a commercial text. It is broken, and only four columns of numerals remain. The first column on the right is headed "Its Name," and the fifteen entries in this column are translated as "First, Second, Third, ..., Fifteenth." The second and third columns are headed "Calculated Diagonal" and "Calculated Width," respectively. The reference is clearly to a triangle or rectangle. The heading of the first column on the left is too badly damaged to be read, and another column to the left of this has been broken off and lost. It is a reasonable hypothesis that the triangles are integral right triangles. If the entries in the second and third columns are b and c, then those in the surviving left hand column are $(c/a)^2$, then, in 11 of the 15 lines,

$$a^2+b^2=c^2$$

This makes the hypothesis practically certain; the four exceptions are undoubtedly mistakes on the part of the calculator. Three of the four mistakes (the second, ninth, and thirteenth lines) admit of a unique correction; the fifteenth line can be corrected in either of two ways.

When the table is rearranged into a format comparable to the previous table, it becomes the table below. The two columns on the extreme right give the values of p and q associated with the triangles by Euclid's formulae. Neugebauer gives convincing reasons for believing that these formulae were used to calculate the table. His argument is based, not only on the table itself, but on the way it relates to all the other Babylonian mathematical texts.

This discovery of this one tablet (out of a half million) proves, beyond reasonable doubt, that one man knew how to calculate integral right triangles more than a thousand years before Pythagoras. It is therefore a major discovery, but it leaves some questions still unanswered. How widespread was this knowledge? Did the calculator have an understanding audience, or was he an isolated worker like Leonardo da Vinci? Did his work influence his contemporaries, or did it lie forgotten like Leonardo's notebooks? Was his knowledge preserved and somehow transmitted to the classical Greeks? Neugebauer believes that the Pythagorean Theorem

Plimpton 322

nth	a	b	c	a/b	p	q
1st	119	120	169	.991(6)	12	5
2nd	3367	3456	4825*	.9742	64	27
3rd	4601	4800	6649	.9585	75	32
4th	12,709	13,500	18,541	.9414	125	54
5th	65	72	97	.902(7)	9	4
6th	319	360	481	.886	20	9
7th	2291	2700	3541	.8485	54	25
8th	799	960	1249	.8323	32	15
9th	481*	600	769	.801(6)	25	12
10th	4961	6480	8161	.7656	81	40
11th	3	4	5	.7500	2	1
12th	1679	2400	2929	.6996	48	25
13th	161*	240	289	.6708	15	8
14th	1771	2700	3229	.6559	50	27
15th	56	90	106*	.6(2)	9	5

(* indicates author's correction)

was widely known, but on this point his evidence is not strong. There is one other tablet that shows the diagonal of a square, whose length is calculated with the approximate value $305470/(60)^3$ for $\sqrt{2}$ which is good to six parts in ten million. Proclus is of no help in answering these questions. His remark about "those who first observed the truth of this theorem" has already been quoted above, and he does not elaborate on it.

By 1600 B.C., the geometry of the Old Babylonians had become highly arithmetized, much more so than the Egyptian and Indian geometries. But there is no evidence that they had evolved the ideal of points without parts, lines without thickness, or irrational numbers. There is then the gap in the recorded history of geometry until about 400 B.C., when Plato's *Theaetetus* gives a glimpse of classical Greek geometry, with its ideal straight lines already commonplace knowledge, and the irrational numbers beginning to be developed. At the present time, it is not even possible to guess at the intervening history. Since the Academy was not started until after 400 B.C., it certainly had no influence on these events; they had already been forgotten, along with the Mycenaean script. Not knowing of them, Eucleides and Plato did not seek to find the history of irrational numbers and ideal straight lines; instead they sought these fictions as immortal components of the human soul or psyche. This is the earliest and most extreme form of the doctrine of psychomathematical parallelism.

Ideals and the Technology of Triangles

In order not to lose the way among speculations about unrecorded history, it is well to begin with the impact of the Pythagorean Theorem and ideal straight lines on the present time. A simple example will provide a start: if there is a right triangle having side $a=1$ unit and the diagonal $c=2$ units the side b will be $\sqrt{(c^2-a^2)}=\sqrt{3}$ We know that $\sqrt{3}$ is an irrational number, and the table below provides successive approximations for $\sqrt{3}$.

9/5
26/15
1351/780
3650401/2107560

The first is $\sqrt{3}\approx 9/5$, and Figure 10 shows a triangle drawn with this approximation. It is apparent to the unaided eye that the angle at A is not a right angle: the difference is nearly four degrees. The next approximation is $\sqrt{3}\approx 26/15$, and Figure 10 also shows such a triangle. The reader may be able to see that the angle B is not ninety degrees: the difference is slightly more than two-thirds of a degree. Even though it is difficult to see, the young Zeiss, in his first workshop, could have measured this difference. The next approximation to $\sqrt{3}$ is 1351/780. This triangle could not be drawn on paper; paper is too flexible, and even printed straight lines are too coarse. Such a triangle differs from a right triangle by less than one tenth of a second of arc. Later, Zeiss and his men developed instruments for measuring such small angles, and means for producing such triangles out of glass, brass, or steel. They did this with the ideal of Abbe's calculations of irrational numbers as a goal.

The last approximation in the table is the rational approximation $\sqrt{3}\approx 3650401/2107560$ Such a triangle would differ from a right triangle by about one hundred millionth of a second of arc.

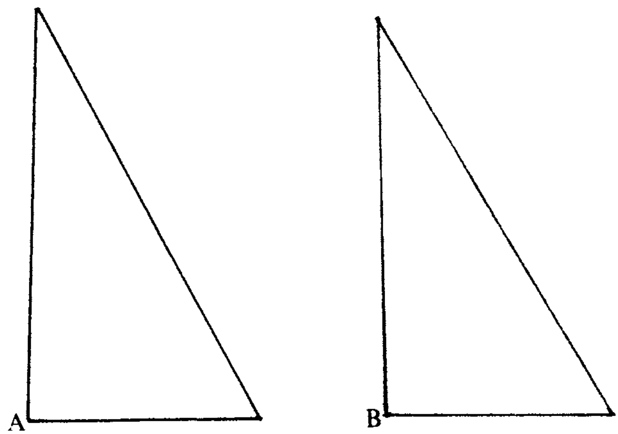


FIGURE 10
Approximations of Right Triangles

No one has yet succeeded in constructing a triangle to these specifications or measuring so small an angle. To do so would require a material that is more rigid and less granular than the finest glass. Enough is known about the nature of matter so that it can be said that no one will ever produce such a material. The ideal of the irrational number is unattainable. From an extreme practical point of view, there is no need for irrational numbers. Yet ideals, even unattainable ideals, are extremely useful, not only to society, but to individual people. Therefore the invention of fictional straight lines and irrational numbers must not be held in contempt, even if it was stimulated by a superstition.

Abbe and Zeiss did not work only with such mathematical ideals. It has been seen that they conducted their business in such a way that their people were somewhat protected from misery. Complete protection from misery is most likely another unattainable ideal. It is not a mathematical ideal, however. Zeiss and Abbe did not postpone their sociological innovations until they were able to measure human misery and construct a mathematical theory of it.

In fact, it is unlikely that they spent a moment thinking about such numerical procedures, and this was not because Abbe was mathematically inept and unimaginative. It is much more likely that they recognized that mathematics, like logic, is inhuman, and that mathematical theories of people are impossible.

Euclid of Alexandria and the Elements

Most of our knowledge of the technical aspects of classical Greek mathematics is derived from the writings of Euclid of Alexandria. Even though he was prolific, not all of his writings have survived. Some earlier writings, by other authors, have also survived, but none are as complete or as detailed as those of Euclid. There were histories of the development of arithmetic, geometry, and astronomy during this period; one was written by Eudemus, who studied with Aristotle. Aristotle and Speusippus wrote about the Pythagoreans; it is most unfortunate that their writings on this subject have not survived, for they might have altered the present low opinion of Pythagorean ideas. Xenocrates wrote histories of science that have already been mentioned. None of these histories have survived, except in fragmentary form. Very likely, most of these writings were available to Euclid, possibly in Alexandria, where the great Library was being assembled during his lifetime. There is some evidence that he studied with Aristotle, and this would most likely have been in Athens. By the time of Proclus, the Library of Alexandria had been destroyed, but many of the early writings may very well have survived elsewhere and have been available to him. Almost nothing is known about the building that housed Plato's Academy, or of its history. Presumably, it housed a library and archives, but whether they were intact in Proclus' day is unknown.

Proclus lists various mathematicians, and then goes on:

> Those who compiled histories bring the development up to this point. Not much younger than these is Euclid, who put together the Elements, collecting many of Eudoxus' theorems, perfecting many of Theaetetus', and also bringing to irrefrangible demonstration the things which were only somewhat loosely proved by his predecessors. This man lived in the time of the first Ptolemy. For Archimedes, who came immediately after the first Ptolemy, makes mention of Euclid: and, further,

> they say that Ptolemy once asked him if there was in geometry any shorter way than that of the elements, and he answered that there was no royal road to geometry. He is then younger than the pupils of Plato but older than Eratosthenes and Archimedes; for the latter were contemporary with one another, as Eratosthenes somewhere says.

From this, it is inferred that Euclid lived about 300 B.C., and that he was primarily a compiler and editor. Eudoxus and Theaetetus were contemporaries of Plato, as has already been mentioned. The popularity of Euclid's compilation may account for the failure of the earlier writings to survive. Proclus greatly admired Plato and Euclid, and it is now believed that this led him to underestimate Theaetetus. Euclid's writings underwent many changes and alterations at the hands of many editors and translators. The effort to restore them to their original form was a major project for nineteenth- century historians and philologists, and is probably not yet completed. The standard English references to this project are the works of T.L. Heath. More recently, the philologist Arpad Szabo has continued this investigation. He has shown that there was great confusion in the terminology used by the mathematicians of the period preceding Euclid; it has already been noted that "mathematics" acquired its present meaning only during Plato's lifetime, and there was no standard dictionary of the Greek language. This makes it easy to understand the problems that confronted later interpreters, and does not simplify the task of restoration.

Euclid wrote very tersely and impersonally, and gave no hint of his motives or personal beliefs. This has invited imaginative interpretations and legends. He wrote about optics and other topics outside mathematics, but these other writings have received relatively little attention. His mathematical writings have enjoyed a popularity that overshadows all his other work. They were soon translated into Latin and Arabic; versions of them are now available in every major language. They have recently been drastically revised, both by eminent mathematicians and by high school teachers writing textbooks for their students. This enduring influence of Euclid's writings is shared with only a few others. In Western Europe, only the Scriptures, and the writings of Plato and Aristotle are comparable. The way in which Plato's writings came to be associated with the Scriptures has already been described; Euclid came to be associated with Plato by reason of another misunderstanding. The conflicting legends about Euclid show that his influence began very early. There is an Arabic legend that his writings were the work of a carpenter named Apollonius. This can only be mythical. It has been seen that all mathematics grew out of the crafts and trades, and that its roots were not severed until about the time of Plato. The significant misunderstanding arose in Rome during the first century A.D., when the influence of Plato on Christianity was just beginning. It was thought that Euclid of Alexandria and Eucleides of Megara were one and the same person. "Euclid" and "Eucleides" are two transliterations of the same Greek name, and it has become conventional to use the different spellings precisely to avoid this confusion. However, this confusion and Euclid's failure to give a clear account of his personal beliefs, led to the conclusion that he held the same beliefs as Eucleides and Plato. This may or may not have been the case. This confusion continued in Europe throughout the Middle Ages. It was not until the time of Columbus that Commandius, a translator of Euclid, was emphatic in distinguishing the two. But the tradition as to Euclid's beliefs had been firmly established, and continues to this day; traces of it are to be found in our dictionaries and elementary textbooks. It is an example of correcting a mistake, once it has been widely accepted.

Proclus' recognition that Euclid was primarily a compiler and systematizer came much earlier. It should focus the attention of scholars on Euclid's predecessors, and their relation to each other. One may hope that such studies will provide a clearer understanding of the Greek contribution to mathematics, but, as yet, much is only conjecture and legend. One general clue to the nature of the Greek contribution is to be found in the original title of Euclid's major work; it did not contain either of the words "mathematics" or "geometry"; it did contain the Greek word *stoicheiota*. When his early admirers wished to express their appreciation of his uniqueness, they did not call him "THE geometer" they called him "THE *stoicheiotist*." In English, the word "letters" may mean either the alphabet or pieces of correspondence; *stoicheiota* had a similar double meaning: the alphabet or a certain literary form (not correspondence). It is therefore conventional to translate the word as "element," even though this also invites confusion (for example, with the chemical elements). In this sense, an element consists of two parts. The first is the theorem or proposition. These words are the modern, conventional translations of the Greek word *protasis*; it seems that it would be more literal to translate it as "enunciation" or "statement." The second essential part of an element is the proof or demonstration of the theorem, the chain of reasons for believing the theorem to be true. The Greek word was *apodeixis* and modern philosophers often use the adjective "apodictic" in place of "demonstrative" or "demonstrable." In what follows, "demonstration" will be preferred to "proof," primarily because it became customary to end an element with the triumphant "Q.E.D." (*quod erat demonstrandum*). Many translators of Aristotle also use "demonstration" rather than "proof." Sometimes "proposition" is used to designate the entire element, not only the theorem. This ambiguity would be sufficient reason for preferring "element," but there is another. Since the Dark Ages, a proposition has been something that may be either true or false, but the demonstration of an element is supposed to make certain that the theorem is not false. While an element is not addressed to a single person, there is every reason to suppose that its writer often submitted it to his friends for criticism. Having written out his arguments, they could be carefully reviewed, and perhaps refuted, by his colleagues.

The replacement of simple dogmatic enunciation by the publication of a complete element is one of the most important intellectual achievements of the classical Greeks. Unfortunately, they were not consistent, and often lapsed into dogmatic assertion. Still, this achievement ranks with Plato's invention of the written dialogue, and the two cannot be considered separately. They were not the only intellectual or literary achievements of the classical Greeks. Thucydides and Xenophon, both contemporaries of Plato, wrote factual accounts, histories, of events in which they had participated, had observed, or of which they had received reliable accounts. The traditional songs of the bards were written out, and Euripides and Aristophanes produced new versions, and even wrote secular plays that were evolved out of earlier religious rituals. One does not detract from these achievements by noting that they rested on solid foundations laid by people who lived thousands of years earlier, who were known to the classical Greeks only by fragments of tradition and legend.

All of these new writers used writing materials more lavishly than ever before, which usually implies that they had wealth as well as leisure. Plato, Thucydides, and Xenophon were involuntarily prevented from continuing to participate in the political and military affairs of their time. Other writers may have stood aside voluntarily, which is a departure from the aristocratic disdain for anything that does not involve the forcible domination of other people. It was the beginning of the intellectual aristocracy, of the intelligentsia.

Plato's dialogues and Euclid's *Elements* have this in common: their writers recorded their soliloquies at some length. In another respect, they are antithetical. In Plato's dialogues, one of the participants is induced to express a personal opinion, whereupon Socrates presents the reasons why this opinion is wrong: this is the essence of Socratic agnosticism. In the *Elements*, the opinion is expressed impersonally, and the reasons for its correctness are given with the same impersonality. Much of Plato's work thus has a negative, skeptical character, while Euclid's is positive and affirmative. Proclus does not list Plato as a writer of elements, and there is no evidence that he was. It is more surprising to find that no geometric theorem is subjected to Socrates' destructive criticism in any of the dialogues. Few people today would dream of provoking an argument about mathematics, much less of emerging victorious. This suggests that mathematics had already acquired its awesome sanctity in those early days, but this may only have been Plato's personal idiosyncracy, which later became widespread as his dialogues and Euclid's *Elements* became central to the European educational system.

Prior to Euclid, and even during Plato's lifetime, there seem to have been lively arguments about geometry and arithmetic. They seem to have been illustrated with diagrams, but these were later considered to be memoranda, concessions to the fallibility of human memory, rather than as an essential part of the reason for believing the theorem. Most of Plato's Dialogues are devoted to non-mathematical subjects; the reasoning is less complex than in the *Elements,* and the arguments are more easily remembered. Their mathematical parts are, as has been seen, parenthetical and illustrative. They invite the audience to adopt a more abstract, impersonal view of the matter under discussion, and to search their souls for ideal concepts similar to the straight line without thickness. Since the evolution of these geometric ideals had required an age-long study of actual cases, it should not be surprising to find that this sudden invitation resulted in surprise and confusion rather than success. Plato's negative conclusion, as summarized in Letter VII, becomes understandable. It is an extreme version of the Pythagorean doctrine of conversation with the divine. It seems to foreshadow the later doctrine of meditation followed by the revelation of truth. Nothing of the much earlier history of the cord and peg, of the origin of arithmetic among the merchants and craftsmen, is recalled. Even the ethical doctrines, such as they were, that made ancient trade possible, are excluded from Plato's aristocratic writings.

This places the *Elements* in perspective against the background of the more general intellectual activity of classical Greece, and has prepared the ground for their more detailed consideration. The theorem of one element can be cited as one of the reasons for believing the theorem of another. This is a convenient simplification: one might also repeat the whole demonstration of the original element. The avoidance of such repetitions is clearly justifiable and efficient. It does involve the danger of circular reasoning; it is therefore necessary to arrange the elements in a serial order, such that no theorem is cited in the demonstration of an earlier one, but only in later elements of the series. This results in a very formal kind of literary work. Unfortunately, this literary genre has received no general name, unless one interprets Heath's phrase, "book of elements" as such; "series of elements" might be better. For the philologist, or even for the nineteenth-century scientist who had some knowledge of Greek, the emphasis on serial order was unnecessary. The letters of the alphabet are arranged in a standard sequence, and the *stoicheion* was derived from an earlier Greek word meaning "to go in a row." It will be simplest to use "element" when a single theorem and its demonstration is meant, and "Elements" for a series of them in a suitable order.

The recognition that Euclid's "Thirteen Books" are a synthesis of earlier Elements makes it certain that these earlier ones were less complete than Euclid's. They may also have contained evidence of the motivations of their writers, if only by the nature of the problems that they considered. Lacking this evidence, one must resort to less direct inference, which may amount only to conjecture. One may begin by considering what is known about the men whose writings Euclid collected and edited. They lived and worked during the period when biographical data concerning others besides kings and generals were just beginning to be preserved, though not with great accuracy. There are usually several versions of every biography, and one must either choose one, or else become involved in tedious controversies. In general, I have adopted a version that is to be found in one of our standard encyclopedias; I have departed from this only when there seemed to be compelling reasons.

According to Proclus, the first writer of elements was Hippocrates of Chios. Like Cos, Chios is an island near the coast of Asia Minor, but farther north, about sixty miles northwest of Samos, and a part of Ionia. Unlike the famous Hippocrates of Cos, Hippocrates of Chios was not a physician; he seems to have been a teacher of mathematics. He lived after Pythagoras and before Plato, about the time that the two Greek systems of numerals were coming into use, and the Greek language was displacing Akkadian in commerce. It appears that, besides writing a series of elements, Hippocrates solved the problem of finding the areas of certain crescents, bounded by arcs of two circles. This involves the solution of quadratic equations, presumably by geometric algebra. The Old Babylonians had previously prepared tables for the numerical solution of some quadratic equations. Whether this is coincidence or whether it implies that Hippocrates knew of the Old Babylonian mathematics, is an unsolved problem. One other legend about Hippocrates has come down to us: he taught mathematics for pay. This seems trivial to us, but the aristocratic Pythagoreans and Platonists considered it deplorable; perhaps this makes the legend certain.

Proclus also mentions Leon and Eudoxus (who have been mentioned above and will become increasingly important later) as early writers of series of elements. Nothing is known of Leon except that he was older than Eudoxus (408-355 B.C.) and younger than Plato (427-346 B.C.). Durant gives the following version of Eudoxus' career, and Dreyer, an abbreviated account that is not incompatible with Durant's. Eudoxus was born on Cnidus, a very small island just south of Cos. It may very well have been the home of a dissident branch of the medical cult on Cos. Eudoxus is said to have left the island at the age of twenty-three, to study medicine at Locri, in Italy; then to study geometry and mechanics with Archytas; and finally to have come to Athens on Archytas' advice. He was very poor and lived at the waterfront, five or six miles from the Grove of Academe, but studied there with Plato for some months. Some say that Plato expelled him from the Academy. Then he returned to Cnidus, and went on to Egypt, where he stayed more than a year. Later he settled for some time at Cyzious, where he established a school. Cyzious was a great port on the southern shore of the Sea of Marmora, near modern Gallipoli. At the age of forty, he moved his school to Athens, teaching (for money) not only mathematics, but ethics, mechanics, astronomy, geography, and meteorology. Like his teacher Archytas, he cited experiment and observation as sources of knowledge. He and Archytas were therefore interested in the obvious changes occurring around them, which Plato considered misleading illusions, distracting one from the search for the eternal, immutable realities beyond the stars. Eudoxus' book on astronomy is said to have been the "greatest in antiquity," and to have stimulated the study of that subject; much of it was preserved by Aristotle. Although its ideas are far from modern, they are not despised by modern astronomers for reasons that will be discussed later. Eudoxus was one of Plato's most formidable intellectual rivals, and certainly did not subscribe to the Eucleidean belief that knowledge is independent of perception. Ultimately he returned to Cnidus and had an astronomical observatory built for himself; perhaps this depleted the savings from his years of teaching. It was not a very good observatory; of course it had no telescope, but neither did Tycho Brahe's. Even so, its instruments must have been primitive, though ingenious; it scarcely rose above the roofs of surrounding houses. But Archimedes would cite his observations, and one of his own pupils is said to have used them to predict a solar eclipse. His series of elements is said to have furnished Euclid with the material for his Book V; if so, they were largely concerned with the geometric theory of ratios.

Theudius of Magnesia is said, by Proclus, "to have put together the elements admirably, making many limited theorems more general." There were two Magnesias: the one a city in Asia Minor about twenty-five miles inland from Miletus, the other was the coastal region of Thessaly. It is not known which was

Theudius' birthplace, or who taught him. He is said to have become a member of Plato's Academy, and his series of elements to have been its mathematics textbook. There were other mathematicians who were said to have worked with Plato and Theudius during these years. Because of the general admiration excited by Plato, it is difficult to be certain that this relation has not been overemphasized. Menaechimus and his brother Dinostratus were presumably Athenians, and are said to have been closely associated with the Academy. Yet Menaechimus is also said to have been a pupil of Eudoxus; one may surmise that Athenaeus of Cyzius was another. It is thus reasonable to suppose that these men were closer to Eudoxus than to Plato. Amyclas of Heracleia is said to have worked with Theudius; there were several cities named Heracleia, and nothing more seems to be known about his birth or early teachers. Theaetetus was an Athenian, but Plato himself states that his teachers were Theodorus of Cyrene and that Theaetetus was already becoming an independent worker at the time of Socrates' execution, long before the Academy was started. Theodorus was older than Plato, but is said to have spent many years in Athens before being forced to return to Cyrene because, according to nineteenth- century historians, "he was an atheist." Cyrene was a city on the North African coast, settled some two centuries earlier by colonists from, of all places, the island of Thera. Within another generation, it would develop its own school of philosophy, about which more will be said later. Plato is said to have visited Theodorus in Cyrene after leaving Megara and to have studied geometry with him, but this is part of a legend that is internally inconsistent.

Heracleides of Pontus is said to have been trained in mathematics at the Academy; Pontus was a province north of Mesopotamia proper, and extended to the southern shore of the Black Sea. Proclus also mentions Hermotimus of Colophon as the "discover of many of the elements"; Colophon was in Asia Minor, an island city of Ionia not far from Miletus. There is no suggestion that Hermotimus was ever with Plato.

Although this list of pre-Euclid mathematicians has been known for a very long time, no historian seems to have commented on its most obvious characteristic. Very few of these men were Athenians, and even fewer are mentioned in Plato's dialogues. It seems certain that not all of them studied at the Academy, or shared Plato's opinions. Yet there is still a general belief that Euclidean geometry originated in Athens, and that Plato's "great love for mathematics" nourished its development. There is no evidence to support this belief. A glance at the map suggests, rather, that there was great interest in mathematics throughout all the Greek-speaking lands of the Mediterranean area, and most likely many cities in which one could obtain good instruction in geometry. The frequent recurrence of Miletus and Samos in the above suggests that this interest may have originated there, and then diffused to the rest of classical Greece. According to legend, when Plato left Eucleides in Megara, he first went to Taras (the modern Taranto in Italy) and there learned the Pythagorean doctrines from Archytas, with whom he maintained a lifelong friendship and correspondence. From there, he went to Locri (also in Italy) and studied astronomy with Timaeus; then he visited Egypt, Syracuse, and Cyrene, though in which order is not made clear. At Cyrene, he studied mathematics with Theodorus. This whole legend has been thoroughly discredited, for reasons that have already been mentioned. Yet it can still be found in recent and respected histories. Perhaps the visit to Archytas is authentic, and may have contributed to the influence of Pythagoreanism that is evident in the dialogues. Archytas may also have been the prototype of Plato's ideal philosopher-statesman. Plato's letters indicate a long friendship with Archytas, and they may have exchanged visits on several occasions. Concerning Plato's own stay in Syracuse and his subsequent enslavement, it has been noted that there are various versions of this part of his biography. However, they agree that this visit to Syracuse was just prior to the incorporation of the Academy. He is said to have visited Syracuse three times, the third time being shortly after Theaetetus' death, and the death of the king, Dionysius I, who had enslaved him. One must strongly discount the supposition that Plato himself made any great contribution, directly or indirectly, to the materials with which Euclid worked. The legend that he inscribed the door of the Academy's building with the warning "Let no one enter here without geometry," seems to have arisen in Roman times, some centuries after Plato lived. It is also difficult to reconcile with the generally accepted belief that Theudius taught geometry at the Academy.

From this biographical material, it is clear that not all of Euclid's predecessors were Athenians, and that it is improbable that even all contemporaries were ever assembled there. They must often have communicated primarily by writing. This is confirmed by the writings of Archimedes, though he lived somewhat later than Euclid, and quotes both him and Eudoxus. Of Archimedes' surviving works, most are *Elements*. Some of them are preceded by what may be described as a letter of transmittal, a personal and explanatory communication to the recipient. There were no publishing houses, and interested individuals made a copy of a manuscript for their own use, or paid a scribe to make a copy.

Aristotle and the Axioms

Keeping these facts in mind, one can form reasonable opinions about the materials that Euclid synthesized into his "Thirteen Books." They were primarily Elements of geometry, but he also used other material. Some of this was obtained from Aristotle; he may also have been somewhat influenced by Plato. Even though there were Elements on astronomy, optics, and mechanics, they were not included in the "Thirteen Books," but Euclid may have included them in his other writings. Since these other writings have not been as influential as the "Thirteen Books," there is a tendency to restrict the term *Elements* to geometry; this was certainly the way in which Proclus used the term. However, the literary form of the *Elements* does not impose any restriction on their subject matter. This was much later recognized by Newton. *Elements* are not even restricted to the exact sciences, any more than a written dialogue is necessarily skeptical and negative. This last was recognized by Galileo and Berkeley. These are all matters that call for investigation, and are not unrelated.

The geometrical *Elements* available to Euclid must have been incomplete, from both his point of view and from ours. One could combine and unify them, and one could enlarge them in two directions. One might find more complex and detailed theorems, and demonstrate them by citing the elements already written. One might also become doubtful of the reasons cited in the demonstration of existing elements; then one would devise demonstrations for simpler and more general theorems, and cite these in new versions of the dubious elements. If the theorem seemed sensible and only the reasoning was dubious, there would be a tendency to bias the new, simpler elements so that they supported the original theorem more effectively. In recent times, mathematicians have admitted this frankly; most of them consider it a legitimate way of generating new ideal concepts. Many of the more recent editorial corrections of Euclid's *Elements* have been of this kind. This is the reverse of negative, skeptical criticism; there is a tendency to call it "positivism."

One may therefore say that the series could be extended both in the direction of simpler elements, and in the direction of more complex elements. This was recognized by Aristotle and his contemporaries, but they did not name the two directions. Today, we call the direction of more detail "deductive" or "demonstrative," and the other direction is called "inductive" or "inferential." We also speak of "deductive logic" and "inductive logic." Deductive logic is usually considered to be more conclusive and certain than inductive logic. It has been noted that extension in the inductive direction is likely to be biased by theorems that one believes to be true and wishes to support. In this respect, inductive logic resembles rhetoric: the objective of rhetoric is to persuade others to accept one's own belief. In Aristotle's time, no word had been coined for logic as we think of it today; this must not be forgotten when modern words are used in discussing ancient writings.

According to Szabo, there was a long debate about the extension of the series in the inductive direction. Some maintained that it could be extended indefinitely, others that it could not. If there were theorems so simple and general that they could not be demonstrated, but yet were known to be true, these would be a sort of absolute knowledge that could be put into words. One may surmise that Plato rejected this view, since he believed that absolute knowledge could be obtained only by rare individuals, by a process resembling revelation. It seems to have been Aristotle who, in his attempt to systematize science, gained acceptance for the idea that there are sentences so simple that everyone knows they are true, that they are self-evident. This difference between the views of Plato and Aristotle is fundamental, although the exact words used above are modern. It is not known how this difference influenced their personal relations.

Since there was this debate about the extension of Elements in the inductive direction, it is remarkable that no one seems to have raised the issue of their extension in the deductive direction. Such an attempt might lead one to theorems as complicated and elaborate that they too cannot be demonstrated. This does not seem to have occurred to anyone until the present century. When it was shown, by Kurt Godel and Alonzo Church, that there are such undemonstrably elaborate theorems, both mathematicians and philosophers were startled. One might expect that psychologists would consider the implications for the doctrine of psychomathematical parallelism, but this has not yet happened. These elaborate theorems are certainly not self-evident.

Turning now to Aristotle's biography: he was born in 384 B.C., in Stagyra, a city just east of Macedonia, which had been settled by colonists from Ionia in Asia Minor. His father was, at some time, physician to King Amyntas II of Macedonia, the grandfather of Alexander the Great. Aristotle's early education may have been directed toward medicine. At the age of eighteen, he went to Athens, but it was not until later that he began to study with Plato at the Academy. It does not seem to be known how long this period lasted. He remained in Athens for twenty years, until Plato's death. Some of his more recent biographers believe that during these years he conducted a school whose curriculum included rhetoric, the subject that Plato despised and which is related to logic. This may have been another reason why he was not elected Plato's successor as head of the Academy. It is usually said that this turn of events disappointed him, and caused him to leave Athens. In any case, he left Athens at about the time of Plato's death, and five years later he became the tutor of the young Alexander of Macedon. Falling out of Alexander's favor, he returned to Athens. It is certain that he then established a school, known as the Lyceum. Its curriculum again included rhetoric. After Alexander's death, Aristotle was accused of impiety, and hastily escaped from Athens to Euboea, where he died soon after.

Aristotle wrote on a great variety of subjects. Some of his writings were in the form of dialogues, others were scarcely more than notes intended for his own use. Some were certainly carelessly worded, and others seem to have recorded changes in his opinions. His writings were edited and compiled, possibly by his students. These versions were later translated into Arabic, and thence into Latin. One of these derivative versions became available to scholars in Italy at about the same time that Fibonacci introduced the Arabic numerals from North Africa. This Arabic edition had been prepared shortly before by the scholar, Averroes, in Spain.

It is generally conceded that Aristotle was the founder or inventor of logic. The writing of elements naturally posed a problem: Is this demonstration valid or invalid? Aristotle attempted to solve this problem by examining the form (syntax or grammar) of the sentence, as well as the meaning of the words or phrases in the sentence. He restricted himself to sentences having the form of a syllogism. He wrote, "A syllogism is an argument in which, certain things being posited the premises, something other than the premises necessarily follows from their being true." This contains no reference to sentences, but it must be remembered that the distinction between a sentence and a proposition had not yet been introduced. In practice, he often used a considerably narrower definition; most of his syllogisms had a form similar to "A is B and B is C; therefore A is C," where all three "A is B," "B is C," and "A is C" are correct sentences; all three are subject-predicate sentences. He recognized only a few variants of this form as valid. According to Averroes, the Greek physician Galen (130-201 A.D.), recognized that other variants were equally valid, and these were added to Aristotle's list. These were usually known as the deductive syllogisms. Aristotle also recognized another, which became known as the inductive syllogism. It has the form

A,B,C,... are all of M, and each of A,B,C,... is an N;
therefore every M is an N.

Note that all the constituents of M are enunciated, and that these may only be some of the constituents of N. Again, one is dealing with three subject-predicate sentences. Somewhat after the time of Galen, the distinction between a sentence and a proposition was introduced into logic. Until the nineteenth century, no other modification of this formal scheme was made; however, Aristotle and his successors considered that logic was more than a formal scheme, and differences of opinion developed in this area. For the present, only Aristotle's views will be examined.

Passages from his writings, (taken from the Introduction to Heath's "Thirteen Books of Euclid's *Elements*"), mention a great variety of topics, expressing opinions concerning them, but all very briefly. It is not a finished literary production, especially when the editorial inserts are deleted. It is the sort of thing a teacher might jot down at the end of a vacation, in anticipation of the start of a new class. It contrasts from the extract from Plato's Letter VII, not only in its lack of polish, but in its general matter-of-factness; only the reference to the "reason dwelling in the soul" has a mystical content. Plato and Aristotle are irreconcilable, "unless indeed it be asserted that any audible speech is an hypothesis."

In the extract referred to above, there is only a passing reference to "the common axioms, so called." In a later passage from the same work he distinguishes between axioms, hypotheses, and definitions. It has been seen that, both before and after Aristotle, there was a tendency to suppose that when a thing has been named or defined, it existed. This tendency is still apparent today; the insistence that existence requires a demonstration, or else an assumption is one of Aristotle's most important achievements.

Historically, equally important is his view that an axiom is "that which it is necessary for anyone to hold who is to learn anything." It is unfortunate that he does not give his reasons for believing that there are axioms. The passage has been interpreted in various ways, usually in the context of "reason dwelling in the soul." It has been held to imply that anyone who doubts an axiom is of unsound mind. It has also been held that there are axioms of religion, and that the soul of anyone who doubts them will be eternally damned. Divested of theology, the assertion that there are axioms in the Aristotelean sense is an assertion about the psychology of learning: there are some things that everyone knows and that do not need to be taught. In this form, the assertion has survived in our dictionaries and our educational system. It may be the reason why many people find it difficult to learn science.

The question of Aristotle's influence on Euclid is complicated by the terminological confusion mentioned above. It is apparent that Aristotle sometimes used the phrase, "common opinion" rather than "axiom." Some of his editors used "common opinion" in the same way. "Common opinion" must mean an opinion that is widespread among people, or even universal. "Common principle" may mean a principle that is common to all demonstrative sciences. There were also other terminological variations in the writings of the time.

According to tradition, Euclid opens his treatment of geometry with seven axioms. This may have been the case with some early edition, but according to the accepted restoration of Euclid's text, he nowhere used the word. The first book opens with a long list of definitions, five postulates, and five common notions. The last are:

Common Notions
1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

It seems clear that Aristotle would have called all of these "axioms," "common opinions," "common principles," or "syllogistic principles." The postulates are:

Postulates
1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.
4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

One need not quibble because the sentences are incomplete; they are intended to assert the possibility of geometric constructions. It seems reasonably certain that Euclid used "postulate" in Aristotle's sense: something "which is rather contrary than otherwise to the opinion of the learner, or whatever is assumed and used without being proved, although matter for demonstration." However, this has not been the generally accepted opinion, and many have considered them to be axioms. In particular, the fifth postulate is equivalent to the Axiom of Parallels that would later become the center of a controversy. It has, however, been noted by many that Euclid never invokes the fifth postulate in a demonstration if he can avoid it. The controversy might have been avoided if the postulate had not become generally accepted as an axiom.

The first few definitions are:

Definitions
1. A point is that which has no part.

2. A line is breadthless length.
3. The extremities of a line are points.
4. A straight line is a line which lies evenly with the points on itself.

Perhaps it is not certain that Euclid's definitions are to be interpreted in Aristotle's sense. If they are, then they do not assert the existence of the things defined. Euclid's points are definitely not pegs, and his lines are not cords. If he does not assert their existence, they may be interpreted as fictional ideals. However, this is not the view that would be adopted by later writers. These definitions would be considered as asserting the existence of points without parts and breadthless lines. This raises Plato's problem: How can people know of the existence of such things? His answer was that they cannot. Later writers seem to have wanted a less negative answer, and supposed that this knowledge was axiomatic, self-evident. It required no teaching or learning. The existence of these ideals was certain, and required no demonstration. This appears to be a remnant of the early confusion of Euclid of Alexandria with Eucleides of Megara. Perhaps Proclus also contributed to this view of the matter. In any event, this view was held until quite recently, and perhaps still has adherents.

Aristotelianism and Christianity

There is a theory that theology and science are incompatible, that they involve different theories of knowledge. Theology is suppose to rely on revelation and faith, science on sense-perception and self-evident common sense. It is also traditional that this conflict became acute at the time of Galileo and Newton. Galileo's persecution is often cited as evidence that theology is authoritarian; similarly, Newton's freedom to publish his opinions in England is considered as evidence that science is democratic and individualistic. As is often the case, an examination of the historical settings in which Galileo and Newton lived shows that this tradition is an oversimplification.

Early Christianity was based on the testimony of the Apostles, on their personal acquaintance with Jesus. They told the story of his sayings and actions, and of the sayings and actions of others. Whether one believes the story or not, it is a simple one, based on the Apostles' reports of their sense-perceptions. Since Jesus and the Apostles were Jews, the Old Testament was incorporated into Christianity as a matter of course. The Apostles and their early converts were neither scholars, scientists, nor theologians. First-hand accounts of the early Christian community that formed after the time of the Apostles are rare. Accounts by non-Christians all express amazement at the Christians' love for one another, at the concern of each for the welfare of all the others. The Christian Church was organized quite early, but the authority of its bishops was limited to excommunication, to depriving a person, temporarily or permanently, of membership in the Christian community.

Alexandria was an early center of Christianity, and its scholars seem to have added some of Plato's doctrines to Christian theology, as has already been noted. During the early Middle Ages, Christianity spread over all of Western Europe. Most people were illiterate and learned about the faith from their local missionary priests; most of whom were also illiterate and learned Christian doctrines by oral instruction. For one reason or another, other local pagan beliefs and rituals were often mingled with Christianity. Among educated persons, some knowledge of Plato was preserved; Aristotle seems to have been remembered only as the originator of formal logic, which was of no interest to ordinary people. Plato's skeptical negativism was converted into a positive belief in divine revelation following prolonged ascetic meditation.

Revelations were often in the form of visions, which were reported as if they were simple sense-perceptions. Thus Plato's doubts concerning the senses as a source of knowledge was converted to, or at least replaced by, that uncritical acceptance of direct sense-perception and hearsay which we call superstition. Under these conditions, it was inevitable that many versions of the Christian doctrine would evolve. By the end of the medieval period, there was a need for an authoritative manual or reference book. Various men attempted to supply this need.

By the thirteenth century, literacy had increased; Fibonacci was writing about the Arabic numerals. Latin translations of Arabic (Moorish) writings became available. These became popular, though only with those whose education included Latin. These scholars might be laymen or churchmen; as yet, not all churchmen were scholars. Aristotle's writings on topics other than formal logic became available. The Dominican brother, Thomas of Aquinas, devoted himself to constructing the system of Christian theology and philosophy, which is now known as Thomism. It ultimately became very influential and a modified version is still influential with Catholic intellectuals. It was not without rivals; the Franciscan brother, Duns Scotus of Oxford, wrote a critique of Thomism that was equally influential at the time. When Thomas of Aquinas encountered a problem for which the Scriptures gave no ready solution, he relied on Aristotle, or rather on Averroes' edition of Aristotle's writings. Ultimately Thomism gave the Aristotelian philosophy a prestige that it might not have otherwise acquired. It seems that Scotus was especially critical of those aspects of Thomism that were derived from Aristotle.

For the present purposes, only two aspects of Thomism are of importance. Firstly, it emphasized, and perhaps exaggerated, Aristotle's doctrine that there are axioms that no right-thinking person will deny. This now reinforced the earlier doctrine of heresy (wrong thinking on the part of a Christian), and made it seem perverse, rather than merely wrong. Secondly, since neither the Scriptures nor Aristotle mention the heliocentric theory of the solar system, Thomism tacitly assumed the geocentric theory.

But Thomism was an intellectual invention, and did not claim to be revealed truth. It was therefore entirely possible for devout Christians to doubt both it and Aristotle. It is only remarkable that these doubts did not become widespread earlier than the sixteenth and seventeenth centuries. During this interval, the secular power of the Roman Church increased greatly, at least in Western Europe. Theoretically, it was centered in the Pope and the Vatican, but it was exercised either by local ecclesiastics or by the local kings. After Charlemange, one king was elected the Holy Roman Emperor and became the Pope's secular and military defender. Theoretically, the objective of this secular power was the elimination of pagan elements from Christianity. This, together with the revival of Roman law and its use of torture as a judicial procedure, resulted in the torture and burning of witches in great numbers. All local pagan gods were identified as the Devil or his minions.

However, in scholarly circles, knowledge of the classical Greek and Latin writings increased. Doubts about Thomism or Aristotelianism persisted or increased. Interest in other aspects of Greek thought (e.g., science, medicine, and mathematics) also increased, and the Vatican did not frown upon it, but often actively participated in furthering this interest. The original writings of Aquinas could be regarded as the "old theology."

Pythagoreanism Becomes Rationalism

While they were not the first to express such doubts, Galileo and Newton are often considered as the leaders of this skepticism, and Galileo certainly expressed it explicitly, though respectfully. He wrote:

> Such is the greatness and authority of Aristotle that it is difficult and dangerous to write against his teachings, and to me in particular since I have always held his wisdom a matter for admiration. Nevertheless, impelled by a zeal for truth, by the love for which, if he were living now, he would be activated, I have not hesitated in the interests of all to state wherein the unshakable foundations of mathematical philosophy force me to dissociate myself from him.

Galileo's attitude toward Aristotle and mathematics was similar to Proclus' attitude toward Euclid and geometry, but most of Galileo's writings are more closely related to those of Pythagoras and Archimedes than to the writings of Aristotle.

Both Newton and Galileo were devout and believed that they were discovering the laws imposed on the universe by God. They did not use this Pythagorean phrase, but they believed they were "conversing with the divine" and learning that there is mathematical order in the world. Galileo wrote: "Truth is written in the Great Book of Nature, but only he can read it who can decipher the letters in which it is written." This is a concise statement of the philosophical doctrine of empiricism. It is very different than the doctrine that truth is written only in the Scriptures; but while this doctrine is still held by some fundamentalist sects, it was not widespread among the scholars of Galileo's time. Neither does Galileo say that truth is written only in the Book of Nature. His formulation of empiricism does raise an important question, however. Leaving personal egotism aside, what people can decipher the Book of Nature? At least two books, Herbert Wendt's *In Search of*

Adam and J. Allen Hynek's *The UFO Experience*, provide adequate documentation for the part that personal bias has played in two different fields of empirical science. The question can also be formulated as: How can truth be distinguished from superstition?

The question is therefore not an idle one. When it is answered that, "The Book of Nature is written in the letters of logic and mathematics," then Galileo's doctrine becomes rationalism. The monk, Roger Bacon, a contemporary and colleague of Duns Scotus at Oxford, seems to have been the first to advance the doctrine of rationalism. He held that mathematics applied to observation is the only way to arrive at knowledge. For this and other "novelties," including gunpowder, he earned the enmity of his fellow Franciscans and suffered a lengthy imprisonment. In those days, long imprisonment resulted in malnutrition and general ill-health, if not in specific disease. Bacon died shortly after being released.

Rationalism is even more closely related to Pythagoreanism than is empiricism. Newton seems never to have made explicit reference to rationalism, but there is evidence that he shared Galileo's philosophy. Statements of his friends confirm his belief in a divine Creator, who has established a relation between Himself and human beings. Newton compared himself to a boy on the shore of the sea of knowledge, who had been fortunate and found a pebble with unusually bright colors. He also said that he made no hypotheses. It is generally considered that he was one of the three or four greatest mathematicians who ever lived, and this seems to have contributed to the spread of rationalism as defined above. However, it is difficult, if not impossible to identify philosophical doctrines; it is only possible to identify individual philosophers. The English philosophers, Locke, Hume, and Berkeley are said to have been empiricists; the Continental philosophers, Leibniz, Kant, and Descartes are said to have been rationalists. Each philosopher seems to have borrowed from all others, and to differ or quarrel with a few others. Both Newton and Galileo, and many of their successors, believed that the laws of nature are fixed and immutable, independent of the people who put them into words. Even though these laws govern the changes that are so obvious in nature, they thus acquire some of the characteristics of Plato's realities. This belief will require further investigation.

Returning to Galileo, tradition places more emphasis on his persecution by the Roman Inquisition than on his achievements. He was a man of attractive personality and an excellent teacher. His lectures brought students from all over Europe to Padua. They returned home and spread the knowledge of his achievements. In short, he became an internationally respected figure. He was not infallible, and was often irascible. He proposed a theory of the tides which is now admitted to be untenable. When a contemporary scholar pointed this out, and proposed a somewhat better theory, Galileo would not admit his error. Instead, he used his literary ability to ridicule his opponent, and refused him the courtesy of debate. This and other tactless discourtesies soon made powerful enemies. It is not unusual for a man to have both friends and enemies, and to be caught in the no-man's land of their conflict. The theory of the tides was a comparatively minor episode. Galileo also thought that the heliocentric description of the solar system could also be read in the Book of Nature. In his enthusiasm, he endeavored to convince others of this. Even his well-educated friends must have been confused; they had never heard of rationalism as Galileo's formulation of this doctrine was not published until later. Some of them were familiar with Pythagoreanism, but Galileo seems to have made no explicit reference to it, so that it is possible that even this connection with familiar ideas escaped them. Neither the Scriptures nor Aristotle, nor even Thomas of Aquinas mention the heliocentric solar system. Knowing that he was a devout Catholic, his friends could only be confused. His enemies could doubt the sincerity of his professed devoutness.

On February 25, 1616, the Roman Inquisition declared that belief in the heliocentric description of the solar system was heretical. We call the heliocentric system "Copernican" because it was Copernicus' book about it that was placed on the Index of Prohibited Books. Yet it has been seen that Archimedes and others had considered the heliocentric theory as an alternative to the geocentric some two thousand years previously. Copernicus had obtained the suggestion by reading various Greek authors. His book had been published more than seventy years earlier, at the end of his life. At that time, Martin Luther had tried to prevent its publication, but Pope Clement VII had sanctioned it. It had circulated freely during the years since then. It was no accident that Galileo was in Rome when the Inquisition reversed Pope Clement's decision; his missionary zeal for the heliocentric theory was well known and he presumably renewed his efforts on its behalf during this visit. On the next day, February 26, he was informed of the Inquisition's action by Cardinal Bellarnini. Bellarnini was a Jesuit, noted for his intellectual interests and moderate views. He and Galileo had had many friendly conversations in the past. However, this was no ordinary conversation: Bellarnini was acting on written instructions from the Inquisition. On March 3, Bellarnini submitted a written report to the Inquisition, stating that Galileo had abjured his belief without protest. This left Galileo, like all others, free to continue the two-thousand-year-old discussion of the heliocentric hypothesis; it was only forbidden to assert it as true: it must be repudiated as false.

Galileo seems to have obeyed this injunction literally, although it is more than likely that he hoped that the decree of heresy would be revoked. His attempts at refutation were certainly not vigorous. His enemies may well have doubted his sincerity, but they were unable to find legal grounds for proceeding against him. Seventeen years passed, and by 1633, Galileo's tactlessness had earned him additional enemies, probably including his former friend, Pope Urban VIII; Bellarnini was no longer alive. Someone added to Bellarnini's report, without bothering to conceal his handwriting, and without bothering to alter Bellarnini's written instructions. According to this blatant forgery, Galileo had been forbidden to discuss the heliocentric system, even in order to refute it. Since he had discussed it openly for years, the forged evidence made it possible to bring him to trial. He was not accused of heresy; the charge was the technical one of having violated Bellarnini's injunction. However, while it was a technicality, this charge was no means trivial as it placed him in danger of torture and execution.

The trial of Galileo immediately became an international incident that no one, perhaps not even its instigators, wished to escalate. Galileo was tried before twelve cardinals, who were both judges and jury. Ten days elapsed between the end of the trial and the reading of the verdict and sentence. Although there are no records of events during these ten days, it seems likely that the French and Florentine ambassadors to the Vatican were active in negotiating a compromise. The fact of the forgery was not made public, although many of the participants in the affair must have been aware of it. Galileo himself cannot have seen the forged document, but his testimony at the trial is consistent with the original version, and with Bellarnini's written instructions. It is inconsistent with the forged addition. He seems to have been confused by the evidence, but unaware of the forgery. Only seven of the twelve cardinals signed the verdict, but this was not made public either. The sentence of imprisonment imposed on Galileo was remarkably light, considering the temper of the time. It was soon commuted to house-arrest in Florence where he was under the protection of the Duke. He continued to work, and his writings defended the heliocentric theory more openly than ever. This is the foundation for the legend that he muttered, "But still, it moves," as he was being led from the courtroom. These later writings were published in Protestant Holland, but the Inquisition took no action against Galileo. He was certainly no nominalist, and believed in the literal truth of the heliocentric theory.

At his home in Arcetri, across the river from Florence, he was allowed to receive unannounced visitors and speak with them in private. Very likely, one of these visitors took his manuscript to Holland for publication. Another visitor was the young English poet, John Milton, to whom we owe our knowledge of the conditions under which Galileo lived. While the compromise prevented an irrevocable break between France and the Vatican, the trial provided Protestants with material for propaganda. Luther's original objection to the heliocentric theory had been forgotten. In England, Milton was an especially active propagandist. He was certainly ignorant of the forgery. His personal sympathy for Galileo was enlisted, and he was sincerely opposed to the Papacy.

The records of the case were preserved in the Vatican Library, but they were not made public until about the end of the nineteenth century. They have now been examined by various scholars. Rudolf Lammel has written a factual biography of Galileo in which the forgery is discussed, as well as the provocative effect of Galileo's vanity and irascibility. It becomes clear that the accusation and trial of Galileo was not based on broad principles, but was more in the nature of a personal vendetta. Many Vatican officials recognized its potential for harming the cause of the Catholic Church, and may have deeply regretted the whole scandalous affair.

Unfortunately, Lammel's book has not been translated into English. G. de Santillano's book, *The Crime of Galileo*, also presents the results of studies of the trial records, but it attempts to demonstrate that broad moral principles were involved. Almost for this very reason, his book is more than fair to Galileo; it glosses over his personal foibles. It does not mention Luther's part in the affair, or the long time that elapsed between the publication of Copernicus' book and the first action by the Inquisition to supress it.

Today, it seems remarkable that the trial assumed such international importance; perhaps this was unexpected even then. The Reformation was in progress, but it had begun before Galileo was born. The heliocentric theory was not at issue. The issues were numerous and complex; for the present purpose it will suffice to enumerate a few of them. During the Renaissance, there was a strong tendency to revert to older religious forms, in which there was no sharp distinction between secular and religious offices and functions. In the upper levels of the Church hierarchy, there was an interest in both science and scholarship, which Plato and his contemporaries had made aristocratically respectable. At all levels, the less ethical incumbents often served their own selfish interests, political and financial, with ruthless cruelty and misuse of power. In the lower levels of the hierarchy, this malfeasance became crass commercialism. Luther, as leader of the reformers, at first directed his attack on the more obvious commercialism; later he called for a general return to the old theology, by which he seems to have meant Thomism. Very soon, however, there was dissention accompanied by physical violence, both among the reformers and those who supported the established Church. Of immediate concern are the political and international consequences of these dissentions.

Without analyzing the matter in detail, it is a fact that England had rebelled against the political power of the Pope, and that it was considered quite possible that France would do the same. Florence was allied with France in a long conflict with the Vatican that had already resulted in military action. The Duke of Florence, and the leaders in France, including its ambassador to the Vatican, were personal friends of Galileo. His trial thus became pivotal in this phase of the Reformation. It is entirely possible that, without the compromise and the light sentence, the relations between France and the Vatican would have become irrevocably strained.

This made the trial and the heliocentric system notorious and good material for Protestant propaganda against the Catholic Church.

Returning to the heliocentric system, and perception as a source of knowledge: the motion of the Earth in its orbit and around its axis is not directly perceptible. This is certainly one reason for the age-long debate about the heliocentric system, and makes it unsuitable as an example with which to refute either the Platonic or the Aristotlelian theories of knowledge. Galileo's use of the telescope for astronomical observations did little to alter this situation. He had obtained a telescope from Holland; today, we would not dignify it with that name, but instead call it half of an opera-glass. He enlarged and improved it until it became serviceable for astronomical observations. With it, Galileo saw the mountains on the Moon, the four major satellites of Jupiter, the rings of Saturn, spots on the Sun, and the stars of the Milky Way. Except for possibly the last, none of these are visible to the unaided eye. Many of his contemporaries (and not only his enemies) were skeptical. If, according to Plato, the unaided senses were unreliable sources of knowledge, what about the telescope? Was it not merely a further source of illusion? Some of the skeptics were afraid to look through the telescope, fearing that it was the work of the Devil. A century earlier, Columbus had encountered the same skepticism. Three centuries later, Abbe and Zeiss encountered a similar, though less stubborn, skepticism when they first produced their improved microscopes. Kepler's invention of the astronomical refractor, Newton's invention of the reflector, as well as increased familiarity with these devices, did much to dispel this skepticism. Yet, even George Berkeley confined his discussion of vision to the unaided eye. Using one of Galileo's telescopes, Aleiphron would have seen the turrets and windows of the distant castle. Consequently, it was easily possible, even in the seventeenth century, to convince oneself that the telescope did not produce illusions; at least, not when it was used to observe terrestrial objects. Since the astronauts landed on the Moon, the truth of the round Earth, and of Galileo's lunar observations, has been demonstrated. The only skepticism that remains concerns observations of objects outside the solar system; professional astronomers are still debating alternative versions of the cosmological theory of relativity, of the red-shifts of the spectra of distant stars, and of quasars. While their debate is conducted in modern words, it is essentially a discussion of the same problems that confused Galileo's contemporaries.

It must also be emphasized, for it is usually not mentioned, that Galileo did not use the telescope to quantitatively measure the apparent motions of celestial objects. For this, he relied on Copernicus, and to some extent on the non-telescopic measurements of Tycho Brahe. It does not detract from his achievement to describe it as a partial confirmation of Thales' ancient hypothesis that celestial objects are made of the same stuff as the Earth.

Galileo's work on falling objects was even further removed from direct perception by the unaided senses. Contrary to popular opinion, he did not perform the famous experiment of dropping two objects from the Leaning Tower of Pisa. This experiment was first mentioned by Galileo's contemporary, Giorgio Coresio, who ascribes it to an earlier Mazzone. It is true that Coresio cites it as evidence against Galileo's conclusions. In a letter to a friend, published only posthumously, Galileo discusses the experiment, but makes no mention of having performed it himself. His own experiments were with blocks sliding down smooth inclined boards. He timed their motion by counting his pulse. After much fumbling, he succeeded in establishing the mathematical theory that describes their motion and its dependence on the degree to which the board is inclined. Free fall was much too rapid to be reliably observed with his methods. He arrived at the law by extrapolation, by calculating what would happen if the board were vertical. This is inference from perception, not perception itself. Moreover, the inference involves mathematics that even Galileo found difficult. It was, even more than the telescope, a new way of aiding the senses to perceive phenomena with greater clarity. At the other extreme, the sliding motion of objects on a horizontal plane was very erratic. Instead of trusting his unaided senses, Galileo again extrapolated from his experiments with inclined boards. He thus concluded that, under ideal circumstances, the block would continue sliding indefinitely with unaltered speed; yet anyone could see that this was not the case in any actual experiment. In the same way, his discussion of the motion of projectiles was not based on direct observation, as is that of an archer or gunner. He again used mathematics to supplement his experiments with sliding blocks. And finally, all of this was contrary to Aristotle. His contemporaries' bewilderment should not occasion surprise.

It is true that theology theoretically placed much emphasis on revelation and meditation as reliable sources of truth concerning the nature of things. For everyday purposes, however, people of those days relied on direct sense-perception, just as we do today. Galileo was relying neither on accepted theology, nor on direct sense-perception. This raised novel and difficult problems that few were prepared to face, much less solve. Galileo's writings show that he had little understanding of the reasons for their bewilderment.

One should now return to Copernicus, and inquire whether he had more convincing reasons for adopting the heliocentric theory. He was Polish, raised by an uncle, who was Prince-Bishop of one of the Polish dioceses. This uncle sent him to Italy, where he received the best education then available. He studied medicine, mathematics, astronomy, law, and theology. Throughout his life, his services as a physician were always available to poor and rich alike. At the age of twenty-eight he lectured in Rome on mathematics and astronomy to enthusiastic and distinguished audiences. The Pythagorean doctrines were being freely discussed there, despite Aristotle's scorn for them. Such wide-ranging discussions were characteristic of the Italian Renaissance in the period preceding the Reformation. Unlike Thomas Aquinas, these scholars were able to read Greek and Hebrew, and thus had better access to the ancient writings. Egyptian and Babylonian, were, of course, not known to them. Copernicus' lectures in Rome preceded the formation of the now famous Accademedia dei Lincei, but only by a few years. Membership in this society and participation in its discussions was restricted to relatively few scholars. Less well-educated people were ignorant of all this activity, or, if it came to their attention, they were offended by it.

Even before this second visit to Rome, Copernicus had been appointed as his uncle's chief assistant in the secular administration of his diocese. He made a few astronomical observations, but his major interest in this field was the calculation of the future positions of the planets. The reason for this interest is not clear, and some commentators think it a mystery. He used the decimal numerals, but not the decimal point; Stevin would not send his greetings to astrologers until a generation later. The increasing precision of astronomical calculations had resulted in complicating Ptolemy's geocentric description and made the calculations more and more laborious. Copernicus tells us that his reading (presumably of Archimedes' *Sand Reckoner*) had made him aware of the heliocentric theory. He therefore recast his calculations in this form and found that they were much simplified. He became convinced that the heliocentric description was the one and only true description of the solar system. He soon finished a manuscript containing his calculations and this conclusion, but delayed its publication for many years. Through conversations and letters, others became acquainted with its contents, and made them public. His friends in Rome urged him to publish, and Pope Clement VII seems to have had no reluctance in concurring. The book was finally published in 1543, the last year of Copernicus' life. Its publisher, in Leipzig, added an unauthorized preface, stating that the heliocentric description was only a means for simplifying planetary calculations. This contradicted the text, and it is generally supposed that the publisher inserted it to protect himself; Leipzig was in Luther's country. He may also have been sincere, and may even have voiced the general opinion of contemporary educated people. Today, the cosmological theory of relativity agrees with the publisher, not with Copernicus; moreover, geophysicists and observational astronomers find the geocentric description more convenient for many of their calculations.

Although Copernicus' book circulated freely for many years, it is therefore unlikely that it relieved its readers of their confusion. In fact, Tycho Brahe, who worked during these years and made better astronomical observations than anyone before, advocated a modified geocentric theory. According to Tycho's theory, the Sun and Moon circle about the stationary Earth, but the planets circle about the moving Sun. The ancient origins of Tycho's system are confused, but various Greek writers seem to have foreshadowed it. Anyone who was sufficiently interested to read Tycho as well as Copernicus and Galileo would certainly be confused: even the contemporary authorities disagreed with each other! Kepler was another contemporary of Galileo; his contributions, both to science and to confusion, will be discussed after Newton's.

Newton was born in 1642, the year of Galileo's death; he was two years older than William Penn. It was a scant generation since the Pilgrims had landed at Plymouth Rock. In England, the turmoil of the Reformation was greater than ever. The Puritan revolution, led by Oliver Cromwell, began at about the time of Newton's birth. Some of its early battles were fought within fifty miles of his home, and the disturbances did not end until he was middle-aged. At the universities, the freedom to discuss non-Aristotlelian philosophers was maintained. This made it possible for Newton to publish his elaboration of the heliocentric theory but, like Copernicus, he did so only at the urging of friends. There was no similar freedom of religious belief. After Cromwell's downfall and the restoration of Charles II to the throne, the Puritan, John Milton, lost his income and died in poverty. William Penn was twice imprisoned. On a third occasion, he defended himself so ably that the jury acquitted him and made the judge angry. This famous case established the English common law that "a judge may not lead a jury by the nose," but it did little for the cause of religious freedom. Under Charles II, imprisonment was not the worst thing thing that could happen to an avowed Quaker. Penn emigrated to America, preferring exile to conformity. Even Cromwell's tolerance of most forms of Puritanism did not extend to Unitarianism. If Newton had allowed his Unitarian convictions be known, he would have become ineligible for teaching and governmental appointments. At the very best, he would have shared Milton's poverty.

Newton's most famous book is the *Mathematical Principles of Natural Philosophy*. Samuel Pepys records that its Latin manuscript was received by the Royal Society on July 5, 1686; Newton was then in his forty-fourth year. The book is much more than an elaboration of the heliocentric system. It is a kind of synthesis of the ideas of Aristotle, Plato, Euclid, and other Greek writers; he made this quite clear in his preface to the first edition. The book is not a mere compilation, however, and contains so much originality that the materials he borrowed from the past are often overlooked.

The *Mathematical Principles* has more or less the same format as Euclid's *Elements*, but it is not so tersely formal. It opens with eight definitions, and three "Axioms, or Laws of Motion." These axioms are:

Law I

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by

> forces impressed upon it
>
> Law II
>
> The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.
>
> Law III
>
> To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

These laws are again contrary to Aristotle, who held that a force is needed to keep an object in motion. Newton explains: "Projectiles continue in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity." It is clear that Newton was borrowing, though not slavishly, from Galileo. This was recognized by Albert Einstein, who attached Galileo's name to Law I. Newton believed that, starting with these three axioms, he could derive both the mathematical formulae for planetary motion, and also his famous law of gravitation. The latter linked the motion of a planet with that of a projectile, much as Galileo had linked the motion of a sliding block with that of a freely falling object. For these demonstrations, Newton used mathematical methods that were then novel, and are now known as the calculus. At the time, there was a controversy as to Newton's priority in inventing the calculus; his rival was G.W. Leibniz. This controversy was very painful to Newton, but it is no longer of much importance; both were somewhat anticipated by Archimedes and many later mathematicians, including Kepler. However, these methods were not widely known, and this made it difficult for Newton's contemporaries to understand the *Mathematical Principles.* This did not lead them to be skeptical of his work, though the reasons for its immediate acceptance by mathematicians are not clear. Many thinkers devoted themselves to understanding and extending his work. Since his use of the calculus made it difficult for even well-educated people to understand the *Mathematical Principles*, there were soon "popular" editions of the book, written in English, that avoided the use of the calculus. Perhaps the first of these was by Henry Pemberton, published in 1728; he had edited the third Latin edition of the *Mathematical Principles* a few years earlier, and the manuscript of his popular version had also circulated for some years. It seems to have had a wide circulation, and its influence on physics textbooks used in the nineteenth-century United States is clearly recognizable.

Today, we can be more critical and inquire how Newton came to enunciate these particular laws or axioms. Since no one had accepted them in earlier times, and Aristotle had even accepted others, it cannot be maintained that they are self-evident. It seems that Newton arrived at them from a consideration of some conclusions reached by Johannes Kepler, that is, by induction or inference.

Kepler was a younger contemporary of Galileo. He was for many years the astrologer at the court of the Emperor Rudolf II, in Prague. In this post he succeeded the Danish observational astronomer, Tycho Brahe. After Rudolf's death, Kepler became astrologer to the famous Wallenstein. Originally, Tycho had been supported by the King of Denmark and had his observatory on the island of Hven. When the Reformation spread to Denmark, he was advised to leave. He then found employment with Rudolf, the Holy Roman Emperor. Kepler fell heir to Tycho's records, which, as has already been noted, contained the most accurate and continuous series of observations then available. Kepler also made observations; he invented the astronomical refractor, which is an improvement of Galileo's telescope. With this, he was able to observe the planetary motions with even greater precision than Tycho had achieved.

Both Tycho and Kepler had to draw horoscopes and publish astrological-astronomical almanacs. To most people of the time, "astrology" and "astronomy" were synonymous; the concept of astronomy for its own sake had not yet developed. While astrology was not officially recognized as part of Christian theology, so many prominent people availed themselves of its predictions that it was scarcely frowned upon. Kepler himself was never persecuted for it, but his mother was accused of witchcraft and was saved from the stake only by Rudolf's intervention. After doing this favor, Rudolf never paid Kepler's salary. Kepler explained astrology as follows:

> Insofar as the soul has the idea of the zodiac...it also feels which planet at which time is under what sign, and measures the angles of the rays that reach the Earth. Insofar, however,

> as it takes up from the rays the divine essence of the geometric figures of the zodiac...and also knows the measure of the angles, it evaluates some (configurations) as congruent or harmonic, others as incongruent.

Kepler also believed that mathematics was the ordering principle of the universe. His specialized writings show that he devoted much time to studying the unprecedented amount of observational material available to him, with the object of finding the mathematical order hidden in it. This is certainly decipherment of the book of Nature, whether it be Pythagoreanism, rationalism, or empiricism. It requires much good will to describe these writings as consisting of theorems and demonstrations; today his logic is considered deplorable. Despite their fantastic quality, it is convenient to speak as if his demonstrations were bona fide; this was certainly Kepler's intent, for he was not a charlatan. Today, only three of his theorems and none of his demonstrations are quoted. The three theorems were not even published simultaneously. They have been selected from among all the others because they represent an advance over all previous heliocentric theories. They are called Kepler's Laws, and are expessed in words as follows.

Kepler's First Law

The planets (including Earth) move about the stationary Sun in elliptic orbits, the Sun being at one focus of the ellipse.

Kepler's Second Law

The line drawn from the Sun to a planet sweeps over equal areas in equal times.

Kepler's Third Law

The squares of the times of revolution of the planets are proportional to the cubes of their mean distances from the Sun.

These laws make it possible to calculate future positions of the planets rather more simply and accurately than with Copernicus' system. Copernicus had supposed that the planets move about the Sun in circular orbits, and with constant speed. Using the Copernican system, therefore, the predicted positions of the planets differ from those observed by appreciable amounts. To improve the accuracy of the predictions, corrective calculations are needed. These are simpler than those required by the Ptolemaic system, yet they are not trivial. Kepler's introduction of elliptical orbits and constant areal speeds resulted in predictions that were more accurate, and required less correction. For some purposes, one can even omit the corrective calculations. Kepler found the first two laws primarily by studying his own and Tycho's observations of Mars; the third law required study of the observations of all the planets. The remainder of Kepler's theorems are no longer quoted because they are of little use in simplifying the planetary calculations.

With these provisos and reservations, it is possible to explain the relation between Newton and Kepler in simple terms. Newton accepted Kepler's three theorems, but was not satisfied with their demonstrations. He therefore extended Kepler's writings in the inductive direction, arriving at the three Axioms or Laws of Motion. Starting from these, he devised more satisfactory demonstrations of the three theorems, as well as demonstrating many other interesting and useful theorems. The evidence for this is of two kinds: the statements of Newton's friends, and the outline of the earlier parts of *Principles of Natural Philosophy*. As has been noted, it opens with definitions. These are followed by some pages of discussion that would be revised and extended by Albert Einstein, but not until more than two centuries later. The Axioms are followed by the demonstration of some theorems (corollaries). Some of these are similar to Galileo's conclusions, others explain experiments by Christopher Wren and others of Newton's fellow members of the Royal Society of London. Following Euclid's outline, only then does Book I open. Its first section is devoted to an exposition of the mathematical method we now know as the differential calculus. Section II opens with:

> Theorem I
>
> The areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable plane, and are proportional to the times in which they are described.

The differential calculus is used in the demonstration of this theorem. The theorem is obviously a carefully worded version of Kepler's Second Law, although there is no explicit mention of either the planets or the Sun. Theorem II is the converse of Theorem I. There follow other theorems and problems; Section III is entitled "The Motion of Bodies in Eccentric Conic Sections." It opens with:

> Problem VI
>
> If a body revolves in an ellipse; it is required to find the law of centripetal force tending to the focus of the ellipse.

It is shown that the solution is the famous law of gravitation: the force is inversely proportional to the square of the distance of the body from the center of force. The relation of this to Kepler's First Law is again clear, though it is not identical to it. Skipping a few pages, one comes to:

> Theorem VI
>
> If several bodies revolve about one common centre, and the centripetal force is inversely as the square of the distance of places from that centre, I say that....

and,

> Theorem VII
>
> The same things being supposed, I say, that the periodic times in ellipses are as of the (3/2)th power of their greater axes.

Again, Theorem VII is essentially Kepler's Third Law. One hardly requires additional evidence to see that Newton chose his axioms so that he would be able to demonstrate Kepler's three laws. It is known that he had solved Problem VI several years before the *Principles of Natural Philosophy* was written.

One will be curious about the way in which he made this choice of axioms; the record is not clear. The legend of the falling apple may contain a seed of truth. It may have been a lucky guess, or it may have been a matter of trial and error. It is known that he at first rejected the inverse square law of gravitation because the currently accepted value for the distance between the Earth and Moon was very inaccurate. In suggesting trial and error, it is not implied that anyone with patience could have done it. Newton was not only extraordinarily creative, but he must have systematized and been perfectly familiar with the differential calculus before he could have solved Problem VI.

The Intellectual Problems of the Seventeenth Century

The doctrines of empiricism and rationalism are now accepted uncritically by most educated people. It has been seen that this was not possible in the seventeenth century, when these doctrines first began to spread through Europe. They were, perhaps, an integral part of the theological confusion of the times, but most people were so preoccupied with the theological problems that they were unaware of rationalism. The struggle for religious freedom was essentially an egoistic one: it concerned the right of all persons to make their own decisions as to the truth or falsehood of certain beliefs, rather than having these decisions made by persons in authority. The empirical investigation of new phenomena and the intellectual efforts required by rationalism were relatively safe occupations, much less dangerous than involvement in the politico-religious anarchy of the time. This retreat to safety has been recognized by some historians, but not in a derogatory way: Galileo and Newton were not the stuff that martyrs and heroes are made of. However, all members of the intellectual community were conscious of the religious conflicts, and gave some attention to them, in private if not in public.

Even if one avoids passing moral judgement on these escapists, their activities often seem ridiculous to us. A popular theory held that the powdered ashes of a murderer's bones were sprinkled in a circle, a frog placed within the circle would not be able to escape. One evening, after a good dinner, the Fellows of the Royal Society put the matter to a test. The frog cleared the magic circle in a single bound and fled to the darkest corner of the room. This incident is remarkable only for the throughness with which this theory was investigated. The early volumes of the Royal Society's Transactions contain other items, less adequately investigated.

The mention, in an extract from the Royal Society's Transactions, of the transmutation of iron into silver is also of significance.

Books on alchemy and astrology were available and carefully studied by the ablest scientists of the time. Kepler's influence on Newton has already been mentioned. Kepler himself compared astrology to a dung heap in which precious jewels are buried. This is often thought to have been a defense against any possible charge of heresy. It may also have been sincere: Kepler would, of course, have considered his own writings to be jewels. Robert Boyle's choice of "Sceptical Chymist" as the name of one of his books is further evidence of the scientific temper of the time. Newton and Leibniz, rivals in the invention of the calculus, shared this interest in astrology and alchemy. Newton seems always to have found experimental work a welcome relief after lengthy periods of concentration on mathematics and theory. Some biographers have represented his alchemical work as mere recreation; but it seems to have been as significant to him as was his other work. He did not publish most of his voluminous records, and very little on chemistry or alchemy. Yet his library contained many alchemical books and they show signs (in the form of marginal notes and turned-down page corners) of serious study. A letter to a friend who was about to start a journey through Europe has been the subject of much speculation. Each of his biographers has used it to support his special view of Newton's character and personality. In the letter, Newton asks his friend to look for evidence of the transmutation of metals in the mines and smelters of the Continent. The knowledge of metallurgy which he obtained through these studies became very valuable to Newton in later years, when he became Master of the Mint.

In summary, the seventeenth century scientific community was not as glib as ours in making the distinction between superstition, theory, and fact. We tend to ignore the fact that the Book of Nature is not written in a single alphabet. Perhaps it is more accurate to say that we hope that it is all written in the language of mathematics and logic, and that developing these sciences, a uniform version of the Book will be obtained. For practical purposes, it must be remembered that the characters of some of its chapters are geologic strata and their fossils; others are written in buried cities and human bones, and the letters of other chapters are living things, including people. This makes the Book confusing to read, and our hope of a uniform text was born in seventeenth-century empiricism, which had not yet crystallized into the rationalist faith. Paradoxically, the alphabet of some chapters of the Book of Nature consists of human languages. This was not ignored by our predecessors, and their investigation of these chapters is closely related to the evolution of empiricism into rationalism. Many of these linguistic problems had roots in much earlier times. When Greek displaced Babylonian as the international language of commerce and diplomacy, this was complacently accepted as a barbarian acknowledgement of Greek superiority, not worthy of record. When the needs of trade consolidated the Greek dialects into a single language, the aristocratic philosophers seem to have continued to speak and write in their native dialects. When Latin displaced Greek, the Romans seem to have considered this a matter of administrative convenience; Roman philosophers continued to pay a barbarian's tribute to their Greek predecessors. In medieval times, Latin became a dead language, but the internationalism of the Christian Church preserved it, in a modified form. The revival of interest in Greek and Latin writings during the Italian Renaissance was uncritical, and again gave the Greeks the barbarian's respect, which they had taken as their natural due. But the Greek spoken during the Renaissance would have been unintelligible to Plato, and the Latin unintelligible to Caesar.

For a time, these "dead" languages were the international medium through which scholars and diplomats communicated. They still retain this international character, but their usefulness has slowly diminished over time. The various European languages and dialects evolved from their aboriginal forms, until, by the seventeenth century, they were not much different than they are today. The Roman military domination exerted some influence on the early evolution of these languages, more so in Southern than in Northern Europe, and left some mangled Latin words behind. Other phenomena of greater importance began to appear very early: "Beowulf" and other literary works were written in Anglo-Saxon during the eighth century; the "Domesday Book" was written in Norman French at the command of William the Conqueror. In the fourteenth century, Dante, Boccaccio, and Petrarch were writing in Italian. By the seventeenth century, Shakespeare had written his plays and they were already being translated into other languages. Galileo wrote both in Latin and in Italian, Descartes wrote in Latin and French. Printers in Holland set Galileo's Italian manuscript in "Roman" type, which was used for all the languages of Western Europe except German. Various national academies of science and philosophy had been founded, and the proceedings were published in the local language. Finally, the increasing number of new discoveries and inventions required that the vocabularies of all languages be increased. The new words were manufactured out of bits and pieces of Latin and Greek words, and acquired an international character. A popular misunderstanding of this process is reflected in the modern adage that "the Greeks had a word for it." Actually, the Greek language of Plato's time had a relatively small vocabulary. These linguistic phenomena excited the curiosity of seventeenth-century scientists as much as physical and chemical phenomena. Newton was no exception.

Since Newton has had a great influence on us, it will be well to consider how the seventeenth century influenced him. He habitually wrote in English and left an abundance of unpublished manuscripts and letters as evidence of this. His book on optics was first printed in English. The Latin manuscript for the first edition of the *Mathematical Principles of Natural Philosophy* was prepared with the assistance of a nephew, but this does not demonstrate that he knew only the English language. His library contained Bibles in Hebrew, Greek, Latin, and Syriac, as well as polyglot editions. Many of his manuscripts that are usually described as theological were actually critical and comparative studies of scores of New Testament texts. In these works, Newton anticipated the methods used by modern scholars in restoring the original version of ancient texts that have undergone many editions and translations. His work in this field was biased by the hypothesis that the earliest people were monotheists, and that polytheistic religions were corruptions of the original faith. Other Newtonian manuscripts are attempts at a chronological history; the Bible and astronomy were among the few sources of data that he considered to be reliable. He seems never to have been satisfied with any of his chronological tables; an unauthorized abstract of one of them was published, much to Newton's distress. Frank E. Maurel has given an account of this phase of Newton's studies, together with an extensive bibliography.

A number of linguistic problems were isolated and formulated more or less clearly. One was the adoption of a standard spelling for each language; France attempted to solve this by fiat, and the Academie des Inscriptiones was established to formulate and issue these edicts. In the course of time, other countries have followed this example in various modified forms, such as the present-day Oxford English Dictionary. A more theoretical effort was to make spelling conform to pronunciation. There are many difficulties to overcome before this is possible. Each adult person has his own voice, which is almost as individual as his fingerprints. This voice, however, is influenced by the dialect of homeland and instruction, as well as by the structure of the vocal organs. However, certain positions of tongue and jaw, the inhalation and exhalation of air, do have a general correspondence to the sounds of a language. Early phoneticists found it difficult to observe, much less describe, these processes of speech. A notebook of Isaac Newton's, dating from about the time that he entered Trinity College at Cambridge, has been published recently. It was not copied from other works, although reading may have influenced it. He does not give a complete description of the formation of the sounds of speech, but his classification is fairly good and there are a few descriptive sentences: e.g., "To ye labialls may be added ye jarring of ye lips caused by shutting ye lips and forcing ye breath through yon." As has been confirmed by many others, there are not enough letters in our alphabet to make it possible to spell English phonetically. Consequently, Newton borrowed from both the Greek and Hebrew alphabets, and invented other signs for himself. While this phonetic work was clearly left unfinished, it does exhibit the range of Newton's interests, and may deserve more study than it has received.

Another problem that attracted Newton at about this time was that of a "universal character." This was clearly formulated, some fifteen years earlier, by Robert Boyle. Boyle wrote: "And truly, since our arithmetical characters are understood by all the nations of Europe, the same way, though several people express that comprehension with its own particular language, I conceive no impossibility that opposes the doing that in words that we see already done in numbers." In fact, one part of this complex problem had already been solved in China. Documents written in North China could be read and understood in South China, although when read by a Southerner, the Northern writer would not understand the spoken words. This was known in seventeenth century Europe from the accounts of travelers who had reached the Orient. The Florentine merchant, Francesco Carletti, spent several years in China and Japan. As befitted his time and nationality, he was a shrewd observer, not too credulous, and he had an interest in languages. He confirms the above and adds that Japanese scholars could read Chinese books, although they could not understand spoken Chinese. He also mentions phonetic alphabets with about forty letters used for writing Japanese and Chinese. His narrative contains evidence that he had made a considerable effort to understand these linguistic matters. He describes the huge profits that were made by the East India Company and others. He does not become emotional when he describes the inhumane and dishonest methods, which included murder, by which these profits were made. However, when he was deprived of his own profits by a cruel and bloody act of piracy by Dutch ships, his complaints are loud and long.

Another problem, that of inventing a language whose grammar and spelling are as simple and exact as the rules of arithmetic but with a vocabulary large enough to deal with abstract concepts, is more difficult. This problem has had a great influence on the development of mathematics and logic, and now influences our computer technology. Very little progress could be made toward a solution until the distinction between grammar and meaning, syntax and semantics, had been made clear. It has been seen that the Europeans learned this from ancient Sanskrit writings; the great importance of this was not recognized until very recently. Unaware of this distinction, Boyle and many of his successors could only appeal to a vague form of the doctrine of psychomathematical parallelism.

Newton, not yet twenty, wrote two drafts (both unfinished) of a paper to be entitled "Of an Universal Language." This manuscript has also been published only recently. It seems to have been his plan to construct this language according to "philosophical notions," thus making it a tool for scientific and philosophical discussions. However, the implications of these drafts have not been investigated sufficiently to make it possible to summarize them. Although they were left unfinished, there is evidence that Newton's interest in linguistics continued throughout his life. Only a part of a much later philological manuscript, "Concerning the Language of the Prophets," has been published.

These examples do not exhaust the range of problems that interested and bewildered the scientists of the time of Newton, but will serve as guides while we investigate some of our inheritance, both good and bad, from this remarkable century.

The Rise and Decline of the Axioms

Perhaps the seventeenth century can be described as one of enthusiastic intellectual confusion. This enthusiasm and some of the confusion has persisted until now, but by the eighteenth century, several trends can be distinguished. One is the elaboration and extension of empiricism and rationalism for the production of new knowledge. Another is the effort to answer some of the questions stirred up by these doctrines. The second of these trends will be investigated first, since some of the later stages of the first have already been discussed in connection with the Industrial and Scientific Revolutions.

One of the most influential philosophers who tried to solve the problem connected with empiricism and rationalism was Immanuel Kant. The best known of his writings is the *Critique of Pure Reason*, which can be seen as a defense of Aristotle and Newton, though Kant's own creative ability made it more than that. Immanuel Kant was born in 1724, in Königsberg, East Prussia. His grandfather was Scottish, probably having emigrated to escape the religious disturbances at home; his father was a master leather worker or saddler, and not wealthy. The family was deeply, though not orthodoxly, religious. Like many such families, they made it possible for Immanuel to attend the local university. At the age of thirty-one he joined its teaching staff, on which he remained for nearly fifty years. During this time, his opinions changed and matured; he wrote the *Critique of Pure Reason* at the age of fifty-seven. In the preface to the second edition he says that no essential change has been made in the logical science of Aristotle. He notes that Copernicus and Newton had based their work on principles that are contrary to perception (e.g., the motion of the earth is not perceptible by the unaided senses). However, he says, these principles are known with certainty. Elsewhere, he dismisses "the good Berkeley's" doctrine that things exist only because they are being perceived. He vigorously denies David Hume's contention that the relation of cause and effect is merely the perception of an oft-repeated coincidence in space and time which evokes the expectation of the effect whenever the cause is perceived. In modern terms, Hume's relation of cause and effect is called a conditional reflex. The classic experiment in this field was performed by the Russian scientist Ivan Pavlov. The subjects in this experiment were dogs. He rang a bell just before feeding the animals. After repeating this procedure several times, he found that the dogs would begin to salivate at the sound of the bell, even if their food was delayed. Both Kant and Hume would have argued in this case that the sound of the bell caused the dogs to salivate. In other words, Kant would agree that every particular case of cause and effect is a conditional reflex, in the sense that all learning by experience is a conditional reflex. However, he maintained that the general principle of cause and effect somehow exists in nature, independent of the existence of people who can, in Galileo's words, decipher the Book of Nature. Furthermore, according to Kant, people are born with a knowledge of the principle of cause and effect; it is one of the principles that they use to interpret their sensations and convert them into perceptions and knowledge. Since it is impossible to question a young infant about such abstract matters, Kant had to resort to indirect methods for demonstrating the validity of his opinions. He maintained that cause and effect is a necessary relation, axiomatic and not arising out of experience. Mathematical theorems provide other examples of truths that are certain, but are not obtained from experience or perception. In this, Kant followed Aristotle and thus contributed to the doctrine of psychomathematical parallelism. In the Introduction to the second edition of the *Critique of Pure Reason*, he summarized his views as follows: "There can be no doubt that all our knowledge begins with perception...[even so, our knowledge] does not all spring from perception...It is at least worthwhile to investigate the question, whether...there is knowledge that is independent of experience and the impressions of the senses." At the end of the book, he thought that he had successfully answered this question in the affirmative: such knowledge is certain, even though it cannot be demonstrated either theoretically or by sensory experience. This knowledge corresponds to Aristotle's axioms, but Kant called it "knowledge a priori"; modern dictionaries call it intuitive or self-evident knowledge, just as the axioms are said to be self-evident.

One is passive in perception, but active and spontaneous in the intuitive appraisal and imaginative construction of the objects perceived. Space and time are a priori objects; Kant wrote, "geometric principles are always apodictic, i.e., combined with awareness of their necessity. Principles of this kind cannot be empirically perceived or derived from perception." (Here one has an example of the modern technical use of "apodictic," in contrast to the original Greek meaning of demonstration, proof, or derivation.) Had Kant confined himself to such esoteric matters, his book would have received attention only in academic circles. However, he maintained that the existence of God, though certain, is known a priori, and cannot be proven. Since politically powerful theologians believed that they could construct such a proof, this nearly cost him his career.

Beginning almost at the time of Aristotle, an increasing number of axioms, undemonstrable but asserted to be certainly true, were proposed. Newton's three laws of motion were only the most recent additions to this list. Kant's writings proved to be their ablest philosophical defense, possibly because they are scarcely mentioned in the *Critique of Pure Reason* and the reader is left to make his own application of Kant's principles to them. If he had made a special effort to defend Newton, someone would surely have pointed out that Aristotle had denied his axioms of force and motion. Aristotle had said that an axiom must be held by anyone who is to learn anything. Kant made this more explicit; he maintained that anyone who denied the truth of an axiom would become entangled in absurd contradictions. Any mention of Aristotle's writings about motion and force would have demolished Kant's defense of Newton's axioms. By evading the issue, Kant (unintentionally, most likely) avoided the fate of the Royal Society's seventeenth-century frog, and kept the discussion on a more dignified level.

It is easily seen, however, that if it could be shown that even one accepted axiom could be denied without leading to logical contradictions, Kant's whole philosophical system would be demolished. This was an implicit challenge, and no axiom was better suited for attack than Euclid's Fifth Postulate. The astronomer Ptolemy, and Proclus after him, attempted to demonstrate the validity of the Fifth

Postulate. Possibly even Euclid attempted it, but in hindsight it is now thought that he showed genius in not trying it. Euclid's editors and revisors changed the wording, and by the time of Kant, the Fifth Postulate was generally known as "Axiom XI: The Axiom of Parallels." A typical revised form is:

> Through any point in a plane, there goes exactly one straight line that does not intersect a given straight line which lies in that plane and does not go through the point.

The words "exactly one" are to be emphasized.

The discussion of this axiom occurred in two phases. During the first phase, its validity was not disputed, and all efforts were devoted to "proving" it, to show that it was actually a theorem, derivable from Euclid's other axioms. Few, if any, have read the mountain of literature generated by these efforts. By the time of Kant, it was virtually certain that it could not be so derived, and that it was an axiom in the strict sense, not demonstrable, yet true. In the second phase, its truth was challenged, and it was shown that no contradictions arose if one denied it. One need not be crazy or even illogical to deny it. By 1904, Oswald Veblen's revision of Euclid's geometry had reduced it to "Assumption XIII," which could be accepted or rejected as the reader pleased.

The transition between the two phases occurred in a somewhat dramatic manner. J.K.F. Gauss, a German, and Wolfgang Bolyai, a Hungarian, were fellow students at the University of Göttingen. Gauss remained there and became a mathematician, physicist, and geodesist, whose ability and versatility can only be compared with that of Archimedes or Newton. Bolyai became a professor of mathematics, physics, and chemistry at a small Hungarian college; he was also a poet and playwright. As students, Gauss and Bolyai must have discussed the problem of demonstrating the axiom of parallels; later both worked on it and corresponded about it. Gauss was always able to find a flaw in the demonstrations, not only in his own work, but also in that of his friend. Janos Bolyai was the son of Wolfgang; by the age of fourteen his father had already taught him the calculus. He then studied at the University of Vienna, after which he had to do a term of military service. He relieved the tedium of army life by working on the problem of parallels. He wrote a series of elements in which the words "exactly one" in the axiom were essentially replaced by "two or more." When this work was polished and scrutinized for flaws, it was published as an appendix to a book of his father's. With youthful enthusiasm, he called it "the absolutely true science of space." But he had shown only that the axiom of parallels could be denied without being absurd. Although he did not realize it, he had demolished Kant's work. Unfortunately, others had already done much the same thing, and he became so discouraged that he published little more.

One of those who had anticipated him was his father's friend, Gauss, who did not publish his own notes. The other was a previously unknown Russian mathematician, N.I. Lobatchevsky, who was at the University of Kazan (on the Volga, about 450 miles east of Moscow). Lobatchevsky's work was published at a date variously given as 1826, 1829, or 1835. The writings of Lobatchevsky and Gauss were sufficiently different to make it certain that neither man knew of the other's work until later. Lobatchevsky continued his work, and published a shorter and more mature account in 1855, which he entitled "Pangeometry."

In 1854, Bernhard Riemann, who had studied with Gauss and others, applied for an instructorship at the University of Göttingen. To show his fitness for the position, he had to lecture before the entire faculty about his own work. His lecture was "On the Hypotheses Which Lie at the Foundations of Geometry." While his methodology was not that of Bolyai and Lobatchevsky, one can say that he replaced "exactly one" in the axiom with "not one." It is said that as Gauss was leaving after this lecture, a colleague asked his opinion of the work, and that he answered, "I wish I had done it myself." Although Riemann died less than twelve years after the lecture, his influence on nineteenth-century mathematics was second only to that of Gauss. Every one of his few papers pioneered a new field or a new method. This first lecture prepared the way for the non-Euclidean, non-Newtonian geometries and cosmologies proposed by Einstein and other twentieth-century thinkers.

Pacifism and the Elements of Logic

It might be supposed that all this would have discredited Kant's doctrines, but this was not so. They could be defended in various ways. It could be held that the axiom of parallels is not a true axiom: many people have believed that it is not. Then, if it is an axiom, it might be that Bolyai, Lobatchevsky, and Riemann had not continued their series of elements far enough: contradictions might have appeared if they had extended their Elements in the deductive direction. It could also be maintained that parallel lines are unnecessary; the argument is similar to that given above in the chapter "Ideals and the Technology of Triangles." No one can construct infinitely long straight lines. And it might be that even Aristotle's logic was too primitive. Kant had remarked that no essential change had been made for two millenia. A close examination of Euclid's work soon shows that his proof of the very first theorem in his Elements contains a fallacy. Zeno had "proven" that Achilles could not overtake a tortoise. This proof had been debated for centuries, but the fallacy had not been explained until just about the time in which Kant was writing. The Bolyai-Lobatchevsky Elements would reveal the contradictions that were expected to follow from the denial of an axiom.

An overhaul of the science of logic was long overdue. Newton's competitor in the invention of the calculus, Leibniz, had speculated that many of the inconclusive philosophical debates were due to the looseness of the laws of grammar in the natural languages. If it were possible to construct an artificial language, whose grammatical rules were as simple as the rules of arithmetic, philosophy might become an exact science. Robert Boyle had anticipated this in his remark about a "universal character", and Newton's "universal language" was an incomplete attempt. But some additional progress had been made by George Boole. Boole's father was a tradesman and amateur mathematician who imparted his enthusiasm to his son at an early age. George Boole's creativity has increasingly influenced both mathematics and logic since the mid-nineteenth century. He was ably assisted by his wife, though she took no credit for herself. The fundamental step beyond Aristotle's "If A, then B" required a precise definition of the word "if": once this step was taken, the way to a mathematized language (or logic) was open. The Boolean definition of "If A, then B" is "Not A and/or B." Two items require comment: one is the introduction of the non-exclusive "and/or" which has by now become commonplace in our ordinary language. The other is the prominence given the word "not." So long as A and B are interpreted as sentences there is no formal or syntactic problem. Suppose A is the sentence "Apples are fruits," then *Not A* is the sentence "Apples are not fruits." But when the medieval distinction between a sentence and a proposition is introduced, the problem becomes more difficult. Both Sigmund Freud and Bertrand Russell have pointed out that it then becomes a problem of meaning and psychology, of semantics. Both Freud and Russell analyzed it in the same way (though they disagreed on so many other matters). Suppose I open the refrigerator and say "There is no(t) apple here": that means "I expected that there would have been an apple here; I am disappointed." This is why ancient mathematicians, before irrational numbers had been invented, did not, and could not, write "The ratio of the diagonal of a square to its side is not a rational number." Neither Boyle nor Newton (nor the Booles, most likely) were aware of the distinction between syntax and semantics. All of this seems very simple and innocuous; it is surprising that Freud has been so violently ridiculed for advancing this psychological doctrine.

Once the relation between "if" and "not" (and between "not," "all," and "some") had been clarified, Boolean algebra became possible. Today it is fundamental to both logic and computer technology. Its only shortcoming as a language is the small size of its vocabulary. Other workers soon began to remedy this failing; especially noteworthy are the works of Peano and Frege. The latter extended Boole's work until he thought that he had shown that arithmetic is merely a branch of logic. When Bertrand Russell first read these works in 1900, he was elated for he had long been looking for a reason to believe that mathematics is true. He immediately planned a much more ambitious work in which all mathematics would be shown to be a subset of logic. His elation soon turned to frustration and despair, from which he did not recover for some time. Frege and others believed that they had demonstrated Archimedes' Axiom which states that there is no largest whole number. Frege's scheme was as follows. An apple is a thing, apples are a class of things. Fruits are a class that has apples, cherries, bananas, plums, dates, (and so on) as members. Therefore there are classes whose members are themselves classes. It was then shown that the class of all classes has an infinite number of members: Archimedes' Axiom was thus proven. In examining this demonstration, Russell was led to consider those classes that are members of themselves. He defined N as the class of all classes that are not members of themselves. Now, is N a member of itself? If it is, it isn't; if it isn't, it is.

Russell's frustration stemmed from the fact that he thought this to be a triviality, but unless he could eliminate it, no philosopher would accept the thesis that mathematics is simply a part of logic. Further examination of the work of Bolyai and Lobatchevsky might disclose a similar contradiction. After what seems like an unduly long time, and much mental anguish, Russell found the simple solution to the paradox: classes of classes are different from classes of things. No class can be a member of itself; but this also made the demonstration of Archimedes' Axiom impossible. The elation was gone, but with ten years of grim determination and the assistance of Alfred North Whitehead, three volumes of a work called the *Principia Mathematica* were finally published.

The *Principia Mathematica* is essentially a series of elements, the most ambitious ever published. It begins with the axioms of logic, and all of mathematics is then deduced from them; all except that there is no largest whole number. Since mathematicians will not agree to stop talking about infinity, it was necessary to introduce Archimedes' Axiom, or some equivalent axiom that is obviously not an axiom of logic. The book is difficult to read for it is written in an artificial language, the "Language of the *Principia*." It is not too much to say that it realized Boyle's and Newton's ideal of a "universal character" with a fairly large vocabulary. A special font of type had to be cast, and typesetters specially trained before the *Principia Mathematica* could be printed. Later, Russell wrote *An Introduction to Mathematical Philosophy*, that can be enjoyed by anyone who has retained an interest in high school mathematics. Like Boyle and Newton, Russell and Whitehead hoped that philosophy could be made into an exact science.

Many superlatives have been written about the *Principia Mathematica* ("colossal," "monumental," and so on), but it was, after all, the work of two young men who both were to accomplish much more during their lives. Russell's life was especially long and dramatic; his autobiography fills three volumes, of which only a part of one chapter is devoted to the *Principia Mathematica*. Having been so influential in the founding of mathematical philosophy, it might be expected that he would have kept all of his opinions to himself until they had been proven mathematically. Some of his followers have shown this tendency, and it is even more prevalent among scientists who have not studied the *Principia Mathematica*. Russell, however, did nothing of the sort, as is evidenced by his biography. He took an active role in bringing about change in our society, and his influence continues today. This will be investigated before returning to the investigation of mathematics. If one of the founders of mathematical philosophy did not proceed to elaborate on the doctrine of psychomathematical parallelism, this is an important datum to be included in the present investigation.

Both Russell's mother and father were members of large and ancient British noble families, the Stanleys and the Russells. No doubt, many of his ancestors were nonentities, but many were active in public affairs. During the latter part of the nineteenth century, all of his close relatives were strong individualists, often radicals. His maternal grandmother has already been mentioned. Her immediate family was much given to excited debates on all the social and intellectual issues of the day. Russell's father ruined his political career at a private party, where he spoke "not unfavorably" about birth control; worse things were whispered about Bertrand's mother. His paternal grandmother died young, and his grandfather then married a younger woman, who was noted as much for her beauty as for her shyness on public occasions. Shortly after his marriage, grandfather Russell became Prime Minister of Britain. He respected his young wife's ethical opinions, and allowed them to influence his decisions. This was disconcerting to his colleagues; they ruefully named her "Deadly Nightshade." She gave Bertrand a Bible in which she had written,"Thou shalt not follow a multitude to do evil.'' No one can read his autobiography and doubt that this commandment dominated his whole life.

When Bertrand was not yet four, both his mother and his father had died. His father's will had nominated two men, both avowed atheists, as guardians for his sons. Presumably after discussions with Queen Victoria, the Lord Chancellor declared this portion of the will invalid, and that the boys were wards of the court. The former Prime Minister and ''Deadly Nightshade'' were made responsible for raising the boys. Most of the responsibility for Bertrand (the younger by nine years) fell on her and a succession of governesses and tutors. She was a strict disciplinarian, though far from orthodox and unchangable, and would allow no one to discuss opinions that differed from her own current views; most tutors were dismissed after a few months. When the young Bertrand visited his boisterous Stanley relatives, he found himself confused and tongue-tied. Since his older brother was mostly away at school, Bertrand led a lonely and bookish life, with few friends his own age. He read books on theology, ethics, mathematics, and science. He found no unanimity of views in these books, and thus began debating the issues by himself. Some of these soliloquies were recorded in a diary. He considered it obvious that the happiness of all mankind should be the goal of all planning, and was surprised to find that some people disagreed. Like many well-known mathematicians, he had difficulty memorizing the multiplication table.

The summer that he was eleven, his older brother (home from school) began teaching him geometry. At first, he felt that he had reached certainty, only to find that Euclid's demonstrations depend on unproven premises. Still, mathematics provided him with problems that he could solve without the painful doubts of his own competence that were left after he had worked on theological and philosophical problems. At the age of sixteen, he was sent to a ''cram-school'' to prepare him for the university. Here, for the first time, he was placed in a group of boys, mostly older than himself. He found it very disagreeable. Arriving at Cambridge, his entrance examination in mathematics was read by Alfred North Whitehead. Recognizing his ability, Whitehead saw to it that Russell met everyone worth knowing, and got him elected to a semi-secret society that debated all kinds of topics, social and political as well as philosophical. The reason for the closed meetings and undisclosed membership of this society is obvious. Russell majored in mathematics but he devoted his fourth year to a concentrated study of moral philosophy, his minor.

Russell became engaged to, and ultimately married, Alys Smith, an American Quaker. At the home of her parents, he became acquainted with Sidney and Beatrice Webb, who were among the leaders of the British socialist movement. After their marriage, the Russells traveled extensively and spent some time in Germany, where they attended the meetings of the socialists. This so disturbed the German government that it protested to the British Embassy. Returning to England, Russell published his first book, in which he discussed German socialism. Soon afterward, he published a revised version of his dissertation, which was entitled "An Essay on the Foundations of Geometry." This essay shows that he was then strongly influenced by Kant's ideas, and he sought to rescue them. In 1900, he became acquainted with the work of Peano and Frege, published another book on mathematics, and began to plan the *Principia Mathematica*. He and his wife were staying with the Whiteheads. Mrs. Whitehead suffered from frequent heart attacks. The sight of her agony affected Russell very strongly. During one of the attacks he took her young son, who was frightened, for a walk. On this walk Russell underwent an intense emotional upheaval. In five minutes, he says, he became a completely different person. Having been an imperialist, he became a pacifist; having cared only for the exactness of mathematics and philosophy, he discovered within himself an intense interest in people, especially children, and in beauty.

Such emotional crises are common among able mathematicians, and often end in suicide. Fortunately, neither Newton nor Russell chose that way out. Anyone who has engaged in intensive mathematical work knows that it is an anodyne for the painful awareness of the uncertainties of life, the dirt and misery of ordinary human existence. Like all pallatives, however, it does not cure the pain, and when it becomes inadequate, the pain is greater because the sufferer has not become accustomed to enduring it.

The intensity of Russell's crisis passed, but its influence remained with him. The remainder of his life will be more understandable if his ultimate and considered conclusion about war is stated at once. Russell believed that most wars are unjustified because, as Gorgias had said long ago, their issues can be resolved by peaceful discussion, without resorting to armed conflict, which is so disastrous to both sides. A very few wars are justified; this is always because evils have been allowed to grow, slowly but unchecked, until they can only be eradicated by force. It would be desirable that this force be applied by a permanent world government, rather than by a temporary alliance of individual nations.

Russell's earlier enthusiasm for mathematics was gone; nevertheless, he and Whitehead doggedly worked at the *Principia Mathematica* for more than a decade, and finally finished three of the four volumes originally planned. Neither man seems to have done much original mathematics afterward, but Russell kept abreast of developments in the field, and often gave systematic series of lectures on mathematics and mathematical philosophy. Even before this work was finished, Russell spoke publically on political issues. He even stood for Parliament on the issue of votes for women. He and his wife were pelted with rotten eggs. Neither the behavior of the Russells, nor that of the egg-throwers, can be cited as evidence for the doctrine of psychomathematical parallelism. Russell's activities are of special interest in assessing the proposal to mathematize the behavioral sciences, in order to enable man to control his own future. Russell was certainly an able mathematician, with wide interests, yet, like Abbe, he propounded no mathematical theories of behavior, but sought to alleviate human misery by practical action. Like Abbe, he held strong views on the ethics of the unearned increment to capital. He had inherited much money from various relatives; he gave it all away, most of it to Cambridge University and its associated Newnham College for Women. Russell proved to be a prolific and popular writer and lecturer, who could usually earn his own living. Yet there were periods when finances were worrisome. When the *Principia Mathematica* was finished (it was not one of his bestsellers), Russell again tried to enter politics, but so unsuccessfully that he accepted an invitation from Newton's Trinity College to be a lecturer on mathematics.

His political work made him personally acquainted with the leaders of the British government. During World War I, he was an outspoken pacifist, and especially critical of conscription and the treatment of conscientious objectors. Under certain circumstances, they were tried by court martial and sentenced to death. Russell easily persuaded Asquith to stop the executions, but he was not able to persuade Lloyd George to improve the treatment of conscientious objectors in prison. Russell founded the No Conscription Fellowship, almost all of whose members ultimately went to prison. He spoke at a public meeting and was mobbed; he was appalled to find that the general public was even more bloodthirsty than its leaders. He wrote pacifistic leaflets and was arrested; as a first offender, he was merely fined. Unable to pay the fine at once, his library was confiscated and sold. Recalcitrant, he repeated the offense and was sent to prison for six months; while there, he wrote the *Introduction to Mathematical Philosophy*. Trinity College cancelled his appointment as a lecturer in mathematics. Harvard University invited him to come to the United States, but he was denied a British passport.

By 1920, the war hysteria had subsided. Though Russell was critical of the treatment of Germany by the Allies, he was sent to Russia as part of an official delegation. He found the visit a nightmare. It formed the basis of his book, *Practice and Theory of Bolshevism*, in which he described and denounced the actions of Lenin and his Party. A later visit to China led him to take an unduly optimistic view of the future of that country.

While continuing to give occasional series of lectures on mathematics and logic, Russell became more and more preoccupied with social and educational problems. He, and later his second wife, again stood for Parliament, again unsuccessfully. He wrote a book, *On Education, Especially in Early Childhood*; it was widely read, and provided an income for some years. He and his wife founded an experimental school, which provided them with more than simple financial worries. It was, however, one of the forerunners of the progressive education movement. Another popular book was *The Conquest of Happiness*, in which Russell considered what an individual could do to achieve a satisfying life, without waiting for a change in the social and economic system.

In 1929, he wrote *Marriage and Morals*, which caused a storm of indignation among the orthodox. Ten years later, it was the basis of a court decision that abruptly canceled his appointment as Visiting Professor of Mathematics at the College of the City of New York. Twenty years later, he was awarded the Nobel Prize for this book. Today, it is almost lost in the welter of books on the subject, most of them much less carefully considered than Russell's.

During this period, he also became concerned about the aftermath of World War I. Fascism and Nazism were even worse disasters than he had anticipated. In 1931, he foresaw the possibility of atomic bombs. This was not as early as Anatole France, but at a time when most physicists agreed with Lord Rutherford that radioactive decay could neither be slowed nor accelerated by human action. In private conversations, some physicists foresaw the time when the atom would provide a new source of energy. None realized how easily or how soon the problem would be solved; most thought of the new energy source as beneficent, very few thought of it as a new and immensely destructive weapon. It is not irrelevant to mention that, almost simultaneously, Michael Arlen wrote his novel, *Man's Mortality*. His knowledge of science was much less than Russell's. He also underestimated the pace of technological development; he did foresee television, radar, walkie-talkies, nuclear energy, large intercontinental airliners, smaller supersonic military aircraft, laser beams used as "death rays," and so on. Of course he did not use these names, but it is easy to identify them now. Like Bellamy, he set his story in 1987; like Russell, he foresaw mass murder and the destruction of cities; like Anatole France, he was ignored. Arlen's prolific imagination, though remarkably accurate in foreseeing technological developments, roused only incredulous smiles in his contemporary readers. He might almost as well have been addressing an Elizabethan audience.

Then Russell wrote *Which Way to Peace?*. When World War II broke out, Russell agreed that while Nazism and Fascism were the disastrous consequences of World War I, they could be eliminated only by an Allied victory. At the end of the war, he had become temporarily respectable. Trinity College reappointed him as a Lecturer; his *History of Western Philosophy* was an international bestseller. He became increasingly concerned about the possibility of nuclear war; after writing *Common Sense and Nuclear War*, he was invited to lecture at the Imperial Defense College. During the Berlin Airlift, he was made a member of a military deputation to Berlin and was horrified by the destruction which the Allies had visited upon Berlin and Dresden, even without using nuclear weapons.

It is difficult to summarize Russell's activities during the remaining twenty years of his life, and impossible to list them all in the space available here. He seemed to become more energetic with the years. He also found more than a few people eager to supplement his own energy and work under his guidance. In 1949, he published *Authority and the Individual*, which may be taken as an outline of his future activities. He shared Gandhi's views on passive resistance and civil disobedience. The Nuremberg Trials had discarded military commands as a justification for individual wrongdoing. Russell felt that there was then a greater need to seek individual liberty than in the past. World War II, despite the defeat of the Nazis and Fascists, had left an encroachment on individual liberty throughout the world, and this could not be ignored; otherwise there would be private lethargy and undue displays of public power in suppressing the individual. Then, thinking he had been unduly pessimistic, he wrote *New Hopes for a Changing World*. World government and an end to wars, especially nuclear war, was a possibility.

Russell spoke and wrote about nuclear disarmament. He was discouraged by the reception given his views by high government officials, and by some of the men who had made nuclear weapons possible. He was encouraged by letters from private individuals and resolved to organize these individuals for more effective action. The result was a long series of committees and other organizations, many of them short-lived. He persuaded many people of various ideologies to sign what has become known as the Einstein-Russell Manifesto against nuclear war. He arranged a congress of Parliamentarians for World Government, which was attended by people from both sides of the Iron Curtain, and this group unanimously passed favorable resolutions. He supported Cyrus Eaton's "Pugwash Conferences" (these were named for Eaton's home in Nova Scotia), which even today provide a forum for personal discussions between individuals of many nations and viewpoints. These activities were successful and received wide and favorable publicity. Russell hoped to create much larger, politically powerful, organizations devoted to these same goals.

In 1960, Russell was joined by Ralph Schoenman, who brought news of the initial successes of sit-down strikes in the United States. In England, there was much dissatisfaction with the granting of a nuclear submarine base to the United States. Russell and Schoenman organized this discontent into a nonviolent sit-down strike in front of the Ministry of Defense, in which about 5000 people participated. On the anniversary of the explosion of the atom bomb over Hiroshima, Russell addressed a crowd in Hyde Park. Knowing that the crowd would be large, Russell and his associates resolved to use loudspeakers, even though these were prohibited by the park's rules. The police therefore interfered, politely enough. The meeting adjourned, and Russell led a march to Trafalgar Square. Despite a thunderstorm, many followed him there. Russell, his wife, and other associates were summoned to trial for inciting civil disobedience. They were all sentenced to two months or more in jail. Out of consideration for their age and health, the sentences of Russell and his wife were commuted to one week. During this week, some of their associates announced a meeting in Trafalgar Square, to be followed by a sit-in before Parliament. Despite a public order forbidding it, this and similar meetings were attended by unprecedented but peaceful crowds. Police brutality against the participants was documented by photographs, and received worldwide publicity. Before the end of the year, five of Russell's associates were indicted under the Official Secrets Act. The jury deliberated at great length, but eventually brought in a verdict of guilty; the defendants were sentenced to eighteen months in jail.

In 1962, Russell reached the age of ninety, and it became obvious that his work could be carried on only by a permanent organization. Ralph Schoenman proposed the Bertrand Russell Foundation, a group organized "for any purpose that would further the struggle against war, armament races, and injustice suffered by oppressed individuals and peoples." The British government refused to incorporate this group, much less grant it tax-exempt status. The lack of a corporate status meant that each member of the Foundation would be liable for all of its debts, a risk that only the most dedicated would take. After much maneuvering, the Bertrand Russell Foundation has finally been incorporated. It usually functions without publicity.

An exception to this occurred in 1966. After some of the Foundation's leaders had personally investigated the Vietnam war, it sponsored the "International War Crimes Tribunal." Although without any legal authority to enforce its decisions, it followed the legal precedents of the Nuremberg Trials. It found many members of the United States Government and its Armed Forces to be guilty of war crimes. Russell personally participated in this, and in a sit-in in front of the U.S. Embassy in London. Russell and the "Tribunal" were ridiculed, especially in England and the United States. Since then, the publication of the secret Pentagon Papers by Daniel Ellsberg, and the My Lai court-martial conviction of Lt. William Calley have confirmed much of the evidence considered by the "Tribunal" and amplified some of it.

Russell was criticized for his activism, his attempts to incite civil disobedience, and he was made to suffer for this. No final conclusion concerning him will be reached without considering the ineffectiveness of the milder methods used by Anatole France, Ignatius Donnely, and Michael Arlen. One will also recall the effectiveness of *Uncle Tom's Cabin*. When Harriet Beecher Stowe was introduced to Abraham Lincoln, he greeted her: "So this is the little woman whose book provoked so great a war!" It appears to be easier to provoke a war than to prevent one. The militarism of Plato, and his diatribe against the pacifist Gorgias continue to influence our present society.

Atoms, Evolution, and Ethics

It has been seen that at least two able mathematicians, Abbe and Russell, abandoned mathematics when they approached the problems of people rather than those of technology and science. They are not the only examples of this phenomenon, which seems worthy of more investigation. It has also been seen that mathematics originated among the tradesmen and craftsmen, but that there was a discontinuity in its development at about the time of Plato and Aristotle. This had two results that might be associated with these two men, even though this is an oversimplification. Plato gave all knowledge, including mathematics, a high status in the aristocratic hierarchy. Since Plato also gave warfare a high status, and the crafts and trades a low status, this somewhat reversed the trend which has been called the sedimentation of knowledge. However, it gave knowledge, especially mathematics, a mystical cast, and left it without roots in human society. It also perpetuated the aristocratic glorification of war and violent political action, both of which are inhumane, and neither of which is productive of knowledge. Ironically, Plato is now classified as a humanist. Aristotle's interests were wider than Plato's, and his attitude less biased by the aristocratic fallacy. For the present, it is sufficient to recall his attempt to humanize knowledge. He referred it, on the one hand, to human experience (including instruction), but on the other, he tried to follow Plato's early view that knowledge is inherent in the human soul or mind (the classical Greeks made little distinction between soul and mind). To reconcile these two opposing doctrines, he introduced the notion of axioms, which ultimately led to the doctrine of psychomathematical parallelism. The humanitarian activities of Abbe and Russell thus amounted to a rejection of the theory of psychomathematical parallelism as well as the high aristocratic value placed on inhumane action. This is not so evident in the activities of their predecessors, such as Galileo and Newton.

A century or so before Plato and Aristotle, Thales and Pythagoras had initiated another trend in mathematics and philosophy that, as has been seen, was more closely related to later views of Galileo and Newton. This trend was also aristocratic in that it made the pursuit of knowledge at least respectable. Pythagoras seems to have given it the very highest status. It was more humane, however, in that it placed a much lower value on violence. It is this second trend that may provide a clue for understanding the phenomenon exhibited by Abbe and Russell.

It is important to remember that both Pythagoras and Plato were pagans. They subscribed to theologies that are no longer current in Europe and the United States. In the case of Plato, Christian commentators have tended to ignore this. In the case of Pythagoras, both Christian and secular commentators tend to ridicule it. Plato's paganism, especially as exhibited in the dialogues, *Timaeus* and *Phaedo*, has an affinity to Christian theology. This is not to say that the teachings of Jesus were in any way similar to those of Plato, but rather that the two are incongruously combined in orthodox Christian theology. The doctrines of Pythagoras were never incorporated into Christian theology, but did evolve into the European philosophies of empiricism and rationalism. This evolution will now be investigated.

Both Plato and Pythagoras believed in the immortality of souls. Plato believed in reincarnation, and Pythagoras believed in transmigration. The doctrine of reincarnation assumes that only people (sometimes not even women) have souls. The doctrine of transmigration assumes that animals as well as people have souls. Ancient myths suggest that trees were sometimes considered to have souls. Both doctrines declare that after death the soul is freed from the body but retains its identity, and later returns to Earth to inhabit another body. Implicit in both doctrines is an ethics of rewards and punishments. Wrongdoing is punished by reincarnation in a worse body, a soul that has done good is rewarded by reincarnation in a better body. Plato's ordering of better and worse bodies is based on his own version of the aristocratic fallacy. His belief about good and evil is confused, and it is not easily found in the dialogues, but he certainly did not believe that war is evil. The ethics of the Pythagoreans is quite as difficult to reconstruct, but they certainly believed that it is evil to kill even animals.

In the generation just before Plato, the doctrine of the transmigration of souls was given a unique and distinct form that is very different from that which has survived in the Orient. In a modified form, it is implicit in the modern theories of evolution of the universe and of man. This point is rarely emphasized. While its origin can be traced to Thales of Miletus, it was given a definitive form by Leucippus and Democritus. Both lived in the Thracian city of Abdera, on the northern shore of the Aegean. Democritus was either a student or an associate of Leucippus; as a third alternative, it has been suggested that Democritus and Leucippus were two names for the same man. Abdera was a colony founded by Miletus to exploit the Thracian gold mines, so that the connection with Thales and his successors is understandable. When Hippocrates left his position as the head of the Asklepian medical cult on the island of Cos, he settled in Abdera and formed a close friendship with Democritus. The two may have met earlier, and in any case, have been congenial. They were the same age, and considerably older than Plato and Aristotle, even ten years older than Eucleides of Megara. They both lived to be very old, and therefore can be called contemporaries of Plato and Aristotle. More importantly, they would have formed their mature opinions independently of Plato. For this reason, it is unwise to allow simple chronology to force one to discuss Plato and Democritus simultaneously.

Democritus is known as the "Laughing Philosopher" and must have been cheerful and light-hearted. He is said to have been amused by people's foibles, though not cynical. Hippocrates seems to have had a more somber personality. Democritus had inherited a fortune from his father which he spent on travel, thus becoming acquainted with many countries, religions, and scholars. Returning to Abdera, he lived modestly and wrote many books. Hippocrates wrote books about medicine, though not as many as were originally attributed to him. Leucippus had proposed an atomic theory that supposed that the universe contained only empty space and indestructible atoms. The atoms move "of necessity." This is the same phrase that appears in Plato's "Ordinance of Necessity." Neither man made the meaning of this very clear, and logicians have debated the matter ever since. For the present purpose, no harm will be done if Democritus' law of necessity is equated to the familiar "law" of cause and effect. Leucippus explicitly states that the atoms are not moved by love and hate, and Democritus says that what we call "chance" or "accident" is merely our ignorance of the details of the event. Plato, on the other hand, equated ignorance first to the soul's failure to remember past incarnations and then to evil. In *Menon*, Socrates says that no one knowingly does evil, but no other explanation of evil is given. Such differences among the ancient writers gave rise to the logical problem of modalities; for the present purpose, it will be convenient to follow Leucippus and Democritus and ignore Plato and the others. The tendency to revere ancient writings and to view their contradictions and confusions as concealed profundities has created much mischief.

Returning to Democritus and his atoms: in the primeval Chaos or Vortex, each atom was separate from the others. Then they collided, combining into stars and worlds, which again collided. Democritus said that the Milky Way is composed of innumerable stars and worlds. The original forms of all things were "of necessity" formed in this way. He elaborated this theory by supposing that there are many kinds of atoms; those of the soul are the noblest, exceedingly small, smooth, and round. We still use the word "noble" in such a chemical sense, gold and platinum being noble metals, helium a noble gas; this is, however, a somewhat old-fashioned mode of speech and may soon become obsolete. The "soul" atoms are not confined to man and the other animals, though there are more of them in living creatures than in inanimate matter. Plants, and even stones contain some "soul" atoms. Soul, Mind, and Vital Essence are one and the same thing. In animals, including people, the "soul" or "mental" atoms are distributed throughout the body. Living things, ranging from plants to people, originated in the moist Earth. This is reminiscent of later theories of evolution. The body owes its shape to the way in which its atoms have arranged themselves under the law of necessity. The atoms do not combine or merge, but move about each other, still of necessity. On death, the body loses its shape, its atoms (including the "soul" atoms) disperse and return to the earth and the atmosphere. Later, through obvious processes, they rearrange themselves into other living beings or inanimate objects. Hence, the structure of the individual soul does not survive the death of its living body. Only its atoms are immortal, and they transmigrate into new material shapes. Continuing, Democritus held that all knowledge comes from the senses, from changes in the body caused by the impact of atoms from the outside. But we can know nothing for certain, since a single atom is not perceptible. Some commentators have said that this is scandalous materialism, atheism, and heresy. Plato was certainly not pleased with it. Others have said that Democritus was the greatest of the ancient philosophers.

Democritus evolved a system of ethics which was definite and humane, and which he seems to have considered a consequence of his cosmology. To us, the connection between cosmology and ethics seems vague, or even illogical or incongruous. It must be emphasized that our notion of logic, and our fear of being inconsistent, stems from Aristotle. Democritus is said to have been a very able and creative geometer, but he wrote no series of elements. He was writing for an audience that was very different from ourselves. Our history and philosophy teachers rarely bring this to our attention. As to ethics, he says that every person should strive for knowledge and happiness. This seems to imply a freedom to decide, to choose between alternatives, that does not follow from the law of necessity. Good actions should be done without expectation of reward; people should be ashamed to do evil. Sensual pleasure brings only a transient happiness; a more lasting though less intense happiness comes from leading a life of moderation. Certainly, happiness does not come from acquiring wealth, or from spending it. Contentment can only be found within oneself. Democritus wrote extensively, though not everything has survived. This summary is only sufficient to show that his doctrine is easily separated into two parts, which may be called the scientific or cosmological and the ethical or moral. The latter is only loosely associated with the former, and they are certainly not connected by a chain of close logical reasoning. Democritus foreshadows much that is to be found in our own times, including the activities of Abbe and Russell, Darwin's theory of evolution, and the atomic theory of physics and chemistry. He astonishes modern readers. Those who take modern science to be unquestionably true marvel at his preview of so much of it.

The authenticated writings of Democritus' friend Hippocrates add to our bewilderment, not only in regard to the connection between science and ethics, but also because he is a much less controversial figure than Democritus. Commentators have only praise for Hippocrates. On the scientific side, he describes nearly fifty medical case histories, recording the patients' initial symptoms, the treatments administered, and the changes in the symptoms during treatment. This is the prototype of modern clinical science. With what has come to be known as scientific honesty, he records that more than half of the patients died, despite the treatment. With the accumulated knowledge of two thousand years, we might be tempted to say "because of the treatment." Hippocrates may have suspected the same. This suggests a possible reason why he left his eminent position as head of the Asklepian cult on Cos, and retired to Abdera. No other explanation has ever been advanced.

Then there is the Hippocratic Oath, which is still fundamental for the professional ethics of medical practice. Again it is not directly related to his science; it may have been older than Hippocrates, but there is internal evidence that he (or one of his contemporaries) added to it in an essential way. The Oath lists certain things that the physician swears not to do. It does not present the reasons for these prohibitions. The performance of abortions and the preparation of contraceptive pessaries are both prohibited. This is the more remarkable because both practices were common among the Greeks of his day. More than that, it was legal to expose unwanted infants, and allow them to die unattended. It is said that in Athens it was becoming customary to expose children near a temple, where they might be found and adopted by kindly people. In Sparta, however, sickness was a crime against the State, and it was still customary to expose weak and sick babies in lonely, barren places where they were almost certain to die. This problem is not brought nearer to a solution by noting that the medical cult of Asklepios was originally Dorian, and that its center was originally in the mainland area that later became Sparta. When the stern Spartan constitution was adopted, members of the cult moved to distant Asia Minor and then to the island of Cos. Others appear to have gone to Crotona and Locri in Italy.

One begins to see that both Democritus and Hippocrates were innovators who did not follow the multitude of their contemporaries. In the following generation, their ideas were elaborated and propagated by Epicurus. Like Pythagoras, Epicurus was born on Samos. He spent a year in Athens at the Academy shortly after Plato's death, and after Aristotle had left. He seems to have learned of Democritus' ideas from the elder Aristippus of Cyrene, who is often confused with a grandson of the same name. The elder Aristippus is supposed to have been a pupil of Socrates, the first to accept money for teaching. After the death of Socrates, he spent some time at Syracuse, at the court of the king who enslaved Plato. Aristippus seems to have been in Athens at the time of Epicurus' first visit to that city. After leaving the Academy, Epicurus studied and taught elsewhere. He later returned to Athens and started a school in a pleasant garden at about the same time that Euclid was starting his school in Alexandria. Unlike the Academy, Epicurus' school was not incorporated as a religious cult. He lived very frugally and charged tuition; apparently the precise amount was determined by individual agreement, based on the student's ability to pay.

The most original part of Epicurus' philosophy is not often emphasized. He held that cosmology is not essential to ethics. He must have known enough of Aristotle's logic to recognize that Democritus' ethics cannot be logically derived from his cosmology. Yet he said that if one must have a cosmology, that of Democritus is preferable. He did teach Democritus' cosmology and even extended it in at least one respect. He taught that the atoms in compounds remained distinct, and they did not fuse or lose their identity. However, they remained close together while moving about. This is very much like the theory of atoms and molecules that was developed during the late nineteenth and early twentieth centuries. One hesitates to ascribe precision to Epicurus any more than to ascribe it to Democritus or Hippocrates. It seems more likely that, two thousand years later, people were stimulated by his writings, either directly or indirectly, and were thus induced to interpret their new experiments in terms of his speculations. In the eighteenth century, some copies of his correspondence and other writings were found at Herculaneum in the "House of Papyri," having been preserved through the centuries by the heavy layer of ash from Mount Vesuvius that had obliterated the ancient town in a few hours. More of Epicurus' writings survived in edited and translated form.

One should pause and ask which cosmology is less good than Democritus'? Epicurus seems to have given no answer; presumably the answer was too obvious to his contemporaries to require explanation. Perhaps, however, it was too dangerous to be too explicit; Epicurus did not have the immunity of the leader of a religious cult. Even so, commentators have persistently maligned and denounced him and his ethics, which is an elaboration and refinement of Democritus' ethics. He did say that the aim of philosophy is not to explain the universe, but to free men from the fear of the gods and death. Epicurus claimed that there are gods, but they do not choose to involve themselves in the relatively inconsequential affairs of human beings. Hippocrates had previously claimed that sickness was not caused by the gods. This is slightly reminiscent of Pythagoras' "conversing with the divine" to obtain knowledge, rather than praying for miraculous favors. However, Epicurus asked that if knowledge does not indeed come from the senses, where then does it come from? He confusingly implied that the knowledge of the existence of the gods is *a priori*, to use Kant's term.

Epicurean ethics is based on the principle that the goal of all men should be the simple and undisturbed joy of being alive; he called this goal *hedone.* We have no single equivalent word in English for this as "hedonism" now has a completely different meaning. In Epicurus' view, many things prevent the achievement of this goal, and many of these are under the individual's control, they do not occur "of necessity." Obvious examples of this include the overindulgence in food and wine. Epicurus was certainly not the "philosopher of the belly" as some have said. Plato's Academy was, at least theoretically, coeducational; Epicurus' school actually was. Moreover, he had a mistress, an Athenian, who he was prohibited as a foreigner from marrying. This contributed to later scandalous stories that are most likely without foundation in fact. Epicurus advocated moderation in all activities: the strength that comes from incessant exercise (a habit among Athenian men) is a virtue in a horse, but not in a man. This is a somewhat aristocratic view, for many of his contemporaries, especially slaves, had need of such strength to earn even the barest necessities of life. For if much food is not conducive to *hedone*, neither is too little and Epicurus did not advocate asceticism. Not everything necessary for *hedone* is material: the love, friendship, and respect of other people is also essential. These can easily be lost if one does evil, if one behaves so as to prevent others from achieving *hedone*. The fear of an enemy is disturbing to one's pleasure in being alive: it is therefore well to behave so as not to make enemies. Envy of another's good fortune, ambition for the unattainable, competitiveness in general; all can and should be avoided, since they are disturbing emotions. There are other obstacles in life which one cannot control, accidental misfortunes of all kinds, including sickness and death. However, these are not eased by complaining or by envying the more fortunate. In particular, death is not avoidable by living in constant dread of it; besides, death might not be as painful as other misfortunes that one has endured. One may even risk one's life to save a loved one, or in war, knowing that the loss of the beloved or the defeat of one's country will make one incapable of achieving *hedone*. All of this is an ethics of personal responsibility, responsibility not only for oneself, but for others as well.

While Epicurus must have been aware of Aristotle's logic, his ethics is not a closely reasoned series of elements. It contains no references to axioms, or to anything resembling Kant's *a priori* knowledge. The *a priori* knowledge of the existence of gods was as irrelevant to Epicurus as Democritus' cosmology. Certainly, Epicurus' ethics contains nothing resembling the principle of psychomathematical parallelism, or do the writings of Democritus or Hippocrates. This is one of the reasons why the work of these men should be considered separately from the mathematical work of classical Greece which culminated in Aristotle and Euclid. Epicurus' philosophy is an ethics of courage and self-control, though not of asceticism and empty humility. It has never had a large following, and it is therefore impossible to say what its unforeseen consequences would have been, especially in our industrial civilization. But it is not contemptible ethics as they have often been said to be. Epicureanism is certainly not a "philosophy of the belly," but Aristippus the Younger transplanted it to Cyrene, and from there it came to pre-Christian Rome. It was gradually debased, and it ultimately did become a philosophy of the belly.

In Rome, the original pure form of Epicureanism was revived by the melancholy poet and reformer, Lucretius. He lived during the early part of the first century B.C., and his long poem, *On the Nature of Things*, expounds both Epicurus' ethics and Democritus' atomic cosmology. He also said that the purpose of philosophy is to free people from the fear of the gods and punishment after death.

Freed from these fears, a nobler ethics of personal responsibility, for oneself and for others, becomes possible. These ideas are very like those expressed in some of Bertrand Russell's writings. Lucretius died young and under suspicious circumstances; the gossipy record leaves one uncertain whether the cause of death was accidental poisoning or murder. Perhaps he was fortunate, for had his writings become widely known and influential before his death, he would certainly have been persecuted.

It seems as if people do not wish to be freed of the fear of gods. But this merely shifts the question: why do people persecute those who seek to free them from this fear? There are certainly no easy answers to this question, but perhaps historical studies can suggest answers.

Greek Religion and Mythology

Lucretius thus provided further evidence that Democritus, Hippocrates, and Epicurus were not following the multitude, especially not in matters of religion. To study the questions just raised one will need to know more about the religious beliefs of their contemporaries; otherwise, one will not be able to understand these innovators. In one of Plato's dialogues, Socrates induces Euthyphron to say that religion is the art of doing business with the gods. Socrates, of course, rejects this, but it is likely that Euthyphron summarized the vague notions of most of his contemporaries. It has been seen that in ancient Rome there was no sharp distinction between religious rituals and secular routines, between priests and public officials charged with preventing cheating in the market place. Religion, ethics, and business were closely linked. From the epic poems of Homer and the tragedies of Aeschylus and Euripides, one also learns that it was believed the world was operated by a group of capricious, sometimes vengeful or even idly malicious gods. These gods intervened directly in the affairs of mortals, causing their favorites to be successful, and others to be unsuccessful. The gods were powerful but all too human in their likes and dislikes, loves and hates. It was possible to do business with them, to buy their help. This is very different from Democritus' cosmos, that runs according to the impersonal law of necessity. One also begins to understand why Epicurus explicitly separated ethics from cosmology, and stated that the gods do not concern themselves with human affairs. For Epicurus, ethics is the art of doing business with people, if "business" is not given a strictly mercantile meaning. One must make the same allowance for Euthyphron. Whether or not one accepts Democritus' and Epicurus' other ideas, one must admit that this clear distinction was a major intellectual achievement, even though it was foreshadowed by Thales and Pythagoras. For Epicurus, it increased, rather than diminished, the grandeur of the gods.

The religious, ethical, and cosmological writings of philosophers and poets are no sure guides to the beliefs of their contemporaries and predecessors. Until the present century, historians based their accounts of classical Greek religious beliefs on the epic poems of Homer and the tragic dramas of Aeschylus and Euripides. As these passed through various editions and translations, their original poetic content was increasingly romanticized. Similarly, Plato was romanticized into a humanist, the inhumane passages in his writings being ignored. This movement culminated in nineteenth-century Romanticism, and is still influential in the semi-historical popular literature of the present day.

In its own time, however, the poetic religion can have been prevalent only among the intellectuals, for whose education these poems and plays were the major textbooks, and to which they constantly referred in their writings and discussions. There is abundant recorded evidence of a darker, superstitious attitude toward the gods. Thus Nicias, the wealthiest slaveowner in Athens, became very powerful politically, but was presumably only semi-literate and certainly superstitious. In Greece at that time, there was no distinction between political and military leadership. Nicias became the leader of a large fleet; he allowed it to be destroyed because he was terrified by a solar eclipse and the warnings of soothsayers and he ignored the approaching enemy fleet. Instead of preparing for the impending battle, he did business with the gods by offering sacrifices. Nicias' attitude can be explained even by an examination of the poetic literature of the time. Homer's *Iliad* describes the participation of the gods in the Trojan War, some gods intervening on one side, some on the other. Poseidon, god of the sea, and Apollo are said to have built the walls of Troy. The goddess Athena is said to have inspired the building of the Trojan Horse, which enabled the Greeks to enter Troy, despite its impregnable walls. Euripides' tragedy, *The Women of Troy*, opens with a conversation between Athena and her uncle, Poseidon. She asks that their feud be ended, and suggests his help against the Greek fleet. Later, the Greek victors burn the city and desecrate the temple of Athena there. Wanting revenge, she wishes to make the homeward voyage of the Greeks "unfortunate." Poseidon readily agrees to infuriate the Aegean waves, Zeus has already agreed to send storms and thunderbolts against the fleet.

The ruins of many large theaters in Greece show that even craftsmen and farmers had an opportunity to become familiar with this literary religion. Dramatic performances, as well as the athletic games that often accompanied them, had a ritual significance. This is attested to by the Sacred Truce that protected travelers on their way to the Olympic Games. But the festivals also had a secular element; commercial fairs were held at the same time. The festivals were held during established periods of a few days, and their season varied from city to city. Both actors and hucksters toured from one city to the next. Many animals were sacrificed to the gods on these occasions, which contributed to the prosperity of the local farmers. The fairs gave everyone a chance to buy foreign trinkets, perhaps also useful animals and artifacts. The expense of the plays and sacrifices was borne partly by the city governments, and partly by private individuals wishing to ingratiate themselves with the general populace as well as with the gods. Surviving inscriptions show that for a period of years, every new play financed by Nicias the Pious received first prize at Athens. By Roman times, these festivals had turned into brutal circuses. It is unlikely that the earlier Greek festivals did much to improve either the intellectual or ethical tone of most of the population. This was one of the reasons why Plato wished to censor the poets and playwrights; he was not inclined to romantic notions of their eloquence, and seems to have considered it immoral to attribute human failings to the gods. In other words, he too was dissatisfied with the contemporary popular religion.

The latter part of the nineteenth century saw archaeology and anthropology evolve out of classical scholarship. One of the early workers in these fields was Jane Harrison. She was born in Yorkshire, England, and educated at Cambridge University. There she became acquainted with many of the most progressive classicists of the time, including J.G. Frazer and Gilbert Murray. She began to study the Greek vases and pottery fragments in the museums, not simply as works of art, but as records of the lives of the people who made and used them. In 1900, at the age of fifty, Harrison became Fellow and Lecturer in Classical Archaeology at Newnham College, and she began to publish the results of her studies. Her first book was entitled *Prolegomena to the Study of the Greek Religion*, and her last book was entitled *Epilegomena to the Study of the Greek Religion*. Despite these forbidding titles, her writings have a freshness that comes from her lack of the customary European condescension toward early and primitive peoples. More recently, W.K.C. Guthrie, who made new translations of many Greek writings, has written a critical summary of the work of Harrison and her successors. The writings of Harrison are technical works, addressed to those who are familiar with classical Greek literature. Robert Graves has written a book for the modern public which generally does not have this familiarity. In his *Greek Myths*, he gives prose versions of closely related parts of this literature before proceeding to comment on them.

An example of the way in which history has been exhumed from legend is provided by the story of Diogenes. The legend asserts that Diogenes lived in a barrel and went about in broad daylight with a lantern seeking an honest man. It now appears that sometimes many people lived in barrels, except that the barrels were large earthenware jars, normally used for the storage of foodstuffs. The most unfortunate class of persons found shelter in empty food jars. When they went in search of food, they took all their meager belongings with them, since they might otherwise be stolen. The legend is therefore a concise, but distorted version of history. Combined with information obtained from decorations on surviving artifacts, the history can be inferred with some certainty.

In the same way, the historical religion of the Greeks has been exhumed or restored. Sacrifices to the gods were an important part of everyday life in classical Greece. The sacrifices were of two kinds. The festive sacrifice was most clearly exemplified by the libation: before drinking a cup of wine, some of it was spilled on the ground. Thus the wine was shared with the earth gods, who were included as participants in the feast. Similarly, when animals were sacrificed, often only parts of them were burnt, so that the sweet-smelling (!) smoke would nourish and delight the gods. The animals' blood ran into gutters; ghosts were thought to come and lap it up, greedily depriving the gods of the nether regions from its benefit. The larger part of the sacrificial animal was roasted and formed the main course of the sacrificial feast. During the large festivals, which were financed by the city and its politicians, the crowd of spectators thus received free meals. The gusto with which these meals were eaten was enhanced by the feeling that the gods were also present at the feast. The other kind of sacrifice was the holocaust, intended to appease an angry deity, to bribe the gods into helping with a proposed project, or to cease punishing a repentant offender. In this case, the entire sacrificial animal was burnt, and there was no feasting. Holocaust sacrifices were often preceded by rites of purification. The rites of purification combined ascetic fasting, laxatives, and ritual baths. In contrast to the festive sacrifice, the holocaust was an occasion filled with anxiety, fear, or even abject terror.

A business transaction involves two parties, even if one of them is a god. There was always doubt whether the god agreed to the proposal or request made by the sacrificer, whether he would cooperate with the sacrificer or work against him in the future. The soothsayers determined the god's will by inspecting the entrails of the sacrificial animals. It was also considered a favorable omen if the animal did not resist slaughter. There is some evidence that sacrificial animals were drugged before being led to the altar.

To those whose religion included the doctrine of the transmigration of souls, the thought of animal sacrifice was as revolting as that of human sacrifice. There is strong evidence that the poetic Greek religion evolved out of an older religion in which human sacrifice was practiced. Homer's *Iliad* and Euripides' *Women of Troy* contain references to human sacrifice. When the Greeks were about to sail for Troy, they were delayed by storms. The soothsayers said that these were caused by the anger of the goddess Artemis, which could be soothed only by the sacrifice of Iphigenia, the daughter of Agamemnon. There are two versions of this story. According to one version, Iphigenia was sacrificed. According to the other version, she was rescued by Artemis herself and transported to the Chersonese (Crimean) Peninsula. There she was made into a priestess with the duty of sacrificing all passing strangers to Artemis. When her brother, Orestes, came in search of her, Iphigenia failed to recognize him at first, and almost sacrificed him in the temple. Another tale of human sacrifice concerns the looting of Troy by the victorious Greeks. Achilles had been killed earlier, and Polyxema, the daughter of the Trojan king Priam, was sacrificed at his tomb.

This story leads naturally to a consideration of burial rituals and attitudes toward the dead. Some archaeologists see ritual burials as a sign of the beginning of family love and affection. This may be true, but Jane Harrison and others have shown that the fear and appeasement of the spirits of the dead was also involved. There are ancient burials in which the corpse was bound with cords; this can be interpreted as the immobilization of his ghost. The fear and appeasement of ghosts persisted well into the classical period, especially among the less educated people. In more recent times in England, the bodies of executed criminals were buried with a stake driven through them, to keep the ghost from wandering. The fear of ghosts is thus very ancient and very persistent. The same can be said of the fear of gods that intervene in human affairs. It may be surmised that these fears originated in dreams and nightmares. Other aspects of the lives of the very earliest people must also have contributed to these fears and beliefs.

It is impossible to name all the gods and demigods, goddesses and demigoddesses, that were worshipped in classical Greece. Academus, whose power did not extend beyond a small grove of trees, is remembered only because of an unusual combination of circumstances. These numerous deities were all related by a confusing, and not always consistent, genealogy. Their functions in running the world were vague and these often overlapped. The rituals and titles of even the major gods varied from place to place. Jane Harrison showed that in one place, Zeus was worshipped as "Zeus Smintheus," which translates literally into "the Great God Mouse." A clue to such local variants, as well as to the confused genealogy, is to be found in the mythical sexual relations between gods and humans. Instead of approaching a woman in the shape of a man, they changed both her and themselves into animals. Thus Zeus seduced Io while in the form of a bull; his jealous wife Hera kept Io in the form of a white heifer, doomed to wander about the world tormented by gadflies. Zeus turned Leda into a swan, and the Gemini, Castor and Pollux, hatched from one of her eggs. Europa, sister of Cadmus, was carried off and raped by Zeus, again in the form of a white bull; she bore King Minos of Crete and King Rhadamanthus of the Cyclades Islands. All of these myths are explained by the hypothesis that the classical Greeks were a conglomerate of many small clans, each having its own totemistic religion. As the clans united politically into larger units, their animal gods were fused with the major gods of Greek myth. A variant of this theory was expounded by Newton; instead of supposing that the clan-gods were totems, he supposed that they were ancient leaders, deified by legend. Very likely, there is truth in both theories. The belief, in classical Greece, that there were satyrs (creatures that were half man, half goat), and centaurs (half man, half horse), suggests that the totemistic religions survived longest in wild, remote regions. The consolidation of the older gods into a "Great God" was a step toward monotheism.

Euripides' last play, *The Bacchae*, was written when Athens had been defeated by Sparta in the Peloponnesian War, and was so impoverished that there was little chance that the play would be performed in the foreseeable future. The play tells a story that was widespread, even in earlier times. Dionysus, god of wine, appears on the stage as a mortal, carrying a whip. He has intoxicated and maddened a crowd of women. Agave, daughter of Cadmus, the former king of Thebes, is among them. (The reader should not try to reconcile this myth with other myths concerning Cadmus). Cadmus himself, or his ghost, leads the crowd of women. In their frenzy, the women range over the mountainside, killing and dismembering cattle with their bare hands. The indignant King Pentheus imprisons Dionysus and those of the women that his men have caught. Dionysus frees himself and the women by causing an earthquake to shatter the prison walls. Pentheus then goes up the mountainside in an endeavor to subdue the riot, but is killed and dismembered by his own mother.

All of Euripides' plays except this one had their premiere during the Athenian festival of the Dionysia, held annually in honor of the wine-god. This festival was conducted more decorously than the wild revel just described, with the performance of plays, both new and old, over a period of several days. But much wine was drunk, and many sacrificial animals were dismembered for the feasts. It is little wonder that many scholars have devoted much time to the elucidation of the problems which the play raises. The least of these is Euripides' motive in writing it. Perhaps he intended it as an allegory of the dismemberment of the Periclean empire by the madness of its rulers, one of whom was Nicias. Euripides' treatment of the characters is original; the plot is older.

J.G. Frazer approached the problem with what would now be called the methods of comparative anthropology. He collected myths from all over the world, and published them and his conclusions in a twelve volume work entitled *The Golden Bough*. He was led, on the one hand, to conclusions about very ancient and primitive societies, and on the other, to conclusions about the pre-Lenten carnivals of modern Europe, with records of the riotous, but ritualistic, Roman Bacchanalia linking the two. More recent anthropologists reject his more sweeping conclusions, but respect his work none the less.

According to Frazer, the earliest non-nomadic agricultural settlements were matriarchal. They were small, and were governed by a Priestess-Queen; her male consort had little authority of his own. The yearly succession of the seasons, sometimes favorable, and sometimes unfavorable, was obvious and explained by various myths. It was the function of the Priestess-Queen to conduct rituals to ensure a favorable harvest. The short, dark days of winter led to a fear that the sickly sun might die and never return. Counting had not yet led people to note the constancy of the solar year, or, at least, not to have faith in its recurrence without human (or divine) intervention. The approximately constant lunar month and its relation to the women's menstrual cycles was more easily recognized. Besides, the maternity of a child is indisputable; its paternity was often doubtful. The role of men in the reproductive process was less obvious and certainly not easily explained. All of this was favorable to the ascendency of women, and the assignment of an inferior role to men. The Earth became a goddess, not a god: she was the Great Mother. Somehow, it became customary to sacrifice the Queen's consort (and perhaps her male children) each year. Their dismembered bodies were scattered over the newly plowed autumn fields to ensure their fertility. The Priestess-Queen then took a new consort, thus magically ensuring the rebirth of the sun. The wastefulness of this ritual must slowly have been recognized. The clan had need of an experienced warlord. What could be more natural than that he should marry the Queen, thus consolidating the two authorities? Instead of sacrificing the King, perhaps an annual surrogate, perhaps a slave, or a subordinate consort of the Queen was sacrificed. Later this would become the sacrifice of an animal. Underlying these changes would be the endeavors of men to assert their rights.

Much has been written about the intellectualism of classical Greece. Yet its religion was one of unreasoning fear, fear of a rabble of moronic gods. Its rituals were magical efforts to avert the anger and the malice of these supposedly brutal, powerful, and invisible idiots. A few poets, architects, and sculptors tried to give the rituals a gloss of beauty and dignity; they were successful only in the wealthier cities and sanctuaries. In the hinterland, the religion must have retained much of its primitive ugliness. The intellectualism of the philosophers must have made them disconcerted with this state of affairs. The old religion was unsuitable for an increasingly large and mercantile people, perhaps even less suitable than the religion of Mesopotamia, and not much better than that of Egypt. It is in this context that the Greek philosophers become understandable. They were faced with three problems. The first was that it was unnecessary (or even impossible) to do business with the gods. The second was to construct an ethics of doing business with people rather than with the gods. The third was to persuade others to accept their heretical beliefs. None of the three was easy, and success was unlikely to be rewarded by the gratitude of the multitude. It is not surprising that some retreated into the calm delights of abstract mathematics, or that Epicurus secluded himself in his garden.

The distinctness of these three tasks accounts for the illogical discontinuity of Democritus' atomic, causal cosmology and his ethics of freedom of choice and the will. In his travels, he must have learned of other religions than the Greek. The hinterland of Abdera was still uncivilized; he might have known of quite savage rituals practiced there. His cosmological speculations provided him with an example demonstrating that the universe could run itself, and did not need to be run by willful gods whose motives were far from admirable. He could therefore afford to ignore them and their supposed angers, or even deny their existence. He could begin the second task, that of constructing an ethics suitable for responsible, self-reliant people seeking to make their lives happier. But he seems to have been convinced that the third, evangelical, task was beyond his ability, and resigned himself to smiling at the follies of frightened people. He was a creative geometer, but he did not write elements. His writings were not constrained by the narrow logic that Aristotle would impose later. Even later, Lucretius also would not condescend to Aristotle's logic.

Perhaps enough has been said to explain the relation of these early cosmologists to their contemporaries, but it can do no harm to review the matter in more general terms. Lucretius explicitly said that it is not the purpose of philosophy to explain the world, but to relieve people of the fear of the gods. Remarkably, many, or even most, people do not want to be relieved of this fear. But this only shifts the question. How did this fear originate, and how has it been transmitted from generation to generation? First, as to its origin: this can only be inferred from the nature of early religions. It must be a transformed fear of the environment, a fear that must have been present before people invented speech, or thought, or soliloquy. This primeval fear of the environment was transformed by the earliest people into the fear of gods: strange and powerful replicas of themselves. The dangers of the world became the dangers of the gods. Second, this world is not a safe place in which to live. Death is inevitable; it sometimes occurs suddenly and horribly, or it is sometimes preceded by a period of misery or pain. It seems to follow that those who deny fear of the gods are denying the obvious, as well as denying the wisdom passed down from previous generations. The veneration of dead ancestors is itself complex, compounded of love and fear. For all these reasons, those who deny fear of the gods (and much more, those who deny their existence or power) seem to be provoking the anger of the gods. Other people therefore repudiate the doubters, lest they too share the consequences of divine anger. Since this repudiation is founded on fear, it readily turns into anger and violence against the repudiated. It is not clear whether, and if so how, this is related to the ancient custom of human sacrifice, and the more recent asceticism that is accompanied by the "mortification" of one's own flesh by inflicting pain and discomfort upon oneself.

This seems to be a more or less adequate account of the efforts of these early philosophers, and of the resistance which their efforts encountered. It does not explain the situation in which we find ourselves today. Our environment is not the primeval one; it is not even the environment of medieval times. It is less dangerous; people tend to live longer. Death from wild animals almost never occurs; many fatal diseases have been eliminated. The accumulated knowledge of past generations, and especially that accumulated during the industrial and scientific revolutions, has made it possible for people to modify their environment. New, and more or less unanticipated dangers have also entered the environment. In spite of these, it is still safer than it was. The great increase in the population of the Earth is evidence for this. Much less effort today is spent on doing business with the gods; some people deplore this, but there has been a separation of secular and religious activities. People have been freed from the fear of the gods to such an extent that they can and do spend much more time doing business with each other. They also devote more time to the acquisition of knowledge. Yet, the results are scarcely those which Democritus and his followers anticipated. This is essentially the problem formulated in the Introduction to this book, and it almost seems that we have not come closer to its solution. The investigation must be continued further.

The Influence of Democritus in Recent Centuries

It has already been noted that the ideas of Democritus did not become widespread in Europe until after the time of Galileo and Newton. The survival of these ideas during the preceding years was only partly due to the survival of Democritus' writings, as well as those of Epicurus and Lucretius. As has already been noted, astrologers and alchemists helped to keep Democritus' ideas alive. It is often said that astrology originated in early Chaldea, and careless historians have even attached the date 3000 B.C. to its origin. It does seem that the ancient Mesopotamians considered the celestial objects (the planets and some major constellations) to be gods; but these were not the major gods. Business with the gods was conducted much as it was in later Greece, with the sacrifice of animals and the inspection of their entrails. This is not proper astrology: worship of the stars is no more astrology than worship of fire is cooking. Astrology rejects animal sacrifice, does not do business with the gods, and, finally, it does not seek omens in the entrails of animals. It does attempt to foretell something of the future through the systematic observation of the planets and the stars. There is no archaeological evidence that the earlier Mesopotamians made systematic astronomical observations. The oldest Mesopotamian record of a solar eclipse dates to 720 B.C. Systematic astronomical record-keeping seems to have originated in Greece at about the time of Pythagoras. The early history of astrology is not clearly recorded, and, as usual, much must be inferred. The major gods were identified with the planets, the minor deities were identified with the constellations. Since "planet" is derived from the Greek word for "wanderer," it may be that the gods originally retained their freedom of action in operating the cosmos. They certainly continued to influence human events. But the motions of the planets are implacable; sacrifices are consequently unnecessary. With the progress of what we now call astronomy, it became clear that the motions of the planets are predictable by mathematical calculations. The gods thus lost their freedom of action and were brought under increasingly strict laws. This brought about a major theological change. Perhaps it may even be called theological progress, though Gilbert Murray has shown that the abstract idea of progress was not understood by any of the Greek philosophers.

As it became increasingly evident that the motions of the planetary gods were implacable, all need for sacrifices disappeared. It is this recognition that constitutes the intellectual and theological advance. The future actions of the gods could no longer be foretold by the inspection of the entrails of animals, nor could they be influenced by sacrifice. However, those who understood these matters could calculate the future motion of the planets, and infer their influence on people by drawing horoscopes. The astrologers could explain these influences to other people. These others could then resist and modify (although not nullify) the effects of the planetary forces by taking suitable actions in their everyday lives. Such ethics as is involved here is a matter of self-interest, not a matter of doing business with the gods. It is not an ethics of rewards and punishments. The final conclusion of astrology is much the same as those of Democritus and Epicurus. Although man is not omnipotent, he need not placate frightful gods, but is more or less free to do business with people rather than divinities.

In the later Roman Empire, astrology was a competitor of Christianity in the attempt to reform the old Roman religion and its rituals; both were prohibited at about the same time. Neither prohibition was successful, but Christianity emerged more triumphant, and has therefore received more attention from historians. It is difficult to be certain, but it is likely that astrology has more adherents today than ever before. It must also be remarked that the early Christians did not concern themselves with either cosmology or the fear of God. Christ brought tidings of cheer and good will, taught that God was benevolent and that people should do business with each other in a spirit of brotherly love. Early Christianity was not fatalistic. God, like people, was to be both reverenced and loved; He demanded no return for His own love of, and good will toward, mankind.

Alchemy appears to have originated in the metallurgical crafts, and was infused with Democritus' ideas of the combination and recombination of the atoms. The astrological and alchemical communities overlapped; this produced a literature in which the two are not easily separated. Magical incantations and (in the popular opinion) witchcraft entered into it. Perhaps it was only to be expected that the intellectual discontents that led to the Reformation should have also have led to a revival of alchemy and astrology. It has been noted that both Tycho Brahe and Kepler received the patronage of the Holy Roman Emperor in return for their services as astrologers, and that the Emperor's interest in pure astronomy was negligible. Kepler, at least, seems to have been sincere in his astrological work. Both Newton and his rival, Leibniz, began their investigations under the influence of alchemical writings. Initially, alchemy influenced the development of modern chemistry, while astrology had more influence on physics and cosmology.

The revival of Democritus' atomic theory during and after Newton's lifetime has a complex history which has never been adequately presented in a systematic way. Newton is properly credited with making Democritus' Law of Necessity more explicit by introducing the mathematical relation between mass, force, and acceleration. It may be inferred that Newton's atoms, like those of Democritus, were devoid of intelligence. He makes no mention of the "soul" atoms; except for his theory of vision, Newton made no effort to construct a mathematical theory of psychology. He had strong religious convictions. He seems to have believed that the Creator not only made the sun and the planets, but that He also placed them in their initial positions, gave them their initial velocities, and then imposed the law of gravitation to govern their future. Thus, Newton did not share Democritus' belief in the evolution of the solar system from an initial Chaos in which the law of gravitation was already active. He does seem to have thought that human society had evolved into its present condition, and that this process of evolution was continuing. However, his efforts seem to have been directed entirely toward establishing a chronology of these changes, not to investigating their psychological dynamics. Perhaps, like his predecessor, Bishop Ussher, he hoped to find a reliable date for the Creation. These remarks regarding Newton's beliefs and hopes are quite speculative, and may need revision when his unpublished writings have been studied more throughly.

Newton's rival in the invention of the calculus was G.W. Leibniz. He was perhaps more "worldly" than Newton. Like

Newton, he was a mathematician, historian, theologian, and linguist, but he was also a poet, jurist, logician, and diplomat. As a young man he tried to solve the problem of Europe (which was already agitating the nations in the seventeenth century) by a mathematical demonstration. He invented the most advanced calculating machine of his day. Robert Boyle's proposal to generalize mathematics into a universal language is sometimes ascribed to him, for he was certainly interested in that project. When about to become a diplomat, he wrote a treatise on the blessings of God, the freedom of man, and the origin of evil. As a logician, Leibniz was occupied with the problem of "necessity," which arose out of differences in the way that Democritus, Plato, and Aristotle had used this word. Because he dissipated his energy over so many fields, he never wrote a systematic account of his mature ideas, but he did leave many notes. Those who first edited them for posthumous publication certainly failed to understand them. Even today, there is as much room for uncertainty as in the case of Newton. It does seem almost certain that he sought to bridge the gap between Democritus' cosmology and ethics by a dualistic theory, according to which every phenomenon is explicable in two ways. The one is scientific, non-vital, causal, and purposeless; the other is ethical, vital, teleological, and purposeful. The ease with which Leibniz could read Greek and Latin makes it most unlikely that he was unaware of the logical gap between Democritus' cosmology and ethics. Leibniz's atoms (he called them monads) are arranged in a hierarchy, from the lowest to the highest. The highest is essentially God. No two atoms are exactly alike, and they do not combine; like Euclid's points, they have no parts, no extension in space. This is very reminiscent of Kepler's explanation of astrology with which Leibniz must have been familiar. The monads are separate but the space between them is filled by the ether, which is "something like light." All of the monads are intelligent, perceptive beings, and they have something like appetite or feeling. Each perceives all the others, and therefore mirrors the entire cosmos; it is itself a microcosm. Each monad is self-active, its present actions being determined, at least partially, by its past. Force is therefore real, and involved in the monad's ability to perceive. Yet the monads also have freedom of choice, and behave purposefully. Thus, they are also the operators of the universe, giving it its purpose.

This summary is confusing, but no more so than the summaries of Leibniz's edited works prepared by others. These do not sound like the works of the ablest logician of his time, of the man who was competent to criticize Aristotle's logic and to set the goal for twentieth- century logic and computer technology. It is possible that editors have combined manuscript fragments which Leibniz would have considered to be preliminary studies, to be revised before combining them into a systematic cosmology and ethics. The confusion cannot be put aside by simply saying that Leibniz was a theologian and mystic as well as a mathematician and scientist. There are echoes of Democritus in his work, but Leibniz was not merely copying him. Leibniz's knowledge of the classical literature would have made this very easy, and he was most likely striving for something more logical, which may have eluded him.

Compared to this, Immanuel Kant's contribution to the revival of the atomic theory and the concept of evolution was very simple. Kant elevated the Principle of Sufficient Reason, the Law of Cause and Effect to the status of an axiom. It was known a priori, no sane person would deny it. Then he restated the idea that the solar system had evolved from the original chaos, adding only that Newton's laws of motion and gravitation were sufficient reason for the solar system evolving as it did (there is hence no need for the Creator to have placed the newly-formed planets in their respective orbits). This is usually known as the Nebular Hypothesis and is attributed to Kant rather than Democritus.

This brings us to the nineteenth century and the work of Laplace, which must be considered in more detail.

Laplace's Mechanical Man

The molecular hypothesis might have escaped the attention of nineteenth-century scientists had it not been taken up by Laplace. Pierre-Simon Laplace was the son of peasants; he was secretive about his early life and little seems to be known about it. He became one of the intellectual group with which Mongé surrounded Napoleon, and was one of the members of the early Ecole Normale, and later, the Ecole Polytechnique. After the banishment of Napoleon, Mongé fell into disfavor with Louis XVIII and had to hide from the police. Laplace, however, managed not only to succeed Mongé as head of the Ecole Polytechnique, but to obtain the title of marquis as well.

Laplace first referred to Kant's nebular hypothesis in a semi-popular book entitled *System of the Universe*. The book, which was published in 1796, was based on lectures which Laplace had given at the old Ecole Normale. He later briefly referred to it in a technical mathematical work entitled *Celestial Mechanics*. In this four-volume work, Laplace greatly extended Newton's theory of mechanics and, in particular, of the solar system. It has had a great influence on all later work in this field, although it was not free from errors and misunderstandings. Some of the theorems in this large work can be interpreted as disproving the nebular hypothesis. Newton had considered it possible for gravity to eventually cause the planets to fall into the sun. In that event, he speculated, there might be a new Creation. Using Newton's theory, Laplace was able to show that the planets would not fall into the sun, so Newton's fear of that particular catastrophe was groundless. But the theory could also be used to calculate the past motions of the planets. This seemed to show that, just as they would continue to revolve around the sun indefinitely, so they had been moving since the beginning. Thus, they had not evolved out of Democritus' primeval Vortex or Chaos. Laplace did not emphasize this, but his mathematics, in various modified forms, continues to plague all those who seek to discover the true origin of the solar system.

Napoleon is said to have teased Laplace about his *System of the Universe.* "You have written this large book about the universe without ever mentioning its Creator"; to this, Laplace is said to have replied, "I had no need for that hypothesis." Newton surely would not have said this. Laplace may have had in mind his demonstration that the solar system is eternal. He later elaborated this epigram into an extreme form of Democritus' opinion that the universe runs itself according to the Law of Necessity. Like Democritus, Laplace considered chance and accident to be illusions, the result of our ignorance. A mathematical theory of probability had originated earlier but Laplace greatly extended it. His work has been so influential that he is sometimes cited as its originator. As early as 1795, he gave semi-popular, non-mathematical lectures on the subject to the prospective teachers studying at the Ecole Normale. These were published under the title, *A Philosophical Essay on Probabilities*, in 1819. It has been reprinted and translated many times since then. In it he explains the essentials of Newton's mathematical system of mechanics in a single sentence! Others have used more words for the same purpose and succeeded less well. In this work he also summarizes ideas of Democritus, Epicurus, and Lucretius, which Laplace evidently received from Leibniz. However, there is one difference. It is explicitly stated that the ability of people to make choices and decisions is an illusion. All human actions are inevitable, they are governed by natural laws such as Newton's laws of mechanics. People are thus mechanisms, not very different from clocks. Then comes the same hiatus that has already been noted in the case of Democritus. Laplace refers to the human mind and its striving for knowledge, its search for truth. One might conclude these strivings are illusions. Mankind will never control its future, but the human mind will inevitably become more and more mathematical. This is an extremely fatalistic doctrine; it does not admit that people have even that small measure of control that astrology assigns to them. It is also an extreme form of the doctrine of psychomathematical parallelism. Finally, one sees again that the object of this eloquent exposition of an inconsistent doctrine is to persuade people that they need not fear the wrath of God.

There are other major differences between this and the ideas of Democritus. There is no reference to "soul" atoms; people are said to differ from animals in their tendency to become more mathematical. Finally, the hiatus is not used for the insertion of a system of ethics. Laplace made no reference to ethics in the entire essay: Part I is devoted to the exposition of the ten principles of probability, and Part II illustrates their application to many problems of daily life. Uncritical readers have sometimes been persuaded that the success of Laplace's theory demonstrates that man is really a machine. The character of the problems that Laplace solves shows that this does not follow; the essay ends with a history of the theory prior to 1816, and this contributes further evidence against the erroneous conclusion.

Chance, Gambling, and Insurance

The critique of Laplace's *Philosophical Essay on Probabilities* may be separated into a number of parts, but it should first be said that his ten principles of probability have survived, with only minor modification, to the present day. They have been applied to a wide variety of problems, ranging through gambling, insurance, business, politics, physics, psychology, genetics, and evolution. In each of these applications, the ten principles are supplemented with other assumptions or hypotheses, which are often not clearly stated. In particular, textbooks on probability are written in general terms, without explicit reference to these other hypotheses, except possibly a disclaimer to relieve the author of responsibility for their correctness. Laplace's persuasive style has therefore influenced theories of all these many topics to a greater extent than is justified, and a critical discussion must consider many of these topics separately. There are only a few criticisms of a general nature.

First, there is historical evidence that Newtonian mechanics is not an essential component of the theory. The French mathematician and theologian, Pascal, solved a quite difficult gambling problem more than twenty years before Newton published his *Mathematical Principles*. A discussion of this problem will reveal another important aspect of the matter. Two gamblers, A and B, deposit equal amounts of money; they agree to toss a coin 100 times. If it falls heads, A scores a point, if tails, B scores. At the end of the 100 tosses, the one whose score is higher will take the entire deposit. But their game is unavoidably interrupted after, say, 37 tosses, at which time A has the higher score. If A then proposes to take the entire stake, B may quite properly object, for if the original agreement had been fulfilled, he might have had the higher score at the end of the game. The issue of equity, an ethical or legal concept, has been raised. The problem was brought to Pascal for a solution. The details of his calculations need not delay us here: authorities agree that they are correct. Two important conclusions follow. First, Newton's theory of the motion of the coin is irrelevant, and second, equity is an essential element of this problem. One is returned to Democritus' ethics, not to his cosmology. This aspect of probability was not discussed by Laplace.

It is also an interesting fact that Pascal was a Jansenist, a member of a puritanical Christian sect. He invented Pascal's Wager to persuade people that it was a good gamble to follow the ascetic life which he advocated. He argued that men had little or nothing to lose in the present, but much to gain in the hereafter. Clearly, Pascal was a God-fearing man; not even the fear of God is inconsistent with the theory of probability. In fact, Pascal brought it back to doing business with God. It is also clear that he introduced several theological hypotheses, both about life in the hereafter and life in the present.

If one abandons the historical approach, the argument is even simpler. There are certainly many events that we cannot foresee, many plans we may initiate without knowing their outcome. It is certain that no person is omniscient: this is sufficient foundation for the most sophisticated mathematical theory of probability. It is a gratuitous assumption that this human ignorance of the future is an illusion, that what is now unpredictable will some day become predictable. Laplace's vast "intelligence" is a fictional ideal; his statement that human intelligence will tend toward this ideal may or may not be true. Even this alternative is not relevant to current transactions between people now living.

Pascal, despite his puritanism, recognized that equity is an essential element in gambling. A general reference to cheating does not tell us whether it occurred in the market place, the casino, or at the racetrack. Betting on the outcome of a horse race is one of the simplest kinds of gambling. In discussing the selection of aediles, it was not necessary to describe the entire Roman agricultural community; one can also discuss the bookmaker in isolation from the rest of the racing community. The bookmaker accepts bets on any horse, whether or not he thinks it will win. He arranges his transactions so that, whatever the outcome of the race, he will have money left over. If few people wish to bet on a particular horse, he gives them long odds. That is, in the event that the horse does win, he will pay them much more than what they deposited with him. Conversely, if many people wish to bet on a particular horse, he gives them short odds. Since he is operating under pressure of time and in an excited crowd, his calculations will not be very precise, but he will be careful that all errors are in his own favor. Some fee is due him for his services, but he will be tempted to charge an exorbitant amount, since his customers have no way to audit his accounts. Only competition with other bookmakers will restrain his greed. He may also cheat by knowingly giving out false information, in order to lower the odds his customers will accept. When horse racing was first brought under legal supervision, bookmaker's accounts were subjected to audit. This resulted in all the evils of an unenforcable law. The bookmaker could operate at a distance from the racetrack, and not register himself as a bookmaker. Since his business was now illegal, he increased his fees to cover the cost of fines and other legal expenses. Today, the legal bookmaker has been replaced by the tote-machine, a somewhat elaborate cash register or simple computer. The machine determines the payment due each winning bettor in such a way as to leave a residue in the till that will cover the reasonable expenses and profit of the operators of the racetrack. In other words, it computes the odds in exactly the same way as an old-time bookie. It is important to understand this. In computing the odds, the machine makes no use of the knowledge available at the time the bets were placed; it does not consult the previous day's racing form, nor does it use any information concerning the horses at the starting time. It makes the calculation using only the number and amounts of bets placed on each horse and the total amount of money in the till when the betting is closed. Naturally, these matters are automatically recorded and are subject to later audit. The machine does not begin its calculations until *after* the race is over; it makes no attempt to predict its outcome. Only the bettors take any risk. The operator of the racetrack is certain to profit, providing that the attendance is not too low.

One does not usually consider life insurance as a form of gambling, and there are fundamental ethical reasons for this. But there are also superficial mathematical similarities between gambling and insurance, and it will be simpler to explain the differences between the two activities if the calculations are considered first. The insurance company is the bookmaker -- an honest one, for its operations are open to audit by legally constituted state and federal authorities. The policy holders are the bettors. A life insurance policy is a long-term contract, and is thus different from the simple ticket issued at the racetrack. The policy requires the insured to make periodic payments, premiums, to the insurer. In return, the insurer agrees that, in the event of the policyholder's death, it will pay certain designated survivors a definite amount of money. Obviously, the policy holder is gambling that he will die before the accumulated premium payments exceed the amount the company will pay to his survivors. Otherwise, he would do well not to make payments to the company, but instead accumulate his savings in some safe way, leaving them to his survivors in his will. No one can determine, long in advance, the date on which someone will die. Some policy holders will "win" their bet, others will "lose." The insurance company must operate so that its payments to "winners" are balanced by its income from the "losers," leaving a net amount to cover its expenses and allow for a reasonable profit. But unlike the tote-machine, it cannot wait until the race with Death is run; it must fix the odds (premiums) long before. To do this, it must consult the equivalent of the racing form, which is called the mortality table. Unlike the tote-machine, it must take a risk. For example, in 1968, past experience had shown that a person then aged 30, might be expected to live an additional 43 years; he might die within one year, but he also might live for an additional 60 years. The premiums on life insurance policies issued in 1968 to persons aged 30 were calculated on this assumption (among others). But five years later, in 1973, more information had accumulated. The conditions of life (including advances in medical science and the increase in the number of cars on the road) had changed. In 1973, a person then aged 30 might be expected to live an additional 43 years and 11 months, an increase of more than two percent over 1968. It would have been inequitable to issue policies in 1973 at the same premium rates as in 1968, and these were therefore revised. The simple calculations of the tote-machine are thus greatly complicated. This has given rise to professional actuaries to make these calculations. It is true that the arithmetic is now done by electronic computers, but it is still a human being who devises the required calculations. As has been noted above, these calculations are still based on Laplace's ten principles; but Newton's mathematical theory of mechanics does not enter into them. The actuaries ignore Laplace's mechanical man.

One could write an impressive book about the difference between the mathematics of the tote-machine and the mathematics of the actuary, but it would still be superficial if it contained no discussion of equity, which is a special part of ethics.

In our world, unexpected, unforeseeable events occur very frequently. It is irrelevant whether this is an illusion or not; the unforeseeable may be caused by the capriciousness of other people, the capriciousness of the gods, or by the inevitable operation of some law of necessity that we do not understand. Some of these events make us happy, some make us unhappy. One must often make a decision, initiate a course of action, before one knows whether it will end happily or unhappily. It is again irrelevant whether or not this is an illusion of free will, of an ability to choose between alternatives. In a word, Laplace's hypothesis that people are mechanisms is as irrelevant to this discussion as the Greek hypothesis that the destinies of men are controlled by Fate. However this may be, it is a condition of our lives that we must take some risks. It does seem at least foolish to take avoidable risks, such as staking one's fortune on a roll of dice or a draw of cards. If one has dependents who will share in one's misfortune, such an action is not only foolish, but irresponsible as well. If ethics is to be based on responsibility for others, such actions are unethical. One can, of course, avoid this conclusion by basing ethics on the promise of divine rewards and punishments, but many who have such ethical beliefs condemn gambling, condemn the taking of avoidable risks. In practice, most people do adopt an ethics of responsibility.

The fundamental difference between horse racing and life insurance is therefore ethical, not mathematical. It is important to be clear about this. The relatives and friends of a person who dies young are saddened; this is unavoidable, and occurs whether or not the deceased had earlier decided to take out a life insurance policy. If the deceased had taken the policy, he now has "won" his bet, and his survivors are spared the additional misery of poverty. The wager was therefore not irresponsible. If a policyholder enjoys an exceptionally long life, the situation is different, for he "loses" his wager. His survivors receive less money than if he had not insured his life. Still his decision was not irresponsible, unless he took out such an excessive amount of insurance that his dependents are left in poverty because of the premium payments. What his survivors lose has been given to others. It is much as though he had donated the premiums to a philanthropic organization. Had he not taken out insurance but made such a donation instead, his action would not have been irresponsible from a broader social viewpoint. He would have accepted responsibility for people not closely related to him. Again, one should suppose that this donation was not excessive.

What has just been discussed is not the whole of ethics, but only that part which involves money, and is usually called equity. Neither has this completed the discussion of gambling: there is an aspect which is neither mathematical nor ethical, but psychological. Like all matters that involve psychology, this is too complex to permit a simple exposition. There are some people who enjoy taking risks: this group includes the inveterate gambler as well as the entrepreneur. There are others who prefer the relative security of working for a salary. With Lucretius, it may be remarked that it is not the present task to explain this difference between people but only to take notice of it, and to elaborate upon it.

It has already been noted that the socio-religious Roman institution of the aediles was a satisfactory way of preventing cheating in the marketplace so long as Rome was small, and business was transacted face-to-face between producers and consumers. In Mesopotamia and Asia Minor, however, the caravans and wholesalers functioned as intermediaries between consumers and producers. An institution such as the aediles could not function effectively in such an environment: the arm of the law was not long enough. The custom of bargaining developed: there were proposals and counterproposals before a transaction was completed. Each party tried to determine the honesty or the dishonesty of the other, using reputation, inadvertent remarks, facial expression, and other indications. The transaction of business became something of a game of skill. But neither party could ever be sure of having gotten the better of the other.

Atoms, Causality, and Politics

It has been noted that the atomic theory first influenced chemistry, and only later became a fundamental part of physics. Atomic chemistry had been well started by the time of Laplace, but this start is overshadowed by later developments, of which he could know nothing. These can scarcely be dismissed as details, but they do not provide a good beginning for a critique. In particular, they are not immediately relevant to the doctrine of causality and that chance, accident, and probability are merely illusions caused by our ignorance. Some fifty years after Laplace, James C. Maxwell, together with others, developed a theory of heat in which they explictily used both of these doctrines. Maxwell makes explicit reference to Lucretius; his description of the difference between atoms and molecules is essentially that of Epicurus, but there is no reference to him. Neither is there any explicit reference to Laplace's theory of probability, although it is used in Maxwell's calculations and explanations. Thomas Huxley, whose lectures were very popular at the time, summarized this theory by saying that heat is a mode of motion of the molecules of matter, and is therefore also a form of energy. The pressure of a gas on the walls of its container is caused by the impact of its molecules on those walls. The molecules do not all move with the same speed, nor in the same direction, and human senses are unable to perceive them individually, or to determine the speed and direction of any one of them. They are too small, and there is an enormous number of them in even the smallest container with which one can experiment. Their motion, however, is governed by Newton's laws of dynamics. Laplace's "vast intelligence," therefore, could comprehend their positions and motions, and then calculate their positions and motions at all other times. This would include their impacts on the walls of the container, and hence the force or pressure on the wall. This would not be constant in time, nor uniform over the walls, since the number and character of the impacts will be different from time to time and place to place. We too could make this calculation, except for two things. One is, that unlike the "vast intelligence," we cannot perceive the configuration and motion of all the molecules at a single instant. The other is that our minds are too sluggish to make the calculation in any reasonable length of time. Because of our ignorance, we therefore resort to calculating probabilities; even this is difficult (or impossible), except in the simplest cases.

In order to illustrate a fundamental difference between these two doctorines, Maxwell conceived of an imaginary experiment: for this purpose, he imagined a small intelligent being (he called it a demon) that could directly perceive the atoms of a gas and their velocity. The demon could make decisions on the basis of this data, open a small door to let rapidly moving atoms through, but keep it closed when a slow moving one approached. It would do the contrary when atoms approached the door from the other side. Thus the gas on one side of the door would become hotter because its atoms move faster, while the gas on the other side would become colder. This difference in temperature could be used to drive a heat engine, and the need for Watt's condenser would be circumvented. The condenser was needed only because of our ignorance and inability to make and implement decisions quickly enough. This is science fiction, of course, but ever since Plato's psychomathematical experiment with Menon's slave, people have become accustomed to accepting such fictional experiments as revealing reality, even though many, including Plato, doubted the evidence of actual experiments.

Fifty years passed before the theoretical physicist, Leo Szilard, pointed out a fallacy in Maxwell's argument. Szilard did not doubt the validity of reasoning from imaginary experiments, but he did point out the fact that Maxwell had neglected the metabolism of his living demon. The metabolism of actual living things, and especially the metabolism of the brain, was then (and is still) poorly understood. Szilard therefore imagined an automaton, whose "metabolism" could be described in mathematical terms. Nevertheless, this automaton could make simple decisions on the basis of data already accumulated, even though it could not foresee the future. This automaton could effectively do everything that Maxwell's living demon was supposed to do. But when its metabolism is taken into account, it could be shown that Watt's condenser is still an essential element in any heat engine.

Szilard's paper is highly mathematical, and is difficult reading even for specialists. However, it has now been widely discussed and often misinterpreted by psychologists and biologists. The paper at first attracted little attention. It did not become widely known until Szilard's countryman and friend, John von Neumann, recognized that the demonstration of the fact that non-living automata could make simple decisions, was a discovery of major significance for the theory and construction of actual electronic computers. It was the successful construction and use of these automata that directed attention to Szilard's pioneering work.

It must be emphasized that automata can make only simple decisions, but it is not quite simple to explain what "simple" means in this context. Fundamentally, each automaton can understand only one language, and that language is an artificial one, with a very strict syntax. It is determined by the people who construct the machine. And the automaton can understand this language only partially: it can determine whether a statement is correct or incorrect. It cannot determine whether it is true, fictional, or false. The automaton must be supplied with data and instructions; basically, both must be written in the one language whose syntax is built into it. These sentences must be written by people, who can tell whether the sentences are true, fictional, or false. But the computer will accept false sentences as readily as true, provided that they are correct. It will reject incorrect sentences, even though people would readily understand the intent of the writer and correct the error. It is therefore an exaggeration to say that a computer understands even the one language that is built into it. The computer will then make decisions and generate sentences expressing them. But the decisions will be based solely on correctness or incorrectness. If the machine is supplied with false data or instructions, its decisions will be false. Even this somewhat lengthy explanation needs some amplification, as any computer scientist will know, but it embodies the essentials of the matter. A technical word has been coined to describe this limitation of the automaton's ability. It is "gigo," which is an acronym for the disrespectful sentence, "Garbage in, garbage out." As a matter of fact, not only automatic computers, but all of the symbolic, artificial languages used by mathematicians and logicians in their calculations are "gigo." This is part of what von Neumann meant when he concluded that the language of mathematics is not the language of the brain. A company that manufactures many computers summarizes this point even more simply. It advertizes that its machines cannot make a businessman's decisions for him; the machines can only provide him with data summarized in a convenient form. In making his decisions, the businessman must take risks: risks as to the future behavior of people, and the risk of "gigo." This company finds it preferable to advertize this fact rather then let its customers discover it at their own expense.

Returning to Leo Szilard, this work on Maxwell's demon was only one of his earliest achievements. He was one of the first, perhaps the first, to understand that nuclear weapons were not only possible, but they could be constructed using the technology available in 1940. He persuaded Albert Einstein not only of this, but also to write a letter to President Roosevelt, informing him of this. This, and the certainty that Nazi Germany would attempt the construction of these weapons, led Roosevelt to initiate the Manhattan Project in the United States.

After World War II, Szilard, like Bertrand Russell, became increasingly concerned with the prevention of nuclear war and the proliferation of nuclear weapons. His writings on this and similar matters are non-mathematical. They are being collected and will soon be published. Like Russell, he recognized the need for an organization to carry on this work. It was first named "The Council for the Abolition of War," and is now known as "The Council for a Liveable World." Ingeniously, Szilard avoided Russell's difficulties with incorporation. The Council does not collect or disburse large funds. Instead, its offices merely monitor the candidates for high political office, and make recommendations to make personal contributions to the campaign funds of those candidates that seem disposed to further the objectives of the Council. This procedure has had enough unobtrusive successes so that the Council's influence in the political community is increasing.

Man is an Animal, but not an Ape

Before attempting to answer the question regarding his future, it is well to pause and ask "What is man?" Man is an animal. When this view was first advanced, its negative aspects were widely emphasized; they are not conducive to conceit. There are also positive aspects that have not yet received adequate emphasis, and it is these aspects that one should keep in mind. There are major anatomical and mental differences between man and the other animals. It will be sufficient to consider those characteristics which differentiate man from the ape. The most obvious are seen in the hands and feet. Man's toes are almost vestigial; the four smaller toes can hardly be moved independently, and the big toe can hardly be flexed. In marked contrast, the thumb can be made to touch the tips of each of the other four fingers of its hand, and all five digits can be moved and flexed independently. Scarcely less obvious are man's lips and tongue. These can modulate the grunts and squeaks of the larynx into the sounds of speech. The tongue and larynx of the ape are too sluggish to make the motions required for speech; at most, apes can roar or chatter by moving the jaw.

Man's brain also differs from that of the ape, although these differences are hidden by the skull. These three differences are not independent; rather, they seem to be the varied expressions of a single character. Hand gestures often accompany speech. Some anatomists believe that communication by gesture preceded communication by speech. The mental effort of counting things that are out of sight is often aided by making the thumb successively contact the tips of the other four fingers. In writing, the pen is held between the thumb and two fingers, and the mental effort of composition is translated into the motion of the pen. Communicable thought, that is intelligent thought, is converted into speech by the movements of the lips and tongue. One sometimes says that speech is thinking out loud. Our kind of thought, again intelligent thought, is a silent soliloquy. In neurophysiological terms, the brain then generates the nerve inpulses that would ordinarily cause motion of the lips and tongue, but these are inhibited. Just what happens to these impulses that do not reach their destination is unknown.

All of this is easily observed in man, and helps to distinguish him from the ape. It all has its counterpart in the brain, but this is not so easily seen. The anatomy and physiology of the brain are complicated and still poorly understood. What follows is a much simplified account of what is known about this correspondence between the anatomy of the brain and the anatomy of the body.

It will only be necessary to discuss the upper part of the brain, which is known as the cerebrum. It is relatively larger in man than it is in the ape, and it diminishes in size as one goes down the evolutionary ladder. Reptiles have practically no cerebrum. The surface of the cerebrum is covered by a layer of gray matter, which is called the cerebral cortex. In man, the cerebral surface is deeply folded. Because of this folding, the surface area of the cortex is much greater than the surface area of the skull that protects it. This enhancement of the cortex is most pronounced in man, less so in the ape, and is absent in more primitive animals. There are also two kinds of cortex: layered, and non-layered. Almost all of man's cortex is layered. In other animals, the two kinds of cortex are about equal in area. The non-layered cortex is involved in the sense of smell, so this, rather than the anatomy of the nose, accounts for man's inferior olfactory ability.

Much effort has been devoted to theories of the process which we call memory; almost none has been devoted to forgetting. We have all forgotten important matters. Many have had the experience of meeting a friend with whom they participated in a memorable event long before. Each believes that he has a clear recollection of the event, but when the two memories are compared, there are found to be significant discrepencies. Memories can not only fade: they can become distorted.

Of special interest for the present is the large fold known as the Rolando fissure. It separates the frontal lobe of the cerebrum from the parietal lobe. The frontal lobe generates electrochemical impulses that are transmitted through nerves which end in the extremities (hands, eyes, feet, etc.). These motor impulses control the movements of the various parts of the body. The parietal lobe receives nerve impulses that are generated by the sense organs (the eyes, ears, nose, etc.).

When the sensory impulses are received by the parietal lobe, they are somehow synthesized, coordinated, or analyzed. The physiology of this process is not understood in any detail, but ultimately, modified impulses pass across the two layers of cortex in the Rolando fissure and reach the frontal lobe. Again, the physiology is not clear, but somehow new motor impulses are generated and transmitted to the moveable parts of the body, where they (unless they are inhibited) release the energy that is needed for movement.

These processes enable the cerebrum to coordinate the movement of the various parts of the body. We are conscious of this ability to control the motions of our body, and call it "will", or "volition". Ordinarily, we are not aware of the components of this ability. When, for example, we reach for a tool with the left hand, transfer it to the right, and then use it, a very complex and well-coordinated series of motions is made. This coordination is accomplished by the constant flow of nerve impulses in both directions. The eyes follow the hand; the many nerve endings in the hand also constantly generate impulses that reach the parietal lobe. Here they are coordinated, cross the Rolandic cortex, and the frontal lobe generates the impulses that initiate the complex movement needed in the next instant.

This kind of control, involving the flow of impulses from sense organs to a control organ and thence to effector organs, is called feedback, or cybernetic control. We are not ordinarily aware of feedback, but if we try to take the action which is described above when blindfolded, we become unsure. Our movements are groping, less coordinated. If we handicap ourselves still more by putting on heavy gloves, our coordination deteriorates further. Thus, anyone can establish that volitional control of the body is by means of feedback. Elaborate neuroanatomic research is needed only to establish the physiology of the process. Such research has shown that the impulses which control various muscles in the body cross the Rolandic cortex at quite specific points. Occasionally, small tumors or blood clots form in this cortex. The victim thus loses volitional control of certain muscles. This loss of control may range from paralysis to constant involuntary twitching. Clinical observation of the symptoms, followed by post-mortem dissection, has made it possible to map the parts of the body on the surface of the Rolando fissure. In such a map, the tongue is mapped onto an area larger than that of the whole head; the lips are mapped onto an area larger than that of the eyes and scalp. The disproportionate enlargement of the thumb and fingers corresponds to the complexity of the movements which they are able to make. The map of the larynx is relatively small, indicating the simplicity of the motions it can make.

We are accustomed to speak of the mind as though it had parts and as though these parts were distinct and quite different from other organs such as the hands, eyes, tongue, ears, and so on. Like many common notions, this requires only elaboration to make it precise. The mind is a *process*, not a substantial thing. That part of mind called *will* has been under discussion in this chapter. It has been seen to be a process that involves those substantial parts of the body that are most characteristic of man: the cerebrum and its cortex, the tongue with its ability to fashion the sounds of speech, and the hands with their ability to create artifacts and to use them. It also involves parts of the body, like the eyes and ears, which are common to man and the other animals.

The will is itself a complex process which involves the transmission of impulses along nerve fibers, their generation and reception, both in the cerebrum and in other parts of the body. Only the transmission of the impulses is well understood; it is an electrochemical process. The other parts of the will are much less well understood. It would be rash to even assert that all the processes taking place in the cerebral lobes are electrochemical in nature. Memory and consciousness are other parts of the mind. There is clinical evidence that they also are processes in which the cerebrum is involved. The nature of the memory process has been the subject of speculation, but no conclusive knowledge has emerged. There has not even been speculation concerning the process we call consciousness. Many professional psychologists discourage such speculations, and none have indulged in them.

It is with this scant knowledge and great ignorance that we approach the question, "Can man's mind control his future?"

The Changing Beliefs of Physical Scientists

Enough evidence has now accumulated to justify some conclusions. When men of proven mathematical creativity become seriously concerned with the problems of people and society, they abandon the mathematical methods of which they are masters. Their actions show that they do not consider that the problems of society are amenable to mathematical theories and calculations. No matter how pessimistic they may be about the future, or how ungratefully their efforts to improve it are received, they do not become fatalistic. Their hope for improving the future of man rests not on inexorable mathematical calculations, but on the ability of people to make decisions, to make plans, and to implement them.

One would like an explanation of this phenomenon. None has been advanced. That is, none which is not an apology for the past and a reaffirmation of Laplace's faith in future improvement. Of course, such a reaffirmation might be all that is needed, but there are many reasons for thinking that the past needs no apology and the future will bring no such improvement. These reasons have never been collected in one place. The following will be an attempt to reassemble most of them and explain them in non-technical terms, without mathematical detail.

The most obvious reason is the failure of Thurston's attempt to construct a mathematical theory of human abilities and behavior. But this is not the most fundamental reason. This is the recognition that there are no self-evident axioms, no *a priori* knowledge. This is not to say that there are no instincts. A chick that has just been hatched in a incubator and has never been in the presence of an adult chicken, will run and walk, eat and drink. Presumably, a human infant would do the same, but this is not knowledge. Experiments show that other behavior of such chicks depends strongly on the presence or absence of adult hens during the early days of their lives. Perhaps this second kind of behavior, normal or abnormal, can be considered to be indicative of knowledge, but it is a very rudimentary form of knowledge. All knowledge comes from perception, but perception is not simple sensation. The sting of an insect or the prick of a needle comes close to being a simple sensation, and the two cannot be distinguished unless one also perceives the presence of the insect or the needle, or later perceives the development of the wound. One may say that perception is the mind's interpretation of sensation. This has been discussed previously in connection with our knowledge of space-filling objects. Because people can speak, the interpretive element of perception can be passed from generation to generation, and changes only slowly with time. But it can, and does, change. This recognition of an interpretive element in perception does not imply that what we perceive exists only while we perceive it, as Berkeley maintained. There are things in the world that are not perceptions. Nor are perceptions limited to the unaided senses. The invention of new artifacts may alter and improve our perceptions. For example, copper and iron, if present in sufficient quantity and close enough at hand, can be distinguished by the unaided senses. But with the aid of a spectroscope, much smaller quantities, or larger amounts at greater distances, can be distinguished. When anything is heated sufficiently, it becomes a luminous vapor. When this is viewed through a spectroscope, it is seen as a set of colored lines, separated from each other. Luminous copper vapor always produces the same set of colored lines; luminous iron vapor produces a different set of lines. Thales had speculated that the sun and stars were composed of ordinary terrestrial matter, but, strictly speaking, he had not perceived this. It was a hypothesis, a fiction. When the spectroscope and the telescope were combined, it became possible to perceive it, for the spectral lines in sunlight are the same as those in the light of terrestrial substances. After improvement of the spectroscope and telescope, it was perceived that the stars are also composed only of terrestrial substances. With his telescope, Galileo had seen mountains on the moon. At that time, most people were unaccustomed to sensory aids, and some refused to believe him. Biologists repeated this refusal when Abbe and Zeiss made better and better microscopes. Only very recently has anyone actually been to the moon. The astronauts' unaided senses perceived what had previously been perceptible only with the aid of telescopes. Since they were there, they perceived it in more detail, for they were able to move about and handle the dust and the rocks. It is also to be noted that not everyone is able to use large telescopes and spectroscopes: their use requires technical education not available to everyone. Yet, the perception of the few can be made available to all through lectures, books, and photographs; this is an obligation on the part of the technically trained. This duty should not be performed perfunctorily, uncritically, or with undue enthusiasm for unwarrented conclusions.

Laplace was justifiably proud of the astronomy that he and his predecessors had developed, but it was by no means as perfect as he thought. It was limited to the motion of the planets and their satellites. It was possible to calculate their past and future positions with some accuracy. The occurrence of eclipses could be predicted for years in advance, but not to a fraction of a second. Old records of solar eclipses could be used to help establish a chronology for ancient history. But the nebular hypothesis remained a hypothesis; it was not possible to calculate the history of the solar system as it evolved from the primeval chaos. It was not possible to perceive the slow changes that should, if the hypothesis is true, still be modifying the solar system, much less those that had occurred long ago and brought it to its present state. In Laplace's time, telescopes were still rather crude; their improvement had to await Abbe's work on the errors of optical images, and the invention of the spectroscope. They were already good enough to verify Democritus' conjecture that the Milky Way galaxy was a concentration of stars. By the end of the nineteenth century, it was perceived that our sun was one of these stars. Although the sun is of paramount importance to us, it is a mediocre star. It is neither large nor unusually hot and bright. It does not occupy a distinguished place in the Galaxy. There are many stars which are not very different form our sun, and it may be conjectured that some may have planets like ours. Then it was found that even our Galaxy is not unique: there are an uncounted number of other galaxies in the universe. These observations or perceptions required several revisions of the nebular hypothesis. Then it was found that the universe contains objects other than stars and galaxies. It was seen that, contrary to Democritus' ideas, collisions between stars are rare events, but stars frequently explode spontaneously, somewhat like enormous nuclear bombs. Combining the knowledge of nuclear weapons with these new observations, it has been possible to construct a semi-mathematical theory of the life-history of a typical star. But almost every year brings new phenomena to the attention of astronomers. These are often unreconcilable with the current cosmologies, and new ones must be invented. It might be expected that, at least, the theory of the origin of the earth and moon would remain relatively unchanged. Much thought and many observations have been devoted to this topic. Since the return of the astronauts with their samples of lunar materials, specialists are more reluctant to be dogmatic than ever before. There is no indication that Laplace's "one formula" that governs everything in the solar system is being elucidated, much less, that one formula governs everything in the universe.

There are still other reasons for doubting that mathematical laws of cause and effect govern everything. Democritus was not very explicit, even about the Law of Necessity that he thought governed the atoms. He did say that the atoms were tiny spheres, indestructable and immortal. Plato imagined the atoms to be polyhedra with flat faces, straight edges, and corners; he was less explicit about their immortality. While everyone seems to have argued that a single atom cannot be perceived with the unaided senses, everyone seems to have agreed that they were objects that existed and moved in space. They were among the things which are not perceptions. Sometime, it might be possible to devise some way of perceiving them. By the beginning of the twentieth century, chemists had learned much about the way in which the atoms combined into molecules and even into ordinary large objects that can be perceived with the unaided senses. After the discovery of radioactivity, it was learned that the atoms are not immortal. It was found that a radium atom undergoes spontaneous changes and eventually becomes an atom of lead. During each of these changes (decays), "rays" of various kinds are emitted. Some of these rays are corpuscular: beta rays are electrons moving with great speed, while alpha rays are helium nuclei moving with high energy. It was at this time that Anatole France foresaw that people would soon build atomic bombs rather than use gunpowder. It was necessary to suppose that atoms are neither immortal nor without parts. Mathematical theories of atomic structure began to be devised.

Only a few skeptical chemists, led by Wilhelm Ostwald, doubted such theories. Then it was found that when a bit of radium was placed in very moist cool air, the radioactivity caused minute water droplets to form, which could easily be seen. Under the simplest conditions, the droplets were arranged in a more or less straight line. The path (or track) of a single helium nucleus or electron was thus made visible as a cloud-track. Even Ostwald ceased to be skeptical. There were other rays, however, the gamma rays produced no track, but when they struck a bit of metal, electron tracks emerged from the point of impact. Newton had believed that light was a stream of particles, and his theory was revised to explain the behavior of gamma rays: the moist air is transparent to them, and hence they leave no cloud-tracks to show where they have passed. Evidence accumulated that seemed to show that gamma rays, like x-rays, are an invisible form of light.

This sounds much more satisfactory than it was, even at the time, in the 1920's. Newton's corpuscular theory of light had not predicted all optical phenomena that would later be observed, and then it could not be modified to calculate them. When a beam of light falls on an opaque object, it casts a shadow, which is something like a single perspective of the object. When the shadow is observed carefully, this description is found to be inaccurate. Under controlled conditions, the shadow of a straight knife-edge is found not to have a sharp boundary. Instead, the regions of light and dark are found to be separated by a region in which there are colored fringes of varying intensity; this phenomenon is called diffraction. Newton and others tried, unsuccessfully, to explain diffraction. While writing his major works, Laplace was convinced that Newton's corpuscular theory of light would ultimately be successful; this was generally believed by all physicists. It was another apology for the past, accompanied by a promise for the future.

Thomas Young was an English physician, physicist, linguist, and a contemporary of Laplace. Young's work on the anatomy and physiology of the eye is still recognized as fundamental. In addition, he diciphered that form of Egyptian script known as demotic, (one of the cursive forms of the hieroglyphs.) Turning his attention to diffraction, he revived a theory of light which had been advanced by Newton's rivals, Leibniz and Huygens. According to this theory, all space is filled with a very tenuous substance, the ether. The ether is inperceptible to the unaided senses, and it has properties that are very different from those of any perceptible substance. It was these paradoxical properties of the ether that led Newton and his followers to reject the hypothesis that it exists. Young elaborated Huygen's theory of light so that the phenomena of diffraction could be described mathematically. Light is a motion of the ether, much as ocean waves are a motion of water. Diffraction is analogous to the bending of ocean swell around the end of a breakwater, or of ripples around a protruding rock. Young's theory was extended by others; it was the foundation upon which Abbe constructed his mathematical theory of the microscope. When the phenomena of electricity and magnetism had been fully explored, James C. Maxwell was able to unify the theory of light with electromagnetism, but only by assigning still more paradoxical properties to the ether. Still later, Albert Einstein introduced a still more radical notion: the ether not only fills all space, it is identical with space. Euclid's axioms were only hypotheses; the mathematical theory of geometry was only approximate. The axioms were not *a priori* knowledge; they were hypotheses inferred from unaided sense-perceptions. They could be replaced by others, differing very little from Euclid's, and the new mathematical theory of space seemed to embrace light, electromagnetism, and the law of gravitation. It seemed as though Laplace's "single formula" was at last being approached.

But unfortunately, it was not possible to include the phenomena of chemistry and radioactivity in this theory; again, one could only hope for future improvements.

Some Conclusions

It is clear from the forgoing chapters that when men, whose mathematical creativity is unquestionable, begin to consider the problems of people and society, they do not try to solve them mathematically. This is of especial importance, since men of lesser mathematical ability continue to try to construct mathematical theories of economics, politics, sociology, and even psychology. Neither Abbe, Russell, nor Szilard explicitly denied Laplace's Credo of Determinism, unless one wishes to infer such a denial from their behavior. It is also important to note that Laplace did not obtain his marquisate by presenting Louis XVIII with a mathematical demonstration. The current version of the atomic theory does deny that chance and probability are merely the result of our ignorance. Only one major physicist, Arthur Holly Compton, used this as an argument for people's free will, freedom to make decisions. Most others have rarely bothered themselves with the logical conclusion that if completely deterministic laws apply to the universe, man is himself an automaton. It has been seen that this strange oversight (or whatever it may be) was initiated by Democritus, who was certainly one of the first, if not the first, to cite the Law of Necessity or Principle of Sufficient Reason. Epicurus denied that there was any connection between cosmology or science and ethics or religion; the vigor with which his ethics have been denounced has obscured this major element in his writings. Leibniz presumably labored over this problem, without coming to a final conclusion. Apart from him, only one other recent mathematician has approached this two thousand year old problem; this is not to say that non-mathematicians have been silent.

Alfred North Whitehead, who collaborated with Bertrand Russell in writing the *Principia Mathematica*, was another of those who abandoned mathematics and approached more general problems by writing in ordinary English. In his book, *The Concept of Nature* his major thesis is that nature is what we observe by sense-perception; we are aware that in this sense-perception there is something which is not thought; this is nature. It can be thought about without thinking that one (or anyone else) is thinking about it. This he calls thinking homogeneously about nature. We can also think about it while thinking that we (or someone else) is thinking about it; this is thinking heterogeneously about nature. Natural science consists entirely of homogeneous thoughts about nature.

Whitehead's thesis, and especially his distinction between homogeneous and heterogeneous thought, has implications that are too important to be ignored. The one is the distinction between nature and natural science, the latter being a system of homogeneous thoughts about nature. Whether or not people have free will does not depend on their homogeneous thoughts about nature. Hence Laplace's deterministic, fatalistic theory of nature, and the more recent atomic theory which assigns to chance or probability a fundamental role in atomic events, are both irrelevant to the matter of free will, and hence to ethics. Compton was in error because he attempted to establish a logical connection between homogeneous and heterogeneous thoughts. Epicurus (however one might value his ethical system) was right in rejecting cosmology (or natural science) as prerequisite to ethics.

The second implication of Whitehead's thesis follows almost immediately from the first: it is that natural science does not exhaust the possibilities for thought about nature, which is not natural science. The importance of Whitehead's thesis is that it leaves room for ethics and morality. In thinking about doing business with people, we can think heterogeneously. We are all parts of nature, but we are also capable of thought. Thus heterogeneous thought, thinking that we and all people are thinking about nature, becomes a possible, almost an essential part of our transactions with people. In some circumstances, for example during a surgical operation on an anesthetized patient, it may be important to think homogeneously about a human being. But the Hippocratic Oath is heterogeneous thought and remains in the back of the surgeon's mind; it was in the foreground when he decided to operate, and will again come forward while the patient convalesces. Thus, Whitehead has unwittingly contributed to our understanding of the personal relations between Democritus, Hippocrates, and Epicurus, three people whose importance for history cannot be doubted. Since

Whitehead was educated in the classical curriculum of nineteenth-century England, this proposition may also be inverted: he may well have been influenced by them.

There is much evidence that early people thought only heterogeneously about nature. Two people bargaining about the exchange of arrowheads and stone knives would each be conscious that the other was thinking about the arrowheads and stone knives which were a part of nature that was being perceived by them. By using shrewder words, by taking more risk that the other would call the deal off, one could get a better bargain. What the other has said may be a clue to what he is thinking and will say next. A stupid person may intentionally display eagerness, a clever person may intentionally hide his eagerness. Shrewdness borders on deceit, but it may pay off. Doing business face to face with other people almost inevitably required heterogeneous thought. Persuasive sentences express heterogeneous thoughts.

Hunting is doing business with animals. One could possibly bargain shrewdly with them. Magic incantations, rituals in which the hunted animal is impersonated and addressed, would be undertaken. When inantimate nature, rain, winds, and floods, were involved, it is understandable that people accustomed only to heterogeneous thought would suppose that they were produced by sentient beings. The origins of the Greek and similar religions that involve doing business with the gods, also becomes understandable. Evidence for this is to be found in the failure of early peoples to make a distinction between religious ritual and secular routine.

Although this is not stated in the Credo, Laplace's other writings show that he believed in the atomic theory, and that the atoms were both immortal and had no parts. "Atom" means indivisible. This was also the belief of his immediate followers. By the beginning of the twentieth century, evidence had accumulated that the atoms were neither simple nor immortal. They were, it was thought, composed of protons, neutrons, and electrons. The explosion of the first atomic bomb was a dramatic confirmation of this theory, though all physical scientists had accepted it years before.

For a time, it was possible to believe that the electrons and protons were indivisible and immortal, though evidence to the contrary had been accumulating since about 1930. After World War II, many new subatomic particles, still often called strange particles, were found. These seemed to be constituents of the protons and electrons; they were certainly not immortal. They appeared, changed, and disintegrated in a remarkably short time, usually in less than one millionth of a second. All of this is still not clearly understood; but there seems to be no way to calculate the lifetime of any single one, any more than it is possible to calculate the exact lifetime of a certain human being. Neither is it possible to calculate the lifetime of any atom of radium. The precise moment at which an atom of radium will disintegrate appears to be a matter of chance. One can measure its probability; one can calculate the probability that it will disintegrate before the end of the year. But one can neither describe the disintegration process nor calculate exactly when it will occur. Of course, one may follow Laplace and maintain that this is merely our temporary ignorance which will inevitably be remedied before long. For a time, some physicists, including Albert Einstein, maintained this belief. An increasingly larger majority of physicists do not believe this, for a reason that can be explained in non-technical terms.

Of course, there is no way of finding out whether Laplace is right or wrong. The Principle of Sufficient Reason is not self-evident. Physicists, and most modern statisticians, no longer accept as valid any argument which depends on it. Some fifty years ago, the astronomer Henry Norris Russell considered this matter in a lecture before a general audience. He concluded that, even if Laplace is right, we might as well behave as responsible people should. And this is the common sense of the matter. It amounts to dismissing the Credo as irrelevant.

But Laplace is not irrelevant. His ideas have not only influenced physicists and chemists, but biologists and psychologists as well. It is in these last two groups that his ideas are most evident today.

The fact of biological evolution was established by Lamarck, Darwin, and Wallace. Laplace's direct effect on these men was not great. They were not mathematicians. But after Mendel's discovery of the laws of inheritance, it became possible to develop a mathematical theory of biological evolution, using Laplace's theory of probability. Mendel's laws are both empirical and probabilistic, so that this use of Laplace's mathematics is fully justified. It is justified, whether chance is a delusion, or inherent in the world.

More of less simultaneously, psychologists began using probability theory to interpret the results of intelligence tests and similar questionnaires. It will be recalled that initially this was because of dissatisfaction with the aristocratic scheme of rank-ordering people by their grade point averages and intelligence quotients. Secondarily, it could be justified by the hope of discoveries as fundamental as Mendel's. After fifty years, this hope has not been realized, but it has not yet been abandoned. More and more elaborate mathematical methods are being used, and larger and larger electronic computers. It has been forgotten that Mendel made his disoveries in a monastery garden, using only the very simplest arithmetic. Even more distressing, all of this effort has caused only minor changes in the scheme of rank-ordering students.

It will be recalled that Thurstone's major hypotheses were that people's abilities are numerically measurable and that these abilities are innate. Both are implications of the Credo. People's abilities are fixed at birth; no conscious effort on their part can change their abilities. Consciousness, if not a delusion, is impotent, irrelevant. This has been generalized by the school of behavioral psychology. Not only abilities, but all of people's behavior is governed by mathematical laws. Conscious effort is irrelevant. Thurstone's original hypotheses are thus made to conform even more closely to the Credo, and in turn, the Credo justifies the use of more elaborate mathematical methods.

Historically, it is an oversimplification to say that Mendel's discoveries fused the theory of biological evolution with the Kant-Laplace theory of the evolution of the solar system. Yet that is what has happened, and the Mendelian laws were essential. The discovery of genes, and their later identification as large chains of DNA molecules, provided a causal explanation of the Medndelian laws, of precisely the kind envisioned by the Credo. What Ardrey has called the accident of the might could be made to seem determined. A conceivable improvement in the theory of fluids and a conceivable improvement in the recording of muscular movements, and the outcome might be calculable by a conceivably improved electronic computer.

Before the discovery of the double helix DNA and RNA molecules, the behavioral psychologists used to say that no definition of life could be framed which would exclude the candle flame. The flame metabolizes oxygen and organic matter; in the absence of external stimuli, it becomes quiescent; given the proper occasion, it reproduces itself. After the discovery of these molecules and their role in the life of plants and animals, it was possible to frame such a definition. The metabloism of the candle flame does not involve DNA or RNA.

But it also became possible to develop theories of the origin of life; that is, of the natural synthesis of the first self-replicating DNA and RNA molecules from non-living matter. It became possible to theorize about the entire prehistory of man, from the primordial matter, to the synthesis of the first self-replicating molecule, to the evolution of plants, animals, and man. Kant and Laplace supposed that the primordial matter of the solar system was unconscious, unintelligent; most recent cosmologists follow their lead. Alfred North Whitehead has pointed out that there are then two, much neglected, problems. At what stage of the evolution of life did portions of that matter become conscious? And, at what stage did these portions, which we call people, become intelligent, or, at least, begin to suffer the delusion of intelligence? This has also been emphasized by the paleontologist Theilhard de Chardin, from a somewhat more theological viewpoint.

It is related that Whitehead once startled an audience by saying that there might be life on the Sun. He went on to explain that he was thinking of beings that had a lifetime measured in microseconds, dimensions measured in kilometers, but were yet able to learn. The astrophysicist Fred Hoyle has elaborated on this theme in his science fiction novel, *The Black Cloud.*

The question can be simplified to this: Is a candle flame conscious? This is clearly a difficult question to answer, but as yet no attempt has been made to explain the consciousness of people. Some psychologists are annoyed by the suggestion that this might be a topic worthy of study; they are also annoyed if asked whether they are unconscious.

The mathematician John von Neumann wrote on many subjects, including self-replicating automata and the theory of automatic computers. The title of the last chapter of his last book is "The Language of the Brain, Not the Language of Mathematics." There are thus signs that the influence of the Credo is waning, at least within the physical sciences. Elsewhere, however, its influence seems to be increasing. Remarkably, the humanities are no exception to this statement. W.H. Simon reports:

> This debut (of the computer) has been nowhere more spectacular than at Princeton, where two departments -- history and music -- lead the world in developing computer applications to their disciplines, and where nearly all other humanities divisions are getting into the act. A survey of these departments gives credence to the prediction made by Edmund Bowles, IBM's humanist-in-residence, that of the common baggage of research tools and techniques required of every graduate student in the humanities.'...The machine rushes forth answers uncontaminated by human consciousness, free from vagueness, ambiguity, prejudice, inconsistancy, and all other infirmities of the human mind.

The idolatry of mathematics is being replaced by the idolatry of the computer.