Title

Ferrite-Cored Solenoidal Induction Coil Sensor for BUD (MM-1667)

Author

Morrison, F.

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FERRITE-CORED SOLENOIDAL INDUCTION COIL SENSOR FOR BUD (MM-1667)

Frank Morrison[1], Alex Becker[1], Ugo Conti[2],

and Erika Gasperikova[1]

[1]Lawrence Berkeley National Laboratory

[2]Conti Consulting Services

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1. OBJECTIVE AND THEORY

We have designed and lab tested a new ferrite cored induction coil sensor for measuring the secondary fields from metallic UXO with the BUD system. The objective was to replace the 5-inch diameter air-cored coils in the BUD system with smaller sensors that would allow the placement of multiple sensors in the smaller package of the new BUD hand-held system.

The small transients from induction currents in a UXO are generated by a pulse of magnetic field, the primary field, generated by a current pulse in a suitable transmitter, usually a multi-turn coil of wire. The receiver for the target transients is usually also a multi-turn circular loop and in an ideal loop the voltage across the terminals of the loop is proportional to the time rate of sensor change of the magnetic field threading the loop. In practice the loop is not ideal and is found to have a distributed intra-wire capacitance, C that leads to the sensor having a finite bandwidth. The finite bandwidth distorts the response of the measuring system to the secondary transient. More importantly the receiver usually ‘sees’ at least some of the primary field which shows up as another transient in the receiver after the primary field is shut off. This system transient can mask the desired target transient. This effect is particularly vexing because very small perturbations to the circuit components of the receiver result in transient changes that can be as big or bigger than the desired target transient.

A practical induction coil sensor is usually made of a number of turns, N, in a circular loop of diameter D (5 inches in the BUD system) and area A. Such a loop will have a calculable inductance, L, and resistance R. There is no analytic form for the distributed capacitance within the windings, C, and this is usually measured on the finished coil.

For a solenoidal coil, with a core of material of relative magnetic permeability, μ, the sensitivity is increased by the multiplier μ_{eff} where this effective permeability is determined by the length to diameter ratio of the core. A solenoidal coil effectively converts a thin wheel-like coil of diameter D to a small diameter cylindrical coil of length *l*. The objective of this study was to build a small solenoid coil about 4 inches long with as good performance as the air-core coil or better.

The fundamental sensitivity or response of an induction coil is the ratio of the voltage output to the magnetic field input in Tesla's. The *emf* induced in the coil is given by Faraday's law via:

$$emf = \mu_{eff} NA \frac{\partial B_0}{\partial t}$$

where B_0 is the incident magnetic field. This *emf*, really a voltage in series with the coil inductance and resistance, drives the current in the series RLC circuit and the output of the device is the voltage measured across C, multiplied by the gain of the amplifier G.

The circuit representation of an induction coil receiver is shown in Figure 1.

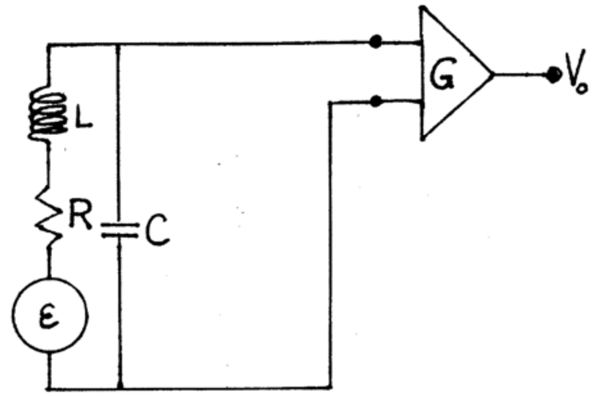

Figure 1. An induction coil receiver circuit.

For a sinusoidal time dependence $e^{i\omega t}$, and at low frequencies, C presents an open circuit and the output voltage V_0 is just:

$$V_0 = \mu_{eff} NA \frac{\partial B_0}{\partial t} = G\mu_{eff} NAi\omega B_0$$

and so the ratio of $\frac{V_0}{B_0}$, the response, is simply given by:

$$\frac{V_0}{B_0} = G\mu_{eff} NAi\omega$$

This is referred to here as the base sensitivity and it is linear in ω. The terms sensitivity, transfer function and response are used interchangeably in this study.

As the frequency increases the voltage across C begins to fall and the response falls. A full solution for the response is:

$$\frac{V_0}{B_0} = G\mu_{eff} NA \frac{i\omega}{1 + i\omega RC - \omega^2 LC}$$

With the definitions of $\omega_0 = \frac{1}{\sqrt{LC}}$ and $\alpha = \frac{R}{2\omega_0 L}$, the response takes on the general form:

$$\frac{V_0}{B_0} = G\mu_{eff} NA \frac{i\omega\omega_0^2}{\omega_0^2 + i\omega 2\omega_0\alpha - \omega^2}$$

or when normalized by ω_0:

$$\frac{V_0}{B_0\omega_0} = G\mu_{eff} NA \frac{i\omega\omega_0}{\omega_0^2 + i\omega 2\omega_0\alpha - \omega^2}$$

The amplitude and the phase of the response are plotted in Figure 2a and 2b respectively as a function of normalized frequency ω/ω_0 (in Hz) for three values of α.

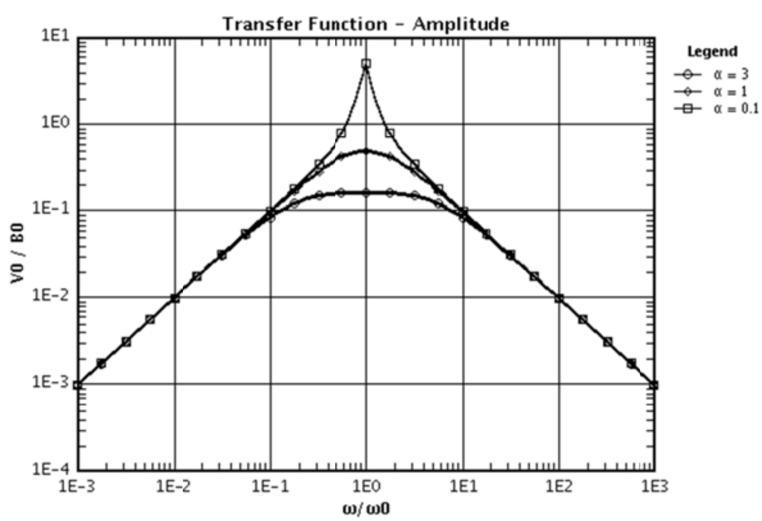

Figure 2a. Amplitude response of an induction coil for α = 0.1, 1, and 3.

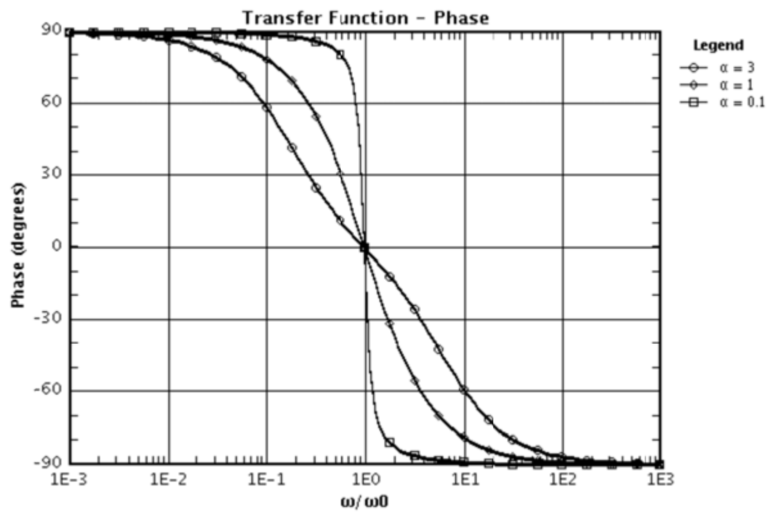

Figure 2b. Phase response of an induction coil for α = 0.1, 1, and 3.

If α is less than one the response curve is sharply peaked at ω_0 (the *resonant* frequency). When α is equal to one the response is said to be critically damped. When α is greater than one the response curve flattens on either side of ω_0 and the response is said to be over-damped. The *bandwidth* of the response function is usually defined by the frequencies at which the response has fallen by three dB, or 0.707 of its resonant value. For the critically damped coil the low, ω_L, and high, ω_H, frequencies are given by $\omega_L = \left(\sqrt{2}-1\right)\omega_0$ and $\omega_H = \left(\sqrt{2}+1\right)\omega_0$, and for the over-damped coil by $\omega_L = \left(\alpha - \sqrt{\alpha^2-1}\right)\omega_0$ and $\omega_H = \left(\alpha + \sqrt{\alpha^2-1}\right)\omega_0$. Finally, the peak response at resonance for the critically damped coil is given by:

$$\frac{V_0\left(\omega_0\right)}{B_0\left(\omega_0\right)} = G\mu_{eff} NA \frac{\omega_0}{2\alpha}$$

We will apply these relationships later when we analyze the responses of actual induction coils.

Note that the damping coefficient, α, is related to the more conventional quality factor Q via $Q = \frac{1}{2\alpha}$. Thus high Q circuits are sharply peaked at resonance, and low Q circuits have smooth broad response curves.

The significance of α is best seen in the time domain response of the above circuit to a step function turn-off of a constant magnetic field of amplitude B_0. The step function response yields the transient that would be observed in the receiver output after abruptly turning off the current in the source. The transient response, normalized by ω_0, is different for different ranges of the parameter α.

For α <1.0:

$$\frac{V_0(t)}{B_0\omega_0} = \frac{F}{\sqrt{1-\alpha^2}} e^{-\omega_0 \alpha t} \sin \omega_0 \sqrt{1-\alpha^2}\ t$$

For α =1.0:

$$\frac{V_0(t)}{B_0\omega_0} = F\omega_0 t e^{-\omega_0 t}$$

For α >1.0:

$$\frac{V_0}{B_0\omega_0} = \frac{F}{2\sqrt{\alpha^2 - 1}} \left[e^{(\sqrt{\alpha^2-1}) - \alpha)\omega_0 t} - e^{-(\sqrt{\alpha^2-1}) + \alpha)\omega_0 t} \right]$$

These three response types are shown schematically in Figure 3.

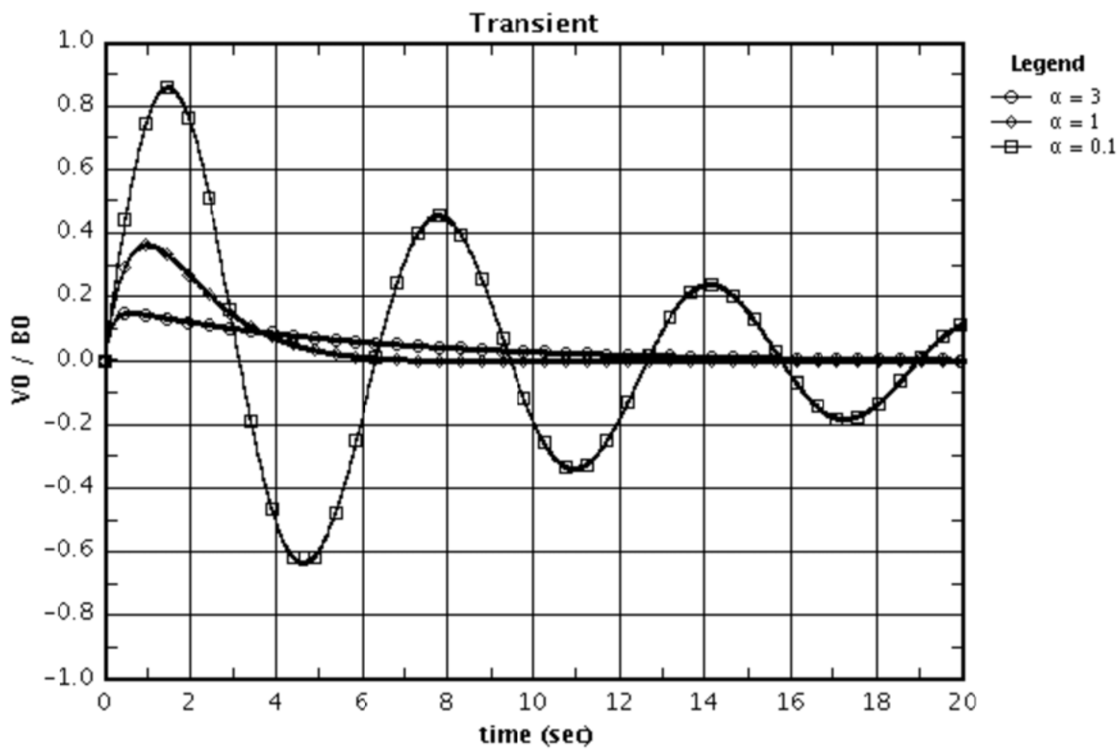

Figure 3. Normalized transient response for $\alpha = 0.1$, 1, and 3.

If α is less than one the response rises and then rings at the resonant frequency, and the oscillations decay with a time constant set by α and ω_0. This response is termed under-damped. When α equals one the transient rises and falls with a time constant of $1/\omega_0$; it is always positive. This response is termed critically damped. Finally if α is greater than one the response rises and then falls very slowly and is always positive.

It is clear that the critically damped receiver has the fastest possible decay with no ringing. The design objective then for any transient receiver is to make ω_0 as high as possible while keeping $\alpha = 1.0$ and have the highest base sensitivity. Maximizing the base sensitivity implies increasing N [the area of the coil is usually constrained by packaging dimensions] but this increases the inductance and the distributed capacitance that lowers ω_0 and therefore lengthens the step function transient.

A lengthy design exercise, largely by trial and error, for the BUD system led to the use of an air-cored coil, 5 inches in diameter, with the following basic parameters (a discussion of the damping resistor, R_d, follows).

5" Air-cored Coil (Critically damped)		
Parameters	**Measured**	**Calculated**
Diameter [cm]	12.7	
Number of turns	330	
NA [m^2]		4.18
Wire: Cu, AWG 32		
R, Coil resistance [Ohms]	142	112
L, Coil inductance [mH]	37.5	58.7
f_0, Resonant frequency [kHz]	73	
C, Coil capacitance [pF]		122
R_d, Damping resistor [kOhms]	9.1	8.5
Sensitivity @ 10 kHz [mV/nT]	0.3	0.26

Table 1. Air-cored coil parameters.

The response was measured by placing the sensor coaxially in the plane of a transmitter loop of diameter 1.0 m with a known number of turns. The magnetic field at the center is then a known function of current. A measurement of the transfer function between the current and output voltage is then easily converted to the response in mV/nT.

For this raw coil the calculated sensitivity at 10 kHz [on the linear, $V \propto i\omega B$, portion of the response curve] is 0.26, just slightly under the measured value. As we will see below in the discussion on inductance and resonant frequency this discrepancy is due to inaccuracy in measuring the effective area for a coil with an appreciable thickness of winding. This coil would be severely under-damped, $\alpha = \frac{R}{2\omega_0 L}$ =0.0041, and would ring under step function excitation. The winding resistance would have to be increased to approximately 30 kOhms to achieve critical damping and this would require a wire too small to use in practical winding.

Damping can be achieved by adding parallel resistor, R_d, to the circuit as shown in Figure 4. The addition of an external damping resistor complicates the circuit analysis but the final result is that R_d must be much larger than R and it is given by the simple formula, $R_d = \frac{\omega_0 L}{2}$. So a parallel damping resistor of 8.5 kOhms is required.

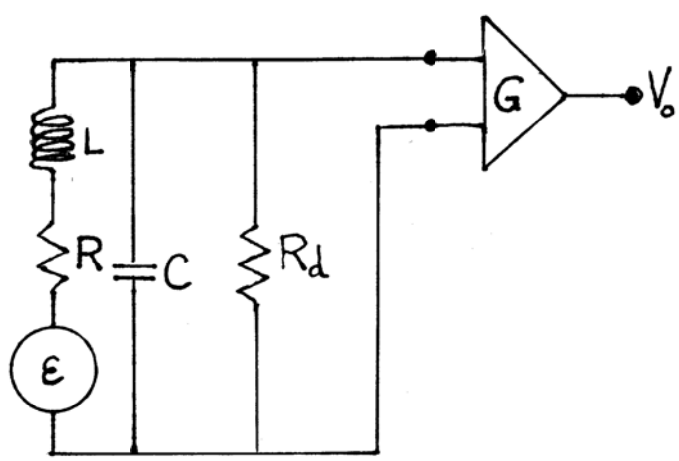

Figure 4. Induction coil receiver with a dumping resistor R_d.

2. FERRITE-CORED COILS

A small solenoid coil with a core of high permeability material can achieve sensitivities similar to those of the air-cored coil with the obvious advantage of occupying less space in any multiple-transmitter multiple-receiver platform. To demonstrate the feasibility of such a sensor we fabricated two small solenoidal sensors with the parameters listed in Table 2.

4" Ferrite-cored Coil (Critically damped)		
Parameters	**Measured**	**Calculated**
Length [cm]	9.62	
Diameter of core [cm]	0.64	
Number of turns	1300	
$\mu_{eff} NA$ [m^2]		6.54
Radius of winding (to mid thickness) [cm]	0.4	
Wire: Cu, AWG 32		
R, Coil resistance [Ohms]	20	18
L, Coil inductance [mH]	95.4	110
f_0, Resonant frequency [kHz]	55	
C, Coil capacitance [pF]		86
R_d, Damping resistor [kOhms]	19	16
Sensitivity @ 10 kHz [mV/nT]	0.175	0.41

Table 2. Ferrite-cored coil parameters.

This sensor has a calculated sensitivity at 10 kHz of 0.41 which is slightly higher than the 5-inch diameter air-cored coil. The measured value is much less than the calculated value probably because the inductance formula for the cored solenoid is approximate and because the intrinsic μ of the ferrite is lower than advertised. The latter would explain why both the sensitivity and inductance calculations are slightly less than the measured values.

It is important to note that the damping is a function of R/L, not just the dissipative R. In the air-cored coil R/L is ~3700 whereas in the ferrite core coil R/L is ~180. The air-cored coil would be expected to have a very sharp resonance peak, and the ferrite core should have a smaller peak by virtue of its smaller R/L value. It is also important to note that despite the higher inductance of the ferrite core coil the capacitance is *less* so that the resonant frequency of the ferrite core coil is about 75% that of the air core coil. For this coil α = 0.00029 (Q=1700) so the coil would be severely under-damped. Critical damping would require a parallel resistor of 16.4 kOhms. In the laboratory tests the value of the damping resistor was found by trial and error in the time domain; R_d was varied manually until ringing just stopped. It can be seen from the tables that the measured and predicted values are close, but in both cases the experimental values were slightly higher than the calculated values, indicating that the test coils were slightly under-damped. Finally the peak value of the response for the critically damped coil is 0.96 mV/nT based on the observed circuit component values; the measured value is about 1.1 mV/nT, which also indicates that the coil was under-damped. The plots of the sensitivity versus frequency for these two coils, damped and under damped are shown in Figure 5.

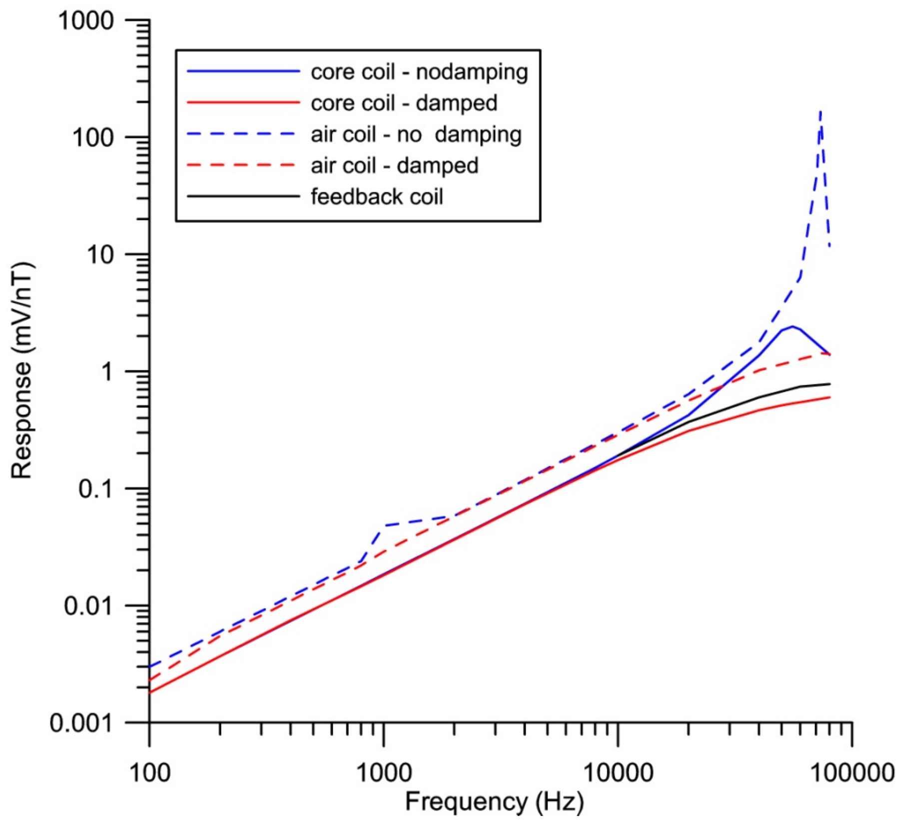

Figure 5. Sensitivities as a function of frequency of air-cored (dashed lines) and ferrite-cored (solid lines) coils.

As discussed, the response is linear to 10 kHz at which point the observed and calculated responses are very close as seen in the above table. The resonance peaks at 55 kHz and 73 kHz are well defined. The calculated Q values for the raw coils do not agree with the observations. In particular, the measured response of the ferrite coil is far lower at the resonance that its calculated Q would suggest. At the moment we have no explanation for the anomalously low Q observed in a ferrite cored coil. It differs from the air core coil in having a permeable core and in having different shielding. If the core had ac losses the Q would be reduced. The ferrite itself is very resistive and would not be expected to have any significant eddy current losses. It is possible that there is a lossy magnetic relaxation phenomenon in the ferrite. In any event the effect aids in critical damping and had no observable effect on step-off transient [i.e. anomalous transients that might be associated with relaxation phenomena in the ferrite].

We have shown that the small compact solenoidal ferrite cored coil can have similar sensitivity to the conventional air core coils without significantly lowering the resonant frequency and consequently, without seriously increasing the step-off transient.

3. THE FEEDBACK COIL

The externally damped sensor shown in the schematic of Figure 4 has some serious practical problems. One is that the damping resistor may add significant Johnson noise to the response, and the second is that the critical damping condition is somewhat unstable; small changes in ω_0 or R [or L due to deformation of the air core loop] cause changes in α, and can therefore move the response from under-damped to damped to over-damped with concomitant changes in the transient just after shutoff. These will add serious 'noise' to the secondary field, and usually very small transient and that is to be measured.

To stabilize the critically damped coil we have used the negative feedback scheme illustrated in the schematic of Figure 6.

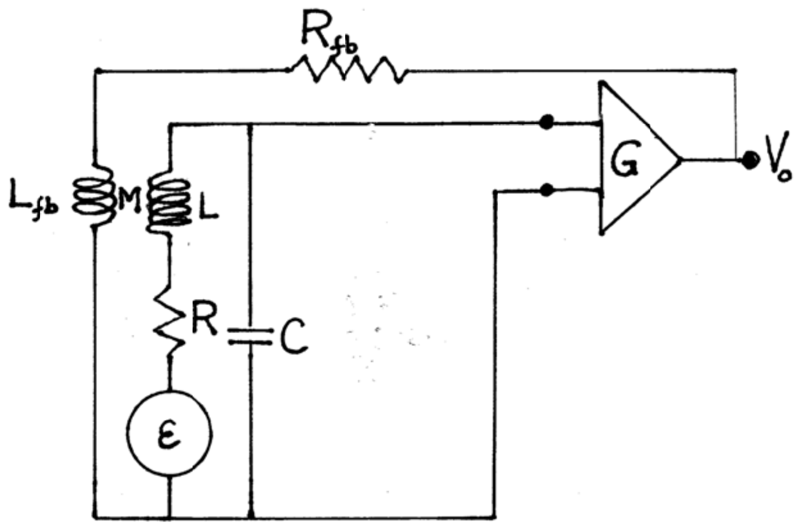

Figure 6. Circuit of critically damped feedback coil.

Here the amplifier output is fed back through the feedback resistor, R_{fb}, to a feedback coil arranged so as to oppose the *emf* developed by the changing external magnetic field through the main coil. The mutual inductance between the main coil winding and the feedback winding is M. The sensor output V_0/B_0 is then reduced around the resonant frequency. The resulting bandwidth is now dependent on the amount of feedback. The derivation of the response of the circuit has been done elsewhere, and we need only extract the pertinent relationships needed to design a feedback coil that is critically damped.

It turns out that the response is exactly the same as the damped coil except that the damping coefficient α is now defined as:

$$\alpha = \frac{R}{2\omega_0 L} + \frac{\omega_0 MG}{R_{fb}}$$

So if α is too small because R may be too small, it can be increased arbitrarily by adding the term $\frac{\omega_0 MG}{R_{fb}}$. There is some flexibility in choosing M, G and R_{fb} so it is easy in practice to achieve any desired value of α. The added importance of this becomes evident when we find that $\frac{\omega_0 MG}{R_{fb}}$ is usually much greater than $\frac{R}{2\omega_0 L}$ and so instabilities in R or L become insignificant and the stability of the critically damped response depends mostly on MG/R_{fb}, all of the terms of which are easily held constant. Since the ferrite coil is potted in a stiff cylindrical shell, L and L_{fb} are already relatively constant compared to their counterparts in the less rigid air-cored coil, so it is only necessary to use a very high quality feedback resistor to achieve exceptional stability. The criteria for selecting the feedback resistor simplifies to:

$$R_{fb} = \omega_0 MG.$$

The mutual inductance is given by:

$$M = k\sqrt{L\,L_{fb}}$$

and the gain, G, for the feedback coil was 100.

A summary of the parameters of the feedback ferrite-cored coil is given in the Table 3.

4” Feedback Ferrite-cored Coil		
Parameters	**Measured**	**Calculated**
Length [cm]	9.62	
Diameter of core [cm]	0.64	
Number of turns	1300	
$\mu_{eff} NA$ [m^2]		6.54
Radius of winding (to mid thickness) [cm]	0.4	
Wire: Cu, AWG 32		
R, Coil resistance [Ohms]	20	18
L, Coil inductance [mH]	95.4	110
f_0, Resonant frequency [kHz]	55	
C, Coil capacitance [pF]		86
R_{fb} ,Feedback resistor [kOhms]	105	122
L_{fb}, Feedback inductance [mH]	0.13	
M, mutual inductance [mH]		3.52
Sensitivity @ 10 kHz [mV/nT]	0.175	0.41

Table 3. Feedback ferrite-cored coil parameters.

The feedback resistance found by trial and error to achieve critical damping was 105 kOhms, suggesting that the feedback sensor was over damped [it is also possible that the measurement of L with an inductance meter was inaccurate]. The response of this feedback coil is included in the response plots of Figure 5 as a solid black line. The observed response at resonance is found to be slightly higher than the response obtained with the external damping resistor and both are over-damped.

In conclusion the small ferrite-cored coil performed as predicted by the circuit model and demonstrated a sensitivity only slightly less than that of the air-cored coil. Both sensors could be improved by further optimization, but the main conclusion is that a small ferrite-cored coil with feedback appears to be the ideal sensor for EM systems requiring stable multiple receivers in a small package.

The remaining issues are the relative noise performance and, for applications requiring that the sensors be placed close together, what coupling occurs between the two sensors.

4. SENSOR NOISE

A separate analysis has shown that the use of a feedback circuit actually reduces the electronic noise of the receiver and, with the above parameters, should result in noise comparable to the air-cored coils. To verify these results we conducted very simple measurements with a spectrum analyzer to estimate the inherent noise levels of the sensors. The initial response measurements showed that the two sensors had almost identical responses.

First we looked at the raw power spectrum of the coil output in the lab. We visually identified frequencies where the spectrum was low. Then we placed two of the sensors side by side (separated by at least one diameter) and observed the coherence spectrum. At those frequencies where both the spectrum and the coherence were low we then calculated the noise of the individual sensor (assuming that both sensors have the same response and noise) via the formula:

$$\text{Noise spectrum}(\omega) = (1\text{-Coh}) * \text{Power spectrum}(\omega)$$

Very few frequencies met the criteria; nevertheless the following sensor noise plot shows noise values that are comparable to more rigorously calculated values reported for the BUD system.

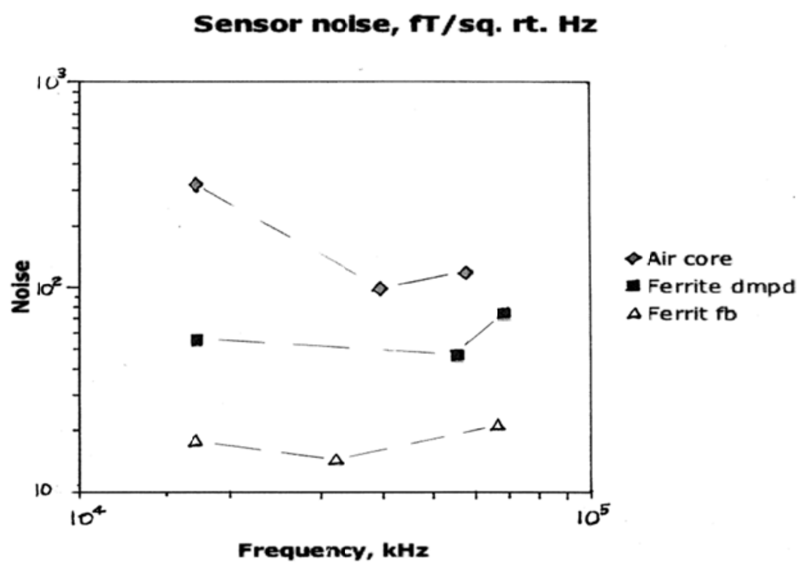

Figure 7. Sensor noise as a function of frequency for air-cored, critically damped ferrite-core, and feedback ferrite-core coils.

Unfortunately the air-cored coils available for this test were not shielded, and this is known to increase the noise by as much as an order of magnitude. In any case the ferrite sensor noise is very low. As expected the feedback coil has less noise than its externally damped counterpart. In passing it is interesting to note that the natural magnetic field noise spectrum in this band varies from 5 to 100 $fT/\sqrt{Hz}$ so that there are conditions when the noise in the BUD system may be from natural sources rather than from the electronics. In conclusion, these crude test results show that the noise levels of the small ferrite-cored coils are more than adequate for use in a BUD style system.

5. PROXIMITY EFFECTS

There is a concern that the feedback coils could couple if placed close together. This could arise because of actual coupling, i.e. mutual inductance, between the separated circuits or because the high permeability cores would somehow distort the field being measured by either coil.

To test the influence of proximity on the sensitivity of the coils we made a small jig to hold the coils in various relative positions. The jig was set in the center of the 1.0 m diameter transmitter loop used to determine the sensitivities described earlier. For this experiment we used the spectrum analyzer to measure the transfer function between the current in the large loop and the voltage output of one of the feedback ferrite-cored coils as a second operating coil was moved closer and closer. The various configurations used for this test are shown in Figure 8. In each case the heavily shaded coil symbol represents the fixed coil whose end was kept centered in the plane of the transmitter loop and on the axis of the loop. The second coil was brought in from a large distance (∞ in the following tables) to a position at a distance x from the fixed coil as indicated. The coil length is L and the separations are in fractions of L.

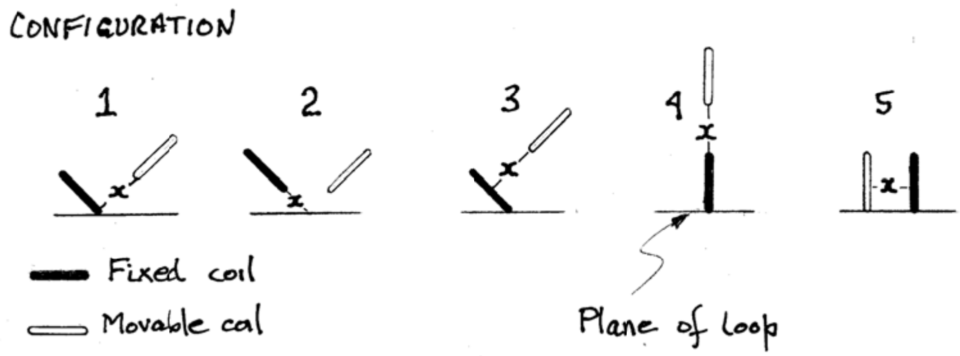

Figure 8. Proximity effect test configurations of two feedback ferrite-core coils.

The following tables show the change in the transfer function as a function of frequency and separation distance x for each configuration as the movable coil is brought into proximity with the fixed coil.

Configuration 1	**x**	**x**	**x**	**x**
Frequency (kHz)	**∞**	**L**	**L/2**	**0**
10	1.38	1.37	1.35	1.31
20	2.66	2.64	2.61	2.54
40	4.64	4.62	4.58	4.51
60	5.57	5.56	5.56	5.53
80	5.88	5.85	5.85	5.85

Table 4a. Transfer function as a function of frequency (f) and separation (x) for proximity test configuration 1.

Configuration 2	**x**	**x**	**x**	**x**
Frequency (kHz)	**∞**	**L**	**L/2**	**0**
10	1.39		1.37	
20	2.69		2.65	
40	4.70		4.65	
60	5.63		5.62	
80	5.91		5.91	

Table 4b. Transfer function as a function of frequency (f) and separation (x) for proximity test configuration 2.

Configuration 3	**x**	**x**	**x**	**x**
Frequency (kHz)	**∞**	**L**	**L/2**	**0**
10	1.38		1.38	1.34
20	2.67		2.66	2.60
40	4.67		4.65	4.60
60	5.61		5.61	5.58
80	5.89		5.87	5.90

Table 4c. Transfer function as a function of frequency (f) and separation (x) for proximity test configuration 3.

Configuration 4	**x**	**x**	**x**	**x**
Frequency (kHz)	**∞**	**L**	**L/2**	**0**
10	1.91	1.92	1.93	2.07
20	3.69	3.69	3.72	3.96
40	6.38	6.39	6.38	6.69
60	7.53	7.56	7.57	7.73
80	(7.82)	(7.86)	7.85	7.91

Table 4d. Transfer function as a function of frequency (f) and separation (x) for proximity test configuration 4.

Configuration 5	**x**	**x**	**x**	**x**
Frequency (kHz)	**∞**	**L**	**L/2**	**0**
10	1.94	1.93	1.86	1.56
20	3.76	3.73	3.63	3.17
40	6.56	6.54	6.46	6.15
60	7.82	7.84	7.80	8.03
80	8.21	8.21	8.27	8.52

Table 4e. Transfer function as a function of frequency (f) and separation (x) for proximity test configuration 5.

It is evident that within the measurement accuracy there is virtually no coil interaction for separations greater than half a coil length. This is somewhat surprisingly true even for end-on configuration 4. The only exception is for two parallel coils, configuration 5, where the separation must be at least one coil length. Another subtle but possibly important observation is that the interactions are notably less at the high frequencies. This could be because for critical damping the feedback is basically bucking the incident field in the core so that the external field is unperturbed as seen from outside.

6. CONCLUSIONS

The conclusions can be summarized by the following observations:

1) A ferrite-cored solenoidal coil of length L can easily be made to have sensitivity and noise level roughly the same as an air-cored coil of a diameter on the same order as L.
2) A ferrite-cored solenoidal coil can easily have a feedback configuration to achieve critical damping.

3) The feedback configuration leads to a very stable response.

4) Feedback ferrite-cored solenoidal coils show very little interaction as long as they are separated by one half their length.

7. ACKNOWLEDGMENTS

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