Title
NUCLEAR INTERNAL MOMENTUM DISTRIBUTION

Author
Wilcox, John M.

Publication Date
1956-07-20

UNIVERSITY OF
CALIFORNIA

Radiation Laboratory

NUCLEAR INTERNAL MOMENTUM DISTRIBUTION

UCRL-3475
Physics Distribution

UNIVERSITY OF CALIFORNIA
Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

NUCLEAR INTERNAL MOMENTUM DISTRIBUTION

John M. Wilcox

July 20, 1956

Printed for the U.S. Atomic Energy Commission

NUCLEAR INTERNAL MOMENTUM DISTRIBUTION

John M. Wilcox

Radiation Laboratory
University of California
Berkeley, California

July 20, 1956

ABSTRACT

The results of McEwen, Gibson, and Duke on nuclear internal momentum distributions are considered. By eliminating a restriction due to assumed background, we have found the observed distribution to be Gaussian with a 1/e value of 22 Mev. The use of momentum distributions in interpreting the results of other experiments is discussed.

NUCLEAR INTERNAL MOMENTUM DISTRIBUTION

John M. Wilcox

Radiation Laboratory
University of California
Berkeley, California

July 20, 1956

McEwen, Gibson, and Duke (1956) (hereafter called MGD) have investigated quasi-elastic collisions of protons with protons bound in nuclei of G-5 emulsions exposed to 925-Mev protons from the Birmingham synchroton. The observed nuclear internal momentum distribution was approximately Gaussian, falling to 1/e at 8 ± 2 Mev,

$$N(p) = e^{-p^2/p_0^2}\ , \text{ where } \ p_0^2/2M = 8 \pm 2 \text{ Mev.}$$

The experimental results were presented (in MGD Fig. 4) as a plot of the p_z component versus the p_x component, where p_x is the component of momentum of the nuclear proton in the same direction as the beam proton, and p_z is a component of momentum of the nuclear proton in a direction perpendicular to the incident direction. The number of events per unit area in this plot is presented as a function of the corresponding energy (of the momentum component in the xz plane) in MGD Figs. 5 (a) and 5 (b). All events whose equivalent resultant momentum corresponds to more than 30 Mev energy have been assumed to be background, and have been excluded from consideration in the results. The precise contribution of background events is difficult to estimate. If one accepts all the events as plotted in MGD Fig. 4, then our Fig. 1 is obtained, which corresponds to MGD Fig. 5 (b).

As suggested by Eq. I of MGD,

$$N(p_x)\, dp_x = e^{-p_x^2/p_0^2}\, dp_x, \qquad \text{I. MGD}$$

the momentum distribution can also be obtained from either the p_x or p_z components separately. In Fig. 2 is plotted the number of components of value p_x per unit p_x, and similarly for p_z. The full line, which represents a Gaussian distribution falling to 1/e at 22 Mev, is in good agreement with the data in these figures, considering the sizable statistical uncertainties.

When one has decided to fit the distribution with a Gaussian and desires to fix the value of p_0^2, which corresponds to the energy at which the distribution falls to 1/e, the following formulas may be used. The mean square value of a single component of momentum is

$$\overline{p_x^2} = \frac{\int_0^\infty p_x^2 e^{-p_x^2/p_0^2}\, dp_x}{\int_0^\infty e^{-p_x^2/p_0^2}\, dp_x} = \frac{1}{2} p_0^2 , \qquad \text{I}$$

and the mean square value of the resultant momentum in the xz plane, $p_{xz}^2 = p_x^2 + p_z^2$, is

$$\overline{p_{xz}^2} = \frac{\int_0^\infty p_{xz}^2 e^{-p_{xz}^2/p_0^2}\, 2\pi p_{xz}\, dp_{xz}}{\int_0^\infty e^{-p_{xz}^2/p_0^2}\, 2\pi p_{xz}\, dp_{xz}} = p_0^2 , \qquad \text{II}$$

and of course the mean square value of the three-dimensional momentum p is

$$\overline{p^2} = \frac{\int_0^\infty p^2 e^{-p^2/p_0^2}\, 4\pi p^2\, dp}{\int_0^\infty e^{-p^2/p_0^2}\, 4\pi p^2\, dp} = \frac{3}{2} p_0^2 \qquad \text{III}$$

As computed numerically from MGD Fig. 4, the values are $\overline{p_x^2}/2M$ = 11.1 Mev (corresponding to $p_0^2/2M$ = 22.2 Mev), $\overline{p^2}/2M$ = 10.8 Mev (corresponding to $p_0^2/2M$ = 21.6 Mev), and $\overline{p_{xz}^2}/2M$ = 22.1 Mev (corresponding to $p_0^2/2M$ = 22.1 Mev). Since p_x and p_z have been computed from the observed emulsion tracks by different methods of analysis, the agreement of the above results is a good check of internal consistency in their experiment. The close agreement of these independent calculations of $p_0^2/2M$ is a strong suggestion that most of the data points are not due to background events, which would make large random contributions to the averaging.

The nuclear internal momentum distribution is frequently used in interpreting other experiments. The use of a Gaussian distribution with a 1/e value of $p_0^2/2M = 22$ Mev and thus a mean energy of 33 Mev is suggested by our interpretation of MGD's results, the experiments by Wilcox and Moyer (1955) on quasi-elastic scattering, and the experiments by Wattenberg et al. (1956) on photodisintegration of quasi deuterons. The degenerate Fermi distribution,

$$N(p) = \text{const for } p^2/2M \leq E_{Fermi}; \quad N(p) = 0 \text{ for } p^2/2M > E_{Fermi}$$

has often been used for this purpose with a Fermi energy of about 21 Mev, partly because of the early theoretical suggestion that nucleons in a nucleus behave like a Fermi gas (Bethe, 1937; Fermi, 1949), and partly for simplicity in performing numerical computations. The use of this distribution could lead to serious error in considering threshold and other processes in which the high components of momentum are important. We can note that the mean energy of a Fermi distribution with Fermi energy of 21 Mev is only 12.6 Mev, and also that 58% of the nucleons in the above Gaussian distribution have energy greater than 21 Mev. As pointed out by MGD, the work of Selove (1956) has indicated that a momentum distribution described by a single Gaussian may underestimate the very high components. However, the distribution proposed by Selove would give a mean energy of 60 Mev, and therefore an appropriate cutoff would have to be introduced.

We wish to thank Drs. McEwen, Gibson, and Duke for communicating their results before publication.

This work was done under the auspices of the U.S. Atomic Energy Commission.

REFERENCES

Bethe, H., 1937 Revs. Modern Phys. 9, 85.

Fermi, E. 1949 Nuclear Physics, Univ. of Chicago Press, 159.

McEwen, J.G., Gibson, W.M., and Duke, P.J. 1956 to be published.

Selove, W. 1956 Phys. Rev. 101, 231.

Wattenberg, A., Odian, A., Stein, P.C., and Weinstein, R. 1956 Bull. Amer. Phys. Soc. II-1, 170.

Wilcox, J.M. and Moyer, B.J. 1955 Phys. Rev. 99, 875.

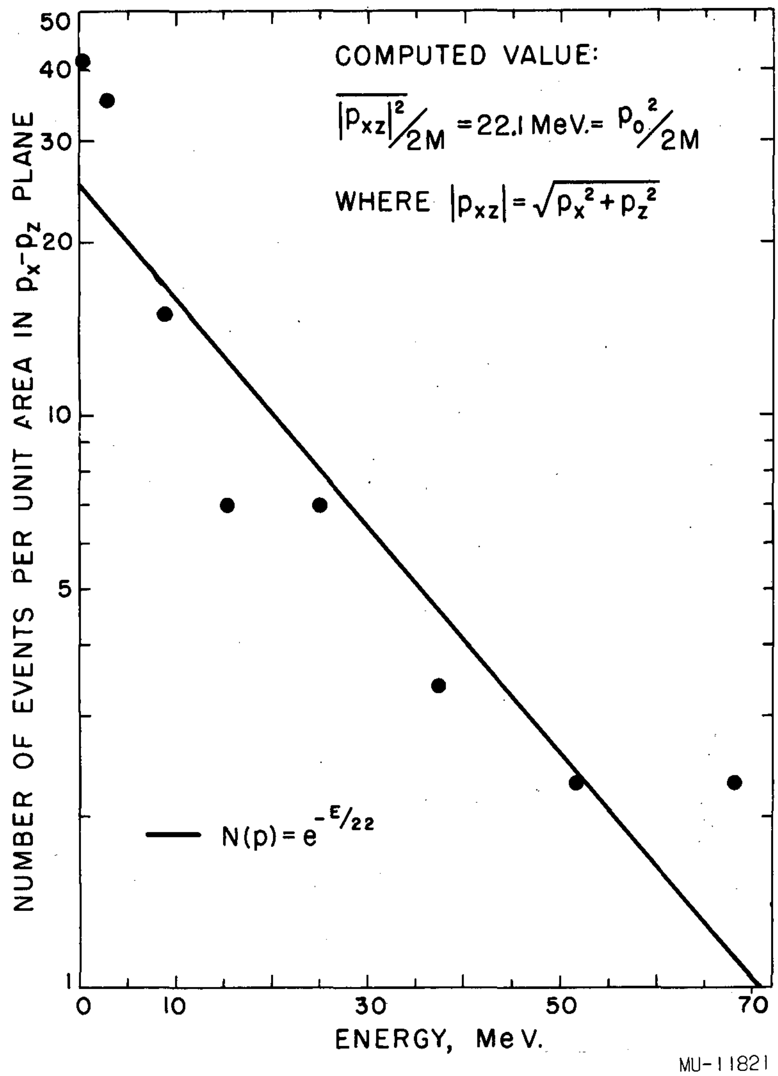

Fig. 1. The number of points per unit area of MGD Figure 4. All points have been included. See MGD for probable errors. The full line is a Gaussian distribution falling to 1/e at 22 Mev.

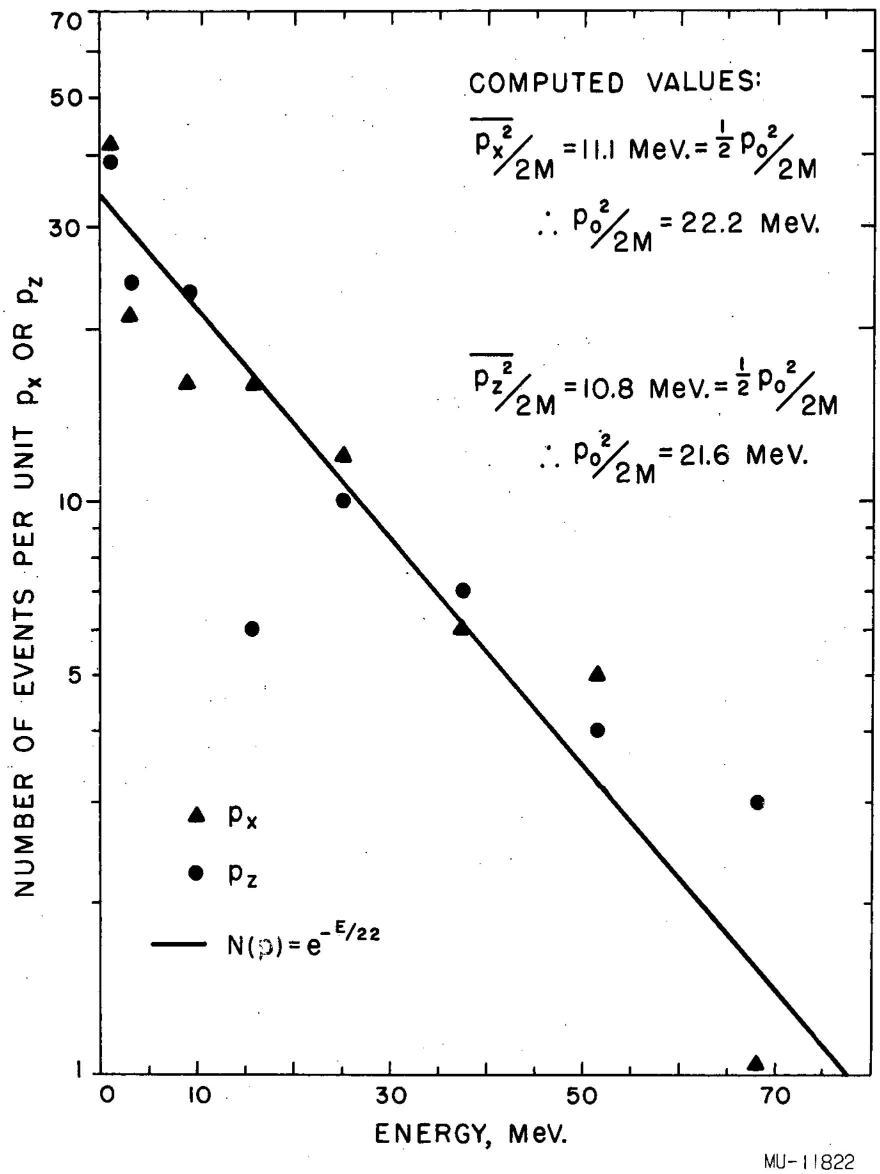

Fig. 2. The number of p_x components per unit p_x, and the number of p_z components per unit p_z. Plotted from MGD Figure 4. See MGD for probable errors. The full line is a Gaussian distribution falling to 1/e at 22 Mev.