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Understanding the Process and Impacts of Price Bargaining and Contracting in High-Tech Supply Chains: A Combination of Empirical and Theoretical Analysis

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A dissertation submitted in partial satisfaction of the

requirements for the degree of Doctor of Philosophy

in Management

by

Wei Zhang

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ABSTRACT OF DISSERTATION

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Doctor of Philosophy in Management

University of California, Los Angeles, 2015

Professor Reza Ahmadi, Chair

Technology advancements in high-tech industries have been the drivers for global economic growth and value creation over the past several decades. In these fast-moving and competitive markets, the process of pricing is complicated and its impacts are multifold. In my dissertation, I study three pricing-related phenomena observed in the semiconductor industry: non-monotonic price-quantity relationship, delayed agreement in negotiation, and price-flexibility-dependent purchase pattern. For each topic, I first analyze a large sales data set obtained from a major microprocessor company to establish the phenomenon, and then I build a theoretical model to explore the underlying rationale and to generate prescriptive insights.

The dissertation of Wei Zhang is approved.

Charles Corbett

Sriram Dasu

Rakesh Sarin

Reza Ahmadi, Committee Chair

University of California, Los Angeles

2015

To my beloved wife and daughter.

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VITA

Wei Zhang obtained his Bachelor's degree in Management and Master's degree in Management from Tsinghua University, Beijing, P.R.China. In 2010, he entered the UCLA Anderson School of Management as a doctoral student. In 2013, he published his master's thesis, titled Contracts for Changing Times: Sourcing with Raw Material Price Volatility and Information Asymmetry, in *Manufacturing & Service Operations Management*.

1 Higher Prices for Larger Quantities? Non-Monotonic Price-Quantity Relations in B2B Markets

Abstract

We study a microprocessor company selling short-life-cycle products to a set of buyers that includes large consumer electronic goods manufacturers. The seller has a limited capacity for each product and negotiates with each buyer for the price. Although purchase quantity is a major factor that influences negotiations, our analysis of their data reveals a non-monotonic price-quantity relationship, namely, that larger purchases do not always result in bigger discounts. While existing theories cannot explain this pattern, we provide a model that is based on practices of this company, which shows that the non-monotonicity is rooted in how sellers value capacity. Sellers have to consider the possibility of selling to other buyers while negotiating. In this setting, expected profit from selling to subsequent buyers need not be concave in the remaining capacity. The value of residual capacity may be initially convex and then concave. Such a value function is sufficient to ensure a non-monotonic price-quantity relationship. We briefly discuss how to determine capacity rationing and the posted price, which also influences negotiations, and how to avoid errors that can stem from assuming an incorrect price-quantity relationship.

[*Keywords*: non-monotonicity; bargaining; B2B market; data-driven; OM-economics interface]

1.1 Introduction

Price and quantity are the most basic economic concepts and the relationship between price and quantity at the market level has been well studied by economists. However, at a firm level, it is still unclear how the price-quantity relation develops in many situations. While conventional wisdom suggests that larger buyers get greater discounts, our empirical observations raise doubts.

This study is based on our interactions with managers of a large semiconductor company. In addition to informing us about industry and firm practices, they provided us with a sales database that spans a three-year period and offered us the opportunity to look into the price-quantity relation at a firm level. We analyze the sales data from fixed-price contracts and observe that, although total payment increases with total quantity in almost all cases, the discount received by a buyer is statistically a non-monotonic function of the buyer's demand share (or relative size) for a product. Specifically, the discount increases with demand share for small quantities. However, as demand share increases, the discount decreases and then increases again. In brief, we observe an N-shaped discount curve.

Industry and Firm Practices

Market Structure. The microprocessor market is intensely competitive, with rapid technological advancements, short product life cycles, and regular pricing activities. Many competing sellers such as Intel, Nvidia, and Advanced Micro Devices (AMD), sell multiple product lines primarily to original equipment manufacturers (OEMs or buyers), such as Hewlett-Packard (HP), Lenovo, and Dell.

Capacity Inflexibility. For sellers to remain competitive, they need production capacity with up-to-date process technology, which requires heavy capital investments. Some sellers like Intel (which is known as an integrated device manufacturer) manufacture products in house, while others like AMD (known as a fabless company) only focus on product design and outsource production to third-party foundries. In both cases, because semiconductor manufacturing facilities are costly and construction lead times are long, capacities are inflexible during a selling season. Sellers allocate available capacity to product lines based on demand forecasts. These forecasts also represent sales commitments from product line managers. Once a commitment is made, these allocations are expensive to change for a number of technical reasons. First, while capacity configuration at a manufacturing facility can be altered, doing so disrupts flows in the manufacturing facility and causes increased inventory and manufacturing cycle times (Karabuk and Wu 2003). Second, semiconductor manufacturing entails significant learning and it takes time for yields to ramp up and for quality to improve. As a result, when facing a supply shortage, it is not only costly but also risky to seek an alternative source. Hence, it is important for sellers' product managers to provide accurate forecasts and to sell according to allocated capacities.

Price Negotiation. Although each product has a posted price, the final price for each buyer is usually set through negotiations. Major buyers are sophisticated, drive hard bargains, and often enjoy higher annual revenues than sellers (Cooper 2008). Buyers know that the marginal production cost of microprocessors is low and sellers are eager to discount prices to fully utilize their capacities. Moreover, buyers can allocate their business among competing sellers. Lacking full pricing power, sellers are unable to use pricing strategies, such as take-it-or-leave-it price schedules or a menu of contracts, and have to engage in negotiations. Once a price is settled, the duration of contract can vary for different buyers and products; price renegotiations happen frequently but not in all cases. In our data set, approximately 45% of the purchases did not involve any renegotiation.

Procurement Quantity. The purchase quantity, however, is normally not a term for negotiation. To produce their products, buyers need other inputs from different suppliers, so it is costly to manipulate purchase quantities from a seller once production plans have been made. In principle, buyers can allocate their requirements among alternative semiconductor firms, but products offered by different sellers are not perfect substitutes. Products differ in technical features and in some cases, the seller's brand image in the consumer market may matter. As a result, buyers' procurement managers prefer to stick to their internal production plans and procure the desired quantity at the best possible price. In summary, buyers determine their purchase quantities based on their production plans prior to negotiating with suppliers, and incur costs if they are unable to procure these amounts and must switch to an alternative seller. Their contracts with sellers typically do not include any commitment or requirement for minimum product purchases.

Technology Upgrades. Another aspect of this industry with a major impact on negotiations is the risk of obsolescence. Sellers are aware that technological advancements from rivals can cause a rapid decline in demand for existing products. Although they are aware of development cycles in the industry and can anticipate when rivals will introduce products, they must constantly consider the likelihood of a demand shock and the possibility of having to salvage inventories (Karabuk and Wu 2003).

In this chapter, we investigate the price-quantity relation in business-to-business (B2B) markets where the product life cycle is short, capacity is inflexible, and prices are set through *one-shot* negotiations. Existing literature provides no clear answers to how quantity affects price in such a setting. Although a vast literature has been built on the Nash bargaining model, it is normally assumed that the size of the pie and the outside options are fixed. This assumption is inappropriate when a seller with a limited capacity sequentially negotiates with a group of buyers. In this dynamic setting, a buyer's purchase quantity influences the outcomes of bargains with subsequent buyers due to changes in residual capacity and information about demand. Thus, the size of the pie and outside options are functions of the buyer's purchase quantity. The negotiated price, as a result, can be a complicated function of purchase quantity.

The sales data we obtained from a major microprocessor firm reveals some compelling instances in which larger-quantity buyers receive higher prices. Driven by this counterintuitive observation, we used a set of linear and nonlinear regression models to control other possible influences on price and still arrive at a non-monotonic relation between price and quantity. We then tested the robustness of the empirical pattern. To gain a deeper understanding of the observed phenomenon as well as provide a theoretical justification, we developed an analytical model that is largely based on the practices of the firm we are studying. Our model suggests that the non-monotonic price-quantity relation is rooted in how the seller values the remaining capacity. In particular, a value function for the remaining capacity that is first convex and then concave is sufficient to lead to a non-monotonic discount curve for a buyer. Further, we show that such a value function can arise quite naturally in practice. Finally, using simulations, we show that the theoretical model can yield the price-quantity curves found in the data set.

Knowledge gleaned on the price-quantity relation will be useful in B2B markets such as the market studied here, where capacity is inflexible and prices are negotiated. Due to the impact of one transaction on subsequent transactions, it is important for the seller to control the capacity allocated to each buyer, if possible. To optimize the trade-off between the profit from the current buyer and that from future buyers, a good understanding of the price-quantity relation is necessary.

This knowledge will also help the seller optimize the posted price, which balances the profit between buyers who choose to take the price and those who choose to bargain.

The rest of this chapter is organized in the following way. We present a brief literature review in Section 1.2. In Section 1.3, we show our empirical observation through linear and nonlinear regressions. We then build a model in Section 4 and analyze the problem in Section 1.5. We discuss the managerial implications of our finding in Section 1.6 and conclude in Section 1.7. All the proofs are in the Appendix.

1.2 Related Literature

Our work is related to B2B pricing, bargaining, and revenue management. In terms of the price-quantity relation, different views exist within the literature. In the operations manangement and marketing literature, quantity-discount pricing policy has been widely studied as a channel or as a supply chain coordination tool (e.g., Kohli and Park 1989; Weng 1995). In these papers, it is assumed that one party will offer the contract in a take-it-or-leave-it fashion and buyers with greater demand receive lower prices. Assuming that one party has full bargaining power simplifies the analysis, but it also ignores prevailing practices in which buyers negotiate. Our study assumes that the seller does not have the power to dictate the price for every buyer.

In the economics literature, Snyder (1998) and Chipty and Snyder (1999) discussed the impact of buyer demand size on price discount. Snyder (1998) showed that when many suppliers compete to sell to one buyer at a time in a repeated game, the price offered to the seller in equilibrium initially increases with buyer size and then decreases with buyer size. However, the result requires that suppliers cooperate and buyers appear sequentially over an infinite horizon, which are both very strong assumptions in supply chains. More importantly, our data exhibits a more complicated price pattern that is not explained by their model. In a very different setting from ours, Chipty and Snyder (1999) showed that a merger enhances (worsens) buyers' bargaining position if the supplier's payoff function is concave (convex) in total transaction size.

Other studies on B2B bargaining have assumed that the size of the pie is given, and researchers have explored how the pie is allocated among channel or supply chain members. While Dukes et al. (2006) and Lovejoy (2010) focused on the impact of channel or chain structure, several scholars consider the impact of bargaining sequence and coalitions. In an assembly chain setting, Nagarajan and Bassok (2008) considered suppliers who form multilateral bargaining coalitions and compete for a position in the bargaining sequence. Nagarajan and Sosic (2008) studied the stability of coalition in assembly models. We supplement this branch of the literature by considering a seller that sequentially negotiates with a group of buyers, by investigating the impact of a buyer's relative quantity size both empirically and analytically, and by discussing the plausibility and possible implications of the non-monotonic discount curve.

Our research is also related to revenue management. Kuo et al. (2011) was the first paper to study revenue management for limited inventories when buyers negotiate. They considered a dynamic setting with fixed compositions of price-takers and bargainers and assumed that each buyer only buys one unit of the product and that the posted price is updated frequently. The authors characterized the optimal posted price and the resulting negotiation outcome as a function of inventory and time. They also showed that negotiation is an effective tool to achieve price discrimination. In contrast, our study considers a dynamic, capacity-rationing problem in a B2B market in which buyers request different quantities and quantities influence prices. Our work is also related to research on dynamic and stochastic knapsack problems that study the optimal admission or pricing policies with limited capacity. While early studies such as Gallego and van Ryzin (1994) and Kleywegt and Papastavrou (1998) showed that the optimal expected revenue is concave in capacity if all demands require the same amount of resources, Kleywegt and Papastavrou (2001) showed that concavity does not hold in general when demands are heterogeneous, which provides support for our analysis. However, Kleywegt and Papastavrou (2001) focused on characterizing the conditions under which concavity holds, and we focus on characterizing the property of the value function under which the price-quantity relation is non-monotonic.

To summarize, our study makes the following contributions. First, we provide an empirical analysis that reveals the existence of a non-monotonic price-quantity relation in B2B markets. Second, we construct an analytical framework to investigate this phenomenon and find a plausible explanation. Third, we show a simple and sufficient condition for the price-quantity relation to be non-monotonic.

1.3 Empirical Observation and Analysis

The data provided by a major global semiconductor company for use in this study encompasses 3,826 products and 251 buyers over a three-year period. Each record in the data set consists of customer ID, product ID, product category, product brand (subcategory), sales territory, bill quantity, bill value in USD, unit price, and date of transaction. The products sold include central processing units (CPUs), graphics processing units (GPUs), and embedded chips, among others.

1.3.1 Preliminary Observations

Because large orders normally provide economies of scale and are often placed by big buyers, we generally expect buyers who buy large quantities to receive lower prices than those who buy less. As we examined the data, though, we observed some larger-quantity buyers paying higher prices. In particular, if we ranked buyers for a product according to their total purchase quantities, we found that in about 26% of the cases, a buyer pays a higher average price than a neighboring, smaller-quantity buyer. These pricing "anomalies" may lead to overall non-monotonic price patterns for some products. We selected five evident examples and summarized them in Table 1.1.

For each product in Table 1.1, we sorted the buyers about equally into three groups—small, medium, and large—according to the total amount of units they purchased. We then calculated the average price each group received by summing the total purchase value and dividing it by the total quantity. Two interesting observations immediately emerged. First, the average price received by those purchasing a medium quantity is less than that obtained by the other two groups.

Table 1.1: Quantity-Weighted Average Price for Three Customer Segments

Product	Category	Number of Customers	Lifespan (year)	Small Amt. Avg. Price	Medium Amt. Avg. Price	Large Amt. Avg. Price
1	Desktop CPU	40	2.77	\$ 58.38	\$ 55.32	\$ 58.78
2	Desktop CPU	5	0.98	\$ 29.13	\$ 27.45	\$ 45.01
3	Desktop CPU	6	0.90	\$ 27.54	\$ 25.33	\$ 28.46
4	Desktop CPU	11	0.86	\$ 92.50	\$ 91.06	\$ 93.43
5	Memory	9	0.87	\$ 2.59	\$ 2.47	\$ 2.51

Second, the average price paid by large-quantity buyers was greater than that paid by either of the other groups, with the exception of memory chips. These "anomalies" motivated us to conduct a more systematic analysis of the marginal relationship between price and quantity to see if the non-monotonicity generally exists when we control for most other factors.

1.3.2 Data Preparation

Fixed-Price Contracts

As a buyer normally purchases a product through multiple transactions over time, the price may be renegotiated. In this chapter, we focus on purchases in which prices are fixed over the entire product life cycle. We say that such a set of transactions are made under a *fixed-price* contract. The analysis for *fixed-price* contracts or *one-shot* price bargaining is simpler than that for repeated negotiations.

Let I and J be the indices of buyer and product, respectively. Let $\mathcal{T}_{ij}$ be the set of dates at which buyer i purchased product j. Let q_{ijt} and p_{ijt} denote the transaction quantity and price for customer i and product j at time $t \in \mathcal{T}_{ij}$. We define an *instance* θ_{ij} as the set of transactions related to buyer $i \in I$ and product $j \in J$; i.e., $\theta_{ij} := \{(t, q_{ijt}, p_{ijt}) : t \in \mathcal{T}_{ij}\}$. As stated earlier, in this industry it is a common practice for the buyer to determine the purchase quantity prior to entering a negotiation. For fixed-price contracts, we have $p_{ijt} = p_{ij}$ for all $t \in \mathcal{T}_{ij}$, and the life-cycle purchase quantity is the target for the price bargaining. Hence, we focus on the relationship between the total purchase quantity, $TQ_{ij} = \sum_{t \in \mathcal{T}_{ij}} q_{ijt}$, and the fixed price p_{ij} for such instances. Of course, transaction-level data may contain information about factors that impact negotiations, such as when a purchase starts and how long it lasts. We therefore use the transaction-level data to construct measures for these factors.

Normalization

Widely varying prices and market sizes for different products compel us to normalize the data to the same scale. Prices of the 425 brands (or product subcategories) range from several dollars to more than $100 per unit. Thus, an observed price that is the lowest for one product may be higher than any observed price for another product. For that reason, in place of the price and total quantity, we use two ratio metrics: (1) effective discount rate (ED) and (2) a power transformation of demand share ($PTDS$). We define

$$
\begin{aligned}
ED_{ij} &:= 1 - p_{ij} / \max_{i' \in I, t \in \mathcal{T}} p_{i'jt}; && (1.1) \\
PTDS_{ij} &:= \left(TQ_{ij} / \sum_{i' \in I} TQ_{i'j} \right)^{\gamma}, && (1.2)
\end{aligned}
$$

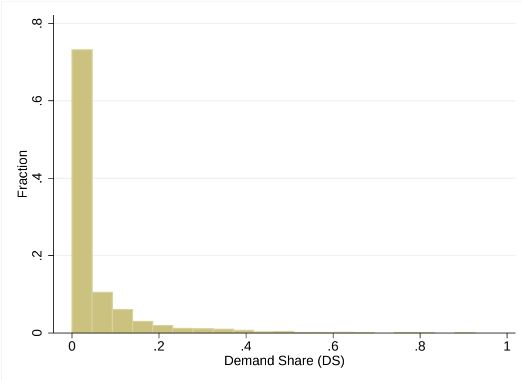
where $\mathcal{T} := \cup \mathcal{T}_{ij}$ and $\gamma \in (0, 1)$. Both variables have the range $[0, 1]$. ED is a measure of price level relative to the highest price ever paid for the product. Note that the posted prices of the seller who offers this data set are very stable, usually lasting for more than a year, so for a fixed-price contract the nominal discount rate is almost constant over time and ED is very close or equal to the nominal discount rate. $PTDS$ is a monotone transformation of demand share (DS), a measure of total quantity relative to the market size of the product. The advantage of DS is that it simultaneously controls the mean and the variation across different products.[1] However, the demand shares in a large amount of instances are concentrated around zero (as shown in Fig. 1.1 on the left) due to the 20-80 rule: 80% of customers contribute only 20% of sales. Therefore, it is difficult to examine how quantity affects price discount in the majority of instances if we use DS. To avoid such a shortcoming, we take a power transformation of DS so that its empirical distribution is more spread out (as shown in Fig. 1.1 on the right) and the range $[0, 1]$ is preserved. Note that maintaining the unit range makes it convenient to interpret the results. As shown in Table 1.2, the distribution is the most spread out when γ takes a value from 0.25 to 0.35, but $\gamma = 0.25$ leads to a distribution that is more normal. In this chapter, we focus on the fourth-root transformation (i.e., $\gamma = 0.25$). Later, to check the robustness, we will also show the results for γ =0.15, 0.2, 0.3, and 0.35. Thus, our objective reduces to identifying the relation between ED_{ij} and $PTDS_{ij}$ while controlling for other factors.

Data Filtering

The data set corresponds to a three-year time period from January 1, 2009 to March 25, 2012. Instances that started prior to January 1, 2009, and those that lasted beyond March 25, 2012, have missing data (or are truncated). This truncation effect may cause a negative correlation between

[1] Note that how the demand share of a customer evolves over time is irrelevant for our analysis. We only use the *ex-post* demand share as a normalized quantity, which is constant over time.

Figure 1.1: Histograms for the selected subset of fixed-price contracts.



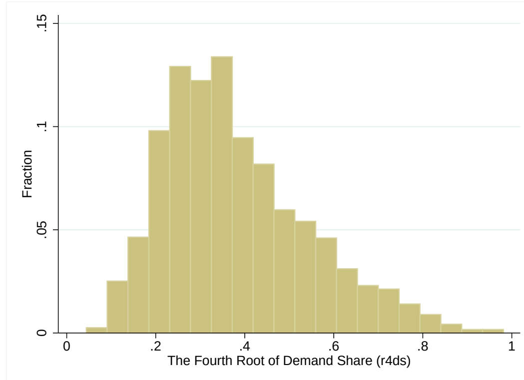


Table 1.2: Measuring the Distributions of Power-Transformed Demand Share

	$DS^{0.15}$	$DS^{0.20}$	$DS^{0.25}$	$DS^{0.30}$	$DS^{0.35}$	$DS^{0.40}$	$DS^{0.45}$
St.Dev.	0.1409	0.1556	**0.1624**	**0.1642**	**0.1626**	0.1589	0.1539
W	0.9873	0.9743	0.9572	0.9365	0.9131	0.8875	0.8604

Note. The Shapiro-Wilk W statistic measures the straightness of the normal probability plot of a variable; larger values of W indicate better normality.

price and quantity. Prices in the microprocessor market are decreasing over time, so instances that started early and were truncated will appear to have smaller total quantities with higher prices than subsequent instances. To mitigate this truncation effect, we focus on instances with an observed starting date at least one quarter later than January 1, 2009, and an observed ending date at least one quarter earlier than March 25, 2012. There are 6,573 instances (about 53%) that satisfy such criteria. Furthermore, to focus on regular purchases but not transactions for one-time substitutions or downgrading, which are entailed by other purchases and are different in nature, we drop another 312 instances that have only one purchase record. Finally, products that have an extremely small number of buyers are often customized and likely to follow a different selling process. In addition, such products tend to have extreme-demand-share buyers as well as narrow price dispersions, which could create a false correlation that large buyers get small discounts. Hence, to avoid this lack of reference, we drop another 1,551 instances and consider products that have more than three buyers. In this way, we obtain a subset of the data with 2,346 fixed-price instances and 2,364 price-renegotiated instances.

Table 1.3: Summary Statistics for the Selected Instances

	Fixed-Price Instances				Price-Renegotiated Instances			
Variables	Mean	S.D.	Min	Max	Mean	S.D.	Min	Max
ED	.1126	.1781	0	.9986	.1956	.1735	4.1e-5	.9986
r3ds	.2910	.1635	.0162	.9751	.3550	.1946	.0287	.9969
r4ds	.3818	.1624	.0440	.9811	.4441	.1861	.0679	.9977
r5ds	.4564	.1556	.0822	.9848	.5151	.1739	.1163	.9981
Cbase	19.66	13.43	4	48	21.73	14.80	4	48
TSQ	8.37e5	2.35e6	826	3.06e7	8.56e5	2.11e6	826	2.31e7
Herf	.2593	.1787	.0561	.9815	.2269	.1579	.0561	.9936
lndod	3.9237	1.8581	0	6.9527	3.0569	1.8986	0	6.7286
lndrt	4.5246	1.3531	0	6.8977	5.6237	.7545	2.0794	6.8977
M3	.2928	.4552	0	1	.3054	.4607	0	1
CapL	.4614	.3052	.1761	1.176	.6058	.3411	.1765	1.1765
Cshr	.0516	.1038	2.24e-6	.7897	.0924	.1581	8.19e-6	.7897
Vrate	.3112	.2506	0	.9375	.6823	.2291	.0541	1
N	2,346				2,364			

1.3.3 Variables

Aside from the demand share, other variables may also influence a customer's discount. According to the generalized Nash bargaining model (Nash 1950; Roth and Malouf 1979), these variables fall into three broad categories: the seller's outside options, the buyer's outside options, and their respective bargaining powers. As far as we can imagine, the seller's outside options are affected by production cost, salvage value, buyer-side competition, time of purchase, capacity or inventory level, and demand uncertainty. The buyer's outside options are affected by the value of adopting a different product, seller-side competition, posted price, and time of purchase. Bargaining powers are affected by the value of the business relationship, the bargaining skills of salespersons and procurement managers, and the buyer's reputation for committing to a forecast. In the following, we explain the variables included in our regression. Table 1.3 provides the summary statistics for the portion of data we use, and Table 1.4 shows the correlation among the variables.

Power transformation of Demand Share. We first focus on the relationship between *ED* and the fourth root of demand share (*r4ds*). Later, we will consider other power transformations of demand share (e.g., $DS^{0.15}$, $DS^{0.2}$, $DS^{0.3}$, and $DS^{0.35}$) to check the robustness.

Cbase. This variable counts *the total number of buyers* for a product, and is thus a measure of a product's popularity and the buyer-side competition.

TSQ. The total sales quantity of a product, which also implies the popularity of the product.

Herf. This is the *Herfindahl Index* for the demand structure of a product, which measures the degree of demand concentration. It is calculated as the square root of the sum of the square of demand shares across all the buyers (Weinstock 1982).

lndod. The discount received by a buyer for a product is related to the time when the buyer starts to purchase, because effective prices (or price-performance ratio) in the semiconductor industry are decreasing over time in general. The later a buyer arrives for a product, the greater discount (relative to the highest price) he may obtain due to better outside options. Hence, to capture such a time effect, we use the logarithm of *days of delay*, which is calculated as the difference in number of days between the starting date of an instance and the first date that the product was ever purchased. Note that, as relative measures, *lndod* and *ED* are compatible.[2] In addition, *lndod* is also a measure of demand uncertainty, because uncertainty is resolved over time.

lndrt. The discount may be also related to the rate of purchase given the same total quantity, so we also control the logarithm of *duration* of an instance. It is calculated as the number of days between the first date and the last date of an instance. We can see that the life span of an instance in the data set is fairly short, with an average duration of 232 days.

M3. Another dimension of the time of purchase is related to the seller's financial cycle. It is well known that the *end-of-quarter effect* may bring buyers an edge in the bargaining. We introduce *M3* as a binary variable with a value equal to 1 if the date of the price negotiation is in the third month of a quarter and 0 otherwise.

CapL. The remaining capacity level at the time of price negotiation is a consideration for both the seller and the buyer. However, we do not have information about the capacity level. We approximate the total capacity level using the total sales of a product divided by the semiconductor industry capacity-utilization rate (about 85%),[3] and the available capacity level for a buyer using the difference between the total capacity level and the cumulative contracted sales prior to this

[2] Another candidate may be the days of delay relative to the introduction date of a product. Compare two scenarios. In scenario I, the first customer of a product delays his purchase 0 days and receives the highest price among all the customers. In scenario II, the first customer delays his purchase 100 days and also receives the highest price among all the customers. Other customers delay their purchases one day after the first customer's purchase, and all receive the same effective discount. However, the absolute delay for other customers is one day in scenario I and 101 days in scenario II, disproving the effectiveness of this alternative measure.

[3] http://www.semiconductors.org/industry_statistics/industry_statistics/, accessed March 2015. We obtain very similar results if we use random utilization rates (e.g., a normally distributed random variable with mean 0.85 and standard deviation 0.1, capped by 0 and 1).

buyer. We then use $CapL$ =(available capacity level)/(total sales) as a control variable.

Cshr. A buyer that contributes to a large portion of the seller's overall sales can have significant bargaining power. To capture the bargaining power of a buyer in this regard, we calculate *Cshr* (i.e., customer share) as 100 times the total quantity purchased by a buyer over the observed period divided by the seller's total sales volume across all products and the observed period. This measures the value of a buyer to the seller. Note that using the total purchase value as a control variable may cause an issue of endogeneity as the value depends on the price discount.

Vrate. Since we use the highest price paid for a product as the approximation of the posted price, the larger the price variation of a product, the greater the computed *ED*. Hence, we should control the price variation of a product. However, prices depend on discounts received by buyers. To avoid the endogeneity issue, we use the *fraction of price-renegotiated instances* to measure a product's price variation, given that buyers with renegotiable-price contracts are more likely to get the posted price at the beginning and price variations are larger than with fixed-price contracts. In addition, *Vrate* measures the uncertainty of the product's value, because price renegotiations normally happen when uncertainties are resolved. For fixed-price contracts, discounts are likely to be larger when uncertainties are higher.

Product-line (or brand) fixed effect. To capture product-line-specific impacts such as the seller-side competition, production cost and salvage value, we use binary variables for the major 16 brands that have at least 100 observed instances in the original data set. It is important to note that the seller's salespeople are organized by product lines. Hence, negotiated prices of a product are all subject to the same impact from the bargaining ability of a salesperson or a group of salespeople. In other words, the impact of salesperson ability is product-line-specific and thus can be captured by the product-line fixed effect.

Buyer fixed effect. Apart from purchasing value, a buyer's bargaining power is also affected by unobservable factors, such as the experience of the procurement manager and the reputation for honoring a commitment. Hence, we use binary variables to control the buyer fixed effect for the 10 major buyers in terms of total purchase value with the seller and use others as the reference.

Quarter fixed effect. To capture the industry dynamics that are cyclical within a financial year, we use the first quarter as the reference and binary variables for the other three quarters.

Location fixed effect. The degree of market competition on both the buyer and seller sides may

Table 1.4: Correlation Matrix for the Selected Fixed-Price Instances

	ED	*r4ds*	*Cbase*	*TSQ*	*Herf*	*lndod*	*lndrt*	*M3*	*CapL*	*Cshr*	*Vrate*
ED	1.00										
r4ds	0.06	1.00									
Cbase	-0.22	-0.42	1.00								
TSQ	0.02	-0.15	0.10	1.00							
Herf	0.15	0.07	-0.63	-0.09	1.00						
lndod	0.30	-0.13	-0.22	0.07	0.24	1.00					
lndrt	-0.16	0.14	-0.01	0.06	0.00	-0.11	1.00				
M3	0.03	0.08	-0.13	-0.01	0.05	0.02	-0.09	1.00			
CapL	-0.16	0.38	-0.07	-0.07	-0.11	-0.65	0.12	-0.00	1.00		
Cshr	0.22	0.29	-0.32	-0.01	0.20	-0.02	0.10	0.05	0.13	1.00	
Vrate	0.36	-0.18	0.05	-0.02	-0.04	0.24	-0.13	0.01	-0.21	0.05	1.00

depend on the location. We use binary variables for nine of the ten recorded sales territories, such as greater China and North America. Additionally, location may also be an indicator of cost level and demand uncertainty.

Interaction effect. The impact of capacity level may interact with time elapsed. The likelihood of a technology shock occurring increases over time after a product is introduced; once a shock occurs, the seller may have to salvage the remaining capacity. Hence, the capacity has less and less value as time elapses and we thus include the interaction between *CapL* and *lndod*.

Though we try to control for as many variables as possible, we still confront the "omitted variable" problem due to a lack of information. Thus, consistent estimators can be obtained only when the omitted variables are uncorrelated with our regressors. The factors we do not control for here are the net cost of switching to an alternative product and contract terms other than price and quantity for a buyer. Later, we will show that under certain mild assumptions, the estimated coefficients are just the scaled true marginal effects when these two factors are relevant but missing. Hence, the shape of the price-quantity relationship will be preserved. Last, note that we consider only the instances with one-shot bargaining (or fixed-price contract) for new products, and thus it is reasonable not to consider any reference effect from previous prices or discounts.

1.3.4 Regression Analysis

In this section, we try to identify the empirical relationship between the effective discount and

Table 1.5: Summary Statistics and Regression Results for Demand Share Segments

Model	Segmt.	Range of $r4ds$	Summary Stats. Mean	S.D.	Obs.	Regression Coef.	Robust S.E.	P value
(i)	1	0˜0.2	.1581	.0313	235	-	-	-
	2	0.2˜0.35	.2772	.0423	916	.0141	.0097	0.149
	3	0.35˜0.5	.4142	.0433	683	.0245	.0111	0.028
	4	0.5˜0.65	.5669	.0406	335	.0428	.0153	0.005
	5	0.65˜0.8	.7187	.0439	145	.0348	.0178	0.052
	6	0.8˜1	.8625	.0459	32	.0933	.0312	0.003
(ii)	1	0˜0.15	.1226	.0199	85	-	-	-
	2	0.15˜0.25	.2087	.0267	423	.0023	.0156	0.885
	3	0.25˜0.35	.2993	.0289	643	.0031	.0159	0.843
	4	0.35˜0.45	.3949	.0293	518	.0171	.0168	0.308
	5	0.45˜0.55	.4965	.0286	291	.0145	.0183	0.429
	6	0.55˜0.65	.5922	.0285	209	.0409	.0206	0.048
	7	0.65˜0.75	.6977	.0303	106	.0194	.0230	0.398
	8	0.75˜0.85	.7905	.0269	55	.0425	.0257	0.098
	9	0.85˜1	.8987	.0374	16	.1223	.0476	0.010

demand share in two steps. The first step is to explore the underlying pattern by segmenting the demand share and computing the average effective discount received by buyers in each segment. Based on the observed pattern, if one exists, we obtain a reasonably well-fitted functional form through piecewise polynomial (spline) regression in the second step. Our analyses in the two steps are both necessary and complementary. The first step provides us with information about the shape of the function, the possible location(s) of the knot(s), and the order of polynomial functions we need. The second step allows us to test the statistical significance of the functional form.

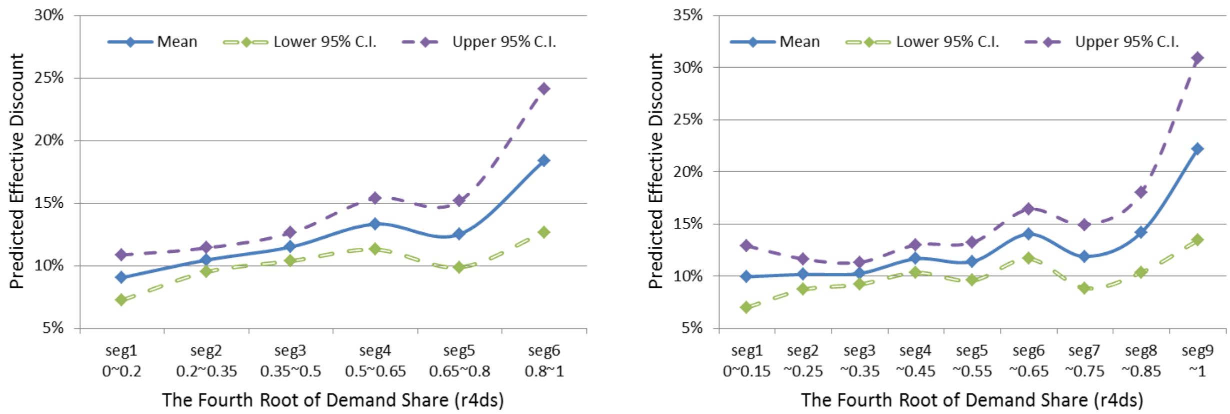
Average Discounts by Segments

For robustness, we consider two different ways of segmentation, the details of which are given in Table 1.5. In model (i), we divide the instances into six segments according to $r4ds$. We use wider ranges for the first and last segments in order to include more instances in the "tails." In model (ii), we use nine segments.

Incorporating the aforementioned variables, we run a regression for each model and the results are summarized in Table 1.5 and Table 1.7.[4] We can see that in both models the marginal impact

[4]We use the *regress* command with the *robust* option in STATA. With the option, STATA estimates the standard errors using the Huber-White sandwich estimators. The produced standard errors (called robust standard errors) can effectively deal with minor problems regarding normality, heteroscedasticity, and some observations that exhibit large residuals. The point estimates of the coefficients are exactly the same as those in ordinary OLS.

Figure 1.2: Average marginal impact of demand share in model (i) and (ii).



Note. In both graphs, all other covariates take their mean values.

of demand share on discount displays a similar non-monotonic pattern. In model (i), the estimated coefficient increases with demand share for the first four segments, then decreases in segment 5, and increases again in segment 6. In model (ii), the coefficient increases until segment 6, then decreases in segment 7, and increases again. These results indicate that the discount is likely to be an N-shaped function of demand share. In Figure 1.2, we plot the average discount received by each segment and the smooth line connecting them. In both graphs, all the other controlled variables take their mean values.

Although the smooth lines look like an "N" in both graphs of Figure 1.2, it is still difficult to tell how significantly the underlying shape is an N based solely on the results we have obtained so far. Rather, what we can learn is that it may be inappropriate to use a simple monotone function or a polynomial function to describe the shape given the irregular pattern; that is, the shape is more likely to be a combination of a monotonically increasing curve and a V-shaped curve, which are easier to fit by polynomial functions separately. In the next step, we propose a piecewise polynomial function to fit the data and test the significance of the shape.

Piecewise Polynomial Regression

To reduce the number of parameters while maintaining adequate flexibility, we use a two-segment, quadratic function with an unknown knot to fit the data in model (iii). Hence, we will let the data decide whether the function is linear or quadratic in each segment and where the two

Table 1.6: Results of the Piecewise Polynomial Regressions

	Model (*iii*)			Model (*iv*)		
	Coef.	Robust S.E.	P value	Coef.	Robust S.E.	P value
a_1	0.2325	0.1042	0.026	0.1030	0.0319	0.001
a_2	0.2956	0.2365	0.212	-	-	-
a_3	-0.4354	0.2093	0.038	-0.3422	0.2001	0.087
a_4	2.0382	0.8610	0.018	1.9251	0.9564	0.044
B	0.5668	0.0309	0.000	0.5794	0.0449	0.000
	Model (*v*)			Model (*vi*)		
	Coef.	Robust S.E.	P value	Coef.	Robust S.E.	P value
a_1	0.1857	0.1101	0.092	0.1355	0.0973	0.164
a_2	0.2477	0.2805	0.377	0.0861	0.1726	0.618
a_3	-0.1252	0.1759	0.477	-0.3958	0.3127	0.206
a_4	0.7575	0.5609	0.177	2.6182	1.4140	0.064
B	0.5050	-	-	0.6286	-	-

smooth lines are connected. We let $r4ds_0 = B$ denote the location of the knot to be estimated, and we run a least-square regression with the following nonlinear model:

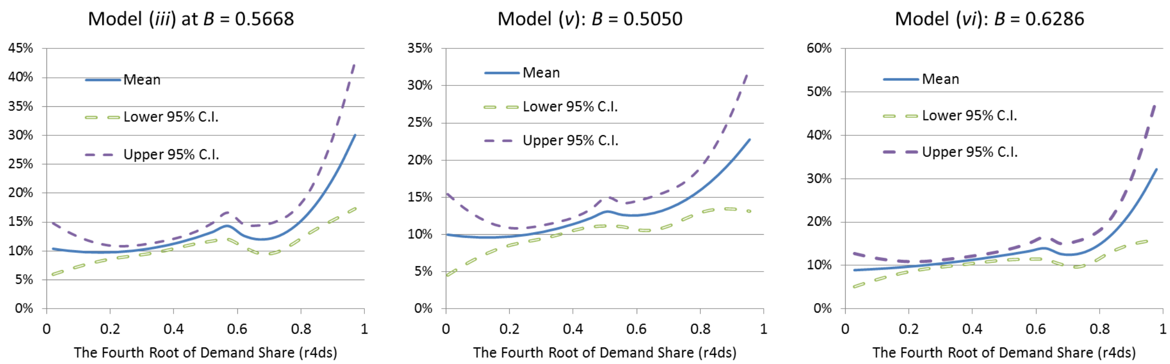
$$ED = a_1 \cdot (r4ds - B)_- + a_2 \cdot (r4ds - B)_-^2 + a_3 \cdot (r4ds - B)_+ + a_4 \cdot (r4ds - B)_+^2 + b'X + \epsilon, \quad (1.3)$$

where $x_- = \min\{x, 0\}$, $x_+ = \max\{x, 0\}$, X is the vector of controlled covariates (including the constant), and ϵ is the error term. We report the estimated parameters, *cons.*, a_1 to a_4, and B, in Table 1.6, and b in Table 1.7.

We can see from Table 1.6 that a_1 is significant (at the 5% level) but a_2 is not, meaning that a linear relationship is significant in the first (left) segment. In addition, both a_3 and a_4 are significant (at the 5% level), meaning that a quadratic relationship is significant in the second (right) segment. Because $B = 0.5668$ is highly significant (at the 0.1% level), the relationship thus cannot be described by a single linear or quadratic function. If we assume in advance in model (*iv*) that the relationship is linear in the first (left) segment and quadratic in the second (right) segment, we will get similar results for all the parameters. Note that the minimum of the quadratic curve is achieved at $r4ds = -\frac{a_3}{2a_4} + B \approx 0.1 + B > B$, meaning that discount first decreases with demand share and then increases in the segment. Hence, we can now claim quite confidently that the empirical relationship between discount and demand share is indeed N-shaped.

To check the sensitivity of the estimated shape to the location of the knot, we run two linear

Figure 1.3: Average marginal impact of demand share: sensitivity to knot location.



Notes. The lines show the predicted marginal effect of $r4ds$ on ED, with other covariates taking their mean values.

regressions based on model (iii) with $B = 0.5668 \pm 2 \times 0.0309$. We call the two regressions model (v) and (vi), respectively, and we plot the predicted average effective discount agains $r4ds$ in Figure 1.3. Although setting a biased knot could smooth out the decreasing part of the curve, we still observe N-shaped curves with both model (v) and (vi).

Finally, we find that the predicted discount decreases with the number of buyers for a product (or product popularity), increases with a buyer's business size with the seller,[5] increases with the number of delayed days (or time passed), and increases with the remaining capacity level. Additionally, impacts from time that has elapsed and capacity level influence each other in a negative way. In other words, the impact of capacity level deteriorates over time, and the impact of time delay decreases with capacity level. It is interesting to find that the effective discount is not significantly correlated with the end-of-quarter effect and the demand concentration rate for the fixed-price contracts. The reasons may be that fixed-price contracts entail long-term considerations and that demand concentration is not a good measure for product popularity for this type of contract.

1.3.5 Further Discussions

Simultaneity. Regarding our regression models, one possible concern is that they suffer simultaneity between price and quantity; i.e., not only is discount affected by demand share but demand

[5] $Cshr$ and customer fixed effect are colinear and $Cshr$ will be significant if customer fixed effect is not included.

Table 1.7: Summary of Regression Results

Variables	(*i*) *ED*	(*ii*) *ED*	(*iii*) *ED*	(*iv*) *ED*	(*v*) *ED*	(*vi*) *ED*
$f(r4ds)$	Tab. 1.5	Tab. 1.5	Tab. 1.6	Tab. 1.6	Tab. 1.6	Tab. 1.6
Cbase	-1.13e-3***	-1.21e-3***	-1.14e-3***	-1.15e-3***	-1.11e-3***	-1.16e-3***
	(3.89e-4)	(3.88e-4)	(3.91e-4)	(3.92e-4)	(3.92e-4)	(3.92e-4)
TSQ	8.41e-10	5.90e-10	6.42e-10	9.34e-10	7.33e-10	7.87e-10
	(1.26e-9)	(1.31e-9)	(1.29e-9)	(1.27e-9)	(1.30e-9)	(1.28e-09)
Herf	-.0008	-.0059	-.0044	-.0053	.0005	-.0052
	(.0271)	(.0270)	(.0271)	(.0272)	(.0272)	(.0272)
lndod	.0382***	.0382***	.0384***	.0385***	.0382***	.0384***
	(.0047)	(.0048)	(.0047)	(.0047)	(.0047)	(.0047)
lndrt	-.0049	-.0049	-.0049	-.0051	-.0050	-.0051
	(.0039)	(.0040)	(.0039)	(.0039)	(.0040)	(.0040)
M3	-.0013	-.0007	-.0013	-.0014	-.0014	-.0014
	(.0074)	(.0074)	(.0074)	(.0074)	(.0074)	(.0074)
CapL	.0873***	.0866***	.0855***	.0865***	.0846***	.0854***
	(.0225)	(.0225)	(.0225)	(.0224)	(.0225)	(.0225)
Cshr	.2129	.2221	.1975	.2095	.2035	.2087
	(.1718)	(.1734)	(.1726)	(.1716)	(.1720)	(.1719)
Vrate	.1931***	.1919***	.1936***	.1938***	.1942***	.1932***
	(.0127)	(.0127)	(.0127)	(.0127)	(.0127)	(.0127)
*CapL**lndod*	-.0289***	-.0293***	-.0296***	-.0297***	-.0293***	-.0298***
	(.0057)	(.0058)	(.0057)	(.0057)	(.0057)	(.0057)
Constant	-.1136***	-.0994***	-.0805***	-.0902***	-.0706**	-.0588*
	(.0329)	(.0353)	(.0346)	(.0328)	(.0323)	(.0343)
Brand F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Buyer F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Quarter F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Location F.E.	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.3817	0.3824	0.3835	0.3830	0.3820	0.3829
Observations	2,346	2,346	2,346	2,346	2,346	2,346
Sample	Fixed Price	Fixed Price	Fixed Price	Fixed Price	Fixed Price	Fixed Price

Notes. Robust standard errors are in parentheses. *p<0.1; **p<0.05; ***p<0.01. F.E.: Fixed Effect.

Table 1.8: Test for Simultaneity with Instrumental Variable

	$\gamma_{I,2}$	$\gamma_{I,3}$	$\gamma_{I,4}$	$\gamma_{II,2}$	$\gamma_{II,3}$	$\gamma_{II,4}$	$\gamma_{III,2}$	$\gamma_{III,3}$	$\gamma_{III,4}$
Estimate	-.0051	.0308	.0077	.0007	.0003	-.0053	.0004	.0025	-.0048
Robust S.E.	.0091	.0091	.0094	.0084	.0082	.0087	.0084	.0082	.0087
P value	0.574	0.001	0.413	0.933	0.969	0.543	0.966	0.764	0.583

share may also be affected by discount. This is normally a valid concern, especially for the aggregate market price and demand. However, this is not valid at such a micro level in our problem, because each buyer's production plan is determined in advance and the production needs inputs other than microprocessors from different suppliers so that it can be costly for buyers to manipulate their demand. To verify this industry practice with our data, we pick the *Quarter* from which an instance started in a year as an instrumental variable (IV). *Quarter* is a categorical variable used to control for time fixed effects in our previous regressions. Notice that *Quarter* can directly influence *ED* given the cyclical nature of the semiconductor industry, but it can hardly relate to demand share directly. It is very unlikely that buyers of a particular demand share prefer to start a purchase from a particular quarter or are required to do so. By running regressions $ED = a_I \cdot r4ds + \sum_{t=2}^{4} \gamma_{I,t} Z_t + b_I' X + \varepsilon_I$ and $r4ds = a_{II} \cdot ED + \sum_{t=2}^{4} \gamma_{II,t} Z_t + b_{II}' X + \varepsilon_{II}$, wherein Z_t is the dummy for quarter t and X is the vector of all other covariates, we find that $\gamma_{I,3}$ is highly significantly positive (as shown in Table 1.8) but none of $\gamma_{II,t}$ is significant. Hence, the validity of *Quarter* being an IV is supported. Next, we run regression $r4ds = \sum_{t=2}^{4} \gamma_{III,t} Z_t + b_{III}' X + \varepsilon_{III}$ and we find that none of $\gamma_{III,t}$ is significant. Suppose that *r4ds* depends on *ED* at least linearly: $r4ds = a_{II} \cdot ED + b_{II}' X + \varepsilon_{II} = a_{II} \cdot a_I \cdot r4ds + a_{II} \cdot \sum_{t=2}^{4} \gamma_{I,t} Z_t + (a_{II} \cdot b_I' + b_{II}')X + \varepsilon_{IV}$, which leads to $r4ds = \frac{a_{II}}{1 - a_{II} \cdot a_I} \cdot \sum_{t=2}^{4} \gamma_{I,t} Z_t + b_{III}' X + \varepsilon_{III}$. If $a_{II} \neq 0$, we should have observed a reasonably significant $\gamma_{III,3}$ given the highly significant $\gamma_{I,3}$. However, this is not supported by the data. Therefore, the data suggests that ***price is not driving quantity***.

Piecewise Polynomial vs. Ordinary Polynomial. We know that a polynomial of degree three can also generate an N-shaped curve. However, a degree-three polynomial is concave on the left of "N," meaning that discount increases rapidly with demand share for very small buyers. According to our empirical observation, the discount curve should be linear or convex first on the left of "N," so it is difficult to have a good fit with our data for a degree-three polynomial. Although polynomials of

Table 1.9: Regressions with Alternative Transformations of Demand Share

	(vii)	*(viii)*	*(ix)*	*(x)*
a_1	.2553**	.2406**	.2262*	.2334*
	(.1080)	(.1050)	(.1188)	(.1275)
a_2	.3591	.3178	.2764	.2935
	(.2406)	(.2272)	(.2648)	(.3061)
a_3	-.6497*	-.5111*	-.3915*	-.3474*
	(.3605)	(.2854)	(.2246)	(.2039)
a_4	4.5500*	2.8018*	1.6499*	1.3247*
	(2.4114)	(1.4686)	(.8594)	(.6927)
B	.7159***	.6352***	.5104***	.4517***
	(.0265)	(.0317)	(.0388)	(.0412)
Transformation	$DS^{0.15}$	$DS^{0.2}$	$DS^{0.3}$	$DS^{0.35}$

Notes. Robust standard errors are in parentheses.
*p<0.1; **p<0.05; ***p<0.01.

high-enough degree can approximate any shape of curve, they have potential problems of overfitting and multicollinearity. In contrast, piecewise polynomials of lower degrees are capable of offering adequate flexibility, while having fewer parameters.

Alternative Transformations of Demand Share. To check the robustness of power transformations of demand share, we run four additional nonlinear-piecewise-polynomial regressions using $DS^{0.15}$, $DS^{0.2}$, $DS^{0.3}$, and $DS^{0.35}$, respectively, in place of $r4ds$ in (1.3). The results are summarized in Table 1.9. In all four models, we can get similar non-monotonic curves composed of a linear piece on the left and a quadratic piece on the right. Hence, power transformations of demand share not only preserve the unit range, but also offer a robust way to study the price-quantity relationship.

Omitted Variables. Note that we ran linear regressions in models (i), (ii), (v), and (vi). We also find that the results of linear regressions are very similar to what we obtained from nonlinear regressions, so here we focus on the discussion of linear regressions for simplicity. Let $f(DS)$ denote the vector of demand-share-related variables, X the vector of other covariates included in our model, and $Z = (Z_1, Z_2)'$ the vector of those not included. As discussed earlier, Z_1 is the buyer's net cost of switching to an alternative product, and Z_2 is the value of other contract terms for the buyer. We will show in Section 1.4.2 that the demand-share-dependent switching cost is not supported by the data, so it seems plausible to assume that Z_1 is uncorrelated with $f(DS)$. Hence, $Z_1 = b_{z1}' \cdot X + \varepsilon_{z1}$, wherein b_{z1} is a constant vector and ε_{z1} is a random variable that is independent of $f(DS)$. Next, price discount and other contract terms, if any, are determined through the same bargaining process, so $Z_2 = b_{z2}\left(\gamma_{ds}' \cdot f(DS) + \gamma_x' \cdot X + \gamma_{z1} \cdot Z_1\right) + \varepsilon_{z2}$. Therefore, if ED is determined by a linear model that $ED = \gamma_{ds}' \cdot f(DS) + \gamma_x' \cdot X + \gamma_z' \cdot Z + \varepsilon_1$, wherein $\gamma_z = (\gamma_{z1}, \gamma_{z2})'$. Defining $\gamma_0 = (1 + \gamma_{z2} b_{z2}) \cdot (\gamma_x' + \gamma_{z1} b_{z1}')$ and plugging Z into ED, we get

$$ED = (1 + \gamma_{z2} b_{z2}) \cdot \gamma_{ds}' \cdot f(DS) + \gamma_0' X + \varepsilon_0. \tag{1.4}$$

We can see that the true marginal effect of $f(DS)$ is just scaled if we regress ED against $f(DS)$ and X, and thus the underlying non-monotonic pattern will not be affected.

Our observations from the regressions are particularly interesting in that they are somewhat inconsistent with the conventional wisdom—and our intuition—which says that buyers with larger quantities should receive lower prices. While our intuition is correct for small- and large-quantity buyers, we are not aware of a discount valley for medium-sized buyers.

Why do we see such a discount curve? The literature offers no theory that can explain our observation. Although existing theories can predict increasing or V-shaped discount curves, they work under different premises and thus can never be combined to generate other discount curves. If the seller is powerful and offers quantity discounts to coordinate the supply chain, we should observe an increasing discount curve (e.g., Weng 1995). In a cooperative setting, we can obtain an increasing discount curve if the buyer and the seller use Nash bargaining to set the price (e.g., Kohli and Park 1989). According to Snyder (1998), when competing suppliers cooperate in a repeated game, we should observe a discount curve that decreases first and increases later with buyer size. Unfortunately, none of these models is general enough to explain our observation from the data and thus we do not yet fully understand the mechanism of price negotiation in B2B markets.

Our empirical finding has important implications for both buyers and sellers. Because larger quantities may not lead to lower prices, it may not be wise for a buyer to increase the purchase size. However, there should not be an arbitrage opportunity for a buyer, because the total purchase cost still increases with quantity.[6] For the seller, it may be imperative to rethink posted pricing

[6]We find that in only 7% of instances across the entire data set, a buyer pays less in total than another buyer of

and capacity rationing, given this non-monotonic relationship between price and quantity. We will continue discussion in the following sections, in which we try to use an analytical model to understand and explore the rationale behind our observation.

1.4 Modelling and Verification

In order to study the rationale behind the empirical observation, we build an analytical model in this section. We then verify with the data that the non-monotonicity is rooted in how the seller values capacity by comparing different models in terms of goodness-of-fit as well as the statistical significance of non-monotonicity in each model.

1.4.1 The Model

Consider a seller $\mathcal{A}$ (she) that sells a new product to a group of OEMs. We assume that $\mathcal{A}$ has a fixed capacity κ due to a long production lead time and a short product life cycle. The selling starts at time 0 and ends when there are no more buyers or the capacity is sold out or salvaged. There are M potential buyers who arrive stochastically. We consider a general *non-homogeneous Poisson arrival process* wherein the arrival rate of the i-th buyer is $\lambda_i < \infty$ for $i = 1, 2, \cdots, M$. For simplicity, we assume that the arrival process is only determined by market characteristics and is independent of buyer identities or the history of the arrival process.

As stated earlier, in the semiconductor industry, technological advancements of competing products will lead to obsolescence of the focal product. When such a technological shock happens, potential buyers will change their adoption decisions. However, for buyers that have already adopted the product and integrated it into their product designs, the switching cost will be high; thus, existing buyers will continue their purchase until they phase out products that use the focal product. We assume that the arrival time of the technological shock is exponentially distributed with rate $\lambda_0 < \lambda_i$ for any i, and that seller $\mathcal{A}$ salvages the remaining capacity at marginal value s when the shock arrives. Let $\delta_i \in (0, 1)$ represents the probability of the shock arriving after the $(i-1)$-th and prior to the i-th buyer. Using the memoryless property of exponential distribution, we can check that $\delta_i = \frac{\lambda_0}{\lambda_0 + \lambda_i}$.

the same product who buys less. This number is only 4% among fixed-price instances.

Let D_i denote the demand of the i-th buyer and Q_i the capacity allocated to buyer i. We assume that buyers accept partial fulfillment as long as $Q_i \in [\eta D_i, D_i]$, where $\eta \in (0, 1]$ is plausibly an industry standard that is exogenous and identical for all the buyers. Hence, if $\eta < 1$, seller $\mathcal{A}$ faces a dynamic capacity management problem wherein she must decide the degree of fulfillment ρ_i for buyer i in order to maximize the total expected revenue. Let K_i be the total available capacity when buyer i arrives, so K_i/D_i is the maximum level of fulfillment and $\rho_i \in [\eta, \min\{1, \max\{\eta, K_i/D_i\}\}]$. Note that ρ_i is not relevant if $K_i/D_i < \eta$.

Demand is unknown to the seller *ex ante* but is exogenously given because each buyer's production plan is determined in advance and the production needs inputs from different suppliers so that it is costly for buyer i to manipulate D_i. Although demand may not be exogenous from a buyer's point of view, that is not a concern of this study. From the seller's point of view, demand can be correlated and thus the distribution of each oncoming demand is history-dependent because buyers may be subject to the same demand shock and (or) competition in the same market. We define "history" as the set of information that is revealed to the seller. Let $\psi(t)$ denote the history up to time t, and ψ_i the history up to the arrival of buyer i. We assume that D_i follows distribution function (cdf) $F(\cdot|\psi_{i-1})$, where $\psi_0 = \emptyset$. For the purpose of analysis, we make the following technical assumptions: (1) that the expectation of the demand from a buyer is always finite, and (2) that there exists a lower envelope for the possible forms of F.

Assumption 1. *$\int_0^{+\infty} D dF(D|\psi) < \infty$ for any $\psi \in \mathcal{H}$, where $\mathcal{H}$ stands for the set of all possible histories.*

Assumption 2. *There exists an increasing and continuous function $F_0(\cdot)$ on $[0, +\infty)$ such that (i) $F_0(0) = 0$, (ii) $F_0(+\infty) = 1$, and (iii) $F_0(x) \leq F(x|\psi)$ for any $x \in [0, +\infty)$ and $\psi \in \mathcal{H}$.*

The sequence of events with the i-th buyer is modeled as follows. (1) Buyer i arrives at t_i and proposes an acceptable range $[\eta D_i, D_i]$ for quantity. (2) Seller $\mathcal{A}$ decides ρ_i. (3) Buyer i stays if $\rho_i \geq \eta$ and leaves permenently if otherwise. (4) If buyer i stays, they settle the transaction price w_i for quantity $Q_i = \rho_i D_i$ through *Nash bargaining*, in which information is assumed to be symmetric for simplicity.

Let β_i denote the exogenous, relative bargaining power of buyer i against seller $\mathcal{A}$. It captures exogenous factors such as bargaining skills and net cost of keeping a long-term relationship. We assume that β_i is known given the identity of buyer i. Conditional on history ψ, the bargaining power β of a potential buyer follows distribution $B(\cdot|\psi)$. The *generalized Nash bargaining model* predicts that if player j's payoff and outside option for the focal transaction are $\Pi_j(w)$ and d_j given the transaction price w, where $j \in \{\mathcal{A}\} \cup \{1, 2, 3, \cdots\}$, then the bargaining results in price $w^* = \arg\max_w (\Pi_i(w) - d_i)^{\beta_i} \cdot (\Pi_A(w) - d_A)^{1-\beta_i}$. In particular, if $\Pi_i(w) - d_i + \Pi_A(w) - d_A$ is independent of w, then w^* splits the pie between the buyer and the seller in proportion to their respective bargaining powers.

For buyer i, let r_i and r'_i denote the profit margins before subtracting the cost of the product purchased from seller $\mathcal{A}$ and an alternative supplier, respectively, p the posted price for $\mathcal{A}$'s product, $\tilde{c}_i$ the marginal cost of buying from the alternative, and Q'_i the quantity available from the alternative. In addition, let $l_i = \frac{Q'_i}{Q_i}$. Accordingly, the total payoff is $\Pi_i(w_i) = (r_i - w_i) \cdot Q_i$ and the outside option is

$$\begin{aligned} d_i =& \max\{Q_i \cdot (r_i - p), Q'_i \cdot (r'_i - \tilde{c}_i)\} \\ =& Q_i \cdot \max\{r_i - p, l_i \cdot (r'_i - \tilde{c}_i)\} \\ =& Q_i \cdot [r_i - \min\{p, r_i - l_i \cdot r'_i + l_i \cdot \tilde{c}_i\}] . \end{aligned} \tag{1.5}$$

Let $\bar{c}_i = r_i - l_i \cdot r'_i + l_i \cdot \tilde{c}_i$ represent the net marginal cost of buying from the alternative supplier in order to keep the same margin r_i. If $\bar{c}_i > p$, it is not credible for buyer i to switch, so the outside option is to buy from seller $\mathcal{A}$ at the posted price. This is possible because products are not perfectly substitutable, and $\bar{c}_i$ includes switching costs such as searching, redesigning, damage to the brand image, and so on. We assume that $\bar{c}_i$ is unknown to the seller *ex ante* but will be revealed during the negotiation. For a potential customer who has not arrived, $\bar{c}$ follows distribution $G(\cdot|\psi)$ given history ψ.

For seller $\mathcal{A}$, let $V(K, p, \psi(t))$ represent the expected revenue obtained *after* time t given remaining capacity K, posted price p, and history $\psi(t)$. Therefore, when bargaining with buyer i, seller $\mathcal{A}$ has expected payoff $\Pi_A(w_i) = w_i Q_i + V(K_i - Q_i, p, \psi_i)$ and outside option $d_A = V(K_i, p, \psi_i) \cdot \mathbb{I}\{\bar{c}_i \le p\} + [pQ_i + V(K_i - Q_i, p, \psi_i)] \cdot \mathbb{I}\{\bar{c}_i > p\}$, where $\mathbb{I}\{\cdot\}$ is an indicator function. In addition, we assume that $\bar{c}_i \ge c_L > s$ for every i so that the bargaining always has a solution (i.e., the highest price a customer would like to pay is higher than the marginal value for the seller). Hence, we have the following lemma, which determines whether buyer i pays the posted price or engages in the price bargaining. Notice that β_i and $\bar{c}_i$ are known when the buyer arrives and are thus taken as certain in the bargaining.

Lemma 1.1. *If $\bar{c}_i > p$, buyer i pays the posted price; i.e., $w_i = p$. If $\bar{c}_i \leq p$, the Nash bargaining results in*

$$w_i = \beta_i \cdot \frac{V(K_i, p, \psi_i) - V(K_i - Q_i, p, \psi_i)}{Q_i} + (1 - \beta_i) \cdot \bar{c}_i. \tag{1.6}$$

Lemma 1.1 simply says that *the chance of a buyer engaging in a price negotiation increases with the posted price p.* Hence, the higher the posted price, the more bargainers. It also says that the negotiated price is a function of the available capacity and transaction quantity. Based on Lemma 1.1, we know that as long as $\partial V/\partial K \geq 0$, our model satisfies the property that larger quantities entail larger total payments.[7] In order to understand how w_i is affected by K_i and Q_i, we need to know more about value function V as well as other factors.

1.4.2 Source of Non-Monotonicity

In our model, we propose that buyer bargaining power β and net switching cost $\bar{c}$ are not the source of price-quantity non-monotonicity and thus assume for simplicity that they are independent of purchase quantity Q. To verify our conjecture, we compare three different models by running nonlinear regressions. To proceed, first note that from Lemma 1.1 we have

$$w_{ij} = \mathbb{I}\{\bar{c}_{ij} \geq p_j\} \cdot p_j + \mathbb{I}\{\bar{c}_{ij} < p_j\} \cdot [\beta_{ij} \cdot \triangle\, \hat{v}_{ij} + (1 - \beta_{ij}) \cdot \hat{c}_{ij}] \cdot p_j, \tag{1.7}$$

where $\triangle\, \hat{v}_{ij} = [V(K_{ij}, p_j, \psi_{ij}) - V(K_{ij} - Q_{ij}, p_j, \psi_{ij})] / (p_j Q_{ij})$ and $\hat{c}_{ij} = \bar{c}_{ij}/p_j$. Accordingly, we can derive the discount received by buyer i for product j:

$$1 - \frac{w_{ij}}{p_j} = \mathbb{I}\{\hat{c}_{ij} < 1\} \cdot [1 - \beta_{ij} \cdot \triangle\, \hat{v}_{ij} - (1 - \beta_{ij}) \cdot \hat{c}_{ij}]. \tag{1.8}$$

Now we can see that three factors can possibly contribute to the non-monotonicity we are after:

[7] It is easy to check that $\partial(w_i Q_i)/\partial Q_i = \beta_i V'_K(K_i - Q_i, p, \psi_i) + (1 - \beta_i) \cdot \bar{c}_i$.

$\hat{c}_{ij}$, β_{ij}, and $\triangle\ \hat{v}_{ij}$. Hence, we consider three different models. In preparation, we define $\phi(x) = a_1 \cdot (x-B)_- + a_2 \cdot (x-B)_-^2 + a_3 \cdot (x-B)_+ + a_4 \cdot (x-B)_+^2$, which is the piece-wise polynomial function we used to capture the non-monotonicity in the empirical analysis. In model (I), we assumes that the non-monotonicity is rooted in how the seller values capacity. In particular, $\hat{c} = b_c' \cdot X_c$, $\beta = b_b' \cdot X_b$, and $\triangle\ \hat{v} = \phi(r4ds) + b_v' \cdot X_v$. In model (II), we assume that the non-monotonicity is originated from the net switching cost. In particular, $\hat{c} = \phi(r4ds) + b_c' \cdot X_c$, $\beta = b_b' \cdot X_b$, and $\triangle\ \hat{v} = b_v' \cdot X_v$. In model (III), we assume that the non-monotonicity is due to quantity-dependent bargaining power. In particular, $\hat{c} = b_c' \cdot X_c$, $\beta = \phi(r4ds) + b_b' \cdot X_b$, and $\triangle\ \hat{v} = b_v' \cdot X_v$. Note that although the three models have the same components, they are different in structures. Regarding other explanatory variables, we use $lndod$ and 10 major brand names for X_c, $Cshr$ and $Vrate$ for X_b, and $Cbase$, $lndod$, $CapL$, and three quarters for X_v. At last, we run three nonlinear regressions based on the following equation:

$$ED_{ij} = \mathbb{I}\{\hat{c}_{ij} < 1\} \cdot [1 - \beta_{ij} \cdot \triangle\ \hat{v}_{ij} - (1-\beta_{ij}) \cdot \hat{c}_{ij}] + \hat{\epsilon}_{ij}. \tag{1.9}$$

The results are summarized in Table 1.10. Notice that the non-monotonicity is statistically significant only in model (I). In addition, given the same number of parameters or degree of freedom (d.f.), model (I) has the highest R^2 and the lowest sum of squared residuals (SS). If we can assume that $\hat{\epsilon}$ is normally distributed, we can use Akaike's Information Criterion (AIC) (Akaike 1981) to compute the evidence ratio (i.e., how much more likely) of one model against another. We first compute the corrected AIC value defined by

$$AIC_C = N \cdot \ln\left(\frac{SS}{N}\right) + \frac{2 \cdot K \cdot N}{N-K-1}, \tag{1.10}$$

where N is the number of observations and K is the number of parameters in the model plus one. Next, we can obtain the evidence ratio defined by

$$\text{Evidence Ratio} = \frac{\text{Probability that model (I) is correct}}{\text{Probability that model (II) is correct}} = \exp\left(\frac{AIC_C^{(II)} - AIC_C^{(I)}}{2}\right). \tag{1.11}$$

Accordingly, we know that model (I) is 2.26×10^{15} times more likely against model (II) and 239 times more likely against model (III) to be the correct one. In other words, the evidence is overwhelmingly in favor of model (I). Therefore, combining all the results, we conclude that model (I) is the correct model among the three.

Regarding other possible models, the most plausible is the combination of model (II) and (III). However, there will be a serious collinearity problem when we let β depend on $r4ds$ in (II) or let

Table 1.10: Selected Regression Results for the Three Alternative Models

	(I)	(II)	(III)
a_1	-0.6508***	-0.2475*	1.0113**
	(0.2148)	(0.1354)	(0.4718)
a_2	-1.0167**	-0.3875	1.1682
	(0.4417)	(0.3095)	(0.9095)
a_3	1.8758***	0.4899	-1.5467
	(0.5942)	(0.3113)	(1.5779)
a_4	-8.7269***	-2.0765	5.0896
	(2.7016)	(1.2606)	(6.7346)
B	0.5865***	0.5795***	0.6297***
	(0.0196)	(0.0397)	(0.0474)
R^2	0.5140	0.4991	0.5117
d.f.	27	27	27
SS	50.6148	52.1636	50.8517
AIC_C	-8945	-8874	-8934

Notes. Standard errors are in parentheses. SS: sum of squared errors. $^*p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$.

$\hat{c}$ depend on $r4ds$ in (III). Hence, more complicated models cannot provide better explanations. Lastly, if we add $r4ds$ as a linear part of $\hat{c}$ in model (I), the estimated coefficient is not significant, so it is reasonable to assume that $\hat{c}$ is not correlated with $r4ds$. Note that although $l_i = \frac{Q_i'}{Q_i}$, it is the industry standard to fill at least 90% by any seller according to our interaction with practitioners, so $Q_i' \approx Q_i$ in most cases.

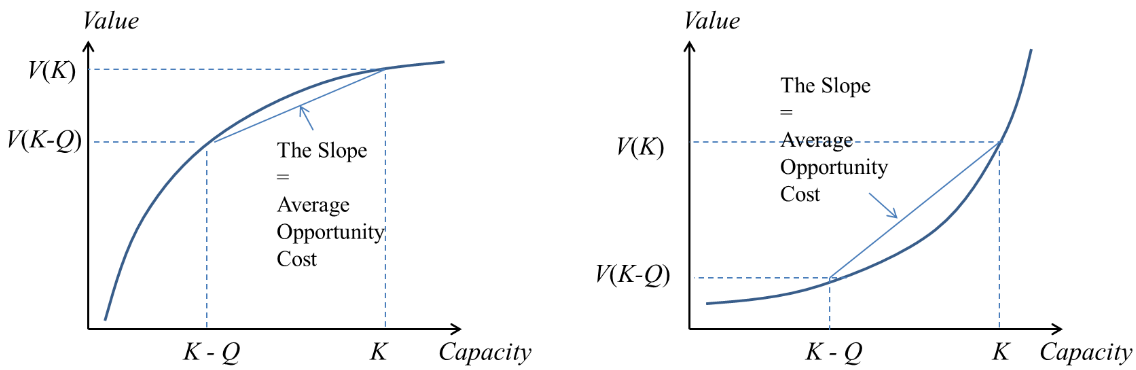
1.5 Theoretical Analysis

In this section, we first derive a sufficient condition on the value function for the price-quantity curve to be non-monotonic. We then analyze the seller's problem, formulate the value function, and investigate its property. Finally, we try to simulate the price curve given a certain form of the value function.

1.5.1 A Sufficient Condition for Non-Monotonicity

We assume V is a non-decreasing and twice-differentiable function of capacity K. To simplify the notation, we write $V(K, p, \psi_i) = V_i(K)$. We know that how the price w_i changes with quantity

Figure 1.4: The intuition behind proposition 1.1.



Q_i depends on the sign of the first-order derivative of w_i in (1.6) with respect to Q_i:

$$\begin{aligned} \frac{\partial w_i}{\partial Q_i} &= \beta_i \cdot \frac{Q_i \cdot V_i'(K - Q_i) - [V_i(K) - V_i(K - Q_i)]}{Q_i^2} \\ &= \frac{\beta_i}{Q_i}\left[V_i'(K - Q_i) - \frac{V_i(K) - V_i(K - Q_i)}{Q_i}\right]. \end{aligned} \tag{1.12}$$

Given that $\frac{\beta_i}{Q_i} > 0$, the sign of $\frac{\partial w_i}{\partial Q_i}$ depends on that of $V_i'(K - Q_i) - \frac{V_i(K) - V_i(K-Q_i)}{Q_i}$. A simple examination leads us to the following proposition.

Proposition 1.1. *If $V_i(x)$ is concave for any $x \in [0, K]$, then w_i increases with Q_i. If $V_i(x)$ is convex for any $x \in [0, K]$, then w_i decreases with Q_i.*

Although the above results are simple, they are surprising. Our initial intuition is that the value function should be concave and the price should decrease with quantity. However, in order to have the quantity discount, our simple model requires the value function to be convex. Figure 1.4 illustrates the intuition behind Proposition 1.1. Note that given buyer i's outside option and the bargaining power, w_i depends on the seller's average opportunity cost of selling Q_i units. We can see that as Q_i increases, the average opportunity cost increases if the value function is concave and decreases if the value function is convex. Based on this observation, we suspect that the value function may not be simply convex or concave, which may be the reason for a non-monotonic price-quantity relation. In fact, we can show that a simple combination of convexity and concavity for the value function will generate a non-monotonic price-quantity curve.

Proposition 1.2. *If there exists $x' \in (0, K)$ such that $V_i(x)$ is strictly convex for $x \in [0, x']$, strictly concave for $x \in [x', K]$, and $V_i'(0) < V_i(K)/K$, then there exists $x'' \in (0, K)$ such that w_i increases with Q_i for $Q_i \in [0, K - x'']$ and decreases with Q_i for $Q_i \in [K - x'', K]$.*

Proposition 1.2 provides us with a sufficient condition for the price-quantity relation to be non-monotonic. We call such a property *convex-concave.* Actually, it is reasonable to expect the value function to be convex-concave or *S-shaped.* When the capacity is very low, the seller is unlikely to fulfill any buyer's need and will have to salvage the capacity. As the capacity increases, it becomes more and more likely that the capacity is sufficient to satisfy more buyers' needs. When the capacity is very high, it may exceed demand and the seller may have to salvage a portion. We can infer from Figure 1.4 that when the value function is convex-concave and the capacity level is high enough, the average cost of selling Q units for the seller first increases and then decreases with Q, which leads to a non-monotonic price-quantity relation.

However, this result alone is not satisfactory, because it cannot explain the pricing pattern we observe in the data. The discussion in the previous paragraph is based on perturbing the purchase quantity of a single buyer with a fixed-value function. Notice that the seller may update the estimation of future demand based on the demand of the current buyer. Thus, the shape of the value function may be different for buyers with different purchase quantities, which may explain the empirical pattern. In the following, we try to verify our conjecture by formulating and analyzing the value function.

1.5.2 Formulating the Value Function

Assume that K units of capacity is available after the $(i-1)$-th buyer leaves. Let us consider the value of this remaining capacity in four cases that constitute the sample space. First, the leftover capacity will be salvaged with probability δ_i. Second, if buyer i arrives, the buyer walks away immediately if the capacity is insufficient. Hence, if $K < \eta D_i$, the seller's expected revenue at t_i is $V_i(K)$. Third, if $K \geq \eta D_i$ and $\bar{c}_i > p$, the expected revenue is $pQ_i + V_i(K - Q_i)$. Fourth, if $K \geq \eta D_i$ and $\bar{c}_i \leq p$, the expected revenue is $(1 - \beta_i)\bar{c}_i Q_i + \beta_i [V_i(K) - V_i(K - Q_i)] + V_i(K - Q_i)$.

Note that $Q_i = \rho^* (K, D_i, \bar{c}_i, \beta_i, \psi_i) \cdot D_i$ can differ for different parameter values. As a result,

$$
\begin{aligned}
V_{i-1}(K) = \delta_i \cdot s \cdot K + (1-\delta_i) \cdot \Bigg\{ & \int_{K/\eta}^{+\infty} V_i(K)\, dF(D|\psi_{i-1}) \\
& + \int_0^1 \int_0^{K/\eta} \int_p^{+\infty} [p\rho^* D + V_i(K - \rho^* D)]\, dG(\bar{c}|\psi_{i-1})\, dF(D|\psi_{i-1})\, dB(\beta|\psi_{i-1}) \\
& + \int_0^1 \int_0^{K/\eta} \int_{c_L}^{p} [\beta V_i(K) + (1-\beta) V_i(K-\rho^* D)]\, dG(\bar{c}|\psi_{i-1})\, dF(D|\psi_{i-1})\, dB(\beta|\psi_{i-1}) \\
& + \int_0^1 \int_0^{K/\eta} \int_{c_L}^{p} (1-\beta)\, \bar{c} \rho^* D dG(\bar{c}|\psi_{i-1})\, dF(D|\psi_{i-1})\, dB(\beta|\psi_{i-1}) \Bigg\}. \qquad (1.13)
\end{aligned}
$$

Apparently, this is a complicated function and it is not obvious how $V_{i-1}(K)$ is affected by various parameters. Hence, we try to derive approximations for the value function in two cases: one with a finite buyer group and the other with a very large buyer group (i.e., $M \to \infty$).

1.5.3 An Approximation with Finite M

For this approximation, we begin with the last (i.e., the M-th) buyer. Given that no more selling opportunities will exist after the last buyer, we have $V_M(K) = sK$. From Lemma 1.1, we know that the price for the M-th buyer is $w_M = p$ or $w_M = \beta_M \cdot s + (1-\beta_M) \cdot \bar{c}_M$. Thus, we have $w_M > s$ and the seller should sell as much to the last buyer as possible; i.e., $\rho_M^* = \min\{1, K/D_M\}$. Plugging $V_M(K) = sK$ and $Q_M = K \wedge D_M$ into (1.13), we can obtain

$$
V_{M-1}(K) = sK + (1-\delta_M) \cdot \nu(p, \psi_{M-1}) \cdot \left[\int_0^K DdF(D|\psi_{M-1}) + \int_K^{K/\eta} KdF(D|\psi_{M-1}) \right], \qquad (1.14)
$$

where

$$
\begin{aligned}
\nu(p, \psi_{M-1}) &= p - s - \int_0^1 \int_{c_L}^{p} (p - (1-\beta)\, \bar{c} - \beta s)\, dG(\bar{c}|\psi_{M-1})\, dB(\beta|\psi_{M-1}) \\
&= p - s - \mathbf{E}_{\beta,\bar{c}} \left[(p - (1-\beta)\bar{c} - \beta s) \cdot \mathbb{I}\{\bar{c} \le p\} \,|\psi_{M-1} \right]. \qquad (1.15)
\end{aligned}
$$

Although in general $V_{i-1}(K)$ in (1.13) is not a separable function for the three random variables, β, $\bar{c}$, and D, we find in (1.14) that D can be multiplicatively separated from β and $\bar{c}$. Basically, $\nu(p, \psi_{M-1})$ is only related to β and $\bar{c}$, and it measures the expected margin obtained from a buyer above the salvage value. Note that $\nu(p, \psi_{M-1})$ is finite because $\beta \in [0,1]$ and $\bar{c} \in [c_L, p]$. Furthermore, under the assumption of finite demand expectation, we have that $\int_K^{K/\eta} KdF(D|\psi_{M-1})$

approaches zero as K increases, so this term can be ignored when $K \to +\infty$.

Lemma 1.2. $\lim_{K\to+\infty} \int_K^{K/\eta} K dF(D|\psi) = 0$ *given that* $\int_0^{+\infty} DdF(D|\psi) < +\infty$.

Therefore, based on (1.14), for any scalar $\lambda \in (0,1)$, we can easily have a lower bound $\hat{V}_{M-1}(K,\lambda)$ for $V_{M-1}(K)$:

$$V_{M-1}(K) \geq sK + (1-\delta_M) \cdot \nu(p, \psi_{M-1}) \cdot \int_0^{\lambda K} DdF(D|\psi_{M-1}) = \hat{V}_{M-1}(K,\lambda). \tag{1.16}$$

We can see that the shape of $\hat{V}_{M-1}(K,\lambda)$ (i.e., how $\hat{V}_{M-1}(K)$ changes with K) depends on the shape of distribution $F(\cdot|\psi_{M-1})$. If $F(\cdot|\psi_{M-1})$ is exponential, then $\hat{V}_{M-1}(K,\lambda)$ is increasing and concave in K. If $F(\cdot|\psi_{M-1})$ is normal, then $\hat{V}_{M-1}(K,\lambda)$ is S-shaped. Using the result for the last buyer, we move on and consider $V_{i-1}(K)$ in general. We derive an upper and a lower bound for $V_{i-1}(K)$ and we present the results in Theorem 1.1.

Here is the key idea of the proof for the lower bound. First, setting $\rho = 1$ in (1.13) leads to a lower bound of $V_{i-1}(K)$. We then utilize the fact that the lower bound is a separable function for D and we iteratively plug the lower bound into (1.13). We complete the proof by induction. The proof for the upper bound is similar. An essential tool we use is the law of iterative expectation, based on which we have $\mathbf{E}_{\beta,\bar{c}}[\nu(p,\psi_{i-1})|\psi_{i-2}] = \nu(p,\psi_{i-2})$ and $\int_0^{+\infty}\int_0^{\lambda K} DdF(D|\psi_{i-1})\, dF(D'|\psi_{i-2}) = \mathbf{E}[\mathbf{E}[D\cdot \mathbb{I}\{D \leq \lambda K\}|\psi_{i-1}]|\psi_{i-2}] = \int_0^{\lambda K} DdF(D|\psi_{i-2})$.

Theorem 1.1. *For any* $1 \leq i \leq M$ *and* $K \geq 0$,

$$V_{i-1}(K) \geq sK + \left(1-\delta_i^l\right) \cdot \nu(p,\psi_{i-1}) \cdot \int_0^{K\cdot\lambda^{M+1-i}} DdF(D|\psi_{i-1}), \tag{1.17}$$

$$V_{i-1}(K) \leq sK + (1-\delta_i^u) \cdot \nu(p,\psi_{i-1}) \cdot \int_0^{K/\eta} DdF(D|\psi_{i-1}), \tag{1.18}$$

where $\delta_i^l = \delta_i - (1-\delta_i)\left(1-\delta_{i+1}^l\right) F_0(K-\lambda K)$, $\delta_i^u = \delta_i - (1-\delta_i)\left(1-\delta_{i+1}^u\right)$, *and* $\delta_M^u = \delta_M^l = \delta_M$.

Because $\lim_{K\to+\infty} F_0(K-\lambda K) = 1$, we have $\lim_{K\to+\infty} \delta_i^l = \delta_i^u$. Therefore, the upper and lower bounds converge as K goes to infinity. Both the upper and lower bounds take a functional form similar to $\hat{V}_{M-1}(K,\lambda)$, and it is reasonable to expect that $V_{i-1}(K)$ is similar to the bounds as long as they are close enough. We may also conclude that the shape of the value function is largely dependent on the demand distribution of the next buyer. However, the gap between the upper and lower bounds increases with M. Hence, the bounds will perform well when M is not extremely large. Otherwise, it may be useful to get bounds that are independent of M.

1.5.4 An Approximation with Infinite M

In this case, we assume that the buyer arrival rate is constantly λ_b. Hence, the probability of ending the selling process is constantly $\delta = \frac{\lambda_0}{\lambda_0+\lambda_b}$. Before we analyze $V_{i-1}(K)$, note that we can write it as $\mathbf{E}\left[\mathcal{R}\left(K,\{D_n,\beta_n,\bar{c}_n,t_n-t_{n-1}\}_{n=i}^{\infty}\right)|\psi_{i-1}\right]$, where $\mathcal{R}$ is the total revenue, which is a function of the future demand, buyer bargaining power, net marginal cost of buying outside, and arrival times. Similarly, $V_i(K) = \mathbf{E}\left[\mathcal{R}\left(K,\{D_n,\beta_n,\bar{c}_n,t_n-t_{n-1}\}_{n=i+1}^{\infty}\right)|\psi_i\right]$. Based on our assumptions, $\{D_n,\beta_n,\bar{c}_n,t_n-t_{n-1}\}_{n=i}^{\infty}$ and $\{D_n,\beta_n,\bar{c}_n,t_n-t_{n-1}\}_{n=i+1}^{\infty}$ are statistically equivalent given information ψ_{i-1}. Thus, using this condition and the law of iterative expectation, we get

$$\mathbf{E}\left[V_i(K)|\psi_{i-1}\right] = \mathbf{E}\left[\mathcal{R}\left(K,\{D_n,\beta_n,\bar{c}_n,t_n-t_{n-1}\}_{n=i+1}^{\infty}\right)|\psi_{i-1}\right] = V_{i-1}(K). \tag{1.19}$$

Leveraging this property of the value function, we obtain an upper bound and a lower bound for $V_{i-1}(K)$. In preparation, let

$$H_i(K) = \left[p\left(1-G\left(p|\psi_{i-1}\right)\right) + \left(1-\mathbf{E}\left[\beta|\psi_{i-1}\right]\right)\int_{c_L}^{p}\bar{c}dG\left(\bar{c}|\psi_{i-1}\right)\right]\cdot\int_0^{K/\eta} DdF\left(D|\psi_{i-1}\right), \tag{1.20}$$

which is an *approximate measure* for the expected revenue obtained from the i-th buyer. Let

$$h_i(K) = \mathbf{E}\left[V_i\left(\left[K-D_i\right]^+\right)|\psi_{i-1}\right]/V_{i-1}(K). \tag{1.21}$$

It is easy to see that $h_i(K) \in [0,1]$. Now we can introduce the following theorem.

Theorem 1.2. *For any $i \geq 1$ and $K \geq 0$, we have*

$$\frac{s\cdot K + \frac{1-\delta}{\delta}\cdot H_i(K)}{1+\frac{1-\delta}{\delta}\cdot[1-h_i(K)]\cdot\left(1-\mathbf{E}\left[\beta|\psi_{i-1}\right]\cdot G(p|\psi_{i-1})\right)} \leq V_{i-1}(K) \leq s\cdot K + \frac{1-\delta}{\delta}\cdot H_i(K). \tag{1.22}$$

If $\frac{\partial}{\partial K} V_i(K) \geq s$ for any $i \geq 1$, $K \geq 0$, and ψ_i, then $\lim_{K\to\infty} h_i(K) = 1$.

Let U_{i-1} and L_{i-1} be the upper and lower bounds in (1.22), respectively. We have that $L_{i-1} = U_{i-1}/(1+Z_i)$, where $Z_i = \frac{1-\delta}{\delta} \cdot [1 - h_i(K)] \cdot (1 - \mathbf{E}[\beta|\psi_{i-1}] \cdot G(p|\psi_{i-1}))$. We can see that the percentage gap, $\frac{U_{i-1}-L_{i-1}}{U_{i-1}} = \frac{Z_i}{1+Z_i}$, goes to zero as $K \to \infty$. The absolute gap, $U_{i-1} - L_{i-1} = \frac{Z_i}{1+Z_i} \cdot U_{i-1}$, also goes to zero if $s = 0$. When $s > 0$, the size of the absolute gap depends on $Z_i \cdot K$.

The condition $\frac{\partial}{\partial K} V_i(K) \geq s$ should be satisfied by definition, because we assume that the seller can always salvage the capacity at marginal value s, and thus s should be the lowest marginal value for $V_i(K)$. This means that *the bounds will perform particularly well at the beginning of the selling season*, when the capacity is relatively large compared with the average buying quantity. The gap is also decreasing in δ, $\mathbf{E}[\beta|\psi_{i-1}]$, and $G(p|\psi_{i-1})$. In other words, the bounds are closer to the true value function when the leftover capacity is more likely to be salvaged, buyers are more powerful on average, and buyers are more likely to engage in price bargaining.

1.5.5 Discussion

Note that $H_i(K)$ can be written in the form of $a' \cdot \int_0^{K\cdot a''} D dF(D|\psi_{i-1})$, where a' and a'' are parameters independent of K. Hence, the upper and lower bounds given by Theorem 1.1 and 1.2 can all be written in the form of $a \cdot S + a' \cdot \int_0^{K\cdot a''} D dF(D|\psi_{i-1})$, where $a, a',$ and a'' are parameters independent of K. Moreover, we learn from Eq. (1.14) that the value function of a single-period problem takes a similar form. Therefore, we are basically approximating the value function by a single-period problem in which the seller treats the next buyer as the last one. This is very likely to be the mental heuristic used by a salesperson. More importantly, the approximations in both cases suggest that the shape of the value function depends much on the demand distribution of the next buyer, which supports our conjecture in Section 1.5.1. We find that if the demand is normally distributed, we then have a *convex-concave* value function as described in Proposition 1.2.

Proposition 1.3. *If the demand is normally distributed, then the bounds are all convex-concave.*

Other distributions—for example, any unimodal distribution—may also generate an S-shaped value function, and it is quite natural to expect a unimodal demand distribution.

1.5.6 Plotting the Price Curve

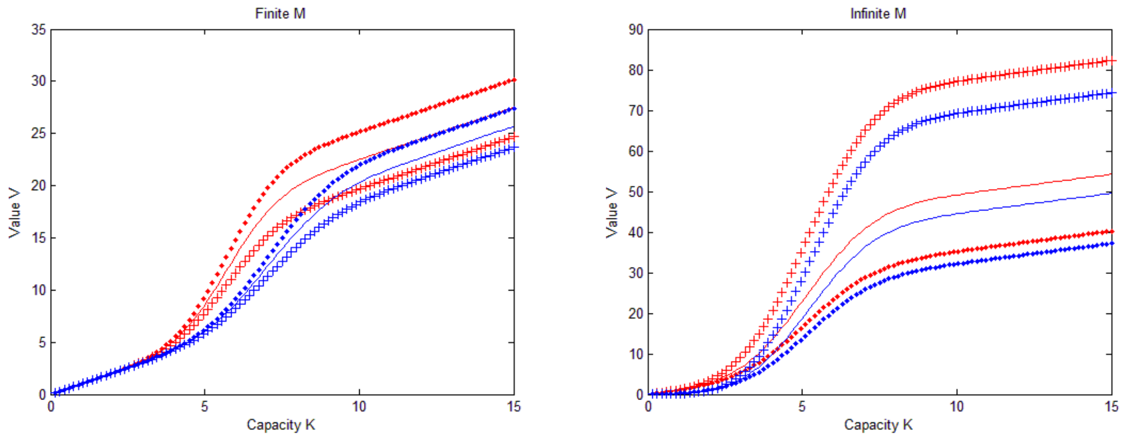
From (1.6) we know that the negotiated transaction price w_i is a linear combination of both parties' outside options. The seller's outside option is the average opportunity cost of selling Q_i, which depends on Q_i, capacity K_i, and the value function. In this section, we first investigate the performance and the shape of the bounds given a normal demand distribution and then try to plot the price curve using different parameter settings.

Without loss of generality, we consider negotiating with buyer i for quantity Q_i at time t_i. As in the regression models, we control for capacity level, bargaining power, posted price, demand uncertainty, as well as all the other buyer-, product-, and market-related factors. In the base case, we set $K = 10$, $\beta_i = 0.8$, $\mathbf{E}\left[\beta|\psi_i\right] = \beta_i$, $p = 8$, $s = 1$, $c_L = 6$, $\bar{c}_i = 7$, $G\left(\bar{c}|\psi_i\right) = 1 - \exp\left(c_L - \bar{c}\right)$, and $\eta = 0.9$. We assume that arrival rates satisfy $\frac{\lambda_i}{\lambda_0} = \frac{M-i}{i^2}$, which means that buyer arrival rate is linear in the number of potential buyers and the technology-shock arrival rate increases quadratically in the number of buyers that have arrived. In addition, we assume that the demand of the next buyer is normally distributed with mean μ and standard deviation $\sigma = \mu \cdot C_V$, where $C_V = 0.25$ is a constant. When observing Q_i, the seller updates belief and set $\mu = \min\left\{12, 16 - s_m \times Q_i\right\}$ where s_m captures the market structure—larger s_m means more concentrated demand. This way of updating means that if buyer i is very large, the rest of the buyers are likely to be small, especially when the seller knows in advance the market structure and the identities of the buyers. F_0 is normal distribution with mean 12 and standard deviation $12 \cdot C_V$. Finally, we set $h_i(K) \approx 1 - s \cdot \mathbf{E}\left[D_i|\psi_{i-1}\right] / V_{i-1}\left(K\right)$. See the proof of Theorem 1.2 for justifications.

In Figure 1.5, we present three numerical examples of the bounds for both finite and infinite M. We can see that the bounds for finite M perform better with smaller M; the performance of the bounds for infinite M depends on the assumption of δ, and they work better with larger δ. In both cases, the bounds are S-shaped given the normal demand distribution.

In Figure 1.6, we use the upper bounds in each case (of finite vs. infinite M) as the approxima-

Figure 1.5: Illustrations of bounds for the value function.



Note. The three sets of lines illustrate the upper and lower bounds of V_i for different M and δ. In the case of finite M: "+" for $M - i = 3$; "–" for $M - i = 4$; "•" for $M - i = 5$. In the case of infinite M: "+" for $\delta = 0.2$; "–" for $\delta = 0.3$; "•" for $\delta = 0.4$.

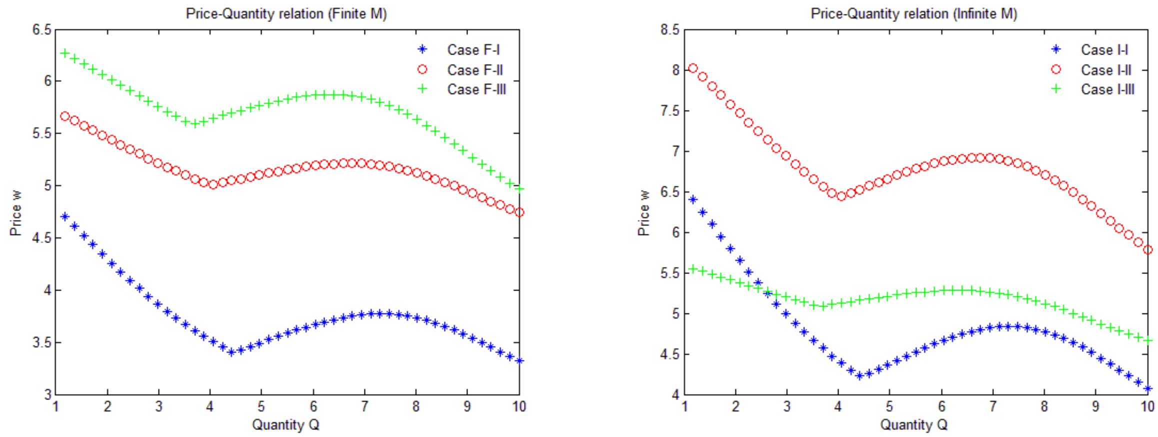
tion of the value function and generate the negotiated price for three different scenarios. With an S-shaped value function, we obtain a price-quantity curve in all scenarios that is reversed-N-shaped, which is consistent with our empirical observation.

1.6 Managerial Implications

So far, we have found the existence of a non-monotonic price-quantity relation in the microprocessor market and have established its plausibility by building a theoretic model and generating a pattern that is consistent with our empirical observation. In addition, we compared our model with other possible models in terms of goodness-of-fit to the data and found that our model is much more likely to to be the correct one. According to our model, the reason some buyers are receiving lower discounts than who buy less is simple: large buyers accelerate the selling process and small buyers are helpful in finishing the residual capacity. However, satisfying mid-sized buyers is costly because after doing so, it would be unlikely to satisfy the next buyer and profit from the remaining capacity.

In this section, we discuss the managerial implications for the seller to allocate the capacity given such a non-monotonic price-quantity relation. Basically, it is in the seller's best interest to avoid mid-sized transactions by increasing or decreasing the capacity allocated to these buyers. Also,

Figure 1.6: Model-generated price-quantity relation.



Note. Case F-I: $\beta_i = 0.8$; $M = 3$; $s_m = 0.9$; $C_V = 0.2$. Case I-I: $\beta_i = 0.8$; $\delta = 0.4$; $s_m = 0.9$; $C_V = 0.2$.
Case F-II: $\beta_i = 0.5$; $M = 3$; $s_m = 1$; $C_V = 0.25$. Case I-II: $\beta_i = 0.5$; $\delta = 0.4$; $s_m = 1$; $C_V = 0.25$.
Case F-III: $\beta_i = 0.5$; $M = 5$; $s_m = 1.1$; $C_V = 0.3$. Case I-III: $\beta_i = 0.5$; $\delta = 0.6$; $s_m = 1.1$; $C_V = 0.3$.

when decisions such as posted prices are made *ex ante*—especially those that are not determined by salespeople but are related to the price-quantity relation—the seller should be aware of the non-monotonicity and should not underestimate the price paid by these buyers.

1.6.1 Dynamic Capacity Rationing

The first implication suggests that the seller should control the capacity that is allocated to each buyer when the flexibility exists. Given the complexity of the value function and the price-quantity relation, it is not immediately clear whether the seller should increase or decrease the transaction quantity. Based on our model, we derive a simple rule for deciding the quantity in the following proposition.

Proposition 1.4. *The seller should increase Q_i if $\bar{c}_i > V_i'(K - Q_i)$ and decrease if $\bar{c}_i < V_i'(K - Q_i)$.*

The above result suggests that the rationing decision depends on inventory level, quantity, demand distribution, and the buyer's effective procurement cost of an alternative product. If we hold $\bar{c}_i$ constant, then quantity reduction is most likely to happen if $K - Q_i$ is close to the mean of the demand from the next buyer; otherwise, the seller should sell as much as possible. The logic is straightforward: to avoid losing the next major buyer due to insufficient capacity. Mathematically, due to the shape of the value function, it is more likely to have high $V_i'(K - Q_i)$ when $K - Q_i$ is neither too high nor too low. On the other hand, if we hold $V_i'(K - Q_i)$ constant, then quantity reduction is more likely to happen when the buyer has a higher outside option (and thus lower $\bar{c}_i$). The logic is clear: reserve the capacity for buyers who pay higher margins. Overall, the lesson is that a decision cannot be solely based on the capacity level or a buyer's quantity and that incorrect assumptions on the value function lead to suboptimal decisions.

1.6.2 Posted-Price Optimization

We learn from Lemma 1.1 that the posted price determines not only the price a buyer pays but also the number of price-takers. A price that is too low undercuts the seller's profitability; a price that is too high encourages more buyers to engage in bargaining. Hence, the posted price is an important trade-off. Our model can be used by sellers to optimize the posted price while considering such a trade-off and a potentially non-monotonic price-quantity relationship. The optimal price is $p^* = \arg\max_p V_0(\kappa, p)$.

1.6.3 Implications for Other Industries

There are other industries that resemble the semiconductor industry in terms of the key features. For example, in the travel industry, airline companies and hotels have limited capacities for a particular flight or date and these capacities should be sold within a limited period. Customers often include bulk buyers such as travel agencies and resellers who may purchase different quantities and prices are also normally negotiated. Hence, the implications of our study may carry over to such businesses. Other examples may include movie theaters, art performances, and sports events among others, if capacities are sold to agencies or resellers. The main differences between the semiconductor industry and others are that the obsolescence date is stochastic for the former and deterministic for the latter and that purchase quantities may be subject to negotiations in other industries. However, other industries have equivalent stochastic obsolescence dates as long as the inter-arrival time of bulk buyers is stochastic (i.e., the seller is not sure if another buyer will come along before the capacity is salvage). In addition, even if the quantity is subject to negotiation, we may still observe a non-monotonic price-quantity relationship in other industries, because given a fixed capacity, selling mid-sized quantities is still the most costly for the seller. Theoretically, Lemma 1.1 and Eq. (6) still hold even if both price and quantity are determined in Nash bargaining. Therefore, the formulation of the value function is unchanged, with the exception of the quantity $Q_i = \rho_i^* D_i$, which is determined by the negotiation and does not affect subsequent analysis for the bounds and shape of the value function.

1.7 Concluding Remarks

In this data-driven research, we study the price-quantity relation in B2B markets where the product life cycle is short and prices are set through one-shot negotiations. Using data from the microprocessor market, we found that, statistically, the transaction price can be a non-monotonic function of the transaction quantity. Contrary to our intuition, larger quantities—in a certain range—can actually lead to higher prices through negotiations. We showed the robustness of this statistical result with multiple linear regression models. While existing theories cannot explain our observation, we built an analytical model that allows us to dig into this phenomenon and understand the rationale behind it.

Our analysis reveals that it is fairly plausible for the price-quantity relation to be non-monotonic. One sufficient condition for a non-monotonic price-quantity curve is the value function of the seller being first convex and then concave in capacity. Although we normally assume that the value function is increasing and concave in capacity, our model shows that this need not be true in B2B markets. Instead, if the demand is normally distributed—which is often the case—the value function is likely to be convex-concave. More importantly, we found that a convex-concave value function is enough to explain our empirical observation: an N-shaped discount curve. We confirmed this finding by generating a price-quantity curve that is reversed N-shaped, using our model and the assumption of normally distributed demand.

Such a non-monotonic price-quantity relation has useful implications for a seller regarding dynamic capacity rationing (or revenue management) and posted pricing. The model can be extended in many ways to further study these issues. Ultimately, this study provides a new theory of non-monotonic price-quantity relations in B2B markets of short life-cycle products. Though price bargaining is complicated in practice, our model shows that the shape of the value function could be a sufficient explanation for the non-monotonic relation. We leave it to future researchers to explore other perspectives on this issue.

Appendix

Proof of Lemma 1.1

If $\bar{c}_i > p$, then $\Pi_i(w_i) - d_i = (p - w_i)\, Q_i$ and $\Pi_A(w_i) - d_A = (w_i - p)\, Q_i$. Hence, it must be that $w_i = p$. If $\bar{c}_i \leq p$, $\Pi_i(w_i) - d_i = (\bar{c}_i - w_i)\, Q_i$ and $\Pi_A(w_i) - d_A = w_i Q_i + V_i\,(K_i - Q_i, p) - V_i\,(K_i, p)$. Hence, $\Pi_i(w_i) - d_i = (\bar{c}_i - w_i)\, Q_i = \beta_i\,(\Pi_i(w_i) - d_i + \Pi_A(w_i) - d_A) = \beta_i\,(\bar{c}_i Q_i + V_i\,(K_i - Q_i, p) - V_i\,(K_i, p))$, which results in (1.6). ■

Proof of Proposition 1.1

Suppose $V_i\,(x)$ is concave for any $x \in [0, K]$. Thus, we have $V_i'(x) < V_i'\,(K - Q_i)$ for any $x \in [K - Q_i, K]$. As a result, we have

$$\frac{V_i(K) - V_i\,(K - Q_i)}{Q_i} = \frac{1}{Q_i}\int_{K-Q_i}^{K} V_i'(x)dx < \frac{1}{Q_i}\int_{K-Q_i}^{K} V_i'\,(K - Q_i)\, dx = V_i'\,(K - Q_i)\,.$$

Similarly, we can get the result for $V_i\,(x)$ being convex for any $x \in [0, K]$. ■

Proof of Proposition 1.2

Let $L(x) = \frac{V_i(K) - V_i(x)}{K - x}$ and we have $L(0) > V_i'(0)$ and $\lim_{x \to K} L(x) = V_i'(K)$. By continuity, there exist $x'' \in (0, K)$ such that $L(x) > V_i'(x)$ for $\forall x \in [0, x'')$. We claim that $x'' < K$ and $L(x) \leq V_i'(x)$ for some $x \in [x'', K]$. Suppose this claim is not true. We have $L'(x) = \frac{1}{K - x}\,[L(x) - V_i'(x)] > 0$ for $\forall x \in [0, K]$ and thus $L(x)$ is strictly increasing on $[0, K]$. In addition, because $V_i\,(x)$ is concave on $[x', K]$, we have $V_i'\,(x) > V_i'\,(K) > L(x)$ on $[x', K)$, which is a contradiction.

Now let $x'' = \min\{x \in [0, K) : L(x) \leq V_i'(x)\}$. By continuity, we must have $L(x'') = V_i'(x'')$, which indicates $L'(x'') = 0$. Suppose $x' < x''$. Concavity requires that $V_i'\,(x) > V_i'(x'') = L(x'') > L(x)$ on (x', x''), which is a contradiction. Hence, $x' \geq x''$. Now, suppose $\exists x_0 \in (x'', K)$ such that $L(x_0) > V_i'(x_0)$. We consider two cases: (I) $x' \leq x_0$ and (II) $x' > x_0$. In case (I), $V_i'(x)$ decreases for all $x > x_0$ by concavity. However, $L(x)$ increases as long as $L(x) > V_i'(x)$. In order to have $\lim_{x\to K} L(x) = V_i'(K)$, we need that $L(x)$ decreases while $L(x) > V_i'(x)$, which is a contradiction. In case (II), we have that $x'' < x_0 < x'$ and $V_i'(x)$ increases for all $x \leq x_0$ by convexity. However, $L'(x'') = 0 < V_i''(x'')$, so by continuity there exist $\epsilon > 0$ such that

$$\frac{L(x''+\epsilon) - L(x'')}{\epsilon} < \frac{V_i'(x''+\epsilon) - V_i'(x'')}{\epsilon}.$$

Therefore, in order to have $L(x_0) > V_i'(x_0)$, we need that $L(x)$ increases while $L(x) < V_i'(x)$, which is a contradiction. As a result, $L(x) \leq V_i'(x)$ for all $x \in [x', K)$. Suppose $L(x) = V_i'(x)$ for all $x \in [x', K)$. We then have $L(x) = \lim_{x\to K} L(x) = V_i'(K)$ for all $x \in [x', K)$, but $V_i'(x) > V_i'(K)$ for some $x \in [x', K)$ by concavity, which is a contradiction. The result follows. ■

Proof of Lemma 1.2

First,

$$\int_0^{+\infty} DdF\left(D|\psi\right) = \int_0^{K} DdF\left(D|\psi\right) + \int_K^{K/\eta} DdF\left(D|\psi\right) + \int_{K/\eta}^{+\infty} DdF\left(D|\psi\right) < \infty$$

for any $K > 0$. Second, $\int_0^{+\infty} DdF\left(D|\psi\right) = \lim_{K\to\infty} \int_0^{K} DdF\left(D|\psi\right)$. Thus,

$$\lim_{K\to\infty} \int_K^{K/\eta} DdF\left(D|\psi\right) = \lim_{K\to\infty} \int_{K/\eta}^{+\infty} DdF\left(D|\psi\right) = 0.$$

Furthermore, $\int_K^{K/\eta} KdF\left(D|\psi\right) \leq \int_K^{K/\eta} DdF\left(D|\psi\right)$, so $\lim_{K\to\infty} \int_K^{K/\eta} KdF\left(D|\psi\right) = 0$. ■

Proof of Theorem 1.1

Part I. Let's start with the proof of the lower bound. To prove by induction, we suppose for $i < M$ that

$$V_i\left(K\right) \geq sK + \left(1 - \delta_{i+1}^{l}\right) \cdot \nu(p, \psi_i) \cdot \int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right).$$

Now, using (1.13), we have

$$
\begin{aligned}
V_{i-1}(K) \geq & sK + (1-\delta_i) \cdot \\
& \left\{ \left(1-\delta_{i+1}^l\right) \cdot \mathbf{E}_{\beta,\bar{c}}\left[\nu(p,\psi_i)|\psi_{i-1}\right] \cdot \int_{K/\eta}^{+\infty} \int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \right. \\
& + \left(1-\delta_{i+1}^l\right) \cdot \mathbf{E}_{\beta,\bar{c}}\left[\nu(p,\psi_i)|\psi_{i-1}\right] \cdot \int_0^{K/\eta} \int_0^{(K-D')^+\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \\
& \left. + \nu(p,\psi_{i-1}) \cdot \int_0^{K/\eta} D'dF\left(D'|\psi_{i-1}\right) \right\} \\
= & sK + (1-\delta_i) \cdot \nu(p,\psi_{i-1}) \cdot \left\{ \left(1-\delta_{i+1}^l\right) \cdot \int_0^{+\infty} \int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \right. \\
& - \left(1-\delta_{i+1}^l\right) \cdot \int_0^{K/\eta} \int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \\
& + \left(1-\delta_{i+1}^l\right) \cdot \int_0^{K/\eta} \int_0^{(K-D')^+\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \\
& \left. + \int_0^{K/\eta} D'dF\left(D'|\psi_{i-1}\right) \right\} \\
= & sK + (1-\delta_i) \cdot \nu(p,\psi_{i-1}) \cdot \left\{ \left(1-\delta_{i+1}^l\right) \cdot \int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right) \right. \\
& - \left(1-\delta_{i+1}^l\right) \cdot \int_0^{K/\eta} \int_{(K-D')^+\cdot\lambda^{M-i}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \\
& \left. + \int_0^{K/\eta} D'dF\left(D'|\psi_{i-1}\right) \right\}.
\end{aligned}
$$

Here for the first equality, we add and subtract $\left(1-\delta_{i+1}^l\right) \cdot \int_0^{K/\eta} \int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right)$ in the curly braces. Further, we have

$$
\begin{aligned}
& \int_0^{K/\eta} \int_{(K-D')^+\cdot\lambda^{M-i}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \\
& \leq \int_0^{+\infty} \int_{(K-D')^+\cdot\lambda^{M-i}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_i\right) dF\left(D'|\psi_{i-1}\right) \\
& = \mathbf{E}\left[\mathbf{E}\left[D \cdot \mathbb{I}\left\{(K-D')^+ \cdot \lambda^{M-i} \leq D \leq K \cdot \lambda^{M-i}\right\}|\psi_i\right]|\psi_{i-1}\right] \\
& = \mathbf{E}\left[D \cdot \mathbb{I}\left\{(K-D')^+ \cdot \lambda^{M-i} \leq D \leq K \cdot \lambda^{M-i}\right\}|\psi_{i-1}\right] \\
& = \int_0^{+\infty} \int_{(K-D')^+\cdot\lambda^{M-i}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right) dF\left(D'|\psi_{i-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \int_0^{K\cdot\lambda^{M-i}} \int_{K-D/\lambda^{M-i}}^{+\infty} DdF\left(D'|\psi_{i-1}\right) dF\left(D|\psi_{i-1}\right) \\
&= \int_0^{K\cdot\lambda^{M-i}} D\cdot\left[1-F\left(K-D/\lambda^{M-i}|\psi_{i-1}\right)\right] dF\left(D|\psi_{i-1}\right) \\
&\le \int_0^{K\cdot\lambda^{M-i+1}} D\cdot\left[1-F\left(K-D/\lambda^{M-i}|\psi_{i-1}\right)\right] dF\left(D|\psi_{i-1}\right) + \int_{K\cdot\lambda^{M-i+1}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right) \\
&\le \left[1-F\left(K-\lambda K|\psi_{i-1}\right)\right]\cdot \int_0^{K\cdot\lambda^{M-i+1}} DdF\left(D|\psi_{i-1}\right) + \int_{K\cdot\lambda^{M-i+1}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right) \\
&\le \left[1-F_0\left(K-\lambda K\right)\right]\cdot \int_0^{K\cdot\lambda^{M-i+1}} DdF\left(D|\psi_{i-1}\right) + \int_{K\cdot\lambda^{M-i+1}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right).
\end{aligned}
$$

Here, we first extend the range of integral for D' to $[0,+\infty)$ given that D and F are both positive. Next, we rewrite the double integral as iterated expectations. Third, we apply the law of iterated expectations. Fourth, we write the expectation as a double integral again. Fifth, we change the sequence of integral and then simplify the expression in the next step. Seventh, we split the integral into two parts and apply $F\left(K-D/\lambda^{M-i}|\psi_{i-1}\right)\le 1$ for the part from $K\cdot\lambda^{M-i+1}$ to $K\cdot\lambda^{M-i}$. Eighth, given $0\le D\le K\cdot\lambda^{M-i+1}$, we have $F\left(K-\lambda K|\psi_{i-1}\right)\le F\left(K-D/\lambda^{M-i}|\psi_{i-1}\right)$. Finally, by assumption, we have $F_0\left(K-\lambda K\right)\le F\left(K-\lambda K|\psi_{i-1}\right)$. Now we can write

$$
\begin{aligned}
V_{i-1}\left(K\right) \ge & sK + \left(1-\delta_i\right)\cdot\nu(p,\psi_{i-1})\cdot\left\{\left(1-\delta_{i+1}^l\right)\cdot\int_0^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right)\right. \\
& -\left(1-\delta_{i+1}^l\right)\cdot\int_{K\cdot\lambda^{M-i+1}}^{K\cdot\lambda^{M-i}} DdF\left(D|\psi_{i-1}\right) \\
& -\left(1-\delta_{i+1}^l\right)\cdot\left[1-F_0\left(K-\lambda K\right)\right]\cdot\int_0^{K\cdot\lambda^{M-i+1}} DdF\left(D|\psi_{i-1}\right) \\
& \left.+\int_0^{K/\eta} DdF\left(D|\psi_{i-1}\right)\right\} \\
= & sK + \left(1-\delta_i\right)\cdot\nu(p,\psi_{i-1})\cdot\left\{\left(1-\delta_{i+1}^l\right)\cdot F_0\left(K-\lambda K\right)\cdot\int_0^{K\cdot\lambda^{M-i+1}} DdF\left(D|\psi_{i-1}\right)\right. \\
& \left.+\int_0^{K/\eta} DdF\left(D|\psi_{i-1}\right)\right\} \\
\ge & sK + \left[1-\delta_i+\left(1-\delta_i\right)\cdot\left(1-\delta_{i+1}^l\right)\cdot F_0\left(K-\lambda K\right)\right]\cdot\nu(p,\psi_{i-1})\cdot\int_0^{K\cdot\lambda^{M-i+1}} DdF\left(D|\psi_{i-1}\right) \\
= & sK + \left(1-\delta_i^l\right)\cdot\nu(p,\psi_{i-1})\cdot\int_0^{K\cdot\lambda^{M+1-i}} DdF\left(D|\psi_{i-1}\right).
\end{aligned}
$$

Part II. Let's now check the proof of the upper bound. First, for $i=M$, we easily have

$$
V_{M-1}\left(K\right)\le sK+\left(1-\delta_M\right)\cdot\nu(p,\psi_{M-1})\cdot\int_0^{K/\eta} DdF\left(D|\psi_{M-1}\right)
$$

according to (1.14). Then suppose we have the upper bound for $V_i\left(K\right)$: $sK+\bar{V}_i\left(K\right)$. Accordingly, we can use this in (1.13) to get

$$
\begin{aligned}
V_{i-1}(K) \leq \delta_i sK + (1-\delta_i) \cdot & \\
& \left\{ \int_0^1 \int_{K/\eta}^{+\infty} \int_{c_L}^{+\infty} \left[sK + \bar{V}_i(K) \right] dG(\bar{c}|\psi_{i-1}) \, dF(D|\psi_{i-1}) \, dB(\beta|\psi_{i-1}) \right. \\
& + \int_0^1 \int_0^{K/\eta} \int_p^{+\infty} \left[p\rho^* D + s(K - \rho^* D) + \bar{V}_i(K) \right] dG(\bar{c}|\psi_{i-1}) \, dF(D|\psi_{i-1}) \, dB(\beta|\psi_{i-1}) \\
& + \int_0^1 \int_0^{K/\eta} \int_{c_L}^{p} \left[sK + \bar{V}_i(K) - (1-\beta) s\rho^* D \right] dG(\bar{c}|\psi_{i-1}) \, dF(D|\psi_{i-1}) \, dB(\beta|\psi_{i-1}) \\
& \left. + \int_0^1 \int_0^{K/\eta} \int_{c_L}^{p} (1-\beta) \bar{c}\rho^* D dG(\bar{c}|\psi_{i-1}) \, dF(D|\psi_{i-1}) \, dB(\beta|\psi_{i-1}) \right\} \\
& = sK + (1-\delta_i) \cdot \left\{ \mathbf{E}_{\bar{c},D,\beta} \left[\bar{V}_i(K) \, | \psi_{i-1} \right] \right. \\
& + \int_0^1 \int_0^{K/\eta} \int_p^{+\infty} (p-s)\rho^* D dG(\bar{c}|\psi_{i-1}) \, dF(D|\psi_{i-1}) \, dB(\beta|\psi_{i-1}) \\
& \left. + \int_0^1 \int_0^{K/\eta} \int_{c_L}^{p} (1-\beta)(\bar{c}-s)\rho^* D dG(\bar{c}|\psi_{i-1}) \, dF(D|\psi_{i-1}) \, dB(\beta|\psi_{i-1}) \right\} \\
& \leq sK + (1-\delta_i) \cdot \left\{ \mathbf{E}_{\bar{c},D,\beta} \left[\bar{V}_i(K) \, | \psi_{i-1} \right] + \nu(p, \psi_{i-1}) \cdot \int_0^{K/\eta} D dF(D|\psi_{i-1}) \right\} \\
& = sK + (1 - \delta_i + (1-\delta_i)(1-\delta_{i+1}^u)) \cdot \nu(p, \psi_{i-1}) \cdot \int_0^{K/\eta} D dF(D|\psi_{i-1}) .
\end{aligned}
$$

Note that we use the fact that $\int_0^{(K-\rho^* D)/\eta} DdF(D|\psi_i) \leq \int_0^{K/\eta} DdF(D|\psi_i)$ for the first inequality. We use that $\rho^* \leq 1$ for the second inequality given that $p > s$ and $\bar{c} > s$.■

Proof of Theorem 1.2

Let $G_{i-1}(\bar{c}) = G(\bar{c}|\psi_{i-1})$, $F_{i-1}(D) = F(D|\psi_{i-1})$, and $B_{i-1}(\beta) = B(\beta|\psi_{i-1})$. We first divide both sides of Eq. (1.13) by $1-\delta$ and add to the right side

$$
\begin{aligned}
\int_0^{K/\eta} V_i(K) \, dF_{i-1}(D) - \int_0^1 \int_0^{K/\eta} \int_p^{+\infty} V_i(K) \, dG_{i-1}(\bar{c}) \, dF_{i-1}(D) dB_{i-1}(\beta) & \\
- \int_0^1 \int_0^{K/\eta} \int_{c_L}^{p} V_i(K) \, dG_{i-1}(\bar{c}) \, dF_{i-1}(D) dB_{i-1}(\beta) = 0. &
\end{aligned}
$$

We obtain

$$
\begin{aligned}
\frac{V_{i-1}(K)}{1-\delta} &= \frac{\delta}{1-\delta}\cdot s\cdot K+\mathbf{E}\left[V_i(K)\,|\psi_{i-1}\right] \\
&+ \int_0^1\int_0^{K/\eta}\int_p^{+\infty}\left[V_i(K-\rho^*D)-V_i(K)\right]dG_{i-1}(\bar{c})\,dF_{i-1}(D)dB_{i-1}(\beta) \\
&+ \int_0^1\int_0^{K/\eta}\int_{c_L}^{p}(1-\beta)\left[V_i(K-\rho^*D)-V_i(K)\right]dG_{i-1}(\bar{c})\,dF_{i-1}(D)dB_{i-1}(\beta) \\
&+ p\int_0^1\int_0^{K/\eta}\int_p^{+\infty}\rho^*DdG_{i-1}(\bar{c})\,dF_{i-1}(D)dB_{i-1}(\beta) \\
&+ \int_0^1\int_0^{K/\eta}\int_{c_L}^{p}(1-\beta)\,\bar{c}\rho^*DdG_{i-1}(\bar{c})\,dF_{i-1}(D)dB_{i-1}(\beta).
\end{aligned}\tag{1.23}
$$

Because $\mathbf{E}\left[V_i(K)\,|\psi_{i-1}\right]=V_{i-1}(K)$, $V_i(K-\rho^*D)\le V_i(K)$, and $\rho^*\le 1$, we have $\frac{\delta}{1-\delta}\cdot\left[V_{i-1}(K)-s\cdot K\right]\le 0+0+H_i(K)$, which results in $V_{i-1}(K)\le s\cdot K+\frac{1-\delta}{\delta}\cdot H_i(K)$.

To derive the lower bound, we start from (1.23) and use the optimality of ρ^*. We have

$$
\begin{aligned}
&\int_0^1\int_0^{K/\eta}\int_p^{+\infty}\left[p\rho^*D+V_i(K-\rho^*D)\right]dG_{i-1}(\bar{c})\,dF_{i-1}(D)dB_{i-1}(\beta) \\
\ge &\int_0^1\int_0^{K/\eta}\int_p^{+\infty}\left[pD+V_i\left([K-D]^+\right)\right]dG_{i-1}(\bar{c})\,dF_{i-1}(D)dB_{i-1}(\beta).
\end{aligned}
$$

Similarly, we apply this logic to the case of $\bar{c}_i\le p$. As a result, we get

$$
\begin{aligned}
\frac{\delta}{1-\delta}\cdot\left[V_{i-1}(K)-s\cdot K\right] &\ge (1-\mathbf{E}[\beta|\psi_{i-1}]\cdot G_{i-1}(p))\cdot\int_0^{K/\eta}\left[V_i\left([K-D]^+\right)-V_i(K)\right]dF_{i-1}(D) \\
&\quad +H_i(K) \\
&\ge H_i(K)-(1-\mathbf{E}[\beta|\psi_{i-1}]\cdot G_{i-1}(p))\cdot\left[1-h_i(K)\right]V_{i-1}(K).
\end{aligned}
$$

For the last inequality above, we use the fact that $V_i\left([K-D_i]^+\right)\le V_i(K)$ and

$$
\begin{aligned}
&\int_0^{+\infty}\left[V_{i-1}(K)-V_i\left([K-D]^+\right)\right]dF_{i-1}(D) \\
\ge &\int_0^{K/\eta}\left[V_{i-1}(K)-V_i\left([K-D]^+\right)\right]dF_{i-1}(D).
\end{aligned}
$$

Hence, we have $V_{i-1}(K)\ge\left(sK+\frac{1-\delta}{\delta}\cdot H_i(K)\right)/\left(1+\frac{1-\delta}{\delta}\cdot(1-\mathbf{E}[\beta|\psi_{i-1}]\cdot G_{i-1}(p))\cdot[1-h_i(K)]\right)$.

To show $\lim_{K\to\infty}h_i(K)=1$, we need to check two cases: $s=0$ and $s\ne 0$. If $s=0$, then we know from the upper bound that $V_i(K)$ is bounded by $H_{i+1}(K)$, which is bounded as $K\to\infty$. In this case, we can show that from the ψ_{i-1} point of view, $V_i\left([K-D_i]^+\right)$ converges in probability to $V_i(K)$ as $K\to\infty$. To this end, note that given any ψ_i both $V_i\left([K-D_i]^+\right)$ and $V_i(K)$ are increasing in K but are bounded. Hence, they converge to the same limit $\bar{C}(\psi_i)$, and for any $\epsilon>0$, there exist $K(\psi_i) < \infty$ such that $|V_i(K) - V_i\left([K - D_i]^+\right)| < \epsilon$. Because $K(\psi_i)$ is finite, there exist $K^*_{\epsilon,\xi} < \infty$ for any $\xi > 0$ such that $\Pr\left\{K^*_{\epsilon,\xi} < K(\psi_i)\,|\psi_{i-1}\right\} < \xi$. In other words, for any $\epsilon > 0$ and $\xi > 0$, there exist $K^*_{\epsilon,\xi} < \infty$ such that for $K \geq K^*_{\epsilon,\xi}$ we have

$$\Pr\left\{V_i(K) - V_i\left([K - D_i]^+\right) > \epsilon|\psi_{i-1}\right\} < \xi.$$

Therefore, $V_i\left([K - D_i]^+\right)$ converges in probability to $V_i(K)$ and thus

$$\mathbf{E}\left[V_i\left([K - D_i]^+\right)|\psi_{i-1}\right] \to \mathbf{E}\left[V_i(K)\,|\psi_{i-1}\right].$$

If $s \neq 0$, then $V_i(K)$ is unbounded. However, we know from the upper bound that $V_i(K) - sK$ is bounded by $H_{i+1}(K)$. Because $\frac{\partial}{\partial K}V_i(K) \geq s$, we have that $V_i(K) - sK$ is increasing in K. As a result, $\tilde{V}_i(K) = V_i(K) - sK$ converges to a limit. Applying the same logic as for the case of $s = 0$, we know that $\mathbf{E}\left[\tilde{V}_i\left([K - D_i]^+\right)|\psi_{i-1}\right] \to \mathbf{E}\left[\tilde{V}_i(K)\,|\psi_{i-1}\right]$. Accordingly, we have

$$\mathbf{E}\left[V_i(K) - sK - V_i\left([K - D_i]^+\right) + s\,[K - D_i]^+\,|\psi_{i-1}\right] \to 0.$$

Since $\mathbf{E}\left[sK - s\,[K - D_i]^+\,|\psi_{i-1}\right] = \mathbf{E}\left[s \cdot \min\{K, D_i\}\,|\psi_{i-1}\right] \to \mathbf{E}\left[sD_i|\psi_{i-1}\right] < \infty$, we know that $\mathbf{E}\left[V_i(K) - V_i\left([K - D_i]^+\right)|\psi_{i-1}\right] \to \mathbf{E}\left[sD_i|\psi_{i-1}\right]$. Therefore,

$$\begin{aligned}
h_i(K) &= \frac{\mathbf{E}\left[V_i\left([K - D_i]^+\right)|\psi_{i-1}\right]}{\mathbf{E}\left[V_i(K)\,|\psi_{i-1}\right]} \\
&= 1 - \frac{\mathbf{E}\left[V_i(K) - V_i\left([K - D_i]^+\right)|\psi_{i-1}\right]}{\mathbf{E}\left[V_i(K)\,|\psi_{i-1}\right]} \\
&\to 1. \blacksquare
\end{aligned}$$

Proof of Proposition 1.3

Let the probability density function be $f(x) = a \cdot \exp\left(-\frac{(x-b)^2}{2c}\right)$. Note that the critical component in all the bounds is $\int_0^{K/\eta} D_i dF\left(D_i|\psi_{i-1}\right)$ and $\int_0^K D_i dF\left(D_i|\psi_{i-1}\right)$. Without loss of generality, we focus on $A(K) = \int_0^K x dF(x)$. The second-order condition gives $\frac{\partial^2 A}{\partial K^2} = f(K) + K \cdot f'(K)$. It is easy to check that $f'(x) = -\frac{x-b}{c} \cdot f(x)$. Hence, we have $\frac{\partial^2 A}{\partial K^2} = f(K) \cdot \left[1 - \frac{K(K-b)}{c}\right]$, which has zero points $K^*_{1,2} = \frac{b \pm \sqrt{b^2+4c}}{2}$. It is clear that only one non-negative zero point exists because $c > 0$. Therefore, $A(K)$ is convex-concave. Given sK is linear, we know that the bounds are all convex-concave. $\blacksquare$

Proof of Proposition 1.4

Note that $\Pi_A(w_i) = w_i(Q_i) \cdot Q_i + V_i(K - Q_i)$, where $w_i(Q_i)$ is given by (1.6). The first-order condition gives $\Pi_A(w_i)' = (1 - \beta_i) \cdot [\bar{c}_i - V_i'(K - Q_i)]$. The result follows. ■

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2 Now or Later? Selling to Strategic Bargainers with Limited Capacity in Business-to-Business Markets

Abstract

In this study we offer a new theory for the common observation of delay of agreements in business-to-business (B2B) markets by studying a strategic bargaining model wherein the seller and buyers negotiate over the price of a new product, possess symmetric information, and use different bargaining tactics. When proposing a counter-offer, a buyer threatens to wait and come back later before a deadline if the seller rejects. As a countermeasure, the seller threatens to sell part of the limited capacity to price-takers who may arrive later as a stochastic shock. We characterize the subgame perfect equilibrium, and we find that (1) it is credible for the buyer to wait until the last minute when the gain from trade is not high and that (2) it is optimal for the seller to reject any counter-offer and encourage the buyer to wait when switching is credible for the buyer (i.e., the buyer is willing to pay a low price) and when the capacity level is not sufficiently high. If the seller does not know when to encourage the buyer to wait and always settle the price at the beginning, our numerical study shows that the seller can lose more than 10% of its revenue. We also study how to optimize the posted price given the anticipated equilibrium of bargaining and show that incorrect anticipations regarding the timing of agreement lead to ineffective prices, which can be 8% lower than the optimal price.

[*Keywords*: business-to-business; price bargaining; dynamic game]

2.1 Introduction

In many business-to-business (B2B) markets, such as the markets for airplanes (Garvin 1991), medical devices (Grennan 2013), and microprocessors (Zhang et al. 2015), prices are normally determined by negotiations. In order to obtain a good price, firms use various bargaining tactics such as waiting and threatening to walk away. However, while there have been numerous efforts in the academic literature that study how various market structures, business strategies, and prior decisions affect bargaining position and profit allocation, fewer efforts have been taken to study how to make tactical decisions such as when to propose, reject, and accept counter-offers in the process of strategic bargaining.

Although traditional strategic bargaining models that build on Robinstein (1982) predict immediate agreement, a common tactic used by buyers to force the seller to accept lower prices is strategic waiting. It is useful for buyers when they believe that waiting or delaying the negotiation, although might be costly for themselves, can bring more benefits by resolving uncertainties. In some situations wherein information is asymmetric, buyers can carry out waiting and signal their low willingness to pay or "patience" (e.g., Admati and Perry 1987 and Cramton 1992). In many other situations wherein information is symmetric, the intuition is that actual waiting is not necessary because the consequence can be rationally expected and buyers just need to give out the threat of waiting if it is credible. However, waiting and delay of agreement are frequently observed in many practical situations where information should be symmetric (Friedenberg 2014). In the following, we present the evidences we obtained from the semiconductor industry.

In the microprocessor market, there are multiple competing sellers, such as Intel, Nvidia, and Advanced Micro Devices (AMD), selling multiple product lines primarily to large original equipment manufacturers (OEMs or buyers), such as Hewlett-Packard (HP), Lenovo, and Dell. Major buyers have significant bargaining power and they interact with the major sellers repetitively for multiple generations of products. The market structure is stable in a reasonably-long time frame, so they know each other's outside option quite well except for the uncertainties.

The seller often has to commit and reserve certain levels of capacity for each of the major buyers, but the purchase quantities proposed by the buyers later on can be different. Once the quantity is given, it is plausible to assume that information is symmetric between a buyer and a seller in the

Table 2.1: Summary Statistics for the 130 Observations

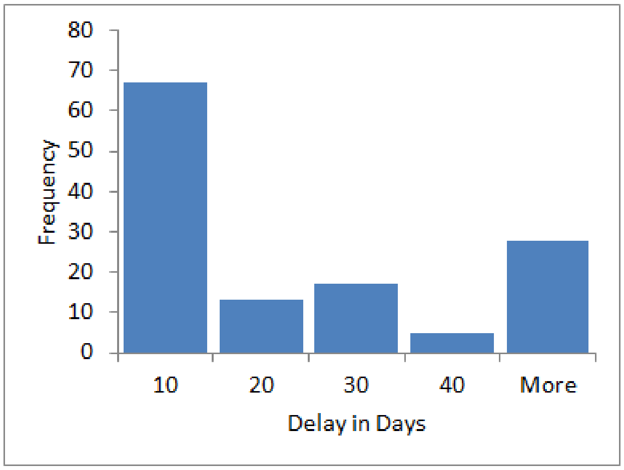
	Mean	Median	S.D.	Min	Max
Qty. of Free Sample	8.18	2	18.77	1	163
Total Purchase Qty.	3.01E6	1.22E5	9.45E6	155	8.84E7
Highest Price (US$)	143.25	84	216.42	7	1469
Waiting Time (days)	69.05	7	154.14	7	892

price bargaining. Other than the major buyers, some small buyers may arrive occationally and pay a high price due to weak bargaining power. The occasional demand serves as an outside option for the seller and thus strengthens the seller's bargaining position. However, it is unknown to both the buyers and the seller when the occasional demand will occur, and the likelihood of occurrence is learnt over time. Hence, waiting can be risky for the buyers, because the seller will have a better position if the occasional demand occurs and worse if otherwise, but there shouldn't be any extra cost for buyers to wait unless productions have to begin.

For this research, we interacted with managers of a major semiconductor company and obtained a data set that encompasses sales record of this company over a three-year period from early 2009 to early 2012. During this period, we have 130 observations that a customer arrived and received a few free samples before the price was finally settled and large quantities were purchased. In other words, in such an observation, the first sales record consists of a zero price and a small quantity, and starting from the second record the customer paid a positive price. This means that the customer arrived but did not settle the price immediately. Normally, the price negotiation starts when the free samples are offered and we define the waiting time or delay of pricing as the time interval between the first and the second sales records. In Table 2.1 we show some summary statistics for these observations, and in Figure 2.1 we show the distribution of waiting time. We can see that the average and the median waiting time before the price is settled are 69 and 7 days, respectively. Given that the major customers had already seen the prototypes in the design stage, the waiting indicates that the agreement is delayed. Note that such delays of pricing are not observed in other purchases.

Why waiting and delay exist for negotiations with symmetric information and how decisions should be made by firms in these situations? To answer these questions, we study in this chapter a strategic bargaining model with symmetric information in a market environment that is motivated

Figure 2.1: Histogram of waiting time.



by the semiconductor industry. While considering that buyers use the tactic of waiting, we study a particular countermeasure for the seller: when a buyer threatens to wait, the seller threatens to sell a portion of the capacity if the occasional demand occurs. We focus on situations wherein buyers and the seller negotiate over the price only once. Using this model, we learn that waiting benefits the buyer when the product is not very profitable such that switching to an alternative product will lead to a loss for the buyer. Hence, waiting is credible in those cases, because obtaining a lower price is very important. Interestingly, buyers' waiting can sometimes benefit the seller as well, becaues their waiting awards the seller the opportunity to gain higher profit and thus better utilize the capacity. When the seller prefers to settle the price later and waiting is credible for a buyer, the seller will reject any counter-offer and encourage the buyer to wait and thus waiting will actually happen.

Our results are important not only because they tell the seller how to negotiate with buyers in different situations, but also because they show the seller what to expect *ex ante*. When launching the new product, the seller often needs to set a posted price for the price-takers. We find that the optimal posted price depends on the timing of negotiation with the major buyers and thus failing to anticipate the right outcome leads to ineffective pricing. In addition, our model is able to predict a non-monotonic price-quantity relationship which is empirically identified by Zhang et al. (2014).

The rest of this chapter is organized in the following way. We present a brief literature review in Section 2.2. In Section 2.3 we construct a dynamic model of B2B price bargaining, and we analyzed this model and obtain the main results in Section 2.4. We then explore the property of the negotiated price in equilibrium in Section 2.5, and we show that our theory can offer an explanation for the empirical observation of non-monotonic price-quantity relationship documented in the recent literature. Later in Section 2.6, we discuss how to optimize the posted price for the seller and the consequences of failing to anticipate the result of price negotiation. In Section 2.7, we study a modified model wherein the seller allocates her capacity on the go to multiple major customers. Lastly, we conclude the chapter in Section 2.8. All the proofs are in the Appendix.

2.2 Related Literature

This study is mainly related to three streams of research in the literature. The first stream is about bargaining with delay of agreement, the second is multilateral bargaining in supply chains, and the third is posted pricing with the presence of both price-takers and bargainers.

Delays in reaching agreement are frequently observed in practice and studied in the literature. Cramton (1984) is among the earliest to study the phenomenon that trade often occurs after costly delay, and it is suggested that the need for learning each other's valuation under incomplete information results in rejections and thus the delay. Later, Admati and Perry (1987) and Cramton (1992) proposed slightly different models in which it is assumed that bargainers can signal the strength of their bargaining positions by delaying prior to making an offer. In the latest paper of this stream, Feng et al. (2014) predict delay in price-quantity contract settlement in supply chains where the demand information is known only to the buyer. In all these papers, there is an infinite horizon, and the delay of agreement is driven by information asymmetry. Roth et al. (1988) considered bargaining with deadlines and they documented some experimental evidences of last-minute agreements, which initiated further theoretical work. Based on this observation, Ma and Manove (1993) proposed a model of continuous-time, alternating-offer with a deadline and symmetric information. They assume that players can decide when to make an offer or counter-offer, but only after an exogenous, random delay due to information transmission and processing. Their model has a symmetric Markov-perfect equilibrium, unique at almost all nodes, in which players adopt strategic delay early in the game, make and reject offers later on, and reach agreements late in the game if at all. Notice that the player who makes an offer closer to the deadline is less likely to be rejected, so the delay in their model is driven by the desire to obtain a stronger bargaining power. We also consider a continuous-time, alternating-offer game with a deadline and symmetric information. However, our model has a different game structure, in which the bargainer can make a counter-offer immediately and threaten to wait if the seller rejects. As a countermeasure, the seller can threaten to sell part of the capacity to price-takers whose arrival is random and shapes the value of the capacity in a way. We show that sometimes it is credible for the bargainers to wait and it is also better for the seller to reject the bargainers and encourage them to wait till the last minute. Hence, our theory adds to the literature that explains the last-minute agreements.

Although our base model focuses on one-on-one bargaining in the B2B market, we also consider bargaining with multiple bargainers in a modified model. The literature of multilateral bargaining in B2B markets or supply chains is emerging in recent years and this stream of research normally study how the profit is allocated among supply chain members. While Dukes et al. (2006) and Lovejoy (2010) focus on the impact of channel or chain structure, several scholars consider the impact of bargaining sequence and coalitions. In an assembly chain setting, Nagarajan and Bassok (2008) consider suppliers who form multilateral bargaining coalitions and compete for a position in the bargaining sequence. Nagarajan and Sosic (2008) study the stability of coalition in assembly models. In a retail setting, Aydin and Heese (2014) study an assortment problem of a retailer that engages in simultaneous bilateral negotiations with all manufacturers for a given assortment. Different from the sequential models, in which bargaining power is manifested in the position in the sequence, simultaneous bilateral negotiation models focus on the static equilibrium in which the contribution of a bargainer to the entire system determines the profit allocation. Zhang et al. (2014) consider a sequential, bilateral negotiation model in a B2B setting where a seller allocates a limited capacity to a group of buyers sequentially and settles the price by Nash bargaining. In their model, firms do not bargain over a single pie, but the size of their respective pies is influenced by other firms through capacity allocation. In this study, we adopt a simultaneous bilateral bargaining model, and we supplement this branch of the literature by investigating the impact of the relative size of a customer's purchase quantity and offering an explain to the non-monotonic price-quantity relationships empirically identified in Zhang et al. (2014).

Price-takers and bargainers coexist in many B2B and retail markets, and it is important for sellers to take both types into consideration when optimizing the posted price. Gill and Thanassoulis

(2009) study a static model wherein they find that an increase in the proportion of bargainers can lower the consumer surplus overall. In contrast, Kuo et al. (2011) study a dynamic model wherein they focus on the dynamic pricing problem and the role of limited inventory when a firm sells to both price-taking and haggle-prone customers. Our study focuses on selling to a major business customer and the role of the price-takers is to serve as the outside option for the seller, shape the value of the limited capacity, and help the seller carry out the threat. Although our model is dynamic, we consider a static pricing problem.

2.3 A Dynamic Model of B2B Price Bargaining

In our base model, we consider the simplest case of bargaining with a single buyer with a reserved capacity. In Section 2.7, we will extend the model to consider bargaining with multiple buyers who share the seller's capacity. We devised the model to capture the process of price bargaining in supply chains with the following features: capacity is limited; the seller and buyers learn over time the value of the capacity; buyers can threaten to wait if their counter-offers are rejected; and the seller can threaten to reduce the reserved capacity for a buyer by selling to other buyers. Hence, the seller and buyers use different tactics in the bargaining. Using this framework, we try to understand the dynamics of the bargaining process and the time it takes to settle the price in supply chains. We consider a one-shot bargaining event because multiple transactions and repeated bargaining will unnecessarily complicate the analysis. In fact, one-shot price bargaining is common in practice; for example, when fixed-price contracts are used.

Seller $\mathcal{A}$ (she) introduces the new product to a group of OEM buyers and sets the posted price p_A. Based on forecasts provided by different customers, $\mathcal{A}$ builds up a finite capacity. In this base model, the seller reserves K units of the capacity for $\mathcal{C}$ (he), a major customer, who agrees to adopt this product. At time 0, customer $\mathcal{C}$ approaches seller $\mathcal{A}$ and asks for Q units of this new product. For simplicity, we assume $\mathcal{C}$ accepts partial fulfillment of his order and $\mathcal{A}$ tries to fulfill the order as much as possible with the reserved capacity.

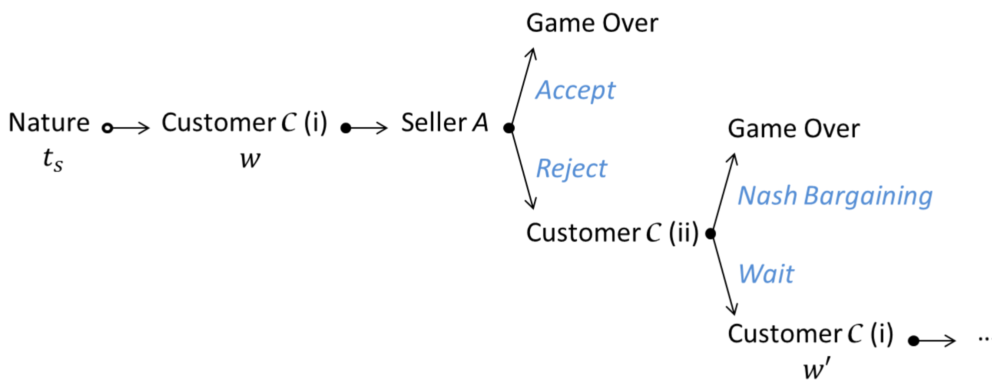
We denote r as $\mathcal{C}$'s marginal payoff before subtracting the procurement cost. As an outside option, $\mathcal{C}$ could also adopt an alternative product from a different seller. Let $\bar{c}$ represent the net marginal cost of buying from the alternative supplier in order to keep the same margin r. However, we assume that $\mathcal{C}$ could only adopt one product. In other words, if $\mathcal{C}$ accepts partial fulfillment of his order, $\mathcal{C}$ could not purchase alternative products from other sellers to fill the gap.

For simplicity, we assume that only the price is negotiated and that the only uncertainty in the price negotiation is the value of $\mathcal{A}$'s capacity. In addition, we assume that all information between $\mathcal{A}$ and $\mathcal{C}$ is symmetric, which is reasonable if they deal with each other repetitively. The negotiation starts with $\mathcal{C}$ proposing a counter-offer w. As a common tactic, $\mathcal{C}$ threatens to walk away from the negotiation and wait if the counter-offer is rejected by $\mathcal{A}$. We assume that $\mathcal{C}$'s production schedule requires him to finalize the purchase quantity by time $T > 0$, so he has some flexibility to manipulate regarding when to close the deal and thus his threat is *possible*. However, his threat is *credible* only when he has the best incentive to carry it out. For example, the customer expects to get a lower price later. With the counter-offer w, seller $\mathcal{A}$ then chooses to accept or reject. As a countermeasure, $\mathcal{A}$ threatens to sell up to s units of the reserved capacity to price-taking customers if $\mathcal{C}$ waits. Notice that this threat is always credible because $\mathcal{A}$ could always obtain higher revenue from price-taking customers.

However, selling the reserved capacity to price-takers is not always possible. For simplicity, we assume that it happens only when a demand shock occurs. The demand shock is realized at time t_s, following a general probability density function $\lambda(t)$ with support $[0, \infty)$. Let $\Lambda(t) := \int_0^t \lambda(x)dx$ be the cumulative distribution function and $\bar{\Lambda}(t) := 1 - \Lambda(t)$ be the complement. Accordingly, $\Lambda(t) = \Pr\{t_s \leq t\}$ represents the probability of having the demand shock by time t. The existence of demand shock, of course, should depend on p_A. However, rather than scaling $\lambda(t)$, we model the dependence of demand on price by scaling the time horizon. Denoting the original time t_o, we define $T(p_A) := T_o \cdot (1 - \alpha p_A)$ and $t(p_A) = T(p_A) \cdot \frac{t_o}{T_o}$ for any p_A. Notice that, T is defined once the posted price is known at time 0, so the model is well specified. By doing so, we maintain the basic logic: as p_A increases, $\Lambda(t)$ decreases. In other words, the time at which the demand shock is realized is stochastically decreasing in p_A. Also, we avoid assuming an explicit functional form for λ, which allows us the flexibility to extend the model to bargaining with multiple customers. For convenience, we will write $\Lambda(t)$ as Λ_t.

Figure 2.2 lays out the sequence of events. Before $\mathcal{C}$ bargains with $\mathcal{A}$ for the price, they agree on the transaction quantity, which is $Q \wedge K$ if the demand shock has not occurred and is $Q \wedge (K - s)$ otherwise. Note that $x \wedge y = \min\{x, y\}$ and $s \leq K$ by definition. At any particular time t, if $\mathcal{C}$'s

Figure 2.2: Sequence of events.



counter-offer is rejected and he does not wait, we assume that the two parties will be engaged in a Nash bargaining (Nash 1950) and the bargaining solution will be given by $\bar{p}_t = \arg\max_p (d_t^C - p)^\gamma (p - d_t^A)^{1-\gamma}$ where $\gamma \in [0,1]$ is $\mathcal{C}$'s relative bargaining power and d_t^i denote the disagreement price for $i \in \{\mathcal{A}, \mathcal{C}\}$ at time t. Actually, the assumption of a Nash bargaining solution is plausible, even in our strategic setting. According to Binmore et al. (1986), a Nash solution can approximate the equilibria of strategic models when the interval between consecutive counteroffers approaches zero, which—because we use a continuous time model—is the case here. However, if $p_A \leq \bar{c}$, it is not credible for $\mathcal{C}$ to switch (i.e., $d_t^C = d_t^A = p_A$) and thus he has to pay the posted price.

If the capacity reserved for $\mathcal{C}$ has not been sold out by time T, seller $\mathcal{A}$ has a number of salvage options. She could sell the capacity to customers who arrive afterwards, or she could downgrade the product to supplement her supply of lower grade products.[1] Let $\pi(x)$ denote the salvage value of $\mathcal{A}$'s residual capacity x after time T. In general, we assume that $\pi(x)$ is increasing in x. To make sure there exists a Nash bargaining solution, we assume $\pi(K) \leq p_A \cdot K$.

2.4 Model Analysis

The objective of our analysis is to characterize the conditions for $\mathcal{C}$'s threat to be credible, obtain the best response of $\mathcal{A}$ to $\mathcal{C}$'s counter-offer, and derive the negotiated price in equilibrium. We assume that both $\mathcal{A}$ and $\mathcal{C}$ aim to maximize their respective expected payoffs.

[1]This is normal in semiconductor and high-tech supply chains.

2.4.1 The Bargaining Process

Given that customer $\mathcal{C}$ could walk away and wait during the bargaining process, he may return with a counter-offer at any time. Thus, suppose $\mathcal{C}$ starts or resumes the bargaining process at time t. First of all, if the demand shock has already occurred ($t_s \leq t$), then it is automatically credible for $\mathcal{C}$ to wait from t to T, because $\mathcal{A}$'s reserved capacity will no longer change. Since time discounting is negligible and both parties' outside options are fixed in this case, it is equivalent to bargaining at time T. Without loss of generality, we assume that $\mathcal{C}$ will buy at time T. Let $p_h := \bar{p}_t|_{t_s \leq t}$ denote the Nash bargaining solution, or NB price, at time t given that $t_s \leq t$. Note that $p_h = p_A$ if $p_A \leq \bar{c}$, and $p_h = \gamma d_t^A|_{t_s \leq T} + (1-\gamma)\bar{c}$ if $p_A > \bar{c}$, where $d_t^A|_{t_s \leq T}$ stands for the disagreement price for $\mathcal{A}$ at time t given $t_s \leq T$.

Next, we consider the case in which demand shock has yet to occur ($t_s > t$). Suppose there exists a subgame perfect equilibrium (SPE) such that *at any time* $t \in [0, T]$, $\mathcal{C}$ *pays* $\omega(t)$ *if* $t_s > t$, *and* $\mathcal{C}$ *can credibly wait from* t *to* $\tau(t)$ *in case of rejection.* Given $W := \{\omega(t) : t \in [0, T]\}$, the price trajectory in SPE, let $J_z^t(W)$ denote $\mathcal{C}$'s expected payoff of buying at time $z \geq t$ when he is at time t and $t_s > t$. Thus, we have

$$\tau(t) = \sup \left\{ \arg \max_{z \in [t,T]} J_z^t(W) \right\}, \tag{2.1}$$

which is a mapping from $[0, T]$ to $[0, T]$. Given $\Gamma := \{\tau(t) : t \in [0, T]\}$, we can compute W accordingly. Hence, an SPE exists if there exists a fixed "point" for $\Gamma(W)$ or $W(\Gamma)$. In the following, we derive $W(\Gamma)$ and then $J_z^t(W)$.

At any time t, if $\tau(t) = t$, then it is not credible for $\mathcal{C}$ to wait and we have that $\omega(t)$ equals the NB price given $t_s > t$: $\bar{p}_t|_{t_s > t}$. However, if $\tau(t) > t$, then $\mathcal{C}$ would offer the lowest price that is acceptable for $\mathcal{A}$. Recursively and compactly, we have (see the appendix for the derivation)

$$\omega(t) = \begin{cases} \bar{p}_t|_{t_s > t}, & \tau(t) = t; \\ \left(1 - \Lambda_{\tau(t)}^t\right) \omega\left(\tau(t)\right) + \Lambda_{\tau(t)}^t \left[p_h + (p_A - p_h)\, \theta(K)\right], & \tau(t) > t, \end{cases} \tag{2.2}$$

where $\Lambda_\tau^t := \Pr\{t_s \leq \tau | t_s > t\} = \frac{\Lambda_\tau - \Lambda_t}{1 - \Lambda_t}$ is the Bayesian updated distribution of the demand shock given $t_s > t$, and $\theta(K) := 1 - \frac{Q \wedge (K-s)}{Q \wedge K} \in [0, 1]$ represents the percentage of $\mathcal{C}$'s procurement quantity that is threatened by the demand shock.

Now, we show how to compute $\bar{p}_t|_{t_s>t}$. If $p_A \le \bar{c}$, we have $\bar{p}_t|_{t_s>t} = p_A$; if $p_A > \bar{c}$, the Nash bargaining leads to $\bar{p}_t|_{t_s>t} = \gamma d_t^A|_{t_s>t} + (1-\gamma)\bar{c}$. To obtain $d_t^A|_{t_s>t}$, we first derive that $d_T^A|_{t_s>T} = \frac{\pi(K)-\pi\left((K-Q)^+\right)}{K\wedge Q}$ and $d_T^A|_{t_s\le T} = \frac{\pi(K-s)-\pi\left((K-s-Q)^+\right)}{(K-s)\wedge Q}$. Note that $x^+ := \max\{0,x\}$. Second, if customer $\mathcal{C}$ walks away at any time t, the seller will do the following: sell up to s units of the reserved capacity if the demand shock occurs and salvage the residual capacity at time T. As shown in the appendix, the lowest price the seller would accept is

$$d_t^A|_{t_s>t} = \Lambda_T^t \left[\theta(K)p_A + (1-\theta(K))\, d_T^A|_{t_s\le T}\right] + \left(1-\Lambda_T^t\right) d_T^A|_{t_s>T}. \tag{2.3}$$

Next, we define $p_l := \bar{p}_T|_{t_s>T}$. After obtaining $W(\Gamma)$, we proceed to get

$$J_z^t(W) = \Lambda_z^t (r-p_h)(Q\wedge(K-s)) + (1-\Lambda_z^t)[r-\omega(z)](Q\wedge K). \tag{2.4}$$

It is the weighted sum of two possible outcomes, which depend on whether demand shock occurs while $\mathcal{C}$ waits. By definition, the threat to wait at t is credible if and only if $\tau(t) = \sup\left\{\arg\max_{z\in[t,T]} J_z^t(W)\right\} > t$. Before we solve $\mathcal{C}$'s optimal timing problem and the SPE in the next section, we offer a preliminary observation.

Proposition 2.1. *NB price $\bar{p}_t|_{t_s>t}$ is decreasing in t if $\theta(K)p_A + (1-\theta(K))\, d_T^A|_{t_s\le T} > d_T^A|_{t_s>T}$, increasing in t if $\theta(K)p_A + (1-\theta(K))\, d_T^A|_{t_s\le T} < d_T^A|_{t_s>T}$, and time-invariant if otherwise.*

Intuitively, it is not credible for $\mathcal{C}$ to wait if $\bar{p}_t|_{t_s>t}$ is increasing or invariant in t, considering the possibility of a demand shock that depletes $\mathcal{A}$'s capacity. It can be checked that $\theta(K) = 0$ if $K > s+Q$. Hence, if $\mathcal{A}$ has sufficient capacity and $d_T^A|_{t_s\le T} > d_T^A|_{t_s>T}$, then $\bar{p}_t|_{t_s>t}$ is decreasing in t and it may be credible for $\mathcal{C}$ to wait. Otherwise, p_A will play a role in determining the dynamics of $\bar{p}_t|_{t_s>t}$. If capacity is moderate but p_A is high, it may also be credible for $\mathcal{C}$ to wait.

2.4.2 The Customer's Problem

Since we assume $\mathcal{C}$'s purchase quantity is fixed, his objective is to maximize his expected payoff, $J^t_\tau(W)$, given P and W by optimizing τ. Assuming $\omega(t)$ is differentiable, we have the first order derivative

$$\frac{d}{d\tau}J^t_\tau(W) = \frac{\lambda_\tau \cdot (Q \wedge K)}{1-\Lambda_t}\left[\omega(\tau) - \omega'(\tau)\cdot\frac{1-\Lambda_\tau}{\lambda_\tau} - [1-\theta(K)]\cdot p_h - \theta(K)\cdot r\right]. \tag{2.5}$$

If the maximum of $J^t_\tau(W)$ occurs at $\tau^* \in (t,T)$, then we must have $\frac{d}{d\tau}J^t_\tau(W)|_{\tau=\tau^*} = 0$. If multiple maximizers are present in (t,T), then by definition the supremum of the set of solutions, τ^*, is the waiting destination. Since $\frac{\lambda_\tau\cdot(Q\wedge K)}{1-\Lambda_t} > 0$, the first order condition can be reduced to $\omega(\tau) - \omega'(\tau)\cdot\frac{1-\Lambda_\tau}{\lambda_\tau} = [1-\theta(K)]\cdot p_h + \theta(K)\cdot r$, which indicates that τ^* is irrelevant of t. If the maximizer is not in (t,T), then it is either $\tau(t) = t$ or $\tau(t) = T$. In any case, as long as it is not optimal to buy now, τ^* is irrelevant of t. In other words, $\frac{d}{dt}\tau^* \equiv 0$. Thus, if any offer is rejected at any time $t < \tau^*$, $\mathcal{C}$ would wait until τ^*. Although it is possible that for certain $t' > \tau^*$ we have $\tau(t') > t'$ when $J^t_\tau(W)$ is not monotone in τ, it is not relevant when we stand at $t \leq \tau^*$. Incorporating the above observations, we obtain the following important result.

Theorem 2.1. *In any SPE, $\frac{\partial\omega(t)}{\partial\tau^*} = 0$ and thus $\omega(t) \equiv \bar{p}_t|_{t_s>t}$.*

The theorem says that $\mathcal{C}$ always pays the NB price no matter when he buys and he cannot gain advantage by waiting. Based on this theorem, we obtain $W := \{\omega(t) : t \in [0,T]\}$, and can solve for τ^* by plugging W into (2.5). In particular, we have $\frac{d}{d\tau}J^t_\tau(W) = \frac{\lambda_\tau}{1-\Lambda_t}\cdot(Q\wedge K)\cdot\theta(K)\cdot(p_A \wedge \bar{c} - r)$ when we plug $\omega'(\tau)$ into (2.5). Accordingly, we characterize SPEs as follows.

Corollary 2.1. *If $K \geq Q+s$, then $\frac{d}{d\tau}J^t_\tau(W) = 0$ and thus $\tau(t) = T$; if $K < Q+s$ and $p_A \wedge \bar{c} < r$, then $\tau(t) = t$; if $K < Q+s$ and $p_A \wedge \bar{c} \geq r$, then $\tau(t) = T$.*

The result shows that if $\mathcal{A}$'s capacity is absolutely sufficient (i.e., $K \geq Q+s$), then $\mathcal{C}$ can credibly wait at any time. It is because the capacity that is needed by $\mathcal{C}$ is not under any threat.

Mathematically, when $K \geq Q + s$ we have $\frac{d}{d\tau} J^t_\tau(W) = 0$ and $\mathcal{C}$'s expected purchase cost is a constant ex ante. Otherwise, the credibility of $\mathcal{C}$'s threat largely depends on the relative values of the posted price p_A, the marginal cost of the alternative product $\bar{c}$, and the marginal revenue r. If the end product is very profitable for the customer, then the threat of waiting is never credible because the customer would have huge loss if the capacity is reduced. If the product is not very profitable such that the profit would be negative when paying the posted price or when switching to the alternative product, then getting a greater discount is so important that waiting till the last minute is credible. In short, the SPE depends greatly on the capacity level and the level of the customer's profitability.

2.4.3 The Seller's Problem

Given the SPE, seller $\mathcal{A}$ then posts p_A at $t = 0$ and decides on how to react to $\mathcal{C}$'s initial counter-offer in order to maximize her expected revenue in equilibrium, Π_A. In this section, we analyze the best reaction to $\mathcal{C}$'s counter-offer. Let Π^0_A and Π^T_A denote $\mathcal{A}$'s expected revenue when $\mathcal{C}$ decides to buy at time 0 and T, respectively, given $\{K, Q, s\}$. We have

$$
\begin{aligned}
\Pi^0_A &= \bar{p}_0 \cdot K \wedge Q + \Lambda_T \cdot \left[p_A \cdot s \wedge (K-Q)^+ + \pi\left((K-s-Q)^+\right)\right] \\
&\quad + \overline{\Lambda_T} \cdot \pi\left((K-Q)^+\right), \text{ and} \qquad (2.6) \\
\Pi^T_A &= \Lambda_T \left[p_A \cdot s \wedge K + p_h \cdot Q \wedge (K-s) + \pi\left((K-s-Q)^+\right)\right] \\
&\quad + \overline{\Lambda_T} \left[p_l \cdot K \wedge Q + \pi\left((K-Q)^+\right)\right]. \qquad (2.7)
\end{aligned}
$$

Through algebraic manipulations, we find the following important result:

$$
\Pi^0_A - \Pi^T_A = \begin{cases} 0 & \text{if } p_A \leq \bar{c}; \\ (\bar{c} - p_A) \cdot (1-\gamma) \cdot \Lambda_T \cdot (K \wedge Q) \cdot \theta(K) & \text{if } p_A > \bar{c}. \end{cases} \qquad (2.8)
$$

Therefore, we always have $\Pi^0_A \leq \Pi^T_A$, which is surprising. In other words, within our modelling framework, $\mathcal{A}$ should never strictly prefer that they close the deal at time 0. The intuition is that

Table 2.2: The Time Preference versus the Credibility of Waiting

Scenarios	$p_A \leq \bar{c}$ or $K \geq s+Q$	$p_A > \bar{c}$ and $K < s+Q$
$p_A \wedge \bar{c} < r$	$\Pi_A^0 - \Pi_A^T = 0$ and $\tau(0) = 0$ Best action for $\mathcal{A}$: Negotiating at $t = 0$	$\Pi_A^0 - \Pi_A^T < 0$ and $\tau(0) = 0$ Best action for $\mathcal{A}$: Negotiating at $t = 0$
$p_A \wedge \bar{c} \geq r$	$\Pi_A^0 - \Pi_A^T = 0$ and $\tau(0) = T$ Best action for $\mathcal{A}$: Accepting $\omega(0)$ or negotiating at $t = T$	$\Pi_A^0 - \Pi_A^T < 0$ and $\tau(0) = T$ Best action for $\mathcal{A}$: Rejecting any counteroffer and negotiating at $t = T$

splitting the entire pie (i.e., the total expected payoff generated by the full capacity) *at time 0 is worse for $\mathcal{A}$ than splitting the residual pie* (i.e., the total expected payoff generated by the residual capacity) *at time T*. In particular, $\mathcal{A}$ would prefer that $\mathcal{C}$ waits until T if $\Pi_A^0 < \Pi_A^T$, which is possible when $p_A > \bar{c}$ and $\theta(K) > 0$. This is because $\mathcal{C}$ would not pay more than $\bar{c}$, and $\mathcal{C}$'s waiting awards $\mathcal{A}$ the opportunity to gain higher profit and thus better utilize the capacity. Furthermore, it is possible to let $\mathcal{C}$ wait when his threat is credible. Therefore, when $\Pi_A^0 < \Pi_A^T$ and $\tau(0) = T$, the best reaction for seller $\mathcal{A}$ is to reject $\mathcal{C}$'s initial counteroffer and encourage him to wait. We restate this result in the following Corollary and summarize all the results in Table 2.2.

Corollary 2.2. *When $p_A > \bar{c} \geq r$ and $K < s + Q$, the best reaction for seller $\mathcal{A}$ is to reject $\mathcal{C}$'s initial counteroffer and encourage him to wait.*

Now we know that the credibility of customers' threat to wait can actually sometimes benefit the seller. To help the seller exploit this result, we explicitly provide the bargaining tactics in Table 2.2, which points out the best actions for the seller to take in different situations.

- When the end product is very profitable for $\mathcal{C}$ (i.e., $p_A \wedge \bar{c} < r$), it is not credible for him to wait and thus it is not possible for $\mathcal{A}$ to ask him to wait. As a result, the seller should engage in the price bargaining immediately.

- When the end product is not very profitable and switching is costly for $\mathcal{C}$ (i.e., $r \leq p_A \leq \bar{c}$), it is credible for him to wait and $\mathcal{A}$ can either accept $\omega(0)$ as the initial counteroffer or wait till the last minute.

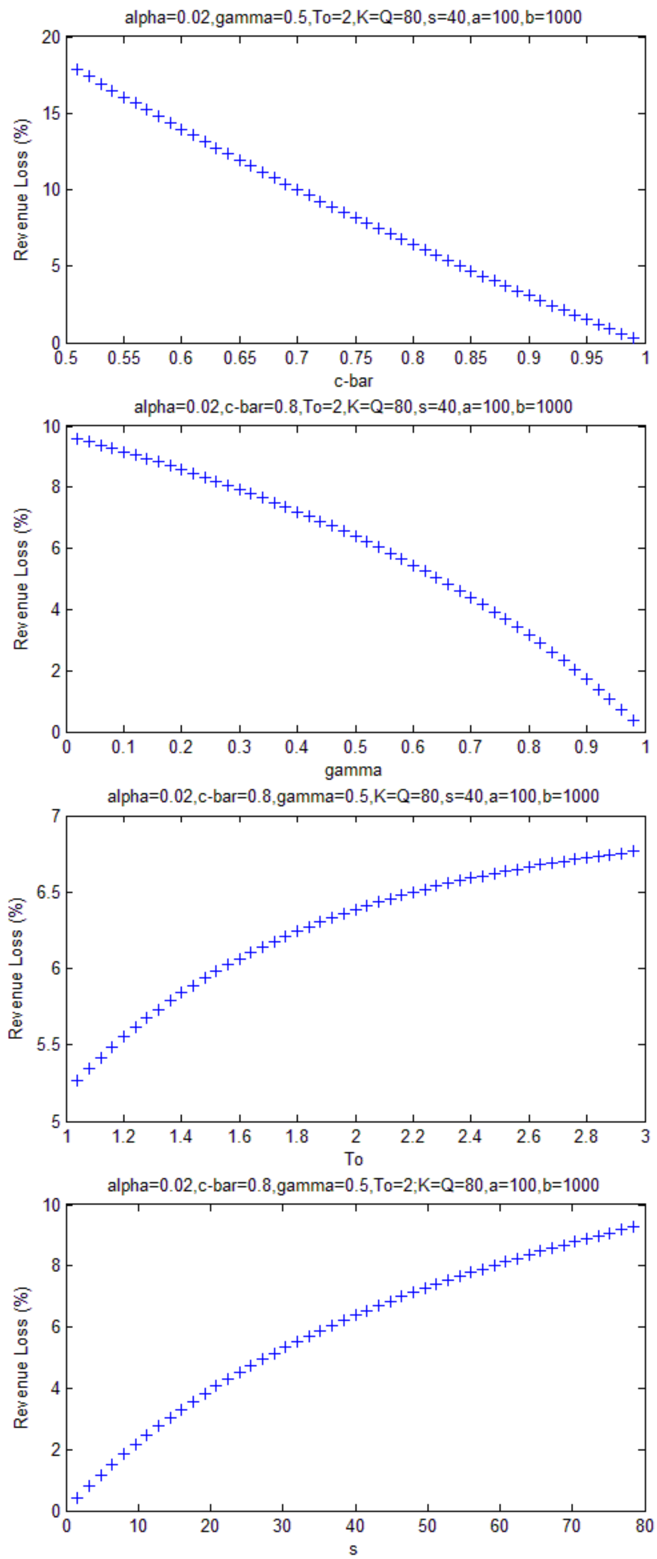
- When the reserved capacity is not absolutely sufficient (i.e., $K < s + Q$), the end product is not very profitable, and switching is not costly (i.e., $r \leq \bar{c} < p_A$), it is credible for $\mathcal{C}$ to wait and $\mathcal{A}$ should reject any counteroffer and wait till the last minute. In this case, if the seller does not encourage the buyer to wait but settle the price at the beginning, we show by numerical studies that the seller can lose more than 10% of its revenue. The results are summarized in Figure 2.3. We can see that the loss decreases with $\bar{c}$ because the larger $\bar{c}$ the higher price $\mathcal{C}$ pays at the beginning. The loss also decreases with γ because the larger γ the more the revenue depends on $\mathcal{A}$'s own outside options but not the negotiated price. In addition, the loss increases with T_o and s, because the larger the values the higher expected revenue $\mathcal{A}$ can obtain from the price-takers.

Notice that the results presented here are important not only because they tell the seller how to negotiate with the customer in different situations, but also because they show the seller what to expect *ex ante*. As we will see later, the optimal posted price depends on the timing of negotiation and thus failing to anticipate the right outcome leads to ineffective pricing.

2.5 Non-Monotonic Price-Quantity Relation

In this section, we explore the property of the negotiated price in equilibrium. Zhang et al. (2015) point out that the negotiated price can be a non-monotonic function of the purchase quantity in B2B markets. They find some empirical evidences in the semiconductor industry and they offer a model to explain that phenomenon. They consider the case wherein the seller's capacity is not reserved for any specific customer and the seller negotiate with customers sequentially. Here, we

Figure 2.3: Revenue loss for suboptimal bargaining strategy.



show that our model can predict a similar price-quantity relation, but we consider the case wherein there are reserved capacities for the major customers and the negotiations are thus independent of each other. In fact, both situations exist in the semiconductor industry.

2.5.1 Decomposition of the Negotiated Price

According to Theorem 2.1 and Corollary 2.1, customer $\mathcal{C}$ would either buy at $t = 0$ or $t = T$ in equilibrium. Hence, there could be three possible outcomes for the negotiated price: $\bar{p}_0|_{t_s>0}$, $\bar{p}_T|_{t_s>T}$, and $\bar{p}_T|_{t_s\leq T}$. In the following, we check how each of them changes with Q. For simplicity, we assume that the value of the residual capacity $\pi(\cdot)$ is increasing concave. However, notice that concavity is not required to obtain any of the previous results.

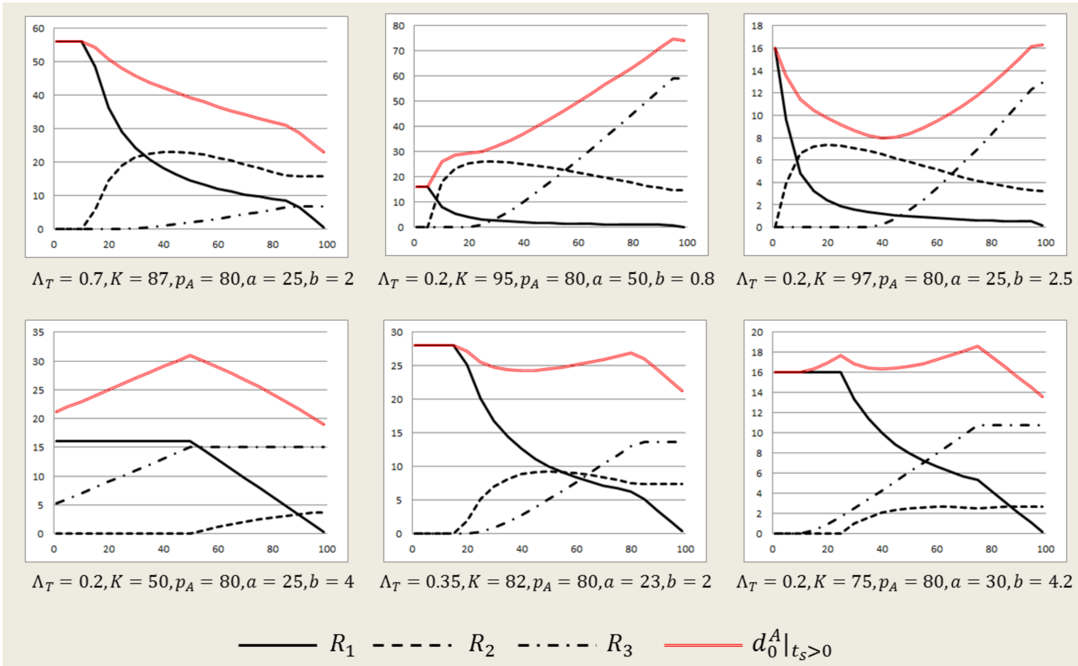
We use the following parameter setting. The reserved capacity level K is fixed. To make sure that the seller's threat is effective, we assume she picks an s such that $K < Q + s$. (Note that we assume the total size of the demand shock is large enough.) We set $Q + s = 100$. In addition, we set $p_A > \bar{c}$ and $\pi(x) = \max_p p \cdot \min\{x, a - bp\}$. In preparation, we define

$$
\begin{aligned}
R_1 &= \Lambda_T \cdot p_A \cdot \theta(K) = \Lambda_T \cdot p_A \cdot \frac{K \wedge Q - (K - s) \wedge Q}{K \wedge Q}; && (2.9)\\
R_2 &= \Lambda_T \cdot d_T^A|_{t_s \leq T} \cdot [1 - \theta(K)] = \Lambda_T \cdot \frac{\pi(K - s) - \pi((K - s - Q)^+)}{K \wedge Q}; && (2.10)\\
R_3 &= (1 - \Lambda_T) \cdot d_T^A|_{t_s > T} = (1 - \Lambda_T) \cdot \frac{\pi(K) - \pi((K - Q)^+)}{K \wedge Q}. && (2.11)
\end{aligned}
$$

According to (2.3), $d_0^A|_{t_s>0}$ can be expressed as the sum of R_1, R_2, and R_3. Intuitively, if the bargaining breaks down and customer $\mathcal{C}$ walks away, there could be three possible outcomes for the amount of capacity $K \wedge Q$ that could be sold to customer $\mathcal{C}$. First, if the demand shock occurs, a fraction $\theta(K)$ of it will be sold to the price takers. Second, if the demand shock occurs, a fraction $1 - \theta(K)$ of it will be salvaged at time T. Third, if the demand shock does not occur, all of it will be salvaged at time T. The expected revenue obtained from each outcome is R_1, R_2, and R_3, respectively. As a result, we can write

$$\bar{p}_0|_{t_s>0} = (1 - \gamma) \cdot \bar{c} + \gamma \cdot (R_1 + R_2 + R_3). \tag{2.12}$$

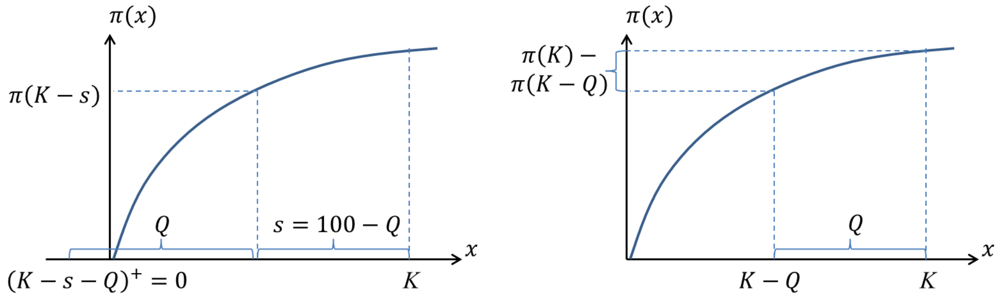
Figure 2.4: The impact of purchase quantity Q.



Therefore, how $\bar{p}_0|_{t_s>0}$ is influenced by Q depends on how R_1, R_2, and R_3 change with Q. In Figure 2.4, we plot $d_0^A|_{t_s>0}$, R_1, R_2, and R_3 against Q. We can see that R_1 is decreasing in Q. This is intuitive because as Q increases, the seller can use a smaller s to create a sense of "scarcity" for her threat and the revenue from the price takers will be lower. Next, according to Figure 2.4, *R_2 and R_3 are not decreasing in Q, which causes the non-monotonicity of the negotiated price.* The reason is that π is increasing and concave: as the residual capacity increases, the seller has more and more flexibility to optimize the salvage value, but only up to a certain scale. Hence, as Q increases, the marginal salvage value diminishes and the seller's opportunity cost in this dimension increases. Figure 2.5 illustrates the effect of Q on $\pi(K-s) - \pi((K-s-Q)^+)$ and $\pi(K) - \pi((K-Q)^+)$ while holding $Q + s = 100$ constant.

Now let's consider the negotiated price at time T. Under the assumption of $K < s + Q$, we have $\bar{p}_T|_{t_s>T} = (1-\gamma)\cdot\bar{c} + \gamma\cdot R_3/(1-\Lambda_T)$ and $\bar{p}_T|_{t_s\le T} = (1-\gamma)\cdot\bar{c} + \gamma\cdot\pi(K-s)/(K-s)$. Hence, we know immediately that $\bar{p}_T|_{t_s>T}$ is increasing (because R_3 is increasing) in Q and $\bar{p}_T|_{t_s\le T}$ is decreasing in Q. They can actually be viewed as special cases of $\bar{p}_0|_{t_s>0}$. This also suggests that only transactions for which the prices are negotiated at the beginning can display a non-monotonic

Figure 2.5: Illustration of increments in salvage value.



price-quantity relation.

2.5.2 Explorations

If our model is correct, we can use it to explore possible price-quantity relations by setting different values for $\{\Lambda_T, K, p_A, a, b, \gamma\}$. After extensive numerical experiments, we show in Figure 2.4 that six patterns describe the price curve: (1) decreasing, (2) increasing, (3) V-shaped, (4) Λ-shaped, (5) $\backsim$-shaped, and (6) M-shape.

The six curves are all linear combinations of R_1, R_2 and R_3. Key factors determining the final shape of the curves are K (capacity level), Λ_T (probability of the demand shock occuring prior to T) and π (salvage value determined by a and b). Specifically, K determines the location of the "kinked" points on R_1, R_2 and R_3, while Λ_T and π determine the relative weight of each component. The irregularities complicate the bargaining process and make the bargaining outcome hard to predict without an analytic model.

Considering the intuition behind these patterns, we notice that Λ_T literally represents the probability of the demand shock. It is actually a measure of the number of potential customers for the product; the greater number of potential customers, the greater the probability of demand shock during the selling season. As illustrated by the first graph in Figure 2.4, when Λ_T is high ($= 0.7$) and π is small, responding to demand shock is the major outside option for the seller. In the second graph, Λ_T is low ($= 0.2$) but π is large, so R_3, the value of selling to the salvage market after T, dominates. In the third graph, both Λ_T and π are small, so that neither of the two sources of revenue dominates, and the mixed effect generates a non-monotonic curve. The explanations for the other three patterns are similar but more complex because the effect of K is involved.

2.6 Posted Price Optimization

Given that the output of the negotiation could be complicated, it should be difficult to make decisions that would influence the negotiated price. In this section, we first solve for the optimal posted price in the base model, in which the capacity is reserved for the major customer. Later, we will modify the model to consider the case of pricing and bargaining without capacity reservation in which negotiations with different customers will be interconnected.

Consider that there are n major customers who would like to adopt this product. Certain level of capacity is reserved for each of them by seller $\mathcal{A}$. We assume that the product is introduced to the customers at the same time, but they have different deadlines. Let $T_i\left(p_A\right)$ denote the deadline of customer $i \in N$, where N stands for the set of customers. Define $T\left(p_A\right) = \min\left\{T_j\left(p_A\right) : j \in N\right\}$ and $\beta_i = T_i\left(p_A\right)/T\left(p_A\right)$. We assume that α, the price sensitivity of the price-takers, is identical for all the customers so that β_i is independent of p_A. In addition, let $T_o = T\left(p_A\right)/\left(1-\alpha p_A\right)$, which means the earliest actual deadline.

We focus on the case wherein the cost of the alternative product is low enough for the major customers so that they never take the posted price. We know from our analysis that each customer would settle the price at either the beginning or the deadline in equilibrium. In this case, the seller's expected revenue from a customer i can be written as $C_i^I + C_i^{II} \cdot \Lambda_{T_i} + C_i^{III} \cdot p_A \cdot \Lambda_{T_i}$, where C_i^I, C_i^{II}, and C_i^{III} are constants that are independent of p_A. Particularly, it is easy to check that $C_i^{II} < 0$ and $C_i^{III} > 0$ for any $i \in N$. Then, by restricting Λ and the deadlines, we can produce the following result.

Proposition 2.2. *If $\Lambda(t)$ is log-concavae in t and $\beta_i \approx 1$ for any i, then p_A^* closely approximates the optimal price and it uniquely solves*

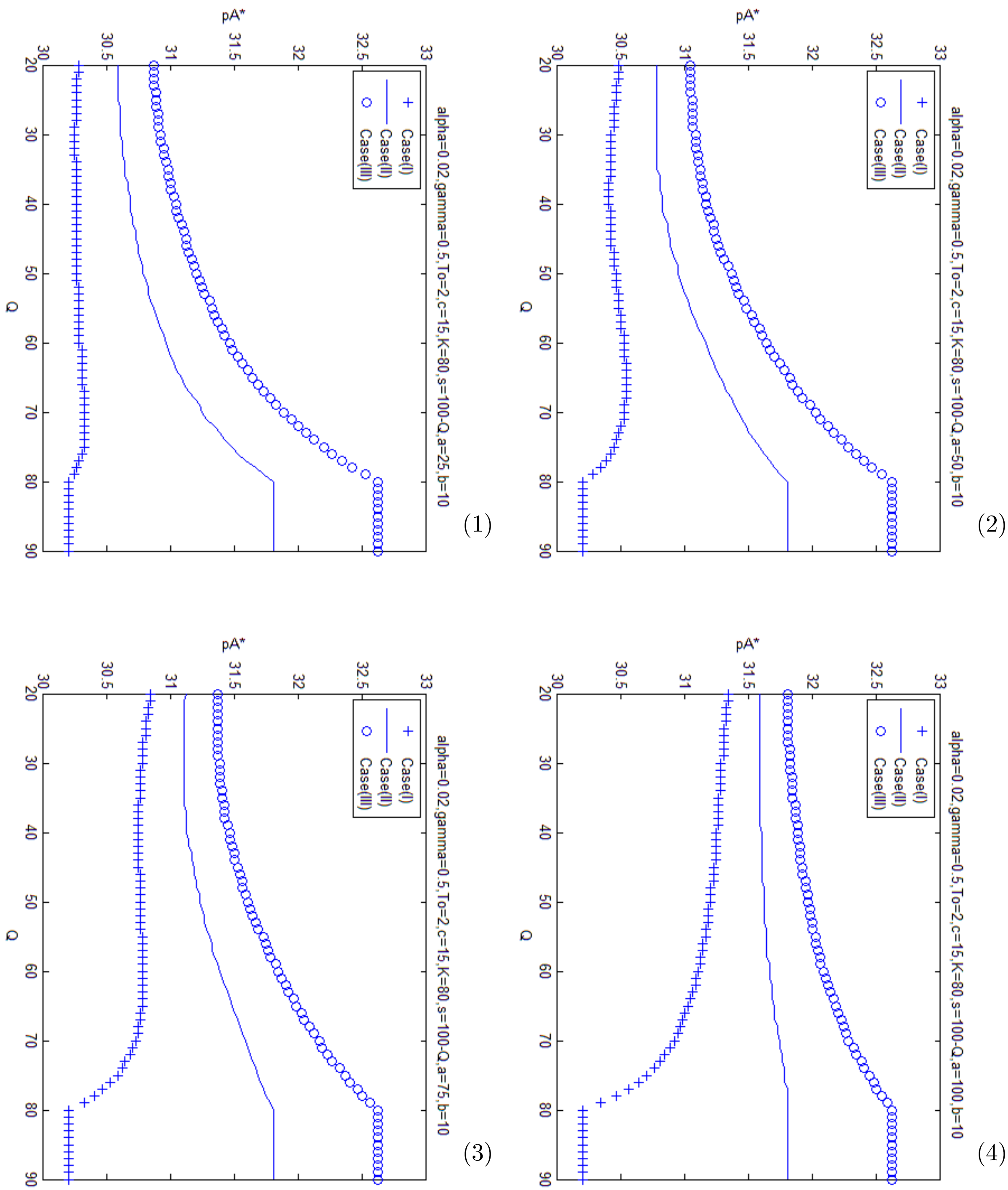
$$p_A = \frac{\Lambda\left(T(p_A)\right)}{\lambda\left(T(p_A)\right)} \cdot \frac{1}{\alpha T_o} - \frac{\sum C_i^{II}}{\sum C_i^{III}}. \tag{2.13}$$

The condition of $\Lambda(t)$ being log-concave in t is actually not very restrictive. Many common distributions such as normal, logistic, uniform, and exponential distributions satisfy this condition. Although different customers may have different production plan and deadlines for price bargaining, the above result is useful for us to generate managerial insights.

What can we learn from this model? How could things go wrong if managers mistakenly estimated the negotiated prices? How would the posted price be affected? Here we numerically explore how the optimal posted price should be when customers settle their prices at different time points. For simplicity, we assume that there are two major customers with symmetric characteristics, and we consider three cases for the timing of negotiation: (I) both prices are negotiated at $t = 0$; (II) one price is negotiated at $t = 0$ and the other is negotiated at $t = T$; (III) both prices are negotiated at $t = T$. We also consider four different cases for the residual value function $\pi(x) = \frac{1}{b} \cdot \left(a - x \wedge \frac{a}{2}\right) \cdot \left(x \wedge \frac{a}{2}\right)$ by setting four different values for a which measures the size of the residual market. In addition, we assume that Q is known when setting the price, we can view the optimal price as "hindsight-optimal". We assume that the capacity level relative to the total potential demand is fixed; i.e., $K/(Q+s) = 0.8$. The results of our numerical study are summarized in Figure 2.6. We have the following observations.

First of all, the optimal posted price should depend on the purchase quantity of the bargainer and the timing of agreement. In particular, the quantity can impact the optimal posted price in different ways, depending on when the agreements are reached. If any of the major buyers wait, the optimal posted price increases with the quantity; otherwise, the price should decrease with the quantity in general, although sometimes the relation could be non-monotonic. As price-takers become less and less important, the seller should focus more on the price bargaining. Recall that the seller splits the entire pie with the bargainer if they bargain at time 0. If the bargainer doesn't wait, the seller should reduce the price as s shrinks in order to maximize the pie. If the bargainer waits, the seller should increase the price in order to maximize the expected revenue from price-takers. In addition, note that only the residual capacity will be sold to the price-takers if the bargainer makes the purchase at the beginning; otherwise, more capacity will be sold to the price-takers. Hence, the optimal price for case (II) or (III) should be higher than in case (I), although the chance of demand shock will be reduced. From the results, we can learn that failing to anticipate the right timing of negotiation will lead to ineffective pricing. In particular, if the seller doesn't notice that she can

Figure 2.6: The optimal posted price versus purchase quantity.



benefit from customers' waiting and always anticipates settling the price at time 0, then she may underprice the product.

2.7 Selling to Multiple Bargainers

The base model is reasonable when there is only one major customer or when the seller reserves capacity for each major customer prior to the negotiations. Now, we will consider a scenario in which the seller allocates her capacity on the go to multiple major customers. In this case, we assume that the seller does not rely on the price-takers to implement her threat; instead, she can utilize the competition for the limited capacity among the major customers. Hence, customers have to take into account other customers' strategies when bargaining with the seller. We will characterize a Nash equilibrium in this scenario.

We know from the base model that credibility of a customer's waiting threat depends on K, $Q + s$, p_A, $\bar{c}$, and r. In this modified model, s represents the total demand from other customers. Therefore, the credibility of threat only depends on the total demand $\mathbb{TD}$ but not individual demand Q. Hence, it is natural to focus on an equilibrium wherein all customers buy at the same time. Now suppose all other customers buy at time τ. It is equivalent to setting $\lambda(t) = 0$ for $\forall t \in [0, \tau) \cup (\tau, T]$ in the base model. Then according to Corollary 2.1, customer i is indifferent to $\forall t \in [0, T]$ if $K \geq \mathbb{TD}$. Or, if $K < \mathbb{TD}$ and $p_A \wedge \bar{c} < r$, then customer i can buy at any $t \in [0, \tau]$, because $\lambda(t) = 0$ for $\forall t \in [0, \tau)$; thus, $\frac{d}{dt} J_t^0 = \lambda_t (Q \wedge K) \theta(K)(p_A \wedge \bar{c} - r) = 0$. If $p_A \geq p_B$, similar arguments suggest that customer i can buy at any $t \in [\tau, T]$. Consequently, we have the following result.

Lemma 2.1. *If all other customers buy at τ, buying at τ is weakly dominant for i in any case.*

Since time discounting is ignored in our problem, the result is identical for any $\tau \in [0, T]$. Without loss of generality, we focus on the equilibrium that all strategic customers buy at time $\tau = 0$, and such a Nash equilibrium exists.

Lemma 2.2. *All customers buying at time* 0 *constitutes a Nash equilibrium.*

In this scenario, the dependence of T on p_A is meaningless because the demand shock is no longer relevant. As a result, we model the dependence of demand on p_A in the following way: the distribution of $\mathbb{TD}$ under p_A' first order stochastically dominates the distribution under p_A'' if $p_A' < p_A''$. Additionally, we assume $\pi(x) = \max_p p \cdot (x \wedge [\mathbb{TD} \cdot (a - bp)])$. Then, the seller's pricing problem can be described as follows:

$$
\begin{aligned}
\max_{p_A} \quad & \quad \mathbf{E}\left[(K \wedge \mathbb{TD}) \cdot \bar{p} + \pi\left((K - \mathbb{TD})^+\right)\right] && (2.14)\\
s.t. \quad f_i & = Q_i/\mathbb{TD}, && (2.15)\\
\bar{p} & = \sum f_i \cdot p_i, && (2.16)\\
p_i & = (1-\gamma_i) \cdot \bar{c} + \gamma_i \cdot w_{-i}^A \quad \forall i \in I_C, && (2.17)\\
w_{-i}^A & = \bar{p}_{-i}\left(1 - \frac{\left[K - \sum_{j \neq i} Q_j\right]^+ \wedge Q_i}{K \wedge Q_i}\right) + \\
& \quad \frac{\pi\left(\left[K - \sum_{j \neq i} Q_j\right]^+\right) - \pi\left([K - \mathbb{TD}]^+\right)}{K \wedge Q_i} \quad \forall i \in I_C, && (2.18)\\
\bar{p}_{-i} & = \sum_{j \neq i} Q_j \cdot p_j / \sum_{j \neq i} Q_j \quad \forall i \in I_C. && (2.19)
\end{aligned}
$$

The expectation in (14) is with regard to $\mathbb{TD}$. Parameter $\bar{p}$ is the average price received by all customers, as defined in (16). Next, (17) describes the Nash bargaining price for customer i, which depends on the bargaining outcomes for all other customers. The seller's disagreement price (w_{-i}^A) in (18) is based on (3), in which $\Lambda_T = 1$ and p_A is replaced by $\bar{p}_{-i}$, the average price received by all other customers. An implicit assumption is that every customer receives the same service level when capacity is insufficient. Together, constraints (16) through (19) determine the Nash equilibrium given p_A. Apparently, there is no closed form solution for this problem, though the optimal price, p_A^*, can be determined using simulations.

In search of managerial insights, we consider a special case concerning two customers. To rule out the effect of exgonenous bargaining power but focus on the impact of demand share, we consider two customers with equal exgonenous bargaining power, γ, but different demand shares, f_1 and

f_2, which are known in advance. Furthermore, we introduce $\Delta \in [0, 0.5)$ to measure the demand-share asymmetry between the two customers and, without loss of generality, let $f_1 = 0.5 + \Delta$ and $f_2 = 0.5 - \Delta$. Therefore, we can write $\bar{p} = f_1 p_1 + f_2 p_2$, and we proceed to see how $\bar{p}$ is affected by Δ and $\kappa = K/\mathbb{TD}$, the measure of capacity level. Based on our formulation, the space of capacity level can be divided into three segments given f_1 and f_2:

- High level is defined as $\kappa \geq 1$, which means the capacity is sufficient to satisfy both customers.
- Medium level is defined as $\max\{f_1, f_2\} \leq \kappa < 1$, which means the capacity is sufficient to satisfy the larger customer.
- Low level is defined as $\kappa < \max\{f_1, f_2\}$, which means the capacity can not satisfy the larger customer.

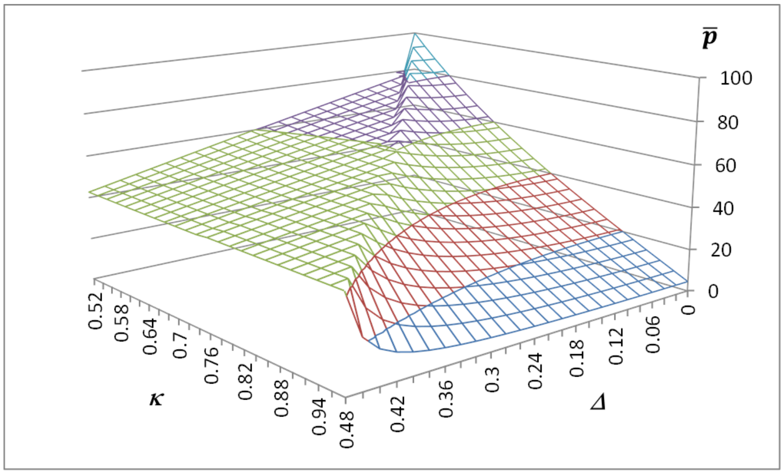
As we show, the impact of Δ on $\bar{p}$ is closely related to κ.

Proposition 2.3. *If $\kappa \geq 1$, then $\bar{p}$ is decreasing in $\Delta \geq 0$.*

This result says that, *when the manufacturer's capacity level is high, she can benefit from customer demand share symmetry.* This is because the salvage value function is increasing concave, and any increase in asymmetry in customer demand shares will drag down the value of the manufacturer's outside option.

Next, we check the impact of Δ on $\bar{p}$ in the cases of medium and low capacity levels; closed-form solutions for the equilibrium prices (p_1^e and p_2^e) are provided in the appendix. Since it is difficult to see analytically how $\bar{p}$ changes with Δ when the capacity level is medium or low, we use computational study to explore the relationships among $\bar{p}$, Δ, and κ. In Figure 2.7, the 45 degree line corresponds to $\kappa = f_1$. It can be seen that $\bar{p}$ strictly decreases with Δ when $\kappa < f_1$ but increases with Δ when $\kappa \geq f_1$. In other words, *demand-share asymmetry benefits the manufacturer when her capacity level is medium, while symmetry benefits her when her capacity level is low.* Changes

Figure 2.7: Relationship among $\bar{p}$, Δ, and κ.



in (18), which describes the manufacturer's outside option, cause the directional changes in the impact of Δ. The reason for this is that there generally is a kinked point on the price curve at $f_1 = \kappa$, and the price received by customer 1 (p_1^e) decreases faster with f_1 when the capacity level is low than when the capacity level is medium. Meanwhile, the price received by customer 2 (p_2^e) generally increases as f_2 decreases. Consequently, the increase of p_2^e has larger impact on $\bar{p}$ when $\kappa \geq f_1$, while the decrease of p_1^e has larger impact on $\bar{p}$ when $\kappa < f_1$.

Knowing that customer demand-share asymmetry can benefit or harm the seller under different capacity levels, we discuss the effect of asymmetry on the seller's optimal price, $p_A^*(\Delta)$. We know that in this modified model, p_A only affects the total demand and thus equivalently the level of capacity. According to Figure 2.7, we know that $\bar{p}$ in general decreases in the capacity level, regardless of Δ. Hence, the optimal posted price does not depend on demand-share asymmetry.

2.8 Summary and Insights

In this study, we construct a model to study the dynamics of price negotiation in B2B markets where the seller and buyers use different bargaining tactics. In particular, buyers can make counteroffers and threaten to wait if their prices are rejected by the seller; as a countermeasure, the seller can threaten to sell the capacity to other buyers. We consider two different versions of the model with the key difference being whether the seller reserves the capacity for each major customer. We solve the subgame perfect Nash equilibrium for the dynamic game and characterize the credibility of buyers' threat of waiting as well as the negotiated price in different situations. Further, we analyze the seller's payoff in various situations and obtain the optimal bargaining strategy for the seller. In addition, we use the model to show that both the negotiated price and the optimal posted price can be a non-monotonic function of the customer's purchase quantity. By extending the model to the case of selling to only two customers that are both bargainers, we showed that asymmetry in customer size could either benefit or harm the seller, depending on her capacity level.

Theoretic Contribution

Our model is different from the traditional cooperative bargaining models and strategic bargaining models. Cooperative models are static and unable to capture the process of negotiation. Traditional strategic models incorporate the process of negotiation, but they normally predict immediate agreements given symmetric information; furthermore, they normally do not incorporate strategic threats commonly used in practical negotiations. In contrast, our bargaining model allows us to capture the asymmetric threats used by different parties in a dynamic bargaining process. The model predicts delay of agreements without assuming information asymmetry or exogenous delay of counteroffers. In the traditional alternating-offer models, a player has to wait till the next period to propose a counteroffer. Exogenous delay of counteroffers can lead to trivial delay of agreements because the player who proposes the counteroffer at the last moment has full bargaining power. However, we assume that it takes no time to propose a counteroffer unless the buyer decides to wait purposely, so the delay of agreements in our model is purely due to customers' threats to wait.

Implications for Bargaining Strategy

We learn from our model that the credibility of customers' threat to wait can actually sometimes benefit the seller. This is true when (1) the reserved capacity is not absolutely sufficient (i.e., not sufficient to satisfy both the focal buyer and the occasional demand), (2) the end product is not very profitable for the customer, and (3) switching is not costly. In this case, it is credible for the customer to wait and the seller should reject the counteroffer and encourage the customer to wait till the last minute. In this way, the seller can obtain higher expected payoff. Although the result is surprising, it is reasonable in that the three conditions indicate that it is a "mismatch" for the seller and the customer: the product is not profitable for the customer and the customer does not rely heavily on the seller. Knowing such results can make a huge difference for sales managers. In particular, if the seller does not know when to encourage the buyer to wait and always settle the price at the beginning, we show that the seller can lose more than 10% of its revenue.

Implications for Posted Pricing

We show that the optimal posted price depends on the timing of negotiation and thus failing to anticipate the right outcome leads to ineffective pricing. In particular, if the seller doesn't notice that she can benefit from customers' waiting and always anticipates settling the price at time 0, she may underprice the product. As shown by our numerical examples, the price can be set at 8% lower than the optimal level. In addition, given that the price-quantity relationship can be non-monotonic, assuming a simple structure can cause serious problems for the seller when making important decisions *ex ante*.

Future Research

In future research, it might be interesting to model customers' cost of switching as a stochastic process, which is a common consideration when competing sellers may drop prices and new technology may be frequently launched. Another interesting problem is that whether the seller should set the posted price and let buyers offer the first price that sets the anchor for the negotiation. There have been different opinions in practice regarding this issue, it might be a fruitful area for research.

Appendix

Derivation of $\omega(t)$ Given $\tau(t) > t$

If $\mathcal{A}$ accepts $\mathcal{C}$'s offer w at t, then

$$\Pi_A^{accept} = (K \wedge Q)w + \hat{V}\left[(K-Q)^+\right],$$

wherein $\hat{V}\left[(K-Q)^+\right] = \Lambda_\tau^t \left[s \wedge (K-Q)^+ p_A + \pi\left((K-s-Q)^+\right)\right] + \left(1-\Lambda_\tau^t\right) \cdot v_\tau\left((K-Q)^+\right)$. If $\mathcal{A}$

rejects, then $\mathcal{C}$ would wait until τ and a deal is made in equilibrium. At t,

$$\begin{aligned}\Pi_A^{reject} &= \Lambda_\tau^t\left[(s\wedge K)\cdot p_A + Q\wedge(K-s)^+p_h + \pi\left((K-s-Q)^+\right)\right]\\ &\quad + \left(1-\Lambda_\tau^t\right)\left[Q\wedge K\cdot\omega(\tau) + v_\tau\left((K-Q)^+\right)\right].\end{aligned}$$

We get $\omega(t)$ by solving out w from $\Pi_A^{accept} = \Pi_A^{reject}$. ■

Derivation of $d_t^A|_{t_s>t}$

If A accepts C's offer w at t, then $\Pi_A^{accept} = (K\wedge Q)w + \bar{V}\left[(K-Q)^+\right]$, wherein $\bar{V}\left[(K-Q)^+\right] = \Lambda_T^t\left[s\wedge(K-Q)^+p_A + \pi\left((K-s-Q)^+\right)\right] + \left(1-\Lambda_T^t\right)\pi\left((K-Q)^+\right)$. If A rejects, then the bargaining breaks down and $\Pi_A^{reject} = \Lambda_T^t\left[s\wedge K\cdot p_A + \pi\left((K-s)^+\right)\right] + \left(1-\Lambda_T^t\right)\pi(K)$. We get $d_t^A|_{t_s>t}$ by solving out w from $\Pi_A^{accept} = \Pi_A^{reject}$. ■

Proof of Proposition 2.1

$\bar{p}_t|_{t_s>t}$ is linear in $d_t^A|_{t_s>t}$, and how $d_t^A|_{t_s>t}$ is related to t can be easily obtained by looking at (2.3) and noticing that Λ_T^t is decreasing in t. ■

Proof of Theorem 2.1

By (2.2) and $\tau(t) = \tau^* = \tau(\tau^*)$, we have $\omega(t) = \left(1-\Lambda_{\tau^*}^t\right)\bar{p}_{\tau^*}|_{t_s>\tau^*} + \Lambda_{\tau^*}^t\left[p_A - p_A^\delta\left(1-\theta(K)\right)\right]$. Taking the first order derivative of $\omega(t)$ with respect to τ^*, we have

$$\frac{\partial\omega(t)}{\partial\tau^*} = -\frac{\lambda_{\tau^*}}{1-\Lambda_t}\bar{p}_{\tau^*}|_{t_s>\tau^*} + (1-\Lambda_{\tau^*}^t)\frac{\partial\bar{p}_{\tau^*}|_{t_s>\tau^*}}{\partial\tau^*} + \frac{\lambda_{\tau^*}}{1-\Lambda_t}\left[p_A - p_A^\delta\left(1-\theta(K)\right)\right].$$

We know that $\bar{p}_t|_{t_s>t} = (1-\gamma)\cdot p_A + \gamma\cdot d_t^A|_{t_s>t}$. Taking the first order derivative of $\bar{p}_t|_{t_s>t}$, we get

$$\begin{aligned}\frac{d\bar{p}_t|_{t_s>t}}{dt} &= \gamma\frac{\lambda_t(\Lambda_T-1)}{(1-\Lambda_t)^2}\left[\theta(K)p_A + (1-\theta(K))\,d_T^A|_{t_s\le T} - d_T^A|_{t_s>T}\right]\\ &= \frac{\lambda_t}{1-\Lambda_t}\left[\gamma(\Lambda_T^t-1)\left(\theta(K)p_A + (1-\theta(K))\,d_T^A|_{t_s\le T}\right) + \gamma(1-\Lambda_T^t)d_T^A|_{t_s>T}\right]\\ &= \frac{\lambda_t}{1-\Lambda_t}\left[\gamma d_t^A|_{t_s>t} - \gamma\left(\theta(K)p_A + (1-\theta(K))\,d_T^A|_{t_s\le T}\right) + (1-\gamma)p_A - (1-\gamma)p_A\right]\\ &= \frac{\lambda_t}{1-\Lambda_t}\left[\bar{p}_t|_{t_s>t} - \gamma d_T^A|_{t_s\le T} - \theta(K)\left(\gamma p_A - \gamma d_T^A|_{t_s\le T}\right) - (1-\gamma)p_A\right]\\ &= \frac{\lambda_t}{1-\Lambda_t}\left[\bar{p}_t|_{t_s>t} - p_h - \theta(K)\left(p_A - (1-\gamma)p_A - \gamma d_T^A|_{t_s\le T}\right)\right].\end{aligned}$$

Therefore, $\frac{\partial \omega(t)}{\partial \tau^*} = \frac{\lambda_{\tau^*}}{1-\Lambda_t}\theta(K)\left[(1-\gamma)p_A + \gamma d_T^A|_{t_s \le T} - p_h\right] = 0$. Then $\omega(t) = \lim_{\tau^* \searrow t} \omega(t) = \bar{p}_t|_{t_s > t}$. ■

Proof of Corollary 2.1

According to (2.2), we have $\omega'(t) = \frac{\lambda_t}{1-\Lambda_t}\left[\omega(t) - (1-\theta(K))\,p_h - p_A\theta(K)\right]$, and thus

$$\frac{d}{d\tau} J_\tau^t(W) = \frac{\lambda_\tau}{1-\Lambda_t}(Q \wedge K)\theta(K)(p_A - R_s).$$

The results follow immediately. ■

Proof of Proposition 2.2

Given $\beta_i \approx 1$, $\mathcal{A}$'s total expected revenue can be approximated by

$$\Pi_A = \sum C_i^I + \sum C_i^{II} \cdot \Lambda(T(p_A)) + \sum C_i^{III} \cdot p_A \cdot \Lambda(T(p_A)).$$

Then we take the first order derivative with respect to p_A and get

$$\begin{aligned}
\frac{\partial \Pi_A}{\partial p_A} &= -\sum C_i^{II} \cdot \lambda(T(p_A)) \cdot \alpha T_o + \sum C_i^{III} \cdot \Lambda(T(p_A)) \\
&\quad -\sum C_i^{III} \cdot p_A \cdot \lambda(T(p_A)) \cdot \alpha T_o \\
&= \lambda(T(p_A)) \cdot \left[\frac{\Lambda(T(p_A))}{\lambda(T(p_A))} \cdot \sum C_i^{III} - \sum C_i^{II} \cdot \alpha T_o - \sum C_i^{III} \cdot p_A \cdot \alpha T_o\right]. \\
&= \lambda(T(p_A)) \cdot G(p_A)
\end{aligned}$$

Suppose there exists p_A^* that solves $G(p_A) = 0$. Because Λ is log-concave, we know $\frac{\Lambda(T(p_A))}{\lambda(T(p_A))}$ is decreasing in p_A. As a result, $G(p_A) > 0$ when $p_A < p_A^*$ and $G(p_A) < 0$ when $p_A > p_A^*$. Therefore, p_A^* is unique and thus Π_A is quasi-concave given that $\lambda(T(p_A)) > 0$. The result follows. ■

Proof of Lemma 2.1

As stated in the text, if $K < \mathbb{TD}$ and $p_A < p_B$, then $\frac{d}{dt}J_t^0 = 0$ for $\forall t \in [0,\tau)$, so by continuity of J_t^0 we have $J_t^0 = constant$ for $\forall t \in [0,\tau]$. If $K < \mathbb{TD}$ and $p_A \ge p_B$, then $\frac{d}{dt}J_t^0 = 0$ for $\forall t \in (\tau, T]$. Again, by continuity of J_t^0 we have $J_t^0 = constant$ for $\forall t \in [\tau, T]$. The result follows. ■

Proof of Lemma 2.2

According to Lemma 2.1, customer i has no incentive to deviate given all other customers buy at

$\tau = 0$. This is irrelevant to customer's demand size, so the argument applies to all customers. ■

Proof of Proposition 2.3

Notice that we can write $\pi(X) = \mathbb{TD} \cdot \mathbb{C}(x)$, for which $\mathbb{C}$ is an increasing concave function with constant parameters and $x = X/\mathbb{TD}$. Given $\kappa \geq 1$, we have $w_{-i}^A = [\mathbb{C}(\kappa - f_{-i}) - \mathbb{C}(\kappa - 1)]/f_i$. Hence, $p_i = (1-\gamma) \cdot p_A \wedge p_B + \gamma \cdot [\mathbb{C}(\kappa - f_{-i}) - \mathbb{C}(\kappa - 1)]/f_i$. Because $f_1 + f_2 = 1$, we have $\bar{p} = f_1 p_1 + f_2 p_2 = (1-\gamma) \cdot p_A \wedge p_B + \gamma \cdot [\mathbb{C}(\kappa - f_1) + \mathbb{C}(\kappa - f_2)] - 2\gamma \cdot \mathbb{C}(\kappa - 1)$. Due to the concavity of $\mathbb{C}$, for $\Delta > 0$, we have $\mathbb{C}'(\kappa - 0.5 - \Delta) > \mathbb{C}'(\kappa - 0.5 + \Delta)$, so $\partial \bar{p}/\partial \Delta = \gamma \cdot [-\mathbb{C}'(\kappa - 0.5 - \Delta) + \mathbb{C}'(\kappa - 0.5 + \Delta)] < 0$. ■

2-Bargainer Equilibrium Prices for Medium and Low Capacity levels

According to (17) and (18), if $f_1 \leq \kappa < 1$, then we can solve for the equilibrium prices,

$$p_i^e = \frac{f_{-i} \cdot (1-\gamma) \cdot p_A \wedge p_B + \frac{f_{-i}}{1-\kappa} \cdot \mathbb{C}(\kappa - f_{-i}) + \gamma \cdot \mathbb{C}(\kappa - f_i)}{\frac{f_i \cdot f_{-i}}{\gamma \cdot (1-\kappa)} - \gamma \cdot (1-\kappa)}, \quad i = 1, 2;$$

if $0 < \kappa < f_1$, then we have

$$\begin{cases} p_1^e = \frac{(1+\gamma \cdot (\kappa \wedge f_2)/\kappa) \cdot (1-\gamma) \cdot p_A \wedge p_B + \gamma \cdot \mathbb{C}\left([\kappa - f_2]^+\right)/\kappa}{1-\gamma^2 \cdot (\kappa \wedge f_2)/\kappa} & \text{, and} \\ p_2^e = \frac{(1+\gamma) \cdot (1-\gamma) \cdot p_A \wedge p_B + \gamma^2 \cdot \mathbb{C}\left([\kappa - f_2]^+\right)/\kappa}{1-\gamma^2 \cdot (\kappa \wedge f_2)/\kappa} & . \blacksquare \end{cases}$$

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3 Influencing Purchase Patterns in High-Tech Supply Chains: the Effect of Price Flexibility

Abstract

Influencing the pace of adoption is important for a seller that introduces a new, short-life-cycle technology in B2B markets. If buyers adopt too early, they face great demand uncertainty in B2C markets; if they adopt too late, they miss early sales. In either situation, buyers will pass on some of the costs through negotiations, resulting in less-than-optimal benefits to the seller. However, influencing the pace of adoption is difficult for a seller because buyer decisions are correlated by externalities such as seller's cost learning and network effect among the buyers, which cause adoption rush or delay. Besides, in many B2B markets sellers cannot dictate the price over time to control adoption. We propose that sellers can influence and optimize buyer behavior through the structure of contract—i.e., fixed- or renegotiable-price—and we support this by conducting both a causal analysis using data from the semiconductor industry and a theoretical analysis using a game-theoretic model. We find that fixed-price contracts lead to faster adoption in the microprocessor market. However, price flexibility in general can affect the pace of adoption in different ways, and the optimal contract choice depends on the strength of externality, strength of competition, bargaining power, and the number of buyers.

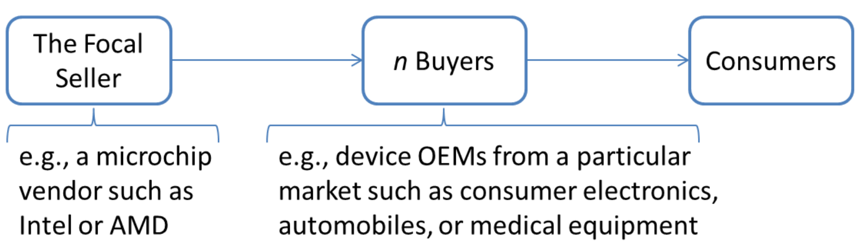
[*Keywords*: product adoption; price flexibility; causal identification; externality; high-tech supply chain]

3.1 Introduction

When it comes to adopting a new technology or component product from a seller that supplies multiple buyers (as illustrated by Figure 3.1), a buyer's decision may bring various externalities to the system, affecting the payoffs and decisions of other buyers. According to previous research, there are at least three types of externality that are related to new product adoption. The first is known as the *learning curve effect* (Adler and Clark 1991; Irwin and Klenow 1994). For example, the unit production cost of semiconductor microchips significantly decreases with the cumulative output. This suggests that buyers could indirectly benefit each other as their adoptions lower seller cost and enable them to bargain for lower prices (Balachander and Srinivasan 1998). Second, the more widely a new component or technology is adopted, the more providers for compatible software, hardware, or services, thus forming a *network effect* (Katz and Shapiro 1985 and 1986), which benefits all the buyers. For example, the more widely a microcontroller is adopted, the more likely other manufacturers are to produce compatible components such as memories and sensors that augment the capabilities of the controller (Yadav and Singh 2004). In addition, many controllers must be programmed after purchase and more application-programming interfaces will support a controller if it is more widely adopted. Third, buyers may be unsure about the benefits and (design, production, and testing) costs associated with adopting a new product and one buyer's decision may influence the beliefs and decisions of others, resulting in an *informational externality* (Bikhchandani et al. 1992; Debo and Veeraraghavan 2009).

When such externalities exist, a buyer's incentive to purchase the focal product over time increases with the cumulative amount purchased by others, thus purchase quantities of different buyers in a given time period will be positively correlated. Although externalities may benefit the

Figure 3.1: The system structure.



seller and buyers, *positive correlation* of buyers' adoptions for a product can be harmful, as it can lead to adoption rush or delay in equilibrium among the buyers, which results in demand-supply mismatches in the business-to-consumer (B2C) market. In particular, demands for high-tech, short life-cycle consumer electronics products are highly uncertain, so OEMs (the buyers) will face great risk if they ramp up productions too early, and the expected mismatch costs may be passed on to the supplier (the seller) through price negotiations. Conversely, if they ramp up productions too late, both the OEMs and the supplier will miss early sales opportunities. In addition, a delay in adoptions potentially undermines sellers' cash flows and their ability to reinvest in R&D. Therefore, sellers should carefully influence buyer behavior when introducing a new product.

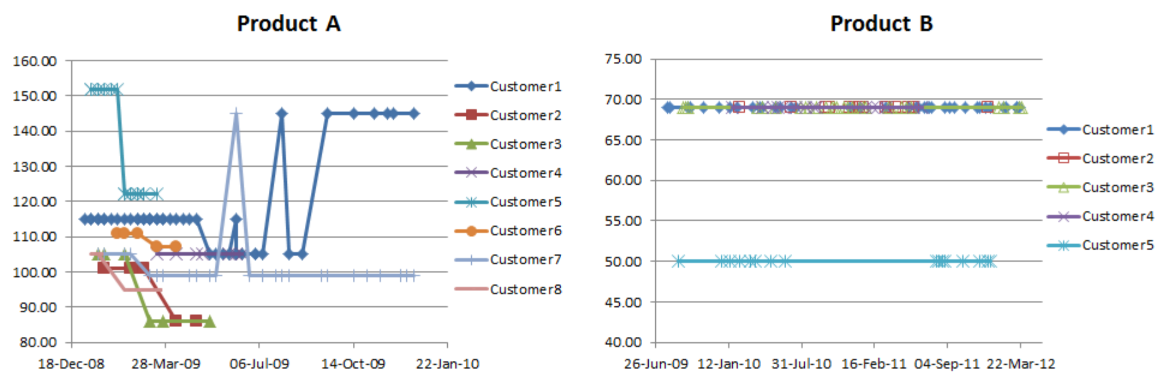
It is well known that in consumer/B2C markets, sellers frequently use intertemporal pricing strategies such as limited-time discounts or free trials (Xiong and Chen 2014) to spark adoption in early stages and control the process over time with price. However, in many business-to-business (B2B) markets, such intertemporal pricing strategies may not be effective because buyers buy large quantities and sellers normally do not have absolute pricing power. Instead of dictating prices over time, sellers in industries such as semiconductor chips (Zhang et al. 2014), medical devices (Grennan 2013), airplanes (Garvin 1991), raw materials (Elyakime 2000), and services (Bajari et al. 2006) have to negotiate with buyers to settle prices. Hence, how sellers can control the pace of adoption in these cases is an open question and we try to answer it in two steps.

In the first step, we interact with a major semiconductor chip company and we analyze the sales data provided by this company. We find that the company is using different types of contracts in terms of price flexibility when selling to downstream OEM buyers. The price flexibility is manifested in contract specifications about how frequently and to what extent the prices can be renegotiated. In Figure 3.2, we show two typical price patterns for semiconductor chips sold to major buyers.[1] We can see that prices for product A were updated one or multiple times across its life cycle, and we refer to these contracts as *renegotiable-price contracts*; for product B, prices for all five buyers were constant across the product life cycle, and we call these contracts *fixed-price contracts*.

Although the company has the conjecture that the price flexibility may influence buyer adoption decisions, the influence has not been verified and it is unclear how the adoption would be affected. To answer these questions, we construct empirical measures for the price flexibility and adoption

[1] Note that buyers possessed different bargaining powers, so they received different prices through price bargaining.

Figure 3.2: Recorded transaction prices for two different products.



Note: Data is obtained from a major microchip vendor.

pattern and we show a correlation. Furthermore, we devise an instrumental variable (IV) and perform a causal analysis to show that price flexibility influenced buyer behavior (i.e., pace of adoption). Based on our observations, we propose that sellers can influence buyers' adoption decisions through the structure of contract; i.e., a fixed- or renegotiable-price contract.

In the second step, given that the price flexibility is a lever, we then study how the seller should best employ it. Using a two-period, game-theoretic model, we analyze the impact of contract choice on buyer behavior as well as on the pace of product adoption in a B2B market with the existence of positive externalities. In this model, we consider the following factors: (*i*) demand uncertainty and demand learning; (*ii*) adoption-independent demand potential; (*iii*) adoption-dependent valuation for the buyers; (*iv*) adoption-dependent cost learning for the seller; and (*v*) price bargaining between the seller and the buyers. Our model helps us understand the incentives associated with different structures of contracts, and our analysis reveals the following interesting but unintuitive results.

- Compared to a renegotiable price, a fixed-price contract can lead to faster adoption in some cases but slower adoption in others.
- The choice between a fixed and renegotiable price depends on the strength of externality, the strength of competition from alternative technologies or products, the relative bargaining power of the buyer, and the number of buyers (i.e., the scale of the "network").

Our work is related to the stream of literature that studies product / technology diffusion-process management. However, the topic of our study, product diffusion-process management in high-tech supply chains through the choice of contract structure, has not been well investigated. Most research on product diffusion is based on the Bass model (Bass 1969). Robinson and Lakhani (1975) conducted the first study of a dynamic pricing problem of a seller who faces a price-dependent demand process that is represented by an extended Bass model. Under a similar framework, Kalish and Lilien (1983) studied optimal government subsidy policies for promoting the adoption of a new energy source. Krishnan et al. (1999) then proposed the generalized Bass model and developed an optimal pricing path that is consistent with empirical data. More recently, Ho et al. (2002), and Kumar and Swaminathan (2003) studied the management of demand and sales dynamics in the new product diffusion process under supply constraint: in their models, a seller can turn down the request of a customer who then either waits or exits the market.

Our study is distinct from this stream of research in the following ways. First, customer behavior is different. While most of the previous research focuses on product adoption in consumer markets and assumes buyers are non-strategic, we focus on business-to-business markets and empirically find that buyer behavior is significantly dependent on the price mechanism and other buyers' decisions. Second, the management lever is different. Previous research normally assumes that the seller has the pricing power and thus could employ intertemporal pricing strategies to influence buyer behavior; in contrast, we consider markets in which prices are subject to negotiations, and we propose contract choice in terms of price flexibility as a lever. Third, the problem focus is different. Previous research uses the Bass model for the demand process and assumes that the market potential is known and fixed; the focus is on problems such as optimal pricing strategy and supply constraints. However, we assume that the demand potential in the end market is unknown and can be learned over time and we focus on how to minimize the cost of demand-supply mismatch in a two-echelon supply chain system, where positive externalities of adoption exist. Finally, we use real data to show that the choice of contract-price flexibility can serve as an effective lever for managing product adoption in high-tech supply chains.

Papers that explicitly consider positive externalities in the product-adoption process normally do not use the Bass model but assume that buyers are strategic. Katz and Shapiro (1986) study the network effect in the product-diffusion process and focus on competitive equilibrium in the market.

Balachander and Srinivasan (1998) study the learning curve effect in new product introduction and focus on the optimal introductory pricing strategy. Our research is different because we consider demand uncertainty and price negotiation and we focus on contract choice. In addition, while the network effect studied in previous research benefits end consumers, we consider network externalities that only impact the buyers (e.g., OEMs), which we call *intermediary good network externality.* Our interactions with sales and procurement managers in high-tech supply chains suggest that thousands of products are introduced into B2B supply chains each year that experience intermediary good network externalities of the kind assumed here.

3.2 Empirical Investigation

In this section, we employ empirical analysis to show that, in an industry where both fixed- and renegotiable-price contracts are frequently used, buyers' product adoption decisions are influenced by contract structure chosen by sellers. To do so, we propose measures that represent contract structure and adoption pattern respectively, and explore the correlation between the measures. Next, we use an instrumental variable to establish the causal relation between contract structure and adoption pattern. In addition, we show that adoption patterns are positively correlated among buyers, which supports the fact that product adoption generates positive externalities.

3.2.1 Data Description

For this study, a major global microprocessor maker supplied sales data encompassing 3,826 products and 251 customers over a three-year period. Each entry of this dataset consisted of a customer ID, product ID, product category, subcategory, sales territory, bill quantity, unit price, and date of transaction. Let I_C and I_P be the indices of customer and product, respectively. We define an *instance* θ_{ij} as the set of purchase (price-quantity) records related to a customer i and a product j, where $i \in I_C$ and $j \in I_P$. Let $\mathcal{T}_{ij}$ be the set of dates at which customer i purchased product j. Then let q_{ijt} and p_{ijt} denote the transaction quantity and price for customer i and

Table 3.1: Instance-Level Summary Statistics

	Mean	S.D.	Min	Max
Initial Price ($)	126	200	0.01	2,625
Total Quantity	59×10^3	266×10^3	101	8.2×10^6
Total Value ($)	1.8×10^6	5.8×10^6	30	132×10^6
Duration (days)	306	266	7	1,179
ACVP†	0.53	16	0	1,284
TDPS‡	0.61	0.26	0.01	1.00
Dist. of ACVP	≤ 0	≤ 0.05	≤ 0.5	Total
# of Instances	4452	6353	9603	9,773

† *ACVP* is a measure of price flexibility. The median is 7.48×10^{-4}.

‡ *TDPS* is a measure of adoption pattern.

product j at time $t \in \mathcal{T}_{ij}$. Hence, an instance is defined as

$$\theta_{ij} := \{(t, q_{ijt}, p_{ijt}) : t \in \mathcal{T}_{ij}\}.$$

Notice that the influence of contract structure extends across the entire life span of an instance, and that any single transaction offers little relevant information. Hence, we examine data aggregated at instance level. In addition, let ω_i be the vector of characteristics that are exogenous for customer $i \in I_C$, and let π_j be the vector of characteristics that are exogenous for product $j \in I_P$. For customers, exogenous characteristics ω include the total purchase value with the seller over the three years (the "size") and the geographic location. For products, exogenous characteristics π may include the number of customers buying the product and the product category. Define $\Omega := \{\omega_i : i \in I_C\}$, $\Pi := \{\pi_j : j \in I_P\}$, and $\Theta := \{\theta_{ij} : i \in I_C, j \in I_P\}$. In this way, the dataset can be summarized as $\mathcal{D} := \{\Omega, \Pi, \Theta\}$.

The instance-level summary statistics are shown in Table 3.1, where ACVP and TDPS are defined later as measures for price flexibility and adoption pattern, respectively. In particular, we have ACVP > 0 if a price change is recorded for an instance; otherwise, we view an instance with ACVP $= 0$ as being under a fixed-price contract. As shown in Table 3.1, nearly half of the instances are with fixed-price contracts. This means that both fixed- and renegotiable-price contracts were frequently used, and provides us with an opportunity to test the impact of contract structure.

3.2.2 Measures

In the following, we propose measures for contract structure and adoption pattern, respectively.

Contract Structure. The dataset does not give information about the underlying contract structure φ for each instance. Thus, we have to uncover φ from the data. However, it is important to note that in practice, contract choice for price flexibility is hardly a binary one. For example, sellers may allow price renegotiations, but restrict the range of price variations or degree of price flexibility by properly designing the contract. Buyers' adoption patterns may depend on the degree of price flexibility. Therefore, we propose a continuous measure for the degree of price flexibility in this study: the adjusted coefficient of variation (CV) of price (ACVP), which is defined as the standard deviation of price divided by the initial price of an instance. Mathematically,

$$\mathrm{ACVP}_{ij} := \frac{\sqrt{\frac{1}{|\mathcal{T}_{ij}|-1}\sum_{t\in\mathcal{T}_{ij}}\left(p_{ijt}-\frac{1}{|\mathcal{T}_{ij}|}\sum_{k\in\mathcal{T}_{ij}}p_{ijk}\right)^2}}{p_{ij(\min\mathcal{T}_{ij})}}. \tag{3.1}$$

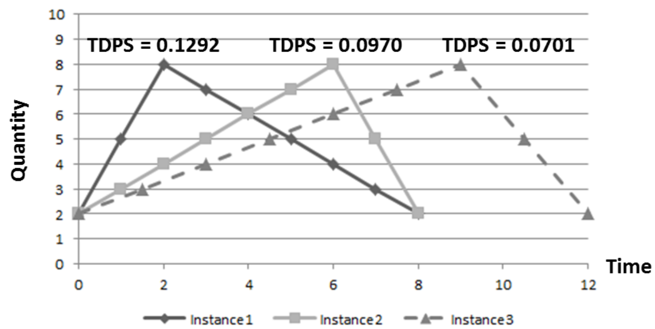
Other possible candidates to measure φ include: (i) the average duration of a price, (ii) the variance of price, and (iii) the CV of price. However, in the Appendix, we point out the problems with these measures, and thus we adopt the ACVP. Note that instances having only one transaction contain no information about the underlying price mechanism, so those instances are deleted from $\mathcal{D}$.

Adoption Pattern. The adoption pattern is basically the pace of adoption, which is manifested in how purchases are made over time. If a buyer decides to slow down the adoption process, the effects are mainly twofold. First, the *duration* of an instance may be prolonged, given the same total amount of purchase. Second, the intertemporal distribution of purchase quantity of an instance will be right-skewed along the time axis (skewed towards the future). To measure the pace of adoption, we introduce the time-discounted percentage sales (TDPS) of an instance, using the first date of the instance as the time reference. Mathematically, we have

$$\mathrm{TDPS}_{ij} \quad := \quad \sum_{t\in\mathcal{T}_{ij}} q_{ijt}\cdot e^{r(\min\mathcal{T}_{ij}-t)} / \sum_{t\in\mathcal{T}_{ij}} q_{ijt}, \tag{3.2}$$

where r controls the time weights of this measure.[2] Notice that the TDPS is equivalent to the Laplace transform of the purchase-quantity distribution over time, so comparing it means ranking quantity distribution in the Laplace transform order. Figure 3.3 illustrates the TDPS meaning.

Figure 3.3: Fictitious instances with different adoption patterns.



In Figure 3.3, the pattern of instance 1 serves as a benchmark. Compared with instance 1, instance 2 has the same total amount of purchase but have large orders placed at a later stage so that the pattern is right-skewed. In instance 3, for the same amount in each purchase as in instance 2, the buyer deferred all the purchases so that the pattern is scaled up along the time axis compared with the second one. Hence, the TDPSs for these three instances are in decreasing order.

3.2.3 Correlation and Causal Analysis

Our goal is to uncover the relation between the unobservable variables—price flexibility and adoption pattern—by investigating the relation between the ACVP and TDPS. We start with four preliminary regressions and the results are summarized in Table 3.2. The variables we control in the regressions include an indicator for fixed-price contracts, the logarithm of the total purchasing value of a customer over the three years (i.e., Cust-Size), the lag of starting date relative to the starting date of the first customer (i.e., Time-Lag), the total purchase quantity of the instance, the average price of the instance, and the number of customers for the product (i.e., Cust-Base).

We can learn from the preliminary regressions that a significant negative correlation exists

[2]In this research, we carefully set $r = 2$ per year to ensure that the TDPS is an effective measure.

Table 3.2: Results of Preliminary Regressions

Variables	(i) TDPS	(ii) TDPS	(iii) ACVP	(iv) ACVP
TDPS	-	-	-0.035*** (0.010)	-0.032*** (0.010)
ACVP	-0.049*** (0.015)	-0.046*** (0.015)	-	-
Fixed-Price	0.150*** (0.006)	0.152*** (0.006)	-0.129*** (0.005)	-0.128*** (0.005)
Cust-Size	-0.011*** (0.001)	-0.012*** (0.001)	0.003*** (0.001)	0.005*** (0.001)
Time-Lag	1.3E-4*** (2.2E-5)	1.6E-4*** (2.1E-5)	5.6E-5*** (1.8E-5)	7.6E-5*** (1.8E-5)
Total-Qty	-8.1E-8*** (1.2E-8)	-	0.4E-9 (1E-9)	-
Avg-Price	-1.6E-4*** (1.4E-5)	-	1.6E-5 (1.2E-5)	-
Cust-Base	-3.8E-4* (2.15E-5)	-	-0.001*** (1.7E-4)	-
Constant	0.779*** (0.028)	0.770*** (0.026)	0.109*** (0.025)	0.041*** (0.023)
Adj. R sq.	0.17	0.15	0.12	0.12
Obs.	6487	6487	6487	6487

Note: Standard errors are in parentheses. *p<0.1; **p<0.05; ***p<0.01.

between the TDPS and the ACVP. The results also imply that if the TDPS of an instance is around 0.61 (the average), then the change of contract structure from fixed-price to renegotiable-price will be associated with a delay of nearly *two month for each purchase*, which is meaningful for semiconductor chips.

Although we try to control as many variables as possible, there are unobservable variables that can influence TDPS and ACVP, such as demand uncertainty, market competition, and buyer preferences. Also the ACVP and TDPS are approximations for the price flexibility and purchase pattern. These factors contribute to low R^2. The focus of our analysis is on uncovering a relation between the price flexibility and purchase patterns but not on developing a prediction model. The low P values establish that the relation is statistically significant.[3]

The following lists alternative casual hypotheses for the correlation between the ACVP and

[3]Note that high R^2 is normally not required for causal analysis (e.g., Ichino and Winter-Ebmer 1999; Glaeser et al. 2004).

TDPS.[4] Notice that the preliminary regressions suggest that the total purchase quantity and the average price of an instance are not correlated with the ACVP, so we do not consider them here.

Hypothesis (I). The contract structure causes adoption pattern; i.e., buyers respond strategically to the degree of price flexibility. For example, it is likely that buyers tend to delay their purchases when prices are renegotiable. If this is the case, contract choice should be a useful lever for sellers to control product-adoption processes.

Hypothesis (II). The purchase pattern of each instance is exogenous, and the seller or the buyer designed the contract structure based on this pattern. For example, the seller may offer a discount to a buyer if product adoption was slow at early stage, or, if a buyer knew that larger quantities would be purchased at later stage, she might prefer a contract with a flexible price so that a lower price could later be negotiated. Another scenario may be that buyers have been strategically controlling their adoption patterns to influence the price regardless of the initial contract choice. Whichever is the case, the degree of price flexibility is caused by the adoption pattern, and *ex-ante* contract choice is not useful.

Hypothesis (III). Price flexibility and adoption pattern are commonly caused by a set of unobservable variables ε (e.g., price flexibility and adoption pattern are jointly determined by buyers, by product-specific market norm, or by a general trend over time).

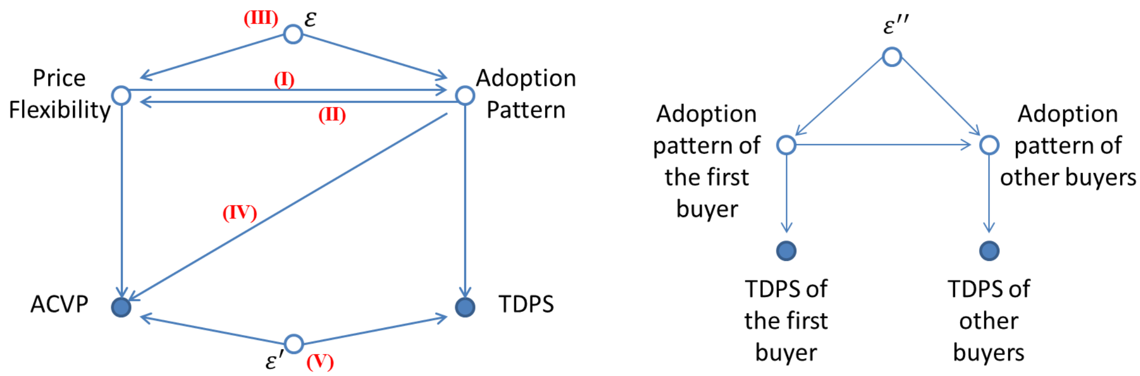
Hypothesis (IV). The measures ACVP and TDPS are commonly influenced by the duration of an instance. The TDPS is by definition dependent on the duration, which is a part of the adoption pattern. However, there is no guarantee that the ACVP is directly affected by the duration of an instance. In addition, it is important to note that the ACVP could be correlated with duration even if there is not a direct link between the two; e.g., they could be commonly caused by the adoption pattern given (II) is true.

Hypothesis (V). The measures ACVP and TDPS are commonly influenced by a set of unob-

[4] Following Reiss (2005), we assume that: Any two variables X and Y are correlated if and only if either (i) X causes Y, (ii) Y cause X, (iii) a common cause Z causes both X and Y, or (iv) any combination of (i)–(iii). Because the ACVP and TDPS are just measures, (i) and (ii) do not apply. Also, we assume the *transitivity* of causal relations holds: For any three variables X, Y, and Z, if X causes Y and Y causes Z, then X causes Z.

servable variables ε', which may include buyer-specific, product-specific, or time-related factors.

Figure 3.4: Possible causal relations.



Note: A circle represents an unobservable variable, a solid dot represents an observable variable, and a directed edge represents a causal relation with the edge pointing from the cause to the result.

All of the possible causal relations are summarized and illustrated on the left of Figure 3.4. Here, we propose an IV that can be used to test the possible causal relations. Thanks to the seller's learning-by-doing and the existence of other possible externalities, buyers may respond directly to adoption decisions of other buyers, especially those who adopt earlier. Even if purchase information is privy to each buyer, it is still very likely that purchase decisions are highly correlated among buyers due to some common causes. As a result, we separate the first buyer of a product from the rest and use the TDPS of the first buyer to generate an instrumental variable to test the relation between the ACVP and TDPS for other buyers of this product. The possible causal relations are shown on the right of Figure 3.4.

3.2.4 Test of Hypotheses

In preparation, we delete data for products with only one buyer and 13 instances with an ACVP greater than 10 (abnormally large). In addition, to avoid the truncation effect in this dataset, we only include instances with an observed starting date at least one month later than the starting date of the dataset and an observed ending date at least one month earlier than the ending date of the dataset. We then run regressions (a), (b), and (c), controlling a set of variables $C = \{$Cust-Size, Time-Lag$\}$. The results are summarized in Table 3.3.

Table 3.3: Tests of Correlations With the Instrumental Variable

Regression	$\hat{\gamma}$	*P-value*
(a) $Y = \gamma_a \cdot X + \mu_a' \cdot C + \epsilon_a$	0.3944	4.9E-232
(b) $Y = \gamma_b \cdot Z + \lambda_b \cdot \mathbb{I}\{Z = 0\} + \mu_b' \cdot C + \epsilon_b$	-0.0486	0.0029
(c) $X = \gamma_c \cdot Z + \lambda_c \cdot \mathbb{I}\{Z = 0\} + \mu_c' \cdot C + \epsilon_c$	0.0442	0.9935

Note: There are 5,652 observations. The P-value for (c) is based on the one-sided test.
X: TDPS for the first buyer of a product.
Y: TDPS for a buyer who is not the first buyer of a product.
Z: ACVP for a buyer who is not the first buyer of a product.

Regression (a). We regress the TDPS of a buyer who is not the first buyer against the TDPS of the first buyer. We have a *positive* correlation with a significant P-value. This means that adoption patterns among the buyers are positively correlated, possibly due to the positive externalities.

Regression (b). We regress the TDPS against the ACVP for a buyer who is not the first one, with the contract type and the total purchase quantity of the instance controlled. We have a *negative* correlation with a significant P-value. This regression is similar to the one with the full dataset, and it suggests the existence of causal relations.

Regression (c). We regress the TDPS of the first buyer against the ACVP of a buyer who is not the first one for that product, with the contract type and the total purchase quantity controlled. We have no significant negative correlation. However, if hypothesis (I) is not true, we would have obtained a negative correlation as discussed in the following.

First, suppose (II) or (IV) or both are true. In other words, the TDPS and the ACVP are commonly influenced by the adoption pattern. In this case, the TDPS of the first buyer (X), the TDPS (Y) and the ACVP (Z) for a buyer who is not the first should be commonly influenced by the adoption pattern of the first buyer or by some factor ε''. Regressions (a) and (b) suggest X and Z should be negatively correlated. However, (c) shows that this is not supported by the data.

Second, suppose (III) or (V) or both are true. As broadly as we can imagine, the unobservable factors ε and ε' can only come from three sources: product-specific factors, buyer-specific factors, or some market- or technology-related trends. We test them in sequence.

- If ε and ε' are product-specific characteristics, such as supply-side or demand-side competition, life-span, grade, or usage among others, then all the buyers buying a product are subject to the same impact; i.e., ε and ε' should influence all the buyers of a product. As a result, X, Y, and Z should be commonly influenced and thus correlated. However, this is not supported by regression (c).

- If ε and ε' are time-related factors, then all the buyers buying in the same period are subject to the same impact. We learn from the data that the average time lag between a buyer's first purchase of a product and the first-ever purchase of that product is less than a quarter. Thus, the first buyer of a product should be viewed as being in the same period as subsequent buyers. As a result, X, Y, and Z should be commonly caused and thus correlated. However, this is not supported by (c).

- Given our previous arguments, ε and ε' can only be buyer-specific characteristics. If this is the case and (I) is not true, then the correlation between the ACVP and the TDPS with total quantity controlled should not exist conditioning on a specific buyer. However, we show that this is not true (in the Appendix). The correlation suggests that contracts were not offered by buyers; otherwise, buyers would have selected contracts to fit their situations, rendering contract structure and adoption pattern commonly caused by buyer type.[5]

Finally, to close the loop and confirm hypothesis (I), we can use the correlation between the ACVPs of the first and a subsequent buyer, which is due to the seller's contract preference, and apply a similar IV method: regress the TDPS of the buyer who is not the first one against the ACVP of the first buyer for that product with the contract type and the total purchase quantity controlled. We obtain a coefficient -0.044 with P-value 0.0735, implying a negative correlation. As a result,

[5]Let q_i denote the purchase profile of buyer i, and $u_i(q_i, \varphi)$ the payoff. If the contract is offered by buyers, then buyer i would choose contract $\varphi_i = \arg\max_{q_i,\varphi} u_i(q_i, \varphi)$, which suggests that φ should be a function of buyer type. Although we cannot directly prove that contracts were indeed offered by the seller, it is very likely the case given that there was no third parties who offered the contract.

we claim that *contract structure influences adoption pattern; i.e., buyers respond to the contract structure by intertemporally shifting their purchase quantities.*

3.2.5 Discussions

To arrive at the above causal relation, we have employed an approach that is different from traditional methods such as direct *regression* of buyer behavior against contract structure. Due to the nature of the problem we are examining, direct regression is not effective. First, direct regression cannot identify causal directions. Second, to estimate causal effects through direct regression, all the possible common causes have to be controlled simultaneously, many of which are unobservable, and we cannot reject the existence of the possible common causes. In addition, it is hard to find an instrumental variable that is independent of all of the unobservable variables. In contrast, our approach is immune to all these concerns. In particular, our approach is to rule out one alternative hypothesis at a time, and thus we do not need to simultaneously test all the alternative hypotheses.

The causal relation we just observed suggests that the contract structure is an effective lever to influence buyer behavior in the product-adoption process, at least for a major seller in the semiconductor industry. Several questions immediately emerge: Does renegotiable price always lead to late adoption? Is faster adoption always preferable? How should the seller pick the contract? To gain a deeper understanding, we develop a model to explore how the price flexibility and contract structure would affect buyer behavior in different situations.

3.3 The Model

Now we build a stylized model and make the following key assumptions. (*i*) The seller makes the choice between fixed- and renegotiable-price contracts. (*ii*) Buyers make quantity decisions after the contract type is determined. (*iii*) Buyers face uncertain demand and they learn about demand over time. (*iv*) The demand potential in the B2C market is independent of product adoption. (*v*) The unit production cost for the seller is decreasing with the cumulative production quantity. (*vi*) Product adoption influences buyers' valuation for the focal product. (*vii*) Prices are negotiated.

Market Characteristics

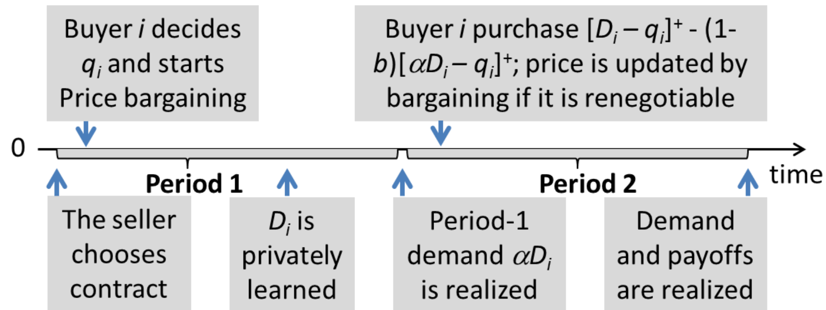
Consider a seller (he) that produces a (component) product X and sells it to n buyers in two periods.[6] Let $N = \{1, 2, ..., n\}$ be the set of the indices for the buyers, and "0" index the seller. The buyers use product X to produce similar but differentiated (or at least branded) final products. Each unit of final product consumes one unit of product X. To make the seller's problem tractable, we abstract from direct competition among the buyers and their pricing problems in the end market. We assume that the total demand potential D_i for buyer i is exogenously given. $\tilde{D} := (D_1, \ldots, D_n)$ is unknow *ex ante*, but follows prior joint distribution F, which is common knowledge. The marginal distribution F_i has a compact support $[0, M_i]$. Demand D_i will be realized over two periods: $\alpha_i D_i$ in period 1 and $(1 - \alpha_i)D_i$ in period 2, where $\alpha_i \in (0, 1)$ for any i is publicly known, plausibly due to known product-diffusion patterns. We model demand learning by assuming that D_i will be observed by buyer i during period 1. Note that brand-choice-based product substitution in the end market is captured by the joint distribution F. However, for the sake of tractability, we ignore stockout-based substitution among the final products. In case of stockout in period 1, a fraction $b \in [0, 1]$ of the unsatisfied demand will be backlogged and satisfied by priority in period 2; the rest will exit the market.

Adoption Decisions

Buyers can purchase product X in both periods. Holding cost for excess purchases is ignored. Let q_i and fq_i denote purchase quantity of buyer i in period 1 and period 2, respectively. Moreover, let d_{it} and s_{it} denote the demand and sales of buyer i in period t, respectively. Then we have $s_{i1} = \min\{q_i, d_{i1}\} = \min\{q_i, \alpha_i D_i\}$ and $s_{i2} = d_{i2}$, because demand is known to i in period 2. In particular, the total demand in period 2 is the sum of backlogged demand and newly realized demand—i.e., $d_{i2} = b\,[\alpha_i D_i - q_i]^+ + (1-\alpha_i)D_i$, where $x^+ := \max\{0, x\}$—and the purchase quantity $fq_i = \left[d_{i2} - [q_i - \alpha_i D_i]^+\right]^+$ just fills the gap between demand and existing capacity for a rational buyer. The following lemma transforms fq_i from a complicated function to the difference of two simple convex functions, and it provides an expression for the amount of excess capacity that is carried over and used in period 2. Note that $x \wedge y := \min\{x, y\}$.

[6] A two-period model sufficiently captures the dynamics of our problem, and the finite horizon reflects that the product has a limited market window, possibly because of advances in technology or changes in consumer tastes. Such a setting is consistent with papers that study product adoption with positive externalities and strategic buyers, e.g., Katz and Shapiro (1986) and Balachander and Srinivasan (1998).

Figure 3.5: Sequence of events.



Lemma 3.1. $fq_i = [D_i - q_i]^+ - (1-b) \cdot [\alpha_i D_i - q_i]^+$ *and* $s_{i2} - fq_i = D_i \wedge q_i - \alpha_i D_i \wedge q_i$.

Externalities

Define S_t as the sum of purchase quantities across all buyers up to period $t = 1, 2$. Hence, we have $S_1 = \sum q_i$ and $S_2 = \sum (q_i + fq_i)$. The purchases result in two types of externalities. First, the seller's marginal production cost c is a decreasing convex function of the cumulative production quantity, which is an increasing function of the total purchase quantity; i.e., unit production cost is $c(S_t)$ in period t, for which $c' \leq 0$ and $c'' \geq 0$. Second, the value of product X for buyer i in period 1 is $v_i(S_1)$, for which $v' \geq 0$ and $v'' \leq 0$,[7] and the value in period 2 is $\rho \cdot v_i(S_2)$, where $\rho > 0$. The value is realized after a final product is sold, and it captures the value of product X in the end market net of the associated production and selling costs. Thus, leftover products will have zero value after period 2. In the base model, we assume $\rho = 1$, and time-dependent value with general ρ will be discussed in Section 3.7.1. For simplicity, we assume S_2 is large enough so that $c(S_2) \approx c(\infty) = c^*$ and $v_i(S_2) \approx v_i(\infty) = v_i^*$. Lastly, we assume that demand potential is independent of S_t. This is true when the network effect is restricted to the buyers,[8] and is also the convention in the literature (e.g., Bass 1969; Ho et al. 2002).

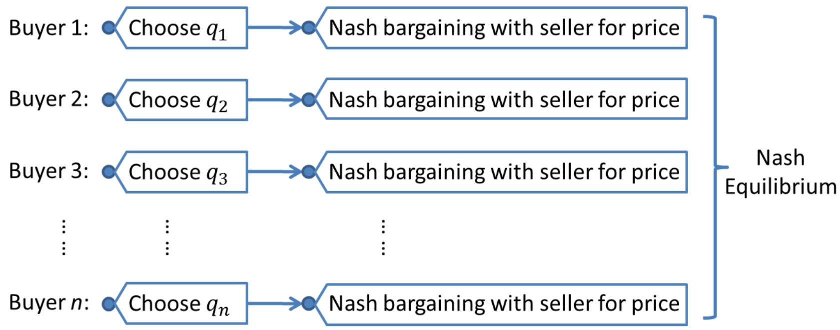
The Bargaining Game

The game proceeds as illustrated in Figure 3.5. At time zero, the seller chooses whether or not

[7]This assumptions about $v_i(\cdot)$ is consistent with the model proposed by Katz and Shapiro (1985), the seminal work on general network effect.

[8]This is particularly true in the semiconductor industry, where the costs of software, hardware, and (manufacturing or testing) services that are compatible with a microchip are usually not the concerns of consumers.

Figure 3.6: Bargaining game in period 1.



to use a fixed-price contract for the product; the type of contract will be applied to all the buyers.[9] In period 1, buyer i decides her quantity q_i and enters a Nash bargaining (NB) with the seller for the price. We assume that the purchase quantity and the bargaining outcome is privy to each buyer (at least for a short period of time), so the multilateral bargaining process can be treated as a simultaneous move game (Cournot-Nash game) among all the buyers. In other words, the seller bargains simultaneously with all buyers who take other buyers' decisions and bargaining results into consideration. This is commonly assumed for bargaining with externalities (e.g., Horn and Wolinsky 1988). However, this game structure is not compatible with informational externality, which typically entails sequential moves. Thus, in our model, the externality with $v_i(\cdot)$ is due to the network effect only.

Once the adoption decisions are made in period 1, product X is integrated into the design of the final products; although it is possible for buyers to switch to alternative components, it will be costly to do so and thus we assume that buyers do not switch unless their negotiations with the seller break down. Let w_i' and w_i'' be the transaction prices for buyer i obtained through Nash bargaining in periods 1 and 2, respectively. If a fixed-price contract is used, then $w_i' = w_i''$; if the price is renegotiable, then w_i'' will be determined by another round of Nash bargaining for each buyer i. Since externalities no longer exist in period 2, buyers do not consider others in period-2 bargaining. The bargaining game in period 1 is illustrated in Figure 3.6.

Let β_i be the relative bargaining power of buyer $i \in N$ when bargaining with the seller. The *generalized Nash bargaining* model predicts that, if firm i's payoff and outside option for the focal

[9]Mixed contract choices will be discussed in the Appendix.

transaction are $V_i(w)$ and θ_i given transaction price w, then the bargaining outcome is a price w^* that maximizes $(V_i(w)-\theta_i)^{\beta_i}\cdot(V_0(w)-\theta_0)^{1-\beta_i}$. In particular, if $V_i(w)-\theta_i+V_0(w)-\theta_0$ is independent of w, then w^* splits the fixed pie between the buyer and the seller in proportion to their respective bargaining powers. In addition, if a renegotiable-price contract is used and the bargaining breaks down in period 2, the buyer has to acquire a substitutable product of value v_o from the outside market at cost c_o (including procurement cost and switching cost such as reconfiguration of production line). For ease of exposition, let $\delta_o := v_o - c_o$, which is a measure for the strength of competition from alternative technologies or products. Note that δ_o can be negative when the switching cost is high.

Payoff Functions

Based on the assumptions laid out so far, buyer i's *ex-post* payoff can thus be formulated as

$$\begin{aligned} u_i =& v_i\left(S_1\right)\cdot s_{i1} - w_i'\cdot q_i + \rho\cdot v_i^*\cdot s_{i2} - w_i''\cdot f q_i \\ =& \left(v_i\left(S_1\right)-w_i'\right)q_i - \left[q_i-\alpha_i D_i\right]^+ v_i\left(S_1\right) + \\ & \left(D_i\wedge q_i - \left(\alpha_i D_i\right)\wedge q_i\right)\rho v_i^* + \left(\rho v_i^* - w_i''\right) f q_i. \end{aligned} \tag{3.3}$$

In the second expression of u_i, the first three components are the payoff generated by the first purchase and the last component is the payoff generated by the second purchase. Given the set of contracts $\mathscr{C} := \{\mathscr{C}_i := (q_i, w_i', w_i'') : i\in N\}$, the seller's *ex-post* payoff can be formulated as

$$u_0 = \sum_{i\in N}\left(w_i' - c\left(S_1\right)\right)q_i + \sum_{i\in N}\left(w_i'' - c^*\right) f q_i, \tag{3.4}$$

which is the sum of total profits in period 1 and period 2. Let $U_i := \mathbf{E}\left[u_i|\mathscr{C}\right]$ denote firm i's ($i\in\{0\}\cup N$) expected payoff at time 0 given $\mathscr{C}$. Note that for buyer i, U_i is not only affected by her own quantity decision q_i but by also other buyers' decisions $q_{-i} := (q_1,\ldots,q_{i-1},q_{i+1},\ldots,q_n)$. To pin down the optimal quantity decisions, we focus on a pure strategy Nash equilibrium among the buyers in this study.

3.4 Model Analysis

3.4.1 Centralized System (CS)

The system payoff is $u_{CS} = u_0 + \sum_{i \in N} u_i$. Note that in our model the contract that generates a higher system payoff also brings a higher payoff for the seller. This is because the buyers and the seller always share the net system payoff proportionally to their bargaining power, given that their outside options are independent of the contract choice and they are risk-neutral. Hence, in this study we consider a seller that aims to maximize the system payoff, and the payoff in a centralized system can serve as a benchmark for her to make the contract choice.

In a centralized system, we assume that the firms optimize the expected system payoff $U_{CS} = \mathbf{E}\left(u_{CS}(q)\right)$ over their own quantities, where $q := (q_1, \ldots, q_n)$ is the action profile. Bargaining is not necessary given that U_{CS} is independent of the transaction prices. To simplify our notation, we introduce:

$$\Delta_i\left(q_i|q_{-i}\right) := C_i^u\left(q_i|q_{-i}\right) - C_i^o\left(q_i|q_{-i}\right); \tag{3.5}$$

$$\begin{aligned} C_i^u\left(q_i|q_{-i}\right) :=& v_i\left(S_1\right) \cdot \left[1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right] + \\ & c^* \cdot \left[b\left(1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right) + F_i\left(\frac{q_i}{\alpha_i}\right) - F_i\left(q_i\right)\right]; \\ C_i^o\left(q_i|q_{-i}\right) :=& c\left(S_1\right) + b\rho v_i^* \cdot \left[1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right]. \end{aligned}$$

We call $\Delta_i\left(q_i|q_{-i}\right)$ the *net mismatch cost*, and it is the marginal impact of q_i on the system payoff when its impact on the externalities is ignored. C_i^u and C_i^o are the expected underage and overage costs for buying one additional unit, respectively. The details for the underage and overage costs in different scenarios are illustrated in the Appendix. After careful algebraic operations on $\partial U_{CS}/\partial q_i$, we get the result in Proposition 3.1.

Proposition 3.1. *In a centralized system, the incentive for adoption in period 1 for any buyer $i \in N$ is composed of three parts: the net mismatch cost $\Delta_i\left(q_i|q_{-i}\right)$, the seller-based externality*

$-c\,(S_1)' \cdot \sum q_j$, *and the buyer-based externality* $\sum v_j\,(S_1)' \cdot \mathbf{E}\,[q_j \wedge \alpha D_j]$. *In particular,*

$$\frac{\partial U_{CS}}{\partial q_i} = \Delta_i\,(q_i|q_{-i}) - c\,(S_1)' \cdot \sum_{j \in N} q_j + \sum_{j \in N} v_j\,(S_1)' \cdot \mathbf{E}\,[q_j \wedge \alpha_i D_j]\,. \tag{3.6}$$

Several observations can be obtained from (3.6). Particularly, we are interested in whether $S_1 = 0$ (called *adoption inertia*; see Jing 2011) or $S_1 > 0$ is obtained in the equilibrium. First, if $c\,(S_1)' = v_i\,(S_1)' = 0$, we have $c(0) = c^*$ and $v_i(0) = v_i^*$; thus, $\Delta_i\,(S_1 = 0) = (1-b\rho)v_i^* - (1-b)c^* \geq 0$ (assuming $v_i(0) > c(0)$). Hence, when externalities do not exist, it is always optimal for the system to purchase in the first period, and thus $S_1 > 0$ is obtained. However, if externalities do exist (i.e., $c\,(0)' < 0$ or $v_i\,(0)' > 0$ or both), the outcome is not obvious. If $b = 1$, then $\Delta_i\,(S_1 = 0) < 0$; if $b = 0$, then $\Delta_i\,(S_1 = 0) > 0$. Therefore, externalities and demand "backlogability" (or customer patience) play major roles. Since this is also true for decentralized cases, we have Corollary 3.1 below.

Corollary 3.1. *Suppose* $\delta_o = 0$. *If externalities do not exist or end customers have zero patience, adoption inertia can never be obtained in equilibrium; if externalities do exist and end customers are perfectly patient, then adoption inertia can be obtained.*

3.4.2 Fixed-Price Contract

Now we look at the situation wherein fixed-price (FXP) contracts are used with all buyers. With an FXP contract, buyers purchase from the seller in both periods once their adoption decisions are made, and there is no more price bargaining in period 2, so we use $w_i := w_i' = w_i''$ to denote the price paid by buyer i. Since the bargaining outcome will determine each firm's payoffs in both periods, the total expected payoff U_i will be the target under negotiation. Thus, for buyer i, the Nash-bargaining-generated price is $w_i^* = \arg\max_{w_i}\,(U_i - \theta_i)^{\beta_i} \cdot (U_0 - \theta_{0,i})^{1-\beta_i}$ given q_i and $\mathscr{C}_{-i}$.

Assume θ_i is constant for all $i \in N$. Note that θ_i represents the highest payoff buyer i could achieve through other ways than buying from the seller in both periods at price w_i^*. Hence, θ_i represents the payoff from the outside market and is a fixed value, which is only a function of the type of buyer i. On the other hand, the seller will have businesses with other buyers in case of breakdown with buyer i, so his outside payoff when bargaining with buyer i is

$$\theta_{0,i}\left(\mathscr{C}_{-i}\right) = \sum_{j \neq i} \left(w_j - c\left(\sum_{j \neq i} q_j \right) \right) \cdot q_j + \sum_{j \neq i} \left(w_j - c^* \right) \cdot \mathbf{E}\left[f q_j \right]. \tag{3.7}$$

Next, it is important to observe from (3.3) and (3.4) that the sum $U_i + U_0$ is independent of w_i. As a result, the Nash bargaining price w_i^* leads to

$$U_i\left(q_i, \mathscr{C}_{-i}\right) = \theta_i + \beta_i\left[U_i\left(q_i, \mathscr{C}_{-i}\right) - \theta_i + U_0\left(q_i, \mathscr{C}_{-i}\right) - \theta_{0,i}\left(\mathscr{C}_{-i}\right) \right], \tag{3.8}$$

for any q_i and $\mathscr{C}_{-i}$. Now with (3.8), we can optimize q_i for buyer i without knowing the explicit functional form of $w_i^*\left(q_i, \mathscr{C}_{-i}\right)$ and have the following result.

Proposition 3.2. *With FXP contracts, the incentive for adoption in period 1 for any buyer $i \in N$ is weaker than in a centralized system. In particular,*

$$\frac{\partial U_i\left(q_i, \mathscr{C}_{-i}\right)}{\partial q_i} \cdot \frac{1}{\beta_i} = \Delta_i\left(q_i | q_{-i}\right) - c\left(S_1\right)' \cdot \sum q_j + v_i\left(S_1\right)' \cdot \mathbf{E}\left[q_i \wedge \alpha_i D_i \right]. \tag{3.9}$$

The right-hand side of (3.9) looks very similar to (3.6). The only difference is that the buyer-based externality is reduced under an FXP (i.e., from $v_i\left(S_1\right)' \cdot \sum_j \mathbf{E}\left[q_j \wedge \alpha_i D_j\right]$ to $v_i\left(S_1\right)' \cdot \mathbf{E}\left[q_i \wedge \alpha_i D_i\right]$) so the buyer has less incentive to purchase in period 1. The intuition is that the seller and buyer i would not benefit from the gain (the increase of product valuation) of other buyers generated by a larger q_i, given the fixed-price contracts with all other buyers. Although the overall incentive—i.e., the first order derivative—is scaled down everywhere by factor β_i with FXP contracts, it has no impact on the optimal q_i.

3.4.3 Renegotiable-Price Contract

With renegotiable-price (RNP) contracts, buyers bargain for the price in both periods. We will refer to this scenario as "RNP." Since the prices determined in period 1 will not affect bargaining in period 2, the bargaining in the first period just focuses on the impact of the first-period purchase. Now we proceed backwards to analyze the two-period bargaining process.

Given the quantity q_i in period 1 and the assumption that D_i is known in period 2, we know that buyer i's purchase quantity in period 2 is $fq_i = [D_i - q_i]^+ - (1-b) \cdot [\alpha_i D_i - q_i]^+$. If she buys from the seller, buyer i can obtain payoff $(v_i^* - w_i'') \cdot fq_i$; otherwise, her outside payoff is $\delta_o \cdot fq_i$. Note that it will be costly for buyer i to produce nothing in period 2 given that she has introduced the product in period 1. For the seller, the payoff obtained with buyer i is $(w_i'' - c^*) \cdot fq_i$, and we assume that the outside payoff for him is zero given $\mathscr{C}_{-i}$. Hence, the generated pie is $(v_i^* - w_i'' - \delta_o) \cdot fq_i + (w_i'' - c^*) \cdot fq_i = (v_i^* - c^* - \delta_o) \cdot fq_i$, which is independent of w_i'', and thus the Nash bargaining price w_i'' splits the pie in proportion to their respective bargaining powers. As a result, we have $w_i'' = \beta_i c^* + (1-\beta_i) \cdot (v_i^* - \delta_o)$, which is independent of w_i'.

We then look at the first-period bargaining problem. In case of a breakdown, the outside payoff of buyer i is θ_i^1, which is a function of her type and is fixed. Given others' quantities q_{-i}, if buyer i purchases q_i units, she obtains value $v_i(S_1) \cdot s_{i1} + v_i^* \cdot \min\{d_{i2}, q_i - s_{i1}\}$ with cost $w_i' \cdot q_i$. Let $U_{i,t}$ denote firm i's expected payoff that is generated by the period-t transaction. We already know $U_{i,2}(q_i)$. If we know $w_i'(q_i, \mathscr{C}_{-i})$, we then have $U_{i,1}(q_i, \mathscr{C}_{-i}) = v_i(S_1) \cdot \mathbf{E} s_{i1} + v_i^* \cdot \mathbf{E} \min\{d_{i2}, q_i - s_{i1}\} - w_i' \cdot q_i$ for $i \in N$. For the seller, we have $U_{0,1}(\mathscr{C}) = \sum_i (w_i' - c(S_1)) \cdot q_i$ and $\theta_{0,i}^1(\mathscr{C}_{-i}) = \sum_{j \neq i} \left(w_j' - c\left(\sum_{j \neq i} q_j\right)\right) \cdot q_j$. Thus, we can see that $U_{i,1}(q_i, \mathscr{C}_{-i}) - \theta_i^1 + U_{0,1}(\mathscr{C}) - \theta_{0,i}^1(\mathscr{C}_{-i})$, the size of the generated pie for the seller and buyer i in period 1, is also irrelevant to w_i'. As a result, the NB leads to

$$U_{i,1}(q_i, \mathscr{C}_{-i}) = \theta_i^1 + \beta_i \left[U_{i,1}(q_i, \mathscr{C}_{-i}) - \theta_i^1 + U_{0,1}(\mathscr{C}) - \theta_{0,i}^1(\mathscr{C}_{-i})\right] \tag{3.10}$$

for any q_i and $\mathscr{C}_{-i}$. Buyer i then decides on q_i based on its impact on the expected total payoff over the two periods; i.e., q_i should maximize $U_{i,1}(q_i, \mathscr{C}_{-i}) + U_{i,2}(q_i)$. Now with (3.10), we can optimize q_i without knowing the explicit functional form of $w_i'(q_i, \mathscr{C}_{-i})$ and have the following result.

Proposition 3.3. *With RNP contracts, the incentive for adoption in period 1 for any buyer $i \in N$ is weaker than with FXP contracts if $\delta_o > 0$; otherwise, the incentive is stronger than with FXP contracts. In particular,*

$$\begin{aligned}\frac{\partial U_i\left(q_i, \mathscr{C}_{-i}\right)}{\partial q_i} \cdot \frac{1}{\beta_i} =& \Delta_i\left(q_i|q_{-i}\right) - c\left(S_1\right)' \cdot \sum q_j + v_i\left(S_1\right)' \cdot \mathbf{E}\left[q_i \wedge \alpha_i D_i\right] \\ & - \frac{1-\beta_i}{\beta_i} \cdot \delta_o \cdot \left[F_i\left(\frac{q_i}{\alpha_i}\right) - F_i\left(q_i\right) + b \cdot \left(1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right)\right].\end{aligned} \tag{3.11}$$

The difference between (3.9) and (3.11) is the second line in (3.11). It is an extra incentive generated by the possible change of price in the second period. Note that $F_i\left(\frac{q_i}{\alpha_i}\right) - F_i\left(q_i\right) + b \cdot \left(1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right) \geq 0$. Hence, the greater δ_o is, the less a buyer wants to buy in the first period, driven by a lower second-period price. In contrast, a buyer wants to buy more in the first period, if δ_o is negative, which could be caused by a high switching cost or low valuation for the outside substitute. Note that δ_o could be a function of S_1 and we will discuss this situation in the extension.

Proposition 3.3 provides an important insight that complements our observation from the data. The data suggests that RNP contracts are associated with significantly slower adoption than FXP contracts; however, our model suggests that this is not always the case. Therefore, the optimal contract choice is not obvious and depends on various factors as discussed in the next section.

3.5 Contract Comparison and Choice

When is a fixed-price contract better than a renegotiable-price contract? We discuss this question based on our analytic model. Denote as q^{CS} the optimal action profile in the centralized system. Denote as q^{FXP} and q^{RNP} the action profiles in the *pure strategy Nash equilibrium* (PSNE) for the two decentralized scenarios. The existence and uniqueness of the PSNEs may not be guaranteed. In the following, we characterize a sufficient condition for the existence of a PSNE in both decentralized scenarios. Define

$$\Lambda\left(q_i, Q_{-i}\right) := -2c'\left(q_i + Q_{-i}\right) - \left(q_i + Q_{-i}\right) \cdot c''\left(q_i + Q_{-i}\right) \qquad \text{and} \tag{3.12}$$

$$\Gamma_i\left(q_i, Q_{-i}\right) := \bar{F}_i\left(\frac{q_i}{\alpha_i}\right) \cdot v_i'\left(q_i + Q_{-i}\right) + \mathbf{E}\left[q_i \wedge \alpha_i D_i\right] \cdot v_i''\left(q_i + Q_{-i}\right), \tag{3.13}$$

where $Q_{-i} := \sum_{j \neq i} q_j$. It is easy to check that $\frac{\partial^2 U_i}{\partial q_i \partial Q_{-i}} \cdot \frac{1}{\beta_i} = \Lambda\left(q_i, Q_{-i}\right) + \Gamma_i\left(q_i, Q_{-i}\right)$ for any $i \in N$ in both decentralized scenarios. Thus, we have the following result.

Theorem 3.1. *If $\Lambda + \Gamma_i \geq 0$ for $\forall i \in N$, then a PSNE exists in both decentralized scenarios.*

The proof of Theorem 3.1 uses the property of a *supermodular game.* The condition is in general easy to be satisfied. Note that $c'(0) \leq 0$ and $v_i'(0) \geq 0$. First, let's focus on Λ, the component related to cost externality. The condition that $\Lambda \geq 0$ is simply that the learning effect satisfies $x \cdot c''(x) + 2c'(x) \leq 0$ for any $x \geq 0$. Without loss of generality, if we assume that $c(x) = c^* + \frac{A_2}{x+A_1}$, where $A_1, A_2 > 0$, then it is easy to check that $x \cdot c''(x) + 2c'(x) = -\frac{2A_1A_2}{(x+A_1)^3} < 0$ for any $x \geq 0$. Second, for Γ_i, we can have a similar result if we assume a similar functional form for v_i and if $S_1 \cdot \bar{F}_i\left(\frac{q_i}{\alpha_i}\right) \geq 2 \cdot \mathbf{E}\left[q_i \wedge \alpha_i D_i\right]$, which is true if buyer i does not take a major share of the demand. Lastly, the condition holds if $\Lambda \geq |\Gamma_i|$, which indicates that the learning curve effect dominates. If $\frac{\partial^2 U_i}{\partial q_i \partial Q_{-i}} \geq 0$ holds for $\forall i \in N$, then it is a supermodular game, in which a PSNE exists.

Although we are unable to analytically show that the equilibria are unique, we conduct extensive numerical tests. We find that the action profile in either a centralized or decentralized case, with any starting point, will converge to the same equilibrium point through best-response iterations. This strongly indicates that the equilibria are unique given the game structure of our model.

Now we assume that the condition $\Lambda + \Gamma_i \geq 0$ for $\forall i \in N$ is satisfied and that q^{CS}, q^{FXP}, and q^{RNP} are unique. Given these assumptions, *the scenario in which the action profile is "closer" to the centralized one is the better one for the system.* By comparing (3.6), (3.9), and (3.11), we get the following results for a seller to make a choice between an FXP and an RNP contract.

3.5.1 Strength of Seller Competition

Recall that δ_o, the marginal payoff of the alternative product in period 2, measures the strength of competition from other sellers in the B2B market: the higher the δ_o, the stronger the competition. It turns out to be an important factor that determines the difference between an FXP and an RNP in terms of early-adoption incentive. Given that it is a supermodular game, we can show that a stronger incentive for every buyer always leads to faster adopton for every buyer, making q^{CS}, q^{FXP}, and q^{RNP} comparable. To proceed, note that for vectors $\hat{q}$ and $\tilde{q}$, we say $\hat{q} < \tilde{q}$ if and only if $\hat{q}_i < \tilde{q}_i$ for every i. A similar definition applies to "$>$", "$\leq$", and "$\geq$" when used for vectors.

Proposition 3.4. *If q^{CS}, q^{FXP}, and q^{RNP} are unique, then (i) $q^{FXP} \leq q^{CS}$; (ii) if $\delta_o > 0$, then $q^{RNP} \leq q^{FXP}$; (iii) if $\delta_o < 0$, then $q^{RNP} \geq q^{FXP}$; (iv) and if $\delta_o = 0$, then $q^{RNP} = q^{FXP}$.*

Now we can compare the system payoff in each situation given different δ_o. For example, an FXP is better than an RNP if $q^{RNP} < q^{FXP} \leq q^{CS}$; this is the case if $\delta_o > 0$ (i.e., it is profitable to acquire a substitute in the outside market in period 2). However, this does not means that an RNP is better if $\delta_o < 0$. For instance, when $\delta_o < 0$—which possible if it is hard to obtain a substitute and costly to exit the market in period 2—we may have $q^{FXP} = q^{CS} < q^{RNP}$, which means an FXP contract is better. In other words, *faster product adoption may not always benefit the seller as well as the buyers.* In sum, an RNP contract is better than an FXP contract only when $\delta_o < 0$, but not too low.

3.5.2 Buyer Bargaining Power

If, in certain cases, the difference between FXP and RNP contracts is small—i.e., the two types of contract lead to similar results—then we need not spend too much time making a choice. Proposition 3.5 tells us that the absolute difference between q^{FXP} and q^{RNP} is decreasing in the bargaining power of every buyer. Hence, a wise contract choice is worthwhile only when buyers are not powerful. The intuition behind this finding is that when buyers are powerful, they can bargain to get a low enough price anyway and thus tactical decisions such as the intertemporal distribution of their purchase quantity does not have much impact.

Proposition 3.5. $\left\|q^{FXP} - q^{RNP}\right\|$ *is decreasing in* β_i *for any* $i \in N$*.*

As buyers become weaker and β_i gets closer to zero, $\left\|q^{FXP} - q^{RNP}\right\|$ increases even faster (due to the $\frac{1-\beta_i}{\beta_i}$ factor in (3.11)). When β_i becomes small enough so that we have either $q^{RNP} \ll q^{FXP}$ or $q^{RNP} \gg q^{FXP}$, then it is very likely that that $\left\|q^{CS} - q^{RNP}\right\| > \left\|q^{CS} - q^{FXP}\right\|$ and an RNP contract is less efficient than an FXP contract. Therefore, as β_i decreases for any $i \in N$, it becomes more and more likely that an FXP is the optimal choice. This is actually not intuitive, because as the seller becomes more and more powerful, he has more incentive to renegotiate and raise the price in period 2. If an RNP contract is used, however, the anticipated price increase would drive weak buyers to buy more in the first period, at the cost of supply-demand mismatch.

3.5.3 Size of Buyer Group

Another interesting factor is n, the size of the buyer group, which is essentially the scale of the "network." To simplify the analysis and to focus on the intuition, we assume for this subsection that the buyers are homogeneous. Let $Z_{FPX}(q) := \frac{1}{\beta} \cdot \frac{\partial U_{FXP}(q)}{\partial q} - \frac{\partial U_{CS}(q)}{\partial q}$ and $Z_{RNP}(q) := \frac{1}{\beta} \cdot \frac{\partial U_{RNP}(q)}{\partial q} - \frac{\partial U_{CS}(q)}{\partial q}$, where $U_{FXP}(q)$ and $U_{RNP}(q)$ represent the expected payoffs of a buyer with FXP and RNP contracts, respectively. Z_{FPX} or Z_{RNP} measures the difference between a decentralized system and the centralized one in terms of incentive to buy in period 1.

We can obtain that $\frac{\partial}{\partial n} Z_{FPX}(q) = \frac{\partial}{\partial n} Z_{RNP}(q) = -\mathbf{E}\left[q \wedge \alpha_i D\right] \cdot \left[v'\left(nq\right) + (n-1) \cdot q \cdot v''\left(nq\right)\right]$. Given n is large and $v(x) = v^* - e^{-x+A}$ or $v^* - \frac{1}{x+A}$, we have $v'\left(nq\right) + (n-1) \cdot q \cdot v''\left(nq\right) < 0$. Thus,

$$\frac{1}{\beta} \cdot \frac{\partial^2 U_{FXP}(q)}{\partial q \partial n} = \frac{1}{\beta} \cdot \frac{\partial^2 U_{RNP}(q)}{\partial q \partial n} > \frac{\partial^2 U_{CS}(q)}{\partial q \partial n},$$

which means that, as n increases, the changes to q^{FXP} and q^{RNP} are more positive than changes to q^{CS}. We know that q^{FXP} is bounded from above by q^{CS}, but q^{RNP} is not. Hence, as the size of the buyer group increases, q^{FXP} will approach q^{CS} but the chance of an RNP contract being the optimal choice decreases. (Note that the RNP contract is optimal only when $q^{RNP} > q^{FXP}$ but q^{RNP} is not too high.) The intuition behind this finding is that v is close to its limit in period 1 when n is large, so the extra incentive from an increase of v—which is the main difference between an FXP and a CS—is small.

3.5.4 Strengths of Externalities

Similarly, we assume for this subsection that the buyers are homogeneous. We have two types of externalities in our model, and the strengths of the externalities are captured by $c'(\cdot)$ and $v'(\cdot)$. If we take $c'(nq)$ and $v'(nq)$ as two variables, then we have

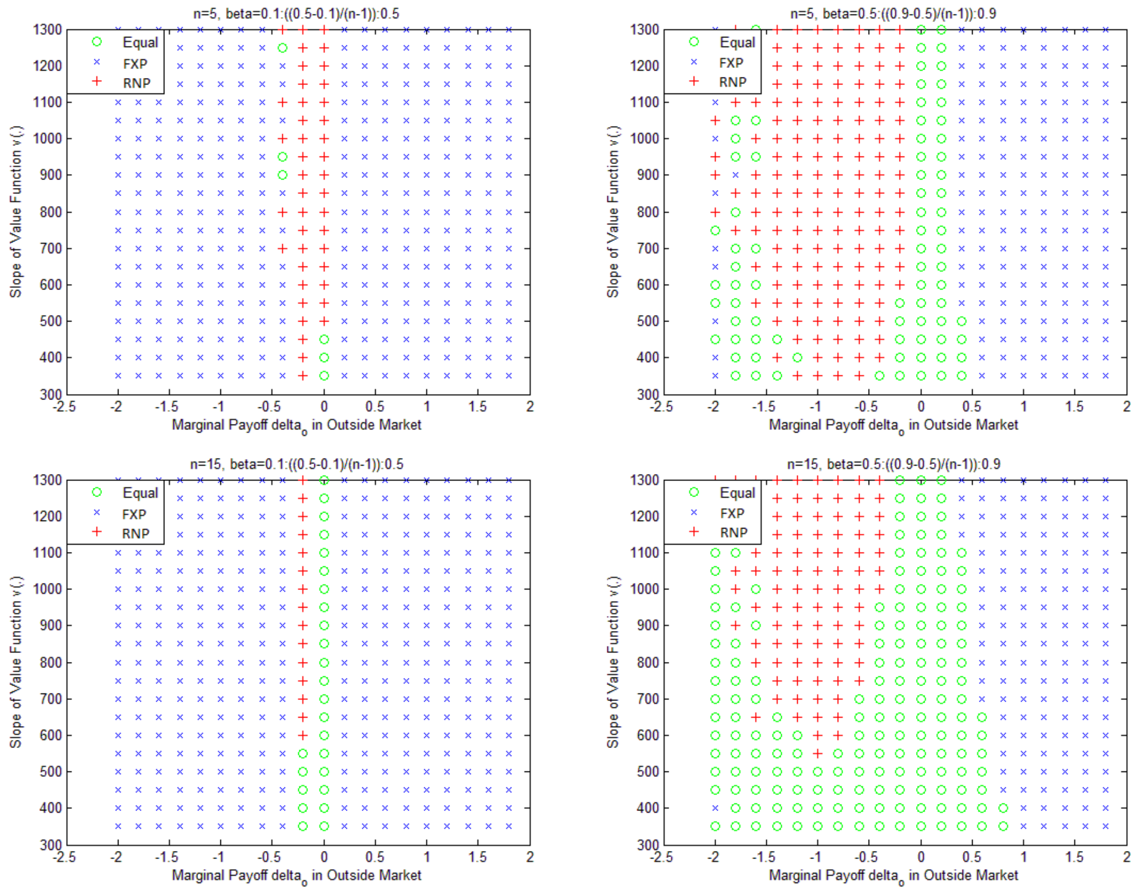
$$\begin{aligned}\frac{\partial Z_{FXP}(q)}{\partial c'(nq)} &= \frac{\partial Z_{RNP}(q)}{\partial c'(nq)} = 0, \text{ and} \\ \frac{\partial Z_{FXP}(q)}{\partial v'(nq)} &= \frac{\partial Z_{RNP}(q)}{\partial v'(nq)} = -(n-1) \cdot \mathbf{E}\left[q \wedge \alpha_i D\right] < 0.\end{aligned}$$

Therefore, we have the following two observations. First, the strength of seller-based externality will have the same impact in all three cases, and we cannot tell how the optimal contract choice is affected. Second, as the strength of buyer-based externality $v'(\cdot)$ increases, the changes to q^{FXP} and q^{RNP} are more negative than changes to q^{CS}; i.e., the sizes of q^{FXP} and q^{RNP} relative to q^{CS} decrease. Given that the RNP is better than the FXP only if $q^{RNP} > q^{FXP}$, this finding means that the chance of the RNP contract being the optimal choice increases with the strength of buyer-based externality. The reason is that the increase of $v'(\cdot)$ amplifies the extra incentive from an increase of v, which is the main difference between an FXP and a CS.

3.5.5 Computational Study

In this subsection, we generate numerical examples to explore the optimal contract choice for the system. We use $c(x) = c_* + \frac{A_2}{x+A_1}$ and $v_i(x) = v_i^* - \frac{B_2}{x+B_1}$, and make them satisfy the condition in Theorem 3.1. In particular, we set $\alpha_i = 0.5, b = 0.8, c^* = 1, v_i^* = 6$, and let the demand follow uniform distribution. As shown in Figure 3.7, the optimal choice of contracting regime depends on (i) δ_o, buyers' marginal payoff in the outside-market, (ii) β_i, buyer bargaining power, (iii) n, the

Figure 3.7: The optimal choice of contract structure.



Note: Each dot ("o", "×", or "+") represents a problem instance with the corresponding parameters. The optimal contract choice for an instance is indicated by the symbol.

size of the buyer group, and (iv) B_2, the slope of value function $v_i(\cdot)$. The results are consistent with the previous discussions.

3.5.6 Discussions

Therefore, if we want to suggest a type of contract for a seller, we shall consider the four dimensions of market characteristics. Note that the marginal payoff δ_o in the outside market is an indirect measure of degree of competition from rivalries within the industry. The other three dimensions (i.e., $v_i'(\cdot)$, β, and n) describe the structure of the downstream market. Moreover, these four metrics also provide us with a framework to predict the contract choice in many different supply chains or industries. Based on our model, we can make the following conjectures. Readers who have related datasets may be able to test them.

- We shall see more RNP contracts in markets where competition is more balanced; i.e., it is not too hard or too easy for buyers to switch to an alternative source. For example, we expect FXPs to be used by AMD when it offers a product that is highly comparable to one of Intel's (i.e., $\delta_o > 0$), or when a unique product is offered (i.e., $\delta_o \ll 0$).
- We shall see more FXPs in markets where buyers are less powerful. For example, if we compare the supply chain of Intel and that of AMD, we expect Intel to use more FXPs.
- We shall see less RNPs in supply chains where a cost-learning effect exists and the buyer group is larger. In other words, if a product is a niche product that is purchased only by a few customers, then it would be very likely that we would observe an RNP.
- We shall see more diversified choices in industries where the role of buyer-based externalities is more significant. For example, the choices of contracting regimes should be more consistent for more basic component products such as memories and hard discs.

3.6 Model Extensions

3.6.1 Time-Dependent Valuation or Cost

Sometimes buyers' valuation for the (component) product is a function of time, regardless of the externalities. This type of valuation occurs because the component product will become obsolete, or because the final product will be obsolete, or the use of the component should be complemented by another critical part, the cost of which is changing over time. We can model this relation by setting a general $\rho > 0$ for period-2 valuation. With such a modification to the model, the incentive for buyers to adopt the product is affected by this time factor. According to (3.5), (3.6), (3.9), and

(3.11), it is easy to obtain that

$$\frac{\partial^2 U_{CS}}{\partial q_i \partial \rho} = \frac{\partial^2 U_i^{FXP}}{\partial q_i \partial \rho} \cdot \frac{1}{\beta_i} = \frac{\partial^2 U_i^{RNP}}{\partial q_i \partial \rho} \cdot \frac{1}{\beta_i} = \frac{\partial \Delta_i}{\partial \rho} = -b \cdot \left[1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right]. \tag{3.14}$$

Hence, the incentive to adopt early decreases with ρ, which is intuitive. In particular, the sensitivity of buyer i's decision to ρ increases with b, α_i, and mean demand ($\int x dF_i(x)$). The flexibility for a buyer to manipulate the purchase intertemporally is captured by b and α_i. Since the impact of ρ is the same for both contracts, such a modification does not lead to qualitatively different results.

3.6.2 Lock-in Effect

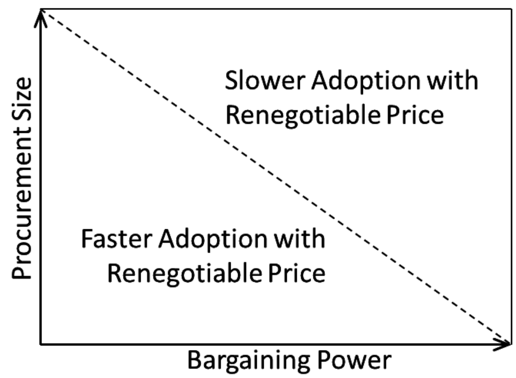
The adoption of a product may not only affect the valuation and cost of this focal product, but also the cost and valuation for the substitute. This is the case when the purchase of the substitute also generates externalities, or the switching cost depends on the adoption of the focal product. In this subsection, we assume that, if the buyer turns to the outside market in period t, then the payoff is a function of q_{-i} in period $t = 1$ and is a function of q in period $t = 2$.

First, it is easy to see that an FXP is not affected by such a modificaiton. This is because there is no negotiation in period 2 and buyer i's quantity decision q_i does not depend on her period-1 outside option θ_i. Note that θ_i represents the highest payoff buyer i could achieve through other ways than buying from seller 0 in both periods. Hence, θ_i could be a function of q_{-i}, but not q_i. As a result, θ_i does not present in $\frac{\partial U_i^{FXP}}{\partial q_i}$.

On the other hand, when an RNP is used, the buyer's outside option in period 2, $\delta_o = v_o - c_o$, could be a function of q, and we have $U_{i,2} = [\beta_i \cdot (v_* - c_*) + (1-\beta_i) \cdot \delta_o(q)] \cdot \mathbf{E}[f q_i]$. Hence,

$$\begin{aligned}\frac{\partial U_i^{RNP}}{\partial q_i} \cdot \frac{1}{\beta_i} &= \Delta_i(q_i|q_{-i}) - c(S_1)' \cdot \sum q_j + v_i(S_1)' \cdot \mathbf{E}[q_i \wedge \alpha_i D_i] \\ &\quad - \frac{1-\beta_i}{\beta_i} \cdot \delta_o(q) \cdot \left[F_i\left(\frac{q_i}{\alpha_i}\right) - F_i(q_i) + b \cdot \left(1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right)\right] \\ &\quad + (1-\beta_i) \cdot \mathbf{E}[f q_i] \cdot \frac{\partial \delta_o(q)}{\partial q_i}.\end{aligned} \tag{3.15}$$

Figure 3.8: The impact of stronger externality associated with outside options.



If we further assume that $\delta_o = a_0 - a_* \cdot S_1$, then

$$\frac{\partial^2 U_i^{RNP}}{\partial q_i \partial a_*} \cdot \frac{1}{1-\beta_i} = S_1 \cdot \left[F_i\left(\frac{q_i}{\alpha_i}\right) - F_i\left(q_i\right) + b \cdot \left(1 - F_i\left(\frac{q_i}{\alpha_i}\right)\right)\right] - \beta_i \cdot \mathbf{E}\left[f q_i\right], \tag{3.16}$$

which indicates that the degree of externality may encourage or discourage early adoption. It is easy to see that for weak (i.e., β_i is small) and small-quantity (i.e., $\mathbf{E}\left[f q_i\right]$ is small) buyers, $\frac{\partial^2 U_i^{RNP}}{\partial q_i \partial a_*}$ is likely to be positive, and negative for strong and large-quantity buyers. Hence, with an RNP, *stronger externality associated with outside options may encourage weak and small buyers to adopt early and discourage large buyers to do so.* (See illustration in Figure 3.8.) We support this result by regressing the TDPS against the ACVP, the $\ln(\text{Size})$ of the buyer, and the total quantity of an instance for RNP contracts (i.e., ACVP>0). We find negative coefficients (with P-values < 0.01) for both the $\ln(\text{Size})$ and total quantity. The reasons are that (i) weak buyers should buy more in period 1 and less in period 2 so that they can avoid paying high price in period 2; and that (ii) for small-quantity buyers the increase of δ_o has relatively small impact on their payoffs, so they suffer less from buying in period 1 than larger-quantity buyers do. Last but not least, as the intensity (a_*) of externality on the outside option increases, δ_o may go from positive to negative, and thus the optimal contract choice may switch.

3.7 Conclusions

In this chapter, we study the product-adoption process management in high-tech supply chains and the contract choice for sellers selling to business buyers. In the B2B market, product adoption generates positive externalities among the business buyers, and the price for a product is subject to negotiations. In the B2C market, the demand is uncertain, is learned by the buyers over time, and is independent of the product adoption in the B2B market. The task of the seller is to manage the adoption of a product in the B2B market at a favorable pace that maximizes the expected payoff.

Given that intertemporal pricing schemes are not viable in the B2B market, we suggest that the seller consider choosing whether to allow the price with a buyer to be renegotiable over time, or to sign a long-term, fixed-price contract with the buyer. Using a dataset supplied by a major global microchip vendor and an approach based on an instrumental variable, we observe a causal relation between price variability and buyer behavior. This suggests that price flexibility is an effective lever to control the product-adoption process, although the optimal choice is not obvious. To understand how the price mechanism affects buyer behavior, we build a simple dynamic game-theoretic model.

Our model shows that the optimal choice of contract depends on (*i*) the source and degree of externality, (*ii*) the strength of sell-side competition, (*iii*) buyer's bargaining power, and (*iv*) the number of the buyers. First, our model predicts more diversified uses of contracts in industries where the role of buyer-based externalities (i.e., network externalities) is stronger. For commodity products such as memories, the contract choices should be more consistent. Second, it indicates less use of fixed-price contracts where the market competition is more balanced (i.e., neither too hard nor too easy to buy from an alternative source). Third, we should use more fixed-price contracts in markets where buyers are less powerful because weak buyers are very sensitive to price changes over time and tend to adopt the product too early or too late. Last but not least, in markets where a learning effect exists and the buyer group is larger, renegotiable-price contracts are less likely to outperform fixed-price contracts because the price will go down significantly and thus buyers do not have an incentive to buy early.

We then extend the model to incorporate time-dependent valuation and externality on outside options. We find that time dependence does not change our results. Also, we find that stronger externality on outside options may encourage weak and small buyers to adopt early and discourage large buyers to do so; as the degree of externality on the outside option increases, the optimal contract choice may switch from a fixed-price to a renegotiable-price contract. Admittedly, there are still limitations for our analyses. In particular, our data is from a single company and it may have limited statistical power. In addition, we only consider intermediary-good externalities that do not benefit end consumers directly. Hence, extending our model to incorporate these limitations could be a fruitful area for future research.

Appendix

The Measure of Contract Type

Here we discuss why we do not choose the three alternative measures. (i) There are three problems with *the average duration of a price*. Firstly, the flexibility measured by the duration of a price is relative to the duration of the instance.[10] Even if we normalize the duration of price and divide it by the duration of instance, there is still a problem. Some instances have a fixed price most of the time, but they have a couple of single transactions where the price has jumped or dropped. Those exceptions only account for a negligible proportion of the duration of an instance but can greatly drag down the average duration of price. Lastly, an average duration of price cannot capture the extent of price changes that is allowed by a contract.

(ii) In comparison, *the variance of price* is more robust to exceptional price deviations and can also capture the extent of price change. The price variance *ex ante* is an increasing function of the flexibility of contract. However, different products have different price levels, so the variance has to be normalized.

(iii) *The coefficient of variation* (CV) *of price* is defined as the quotient of standard deviation of price over the mean. Similar to the problem of price duration, the mean of price is sensitive to some exceptional price jumps or drops that typically occur at the end of the life cycle. In sum, the variance of price is better than the average duration of price; the CV of price is better than the variance of price; and the ACVP improves the CV of price. The ACVP is the best among all these measures.

Test of Buyer-Specific Common Causes

Here we show a contradiction if we suppose that hypothesis (I) is not true and that ε and ε'

[10] For example, instance θ has average duration of price equal to 1 month and instance θ' has an average duration of price equal to 2 months; however, the duration of θ is only 1 month and the duration of θ' is 1 year. In this example, φ of θ' has higher flexibility.

Table 3.4: Test for Major Customers

Customer	ln($size$)	Instances	Coefficient	t Stat	P-value
#1	22.31	159	-0.0175	-0.85	0.4
#2	20.91	203	-0.4277***	-3.73	0.0002
#3	19.52	150	-0.0001	-0.0235	0.9813
#4	21.25	120	-0.0871**	-2.12	0.0357
#5	21.50	386	-0.0008*	-1.8865	0.0600
#6	20.22	161	-0.3643**	-2.0970	0.0376
#7	20.97	149	-0.6560***	-3.7604	0.0003
#8	20.25	146	-0.0804*	-1.9061	0.0586
#9	21.27	163	-0.2508*	-1.8144	0.0715
#10	20.44	214	-0.3522***	-2.7387	0.0067

Note: *p<0.1; **p<0.05; ***p<0.01.

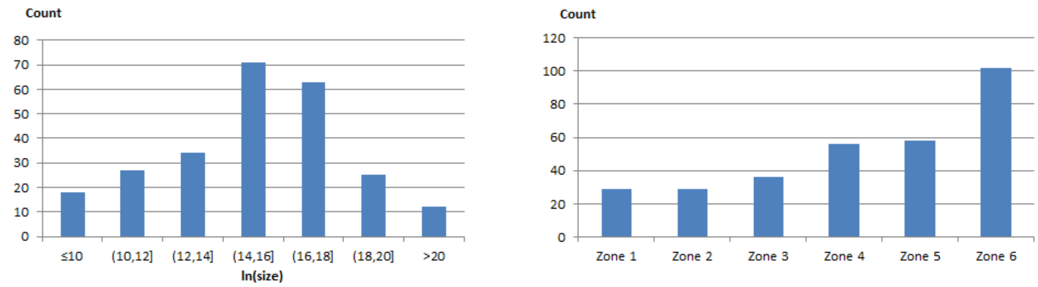
are buyer-specific factors. Ideally, we should test the correlation between the ACVP and TDPS for each individual buyer. However, for medium- to small-sized buyers, we do not have a large enough sample size. Hence, we first focus on the top ten major buyers that have close to or more than 150 instances recorded in our dataset. The results are summarized in Table 3.4. We see that for most (8/10) of these major buyers, the correlation between ACVP and TDPS is still significant with total quantity controlled, which is a strong evidence of these buyers responding strategically to the contract type.

Table 3.5: Test for 10 Customer Groups

Group	ln($size$)	Coefficient	t Stat	P-value
#1	[8.57, 14.62]	0.0656	0.5972	0.5516
#2	[14.62, 16.12]	-0.5975***	-4.8786	0.0000
#3	[16.12, 17.27]	-0.4234**	-2.1792	0.0314
#4	[17.27, 17.64]	-0.5470***	-4.3592	0.0000
#5	[17.64, 18.28]	-0.8911***	-3.8355	0.0002
#6	[18.28, 18.79]	-0.6192***	-3.4272	0.0009
#7	[18.79, 19.32]	-0.5244***	-3.0807	0.0028
#8	[19.32, 19.38]	-0.0007	-0.3124	0.7554
#9	[19.38, 19.58]	-0.5719***	-3.3443	0.0011
#10	[19.58, 20.49]	-0.6373***	-4.9492	0.0000

Note: *p<0.1; **p<0.05; ***p<0.01.

Figure 3.9: Customer distributions on two characteristics.



To test the correlation for other buyers, we control the two buyer-specific characteristics: size and location. We believe that these two characteristics capture some inherent attributes that might lead to the correlation; otherwise, it is hard to imagine that customers would act differently due to some other factors even if they were offered the same contract.[11] As shown on the left in Figure 3.9, we get a distribution that is very close to normal if we plot the distribution of buyers according to the logrithmic function of their total purchasing values (i.e., $\ln(size)$). For location, there are 6 major geographic zones according to the data, and the buyer distribution across the zones is shown on the right in Figure 3.9. We then pick the zone that has the most customers, and equally divide buyers into 10 subgroups according to their sizes so that we have roughly 113 instances in each subgroup. The test results are summarized in Table 3.5. We see that for 8 out of the 10 buyer groups, the correlation between ACVP and TDPS is still significant with total quantity controlled, which further supports our claim that buyers are responding to the contract type.

Proof of Lemma 3.1

First, fq_i cannot take values other than $\left[d_{i2} - [q_i - \alpha_i D_i]^+\right]^+$ given that D_i is known and the purchase price is positive. Suppose the following scenario. If buyer i reduces fq_i, she looses profit; if she increases fq_i, then her cost increases but revenue is unaffected. Hence, the second period

[11] Buyers from different micromarkets might have different behaviors and contract practices, but buyers in the same micromarket tend to purchase similar products. Hence, a micromarket-related characteristic is almost equivalent to a product-specific characteristic. Moreover, we believe that the difference between new and old buyers is not a major concern, because there is no reason for a new buyer to behave differently from an old one in a consistent way regardless of the form of the contract.

purchase quantity should equal period-2 total demand, and thus

$$
\begin{aligned}
fq_i &= \left[b\left[\alpha_i D_i - q_i\right]^+ + (1-\alpha_i)D_i - \left[q_i - \alpha_i D_i\right]^+\right]^+ \\
&= \begin{cases} D_i - q_i - (1-b)\left(\alpha_i D_i - q_i\right) & \text{if } D_i > q_i/\alpha_i \\ D_i - q_i & \text{if } q_i < D_i \leq q_i/\alpha_i \\ 0 & \text{if } D_i \leq q_i \end{cases} \\
&= \left[D_i - q_i\right]^+ - (1-b)\left[\alpha_i D_i - q_i\right]^+; \\
s_{i2} - fq_i &= b\left[\alpha_i D_i - q_i\right]^+ + (1-\alpha_i)D_i - \left[D_i - q_i\right]^+ + (1-b)\left[\alpha_i D_i - q_i\right]^+ \\
&= D_i \wedge q_i - \alpha_i D_i \wedge q_i. \blacksquare
\end{aligned}
$$

Details for the Underage and Overage Costs in Different Scenarios

See Table 3.6 below.

Proof of Proposition 3.1

$$
U_{CS} = \sum_{i \in N} v_i\left(S_1\right) \cdot \mathbf{E}\left[q_i \wedge \alpha_i D_i\right] - c\left(S_1\right) \cdot S_1 + \sum_{i \in N} \rho \cdot v_i^* \cdot \mathbf{E}\left[s_{i2}\right] - c^* \cdot \sum_{i \in N} \mathbf{E}\left[fq_i\right]
$$

It is easy to check that $v_i\left(S_1\right) \partial \mathbf{E}\left[q_i \wedge \alpha_i D_i\right]/\partial q_i - c\left(S_1\right) + \partial\left[\rho v_i^* \mathbf{E}\left[s_{i2}\right] - c^* \mathbf{E}\left[fq_i\right]\right]/\partial q_i = \Delta_i$, where Δ_i is defined in (3.5). Then Eq. (3.6) is obtained. $\blacksquare$

Proof of Proposition 3.2

Table 3.6: System-Wide Underage and Overage Cost for Producing One More Unit in Period 1

Demand	Period 1		Period 2	
Scenarios	Underage	Overage	Underage	Overage
$D_i \leq q_i$	0	$c\left(S_1\right)$	0	0
$q_i < D_i \leq \frac{q_i}{\alpha_i}$	0	$c\left(S_1\right)$	c^*	0
$\alpha_i D_i > q_i$; Exit	$v_i\left(S_1\right)$	$c\left(S_1\right)$	0	0
$\alpha_i D_i > q_i$; Wait	$v_i\left(S_1\right)$	$c\left(S_1\right)$	c^*	ρv_i^*

$$
\begin{aligned}
U_i\left(q_i,\mathscr{C}_{-i}\right)=&\theta_i+\beta_i\left[U_i\left(q_i,\mathscr{C}_{-i}\right)-\theta_i+U_0\left(q_i,\mathscr{C}_{-i}\right)-\theta_{0,i}\left(\mathscr{C}_{-i}\right)\right]\\
=&\theta_i+\beta_i\left[v_i\left(S_1\right)\cdot\mathbf{E}\left[q_i\wedge\alpha_iD_i\right]-c\left(S_1\right)\cdot S_1+\right.\\
&\left.c\left(Q_{-i}\right)\cdot Q_{-i}+v_i^*\cdot\mathbf{E}\left[s_{i2}\right]-c^*\cdot\mathbf{E}\left[fq_i\right]-\theta_i\right].
\end{aligned}
$$

Similarly, $v_i\left(S_1\right)\cdot\partial\mathbf{E}\left[q_i\wedge\alpha_iD_i\right]/\partial q_i-c\left(S_1\right)+\partial\left[v_i^*\cdot\mathbf{E}\left[s_{i2}\right]-c^*\cdot\mathbf{E}\left[fq_i\right]\right]/\partial q_i=\Delta_i$. Then Eq. (3.9) is obtained. ■

Proof of Proposition 3.3

Firstly, $U_{i,2}\left(q_i\right)=\left[\beta_i\left(v_i^*-c^*\right)+\left(1-\beta_i\right)\delta_o\right]\cdot\mathbf{E}\left[fq_i\right]$. Hence,

$$
\begin{aligned}
U_i\left(q_i,\mathscr{C}_{-i}\right)=&\theta_i^1+\beta_i\left[U_{i,1}\left(q_i,\mathscr{C}_{-i}\right)-\theta_i^1+U_{0,1}\left(\mathscr{C}\right)-\theta_{0,i}^1\left(\mathscr{C}_{-i}\right)\right]+U_{i,2}\left(q_i\right)\\
=&\theta_i^1+\beta_i\left[v_i\left(S_1\right)\cdot\mathbf{E}\left[q_i\wedge\alpha_iD_i\right]-c\left(S_1\right)\cdot S_1+\right.\\
&\left.c\left(Q_{-i}\right)\cdot Q_{-i}+v_i^*\cdot\mathbf{E}\left[s_{i2}\right]-c^*\cdot\mathbf{E}\left[fq_i\right]-\theta_i^1\right]+\\
&\left(1-\beta_i\right)\cdot\delta_o\cdot\mathbf{E}\left[fq_i\right].
\end{aligned}
$$

We have $v_i\left(S_1\right)\cdot\partial\mathbf{E}\left[q_i\wedge\alpha_iD_i\right]/\partial q_i-c\left(S_1\right)+\partial\left[v_i^*\cdot\mathbf{E}\left[s_{i2}\right]-c_*\cdot\mathbf{E}\left[fq_i\right]\right]/\partial q_i=\Delta_i$. In addition, $\partial\mathbf{E}\left[fq_i\right]/\partial q_i=F_i\left(q_i\right)-F_i\left(\frac{q_i}{\alpha_i}\right)-b\left[1-F_i\left(\frac{q_i}{\alpha_i}\right)\right]$. Then Eq. (3.11) is obtained. ■

Proof of Theorem 3.1

If $\frac{\partial^2U_i}{\partial q_i\partial Q_{-i}}=\beta_i\left(\Lambda+\Gamma_i\right)\geq 0$ for $\forall i\in N$, it is a supermodular game. A PSNE exists for any supermodular game. ■

Proof of Proposition 3.4

(i) According to (3.6) and (3.9), $\frac{1}{\beta_i}\cdot\frac{\partial U_i^{FXP}}{\partial q_i}|_{q=q^{CS}}=\frac{\partial U_{CS}}{\partial q_i}|_{q=q^{CS}}-\sum_{j\neq i}v_i\left(\sum q_i^{CS}\right)'\cdot\mathbf{E}\left[q_j^{CS}\wedge\alpha_iD_j\right]\leq\frac{\partial U_{CS}}{\partial q_i}|_{q=q^{CS}}$ for $\forall i\in N$. Hence, it is not possible that $q^{FXP}\geq q^{CS}$. Given that it is a supermodular game, it must be that $q^{FXP}\leq q^{CS}$. (ii) According to (3.9) and (3.11), $\frac{\partial U_i^{RNP}}{\partial q_i}|_{q=q^{FXP}}=\frac{\partial U_i^{FXP}}{\partial q_i}|_{q=q^{FXP}}+\left(1-\beta_i\right)\cdot\delta_o\cdot\frac{\partial\mathbf{E}[fq_i]}{\partial q_i}\leq\frac{\partial U_i^{FXP}}{\partial q_i}|_{q=q^{FXP}}$ for $\forall i\in N$ if $\delta_o>0$. Hence, $q^{RNP}\geq q^{FXP}$ is not possible; it must be that $q^{RNP}\leq q^{FXP}$. Similar arguments apply to (iii) and (iv). ■

Proof of Proposition 3.5

First, suppose $\delta_o > 0$. According to Proposition 3.4, $q^{RNP} \leq q^{FXP}$ for any $0 < \beta < 1$. Because $\frac{\partial q_i^{FXP}}{\partial \beta_i} = 0$ and $\frac{\partial q_i^{RNP}}{\partial \beta_i} \geq 0$, we know $\left| q_i^{FXP} - q_i^{RNP} \right|$ is decreasing in β_i. Because it is a supermodular game, we have that $\left| q_j^{FXP} - q_j^{RNP} \right|$ is decreasing in β_i for all $j \neq i$. The result follows. ■

Mixed Contract Choices

Here we consider the case wherein the seller can use different types of contracts with different buyers. Propositions 3.2 and 3.3 suggest that the quantity decision of a buyer does not directly rely on the type of contract used for other buyers; instead, it only depends on the sum of other buyers' purchase quantities in period 1 (Q_{-i}). Hence, it is still a supermodular game if $\Lambda + \Gamma_i \geq 0$ for $\forall i \in N$. Given this assumption, we know from Theorem 3.1 that a PSNE exists no matter what contracts are used. The seller's problem can thus be formulated as follows:

$$\begin{aligned} \max \quad & U_{CS}(q) \\ s.t. \quad & q_i \in \left\{ q_i^{FXP}(q_{-i}), q_i^{RNP}(q_{-i}) \right\}, \quad \forall i \in N \end{aligned}$$

where $q_i^{FXP}(q_{-i})$ solves Eq. (3.9) and $q_i^{RNP}(q_{-i})$ solves Eq. (3.11). This is a highly non-linear problem with hardly any structural properties. Hence, it is an extremely difficult problem if n is large. However, n is usually a small number in reality, so it is possible to test 2^n possible solutions and find the optimal one. More importantly, mixed contract choices are not always desirable; in many cases, it is easy to tell that a dominant contracting regime exists. First, an FXP is a dominant choice if $q^{RNP} < q^{FXP}$, as suggested by Proposition 3.4. Second, an RNP is a dominant choice when $q^{FXP} \leq q^{RNP} \leq q^{CS}$. Lastly, mixed choices may be desirable only when $q^{FXP} < q^{CS}$ and there exists i such that $q_i^{RNP} > q_i^{CS}$.

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