Title
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Toward a dynamical interpretation of hierarchical linguistic structure

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1. Introduction

Hierarchy is one of the most important concepts in the scientific understanding of language. From early structuralist approaches of the mid-1800s, through the paradigm shift to generative grammar in the mid-1900s, and onward to the present, connected-object ("branching") schemas have been used to represent hierarchical constituency relations between linguistic units. Syntactic trees are the most prominent instantiation of such schemas, but similar arboreal schemas have been applied extensively in nearly all domains of linguistic theory. Recently, an alternative conception of hierarchical structure has arisen, which is based upon interactions between waves, or coupled oscillatory systems. This paper presents a synthesis and extension of such approaches, with the aim of advancing a general wave theory of linguistic structure. This wave theory provides a coherent conceptualization of diverse linguistic phenomena, is cognitively plausible, and is a parsimonious theory. Many syntactic and phonological patterns in language can be understood to arise from general principles of wave interaction.

Section 1.1 will consider how hierarchy has traditionally been conceptualized and represented in linguistics, and section 1.2 will describe some previous research which leads the way to a wave-based, dynamical systems perspective on linguistic structure. Section 2 will introduce some general principles regarding how hierarchical structure can be conceptualized with dynamical systems, beginning with systems of two waves, then treating systems of three waves, which can be generalized to more complex systems. This is followed in section 3 by several applications of the theory to phonological and syntactic patterns. These include an examination of how segments interact with syllables and how stress interacts with words, as well as a proposal for dynamical modeling of recursive syntactic patterns and syntactic dislocation phenomena. Section 4 concludes with a discussion of the cognitive plausibility of this approach, and delineates some issues to guide further development of the theory.

1.1 Hierarchical Structure

In this paper we are interested in a structural notion of hierarchy, in which concepts of ranking, relation, and containment play important roles. In order to organize our discussion, it is useful to view this notion of hierarchy as a complex of blended concepts, where ranking is central to hierarchy, and relation and containment (or, constituency) enrich its meaning. The most basic sense of "hierarchy" entails only a ranking or ordering of things, and this stripped down notion will not concern us here. Many examples of solely ranking hierarchies arise in linguistics, such as markedness hierarchies, person hierarchies, or the sonority hierarchy. Such hierarchies do not describe the "structure" of linguistic utterances, and do not aim to represent temporal relations between units.

Structural hierarchy relies fundamentally on the basic notion of the linguistic "unit". Moreover, the notion of the linguistic unit has become conceptually entangled with structural hierarchy. To some extent, these notions originate from early structuralists such as Wilhelm Wundt and Ferdinand de Saussure. In analogy to the physical sciences, structuralists aimed to uncover the "elemental" units of cognition, and to characterize the system of their relations. Early structuralists identified linguistic units such as phonemes, morphemes, words, and sentences as the basic building blocks of linguistic structure. At the core of the structuralist mindset is the idea that there *is*, indeed, a "structure" of language, and so there must correspondingly be mental “objects” out of which that structure is built. This does not mean that that structuralism views linguistic units literally as physical entities, but it does use for conceptual and theoretical purposes a set of metaphors that objectize phonemes, morphemes, words, sentences, etc. It is precisely this objectization (potentiated to a large extent by the permanence of the written word), that allows for the structuralist conception of language in the first place. Because this metaphor is so deeply engrained in linguistic reasoning, it is sometimes difficult to see how fundamental it truly is. Thinking of linguistic units as objects facilitates the connected-object depiction of their relations and enables containment-based reasoning—these are precisely the aspects of hierarchy we are primarily concerned with.

The idea of organizing linguistic units in a hierarchical structure is commonplace in contemporary linguistic approaches, and moreover, the image-schema of connected objects is extensively used for the purpose of representing and reasoning about relations. In examining the history of the use of hierarchical branching structures, Seuren (1998) attributes their origin to the 19th century psychologist Wilhelm Wundt (a progenitor of structuralism), who made prolific use of hierarchical tree-like structures to characterize the relations between linguistic units (cf. Wundt, 1880). These units were conceptualized as the basic mental objects that constitute language, fully in the spirit of structuralist objectization. Seuren further discusses how the essence of this conceptualization, a hierarchical constituent structure, was conveyed through influential early 20th century linguists such as Sapir and Bloomfield to early generativists such as Zelig Harris and Noam Chomsky. Sapir and Bloomfield only very rarely drew tree structures, because there existed a prescriptive norm against the use of diagrams in late 19th and early 20th century humanistic disciplines. Despite this, their writings often clearly describe hierarchical, conceptions of linguistic structure. Following Chomsky (1957), the explicit use of tree diagrams to understand syntactic patterns become commonplace. Hierarchical connected-object structures subsequently infiltrated other linguistic domains such as morphology and phonology, with applicability to prosodic structures such as syllables, feet, phonological words, phonological phrases, etc. Thus the hierarchical conception of linguistic structure, relying fundamentally on the structuralist objectization of linguistic units (i.e. the idea that they are the sorts of things that can be connected), originates over 120 years ago, and has been greatly extended since the generative revolution.

Given the metaphor that linguistic units are objects, the central notion of structural hierarchy is *ranking*. Rank is often conceptualized with orientation and/or size. Units are located with reference to ranked "levels". The levels evoke a vertical orientation and are commonly correlated with the "size" (temporal extent, or information-bearing capacity) of units belonging to a level. It is perhaps curious that, given some set of postulated levels of a ranking hierarchy, there is usually little disagreement about the specific ranking of those levels. Instead, disagreement normally revolves around the details of intra- and inter-level interactions between units, and sometimes involves the question of whether a particular level should be posited to begin with. The idea that there are indeed "levels", that it is sensible to spatially orient units, is rarely questioned.

In structural hierarchies, connection patterns between objects serve to indicate their relatedness, and familial terminology is commonly exploited to characterize relative rank (e.g. mother node, daughter node, sister nodes). Linguists have nearly universally avoided the crossing of lines in connection schemas. There are numerous cases in which the order of spoken words and considerations of constituency conflict, necessitating line-crossing. To resolve this dilemma, Chomsky employed a concept of deep structure, where the order of units does not violate line crossing constraints. In contrast, morpho-phonologists opted to make use of additional spatial dimensions in non-linear autosegmental phonology (cf. Goldsmith, 1976). Reflecting on these approaches, it seems odd that line-crossing has been rejected outright, since in theory the representations are not literally believed to be physical objects. The fact that line-crossing has been so strongly avoided illustrates just how reified the objectization of linguistic units has become.

In many contexts, connection relations also entail containment. Linguists often employ a conventionalized interpretation of the connection patterns in Figure 1 as encoding a containment (or constituency) of C and V segments within syllables (σ), and of NP and VP within S. It is worthwhile to consider how easily the concept of containment can be dissociated from connected-object representations. This can be done by focusing on the spatial separation of the higher-level units from their lower-level associates. In other words, the representation of the syllable can be seen as a distinct object that happens to be associated with other distinct objects, segments. Although the representation does not convey containment in an iconic manner, the convention for inferring containment is straightforward: higher-level units contain the lower-level units that are connected to them. Since containers are also objects, it is quite simple to shift focus between the containment behavior of an object and its connective behavior. Generally speaking, any given linguistic theory either will, or will not, follow the convention that connections indicate containment.

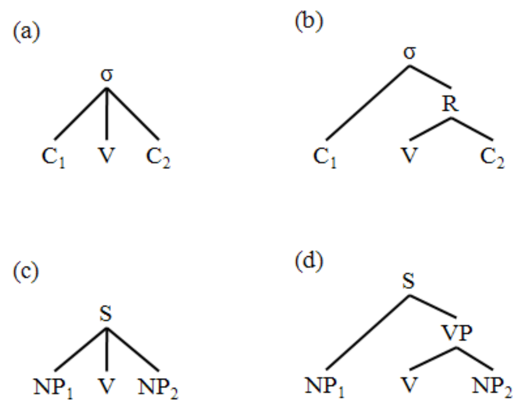


Figure 1. Examples of connected-object representation of hierarchical linguistic structure.

It is noteworthy that some of the most basic questions that arise with connected-object representations regard what the specific units in a hierarchy are, and what the specific patterns of connections are. Figure 1 contrasts (a/c) with (b/d), where the latter posit an additional unit (the rime or VP). A primary consequence of this is that V and C_2, and likewise Verb and NP_2, do not bear the same relation to σ and S as do C_1 and NP_1, respectively. In other words, a symmetry has been broken in (b/d), compared to (a/c). The concept of broken symmetry is extremely important in the wave theory, and as we will see, arises from asymmetries in parameters of wave interaction.

1.2 Dynamical approaches

To a large extent, the wave theory derives from a tradition of dynamical systems modeling of the coordination of movement. Dynamical systems theory involves the use of differential or difference equations to describe the qualitative behavior of complex systems. Of particular relevance here is the use of dynamical systems to understand self-organization in human behavior. Particularly foundational is the work of Herman Haken, who developed the concept of the *order parameter* or *collective variable*. A collective variable is a low-dimensional variable that describes the collective behavior of a self-organized system composed of many individual systems driven far from thermal equilibrium (Haken, 1983, 1993). An often used example is the Rayleigh-Benard experiment. Oil is heated in a pan and forms convection rolls, in which individual molecules are enslaved to an orderly, coherent pattern of motion. Rather than describing the motions of all the individual molecules, the collective variable can be used describe the motion of the system with fewer degrees of freedom. For more background on self-organization and concepts relevant to synchronization in oscillatory systems, the reader is referred to Kelso (1995), Pikovsky et al. (2001), and Winfree (1980).

Dynamical systems theory has been successfully extended to numerous behavioral systems. An early example can be found in work conducted by J.A.S. Kelso and Eliot Saltzman (cf. Saltzman & Kelso, 1983), who, to better understand the coordination of individual effectors, used the concept of *relative phase*. Relative phase is a collective variable that corresponds to the phase difference between oscillatory systems, each of which has its own phase. The phase can be thought of as the angle made by a point moving along a closed trajectory. A key insight in studies of coordinated movement is that some relative phase relations in synchronized systems are more stable than others. In a classic study, Haken, Kelso, & Bunz (1985) found that as movement frequency increases, anti-phase finger wagging becomes unstable and the relative phase of the system undergoes a transition to in-phase mode wagging. Haken, Peper, Beek, & Daffershofer (1996) extended this idea to model phase transitions between frequency-locked modes of drumming performed by skilled drummers. They observed that relatively high-order modes of frequency-locking such as 2:5 and 3:5 became unstable as movement frequency is increased, resulting in phase transitions to nearby lower-order ratios such as 1:3, 2:3, and 1:2.

These findings and their conceptual underpinnings—i.e. relative phase, synchronization, frequency-locking, instability, phase transitions—have found numerous important extensions to speech phenomena. Goldstein & Browman (1988, 1990), Saltzman (1986), and Saltzman & Munhall (1989) advanced the idea that the lexical specification of speech incorporates a target relative phases of gestures. An important example of this is the *c-center effect*, discovered by Browman & Goldstein and incorporated into articulatory phonology, a prominent dynamical theory of phonology (1988, 1990). The c-center effect refers to timing patterns observed between multiple onset consonant gestures and vowel gestures within a syllable, e.g. in CCV syllables like *spa*. In syllables with a single onset C, the beginnings of gestures associated with consonant and vowel are approximately in-phase synchronized (note that consonantal gestures are produced more quickly). However, in syllables with complex CC onsets, the beginnings of the consonant gestures are equally displaced in opposite directions from the onset of the vocalic gesture. Browman & Goldstein (2000) argued that the c-center effect arises from competition between lexically specified relative phases: C gestures are in-phase coordinated with V gestures, but C_1 and C_2 are anti-phased to one another. The compromise between these competing coupling relations is the c-center effect. Saltzman & Nam (2003) simulated competitive coupling using a system of coupled oscillators, or *gestural planning systems*. The gestural planning systems exhibit limit cycle dynamics and their relative phases are governed by potential functions.

Oscillatory dynamical systems have also been applied to the timing of higher-level prosodic units. Port, Cummins, & Gasser (1995) is an early example of this endeavor. O'Dell & Nieminen (1999, 2008) presented a model of interacting syllable and foot oscillators, in order to describe patterns of stressed and unstressed syllable duration, and Barbosa (2002) used coupled oscillators to account for cross-linguistic rhythmic variability. Cummins & Port (1998) and Port (2003) used 1:2 and 1:3 frequency-locking between oscillatory systems to model experimental effects in the timing of feet and phrases. Furthermore, Goldstein, Byrd, & Saltzman (2006) applied dynamical concepts from articulatory phonology to understanding phonological evolution, and Goldstein, Pouplier, Chen, Saltzman, & Byrd (2007) used them to understand error patterns in articulation. A current trend is the development of models that integrate the dynamics of higher-level prosodic and rhythmic systems with the dynamics of gestural planning systems (cf. Saltzman & Byrd, 2003; Saltzman, Nam, Krivokapic, & Goldstein, 2008; Tilsen, 2009).

2. The wave theory

We begin by considering the restricted case of two waves. The first conceptual step is to associate the neural planning of a linguistic unit with an oscillatory, wave-like system. It is important to understand why this idea is cognitively plausible, but we will reserve such considerations for section 4. By "wave-like system" we mean a system of differential equations whose solution can be visualized as a point moving along a closed path. For conceptual parsimony, it is convenient to think of waves as points moving around circles. We can model this motion using polar coordinates, where the dynamic variables are the phase angle (θ) and radial amplitude (r). The simplest version of a wave is the harmonic oscillator described in Eq. (1).

$$\begin{aligned} \dot{\theta}_i &= \omega_i \\ \dot{r}_i &= 0 \end{aligned} \tag{1}$$

Eq. (1) states that the change in phase (the phase velocity) is a constant, ω. The phase angle is wrapped around the interval [0,2π). Hence, when θ = 2π radians, θ is reset to 0. The constant ω determines the frequency which the point will complete a full loop around the circle. By convention, all frequencies will be positive, meaning that the points move counterclockwise. Eq. (1) also states that the radial amplitude does not change. Whatever the initial radial amplitude is, it will remain at that value.

The overly simplistic version of a wave in Eq. (1) lacks a very important quality: its phase cannot be influenced by other waves. To facilitate the description of this influence, it is helpful to define a *relative phase*, φ, which is the phase of one oscillator relative to another. Eq. (2) states that the phase of the *i*th oscillator relative to the phase of the *j*th oscillator is $\theta_j - \theta_i$. Henceforth we will often refer to this relative phase as φ.

$$\phi_{ij} = \theta_j - \theta_i \tag{2}$$

The relative phase can be used to describe how other oscillators influence the phase of a given oscillator. By adding phase coupling forces (F) to Eq. (1) we get Eq. (3). If the net coupling forces are positive they will speed up the motion of the point, if they are negative they will slow it down. Eq. (4) shows that F is the sum of the negative of the derivative of the potentials V, which in turn are defined in Eq. (5).

The potential *V* is a commonly used form of relative phase coupling (which we will henceforth refer to as φ-coupling), derived from an extensively studied model of interacting phase oscillators known as the Kuramoto model (Kuramoto, 1975; Acebron, Bonilla, Vicente, Ritort, & Spigler, 2005). The Kuramoto model is used mostly to investigate the behaviors of systems of many oscillators, often with highly symmetric coupling. When coupling asymmetries are introduced, they are commonly randomly chosen from a noise distribution. This contrasts markedly with the current usage, where each oscillator corresponds to an identifiable behavioral system, and where the coupling asymmetries are determined from principles we will introduce subsequently. These asymmetries are manifested partly in the φ-coupling strength matrix α, in Eq. (4).

$$\dot{\theta}_i = \omega_i + \sum_j F(\phi_{ij}) \tag{3}$$

$$\mathrm{F}(\phi_{ij}) = \sum_j \alpha_{ij} \frac{-dV(\phi_{ij})}{d\phi} \tag{4}$$

$$\mathrm{V}(\phi_{ij}) = cos(\phi_{ij}) \tag{5}$$

The φ-coupling strength parameter α_{ij} does two important things. The magnitude of this parameter corresponds to the strength of the coupling force exerted by system *i* on system *j*. In linguistic applications of this model, usually some systems are uncoupled, which is parameterized as $\alpha_{ij} = \alpha_{ji} = 0$. For any two systems that are structurally associated in a meaningful way, $|\alpha_{ij}| > 0$ and/or $|\alpha_{ji}| > 0$. It is important to keep in mind that α_{ij} represents the strength of the force exerted by wave *i* on wave *j*, and α_{ji} the strength of the force exerted by wave *j* on wave *i*. The second important function of α is to describe the *type* of φ-coupling between systems—i.e. whether the systems are attractively or repulsively phase-coupled. If $\alpha_{ij} > 0$, the phase of *j* is pulled toward the phase of *i*, but if $\alpha_{ij} < 0$, the phase of *j* is pushed away from the phase of *i*.

Another way to look at the situation is that the parameter α allows for a symmetry breaking in a 2-wave system. The most symmetric system is $\alpha_{ij} = \alpha_{ji}$. Symmetry can be broken by $\alpha_{ij} \neq \alpha_{ji}$, meaning that the systems exert different strength forces, or even different types of forces. It turns out, however, that this symmetry breaking mechanism is not sufficient for describing hierarchical structure, and so we will proceed to considering 3-wave systems subsequently.

2.1 Behaviors of 2-wave systems

Consider now a system of two interacting waves, with regard to the branching structure in Figure 2. This branching structure is perhaps the simplest non-trivial branching structure. It tells us that X and Y each form a unit (constituent), and together they form a higher-level unit, Z. Moreover, it tells us that X precedes Y. These are the basic entailments of the representation that we wish to capture with a wave model.

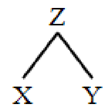


Figure 2. Basic branching structure.

Imagine that each of the lower-level units X and Y is associated with a wave. We have a wave X that describes the *activation* of unit X, and likewise a wave Y that describes the *activation* of unit Y. (We will discuss the concept of activation in section 4, when we consider the neural grounding of the theory). Assume that the waves have approximately the same frequency of oscillation, and unit amplitude. The crucial parameter that describes how waves X and Y interact is α, which determines both (1) the strength of φ-coupling, and (2) the type of φ-coupling, i.e. attractive or repulsive. There are three qualitatively different behaviors that the system can exhibit. These are categorized in Table 1 below. For reasons to be discussed later, we it is sensible to constrain α such that sign(α_{ij}) = sign(α_{ji}). However, for generality, the behaviors described in Table 1 allow for circumstances where this constraint does not hold, e.g. $\alpha_{ij} = 1$ and $\alpha_{ij} = -1.5$—we will refer to such pairs of waves as "conflicted". To accommodate the description of conflicted waves, the symbol α^+ is used to refer to the attractive coupling force exerted by one wave, and α^- to the repulsive force exerted by the other. In this example, the result would be phase repulsion, since the net force ($\alpha^+ + \alpha^-$) is negative.

Table 1.

phase attraction in-phase symmetry	attractive coupling overwhelms repulsive coupling	$\alpha^+ > \alpha^-$
balance/non-interaction phase asymmetry	attractive and repulsive coupling are balanced, or nil	$\alpha^+ = \alpha^-$ OR $\alpha = 0$.
phase repulsion anti-phase symmetry	repulsive coupling overwhelms attractive coupling	$\alpha^+ < \alpha^-$

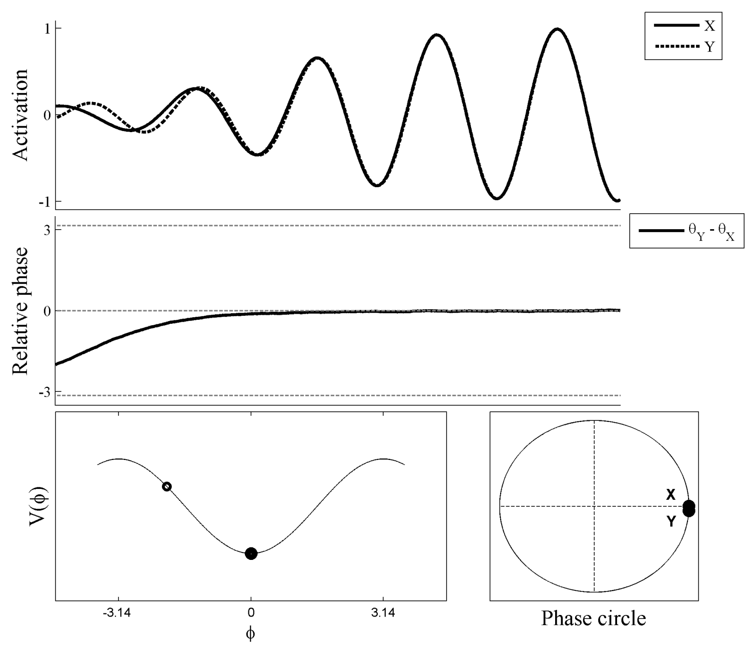
* in a conflicted pair of waves, α^+ refers attractive coupling exerted by one system, and α^- repulsive coupling exerted by the other one.

Figure 3 shows simulation of each of the three possible dynamical behaviors of 2-wave systems. The reader should take note that the simulations incorporate a radial amplitude dynamics that we will later include in the model equations (cf. section 2.3). Hence, activation waves initially have low amplitude and subsequently their amplitude increases until a stable attractor is reached. For current purposes, we are interested primarily in the stabilized dynamics of the systems under consideration. The initial rise in amplitude, a form of transient change, can be ignored. Note that discussion of numerical simulation methodology is provided in the Appendix.

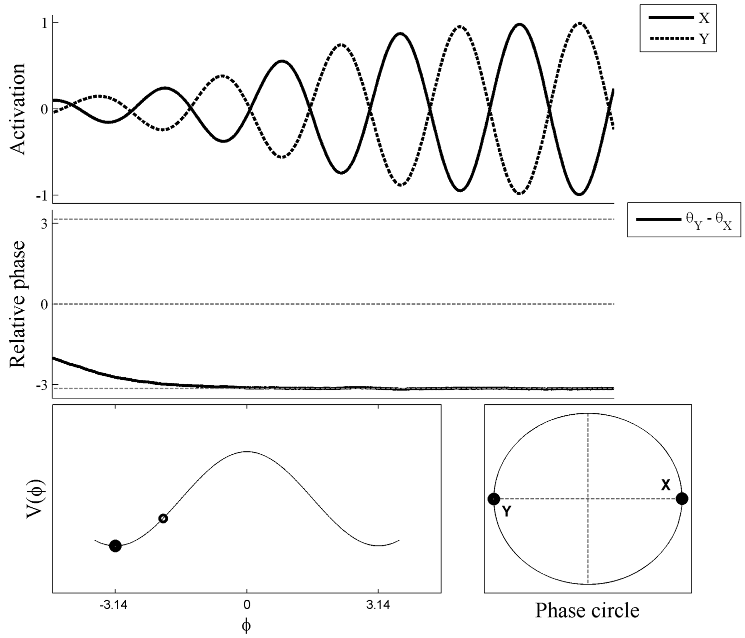
Phase attraction occurs when the net coupling forces between X and Y are positive, which results in a relative phase of 0 between X and Y. Figure 3a shows a simulation of how the activation (-cos θ) and relative phase of two attractively coupled oscillators change over time. Initially, $\varphi_{XY} = 2$, but the mutual phase attraction forces pulls their activation waves together. After the transient dynamics due to the random initial phase conditions have disappeared, the system can be said to have "stabilized" to in-phase synchronization. In contrast, *phase repulsion* describes the situation shown in Figure 3b where the net coupling forces are repulsive; this stabilizes to a relative phase of -π/π between X and Y (anti-phasing).

Phase attraction and phase repulsion are special in that, once the systems have stabilized, they are symmetric with respect to the direction of time. This means that if the flow of time is reversed and the systems evolve "backwards," the in-phase and anti-phase symmetric systems appear the same (disregarding the initial transients). While this is obvious for phase-attraction, it is also the case for phase-repulsion: since the systems are π out of phase, it is ambiguous which one precedes the other.

(a)



(b)



(c)

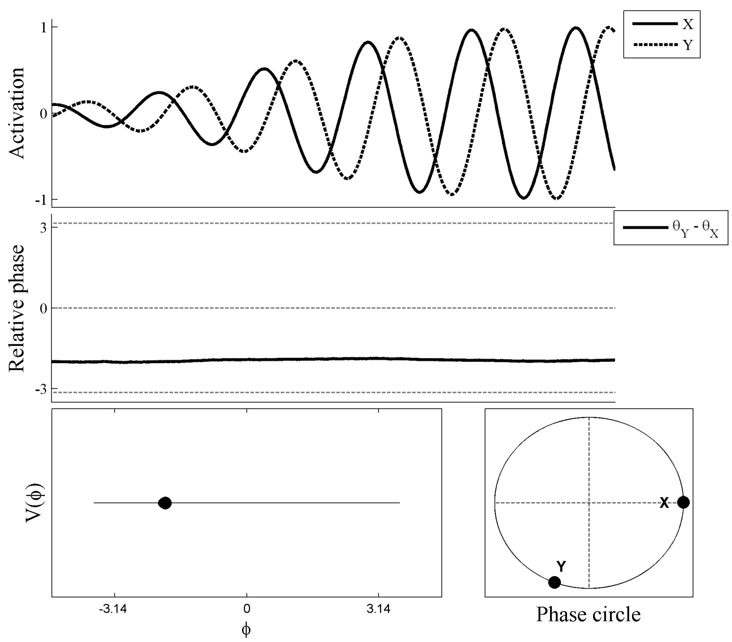


Figure 3. Dynamical behaviors of 2-wave systems. (a) phase attraction, where $\alpha_{XY,YX} > 0$; (b) phase repulsion, where $\alpha_{XY,YX} < 0$; (c) phase balance, where $\alpha_{XY} = -\alpha_{YX}$. In each panel, time plots of activation and relative phase are shown. On the bottom left, the initial (o) and final (●) relative phases are shown in the relative phase potential, V(φ); on the bottom right, locations of stabilized oscillator phases are depicted on the phase circle.

In all cases, phase symmetry (time-reversal symmetry) is initially broken. Whether this symmetry breaking is random, contextually determined, or constitutes a form of lexical memory is an issue we will return to in due time. Regardless of how initial symmetry is broken, phase attraction and repulsion restore it. Only when attractive and repulsive coupling forces are balanced (i.e. the net coupling force = 0) or nil, does the initially broken symmetry remain broken. This can be seen in Figure 3c, where repulsive and attractive coupling forces cancel each other out, resulting in no changes of relative phase. The trivial case where $\alpha_{ij} = \alpha_{ji} = 0$, i.e. no coupling forces are exerted in the first place (not shown), results in the same perseverance of broken symmetry.

Unlike broken objects, broken symmetry in linguistic systems is usually desirable. Indeed, the problem with the 2-wave system is that once phase symmetry is broken, it will be restored unless one of the two conditions for phase balance is met: $\alpha^+ = \alpha^-$ or $\alpha = 0$. The latter is not useful because it means that the systems do not interact at all, and so we would not expect them to exhibit time-reversal asymmetry in the first place. The former condition is awkward because the combined effect of the potential forces is nil, so that the system is effectively behaving as if uncoupled. This behavior is a consequence of the fact that the potential functions themselves are out of phase by exactly π radians (cf. Figure 3c). One of the less desirable aspects of this is that attractive and repulsive force magnitudes have to be the same for balance to arise. Later we will explain why this situation is cognitively implausible, and hence, why conflicted waves do not arise.

Why must phase symmetry remain broken to satisfactorily describe the hierarchical structures in Figure 1? It should be apparent that in-phase symmetry between two systems is not a very useful description of the structures, because these representations imply a temporal ordering of the lower-level systems, X and Y. In-phase symmetry locates activation peaks at the same time. Thus attractive coupling alone is not enough. For less obvious reasons, repulsive coupling is similarly deficient. Here the time-reversal symmetry of X and Y makes their ordering necessarily ambiguous, and their activation peaks are as far apart as possible. The addition of a third wave resolves these problems.

2.2 Behaviors of 3-wave systems

In 3-wave systems, phase symmetry can remain broken without perfectly balanced pairwise coupling forces or nil coupling. Moreover, 3-wave systems can be directly related to the structures in Figure 2, and *n*-wave systems can be used to model any hierarchical linguistic structure.

A straightforward way to accomplish phase balance in 3-wave systems is to posit two sets of coupling relations: first, there is repulsive coupling between X and Y, the units that are associated with the same level; second, there is attractive coupling between Z and X/Y, units that are "connected" between "levels". We thus have a situation in which the attractive and repulsive forces indirectly oppose each other. More specifically, the attractive φ-coupling pulls both X and Y closer to Z, but the repulsive coupling pushes X and Y apart from each other. The result is broken phase symmetry, i. e. relative phases that are *asymmetric* with respect to time reversal. This can be seen by inspection of Figure 4.

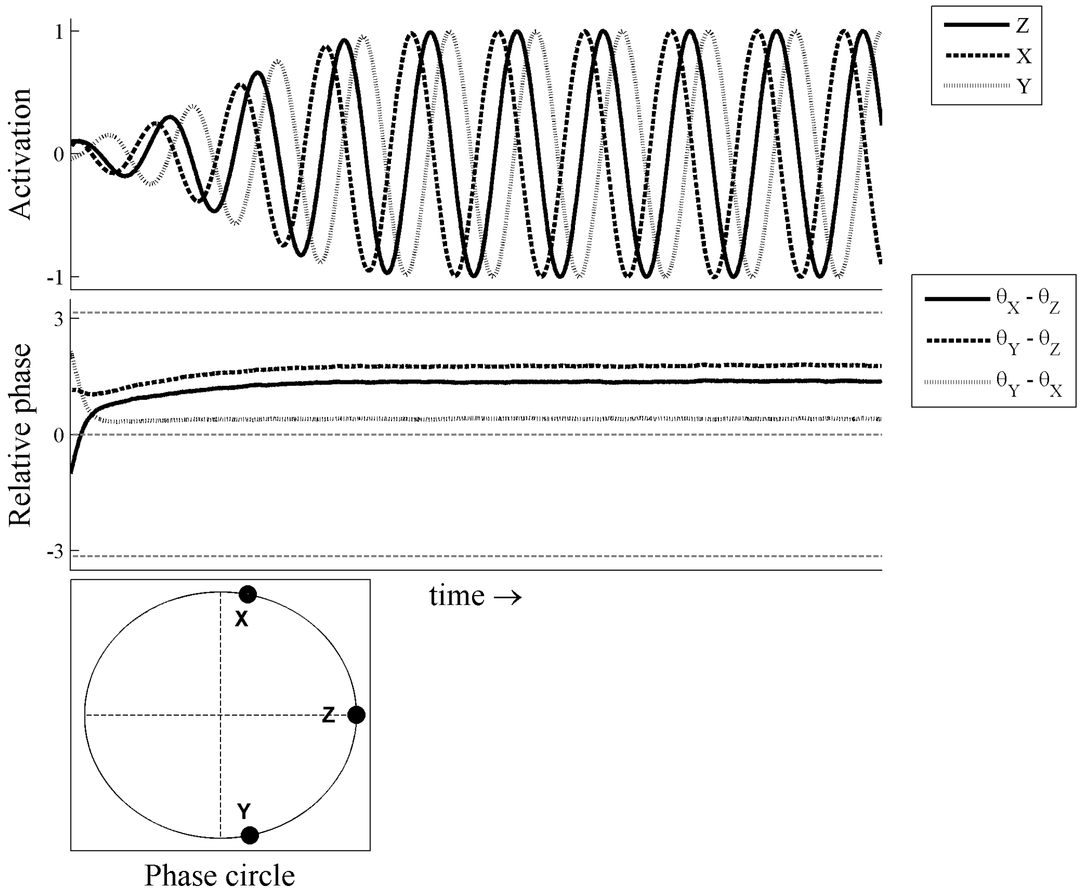


Figure 4. Simulation of 3-wave system with attractive coupling between Z and X/Y, and repulsive coupling between X and Y.

In the forward direction of time, the peak activation of X precedes that of Z, which precedes that of Y. Imagining what this looks like in reversed time, one can see that Y precedes Z, which precedes X. The "precedence" relation here can be established on the basis of the smallest phase difference between successive activation peaks. For this purpose, the system Z (or any system) can be used as a reference point. From the perspective of Z, the peak activation of X comes before the peak of Z. Likewise, the smallest phase difference locates Y after Z. Moreover, X occurs before Y, since the between-peak phase difference is smaller that way. The perseverance of broken time-reversal symmetry between X and Y occurs because the third system Z pulls the phases of X and Y toward itself. This prevents the repulsive force between X and Y from restoring symmetry (i.e. bringing X and Y to a relative phase of π radians). Importantly, whether X precedes Y or vice versa is determined by the initial conditions (in this case), by the transient behavior of the system before stabilization (in more complicated cases), or by stochastic forces. In other words, exactly how the symmetry is broken is a matter of other factors, but the mechanism which preserves this broken symmetry is the interplay between attractive and repulsive coupling forces in the 3-wave system.

This dynamical behavior is precisely the one that was hypothesized by Browman & Goldstein (2000) to explain the timing of consonantal onset gestures relative to a vowel.

They had previously observed that in C_1C_2V sequences, the initiation times of C_1 and C_2 articulatory gestures have been shown to be equally displaced in opposite directions before and after a vowel gesture (Browman & Goldstein, 1988). This observation of motor behavior and several others can be well understood if the systems in CCV structure are coupled as above. We will consider the application of this idea to syllable structure in greater detail later on, and more generally to all types of hierarchical structure. Such 3-wave systems are the primary building blocks for understanding hierarchical structure in the wave theory.

Given that any hierarchical structure of the form in Figure 2 will be modeled dynamically as above, this suggests a couple of generalizations. First, for any pair of coupled systems "on the same level", the coupling forces are repulsive. This is equivalent to saying that two systems of the same type, if coupled, are coupled repulsively. Second, coupled systems "on different levels" are coupled attractively. This amounts to saying that systems of different types, if coupled, are coupled attractively. “Similarity of type" is a relatively underdeveloped aspect of the wave theory. No theory of how to construct similarity criteria is provided here. Generally speaking, it works to posit that lexical items such as Noun and Verb are similar, and that segments C and V are similar. Likewise, N is different from NP, and C and V are different from σ. Some further thoughts on similarity of type are offered in section 4, but for current purposes these assumptions are useful.

We refer to the above generalizations collectively as the *principle of like interaction*: things of similar type (e.g. any number of segments, or syllables, or words, or phrases) are repulsively coupled, and things of different type are attractively coupled. This principle is partly analogous to the electromagnetic interaction between charged particles. It greatly simplifies the parameter space for the wave-based description of a given system, because it follows that $\text{sign}(\alpha_{ij}) = \text{sign}(\alpha_{ji})$. Figure 5 (a) and (b) illustrate how 3-wave systems correspond to basic syllable and sentence structures. Both the CV syllable and the NP-VP sentence can be conceptualized as 3-wave systems. The parameterization of α is entirely parallel between them: C and V segments, being of similar type, are repulsively coupled to one another [C~V]. Likewise, syntactic phrases NP and VP, being of similar type, are repulsively coupled [NP~VP]. In both cases there is a higher-level system, σ or S, which, by means of attractive coupling, binds the repulsively coupled systems together, preserving their broken phase symmetry.

In Figure 5, three different representations are shown. First, there is a “connectionistic” representation, which employs a standard object-connection schema. Second, there is a coupling graph, which represents the pattern of coupling interactions between systems. Coupling graphs are NOT hierarchical structures, and do not conventionally indicate containment/constituency relations. Third, there is a “phase circle representation,” which is a schematized representation of the stabilized relative phases of the systems under consideration. In general, the phase circle is anchored to phase 0 of a particular system (σ or S in the figure). There are two common conventions one might follow: (1) orient the circle like a clock, so that phase 0 is at the top, as in Figure 5; (2) orient the circle with phase 0 being the x-axis. We will use both in this paper, and it should be obvious which one is being drawn upon. The phase circle and coupling graph are often useful in combination. The coupling graph represents coupling forces, but not relative phase relations. Conversely, the phase circle represents relative phases, but not coupling relations.

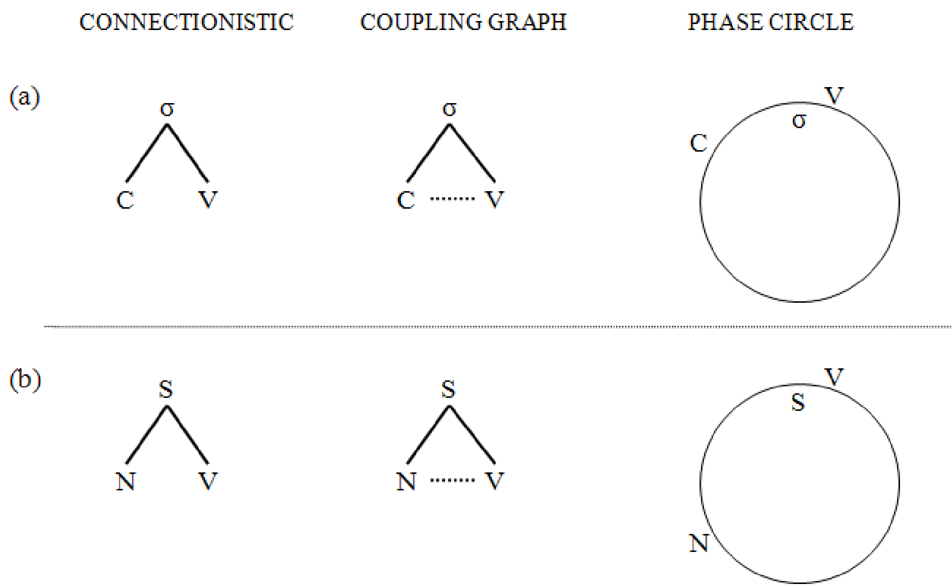


Figure 5. Illustration of how 3-wave systems correspond to basic linguistic structures.

The dynamical reinterpretation of connected-object structure has the consequence that the "sisterhood" relation between nodes often involves inhibitory φ-coupling. The mother-daughter nodal relation normally involves attractive φ-coupling. These follow from the principle of like interaction, which readily extends to *n*-wave systems. For example, Figure 6 shows a CVC and a ditransitive English sentence.

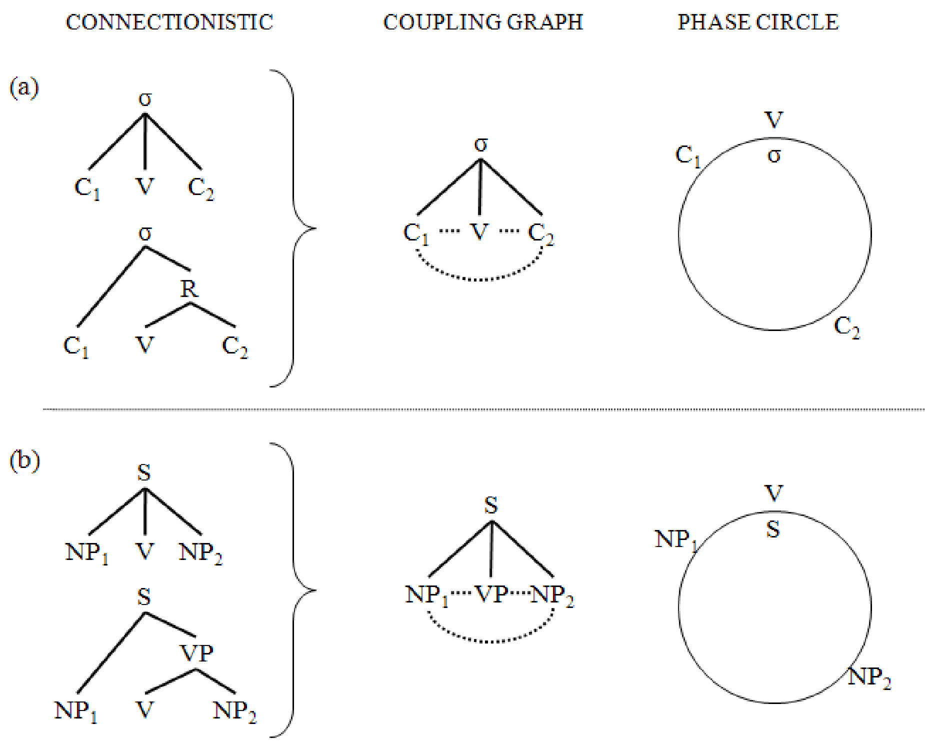


Figure 6. Representations of a CVC syllable and a ditransitive sentence. Standard connectionistic representations of these use 3 levels.

Two alternatives are shown in the connectionistic representations. One is a two-level hierarchy with ternary branching. The other is a three-level hierarchy with binary branching. Most linguistic theories take the latter as the correct one, primarily because the coda/object system seems to exhibit a much closer relation to the vowel/verb system than the preceding one. For example, regarding the syllabic structure, poetic rhyming is determined more crucially by VC than CV. Syllable weight is far more commonly influenced by whether the rime branches (later we will understand this from a new perspective). On the syntactic side, object NPs are generally agreed to be much more closely semantically related to verb meaning than subject NPs, and there are various syntactic manifestations of this.

The dynamical interpretations of the three-level hierarchies in Figure 6 require only two levels, because there are only two different types of systems. Hence, following from the principle of like interaction, all segmental systems in (a) are repulsively phase-coupled, and all phrasal systems in (b) are repulsively phase-coupled. But how, then, does constituency arise from the dynamical model, and what is the dynamical manifestation of constituency?

Somewhat counter-intuitively, phase-proximity of coupled systems does not necessarily correlate with constituency. This can be seen in the phase circle representations above, where systems whose units are sisters in connectionistic representation exhibit larger phase differences from the anchor (vowel/verb) than units that are not immediate sisters. It is theorized that this situation can be understood in the following way: the coda C is much more strongly repulsively coupled to V than the coda C, i.e. $[C_2{\sim}V] >> [C_1{\sim}V]$; at the same time, the coda C is somewhat more strongly attractively coupled to σ than the coda, i.e. $[C_2\text{-}\sigma] > [C_1\text{-}\sigma]$. The analogous asymmetries hold in the syntactic structure. The reader should take a moment to contemplate the net consequences of this situation. The asymmetry in repulsive coupling is greater than the asymmetry in attractive coupling. Therefore a greater phase difference holds between coda C-V than onset C-V. At the same time, the attractive asymmetry indexes what the connected object representation conceptualizes as *constituency* patterns. VC and V-NP are immediate constituents, rather than CV and NP-V, because they more strongly coupled to the higher level σ and S systems. Thus, phase differences do not index constituency; rather, attractive coupling strengths, in addition to phase asymmetries, do so.

However, the fullest embrace of the wave theory rejects the standard notion of constituency altogether, and leads to a new idea of constituency. Because linguistic units are not objects, but waves, they are not the sorts of things that can be contained, or that can be constituent of other things. Rather, they are interacting oscillations. The more strongly a group of oscillations is coupled to another system, the more constituent-like they are. But this does not necessarily make them a "constituent". The standard determinations of constituency are tests such as substitution, coordination, ellipsis, replacement with anaphor, dislocation, etc. We see below how these patterns can be understood by considerations of stability and syntactic wave interaction.

Moreover, there are even deeper reasons why coda systems and object NP systems are associated with stronger repulsive and attractive coupling, which we are not yet prepared to address. To foreshadow just a little, the more basic source of this asymmetry is the *amplitude* of coda and object NP systems. We will later on see that the amplitudes of systems modulate the φ-coupling forces they exert on other systems. For the moment, we operate under the simplified assumption that all amplitudes are constant and equal to 1.

In the dynamical reinterpretation of the three-level hierarchies above, only two levels, or types of systems, were posited. Alternatively, we could have proposed that rime and onset systems are necessary, and we could have stipulated that the VP is a different type of system than the NPs, in which case the principle of like interaction would lead us to three-level structures of the sort in Figure 7. Systems like Rime and VP are inherently ambiguous in this respect—should they necessarily be distinct dynamical systems, or are they accounted for by asymmetries in coupling parameters, as described above?

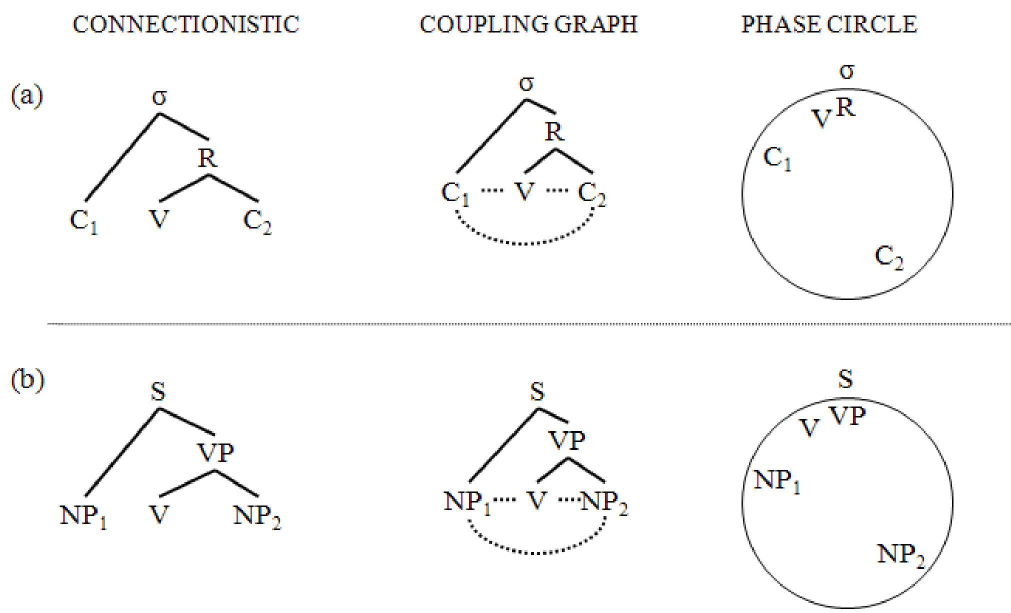


Figure 7. Alternative dynamical model of the three-level hierarchy.

One solution to this question is that both Figure 6 and Figure 7 coupling graph parameterizations are copresent. In other words, codas and vowel nuclei are coupled both to the syllable, and to a rime system. Likewise, we could interpret the VP systems as analogous to the rime: object NP and V are coupled both to VP and to S. A related issue of relevance to this question is the relation between system frequencies. Consider the hierarchical connectionistic structures in Figure 8 below:

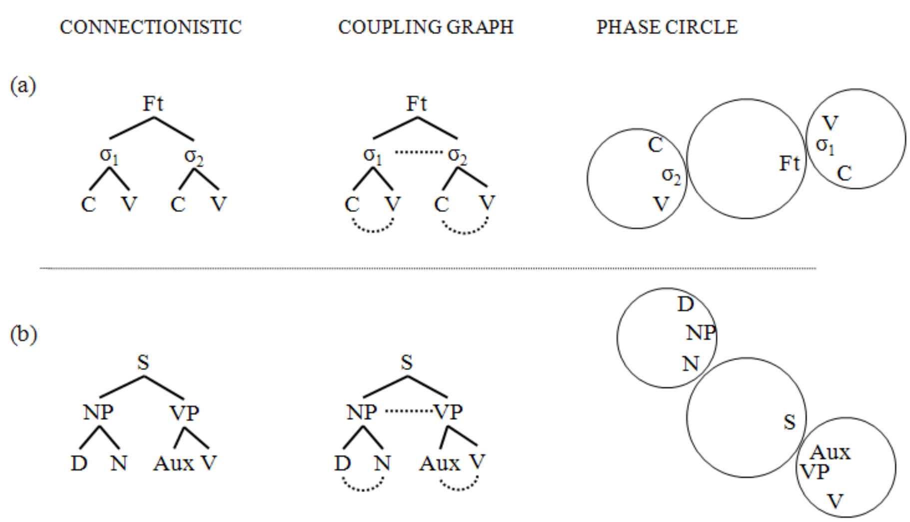


Figure 8. Representations of hierarchical linguistic structure. Connected object representations are shown on the left. Coupling graphs and schematic phase-circles are shown on the right.

In Figure 8, one can see phase circle representations with multiple circles. To correctly interpret these representations, one must understand the conventions behind their use. In such diagrams, there is always a primary phase-circle, which locates a phase 0 of an anchoring system (here we use an x-axis orientation). All other phases on the primary circle are relative to this zero-phase anchor. In many cases, the primary phase circle will become visually crowded if all of the systems of interest are shown on the same circle. In that case, it is convenient to use additional circles. When there is subconstituent structure (i.e. when there are groups of systems sharing a common attractive coupling target), a sensible guideline is to posit a circle for each group of subconstituents (i.e. separate circles for NP- and VP-coupled systems). The phases of the common attractive systems (NP and VP) are represented both on the primary circle and on the secondary circles with which they are associated.

These secondary phase circles are special, however, in that the location of 0 phase is not located at an angle of $0/2\pi$ radians; instead, it is relative to the "tangent point" system which connects it to the primary circle. Consider the σ_2 system, whose relative phase to the σ_1 and Ft approaches π in the limit of no attractive coupling between the Ft and σ_2. The phase of σ_2 is represented correctly on the primary circle, but on the secondary circle, it is shifted $\pm\pi$ radians. To preserve relative phase information between any systems represented on the secondary circle and the system anchoring that circle (the one on the tangent-point of both circles), the phases of the secondary circle are rotated by the phase of the tangent point system. Hence the representation in the secondary circles preserves relative phase information at the expense of mapping consistently from visual angle to absolute phase. This representation is also able to capture wave system amplitude, in the radius of the circles. It does not, however, directly represent system frequencies (which requires an additional factor in the rotation).

The dynamic approach offers another mechanism, beside attractive coupling, to account for the appearance of constituency. Constituency is normally viewed as the *temporal* containment of some units "within" a higher level unit, and this temporal containment is conceptualized spatially, such that a surface or hypersurface encloses objects on a 2D plane or 3D volume, or n-D volume. It is a somewhat different notion of containment that arises in the context of the wave theory. Wave theory sees containment as a consequence of frequency-locking, which implies a repetitive co-occurrence of activation intervals or phases.

Rather than using coupling strength α asymmetries to organize constituents, an alternative manifestation of constituency lies in frequency asymmetries, in the parameter vector ω. The phase circle representations in Figure 8 indicate parameterizations $n\omega_{Ft} \approx \omega_{\sigma}$ and $n\omega_{S} \approx \omega_{xP}$, which is to say that the ratio of Ft and S systems frequencies to their associated σ and xP frequencies is 1:n. In other words, the lower-level system frequencies are harmonically related to higher-level system frequencies. Alternatively, higher-level system frequencies are sub-harmonically related to lower-level ones. Trivial harmonicity of a 1:1 ratio is included here, i.e. $n = 1$. When $n > 1$, it is often the case that $n = 2$. There is also some evidence for situations in which $n > 2$.

By viewing constituency in this manner, we abandon the idea that the temporal order of units necessarily is mapped to an "underlying" sequence. The wave view does not favor viewing systems as "events" that are the sorts of things that can occur in a sequence. Rather, the wave theory takes a different perspective on sequencing, namely that it is a relative phasing of systems. What then determines the relation between observed sequence and wave activation? There are most likely thresholding and suppressive mechanisms, external to the wave theory, which determine when, specifically, an activation wave will drive the execution of its associated content. It is important to understand that the wave theory is a theory of premotor planning dynamics, not the translation of those dynamics to execution of movement. It does, however, provide a ready means for a dynamical model of execution.

The association of constituency with frequency-locking and "phase containment" is important, and depends upon the assuming that all linguistic systems exhibit harmonic oscillations. An analogy here is a plucked string. The length of the string will determine the fundamental frequency of a vibration and all of its harmonics/subharmonics, but other factors determine how much energy is distributed in each harmonic. This leads to a principle of harmonicity and a principle of harmonic interaction. The principle of harmonicity states that all systems oscillate at all harmonics and subharmonics of the fundamental. The principle of harmonic interaction states that attractively coupled systems tend toward frequency-locking.

It is important to see harmonic interaction as a tendency. Any pair of systems will not necessarily exhibit 1:1 frequency-locking, especially when they are initially activated. It is speculated that through nonlinear interactions not treated in the current model, frequencies of strongly coupled systems may approximate toward one another. Thus as transients decay and systems tend toward limiting dynamics, a property of those limiting dynamics is that strongly coupled systems will oscillate at comparable frequencies. This holds true both of attractive and repulsive coupling. Obviously, both the fundamental and harmonics/sub-harmonics will also exhibit frequency-locking.

Given a tendency for frequency-locking between attractively coupled systems, we have an apparent dilemma: how can a system such as the syllable frequency-lock both to a longer timescale system like the foot and to shorter timescale systems like segments? The principle of harmonicity allows us to resolve this dilemma: all systems oscillate at all harmonic and subharmonic frequencies, and thus different frequencies associated with the same system can lock to other systems whose frequencies are disparate. This allows for all coupling relations to be analyzed as 1:1. Consider the syllable, which mediates between segments and feet. It plays a dual role: it interacts with segments/articulatory gestures, and it interacts with Ft systems. We suspect that Ft have most of their energy concentrated at a relatively low frequency, while segments have most of their energy concentrated at a relatively high frequency—we have these suspicions because of the timescales of their behavioral correlates. The syllable has energy concentrated both at foot and segmental frequencies, and therefore interacts non-trivially with both types of systems. Phrasal categories, like syllables, occupy a middle ground in that they mediate between S and word systems. Parallel to syllables, we can view xPs as distributing oscillatory energy between sentential and lexical item (N and V) frequencies.

Using the principles of harmonicity and harmonic interaction in this way enables us to better understand temporal constituency: syllables are "contained" within a foot in the sense that their higher frequency oscillations (which frequency-lock them to gestures/segments) are harmonics of the lower-frequency oscillations that frequency-lock them to feet in a 1:1 ratio. This is schematized in Figure 9 below.

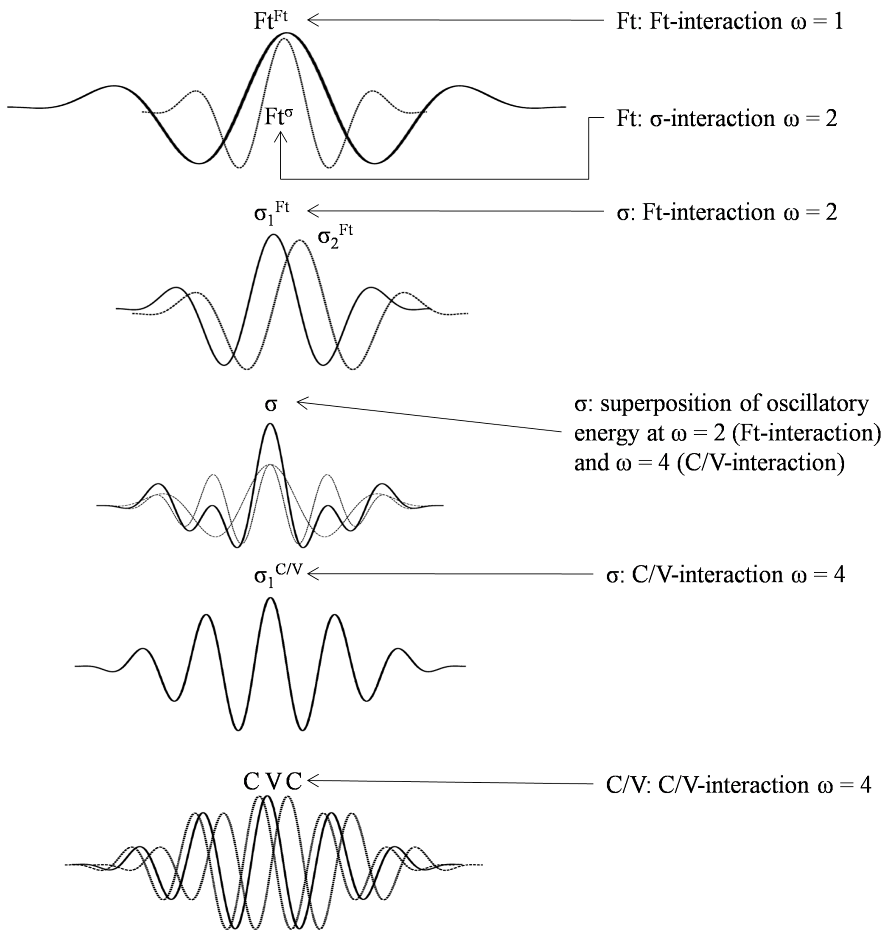


Figure 9. Illustration of how the principle of harmonicity allows for phase coupling across multiple timescales.

A final consideration with regard to harmonicity is that, for any given system, the non-uniformity of the distribution of energy into various harmonics is a dynamical asymmetry. It is speculated that similarity of typehood between two systems requires that both systems exhibit similar energy distribution spectra. In other words, C and V are similar because most of their energy is distributed in relatively high frequencies. Likewise, N and Verb exhibit similar energy spectra. In contrast, C/V and σ are dissimilar because the syllable has a fair amount of energy distributed in both segmental and foot timescales. Thus harmonicity asymmetries are a basis for parametric variation of system types. The most symmetric system in this regard would distribute energy evenly across all harmonics and subharmonics. The way in which spectral energy symmetry is broken (i.e. the shape of the energy spectrum for a given system) is indicative of typehood.

Lastly, note that although the Foot was chosen above as a "fundamental frequency" in Figure 9, there exists no system-wide fundamental frequency. Instead, there are various systems with various frequencies, some of which are more or less important to various coupling interactions than others. The choice of one frequency as a reference is therefore arbitrary, and caution should be taken to avoid the idea that there is a system-wide "clock" which organizes all oscillatory activity. Rather, there are many clocks, which self-organize through frequency-locking interactions, and which thereby give one the impression that a quasi-periodic global clock is operative. The quasi-periodicity of the system (i.e. variation in the apparent global frequency) arises because all systems exhibit variation in frequency and amplitude due to informational, pragmatic, physiological, and memory-related factors. Hence speech is rarely isochronously rhythmic.

2.3 Amplitude coupling and modulation of φ-coupling

A very powerful complication of the model must be incorporated before we can apply it to various case studies. Whereas Eq. (1) posited constant radial amplitude, we will find it broadly useful to adopt a dynamics of radial amplitude. Moreover, this allows for two useful concepts: amplitude coupling (r-coupling) between systems, and amplitude modulation of φ-coupling. The amplitude dynamics defined in Eq. (6) show that the change in radial amplitude is the negative of the derivative of a potential function, which is in turn defined in Eq. (7). The minimum of the radial amplitude potential is $(|k_1|/k_2)^{1/2}$. This value defines a target radial amplitude for a given system. However, the actual radial amplitude is influenced by r-coupling to other systems. A very simple coupling function, shown in Eq. (9), is sufficient for modeling many important patterns. Note that the parameter matrix χ determines the strength of amplitude coupling. As with α and *ω*, theoretical and descriptive mileage can be gained through asymmetries in this matrix.

$$\dot{\theta}_i = \omega_i + \sum_j F(\phi_{ij}) \tag{6}$$

$$\dot{r}_i = \frac{-dA(r_i)}{dr} + \sum_j (\chi_{ij} C_{ij}) + \eta_{r_i}$$

$$\mathrm{A}(\mathrm{r}_i) = \frac{k_{1i}}{2} r_i^2 + \frac{k_{2i}}{4} r_i^4 \tag{7}$$

$$\mathrm{B}(\mathrm{r}_i, r_j) = \left(\frac{r_i}{r_j}\right)^2 \tag{8}$$

$$\mathrm{C}(\mathrm{r}_{ij}) = r_i^2 \tag{9}$$

$$\mathrm{F}(\phi_{ij}) = \sum_j \left(1 + \beta_{ij} B(r_i, r_j)\right) \alpha_{ij} \frac{-dV(\phi_{ij})}{d\phi} \tag{10}$$

The β parameter and B function in Eq. (10), which serve to modulate the strength of φ-coupling, are of great importance. β_{ij} describes the extent to which the relative amplitude of systems *i* and *j* modulates the φ-coupling force exerted by *i* on system *j*. Relative amplitude is defined with the function B in Eq. (8). When β = 0, no amplitude modulation occurs. However, when β > 0, φ-coupling forces are influenced by the amplitudes of the systems involved. The consequence of this is that changes in relative phase patterns (φ-transitions) can occur without any changes in α. In other words, the relative phase between three systems depends not only upon the strength of φ-coupling forces between them, but also their relative amplitudes. It follows that a change in the amplitude of one wave can produce qualitative changes in the dynamics of an entire system. We will see how this works with regard to specific example in the case studies below. For now, we present this idea schematically in Figure 10.

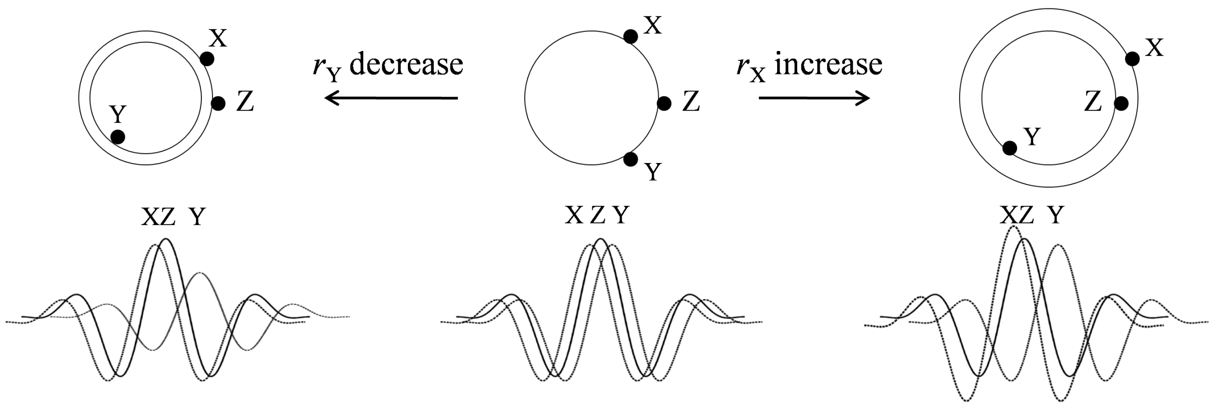


Figure 10. schematic illustration of the effect of amplitude coupling on relative phase. Systems X and Y are attractively coupled to Z and repulsively coupled to one another.

The figure depicts how, given amplitude modulation of phase coupling ($\beta > 0$), changes in relative amplitude can influence relative phase. The phase circle in the middle shows the *r*-symmetric situation, where $r_X = r_Y$, $\alpha_{XY} = \alpha_{YX,}$ and $\alpha_{XZ} = \alpha_{ZX} = \alpha_{YZ} = \alpha_{ZY}$, the result being that X and Y are equally displaced from Z—their phases are mirror reflections across the x-axis of the phase circle. There are two ways in which this symmetry can be broken via β-modulation: either the amplitude of Y relative to X decreases (left), or the amplitude of X relative to Y increases (right). These changes modulate the net coupling strengths (cf. Eq. 10), such that X becomes more strongly attractively coupled to Z than Y is. In other words, the relative phase symmetry can be broken by breaking amplitude symmetry. In many cases, relative phase asymmetries can be altered by changing amplitude asymmetries. We will see below that amplitude modulation of phase coupling strength has much explanatory utility when it comes to linguistic patterns.

3. Applications of the wave theory

3.1 Segment-syllable dynamics

Here we present a partial cross-linguistic typology of segment-syllable coupling patterns. The segment can be conceptualized as a system of gestures, but here we will not concern ourselves with the dynamics of specific gestures associated with a given segment. A "typology" from the dynamical perspective involves a classification of possible qualitatively different stable patterns of relative phase. This enterprise is constrained by the assumption that we can analyze in isolation a single σ-system to which *n* segmental systems are coupled. This is undoubtedly an over-simplification, as segmental/gestural systems belonging to neighboring syllables often interact non-trivially in spontaneous speech. The most relevant free parameters here are the number of segmental systems and the strength and type of φ-coupling (magnitude and sign of α, respectively). Other parameters such as β and χ play an important role in understanding how stress is phased relative to syllables (cf. section 3.2), but to describe hierarchical structural patterns within the syllable, they do not appear necessary.

One of the more interesting questions that arises in this typology is whether the distinction between C and V need be included separately in the parameterization of the system. There has been a wide range of thinking on the question of how to understand the apparent division of sounds into two types. One of the more obvious challenges for this distinction is the relation between glides and their vocalic counterparts (e.g. [w] and [u], [j] and [i]), which in many feature theories have been distinguished solely by the feature ± consonantal, or something comparable to that. This featural distinction could predict that different types of gestures are involved, but a viable alternative is that there is only one type of gesture, and what makes it vowel-like is the strength with which it is φ-coupled to the syllable. For current purposes, we will reconceptualize consonants as systems which are relatively weakly phase coupled to a σ, while vowel segments are more strongly coupled to a σ. Exactly which direction these force differences are manifested in is an open question. It seem reasonable to speculate that V segments exert stronger forces on σ systems than C segments do, but whether the coupling forces exerted by σ systems on V and C differ is less clear.

A related issue is how to conceptualize the role played by the syllable. The syllable, by virtue of its attractive φ-coupling, pulls all segments together. It is possible to conceptualize the syllable as an independent system, and also as something that arises from the activity of segmental systems. Section 4 considers this issue in the context of the neural basis of the theory. Regardless, the "structure" of the syllable results from the interplay between these attractive forces and the repulsive forces between segments. Syllable structure can thus be understood as a consequence of the principle of like interaction. Figure 11 depicts schematic phase-circle representations of a partial typology of syllable structures.

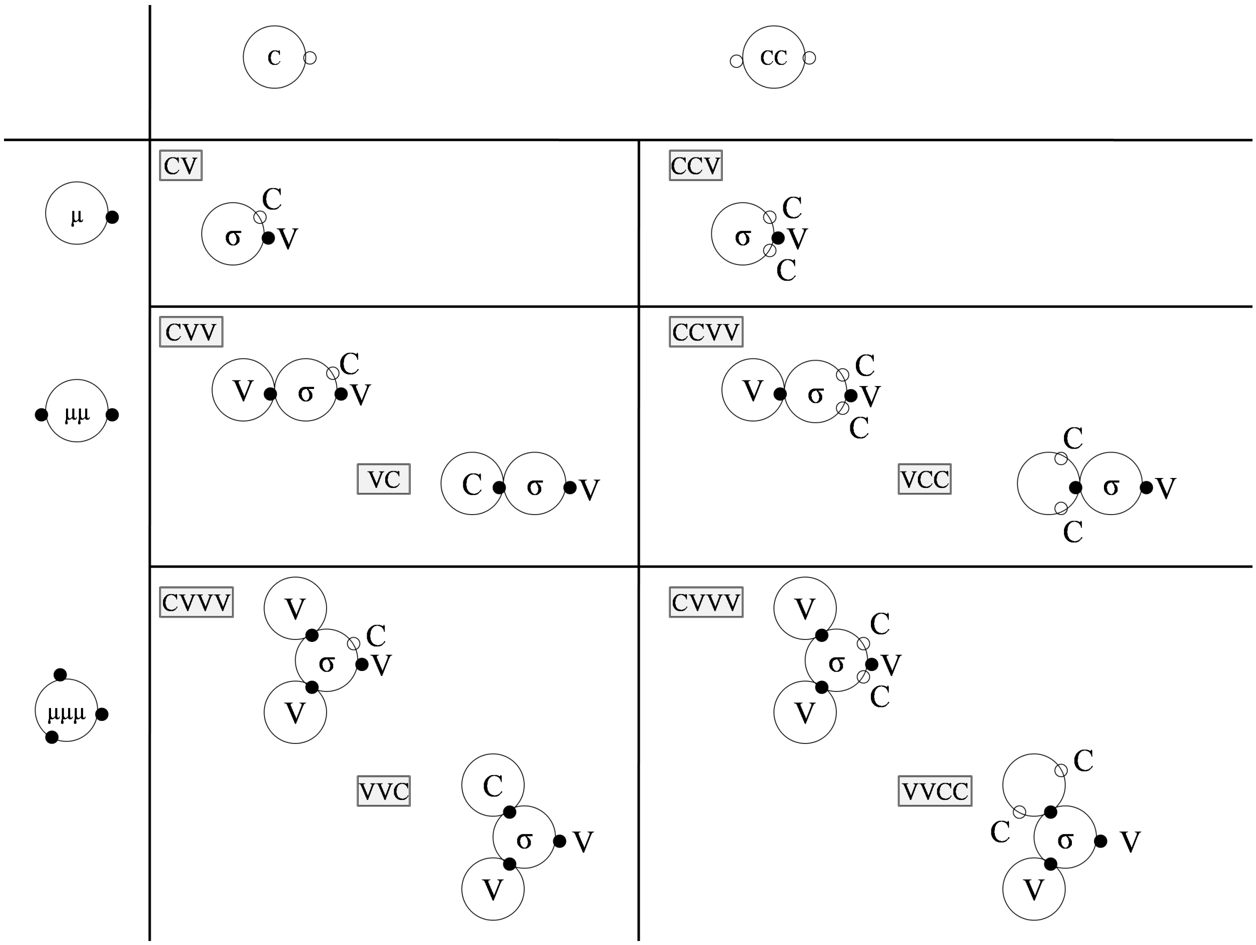


Figure 11. Partial typology of syllables, organized by number of strongly repulsive systems (rows) and number of consonantal systems (columns).

The typology is organized in three dimensions. The two primary dimensions constitute rows and columns in Figure 11. Each row indicates the number of strongly repulsive systems in a syllable. It is speculated that the structural unit of the mora is precisely that: a system which exerts relatively strong repulsive forces on other systems in the syllable. We will henceforth use the term "mora" in this new, wave-theoretic sense. In a one-mora syllable, there is no relatively strong pair of repulsive forces between segments. In a two-mora syllable, there are two moraic systems which exert strong repulsive forces upon one another, the result being waves which stabilize to an anti-phase relation. The repulsive coupling between the systems overrides their attractive coupling to the syllable, and hence the repulsive forces predominate over the attractive ones.

Each column in Figure 11 indicates a number of consonantal systems, which, as explained above, are relatively weakly φ-coupled to σ systems. Conversely, vowel systems are relatively strongly φ-coupled to σ systems. The third dimension of the typology, represented on the diagonals in each cell, indicates whether the consonants are moraic or not. It should be clarified that in the VCC and VVCC structures represented above, CC meta-systems function in tandem as a single repulsive system. It is also possible for each C to constitute its own repulsive systems.

The new sense of mora, as a strongly φ-repulsive segmental system, does not distinguish between segments which influence or do not influence stress assignment. We will consider this issue in section 3.2. A more difficult issue is the question of why no (or very few) languages exhibit onset segments with strong repulsion to vowels. One possibility is that there are nonlinear interactions during the transient phase prior to segmental system stabilization, which break time-reversal symmetry in just that way between weakly repulsive systems (like onset consonants) and moraic ones (like vowels). Exactly how this symmetry breaking occurs is not currently treated by the theory.

3.2 Syllable-stress dynamics

In this section we consider the location of primary and secondary stress in English words of 2, 3, and 4 syllables. There is no generally accepted theory of stress patterns in English, and for any set of rules or constraints that one might formulate, there are numerous exceptions. It is instructive to consider the basics of the influential metrical grid-based approach described in Selkirk (1984), which has provided a basis for more recent hybrid approaches integrating constituency with a metrical grid. In what follows, we first describe how the grid approach works, then present a wave-theoretic interpretation of stress patterns.

In Selkirk (1984) and the vast majority of hybrid theories, it is posited that stress is assigned to English content words from "right-to-left", which is an orthographic metaphor for a preference to assign primary stress to syllables later in the word. It is also posited (albeit in somewhat different terms), that this right-to-left assignment is on the final syllable if heavy, or on the penultimate if the final is not heavy. Secondary stress is assigned to heavy syllables and the word-initial syllable, but rules of destressing, sometimes conditioned by preference for alternating stress patterns, often apply to obscure this. In addition, there are two lexically-specified parameters regarding the final syllable that influence primary and secondary stress assignment: final-C extraprosodicity and final-σ extrametricality. Both of these are ways of allowing stress assignment rules to ignore the final syllable.

To see how these rules play out, let us first consider disyllabic words, which allow for four possible stress patterns. Table 1 subclassifies these patterns and provides examples. Note that the table uses $^1\sigma$ to indicate primary stress, $^2\sigma$ to indicate secondary stress, and $^0\sigma$ to indicate no stress. We will refer to classes of stress patterns as 10 ("one-zero"), 12 ("one-two"), 01, and 02, etc. We will refer to subclasses as 10.a, 10.b, etc. In general, tense vowels are considered to be long (VV), and lax vowels short (V), except word-finally, where tense vowels /i/ and /o/ function as a single V with respect to stress assignment.

Class 10.a adheres to the penultimate stress rule with no need for special treatment of the final syllable because it is not heavy. Class 10.b consists of words with final closed syllables. Since these syllables are heavy, they should receive stress. Because they do not, a mechanism is necessary to allow the stress rules to ignore them. This mechanism can be either extraprosodicity or extrametricality, and in fact, there is no way of distinguishing between the two here. Extraprosodicity renders the final syllable light, causing stress to be assigned to the penult; extrametricality prevents primary or secondary stress from being assigned to the final syllable, in which case the only other target for stress is the penult (here, the initial syllable). In contrast, class 10.c requires extrametricality. Whether or not the final C of VCC is extraprosodic, the syllable is heavy and therefore receives secondary stress. Extrametricality prevents it from being assigned primary stress. In addition, a destressing rule is necessary to remove the secondary stress. Note that the weight of the initial syllable is irrelevant to these considerations, and that there are no 10 patterns with a VV(C) final syllable, since extraprosodicity applies only to consonantal segments.

Classes 12.a-c all retain secondary stress on the final syllable, but do not have primary stress. This is explained by extrametricality: the final syllable is invisible to primary stress assignment. These words differ from those in class 10.c in that no destressing applies, and thus secondary stress is retained. There are some odd gaps in the paradigm here that did not receive mention in Selkirk (1984), or to my knowledge, in any subsequent work. It is curious that the absence of destressing of secondary stress in 12.a-b is associated with a heavy penult. In other words, there are no 12 patterns where the initial syllable is light and the final is VC- or VCC-heavy. There are such patterns when the final is VV(C) heavy (e.g. satire, ally$_N$, mustang). The analysis does not account for this.

Classes 01.a-c involve no extrametricality, and therefore the final heavy syllable receives primary stress. When the initial syllable is heavy, there are two possibilities: either it is destressed, or retains secondary stress. What is noteworthy is that many words with a heavy-heavy pattern can be observed with both destressed and non-destressed variants (e.g. begin, reward, police). This suggests that destressing is not an entirely categorial phenomenon, but rather a matter of gradient parameterization. Classes 21.b-d are parallel to 01.a-c, the difference being that destressing is less common in the former than the latter, but nonetheless variable. Class 21.a is accounted for by the non-application of destressing to the secondary stress assigned to word-initial syllables.

One interesting difference between classes 10/12 and 01/21 is that extrametricality and extraprosodicity only influence the assignment of stress in classes 10/12. In the latter, a destressing rule is needed to account for whether an extrametrical VV(C) receives secondary stress on the surface. However, destressing in unnecessary for extraprosodic VC because this syllable is effectively light and never receives stress. In contrast, for classes 01/21, destressing is necessary to account for secondary stress patterns on initial syllable VX. This is a somewhat awkward situation because the explanatory mechanisms differ so drastically between initial and final syllables: extraprosodicity/extrametricality only apply to final syllables. Destressing applies only to VCC in final syllables, but applies to VX(X) in initial syllables. It is also dissatisfying that there is no deeper explanation for the propensity of heavy syllables to sometimes attract stress and sometimes fail to do so via extrametricality/extraprosodicity.

Table 1. Disyllabic stress patterns

segmental composition	C# ep	σ# em	stress pattern		examples
			$^1\sigma\ ^0\sigma$	**10**	
μ(μ).V				10.a	llama, papa, happy, narrow, motto; data, tuba, seedy
μ(μ).VC	~	~		10.b	common, licit, edit, granite, wallop, Philip, stirrup, syrup, stomach, buttock, Eric, cherub, sheriff, radish, relish, lavish, olive, column, hiccup, gossip, gallop, credit, fidget, vomit, visit, promise, solace, vanish, finish, manage, frolic; hymnal, apron, worship, tulip, handsome, vulgar, solid, balsam, jetsam, harem, sermon, cipher, manner, sphincter, brothel, pilot, carpet, druid, porpoise, zenith, surface, quiet, practice, trespass, canvas, furnish, varnish
μ(μ).VCC	(y)	y		10.c	effort, mollusk, bastard, spinach, monarch, orange, lozenge, honest, modest, modern, haggard, stubborn, challenge, scavenge, warrant, balance, product, annex, affix; tempest, perfect, standard, comfort, culvert, expert, orchard, coward, serpent, giant, moment, gymnast, lantern, Gilbert, cistern, forceps, forward, awkward, earnest,
μ(μ).VV(C)					
			$^1\sigma\ ^2\sigma$	**12**	
μ.VC(C)					
μμ.VC		y		12.a	parsnip, combat, ransack, shindig, nabob, Ahab, nimrod, gonad, monad, humbug, Lakoff, Smirnoff, Aztec, burlap, chaos, mohawk, mayhem, Igor, Oshkosh, kayak, shamrock, kidnap, boycott, highjack, bushwhack
μμ.VCC		y		12.b	convict, anthrax, kiosk, podunk, insect, cobalt, Ozark, Cyclops, Mozart, bombast, acorn, Bogart, sandwich, precinct,
μ(μ).VV(C)		y		12.c	satire, ally, Semite, Hittite, mustang; rotate, quinine, argyle, archive, gargoyle, gentile, profile, senile, gangrene, mangrove, protein, tirade, Moscow, alpine, turbine, bovine, feline, canine, rabbi, bromide, umpire, oxide,
			$^0\sigma\ ^1\sigma$	**01**	
μ(μ).VC			**($^2\sigma\ ^1\sigma$)**	01.a	bizarre, saccade, cigar, attack, Madrid, Quebec, pecan, abyss, gazelle, guitar, Chinook, cadet, garage, equip, maraud, abet, abut, acquit, succumb, appall, caress, harass, amass, attack; begin, regret, Berlin, permit, omit, forget, combat, rebel, excel, possess, surpass, emboss, renege.
μ(μ).VCC			**($^2\sigma\ ^1\sigma$)**	01.b	adept, gestalt, event, corrupt, immense, succinct, direct, lament, accost, arrest, desert, assert, depart, reward, accord, award, record, adorn, dispense, finance; exist, overt, robust, overt, occult, absurd, result, romance$_v$, molest, exert, reward, bombard, return, incense$_v$, condense, enhance, advance, convince
μ(μ).VV(C)			**($^2\sigma\ ^1\sigma$)**	01.c	divine, atone, discreet, attire, lagoon, ravine, immune, cocoon, saloon, buffoon, maroon, balloon, July, surprise, delight, divide, sublime, prestige, cajole; police, esteem, obscene, remote, advice, forlorn,
			$^2\sigma\ ^1\sigma$	**21**	
μ.VC					
μ.VV(C)			**($^0\sigma\ ^1\sigma$)**	21.a	raccoon, bassoon, trapeze, caffeine,
μμ.VC			**($^0\sigma\ ^1\sigma$)**	21.b	combat, montage
μμ.VCC			**($^0\sigma\ ^1\sigma$)**	21.c	torment, robust, usurp
μμ.VV(C)			**($^0\sigma\ ^1\sigma$)**	21.d	boutique, shampoo, obey, canteen, crusade, bamboo, domain, cocaine, champagne, lampoon, pontoon, harpoon, Chinese, Maltese, Burmese, rely, defy

A wave theory account of the above stress patterns has the advantage that it unifies our understanding of weight-based stress assignment with extrametricality, extraprosodicity and destressing. Figure 12 (a) shows a 10 pattern arising from a V.V word. The asymmetry that gives rise to this pattern is that the initial syllable is relatively more strongly φ-coupled to the primary stress (λ), which we view as a harmonic manifestation of the Ft. This asymmetry is bidirectional, such that $\alpha_{\sigma1\lambda} = \alpha_{\lambda\sigma1} > \alpha_{\sigma2\lambda} = \alpha_{\lambda\sigma2}$. Note that the syllable amplitudes and the gestural amplitudes in this simulation are equal between the initial and final syllable. Figure 12 (b) shows what happens when an additional gesture is added to the second syllable. Via amplitude coupling between the gestures and their respective syllables (χ), the final syllable is imbued with additional amplitude (observe the asymmetry in the activation waves). The additional amplitude of the final syllable in turn augments the net phase coupling, via the amplitude modulation of phase coupling (β). Note that this makes use of the β-modulation mechanism that was described schematically in section 2.3. There was no difference in parameterization between (a) and (b), other than the addition of a second segment which is phase- and amplitude-coupled to the final syllable.

Furthermore, when the final segment is rendered "extraprosodic" by diminishing its amplitude coupling to the syllable, this results in a 10 pattern. This can be seen in the contrast between Figure 12 (b) and (c). In (c) the amplitude of the final syllable is not so strongly augmented by amplitude coupling from g_3 (the final segment/gestures), and therefore β modulation does not lead to a situation in which the attractive forces between σ_2 and λ outweigh those between σ_1 and λ. In this way, amplitude coupling is responsible both for weight-based attraction of primary stress and for extraprosodicity/extrametricality-driven failure to attract primary stress.

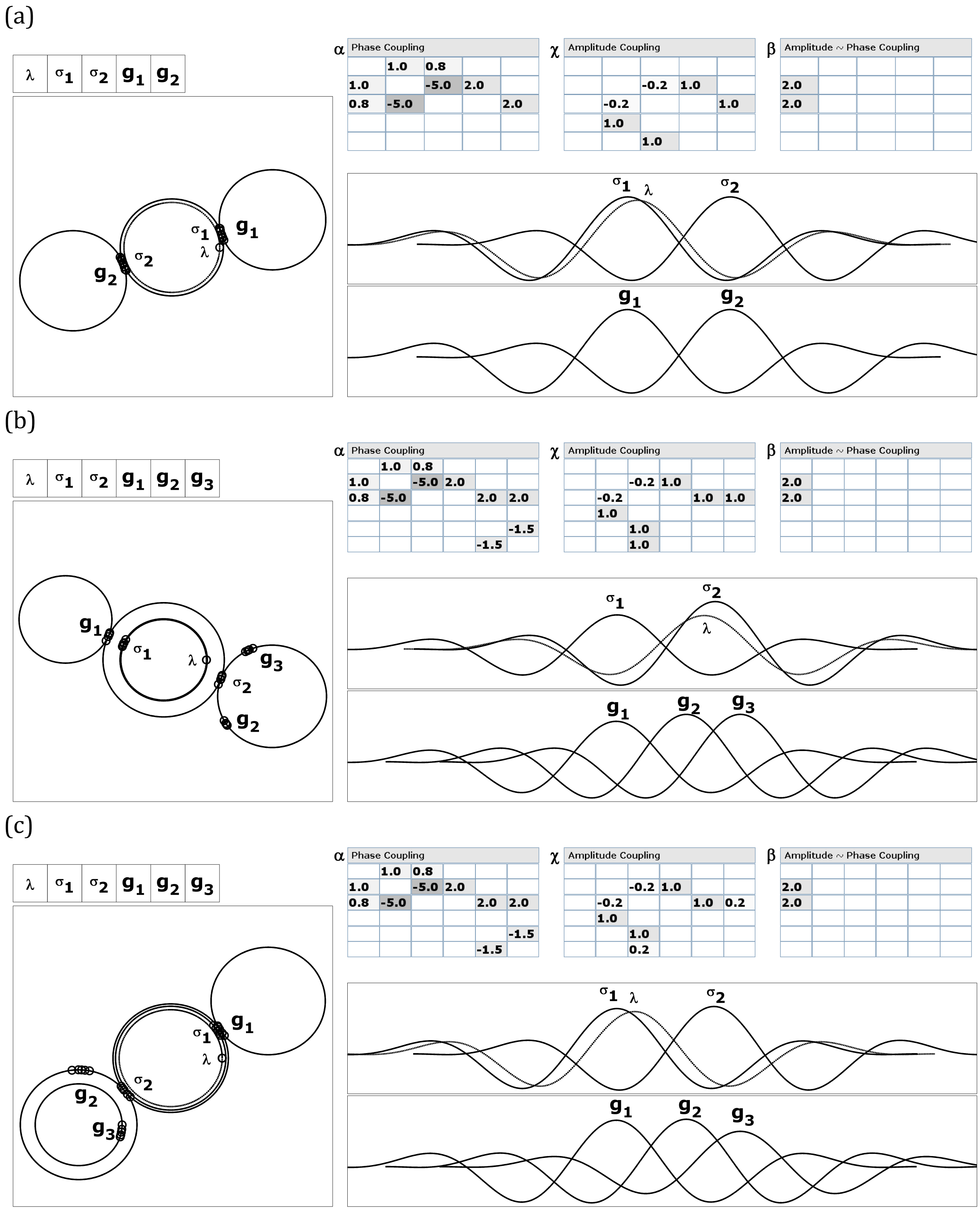


Figure 12. Wave model simulations of how gestural/segmental amplitude can influence two-syllable stress patterns.

r-coupling and β-modulation thus coherently account for how 10 and 01 patterns do or do not arise. With regard to the secondary stress in 12 and 21 patterns, we will depart from the conventional view of secondary stress as a categorial phenomenon, and model it with gradient r-coupling. In that case, lexical and contextual variation in χ can be used to model both destressing and extraprosodicity. In other words, when segments are only weakly r-coupled to their syllables (i.e. low χ values), those syllables do not receive additional amplitude and thus do not attract primary stress. The secondary stress that they may or may not receive is attributable to gradient variation in gestural amplitude. This gradient variation is both lexically determined and contextually influenced—note how the syllables in the word are negatively r-coupled. The effect of this is that amplitude augmentation in one syllable effects diminution in the other. If para-lexical influences can also modulate gestural and syllabic amplitudes, this provides another source of "destressing" effects.

In sum, two-syllable stress patterns can be coherently understood in the wave theory framework, using two parametric asymmetries. First, earlier syllables are more strongly phase-coupled to stress than later ones. This seemingly flies in the face of the generally accepted R-to-L assignment of stress, but from the wave perspective it leads to simplifications. Second, via r-coupling and β-modulation of φ-coupling, both weight-based stress attraction and destressing/extrametricality can be understood as arising from relatively strong and weak χ values, respectively. In addition, other factors can diminish σ amplitude and cause destressing. What makes this view vastly more coherent than the alternative is that both primary and secondary stress patterns are attributable to syllable wave amplitude, which in turn can be influenced by a number of factors.

Now we extend this analysis to trisyllabic stress patterns. As before, we begin by describing a metrical grid approach. Selkirk (1984) attributes 100 patterns to final syllable extrametricality (and extraprosodicity, if the final σ is heavy). Hence the words in classes 100.a-c exhibit antepenultimate stress because their final syllables are invisible to primary stress assignment. The final syllables in class 100.c are assigned secondary stress due to their weight (they are heavy despite final-C extraprosodicity), but destressing renders them stressless. It is noteworthy, though, that many of these words admit a 102 pattern as well; this suggests, as before, that rather than being a categorial phenomenon, secondary stress should be understood with gradient parameterization.

Class 102 likewise exhibits final-σ extrametricality, but not extraprosodicity. Hence primary stress is assigned to the antepenult, but secondary stress is retained on the final syllable and no destressing applies. Classes 100.d and 102.c violate the rules described above, since the penults are heavy and thus should receive primary stress. It is unclear how this is resolved in Selkirk (1984), although Ross (1972) uses a stress shift rule and subsequent destressing of the penult.

By comparing class 010.a patterns to class 100.a patterns, one can see very clearly the effect of extrametricality. Class 010.a involves no extrametricality, and thus primary stress is assigned to the penult. Destressing of initial syllable secondary stress is necessary in this case. Class 010.b final-VC requires extraprosodicity, and final-VCC require extrametricality and destressing. The final-VCC are problematic because their extrametrical status makes the antepenult the target of the main stress rule. This can be avoided if one posits that final /nt/ (lieutenant) and /ns/ (resistance) can be extraprosodic sequences. Class 010.c differs from 010.b in that the penult is heavy, and thus the dilemma in 010.b does not arise. Many class 010.c words exhibit variation between 010 and 012 or 210 patterns. This follows fairly readily from the gradience of secondary stress. Class 210 has been included separately because some 210 words seem more resistant to destressing.

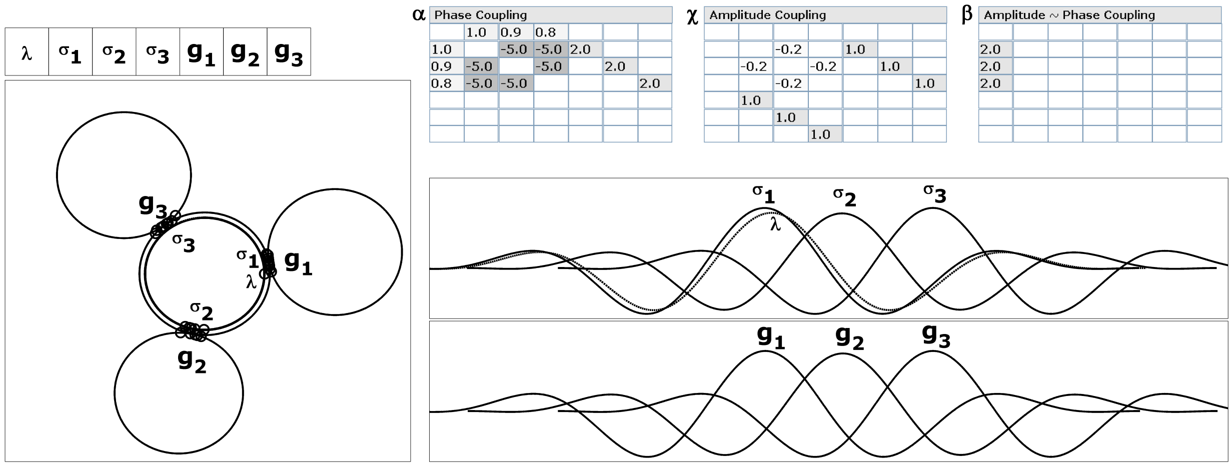
Finally, class 201 exhibits primary stress on a final heavy syllable. An important thing to notice about the words in this class is that non-heavy initial syllables have secondary stress, and this secondary stress is somewhat different from the other instances of secondary stress that we have seen. Secondary stress in 12, 21, 210, 210, and even 102, generally arise because of syllable weight and exhibit some potential for de-emphasis (class 21.a are exceptional in this regard, note that all involve the vowel [æ]). In contrast, the secondary stress in 201 patterns seems to have a much more restricted potential for this variable destressing behavior. Moreover, it does not require a heavy initial syllable, unlike the other ones. Here the secondary stress is attributed not to syllable weight, but to a tendency for rhythmic alternation of prominence.

It is important in this light that class 201 words and class 21 words often change to what have been described as 102 and 12 patterns, when followed by a primary stress (the exact conditions of application are more complicated and variable)—this is the so called "rhythm rule", which is observed in English and some other languages. Phonetic studies of this phenomenon suggest that rather than a "shift" of primary stress, the rhythm rule involves a de-emphasis of the primary stress, resulting in a change of relative prominence within the word.

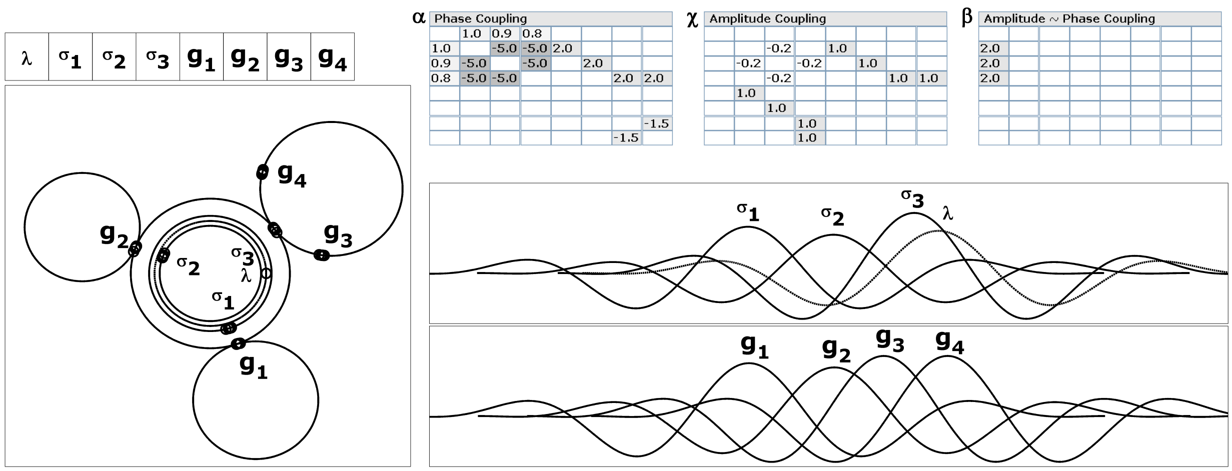
Table 2. Trisyllabic stress patterns

segmental composition	C# ep	σ# em	stress pattern	examples
			$^{1}\sigma\ ^{0}\sigma\ ^{0}\sigma$	
μ.μ.V		y	100.a	Pamela, travesty, tapestry
μ(μ).μ.VC	y	y	100.b	primitive, syllabus, venison, vinegar, jettison, personal, libelous, vigilant, modicum, opium, cinnamon, garrison, denizen, integer, Oliver, capitol, arsenal, funeral, chariot, idiot, abacus, genesis, period, cannibal, limerick, maverick, burial, banister,
μ(μ).μ.VCC	y	y	100.c ($^{1}\sigma\ ^{0}\sigma\ ^{2}\sigma$)	catalyst, president, mendicant, elephant, covenant, element, Everest, gradient, lubricant,
μ(μ).μ(μ).V(X)	y	y	100.d	burgundy, anchovy, document, liturgy, allergy, lethargy, Haggerty, calendar, carpenter, harbinger, messenger,
			$^{1}\sigma\ ^{0}\sigma\ ^{2}\sigma$	
μ(μ).μ.VC(C)		y	102.a	daffodil, parallel, monograph, Amazon, tomahawk, intersect, poetess, analyst, handicap, shishkabob, gallivant, manifest, Ichabod, scalawag, chugalug, Romanoff, Molotov, albatross, orgasm, cataract, catapult, sycophant, Pentecost, unicorn, Abraham, caravan, marathon, metaphor, alcohol, sassafras
μ(μ).μ.VX(C)		y	102.b	criticize, execute, albatross, eulogize, legalize, positron, hurricane, crocodile, dynamite, diocese, artichoke, galvanize, magnify, asinine, bellicose, infantile, devastate, magistrate, barricade, samurai, alkali, alibi, diatribe, sacrifice, chamomile, crocodile, juvenile, paradise, appetite, satellite,
μ(μ).μμ.VX(C)		y	102.c	anecdote, katydid. pollywog, Jackendoff, Palestine, turpentine, philistine, iodine, valentine, columbine, concubine, porcupine, reconcile, enterprise, merchandise, supervise, Algernon, ampersand, Macintosh, Hackensack, balderdash, Vanderbilt, Davenport, meteor, handicap, lollypop,
			$^{0}\sigma\ ^{1}\sigma\ ^{0}\sigma$	
μ.μ(μ).V			010.a	vanilla, banana, spaghetti, Ferrari, pastrami, salami, safari, jalopy, tamale; marina, Electra, aroma, Alaska, bologna, attorney, adobe, Mahoney
μ(μ).μ.VC(C)	y	(y)	010.b	develop, illicit, phlogiston, meniscus, lieutenant, resistance, Ramirez, condition, rebellion, transferal, inhabit, solicit, deposit, embarrass, diminish, abolish, disparage, establish, astonish, imagine, examine, consider, endeavor, semester, disaster, cadaver,
μ(μ).μμ.VX(C)	(y)~	(y)	010.c ($^{0}\sigma\ ^{1}\sigma\ ^{2}\sigma$) ($^{2}\sigma\ ^{1}\sigma\ ^{0}\sigma$)	amalgam, decorum, utensil, betrayal, Wisconsin, abandon, horizon, Uranus, September, colloidal, desirous, defiant, placental, portentous, observant, opponent, goliath, papyrus, Fernandez, behemoth, contrivance, decorum, addendum, defendant, opponent, assailant, bamboozle, remember, cathedral, Wyoming, maneuver; molluscoid, apartment, stalactite, aloha, Achilles, Ulysses, Penobscot,
			$^{2}\sigma\ ^{1}\sigma\ ^{0}\sigma$	
μμ.μ(μ).V(C)				bandana, Montana, Kentucky, Milwaukee, Sandusky, Daytona, epoxy; spumoni, Lombardi, zucchini, chianti, martini, coyote, October, fandango
μμ.μ.VC	y			Nantucket, citation, factitious, clandestine, Pawtucket, skedaddle,
			$^{2}\sigma\ ^{0}\sigma\ ^{1}\sigma$	
μ(μ).μ.VC				majorette, arabesque
μ(μ).μ.VV(C)				Tennessee, buccaneer, Japanese, Portuguese, Javanese, journalese, Siamese, Mozambique, brigadoon, Listerine, Illinois, macaroon, pantaloon, Cameroon,
μ(μ).μμ.VX				acquiesce, guarantee, ascertain, apropos, debonair

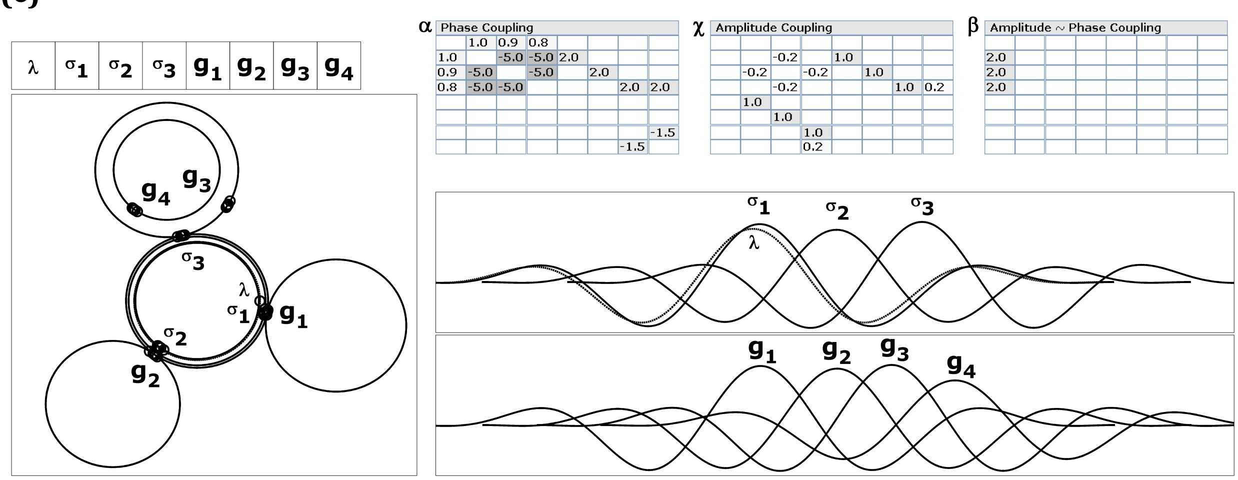
(a)


λ σ1 σ2 σ3 g1 g2 g3
α Phase Coupling
1.0 0.9 0.8
1.0 -5.0 -5.0 2.0
0.9 -5.0 -5.0 2.0
0.8 -5.0 -5.0 2.0
χ Amplitude Coupling
-0.2 1.0
-0.2 -0.2 1.0
-0.2 1.0
1.0
1.0
1.0
β Amplitude ~ Phase Coupling
2.0
2.0
2.0
g3 σ3 σ1 g1 λ σ2 g2
σ1 σ2 σ3 λ
g1 g2 g3

(b)


λ σ1 σ2 σ3 g1 g2 g3 g4
α Phase Coupling
1.0 0.9 0.8
1.0 -5.0 -5.0 2.0
0.9 -5.0 -5.0 2.0
0.8 -5.0 -5.0 2.0 2.0
-1.5
-1.5
χ Amplitude Coupling
-0.2 1.0
-0.2 -0.2 1.0
-0.2 1.0 1.0
1.0
1.0
1.0
1.0
β Amplitude ~ Phase Coupling
2.0
2.0
2.0
g4 g2 g3 σ2 σ3 λ σ1 g1
σ1 σ2 σ3 λ
g1 g2 g3 g4

(c)


λ σ1 σ2 σ3 g1 g2 g3 g4
α Phase Coupling
1.0 0.9 0.8
1.0 -5.0 -5.0 2.0
0.9 -5.0 -5.0 2.0
0.8 -5.0 -5.0 2.0 2.0
-1.5
-1.5
χ Amplitude Coupling
-0.2 1.0
-0.2 -0.2 1.0
-0.2 1.0 0.2
1.0
1.0
1.0
0.2
β Amplitude ~ Phase Coupling
2.0
2.0
2.0
g4 g3 σ3 λ σ1 g1 σ2 g2
σ1 σ2 σ3 λ
g1 g2 g3 g4

(d)

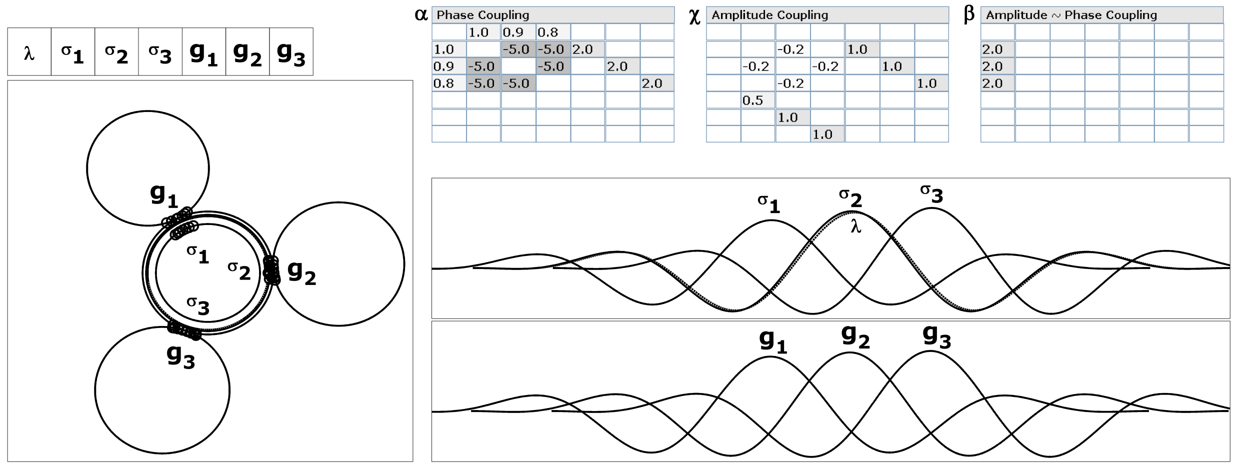


Figure 13. Wave model simulations of several trisyllabic stress assignment patterns. (a) 100 pattern; (b) 201 pattern; (c) 102 pattern; (d) 010 pattern.

The wave theory accounts for trisyllabic stress patterns using the same dynamical mechanisms of r-coupling and β modulation that were used for disyllabic words. In all cases, a parametric asymmetry is broken in α by positing that earlier syllables are more strongly φ-coupled to stress. This results in class 100, the pattern in Figure 13(a), where stress is phased closely to the initial syllable. A further breaking of symmetry is shown in Figure 13(b), where additional segments in the final syllable increase its amplitude. This induces through β modulation forces the attraction of stress to the final syllable, and results in the 201 pattern. The secondary stress, in this view, arises because of inhibitory amplitude coupling between adjacent syllables. The amplitude of the medial syllable and its associated segments is diminished by inhibitory r-coupling ($\chi < 0$) to the initial and final syllable. This leads to a situation in which relatively weak and strong prominence alternates between syllables. Thus rhythmic alternation is understood not as constraint imposed upon a pattern externally, but rather, emerges due to interactions between systems.

If the additional segments in the final syllable are only weakly r-coupled to it, then the pattern of extrametricality/extraprosodicity arises, resulting in 100 or 102 patterns. Figure 13(c) shows a dynamical simulation of this. In most cases, the difference between 100 and 102 admits substantial variation in spontaneous speech. It is also the case that 102 patterns can be seen as a mirror reflection of 201, where inhibitory interactions between neighboring syllables result in alternating relative prominence.

The 010 and 210 patterns arise when the amplitude of the penult is relatively greater than the amplitude of the initial syllable. This is the initial-syllable analog of extrametricality, where initial syllable segments/gestures are relatively weakly r-coupled to the initial syllable. Figure 13(d) shows a wave model simulation of a 010 pattern. The 210 patterns seem somewhat anomalous in this regard, since it must be stipulated that the diminution of r-coupling between the initial syllable and its associated gestures is large enough to allow for rephasing of primary stress, but not so large as to erase the perception of secondary stress prominence. There is an alternative account of 201 patterns, and some

21 patterns, which involves the presence of two foot systems. With two foot systems, there are two λ waves. This is undoubtedly necessary to understand most quadrisyllabic stress patterns, which we consider next.

Table 3 shows several stress patterns involving four syllable words. With the exception of class 0100, it is useful to posit that there are two foot systems involved in these words, and thus two λ/Ft waves that, like syllable waves, are repulsively φ-coupled and inhibitorily r-coupled. The repulsive coupling follows from the principle of like interaction. It accounts for class 2010, class 1020, and class 1002 patterns, with the additional stipulation that one λ wave obtains higher amplitude than the other, due to stronger r-coupling to a word system. It is important to keep in mind the λ wave, which we previously identified with primary stress, is the ω_σ harmonic of a Foot wave. In that case, primary stress is not a system in and of itself, it is the label given to the relatively greatest prominence in the word. This, in turn, is determined by asymmetries in r-coupling and φ-coupling parameters.

Table 3. Quadrisyllabic stress patterns

segmental composition	C# ep	σ# em	stress pattern	examples
			$^0\sigma\,^1\sigma\,^0\sigma\,^0\sigma$	
μ.μ.μ.μ				America
μ(μ).μ.μ.VC(C)		y		Connecticut, inheritance, American, aluminum, Elizabeth, servility, humility
μ(μ).μμ.μμ.VC			$(^2\sigma\,^1\sigma\,^0\sigma\,^0\sigma)$	Napoleon, opprobrium, rhinoceros
			$^2\sigma\,^0\sigma\,^1\sigma\,^0\sigma$	
μ(μ).μ(μ).μμ.VC(C)				Massachusetts, macaroni, fettuccini, pepperoni, carborundum, memorandum, Tallahassee, Mississippi, Cincinnati, kamikaze, Ypsilanti, presentation, emendation, independence, correspondent, armadillo, Amarillo, desperado, Minnesota, Ebenezer, Alexander
			$^2\sigma\,^0\sigma\,^1\sigma\,^2\sigma$	Adirondack, Agamemnon
			$^1\sigma\,^0\sigma\,^2\sigma\,^0\sigma$	
μ.μ(μ).μ(μ).μ				cassowary, Piccadilly, testimony, allegory, category, capillary, territory, ceremony, alimony, melancholy
μμ.μ(μ).μ.μ				Albuquerque, mercenary, parsimony, pumpernickel
μ.μ(μ).μ(μ).VC				Aristotle, caterpillar, haberdasher, helicopter, filibuster, gerrymander, alligator, alabaster, salamander, mollycoddle
			$^1\sigma\,^0\sigma\,^0\sigma\,^2\sigma$	
μ(μ).μ.μ.μμ				catamaran, razzamatazz, rigamarole, thingamabob, gobbledygook, idiolect, heteronym, heliograph, meteorite, alienate, orientate

Hence intrasyllabic hierarchical structure, and also intrafoot structure (i.e. stress assignment), can be understood coherently in a wave framework. The set of principles and their application in this theory is simpler than those that are needed in grid-based approaches or hybrid grid approaches which incorporate branching structure. In the next two sections we consider how the wave theory can be applied to syntactic structure.

3.3 Morphosyntax and recursion in syntactic structure

The hierarchical representation of syntactic phrase structure allows in theory for an infinite number of embedded levels in a hierarchical structure. Processing constraints severely limit recursion in natural language, and depending on the type of recursion involved, only two or three levels of embedding are normally parsible (Christiansen & Chater, 1999). There are several varieties of recursion that we will consider here. These are illustrated in (1a-c). (1a) shows a relatively trivial form of recursion whereby adjectives can be inserted within a noun phrase. (1b) and (1c) show examples of sentential recursion. (1b) shows a less complex variety, tail recursion, which involves attachment of a sentence to the preceding VP. (1c) shows a more complicated variety, known as enter-embedding, where S is embedded within the NP, thereby separating NP and VP sisters in the surface order of constituents.

1. (a) a wise old black swan.
   (b) Al said Bob knows Cy died.
   (c) the rat the cat chased escaped.

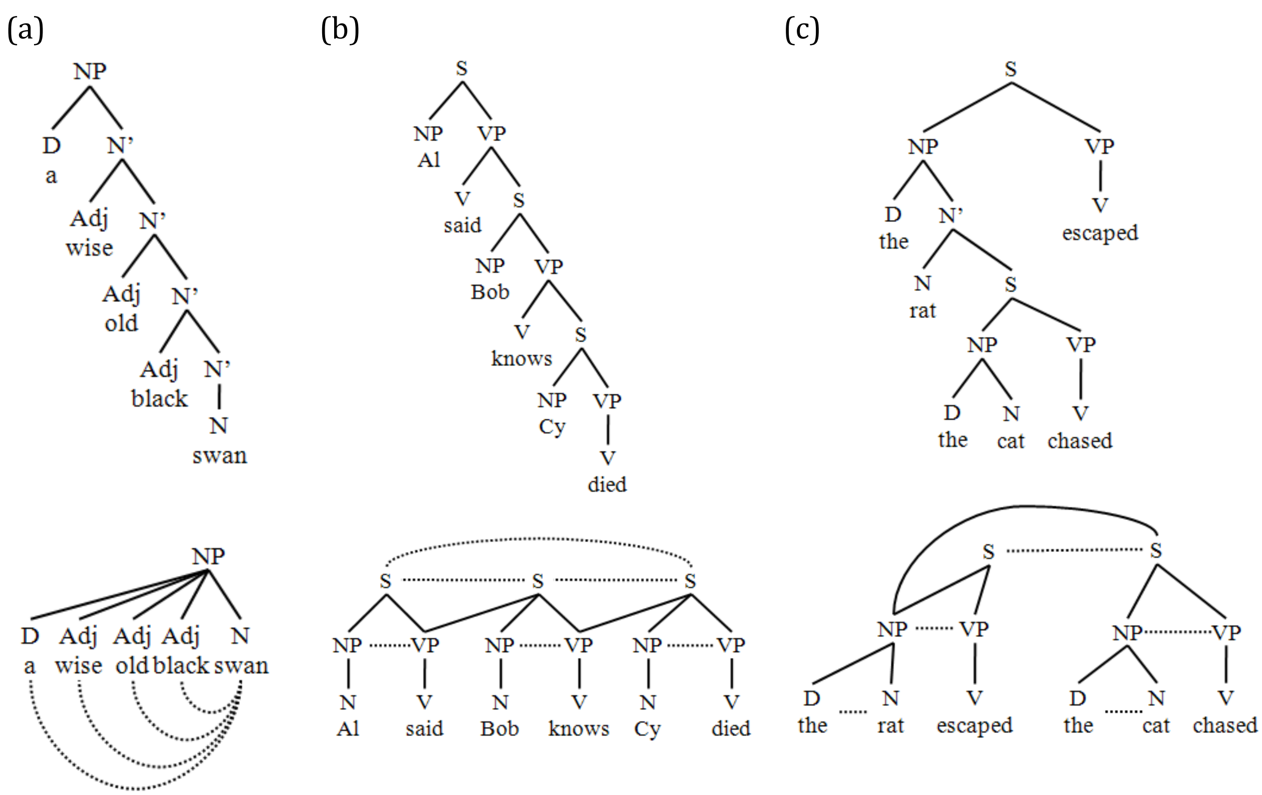


Figure 14. Connected-object schemas and coupling graphs for three varieties of syntactic recursion. (a) Recursion of adjectives within a noun phrase; (b) tail-recursion of sentences within VPs; (c) center-embedding of sentences within NPs.

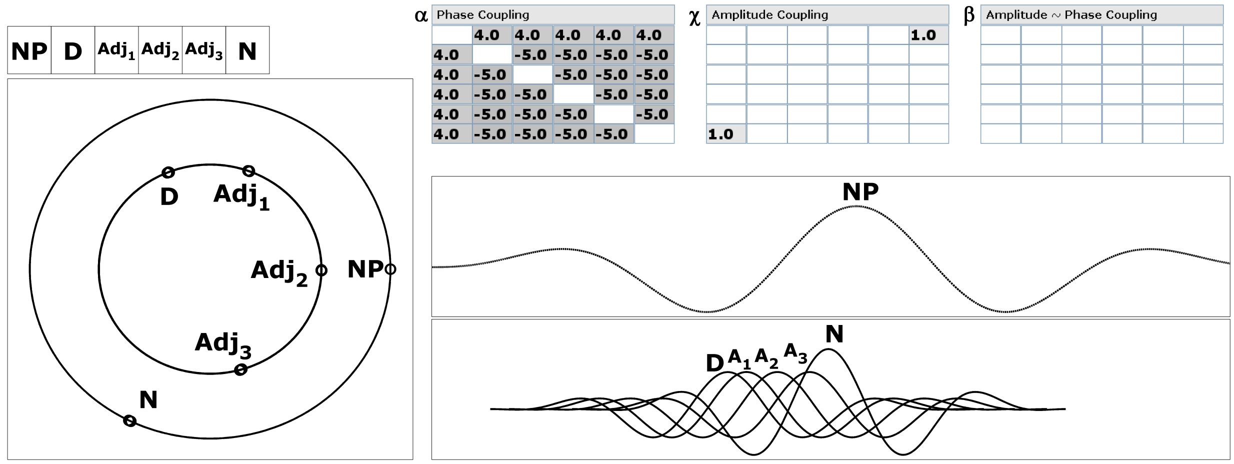
The first two examples of recursion are fairly straightforwardly modeled by the wave theory. Figure 14(a) involves the addition of several Adj systems, which are attractively coupled to the NP. This attractive coupling can be seen as the syntactic manifestation of the semantic relation that they bear toward the NP. At the same time the Adj systems are each repulsively coupled to the N—this is because they are similar types of systems. Furthermore, the Adj systems are repulsively coupled to each other and the determiner (the coupling graph does not show this, but by convention, systems of the same type that are attractively coupled to another system, are repulsively coupled. The result of this parameterization can be seen in Figure 15 (a), where the D, Adj, and N systems are treated as 2nd harmonics of the NP wave. Regardless of how many adjectival systems are added to this structure, all of them will be coordinated relative to the NP phrasal-frequency cycle. The reader should keep in mind that this does not mean, necessarily, that in production they must all be produced within one NP cycle. This merely reflects their premotor/planning organization.

Compared to the coupling graph, the connected object representation of example (1a) contains less information about the semantic relations between units, and some of that information is potentially misleading. For example, it does not very readily represent the semantic association (attractive φ-coupling) between the noun phrase and each of the adjectives. Furthermore, it misleadingly implies that the adjectives exhibit hierarchical relations—other than ordering, it is unclear exactly what follows from this hierarchy of embedded adjectives. In contrast, the coupling graph indicates a semantic relation between the noun phrase and the adjectives via the attractive coupling, and does not implicate hierarchy in the relations between the adjectives.

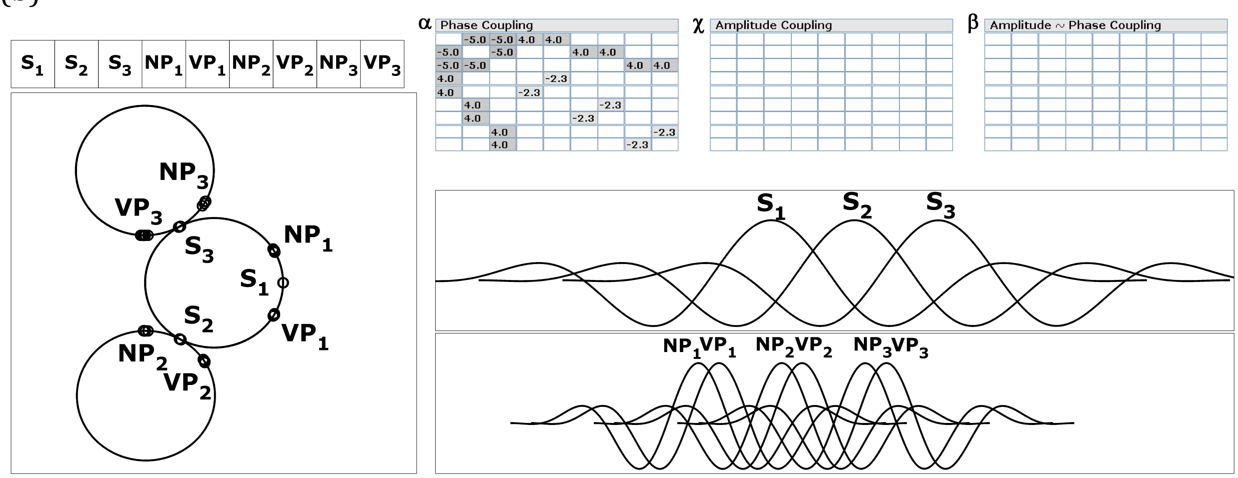
Example (1b) shows recursion of sentences. The connected object expresses the intuition that sentences are subconstituents of VPs. It is doubtful, however, that the hierarchical relations between an S dominating a VP, and an S embedded within a VP, are very similar from a semantic or syntactic perspective. In that regard, the representation is misleading. In the wave theory, bizarre subconstituency relations of this sort are unnecessary. The sentences (or propositions) are themselves distinct entities, which by virtue of the principle of like interaction, are repulsively coupled. The key to capturing the semantic relation between the sentences is that the "embedded" ones are attractively coupled to the appropriate VP systems. This reflects the idea that they provide information about the verbal events (e.g. saying, knowing, dying). Figure 15 (b) shows a simulation of this parameterization of the model.

The more complicated center-embedding recursion in example (1c) can be understood as coupling of an S ("the cat chased") to an NP ("the rat"). The surface order of these systems is not directly the province of the wave theory. Rather, it is speculated to follow from the action of suppressive mechanisms in a model of execution that is not yet developed. However, it is informative to consider how planning dynamics may translate to surface order in cases like these. What appears to happen is that the execution of the main clause VP ("escaped") is suppressed by the embedded clause, and then unsuppressed on a subsequent cycle—the parenthesized S_1 in Figure 15(c). What the wave theory does provide is a new view of the planning dynamics associated with such structures. They differ from non-recursive structures only in that attractive coupling exists between a sentence system and a phrasal system. Attractive φ-coupling relations of this sort are the manifestations of semantic associations.

(a)



(b)



(c)

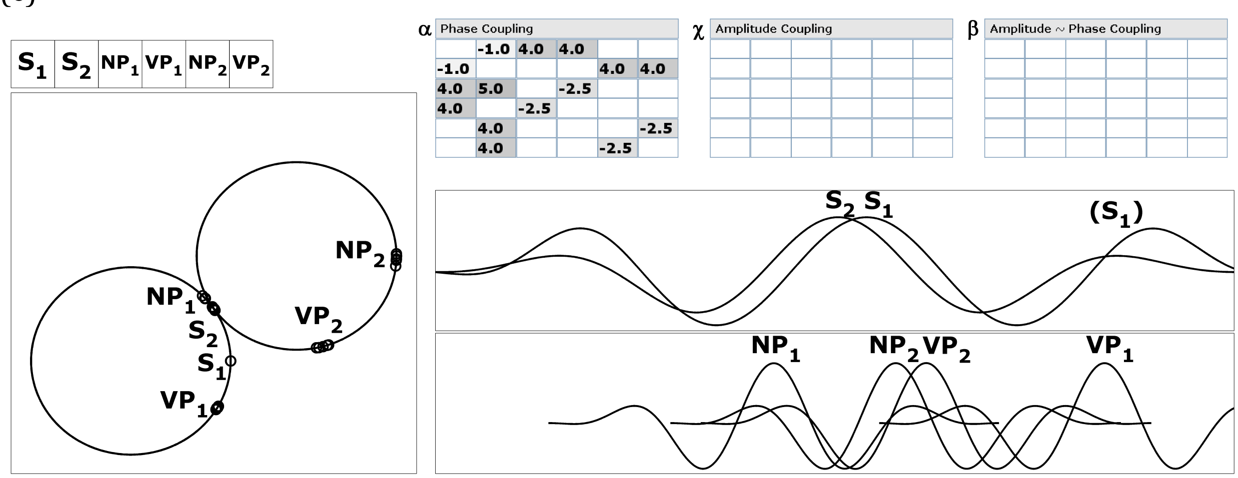


Figure 15. Wave models of recursive syntactic systems. Phase circle representations of stabilized dynamics are shown on the left of each panel. Coupling parameters are shown, as well as activation waves.

Carrying this approach to its logical conclusion, there really is nothing special about recursion, and indeed, recursive structures are not actually recursive. In other words, no special processing mechanism is necessary to formulate and understand center-embedded sentences such as (1c). Instead, they arise from patterns of coupling in which propositions/sentences are attractively coupled to phrases. More specifically, it is not the case that the S "the cat chased" is contained (embedded) within another phrase. Recursive "embedding" and its entailment of containment is arbitrarily imposed by the conventions of the connected object schema. Moreover, the processing constraints arise from cognitive limitations on the number of propositions that can simultaneously be active; because they mutually inhibit each other, only a limited number can be active in planning.

3.4 Ordering: complementation and heavy-xP shift

A basic distinction in the study of phrase structure is the difference between complements and adjuncts. In general, complements are attributed a closer semantic and syntactic relation to their heads than adjuncts. To illustrate this, consider the grammaticality contrast between (2a) and (2b). There are two prepositional phrases associated with the NP: the PP "of syntax" and the PP "at Berkeley". (3) exemplifies corresponding sentences in which the PPs are complement/adjunct within a VP.

(2) a. A discussion of syntax at Berkeley
b. ?? A discussion at Berkeley of syntax

(3) a. X discusses syntax at Berkeley
b. ?? X discusses at Berkeley syntax

In (2) and (3), concepts of "syntax" and "Berkeley" are evoked within a frame of discussion, i.e. these nouns evoke concepts that associate with the event of discussion. Although both concepts hold semantic relations to the event, the relations are different in important ways, regarding the types of information they convey. One concerns a topic of discussion. The other concerns an event location.

How do the semantic relations differ between (2a) and (2b) and the corresponding (3a) and (3b)? The typical answer is that "syntax" is more important to the meaning of discussion than the location, and this is often formalized by use of thematic role assignment. "More important" in wave theory terms implies more strongly coupled to the NP/VP. Higher amplitude systems exhibit stronger coupling through amplitude modulation of phase coupling. The noun system "syntax" is more strongly φ-coupled to the discussion NP/VP than the location (in this case), *because* it exhibits higher amplitude than location. The relatively high amplitude of "syntax" also imbues it with a greater repulsive force between the other N and V systems. (2b) and (3b) are of course possible, if for pragmatic, informational reasons, "Berkeley" is endowed with prominence, i.e. amplitude. This can occur if, through β-interaction, there is stronger coupling between *Berkeley* and the discussion-NP.

Remarkably, when the amplitude of "Berkeley" is very weak relative to the argument system, structures like (4) are possible. This phenomenon is known as heavy-xP shift. Here the argument-xP is more strongly coupled to the NP/VP than the location system, but there are also many more systems/propositions associated with the object, each of which exerts repulsive forces on the N discussion (and vice versa). Individually these repulsive forces are relatively weak, but their combined effect is to greatly increase the inhibitory forces exerted on the N "discussion" (4a) or V "discuss" (4b) by the subconstituents of the argument phrase. Because of this, the argument system can experience a weaker net phase attraction to the discussion-NP. In addition, via r-coupling between the subconstituent systems and the argument PP, the amplitude of the argument PP grows; β-modulation of repulsive φ-coupling between the PPs and the NP/VP results in a relative phase transition. Hence, relatively high amplitude can cause an argument xP to be less closely phased to an event-NP/VP than an adjunct xP.

(4) a. A discussion at Berkeley of the status of trees in syntactic theory.
b. We discussed at Berkeley the status of trees in syntactic theory.

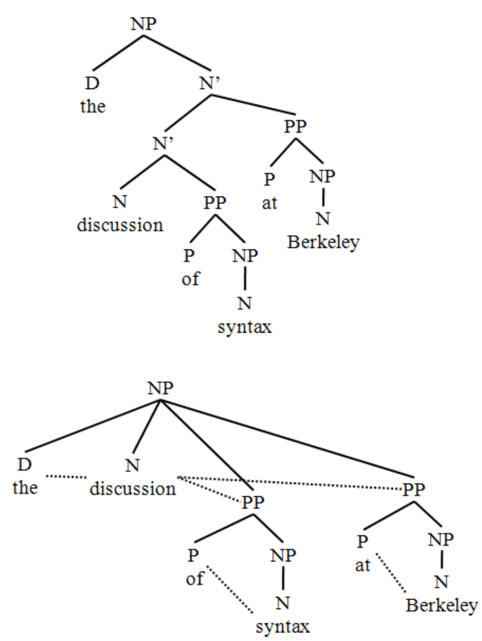


Figure 16. Connected-object and coupling graph representations of the normal argument and adjunct structure in example (2).

A dynamical simulation of heavy-xP shift is shown below. In Figure 17 (a), the PP "of syntax" (PP_1) is more closely phased to the NP than the PP "at Berkeley" (PP_2). In Figure 17 (b), the reverse holds: the location PP_2 is more closely phased to the NP than the object PP_1. In both simulations, α, χ, and β parameters are identical. The only difference is that the amplitude potential parameter k_1—cf. Eq. (7)—is greater in the shifted case (b) than in the unshifted case (a). This corresponds to the idea that the heavy PP is coupled to many constituent systems (i.e. "of the status of trees in syntactic theory") whose cumulative r-coupling greatly increases the amplitude of PP. The manipulation of k_1 is a shortcut for the more theoretically appropriate simulation which would incorporate all of the subconstituent systems, *r*-coupling forces between them and the PP, and β-modulation of the repulsive forces between NP/VP and their argument/adjunct xPs.

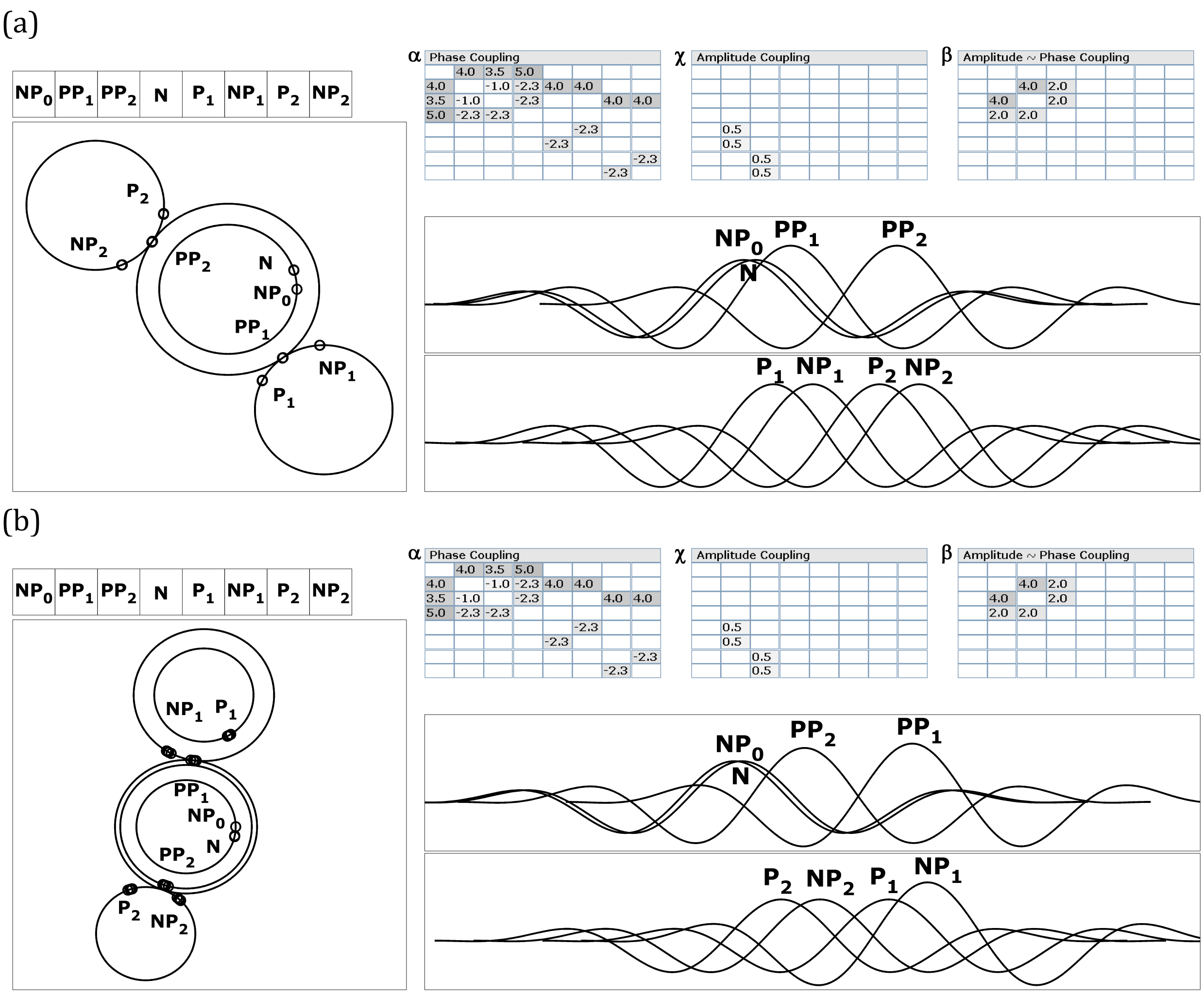


Figure 17. Dynamical simulation of heavy-PP shift in an NP. (a) normal phasing where argument is more closely phased to the NP than the adjunct; (b) shifted phasing where the adjunct is more closely phased to the NP than the argument.

The wave model simulations of syntactic phenomena in sections 3.3 and 3.4 should be viewed as proofs-of-concept only. They do not constitute definitive theoretical claims on exactly how to dynamically parameterize the wave model to understand a given phenomenon. To wit, in the preceding example of heavy xP shift, we saw that there are two mechanisms through which a relative phase transition between NP/VP and argument/adjunct xPs can occur. The first is via the net effect of repulsive interactions between the xP subconstituent systems and the NP/VP. The second is through r-coupling between xP subconstituents and xP, along with β-modulation of repulsive coupling between the xPs and NP/VP. One or both mechanisms may be in operation. The extent to which one or the other is more important remains to be determined, and is likely to be subject to language-specific sources of variation.

4. Development of wave theory

The concepts and their applications presented in the preceding sections represent a first step toward a complete, unified dynamical theory of how phonological and syntactic planning are related to structural hierarchy in language. One of the main advantages of this theory is that it may bridge the gap between linguistic representation and neural representation. It is disappointing how little attention is given to situating linguistic theories in a neurally plausible framework. While most cognitive scientists will acknowledge that cognition is embodied in the nervous system, many state of the art linguistic theories make little or no attempt to connect the representations used in the theory to neural dynamics. The genesis of the wave theory is partly due to a desire to embody linguistic representations themselves. In section 4.1 we consider how the wave theory may be related to neural activity. In section 4.2 we will reiterate the premises of the theory and address a number of issues to direct further development of the theory.

4.1 Neural basis of the wave theory

The wave theory is built upon several assumptions that we have not yet brought to light. Here we will attempt to illuminate these assumptions so as the clarify the reasoning behind the theory.

A key concept in wave theory is activation. This concept cannot be understood without first considering the question: activation of *what*? The *what* is a neural ensemble, which can be thought of as a fuzzy set of interconnected neurons. The ensembles are instantiated multimodally, distributed in both motor and sensory areas. They should not be conceptualized as discrete groups, since they overlap to an unknown extent with related ensembles, and contextual influences may greatly modulate their distribution and overlap at any given moment. Excitatory cortical layer 5 pyramidal neurons are obvious candidates for such neurons, but that does not mean that other types of neurons do not contribute to activation dynamics.

The wave theory assumes that linguistic units can be associated with neural ensembles. This is not to say that anything suggested to be a linguistic unit stands in a one-to-one relation with a neural ensemble. Rather, any unit which the wave theory treats as a "system" should be instantiated as an ensemble. It can be the case, moreover, that some systems are ensembles of ensembles. Exactly how much overlap there is between ensembles, of both same and different timescales and linguistic functions, is currently unknown. One way to interpret hierarchy in light of the neural grounding of the wave theory is to see higher-level systems such as syllables and feet, or syntactic phrases and sentences, as meta-ensembles. In any given instantiation, these meta-ensembles are associated with sub-ensembles specific to particular segments or words. At the same time, the meta-ensembles generically encode a type of systems, such as a syllable or phrase, etc.

Importantly, activation is only an approximation to the integrated spiking, or transient depolarization, of the neurons in an ensemble. The harmonic wave theory employs a gross approximation to the dynamics of ensemble activation, and there are two aspects of this approximation that merit special consideration. The first is the periodic, or wave-like nature of the activation. The second is modeling the activation with harmonic oscillators.

The periodicity of wave activation is motivated by observations of the local field potential in the neocortex, and by consideration of the interaction of excitatory neurons with inhibitory interneurons. In EEG and MEG studies, observed electrical and magnetic fields exhibit transient increases in spectral power in various frequency bands, ranging from very low frequencies (1-4 Hz delta rhythms) to very high frequencies (100-300 Hz high-gamma rhythms). The oscillations whose timescales are most commensurate with behavioral rhythms are 4-8 Hz theta rhythms. 8-12 Hz alpha rhythms have been associated with the activity of inhibitory interneurons, at least in some cortical areas, and 12-20 Hz beta and 30-60 Hz gamma rhythms have been determined to be particularly important to the binding of neural activity across different regions and in the processing of sensory stimuli. What has also been found is that these higher-frequency oscillations are modulated by the lower frequency ones. It is this observation that motivates the use of oscillatory systems in 4-8 Hz theta band as relevant to describing the coordination of linguistically relevant ensembles. For further reading on the functional importance of brain rhythms, the reader can consult Buzsáki (2006), Buzsáki & Draguhn (2004), and Klimesch (1999).

One of the main difficulties in identifying low-frequency oscillations experimentally is their transience and the relatively long time windows necessary to identify low-frequency periodicity. A promising and inspirational source of support for the idea that ensembles exhibit multi-frequency oscillatory activity can be found in the Izhikevich (2003, 2006), and Izhikevich et al. (2004), which models a large population of randomly connected, realistically modeled excitatory and inhibitory spiking neurons with Hebbian learning. In simulations of this model, both low- and high-frequency oscillations emerge.

The second aspect of the wave theory, the use of harmonic oscillators—systems whose motion is circular in the absence of other forces—is undoubtedly a vast oversimplification of the actual integration of excitatory neuronal membrane potentials. In the wave approach used here, the activation of a given system (ensemble) is defined as $A - r \cos \theta$, where A is a normalizing baseline constant. When A is equal to the maximum of $|r|$, the activation ranges from 0 to 2A, and the average activation is A. This means that a system spends about half of each period below its average activation, which is not a very realistic approximation to the integrated spiking of an ensemble. However, the harmonic wave approximation is meant to favor utility and conceptual clarity over realistic ensemble dynamics. The utility of the model lies in the fact that it can simulate phenomena which presumably are more accurately described with nonlinear oscillations. Another observation made by Izhikevich is, however, reassuring in this regard. Izhikevich (2006) reports the formation of "polychronous" neural groups in his model. These are ensembles that spike not simultaneously, but in such a way that the integrated spike rate of the group approximates the positive velocity phase of a wave-like oscillation. This suggests that the continuous wave approximation of spiking activity within an ensemble is not grossly inaccurate.

In sum, the concept of activation is built upon a number of speculative but plausible assumptions about the nature of neural ensembles and the utility of harmonic wave approximations to their integrated dynamics. The dynamical mechanisms from which the wave theory gains the most mileage—attractive and repulsive relative phase coupling, excitatory and inhibitory amplitude coupling, and amplitude-dependent modulation of relative phase coupling forces—can be motivated by consideration of how neural ensembles are likely to interact. To facilitate this it is helpful to envision the ensembles distributed in a 2D field, which is perhaps isomorphic to a flattened deformation of neocortex. Since the ensembles are distributed across many regions, this is patently an oversimplification.

Within a given ensemble, neurons are in-phase synchronized, entailing that connections between excitatory neurons predominate. For any pair of ensembles, if their degree of overlap is relatively small, then the connections between them are likely to be predominately inhibitory. This can seen to follow from considerations of group selection and Hebbian learning (Edelman, 1978). To wit, if some neurons belonging to an ensemble are inhibited when the rest of the ensemble is excited, Hebbian synaptic plasticity will lead to the removal of those neurons from the ensemble. Likewise, when two ensembles are very often mutually excited, they will become integrated into the same ensemble.

It should be no accident that the neural mechanisms behind receptive fields generalize to linguistic systems as well. The principle of like interaction follows nicely from these considerations. The principle holds that systems of the same type are repulsively coupled. By envisioning the sameness of two neural ensembles as correlated with their cortical proximity and low degree of overlap, it follows that similar ensembles are mutually inhibitory. Mutual inhibition implies (1) inhibitory amplitude coupling, and (2) repulsive phase coupling. When one system is most highly active, it exerts the strongest inhibition upon the other; this self-organizes into a state that maximally separates phases of maximum activation, i.e. phase polarization, with a concomitant effect of the inhibitory amplitude coupling.

Likewise, attractive phase coupling and excitatory amplitude coupling are related. When two systems exhibit a high degree of overlap, e.g. a vowel and its associated syllable, or a noun and its associated noun phrase, then their neural interactions are expected to be mutually excitatory. This leads to in-phase synchronization of activation peaks, and increases in activation, modeled in wave theory with radial amplitude.

Taking these observations on attractive and repulsive phase coupling as a whole, we can conclude that for any two systems, phase repulsion never co-occurs with excitatory amplitude coupling, and vice versa, phase attraction never co-occurs with inhibitory amplitude coupling. Parametrically, this implies that $\mathrm{sign}(\alpha_{ij}) = \mathrm{sign}(\alpha_{ji}) = \mathrm{sign}(\chi_{ij}) = \mathrm{sign}(\chi_{ij})$, which is a fairly strong constraint on parametric symmetry. Hence the principle of like interaction follows from neural considerations. Moreover, the considerations discussed above argue that repulsive phase coupling and inhibitory amplitude coupling go hand-in-hand, as do attractive phase coupling and excitatory amplitude coupling. This observation can be formulated as an additional principle: the principle of *phase-amplitude concordance*. Given this principle, it makes sense to posit that the amplitude of a system influences the strength of the φ-coupling forces it exerts upon other systems. This follows naturally from viewing the coupling forces as arising directly from excitatory and inhibitory neural connections.

Finally, although the neural grounding of the wave theory is fairly speculative, it can in principle eventually be confirmed, disconfirmed, or refined through experimentation. In contrast, it is not clear how theories of linguistic structure involving connected object conceptualizations of linguistic units can be experimentally tested in a neurological context.

4.2 Further considerations

In spite of attempts to make the wave theory neurally plausible, it is ultimately only a very simplistic description of neural network dynamics. The primary aim of the theory is to utilize a new set of metaphors to understand linguistic patterns. These metaphors are all related to the idea that a linguistic unit can be conceptualized as a wave, which can in turn be modeled as an oscillatory dynamical system. The utility of the theory arises from this wave metaphor. The theory draws its explanatory power from its allowances for how waves can interact with one another, through phase-coupling, amplitude-coupling, and frequency-locking of oscillations. In this section we will enumerate the assumptions and logical structure of the theory, and then reiterate a number of issues that have arisen herein. The wave theory is based on the following premises:

(1) All "units" of linguistic behavior correspond to activity patterns, i.e. transient wavelike integrated spiking patterns in ensembles of neurons.

(2) The activity pattern of an ensemble is conceptualized as a dynamic "activation" variable, and can be modeled as a dynamical system with phase and radial amplitude components.

(3) The principle of harmonicity: the dynamics of linguistic systems are oscillatory, with energy distributed to varying degrees across harmonically related frequencies.

(4) The principle of harmonic interaction: coupled linguistic systems tend to frequency-lock.

(5) Linguistic systems interact with each other through amplitude coupling (r-coupling, parameterized by χ) and relative phase coupling (φ-coupling, parameterized by α), which is modulated by amplitude (parameterized β).

(6) Amplitude coupling can be excitatory (χ^+) or inhibitory (χ^-).

(7) Relative phase coupling can be attractive (α^+) or repulsive (α^-).

(8) The principle of like interaction: for any two coupled systems, if they are of the same type, they are φ-coupled repulsively, and if they are of different type, they are φ-coupled attractively.

(9) The principle of phase-amplitude concordance: if two systems are inhibitorily r-coupled, they are repulsively φ-coupled; if two systems are excitatorily r-coupled, then they are attractively φ-coupled.

The premises and principles above provide a solid framework for the wave theory, within which dynamical modeling of hierarchical linguistic structure can be productively conducted in phonological and morphosyntactic domains. However, as the theory is in its infancy, a number of outstanding issues remain, which will require resolution in subsequent work.

One such issue is the question of what criteria should be employed to assess similarity of typehood. It was speculated that systems of similar type should exhibit comparable distributions of energy across the frequency spectrum. In other words, phrases are similar to other phrases because their oscillatory energy is concentrated at a frequency that is relatively low. Likewise, segments are similar to other segments because they exhibit spectral energy in a relatively high region of the spectrum. While this is an interesting correlation, it is theory-internal, and there should ideally be theory-external reasons for positing similarity. There are, of course, many such reasons, found throughout the body of linguistic research, which support the idea that these types of units pattern similarly.

Another issue is how systems are transiently activated and deactivated. Prior to the activation phase, when there is negligible perception or planning of a given system, we can assume that the system tends toward its minimal energy state. This does not imply that there are energy quanta, but rather, that in the absence of activation, a given system obtains a negligible amplitude of oscillation that can be approximated as a point attractor. At this negligible amplitude, interaction through coupling approaches zero. In other words, these inactive systems can be pictured in phase space as very tiny non-interacting circles that might as well be points. Activation can be understood as a supercritical Andronov-Hopf bifurcation, whereby a stable point attractor becomes unstable and a stable limit cycle appears. This is brought about by decreasing the parameter k_1 in Eq. (7), which determines the shape and size of the amplitude potential. In the activation phase, the trajectory of the system spirals out to a stable limit cycle. The question of how the bifurcation is triggered remains open. It is presumably during the activation phase that lexical memory or pragmatic, informational factors break time-reversal symmetry to organize relative phases.

Following the transient activation phase, there may be a brief stabilized phase, which is subject to wave-theoretic principles. It may often be the case that stabilization is rarely achieved in any rigorous sense, yet the tendency toward stabilization is what guides the emergence of a hierarchical structure of linguistic systems. Subsequently there follows a deactivation phase, during which the limit cycle shrinks back to a minimal energy point attractor, corresponding to a increase of k_1. It is also possible for more complicated changes in k_1 to occur, although we have not considered them here.

The timing of execution is another concern. The wave theory is a theory of stabilized planning dynamics, but we currently lack a theory of how those planning dynamics are translated into the physical production of speech. As mentioned before, there are undoubtedly suppressive mechanisms that allow for premotor planning without subsequent motor execution. It is likely that any model of such mechanisms will require some sort of thresholding, as well as normalization techniques that may be fairly sophisticated.

It is important, however, that the theory allows for subthreshold premotor oscillatory activity, without necessitating execution. This is interesting in light of the idea that stabilization may not be fully achieved in rapid, unrehearsed speech. Under such conditions, many coarticulatory or omission phenomena can be seen as the byproduct of the failure for planning systems to stabilize. An intriguing possibility is that at the beginning of the activation phase, the relative phases of systems of the same type are in-phase, and hence the burden of ordering falls upon repulsive phase coupling. Without sufficient time for repulsive interactions to manifest, some systems may be masked in execution by other, more strongly activated systems, resulting in omission. Furthermore, this transition could be mirrored in language acquisition: children initially have very weak inhibitory coupling; the task of language learning involves the strengthening of inhibitory connections between neural ensembles.

Another interesting aspect of this model is the potential for system noise to exert qualitative effects on planning dynamics and execution behavior, particularly during the beginning of the activation phase where system amplitudes are low. In all the simulations presented in this paper, a small amount of noise was added to the phase and amplitude velocities, but no conceptual mileage was obtained from this. It is the case, however, that if noise is increased to some extent, relative phase transitions can occur, resulting in altered planning dynamics.

Semantic systems have also been neglected in this paper. It is not clear whether they are subject to the principle of like interaction, primarily because definition of type is less obvious in their case. In utterance planning, semantic systems are likely a primary source of activation for phonological and morphosyntactic ones. They can furthermore be seen as the basis for attractive phase-coupling. Attractive coupling between segments and syllables, between lexical items and phrases, and between phrases and sentences, may be reinforced by virtue of such systems sharing common semantic sources of activation.

To conclude, we reiterate some of the advantages of the wave theory. This approach provides the basis for an understanding of hierarchical linguistic structure that applies across linguistic subdomains, i.e. phonology and morphosyntax. It offers a coherent account of why hierarchical structure emerges, which involves broken symmetries in a relatively small set of parameters. In a couple of cases, it was shown that previously unrelated patterns (e.g. extrametricality and weight-based stress assignment) arise from different values of the same underlying parameter—this simplifies our understanding of such patterns. Moreover, hierarchical structure can be viewed as the consequence of the self-organization of interacting systems, rather than an externally imposed constraint that subserves representation. The theory is cognitively plausible, neurologically testable, and allows for real-time dynamical modeling of planning dynamics.

September 14, 2009

Appendix

Some further information on the parameters used in the dynamical models in sections 2 and 3 is provided here. All simulations were conducted with a 4th order Runge-Kutta algorithm and ran for either 100 or 200 time units, with a time step of $\Delta t = 0.03125$ units. The lowest frequency system in any given simulation had an intrinsic frequency of $\omega=1$, or 2π radians per time unit. Hence the simulations ran fairly long, to ensure that stabilization was reached. Only the first 2 seconds of each simulation are displayed in the activation and relative phase plots in section 2. In section 3, wave activations are defined as $A - r\cos(\theta_i)$, where A is a normalizing activation constant for a group of systems, here the maximum radial amplitude in a given group. In all simulations, unless otherwise specified, initial amplitudes were 0.1 and target amplitudes were 1, as determined by setting $k_1 = -1$ and $k_2 = 1$. The small amounts of phase and amplitude noise that were added to the phase and amplitude velocities were Gaussian distributed with standard deviations of 0.1 radians and 0.1 amplitude units, respectively. In sections 3.3 and 3.4, generalized relative phase coupling—cf. Keith & Rand (1984), Kopell (1988), Saltzman & Byrd (2000)—was used to simulate the effects of the principles of harmonicity and harmonic interaction. A more detailed simulation would model the oscillation of each system at each of a set of harmonically related frequencies; this added complexity was exchanged in favor of a simpler approach in which the phases of n:1, n>1 harmonically interacting frequency-locked systems are multiplied such that they adhere to a common phase circle. In other words, a generalized relative phase is defined as $\varphi^G_{ij} = \theta_j\omega_i - \theta_i\omega_j$. Unless otherwise indicated in a simulation figure, all coupling parameters were 0.

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