Title
PIONIC ATCM 3d-2p LINE SHIFT AND WIDTH FOR 40Ca AND 44Ca AND THE NEUTRON DISTRIBUTION

Author
Kunselman, Raymond.

Publication Date
1970-04-01

Submitted to Physics Letters

UCRL-19267
Preprint
c.2

PIONIC ATOM 3d-2p LINE SHIFT AND WIDTH FOR ^{40}Ca AND ^{44}Ca, AND THE NEUTRON DISTRIBUTION

Raymond Kunselman and Georges-André Grin

April 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

UCRL-19267

DISCLAIMER

Pionic Atom 3d-2p Line Shift and Width for ^{40}Ca and ^{44}Ca, and the Neutron Distribution*

Raymond Kunselman†
Department of Physics, University of Wyoming, Laramie, Wyoming 82070

and

Georges-André Grin*
Lawrence Radiation Laboratory, University of California,
Berkeley, California 94720

ABSTRACT

^{44}Ca was found to show both a smaller shift and width than ^{40}Ca for the 3d-2p pionic atom X-ray transition. From this measurement and an optical model analysis, it can be deduced that the neutron distribution at the surface of the ^{44}Ca nucleus extends significantly beyond the proton distribution.

Ideally, pionic atoms should constitute a good tool for studying the matter distribution in nuclei. However, the difficulty associated with nearly every strong-interaction calculation in nuclear physics, the recourse to models with their limitations and need for parameters, actually makes it less reliable than one would desire. Nevertheless, we decided that a study of a suitable isotope pair could provide useful information, either on the optical model used to describe the π-nucleus interaction, or perhaps on the tail of the neutron distribution in nuclei. The ^{40}Ca-^{44}Ca pair was chosen for study in the form of $CaCl_2$.[1]

The negative pions were stopped in the targets and identified as usual; the X-ray spectrum was observed with a Ge(Li) detector and recorded on a pulse height analyzer.[1] The spectra were taken with a gain stabilizer positioned on the 3d-2p line of Cl. The energy scale was calibrated under approximate beam conditions using radioactive sources; the resolution was 1.8 keV (FWHM) for the 208-keV line of ^{177m}Lu.

We obtained the strong interaction shift by comparing the calculated and measured transition energies. The calculated energy was taken as $E_c = E_k + \delta E_f + \delta E_v$, where E_k is the Klein-Gordon transition energy, δE_f the charge-size correction[2], and δE_v the vacuum-polarization correction[3]. The broadening W_n (Lorentzian) of a line due to the nuclear absorption was obtained from the measured line width W_m, knowing the instrumental resolution W_i (assumed Gaussian), through the formula:[4] $W_n = W_m - (W_i^2/W_m)$, where all widths are FWHM.

The measurement was broken into four runs; the data were analyzed separately, then the results combined. All measurements were consistent with one another and showed the same effect. The 4d-2p Cl line which appeared in the spectrum was used as a consistency check. Figure 1 displays the $^{40}CaCl_2$ and $^{44}CaCl_2$

spectra in the region of interest around the Ca 3d-2p line. The following values were used for that line (in keV): $E_k^{40} = 206.67$, $E_k^{44} = 206.74$ (the difference being due to reduced mass difference), and $\delta E_f = -0.09$, $\delta E_v = 1.11$, for both ^{40}Ca and ^{44}Ca.

The results are as follows:

measured energy	^{40}Ca	209.45 ± 0.14 keV,
	^{44}Ca	209.16 ± 0.10 keV,
shift	^{40}Ca	1.76 ± 0.14 keV,
	^{44}Ca	1.40 ± 0.10 keV,
difference	(44–40)	−0.36 ± 0.17 keV,
width	^{40}Ca	2.29 ± 0.13 keV,
	^{44}Ca	2.07 ± 0.15 keV,
difference	(44–40)	−0.22 ± 0.20 keV.

Taking first the differences for each of the four measurements and calculating the r.m.s. errors gives the following averages:

shift difference	−0.34 ± 0.15 keV,
width difference	−0.23 ± 0.14 keV.

Let us emphasize that these effects (on the line shift and line width) were found in all four runs. As far as the yields were concerned, they were larger by 5-6% in ^{40}Ca than in ^{44}Ca for the 3d-2p, 6f-3d, and 4f-3d lines[1]. However, the targets were $CaCl_2$ and a small isotope effect (mass recoil of the lattice) on the atomic capture ratio might exist[1].

The shift and width being smaller for ^{44}Ca, in regard to ^{40}Ca, is not expected on the basis of trivial deductions. But detailed calculations allow us to explain it and to draw some tentative conclusions on the difference between neutron and proton distributions in the ^{44}Ca nucleus.

spectra in the region of interest around the Ca 3d-2p line. The following values were used for that line (in keV): $E_k^{40} = 206.67$, $E_k^{44} = 206.74$ (the difference being due to reduced mass difference), and $\delta E_f = -0.09$, $\delta E_v = 1.11$, for both ^{40}Ca and ^{44}Ca.

The results are as follows:

measured energy	^{40}Ca	209.45 ± 0.14 keV,
	^{44}Ca	209.16 ± 0.10 keV,
shift	^{40}Ca	1.76 ± 0.14 keV,
	^{44}Ca	1.40 ± 0.10 keV,
difference	(44–40)	−0.36 ± 0.17 keV,
width	^{40}Ca	2.29 ± 0.13 keV,
	^{44}Ca	2.07 ± 0.15 keV,
difference	(44–40)	−0.22 ± 0.20 keV.

Taking first the differences for each of the four measurements and calculating the r.m.s. errors gives the following averages:

shift difference	−0.34 ± 0.15 keV,
width difference	−0.23 ± 0.14 keV.

Let us emphasize that these effects (on the line shift and line width) were found in all four runs. As far as the yields were concerned, they were larger by 5-6% in ^{40}Ca than in ^{44}Ca for the 3d-2p, 6f-3d, and 4f-3d lines[1]. However, the targets were $CaCl_2$ and a small isotope effect (mass recoil of the lattice) on the atomic capture ratio might exist[1].

The shift and width being smaller for ^{44}Ca, in regard to ^{40}Ca, is not expected on the basis of trivial deductions. But detailed calculations allow us to explain it and to draw some tentative conclusions on the difference between neutron and proton distributions in the ^{44}Ca nucleus.

We assumed the nucleon distributions to be described by a Fermi shape:

$$\rho_i = \frac{\rho_{oi}}{1 + \exp\left(\frac{r-c_i}{z_i}\right)} ,$$

where i=n,p for neutron or proton, c_i is the half-density radius, z_i the skin-thickness parameter and ρ_{oi} is given by the normalization

$$\int \rho_i dv = \begin{cases} A-Z & \text{for } i=n \\ Z & \text{for } i=p \end{cases} .$$

For ^{40}Ca the neutron distribution was taken identical to the proton distribution. For ^{44}Ca, the neutron distribution was allowed to vary, in order to fit the observed shift difference. The proton distributions were taken as given by the charge distribution measurements[5]; this includes the effect of the proton size (0.7 F). but as we are interested in differences, it is not important, as long as one remembers that the data we quote for a neutron distribution do not relate to the distribution of the neutron centers.

For the pion-nucleus interaction, we used an optical model[6] and perturbation calculations. The energy shift is then given by:

$$\Delta E = \frac{4\pi\hbar^2}{2m} \left(1 + \frac{m}{N}\right) \left\{ \int \left[(b_o-b_1)\rho_p + (b_o+b_1)\rho_n\right] \psi_\pi^2 \, dv + \int \left[(c_o-c_1)\rho_p + (c_o+c_1)\rho_n\right] |\nabla\psi_\pi|^2 dv \right\} ,$$

where b_o, b_1, c_o, c_1 are parameters related to the π-nucleon interaction, ψ_π is the hydrogen-like pion wave function, m is the meson mass, and N the nucleon mass.

The calculations were made by using the following values for the parameters:

$$b_o = -0.028 \qquad b_1 = -0.10$$

$$c_o = +0.19 \qquad c_1 = +0.16 \text{ [6]}$$

40 Ca $\quad c_n = c_p = 3.60$ F $\quad z_n = z_p = 0.576$ F

^{44}Ca $\quad c_p = 3.68$ F $\quad z_p = 0.567$ F .[5]

With these parameters, acceptable sets of c_n, z_n for ^{44}Ca can then be deduced to fit the measured shift difference (−0.34 keV ± 0.15 keV). For example, with $c_n = 3.68$, $z_n = 0.73 \pm 0.06$ or with $z_n = 0.567$, $c_n = 4.14 \pm 0.16$; the rms radii being respectively 3.94 ± 0.15 and 3.84 ± 0.11, to compare with 3.545 for the protons in ^{44}Ca, and 3.515 for both neutrons and protons in ^{40}Ca. A more realistic solution would probably imply intermediate values for both c_n and Z_n. The condition $\rho_n(r) \geq \rho_p(r)$ over the whole central region would favor a solution with $c_n \approx c_p$, that is $z_n > z_p$. With the values used for the parameters, a normalization factor was introduced in the calculation, the same for both nuclei, in order to get for ^{40}Ca exactly the measured value of 1.76 keV, instead of the 2.26 keV given by the calculation without adjustment.

We could also obtain the measured ^{44}Ca-^{40}Ca shift assuming $\rho_n = \rho_p$ for both isotopes by adjusting the value of either of the optical constants c_1 and b_1, to −0.17 ± 0.11 (instead of +0.16), and −0.25 ± 0.06 (instead of −0.10), respectively. It should be emphasized that the present conclusion does not depend on the exact value of the parameter c_1, which has to be taken from theoretical predictions[6] and lacks experimental support up to now, but mainly on the sign. However, as the agreement between theory and experiment is rather good for the three other parameters, b_o, b_1, c_o[8,9], one can have some confidence in the value assumed for c_1. With $c_1 = 0.00$ instead of +0.16, one would obtain $z_n = 0.66$ for $c_n = 3.68$, and with $c_1 = -0.16$, $z_n = 0.56$ for $c_n = 3.68$.

The reason for the actual shift to be less for ^{44}Ca than for ^{40}Ca, whereas the ^{44}Ca nucleus has more nucleons and is more extended, lies in the fact that the shift for a 2p pion is formed not only of the attractive non-local contribution of the π-nuclear interaction, but also of a repulsive local part. It happens that the change in the local interaction is dominant over the non-local one between ^{40}Ca and ^{44}Ca (see also ref. 7).

A similar study has been undertaken elsewhere for the ^{58}Ni-^{60}Ni pair[10], pointing also to the sensitivity of the difference between the neutron and the proton distributions on the measured shift difference; however, in that case the shift difference was found to be practically zero. Also, if our measured difference had been zero, the corresponding results for ^{44}Ca would have been for the c_n/z_n pair: 3.68/0.60, 3.76/0.567; and if it had been +0.34 keV: 3.68/0.47, 3.37/0.567.

We may also mention that this result of the neutrons extending beyond the protons for ^{44}Ca has been obtained with calculations where the nuclear distributions are built up from single-particle wave functions in shell-model potentials[11] and with Coulomb energy calculations[12]. Furthermore, an experimental work on K^- meson absorption[13] also showed, for a number of medium and heavy elements, the neutrons probably extend beyond the protons, even if a large absorption parameter is considered for the optical model.

In regard to the width which was also found smaller for ^{44}Ca, any calculation is practically out of the question now, mostly because of a lack of theoretical as well as experimental data on the absorption part of the optical model description of the π-nucleus interaction.

In summary, with the above mentioned reservations, we conclude from our experiments that the neutrons (rms radius 3.9 ± 0.15 F) seem to extend appreciably outside the protons (rms radius 3.55 F) at the nuclear surface of ^{44}Ca, if one admits that for ^{40}Ca both proton and neutron distributions are identical (rms radius 3.52 F).

We thank Dr. E. Segrè, Dr. K. Crowe, and Dr. C. Wiegand for their guidance and interest, Dr. R. Pehl for providing excellent detectors, and the 184"-cyclotron crew for their efficient and kind cooperation.

FOOTNOTES AND REFERENCES

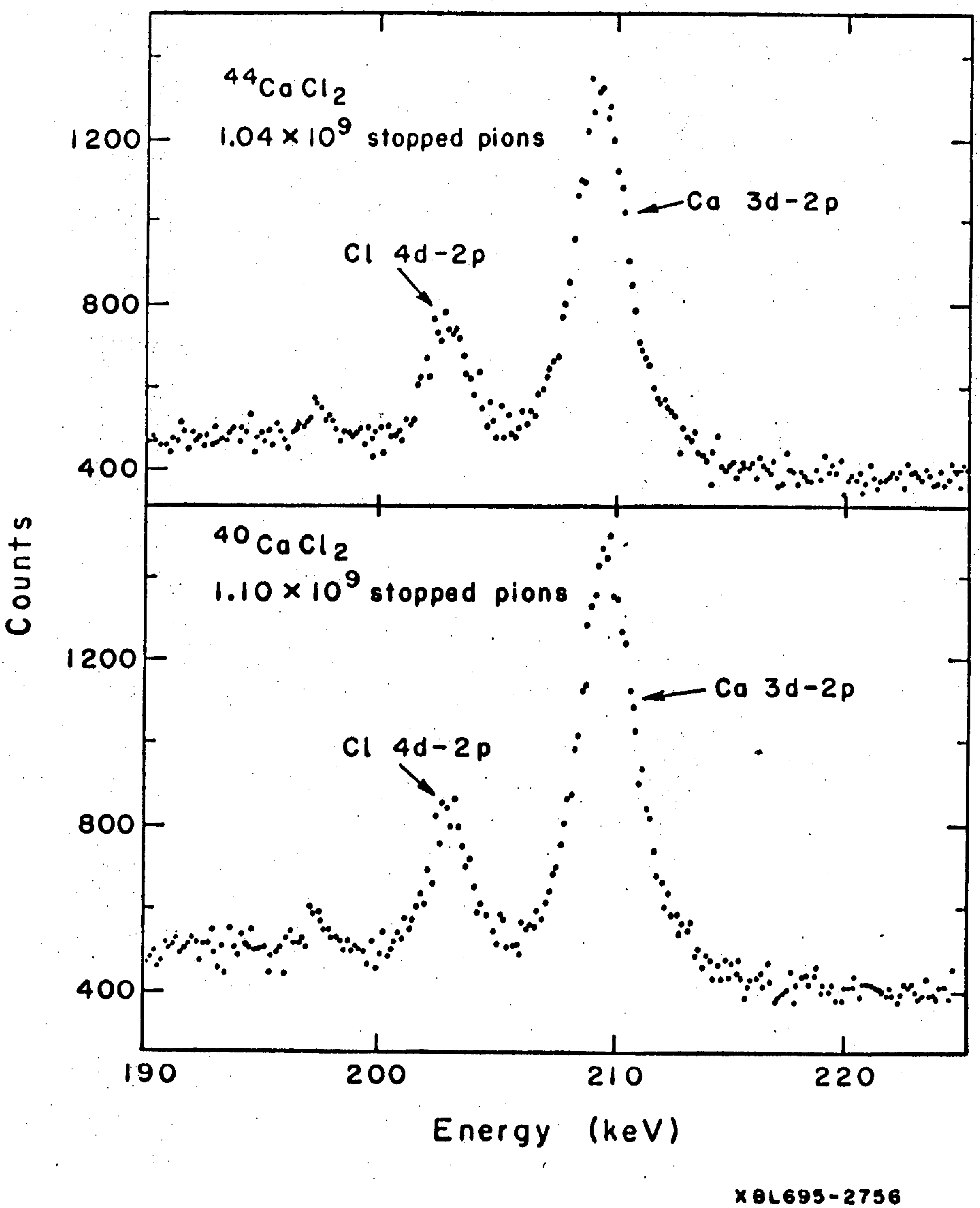
* Work done under auspices of the United States Atomic Energy Commission.

† Part of the work was performed while at the Lawrence Radiation Laboratory, University of California, Berkeley.

‡ On leave from Lausanne, Switzerland with a postdoctoral fellowship, SIN-ETH. Presently at the Swiss Embassy, Washington, D. C.

1. G. A. Grin and R. Kunselman, U. of Calif. Lawrence Rad. Lab. Rep. No. UCRL-18499, (1969); Phys. Letters 31B, 116 (1970).
2. G. E. Pustovalov, Zh. Eksperim. i Teor. Fiz. (U.S.S.R.) 36, 1806 (1959); English translation: Soviet Phys. — JETP 36, 1288 (1959).
3. A. B. Mickelwait and H. C. Corben, Phys. Rev. 96, 1145 (1954).
4. C. W. Allen, Astrophysical Quantities, The Athlone Press, The University of London, London, England, 2nd Edition, p. 82 (1963).
5. R. Hofstadter, G. K. Nöldeke, K. J. van Oostrum, L. R. Suelzle, and M. R. Yearian, Phys. Rev. Letters, 15, 758 (1965).
6. M. Ericson and T.E.O. Ericson, Ann. Phys. 36, 323 (1966).
7. M. Krell and T.E.O. Ericson, Nucl. Phys. B11, 521 (1969).
8. D. A. Jenkins and R. Kunselman, Phys. Rev. Letters, 17, 1148 (1966).
9. D. K. Anderson, D. A. Jenkins and R. J. Powers, ~~to be published.~~ Phys. Rev. Letters 24, 71 (1970).
10. D. A. Jenkins, R. J. Powers, and G. H. Miller, ~~to be published.~~ Phys. Rev. 185, 1508 (1969).
11. L. R. B. Elton, Phys. Rev., 158, 970 (1967).
12. J. P. Schiffer, J. A. Nolen, Jr. and N. Williams, Phys. Letters 29B, 399 (1969).
13. C. E. Wiegand and R. Kunselman, Proceedings of the International Conference on Hypernuclear Physics, Argonne, p. 669 (1969).
    C. E. Wiegand, Phys. Rev. Letters 22, 1235 (1969).

Fig. 1. $^{40}CaCl_2$ and $^{44}CaCl_2$ spectra in the region of the Ca 3d-2p lines.



^{44}Ca Cl$_2$
1.04 × 10^9 stopped pions
Ca 3d-2p
Cl 4d-2p
^{40}Ca Cl$_2$
1.10 × 10^9 stopped pions
Ca 3d-2p
Cl 4d-2p
Counts
1200
800
400
1200
800
400
190
200
210
220
Energy (keV)
XBL695-2756

Fig. 1.