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DOUBLE-POMERON DECOUPLING AND THE RELATION OF EXCLUSIVE TO INCLUSIVE EXPERIMENTS WITH THE DUAL RESONANCE MODEL

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Abstract: The double-pomeron coupling strength in the dual resonance model is found in both the inclusive and exclusive regions by comparison with experiments. Double-pomeron coupling occurs in inclusive experiments in the Mueller diagram for the central plateau region. Its strength can also be bounded from its non-observation in the two-particle to four-particle exclusive experiments. The dual resonance model is used to perform the analytic continuation of a six-point amplitude between these regions. The results show that the coupling strength for two forward pomerons in the exclusive region must be less than $\frac{1}{300}$ of that in the inclusive region. This is experimental evidence for substantial forward double-pomeron decoupling in exclusive processes.

1. Introduction

In order to learn more about the nature of the pomeron singularity, we have examined theoretical models for double-pomeron exchange in two of its experimental occurrences [1]. The first occurrence is in the observation of a central plateau or pionization region in the single-particle spectrum at ISR energies [2]. The pionization cross section has been related by Mueller [3] to an absorptive part of a forward $3 \to 3$ scattering amplitude involving double-pomeron exchange as indicated in fig. 1a. The second occurrence is in the $2 \to 4$ production amplitude in the double Regge region as in fig. 2a. This has been studied for $\pi^- p \to \pi^-(\pi^+\pi^-)\, p$ at 25 GeV/c by Lipes, Zweig and Robertson(LZR) [4]. More recently, others [5] have also searched for double-pomeron exchange in $pp \to p\,(\pi^+\pi^-)\, p$.

In this paper we will study the dual resonance model tree diagrams for the six-point amplitude with the inclusion of pomeron trajectories. The dual resonance am-

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plitude can be analytically continued between the pionization and production region, and we will determine the double-pomeron coupling strengths needed for agreement with each experiment. We find that the coupling strength in the exclusive region must be about $\frac{1}{300}$ of that in the inclusive region. By fixing the double-pomeron coupling strength from the inclusive experiments, we can quantitatively state from the exclusive experiment that double-pomeron exchange must decouple in the forward exclusive process by at least a factor of $\frac{1}{300}$. This experimental evidence agrees with the proof of Finkelstein and Kajantie [6] that double-pomeron coupling must vanish in the forward exclusive process if the pomeron is a Regge pole of unit intercept.

The relation of the absorptive part in $M^2 = (p_a + p_b + p_{\bar{c}})^2$ of the forward six-point function for $a + b + \bar{c} \to a + b + \bar{c}$ to the inclusive single-particle spectra for $a + b \to c + X$ has been derived by Mueller. It was applied to the dual resonance model six-point amplitude by DeTar et al. [7] and others [8]. We present the form suitable for pomerons of intercept one in the forward direction and obtain the double-pomeron coupling strength by fitting to the pionization spectrum.

The six-point dual model may also be used to evaluate the $2 \to 4$ cross section in the double Regge region. Since LZR did not observe double-pomeron exchange we can set an upper limit to the double-pomeron coupling strength in this region. Since the pomeron exchanges should be strongly damped in momentum transfer, we approximate the dual resonance model for pomeron exchanges at zero momentum transfer (forward kinematics) and add phenomenologically the observed exponential damping in pomeron momentum transfers.

The dual diagrams for the six-point function which are present in the double Regge limit with pomeron exchange are shown in figs. 1 and 2. Only the diagram of fig. 1a has an M^2 absorptive part and contributes to pionization in the 3–3 region.

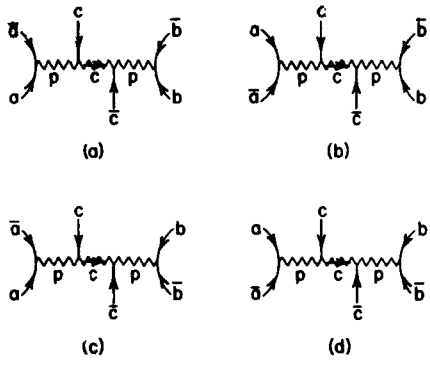

Fig. 1. Dual resonance model diagrams with double-pomeron exchange contributing to B_{tu}. Only (a) has an M^2 discontinuity in the inclusive region.

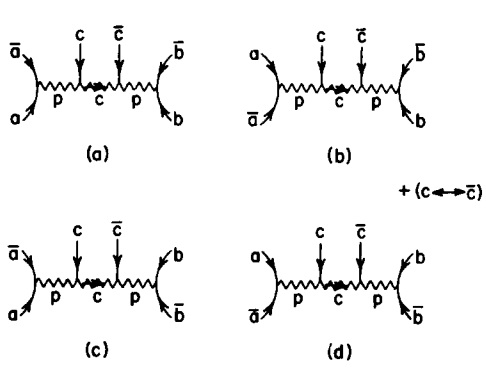

Fig. 2. Dual resonance model diagrams with double-pomeron exchange contributing to B_{st}.

However, this term and the additional signature terms in fig. 1 actually vanish in the analytic continuation to the forward 2–4 region. So the diagrams that give the $2 \to 4$ cross section in the forward limit are those of fig. 2. By the use of crossing, the coupling strength for diagrams of fig. 2 with pion poles in pomeron + pomeron $\to \pi + \pi$ must be the same for the diagrams of fig. 1 for pomeron + $\pi \to \pi$ + pomeron. This connection allows us to compare the coupling strengths in the pionization and production regions. The coupling strength can also be obtained from the residue of the pion pole in the $2 \to 4$ double-pomeron exchange. The value obtained there is consistent with the limit set by the LZR analysis. The inconsistency of the coupling strengths we find arises because the full dual resonance amplitude for double-pomeron exchange does not satisfy the decoupling behavior in the forward exclusive process as required by unitarity [6].

In sect. 2 we evaluate the dual resonance model six-point function in the double-pomeron exchange limit and construct the signature structure. In sect. 3 we evaluate the six-point amplitude in the production region with the forward scattering approximation and show that the diagrams of fig. 1 vanish here. In sect. 4 we compute the production cross section using the diagrams of figs. 1 and 2 and find the strength of the coupling consistent with the LZR experiment. The strength of the coupling consistent with the pionization region is found in sect. 5 where we take the M^2 discontinuity and compare it to the ISR data. Our conclusions and a discussion of the theoretical sources of the discrepancy of the double-pomeron coupling strengths are presented in sect. 6.

2. Dual resonance model for six-point amplitude with double-pomeron exchange

The dual resonance model for the six-point amplitude in the Bardakci-Ruegg [9] formulation gives a completely defined form for the amplitude. In the regions in which c and $\bar{c}$ are close in momentum space ($\alpha_{c\bar{c}}$ not large) and double-pomeron exchange is dominant, the amplitude has contributions from the twelve diagrams in figs. 1 and 2. The pomeron exchanges are in the $a\bar{a}$ and $b\bar{b}$ channels, and the analytic form of the amplitudes are completely specified by the ordering of the external lines.

In the production region we label the momentum as in fig. 3. The correspondence is

$$p_a = p_0, \quad p_{\bar{a}} = -p_1, \quad p_b = p_5, \quad p_{\bar{b}} = -p_4, \quad p_c = -p_2, \quad p_{\bar{c}} = -p_3. \tag{2.1}$$

The double-Regge region has $s_{c\bar{c}} = (p_2 + p_3)^2 \geqslant 4m^2$ fixed and $s_{\bar{a}c}$, $s_{c\bar{b}}$ and s_{ab} positive and becoming asymptotic. This results in $s_{\bar{a}c} s_{c\bar{b}}/s_{ab}$ remaining finite in the physical region.

In the pionization region

$$p_a = -p_{\bar{a}}, \quad p_b = -p_{\bar{b}}, \quad p_c = -p_{\bar{c}} = q, \tag{2.2}$$

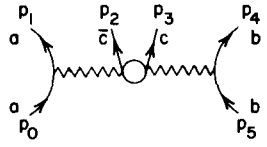

Fig. 3. Labeling of momentum vectors in the production region.

where q is the momentum of the observed particle, so that $s_{c\bar{c}}$ is fixed at zero. Also $s_{a\bar{c}}$, $s_{b\bar{c}}$ and $M^2 = s_{ab\bar{c}}$ become asymptotic with

$$\frac{s_{a\bar{c}} s_{b\bar{c}}}{s_{ab\bar{c}}} = q_\perp^2 + m^2 \tag{2.3}$$

fixed.

Thus to get the double-Regge behavior in both regions we analyze the six-point function in the limit

$$\begin{aligned} &\alpha_{a\bar{c}} \to -\infty\,, \qquad \alpha_{\bar{a}c} \to -\infty\,, \qquad \alpha_{b\bar{c}} \to -\infty\,, \\ &\alpha_{\bar{b}c} \to -\infty\,. \qquad \alpha_{ab\bar{c}} \to -\infty\,, \end{aligned} \tag{2.4}$$

with the following ratios fixed:

$$\frac{\alpha_{\bar{a}c}}{\alpha_{a\bar{c}}}\,, \quad \frac{\alpha_{b\bar{c}}}{\alpha_{\bar{b}c}}\,, \quad \frac{\alpha_{a\bar{c}}\,\alpha_{b\bar{c}}}{\alpha_{ab\bar{c}}}\,, \quad \frac{\alpha_{\bar{a}c\bar{c}}}{\alpha_{\bar{a}c}}\,, \quad \frac{\alpha_{c\bar{c}\bar{b}}}{\alpha_{\bar{c}\bar{b}}}\,. \tag{2.5}$$

We need only present the calculations of the first diagrams in fig. 1a and fig. 2a. This is because the values of the other diagrams, which make up the signature structure, can be found by substituting different variables in the results of the first diagrams, with appropriate continuations around the cuts.

The sum of the diagrams in figs. 1 and 2 is written as

$$T = \frac{1}{\pi^2 s_0}\, G_a G_b G_\pi^2 g_p\, (B_{tu} + B_{st})\,, \tag{2.6}$$

where G_a is the coupling strength of the pomeron to particle a at zero-momentum transfer and g_p is the remaining coupling strength to be fit to various experiments; B_{st} is the sum of diagrams in fig. 2 and st refers to the fact that it has dual poles in the s- and t-channels, as well as the s- and u-channels of pomeron + pomeron $\to \pi^+\pi^-$, while B_{tu} is the sum of diagrams in fig. 1 and has poles in the t- and u-channels of pomeron + pomeron $\to \pi^+\pi^-$. We first calculate the double-Regge limits of the first diagrams in fig. 1a and fig. 2a, B_{tu}^1 and B_{st}^1, respectively, and then discuss the rest of the diagrams through the signature factors.

We begin with B_{tu}^1 of fig. 1a, and the method follows that of ref. [7] but without the restriction to the pionization region *. The six-point amplitude for B_{tu}^1 can be

* We are indebted to C. DeTar for showing us the method for handling these integrals.

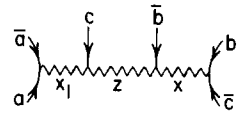

Fig. 4. Bardakci-Ruegg form of the amplitude which gives fig. 1a after performing a dual variable transformation.

obtained from the Bardakci-Ruegg form for $a+\bar{c} \to \bar{a}+c+\bar{b}+b$ (fig. 4) by a dual variable transformation [9]

$$x = \frac{1-x_2}{1-zx_2} ,$$

and gives

$$\begin{aligned} B^1_{tu} = & \int_0^1 \int_0^1 \int_0^1 dx_1 dz dx_2 \, x_1^{-\alpha_{a\bar{a}}-1} z^{-\alpha_{a\bar{a}c}-1} x_2^{-\alpha_{b\bar{b}}-1} \\ & \times (1-x_1)^{-\alpha_{\bar{a}c}-1} (1-z)^{-\alpha_{a\bar{a}\bar{c}}-1} (1-x_2)^{-\alpha_{b\bar{c}}-1} \\ & \times (1-x_1 z)^{-\alpha_{ab\bar{c}}+\alpha_{\bar{a}c}+\alpha_{\bar{b}c}} (1-x_2 z)^{-\alpha_{ab\bar{c}}+\alpha_{b\bar{c}}+\alpha_{a\bar{c}}} \\ & \times (1-x_1 z - x_2 z + x_1 x_2 z)^{\alpha_{ab\bar{c}}+\alpha_{a\bar{a}\bar{c}}-\alpha_{\bar{b}c}-\alpha_{a\bar{c}}} . \end{aligned} \tag{2.7}$$

Since the double-Regge region eq. (2.4) and (2.5) is dominated by

$$x_1 \sim O(-1/\alpha_{a\bar{c}}) , \qquad x_2 \sim O(-1/\alpha_{b\bar{c}}),$$

we substitute

$$x_1 = 1 - \exp(y_1/\alpha_{\bar{a}c}) , \qquad x_2 = 1 - \exp(y_2/\alpha_{b\bar{c}}) , \tag{2.8}$$

and drop terms in $O(1/\alpha_{a\bar{c}})$, $O(1/\alpha_{b\bar{c}})$, to obtain

$$B^1_{tu} = (-\alpha_{\bar{a}c})^{\alpha_{aa}} (-\alpha_{b\bar{c}})^{\alpha_{bb}} \int_0^1 dz \, z^{-\alpha_{a\bar{a}c}-1} (1-z)^{-\alpha_{a\bar{a}c}-1} I , \tag{2.9}$$

with

$$\begin{aligned} I = & \int_0^\infty dy_1 \int_0^\infty dy_2 \, y_1^{-\alpha_{a\bar{a}}-1} y_2^{-\alpha_{b\bar{b}}-1} \\ & + \exp\left[-\frac{y_1}{\alpha_{\bar{a}c}} (z\alpha_{a\bar{c}} + (1-z)\alpha_{\bar{a}c}) - \frac{y_2}{\alpha_{b\bar{c}}} (z\alpha_{\bar{b}c} + (1-z)\alpha_{b\bar{c}}) + y_1 y_2 \frac{\alpha_{ab\bar{c}}}{\alpha_{\bar{a}c}\alpha_{b\bar{c}}} z(1-z) \right] \end{aligned} \tag{2.10}$$

The integrals over y_1, y_2 converge in the limit (2.4) and (2.5) and can be expressed in terms of the confluent hypergeometric function $U(\alpha, \beta; \xi)$ (ref. [11]) (also called Ψ, ref. [12]) yielding

$$B^1_{tu} = \Gamma(-\alpha_{a\bar{a}})\,\Gamma(-\alpha_{b\bar{b}}) \int_0^1 dz\, z^{-\alpha_{a\bar{a}c}-1}\,(1-z)^{-\alpha_{a\bar{a}\bar{c}}-1}$$

$$\times\,[-\alpha_{a\bar{c}}\,z - \alpha_{\bar{a}c}(1-z)]^{\alpha_{a\bar{a}}}\,[-\alpha_{\bar{b}c}\,z - \alpha_{b\bar{c}}\,(1-z)]^{\alpha_{b\bar{b}}} \tag{2.11}$$

$$\times\,(-y)^{-\alpha_{a\bar{a}}}\,U(-\alpha_{a\bar{a}}, -\alpha_{a\bar{a}} + \alpha_{b\bar{b}} + 1\,;\, -y)\,,$$

where

$$y = \frac{[\alpha_{a\bar{c}}\,z + \alpha_{\bar{a}c}(1-z)]\;[\alpha_{\bar{b}c}\,z + \alpha_{b\bar{c}}(1-z)]}{\alpha_{ab\bar{c}}\,z(1-z)}\,. \tag{2.12}$$

The U-function has a cut for negative argument, i.e. y positive, and in the forward 3–3 region for pionization

$$y = \frac{\alpha_{a\bar{c}}\,\alpha_{b\bar{c}}}{\alpha_{ab\bar{c}}\,z(1-z)}$$

is positive and provides the M^2 discontinuity through $\alpha_{ab\bar{c}}$. The duality of fig. 1a shows up in (2.11) by the simultaneous Regge behavior in both $\alpha_{a\bar{c}}$ and $\alpha_{\bar{a}c}$ as well as $\alpha_{\bar{b}c}$ and $\alpha_{b\bar{c}}$. The expression is also unchanged under $(a, b, c) \leftrightarrow (\bar{a}, \bar{b}, \bar{c})$ as can be seen by substituting $z \to 1 - z$.

The other diagrams in fig. 1 can be obtained by the appropriate interchanges of momenta. For example, to evaluate the diagram of fig. 1b we interchange in (2.11) and (2.12)

$$\begin{aligned} &a \leftrightarrow \bar{a}\,, \\ &\alpha_{ab\bar{c}} \to \alpha_{\bar{a}b\bar{c}} \approx -\alpha' s \approx -\alpha_{ab\bar{c}}\,, \\ &\alpha_{a\bar{c}} \to \alpha_{\bar{a}\bar{c}} \approx -\alpha_{a\bar{c}}\,, \\ &\alpha_{\bar{a}c} \to \alpha_{ac} \approx -\alpha_{\bar{a}c}\,. \end{aligned} \tag{2.13}$$

For the pionization region, we take the $\alpha_{ab\bar{c}}$ discontinuity in the $ab\bar{c}$ channel to obtain the single-particle spectrum. This arises from only fig. 1a [7] since the other amplitudes do not contain $\alpha_{ab\bar{c}}$.

In the production region we use the $+\,i\epsilon$ prescription embodied in [13]

$$\alpha_{\bar{a}c} + i\epsilon = -\,e^{-i\pi}\alpha_{\bar{a}c}\,, \qquad \alpha_{\bar{a}\bar{c}} = -\,e^{-i\pi}\alpha_{\bar{a}\bar{c}}\,, \tag{2.14}$$

$$\alpha_{\bar{b}c} + i\epsilon = -\,e^{-i\pi}\alpha_{\bar{b}c}\,, \qquad \alpha_{\bar{b}\bar{c}} = -\,e^{-i\pi}\alpha_{\bar{b}\bar{c}}\,, \tag{2.15}$$

$$\alpha_{ab\bar{c}} + i\epsilon = -\,e^{-i\pi}\alpha_{ab\bar{c}}\,, \qquad \alpha_{\bar{a}\bar{b}\bar{c}} = -\,e^{-i\pi}\alpha_{\bar{a}\bar{b}\bar{c}}\,. \tag{2.16}$$

This involves carefully accounting for the phases in the terms of (2.11) and the crossed terms of fig. 1 as well as for the cuts in y for the different regions of z integration. Defining

$$z_a \equiv \frac{1}{1-\alpha_{a\bar{c}}/\alpha_{\bar{a}c}}, \qquad z_b \equiv \frac{1}{1-\alpha_{\bar{b}c}/\alpha_{b\bar{c}}}, \tag{2.17}$$

we find the result for the sum of the diagrams in fig. 1 in the production region

$$\begin{aligned} B_{tu} &= \Gamma(-\alpha_{a\bar{a}})\,\Gamma(-\alpha_{b\bar{b}})\,(\alpha_{\bar{a}c})^{\alpha_{a\bar{a}}}\,(\alpha_{\bar{b}\bar{c}})^{\alpha_{b\bar{b}}} \\ &\times \int_0^1 dz\, z^{-\alpha_{a\bar{a}\bar{c}}-1}(1-z)^{-\alpha_{a\bar{a}\bar{c}}-1}\,|1-z/z_a|^{\alpha_{a\bar{a}}}\,|1-z/z_b|^{\alpha_{b\bar{b}}} \\ &\times (1+e^{-i\pi\alpha_{a\bar{a}}})(1+e^{-i\pi\alpha_{b\bar{b}}})(-y-i\epsilon)^{-\alpha_{a\bar{a}}}\,U(-\alpha_{a\bar{a}},-\alpha_{a\bar{a}}+\alpha_{b\bar{b}}+1; \\ &-y-i\epsilon) - 2i\,\mathrm{disc}_y[(-y-i\epsilon)^{-\alpha_{a\bar{a}}}\,U(-\alpha_{a\bar{a}},-\alpha_{a\bar{a}}+\alpha_{b\bar{b}}+1;-y-i\epsilon)]. \end{aligned} \tag{2.18}$$

To evaluate the double-Regge limit of the diagram fig. 2a we begin with the Bardakci-Ruegg form [9]

$$\begin{aligned} B_{st}^1 &= \int_0^1\int_0^1\int_0^1 du_1\,du_2\,du_3\; u_1^{-\alpha_{a\bar{a}}-1}(1-u_1)^{-\alpha_{\bar{a}c}-1} \\ &\times u_2^{-\alpha_{a\bar{a}c}-1}(1-u_2)^{-\alpha_{c\bar{c}}-1}\,u_3^{-\alpha_{b\bar{b}}-1}\,(1-u_3)^{-\alpha_{\bar{c}\bar{b}}-1} \\ &\times (1-u_1u_2)^{-\alpha_{\bar{a}c\bar{c}}+\alpha_{\bar{a}c}+\alpha_{c\bar{c}}}(1-u_2u_3)^{-\alpha_{ccb}+\alpha_{c\bar{c}}+\alpha_{cb}} \\ &\times (1-u_1u_2u_3)^{-\alpha_{ab}+\alpha_{\bar{a}c\bar{c}}+\alpha_{cc\bar{b}}-\alpha_{c\bar{c}}}. \end{aligned} \tag{2.19}$$

For the double-Regge limit

$$\begin{aligned} &\alpha_{\bar{a}c}\to-\infty, \qquad \alpha_{\bar{c}\bar{b}}\to-\infty, \qquad \alpha_{ab}\to-\infty, \\ &\kappa \equiv \frac{\alpha_{\bar{a}c}\,\alpha_{\bar{c}\bar{b}}}{\alpha_{ab}}, \qquad \frac{\alpha_{\bar{a}c\bar{c}}}{\alpha_{\bar{a}c}}, \qquad \frac{\alpha_{cc\bar{b}}}{\alpha_{\bar{c}\bar{b}}} \quad \text{fixed}, \end{aligned} \tag{2.20}$$

we substitute

$$u_1 = 1-\exp(y_1/\alpha_{\bar{a}c}), \qquad u_3 = 1-\exp(y_3/\alpha_{\bar{c}\bar{b}}), \tag{2.21}$$

and drop higher-order terms in $O(1/\alpha_{\bar{a}c})$, $O(1/\alpha_{\bar{c}\bar{b}})$ in the integral to obtain

$$B^1_{st} = (-\alpha_{\bar{a}c})^{\alpha_{a\bar{a}}} (-\alpha_{\bar{c}\bar{b}})^{\alpha_{b\bar{b}}} \int_0^1 du_2\, u_2^{-\alpha_{\bar{a}ac}-1} (1-u_2)^{-\alpha_{c\bar{c}}-1}$$

$$\times \int_0^\infty dy_1 \int_0^\infty dy_3\, y_1^{-\alpha_{a\bar{a}}-1}\, y_3^{-\alpha_{b\bar{b}}-1} \tag{2.22}$$

$$\times \exp\left[-\frac{y_1}{\alpha_{\bar{a}c}} (\alpha_{\bar{a}c}(1-y_2) + \alpha_{\bar{a}c\bar{c}} u_2) - \frac{y_3}{\alpha_{\bar{c}\bar{b}}} (\alpha_{\bar{c}\bar{b}}(1-u_2) + \alpha_{c\bar{c}\bar{b}} u_2) + y_1 y_3 u_2 \frac{\alpha_{ab}}{\alpha_{\bar{a}c}\alpha_{\bar{c}\bar{b}}} \right].$$

These amplitudes for fig. 2 do not have discontinuities in $\alpha_{ab\bar{c}}$ and do not contribute to pionization. The y_1 and y_3 integrals may be done as in (2.11) and changing $u_2 = z$

$$B^1_{st} = \Gamma(-\alpha_{a\bar{a}})\,\Gamma(-\alpha_{b\bar{b}}) \int_0^1 dz\, z^{-\alpha_{a\bar{a}c}-1} (1-z)^{-\alpha_{c\bar{c}}-1}$$

$$\times [-\alpha_{\bar{a}c\bar{c}} z - \alpha_{\bar{a}c}(1-z)]^{\alpha_{a\bar{a}}} [-\alpha_{c\bar{c}\bar{b}} z - \alpha_{\bar{c}\bar{b}}(1-z)]^{\alpha_{b\bar{b}}} \tag{2.23}$$

$$\times (-x)^{-\alpha_{a\bar{a}}} U(-\alpha_{a\bar{a}}, -\alpha_{a\bar{a}} + \alpha_{b\bar{b}} + 1; -x),$$

where

$$x = \frac{[\alpha_{\bar{a}c\bar{c}} z + \alpha_{\bar{a}c}(1-z)]\ [\alpha_{c\bar{c}\bar{b}} z + \alpha_{\bar{c}\bar{b}}(1-z)]}{\alpha_{ab} z}. \tag{2.24}$$

In the production region x is positive and we again have a cut from the U-function, which gives rise to cuts in s or α_{ab}. The sum of the diagrams in fig. 2 gives rise to the signature structure for the exchanged pomerons and is carried out analogously to (2.13) – (2.18):

$$B_{st} = (1 + e^{i\pi\alpha_{a\bar{a}}})(1 + e^{i\pi\alpha_{b\bar{b}}}) B^1_{st}(x + i\epsilon)$$
$$- e^{i\pi\alpha_{a\bar{a}}} e^{i\pi\alpha_{b\bar{b}}} (B^1_{st}(x + i\epsilon) - B^1_{st}(x - i\epsilon)). \tag{2.25}$$

For future use, we record that the U-function may be written in terms of the entire function M(ref. [11]) (also called Φ, ref. [12])

$$(-y)^{-\alpha_1} U(-\alpha_1, -\alpha_1 + \alpha_3 + 1; -y) = \frac{\pi}{\sin\pi(\alpha_3 - \alpha_1 + 1)} \left\{ \frac{(-y)^{-\alpha_1} M(-\alpha_1, -\alpha_1 + \alpha_3 + 1, -y)}{\Gamma(-\alpha_3)\,\Gamma(-\alpha_1 + \alpha_3 + 1)} \right.$$

$$\left. - \frac{(-y)^{-\alpha_3} M(-\alpha_3, -\alpha_3 + \alpha_1 + 1, -y)}{\Gamma(-\alpha_1)\,\Gamma(-\alpha_3 + \alpha_1 + 1)} \right\}. \tag{2.26}$$

The discontinuity of this function for $y \geqslant 0$ may be found from this form. The discontinuity cancels the poles in α_1 and α_3 and gives

$$\begin{aligned} &\mathrm{Im}_y \{\Gamma(-\alpha_1)\,\Gamma(-\alpha_3)\,(-y)^{-\alpha_1}\, U(-\alpha_1, -\alpha_1+\alpha_3+1, -y)\} \\ &\quad = -\pi\, \mathrm{e}^{-y}\, y^{-\alpha_1}\, U(\alpha_3+1, -\alpha_1+\alpha_3+1, y)\ . \end{aligned} \tag{2.27}$$

Another formula of interest for double-pomeron exchange [1, 11] is that at $\alpha_1 = 1, \alpha_3 = 1$

$$\mathrm{e}^{-y}\, U(2, 1, y) = (1+y)\, E_1(y) - \mathrm{e}^{-y}\ , \tag{2.28}$$

where E_1 is the exponential integral.

3. Six-point amplitude in the production region

In calculating the amplitude and cross section in the production region we will insert the physical knowledge that momentum transfers associated with pomeron exchange are strongly damped, since this is not present in the dual model. We thus insert the factors $\mathrm{e}^{\Omega s_{a\bar{a}}}\, \mathrm{e}^{\Omega s_{b\bar{b}}}$ with $\Omega = 5\ \mathrm{GeV}^{-2}$ in the amplitude for each pomeron exchange. These phenomenological factors have been used by others for treating pomerons in the dual model. This damping in $s_{a\bar{a}} = (p_a + p_{\bar{a}})^2$, $s_{b\bar{b}} = (p_b + p_{\bar{b}})^2$ motivates the simplifying approximation of taking $s_{a\bar{a}} \approx 0, s_{b\bar{b}} \approx 0$ in the rest of the amplitude. We define invariants for pomeron-pomeron $\to \pi^+\pi^-$

$$\begin{aligned} \tau &\equiv \alpha_{a\bar{a}c} = \alpha'(s_{a\bar{a}c} - m^2)\ , \\ \upsilon &\equiv \alpha_{a\bar{a}\bar{c}} = \alpha'(s_{a\bar{a}\bar{c}} - m^2)\ , \end{aligned} \tag{3.1}$$

where m is the pion mass and α' the slope of the trajectory. Then in the limit $s_{\bar{a}c}$, $s_{\bar{c}b} \to \infty$, $s_{c\bar{c}}$ fixed, and

$$s_{a\bar{a}},\ s_{b\bar{b}} \to 0 + \mathrm{O}\left(\frac{1}{s_{\bar{a}c}}\right) + \mathrm{O}\left(\frac{1}{s_{\bar{c}b}}\right)\ , \tag{3.2}$$

we obtain the forward kinematics:

$$\tau + \upsilon + \alpha' s_{c\bar{c}} = 0\ , \tag{3.3}$$

$$\frac{\alpha_{\bar{a}c\bar{c}}}{\alpha_{\bar{a}c}} = \frac{\alpha_{c\bar{c}b}}{\alpha_{c\bar{b}}} = -\alpha'\, \frac{s_{c\bar{c}}}{\tau}\ , \tag{3.4}$$

$$\frac{\alpha_{\bar{a}c}}{\alpha_{\bar{a}\bar{c}}} = \frac{\alpha_{b\bar{c}}}{\alpha_{\bar{b}c}} = -\frac{\tau}{\upsilon}\ , \tag{3.5}$$

$$\alpha_{\bar{a}c\bar{c}}\, \alpha_{c\bar{c}b} = \alpha_{ab}\, \alpha' s_{c\bar{c}}\ , \tag{3.6}$$

$$\frac{\alpha_{\bar{a}c}\,\alpha_{b\bar{c}}}{\alpha_{ab\bar{c}}} = -\frac{\alpha_{\bar{a}c}\,\alpha_{\bar{b}c}}{\alpha_{ab}} = -\frac{\tau^2}{\alpha' s_{c\bar{c}}} \quad . \tag{3.7}$$

In the contributions of B^1_{tu} and B^1_{st} to the parts of B_{tu} and B_{st} not involving discontinuities, the forward kinematic approximation $\alpha_{a\bar{a}} = 1$, $\alpha_{b\bar{b}} = 1$ allows considerable simplification since the U-function becomes a polynomial [11]

$$U(-1, 1, -y) = -(1+y) \,. \tag{3.8}$$

In the part of B_{tu} not involving the discontinuity the signature factors in (2.18) cancel the poles in (2.11) and the result at $\alpha_{a\bar{a}} = 1$, $\alpha_{b\bar{b}} = 1$ using (3.8) is

$$\begin{aligned} B_{tu} = \quad & \pi^2 \{ \alpha_{ab} B(-\tau+1, -\upsilon+1) + \alpha_{a\bar{c}}\,\alpha_{\bar{b}c} B(-\tau+2, -\upsilon) \\ & + (\alpha_{\bar{a}c}\,\alpha_{\bar{b}c} + \alpha_{a\bar{c}}\,\alpha_{b\bar{c}}) B(-\tau+1, -\upsilon+1) + \alpha_{\bar{a}c}\,\alpha_{b\bar{c}} B(-\tau, -\upsilon+2)\} \\ & - (B^1_{tu}(y + i\epsilon) - B^1_{tu}(y - i\epsilon)) \,, \end{aligned} \tag{3.9}$$

where

$$B(x, y) = \Gamma(x)\,\Gamma(y)/\Gamma(x+y) \,.$$

Now applying the foward kinematics (3.3) – (3.7) we find that the non-discontinuity terms vanish in the forward direction.

The discontinuity of B^1_{tu} arises from the cut of the U-function eqs. (2.11) and (2.12) when $y \geqslant 0$. In the production region, since $\alpha_{a\bar{c}}, \alpha_{b\bar{c}} < 0$ and $\alpha_{\bar{a}c}, \alpha_{\bar{b}c} > 0$ - this occurs for

$$z_a \equiv \frac{1}{1 - \alpha_{a\bar{c}}/\alpha_{\bar{a}c}} \leqslant z \leqslant \frac{1}{1 - \alpha_{\bar{b}c}/\alpha_{b\bar{c}}} \equiv z_b \,, \tag{3.10}$$

or with reversed order when necessary. In the forward approximation, however, from (3.5) we find that $z_a = z_b$ so that the integration over the discontinuity vanishes, provided that the integral is non-singular in the forward limit. This can be shown using (2.11) and (2.27) by first taking the discontinuity and then the forward limit $\alpha_{a\bar{a}} = 1$, $\alpha_{b\bar{b}} = 1$.

Thus we find that the continuation of the B_{tu} term, which gives the M^2 discontinuity for pionization, vanishes in the forward direction in the production region:

$$B_{tu} = 0 \qquad (\text{at } s_{a\bar{a}} = s_{b\bar{b}} = 0) \,. \tag{3.11}$$

Although the B_{tu} term does not vanish in the non-forward directions, it will be smaller than the B_{st} term for some region around the forward direction. Thus we neglect the B_{tu} term with respect to the B_{st} term in calculating the cross section in the production region. The application of crossing in the pomeron + pomeron $\to \pi\pi$ part of the amplitude requires that the B_{st} term have the same coupling constant as the B_{tu} term.

We now evaluate the terms in B_{st} in the forward production region, taking for the trajectory in the $c\bar{c}$ channel $\alpha_{c\bar{c}} = \alpha' s_{c\bar{c}} + \alpha_0$. Using (2.23) and (3.8) for $\alpha_{a\bar{a}} = 1$, $\alpha_{b\bar{b}} = 1$, we can perform the integration and obtain

$$B_{st} = -\pi^2 \{ \alpha_{ab} B(-\tau+1, -\alpha_{c\bar{c}}) + \alpha_{\bar{a}c\bar{c}}\, \alpha_{c\bar{c}\bar{b}}\, B(-\tau+2, -\alpha_{c\bar{c}})$$

$$+ (\alpha_{\bar{a}cc}\, \alpha_{\bar{c}\bar{b}} + \alpha_{\bar{a}c}\, \alpha_{c\bar{c}\bar{b}}) B(-\tau+1, -\alpha_{c\bar{c}}+1) \qquad (3.12)$$

$$+ \alpha_{\bar{a}c}\, \alpha_{\bar{c}\bar{b}}\, B(-\tau, -\alpha_{c\bar{c}}+2)\} - (B^1_{st}(x+i\epsilon) - B^1_{st}(x-i\epsilon)) + c \leftrightarrow \bar{c} .$$

Using the forward kinematics (3.3) – (3.7) we find

$$B_{st} = \pi^2 \frac{s}{s_{c\bar{c}}} \frac{\Gamma(-\tau+1)\,\Gamma(-\alpha_{c\bar{c}})}{\Gamma(-\tau-\alpha_{c\bar{c}}+2)} (1-\alpha_0)\,[\alpha_0(\tau-1)+\alpha_{c\bar{c}}]$$

$$- (B^1_{st}(x+i\epsilon) - B^1_{st}(x-i\epsilon)) + c \leftrightarrow \bar{c} \qquad (3.13)$$

The interchange $c \leftrightarrow \bar{c}$ is accomplished here by interchanging τ and v.

We note that in the case when $\alpha_{c\bar{c}}$ is the pomeron, the intercept is $\alpha_0 = 1$ and the pomeron gives a vanishing contribution to the non-discontinuity terms in B_{st} in the forward direction. Since the $\pi\pi$ channel also receives contributions form the P′ trajectory which will not vanish in the forward direction, we will keep only the P′ in B_{st} as the dominant contribution.

The discontinuity in B_{st} can be taken for $\alpha_{a\bar{a}} \neq 1$, $\alpha_{b\bar{b}} \neq 1$ using (2.27) and then evaluated at $\alpha_{a\bar{a}} = 1$, $\alpha_{b\bar{b}} = 1$ using (2.28)

$$\mathrm{Im}\, B^1_{st}(x+i\epsilon) = -\pi \alpha_{ab} \int_0^1 dz\, z^{-\tau} (1-z)^{-\alpha_{c\bar{c}}-1}$$

$$\times [(1+x) E_1(x) - e^{-x}] + c \leftrightarrow \bar{c} , \qquad (3.14)$$

where in the forward kinematics

$$x = \frac{[\alpha' s_{c\bar{c}}\, z - \tau(1-z)]^2}{\alpha' s_{c\bar{c}}\, z} , \qquad (3.15)$$

We see that this absorptive part in x or s_{ab} does not have a pion pole in the overlapping variable $\tau = \alpha_{a\bar{a}c}$ at $\tau = 0$.

Furthermore, although Im B^1_{st} has a pole at $\alpha_{c\bar{c}} = 1$, the symmetrizations of $c \leftrightarrow \bar{c}$ guarantees that this pole occurs in the S-wave from the daughter trajectory, not in the P-wave. For the case where $\alpha_{c\bar{c}}$ is the pomeron, this result coupled with (3.13) shows that the triple pomeron vertex vanishes in the dual model when all pomerons are at unit angular momentum. This has already been verified for the dual model in the triple Regge inclusive region [14]. Some of the decoupling properties of pomerons in the dual resonance model have also been found by others [15].

In the evaluation of the cross section in the production region we will not include the discontinuity parts since it is sufficient for our purposes to shows that the non-

discontinuity parts of (3.12) by themselves give much too large an exclusive cross section when using the coupling constant found in the inclusive region.

We can independently fix the double-pomeron coupling strength by factorization at the pion pole $\tau = 0$ or $\upsilon = 0$. A pion pole at $\tau = 0$ exists in the four diagrams of fig. 1 and only the first four diagrams of fig. 2, since the interchange of $c \leftrightarrow \bar{c}$ puts the poles into $\upsilon = \alpha_{a\bar{a}\bar{c}}$.

Since the poles are at spin zero, the six-point function splits into a product of four-point functions. The residues of these poles in the double-Regge limit can be evaluated for B_{tu} from (2.9) where the pole occurs from the divergence at $z \to 0$ and integration by parts evaluates the remainder of the integrand at $z = 0$ giving

$$B_{tu} \to -\frac{1}{\tau}(\alpha_{\bar{a}c})^{\alpha_{a\bar{a}}}\,(\alpha_{\bar{b}c})^{\alpha_{b\bar{b}}}\,\Gamma(-\alpha_{a\bar{a}})\,\Gamma(-\alpha_{b\bar{b}})$$
$$\times\,(1+e^{-i\pi\alpha_{a\bar{a}}})\,(1+e^{-i\pi\alpha_{b\bar{b}}})\ . \tag{3.16}$$

The discontinuity term in (2.18) does not possess a pion pole since it is a discontinuity in $\alpha_{ab\bar{c}}$ which is an overlapping variable to $\alpha_{a\bar{a}\bar{c}}$. Similarly we can evaluate the pion pole residue for B_{st} from (2.22) with $u_2 \to 0$

$$B_{st} \to (-\tau)^{-1}\,(\alpha_{\bar{a}c})^{\alpha_{a\bar{a}}}\,(\alpha_{\bar{c}\bar{b}})^{\alpha_{b\bar{b}}}\,\Gamma(-\alpha_{a\bar{a}})\,\Gamma(-\alpha_{b\bar{b}})$$
$$\times\,(1+e^{-i\pi\alpha_{a\bar{a}}})\,(1+e^{-i\pi\alpha_{b\bar{b}}})\ . \tag{3.17}$$

Again the discontinuity terms in α_{ab} does not have a pion pole since α_{ab} overlaps $\alpha_{a\bar{a}\bar{c}}$. The pion poles in (3.16) and (3.17) add and in the forward direction contribute to (2.6)

$$T = \frac{2}{\tau s_0}\,G_a G_b G_\pi^2\, g_p\,(\alpha_{\bar{a}c})\,(\alpha_{\bar{c}\bar{b}})\ . \tag{3.18}$$

By factorization of T into $a + \pi$ and $b + \pi$ forward absorptive parts we find $g_p = \frac{1}{2}$ in the production region.

4. Cross section for two-pion production in the double regge region

From the analysis of the previous section, we use the dominance of the forward amplitude through the B_{st} term to calculate the cross section for $\pi^- p \to \pi^-(\pi^+\pi^-)\,p$. Including the momentum transfer damping in $s_{a\bar{a}}, s_{b\bar{b}}$ with $\Omega = 5\ \mathrm{GeV}^{-2}$ we have in this approximation from (3.13)

$$T \cong G_\pi G_p G_\pi^2\,\frac{g_p}{s_0}\,e^{\Omega(s_{a\bar{a}}+s_{b\bar{b}})}\ \frac{s}{s_0}\ \psi(s_{c\bar{c}},\tau)\ , \tag{4.1}$$

where

$$\psi(s_{c\bar{c}}, \tau) = -\frac{s_0}{s_{c\bar{c}}} \frac{\Gamma(-\tau+1)\,\Gamma(-\alpha_{c\bar{c}})}{\Gamma(-\tau-\alpha_{c\bar{c}}+2)} (1-\alpha_0)\,[\alpha_0(\tau-1)+\alpha_{c\bar{c}}] + c \leftrightarrow \bar{c}. \quad (4.2)$$

We take $\alpha_{c\bar{c}}$ to be the P′ (f^0) trajectory with

$$\alpha_{c\bar{c}} = \tfrac{1}{2} + \alpha' s_{c\bar{c}} + i m_{f^0} \Gamma_{f^0} \left(\frac{s_{c\bar{c}} - 4m^2}{m_{f^0}^2 - 4m^2} \right)^{\frac{1}{2}}, \quad (4.3)$$

where

$$m_{f^0}^2 = 1.6\ \mathrm{GeV}^2 \ , \qquad m_{f^0}\,\Gamma_{f^0} = 0.2\ \mathrm{GeV}^2 \ .$$

The phase space for the cross section is factorized [1] in terms of a pseudo-three-body phase space for the production of a body of mass squared $s_{c\bar{c}} = (p_c + p_{\bar{c}})^2$ times the two-body phase space for pomeron + pomeron → $\pi^+\pi^-$. The pseudo-three-body phase space involves the subenergies $s_1 \equiv s_{\bar{a}c\bar{c}}$, $s_2 \equiv s_{c\bar{c}b}$ and the momentum transfers $s_{a\bar{a}}$, $s_{b\bar{b}}$. The two-body phase space involves $s_{c\bar{c}}$ and $\tau = \alpha_{\bar{a}ac}$. The separation is carried out in detail in ref. [1]. The result for the cross section is

$$\frac{d\sigma}{ds_{c\bar{c}}} = \frac{G_\pi^6 G_p^2 g_p^2}{2(2\pi)^8 s_0^2} I_3 I_2 \ ; \quad (4.4)$$

I_3 is the three-body phase space in which the $s_{a\bar{a}}$, $s_{b\bar{b}}$ integrations over (4.1) can be done exactly giving [16 *, 1]

$$I_3 = \frac{\pi^2}{4s_0^2} \int ds_1\, ds_2\, \frac{e^A \sinh X}{X} \ , \quad (4.5)$$

where

$$A = \Omega \left[s_1 + s_2 - 2s + 3m_a^2 + 3m_b^2 - \frac{1}{s}(m_b^2 - m_a^2)(s_2 - s_1 - m_a^2 + m_b^2) \right], \quad (4.6)$$

$$X^2 = \frac{\lambda(s, m_a^2, m_b^2)}{s^2} \Omega^2 \{\lambda(s, s_1, m_b^2) + \lambda(s, s_2, m_a^2) + 2s(s - s_1 - s_2 - m_a^2 - m_b^2 + 2s_{c\bar{c}}) - 2(s_1 - m_b^2)(s_2 - m_a^2)\} \ , \quad (4.7)$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx \ . \quad (4.8)$$

* The integral is in fact $\frac{1}{4}$ the value given in this reference.

This is integrated over the physical region subject to the additional constraints used by LZR [4] to isolate only the double Regge region

$$s_1 \geqslant 2\ \mathrm{GeV}^2\,, \qquad s_2 \geqslant 4\ \mathrm{GeV}^2\,, \qquad s_3 \equiv (p_{\bar{a}} + p_{\bar{b}})^2 \geqslant 4\ \mathrm{GeV}^2\,; \tag{4.9}$$

I_2 is the two-body phase space which can be factorized from I_3 by taking the forward approximation of $s_{a\bar{a}} = 0$, $s_{b\bar{b}} = 0$ in the limits of I_2 giving

$$I_2 = \frac{\pi}{2\alpha' s_{c\bar{c}}} \int_{\tau_-}^{\tau_+} \mathrm{d}\tau\, |\psi(s_{c\bar{c}}, \tau)|^2\,, \tag{4.10}$$

where

$$\tau \pm = \tfrac{1}{2}\alpha' \left[-s_{c\bar{c}} \pm (s_{c\bar{c}}\,(s_{c\bar{c}} - 4m^2))^{\frac{1}{2}}\right]. \tag{4.11}$$

The integrals I_2 and I_3 are evaluated numerically.

The zero-momentum transfer pomeron couplings G_a, etc., can be evaluated from the optical theorem

$$\sigma_{ab}^{\mathrm{tot}} = \frac{1}{s}\,\mathrm{Im}\, T_{ab}^{\mathrm{el}}\,(s, t = 0) = \frac{G_a G_b}{s_0}. \tag{4.12}$$

With $s_0 = 1\ \mathrm{GeV}^{-2}$, we find from pp and π p total cross sections

$$G_\pi = 6.2\,, \qquad G_p = 10.0\,. \tag{4.13}$$

With the value $g_p = \frac{1}{2}$, the calculated spectrum $\mathrm{d}\sigma/\mathrm{d}s_{c\bar{c}}$ is just bounded by the spectrum found by LZR [4].

The integrated cross section in the LZR region with $g_p = \frac{1}{2}$ is

$$\sigma_{\mathrm{calc}}^{\mathrm{LZR}} = 55\ \mu\mathrm{b}\,, \tag{4.14}$$

which is below the cross section observed by LZR of

$$\sigma_{\mathrm{exp}}^{\mathrm{LZR}} = 90\ \mu\mathrm{b}\,. \tag{4.15}$$

Since most of the LZR cross section was accounted for by (P′, P) and (ρ, P) exchange, this sets an upper limit in the production region

$$g_p \leqslant \tfrac{1}{2}\,. \tag{4.16}$$

5. Double-pomeron coupling strength determined from pionization

We will now compute the single-particle spectrum in the pionization region of $|q^0_{\mathrm{c.m.}}| < O(s^{\frac{1}{2}})$ by using the Mueller optical theorem. The optical theorem relates the $M^2 = (p_a + p_b + p_{\bar{c}})^2$ discontinuity of the forward amplitude for $T_{ab\bar{c} \to ab\bar{c}}$ to the single-particle cross section. In our normalization this is (see ref. [1])

$$\frac{d\sigma}{d^3q/E} = \frac{1}{2s(2\pi)^3} \operatorname{Im}_{M^2} T_{ab\bar{c}\to ab\bar{c}} \ . \tag{5.1}$$

We now compare the M^2 discontinuity of the dual resonance model with the recent ISR measurements of pionization to determine the value of g_p needed to fit the data. The M^2 discontinuity is only contained in the diagram of fig. 1a or the B^1_{tu} term of (2.18) which contains $\alpha_{ab\bar{c}}$, as discussed in sect. 2. This was first computed by DeTar et al. [7]. In the forward amplitude of (5.1) for pionization, the momenta are

$$\begin{aligned} &p_a = -p_{\bar{a}}\ , \quad p_b = -p_{\bar{b}}\ , \quad p_c = -p_{\bar{c}} = q\ , \\ &\alpha_{a\bar{a}c} = 0\ , \quad \alpha_{a\bar{a}\bar{c}} = 0\ , \quad s_{a\bar{a}} = 0\ , \quad s_{b\bar{b}} = 0\ , \\ &\alpha_{a\bar{c}} = \alpha_{\bar{a}c}\ , \quad \alpha_{b\bar{c}} = \alpha_{\bar{b}c}\ . \end{aligned} \tag{5.2}$$

The y-variable occuring in B^1_{tu} in (2.11) is then in the pionization region

$$y = \frac{\alpha_{a\bar{c}}\,\alpha_{\bar{b}c}}{\alpha_{ab\bar{c}}\,z(1-z)} = \frac{\alpha'(q_\perp^2 + m^2)}{z(1-z)}\ , \tag{5.3}$$

and is on the $y \geqslant 0$ cut for all $0 \leqslant z \leqslant 1$. The M^2 or y discontinuity for (2.11) is taken using (2.27), and evaluating it at $\alpha_{a\bar{a}} = 1$, $\alpha_{b\bar{b}} = 1$ gives [7]

$$\operatorname{Im}_{M^2} B^1_{tu} = -\pi\alpha_{ab\bar{c}} \int_0^1 dz\, e^{-y}\, U(2, 1, y)\ . \tag{5.4}$$

Combining this with (2.6) and using the simpler form (2.28) we find for the single-particle spectrum (5.1) in the pionization region

$$\frac{d\sigma}{d^3q/E} = \frac{1}{(2\pi)^4}\frac{\alpha'}{s_0} G_a G_b G_\pi^2 g_p \int_0^1 dz\, [(1+y)E_1(y) - e^{-y}]\ , \tag{5.5}$$

where y is given by (5.3). We note a similarity of this to the $q_\perp^2$ behavior of the multiperipheral model with exponential damping in momentum transfer [1, 17] which is a function of $\kappa = k(q_\perp^2 + m^2)$

$$\frac{d\sigma^{MP}}{d^3q/E} \propto [(1+\kappa)E_1(\kappa) - e^{-\kappa}]\ , \tag{5.6}$$

where k has been determined from data to be $k = 2.7\ \mathrm{GeV}^{-2}$.

We find the value of g_p by matching the prediction at $q_\perp^2 = 0$ from (5.5) with the extrapolated value of the ISR data on $p + p \to \pi^\pm + X$ from the Saclay/Strasbourg collaboration [2] which gives at $q_\perp = 0$:

$$\frac{d\sigma}{d^3q/E}\ (q_\perp = 0) = 140\ \text{mb/GeV}^2\,. \tag{5.7}$$

The integral in (5.5) at $q_\perp = 0$ is 0.90 and using (5.7) with (4.13) we find the intrinsic strength of the double pomeron coupling to be

$$g_p = 150\,. \tag{5.8}$$

This is the same order as the value found from pionization in the multiperipheral calculation [1] since the behavior of the dual model (5.5) is quite similar to (5.6), and the same factorizable coupling constants G_π, etc. have been removed to make the definitions of g_p equivalent. This is two orders of magnitude larger than the coupling consistent with the exclusive region result of eq. (4.16); $g_p \leqslant \frac{1}{2}$.

6. Conclusions

From the discrepancy of a factor of 300 between the coupling strengths needed to satisfy inclusive and exclusive experiments, it is clear that the dual resonance model tree diagrams do not perform a good continuation between these regions. This reduction of the coupling in the exclusive region is similar in magnitude to that found using a multiperipheral model for performing the continuation [1].

Theoretically, the discrepancy arises because neither the dual resonance nor the multiperipheral model used satisfied the requirement that the pomerons in the inclusive $2 \to 4$ process decouple in the forward direction ($s_{a\bar{a}} = 0$, $s_{b\bar{b}} = 0$). For pomerons of unit intercept, complete decoupling in forward double-pomeron exchange is required by unitarity, as shown by Finkelstein and Kajantie [6]. This decoupling could remove the discrepancy since the $2 \to 4$ cross section is strongly dominated near the forward direction due to the fast exponential damping in momentum transfer of the pomeron exchanges. We have given the statement of double-pomeron decoupling a quantitative meaning by fixing its strength in the inclusive region and showing from experiment that it must be at least 300 times smaller in the forward production region.

We note that this discrepancy can be observed simply from the residue of the pion pole giving $g_p = \frac{1}{2}$ compared to $g_p \approx 150$ from pionization. Using an amplitude with only the pion pole as in (3.18) with exponential damping in $s_{a\bar{a}}$, $s_{b\bar{b}}$ gives a cross section which is just consistent with the LZR bound. Therefore the magnitude of the discrepancy is really independent of the intricate details of the dual resonance model results (3.9) and (3.12).

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References

[1] C.H. Mehta and D. Silverman, Nucl. Phys. B52 (1973) 77.
[2] J.C. Sens, 4th. Int. Conf. on high energy collisions, Oxford, 1972.
[3] A.H. Mueller, Phys. Rev. D2 (1970) 2963.
[4] R. Lipes, G. Zweig and W. Robertson, Phys. Rev. Letters 22 (1969) 433.
[5] S. Humble, Daresbury preprint DNPL/P129 (1972);
Yamdagni and Carrilas, Colloquium on multiparticle dynamics, Helsinki, May, 1971;
L. Van Hove, Phys. Letters 28B (1969) 429; Nucl. Phys. B9 (1969) 331.
[6] J. Finkelstein and K. Kajantie, Phys. Letters 26B (1968) 305; Nuovo Cimento 56A (1968) 659.
[7] C.E. DeTar, K. Kang, C. – I. Tan and J. H. Weis, Phys. Rev. D4 (1971) 425.
[8] M.A. Virasoro, Phys. Rev. D3 (1971) 2834.
[9] K. Bardakci and H. Ruegg, Phys. Rev. 181 (1969) 1884.
[10] K. Kikkawa, B. Sakita and M.A. Virasoro, Phys. Rev. 184 (1969) 1701.
[11] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions, NBS Applied Mathematics Series 55 (1964) 504–510.
[12] A. Erdelyi, ed., Bateman Manuscripts, Higher transcendental functions, vol. I (McGraw-Hill, New York, 1953) Ch. VI.
[13] I.T. Drummond, P.V. Landshoff and W.J. Zakrzewski, Phys. Letters 28B (1969) 676.
[14] C.E. DeTar and J.H. Weis, Phys. Rev. D4 (1971) 3141.
[15] R. Brower, private communication.
[16] Chan Hong-Mo, K. Kajantie and G. Ranft, Nuovo Cimento 49 (1969) 157.
[17] D. Silverman and C. – I. Tan, Nuovo Cimento 2A (1971) 489.