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# Spatial Competition, Network Externalities, and Market Structure: An Application to Commercial Banking \*

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## Abstract

I develop a model of spatial competition between multi-branch firms in which consumers value the price of services, spatial proximity to their home branch, and the number of other branches in the firm's network. The model delivers within and across market predictions on the pattern and density of branching, the relationship of concentration to market size, the price-concentration relationship, and price dispersion. I consider the applicability of this model to the commercial banking market for retail deposits. I test the model's predictions by utilizing variation in the timing and extent of within-state branching restrictions on banks and bank holding companies. These restrictions constitute a plausible natural experiment with which to identify the role of spatial externalities on the equilibrium deposit-market structure of commercial banks. I also identify and parsimoniously model difficult-to-resolve conceptual issues in separating loan-side and deposit-side factors in the determination of bank market structure.

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## I Introduction

Since Hotelling [1929], spatial models have informed economists' understanding of the interaction between product differentiation, firms' pricing decisions, and consumer welfare. Within competition policy, these models provide an analytic framework for defining product and geographic markets, as well as for inferring market performance from market structure and firm conduct. Models of firm location in geographic space, including Hotelling's classic model, typically involve horizontal product differentiation. Firms compete by locating close to consumers. Consumers prefer firms in their geographic locality. Typically, as the market demand curve shifts out, a larger number of firms can enter and the equilibrium outcome becomes arbitrarily close to that of a competitive market. Models of vertical product differentiation, by contrast, involve firms choosing different levels of quality, where every consumer prefers a higher quality to lower quality, but consumers differ in their willingness to pay [Tirole (1989)]. In such models of quality competition, a few high quality firms can maintain large market shares even if the market—i.e. demand—grows arbitrarily large [Shaked & Sutton (1982)].

I consider a model where individual firms choose to locate multiples branches in geographic space. Consumers value both whether the firm's branch is close to them as well as the total number of other branches—or geographic locations—the firm has. All else equal, consumers prefer firms with more branches. The model thus contains both features of horizontal and vertical product differentiation. Firms find it advantageous to locate close to consumers, a la Hotelling. But the decision to build the marginal branch involves an externality because it raises consumers' willingness to pay for infra-marginal branches. This branching or location externality characterizes products as diverse as health clubs and wireless internet access, and could provide an explanation for common features of such markets.<sup>1</sup>

In this paper I focus on a particular application of a such a model—deposit competition among commercial banks. I ask to what extent bank competition for deposits explains persistent features of the market structure and size distribution of commercial banks? Since at least Diamond [1984], there is a large literature that explains the structure of banking markets by treating banks as delegated monitors of funds channelled from savers to borrowers. This literature emphasizes the special role of banks in making loans. More recent developments in the literature have begun to ask whether the deposit function could also play a role in explaining bank market structure. Banks compete for deposits by building large networks of branches and ATMs, offering a suite of services, and setting deposit interest

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<sup>1</sup>This model extends Miller [2010] to the case of multi-market competition and derives closed-form solutions for prices, shares, and markups that can be used to calculate the limiting distribution of market shares.

rates and fees. The strategic interaction between competing banks and the distribution of consumer preferences over these services could also shape important features of the banking market.

Understanding the determinants of bank market structure is, in turn, a prerequisite for deciding how to optimally regulate financial institutions. By a variety of measures, the concentration of the commercial banking industry has increased over the last several decades. To the extent that this increase has been driven by the demand for credit on the part of borrowing firms and households, by the demand for state-wide or nation-wide banking services by consumers, or by technological change in the banking industry, it has likely been welfare-improving. To the extent that it has been driven by a desire to become too big to fail, to acquire government-insured sources of funds, or to exert market power, policy makers should be concerned.

The empirical difficulty in identifying the effect of deposit competition is that bank market structure is simultaneously determined by firm and consumer demand for credit, bank lending technology, bank demand for external funds, and consumer supply of deposits and demand for services. This poses a formidable identification challenge. The conceptual experiment requires holding fixed the loan side of the market while varying the ability of banks to compete for deposits. To this end, I utilize the history of state bank-branching restrictions in the United States as a natural experiment to identify the effect of changes in deposit market competition. For reasons that I detail below, these branching restrictions constrained the ability of banks to compete for deposits, but had less of an effect on their lending activity. By lifting these restrictions, states enabled banks to compete for deposits through branch networks.

I use state bank-branching restrictions to test whether the proposed model can explain salient features of deposit market structure. In the model, banks compete by locating close to consumers and offering a deposit rate. The locations that banks can choose constitute a set of neighborhoods that, taken together, constitute a market. (Alternatively, the locations constitute a set of markets that, taken together, constitute a state or region.) Each consumer chooses a bank in its neighborhood. At a given deposit rate, all consumers prefer a bank with locations in many neighborhoods to one with fewer locations.

The model generates the following predictions when branching restrictions are lifted and banks are able to compete by building large branch networks. First, the density of branches should increase within a market and neighborhood. Second, market concentration should increase, while neighborhood concentration should decrease. The intuition for the latter result is as follows. Within a small enough locality or neighborhood, product differentiation through network size allows banks of different scope to co-exist without competing aggressively on price. Without such differentiation, fewer banks can coexist.

Further concentration with a market and concentration across markets should be quite similar.

Third, the negative relationship between market concentration and market size should decrease in magnitude. Put differently, market concentration should increase more in larger markets than in smaller ones. Fourth, with branching there should be a negative association between the size of a bank's branch network and its deposit rate. Consumers should be willing to accept a lower deposit rate from a bank with a large network. This negative association should increase in magnitude with the ability of banks to engage in branch network competition. Fifth, the variance in deposit interest rates should increase with branching. Greater product variety should lead to greater price dispersion.

## II Summary of Results

[To be completed]

## III A Spatial Model of Deposit Competition

I construct an equilibrium model of bank lending and deposit taking where the deposit side determines market structure. In this model consumer heterogeneity in willingness to pay for bank scope gives rise to differing deposit market shares. Deposit market shares are an equilibrium consequence of strategic competition among banks of differing scope, where scope is measured by the size of bank's branch network. The model is based on Miller (2010), but extended to multi-market entry and competition. This family of models was investigated, in some detail, by Shaked and Sutton [1982, 1983, 1988]. A formal description of the game, existence proofs and equilibrium characterization results are contained in Appendix A.

In the first stage of the game, banks sequentially choose to enter a subset of  $N + 1$  distinct neighborhoods or localities. Together these neighborhoods constitute a market. There are enough potential entrants that every neighborhood can contain at least one bank that is in  $0, 1, 2, \dots, N$  other neighborhoods. In other words, each neighborhood can have a bank of every possible scope. Each bank pays a fixed entry cost  $F$  to enter an individual neighborhood. Subsequent to entry decisions, each bank chooses a deposit price  $r^d$  in every neighborhood they enter.

Conditional on bank entry decisions and deposit rates, consumers in each neighborhood choose the bank that maximizes their utility. Each neighborhood contains a continuum of depositors characterized by a preference parameter  $\alpha$  distributed on  $[0, 1]$  according to  $F(\alpha)$ . Let  $j$  index the set of banks who enter neighborhood  $n$  and order these banks by their scope—the total number of neighborhoods they enter. The utility of a consumer in

neighborhood  $n$  when choosing bank  $j$  is given by:

$$u(\alpha, j) = r_{nj}^d + \gamma\alpha\left(\frac{N_{nj}}{N} - 1\right)$$

where  $N_{nj}$  is the number of neighborhoods outside  $n$  in which bank  $j$  has a presence and  $N$  is the total number of outside neighborhoods. I refer to a bank's outside neighborhood presence ( $N_{nj}$ ) as its *outside scope*.<sup>2</sup>

In order to characterize the sub-game perfect equilibrium of this game, I first consider the final-stage deposit competition in a neighborhood  $n$ . As in Miller (2010), the  $J_n$  banks in neighborhood  $n$  are ordered in terms of their outside scope so that  $N_{n1} < N_{n2} < \dots < N_{nJ_n}$ . For a consumer in neighborhood  $n$  to be indifferent between bank  $j$  and  $j - 1$ , it is required that:

$$\alpha_{nj} = \begin{cases} \frac{r_{n1}^d}{\gamma \cdot \left(1 - \frac{N_{n1}}{N}\right)}, & j = 1 \\ \frac{r_{n,j-1}^d - r_{nj}^d}{\gamma \cdot \left(\frac{N_{nj} - N_{n,j-1}}{N}\right)}, & j > 1 \end{cases}$$

It follows that the neighborhood shares in neighborhood  $n$  are given by:

$$s_{nj} = \begin{cases} F(\alpha_{n,j+1}) - F(\alpha_{nj}), & 0 \leq j < J_n \\ 1 - F(\alpha_{n,J_n}), & j = J_n \end{cases}$$

From here forward, I assume that  $F(\alpha)$  is the uniform distribution on  $[0, 1]$  in order to obtain analytic solutions where possible.

Consider a final-stage equilibrium in neighborhood  $n$  in which there exists some consumer who is indifferent between bank  $j$  and  $j - 1 \forall j > 1$ . Each bank  $j$  chooses deposit price  $r_{nj}^d$  to maximize profits:

$$r_{nj}^d \in \arg \max_{r_{nj}^d} (r^l - r_{nj}^d) \cdot s_{nj}(\mathbf{r}_n^d, \mathbf{N})$$

where  $r^l$  is the competitive return on loans and a bank's neighborhood share  $s_{nj}(\mathbf{r}_n^d, N_n)$  is a function of each bank's deposit rate ( $\mathbf{r}_n^d$ ) and outside neighborhood scope ( $N_n$ ). The first order conditions that define the above reaction functions are [Miller 2010]:

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<sup>2</sup>Because all consumers weakly value banks with greater outside scope, the model is one of vertical product differentiation described by Shaked and Sutton (1982, 1983, 1988). Competition across markets also has features of horizontal product differentiation, a la Hotelling, because all consumers choose a bank within their market  $n$ . I rely on well-known results for this class of models to characterize the sub-game perfect Nash equilibria of this game.

$$\begin{aligned}
r_{n1}^d &= \frac{1}{2} \cdot (r^l + r_{n2}^d) \text{ if } r_{n1}^d \geq 0 \\
r_{nj}^d &= \frac{1}{2} \cdot \left( r^l + \frac{(N_{n,j+1} - N_{nj})}{(N_{n,j+1} - N_{n,j-1})} \cdot r_{n,j-1}^d + \frac{(N_{nj} - N_{n,j-1})}{(N_{n,j+1} - N_{n,j-1})} \cdot r_{n,j+1}^d \right), \quad 1 < j < J_n \\
r_{nJ_n}^d &= \frac{1}{2} \cdot \left( r^l + r_{n,J_n-1}^d - \gamma \cdot \frac{(N_{n,J_n} - N_{n,J_n-1})}{N} \right)
\end{aligned}$$

It follows that the final-stage price equilibrium in any neighborhood  $n$  is the solution to this system of equations. From the logic of undifferentiated, Bertrand competition, two banks of the same scope cannot earn positive profits in equilibrium. Therefore, in any given neighborhood, at most  $N + 1$  banks can earn positive profits. It can also be shown that  $r_{nj}^d$  is decreasing in  $j$ . Intuitively, banks of greater scope can offer consumers a lower deposit interest rate.

By backward induction, banks choose an entry strategy anticipating equilibrium profits in each neighborhood that they enter. These profits are a function of the entry decisions taken by other banks. In choosing to enter an additional neighborhood  $n$ , a bank must consider both its profits in neighborhood  $n$  and the change in profits in all (infra-marginal) neighborhoods due to its increase in scope. When choosing to enter an additional neighborhood a bank's profits will increase for every other neighborhood in which this increase in scope does not lead to undifferentiated price competition. For convenience, I restrict the strategy space so that banks must make positive variable profits in each neighborhood they enter. This rules out situations where a bank makes negative variable profits in one neighborhood, but increases overall profits due to its greater scope. Because banks can be indifferent across different entry strategies, the entry equilibrium need not be unique. Nevertheless, these equilibria share important properties.

I consider equilibria of this game where  $F$  can be considered small<sup>3</sup>. By the logic of Bertrand price competition, no two banks of equal scope can coexist in the same neighborhood. Let  $N^*(N_j)$  denote the largest number of banks of outside scope  $N_j$  that can exist in any equilibrium across all  $N + 1$  neighborhoods and let  $[\cdot]$  denote the smallest integer part of the expression in brackets:

$$N^*(N_j) = \left[ \frac{N + 1}{N_j + 1} \right] \quad \forall N_j \in \mathbb{Z}, 0 \leq N_j \leq N$$

It follows that  $N^*(N_j)$  is decreasing in  $N_j$ . The largest number of banks that enter at

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<sup>3</sup>Alternatively, this corresponds to a situation in which demand in each market is large, so that small market shares are sufficient to cover the fixed cost of entry.

least one neighborhood in equilibrium is given by  $\sum_{N_j=0}^N N^*(N_j)^4$ . The number of banks that can enter at least one neighborhood in equilibrium grows arbitrarily large with the total number of neighborhoods ( $N + 1$ ).

The model does not permit an exact calculation of equilibrium market share for each bank. It is still possible, however, to obtain a lower bound for the market share—across all  $N + 1$  neighborhoods—of a bank with outside scope  $N_j$ . Such a bank would obtain its lowest market share in a neighborhood  $n$  where there exists a bank of every possible scope. A lower bound for its share across all  $N + 1$  neighborhoods is therefore given by  $(\frac{N_j+1}{N+1}) \cdot s_{nj}(\mathbf{r}_n^d, N_n)$  where  $N_n = \{0, 1, 2, \dots, N\}$ .

This lower bound can be obtained explicitly from the equilibrium deposit rates in a neighborhood with banks of every possible scope, that is, where  $N_n = \{0, 1, 2, \dots, N\}$ . In Appendix A I show that in such a neighborhood, the interest rate charged by any bank  $j$ , where  $j$  denotes its outside scope, is given by:

$$\begin{aligned} r_j^{d*} &= r^l - \frac{1}{3} \cdot \frac{1}{U_{N-1}(2)} \cdot T_{j-1}(2) \cdot \frac{2\gamma}{N} \\ &= r^l - \frac{\sqrt{3}}{3} \cdot \left[ \frac{(2 + \sqrt{3})^{j-1} + (2 - \sqrt{3})^{j-1}}{(2 + \sqrt{3})^N - (2 - \sqrt{3})^N} \right] \cdot \frac{2\gamma}{N} \end{aligned}$$

where  $U(\cdot)$  and  $T(\cdot)$  are Chebyshev polynomials, respectively, of the first and second type. It can be shown that deposit prices (markups) are decreasing (increasing) in scope. This expression can be used to calculate a lower bound of market share for a bank of any outside scope  $N_j$ . I focus on the case where  $j = N$ , the bank with largest scope and market share ( $s_{N+1}$ ) across all  $N + 1$  neighborhoods. For this bank:

$$\begin{aligned} s_{N+1} &= 1 - \alpha_{N+1} \\ &= 1 - \frac{r_N^{d*} - r_{N+1}^{d*}}{\frac{\gamma}{N}} \end{aligned}$$

I calculate the limiting market share of the largest bank as the number of neighborhoods grows arbitrarily large:

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<sup>4</sup>For example, if  $M = 4$ , then the largest number of banks of outside scope  $\{0, 1, 2, 3, 4\}$ , respectively, is  $\{5, 2, 1, 1, 1\}$ . The largest number of total banks is 10, and the largest number of branches is  $\{5, 2, 1, 1, 1\} \cdot \{1, 2, 3, 4, 5\} = 21$ .



$$\begin{aligned}
\lim_{N \rightarrow \infty} s_{N+1} &= 1 - \frac{2}{3} \cdot \lim_{N \rightarrow \infty} \frac{1}{U_{N-1}(2)} \cdot [T_N(2) - T_{N-1}(2)] \\
&= \frac{2\sqrt{3}}{3} - 1 \approx .15
\end{aligned}$$

The model thus exhibits a characteristic of vertical differentiation models first described by Shaked and Sutton (1982). The market share of the largest  $k$ -firms ( $k$ -firm ratio) has a non-zero lower bound, even if the market grows arbitrarily large. Intuitively, when consumers value scope, there is a limit to the ability of banks to attract consumers from the largest bank by offering a higher deposit rate.<sup>5</sup>

I can now state the empirical predictions that follow from increasing the ability of banks to compete for deposits by establishing networks of branches. These predictions follow from comparing the equilibrium outcome in the model above with one in which banks cannot build branch networks (the no-branch case). While this comparison is stark, it captures the simple logic behind varying the intensity of vertical differentiation in a continuous way.

First, the density of branches should increase within a market and neighborhood. In the no-branch case the equilibrium number of branches per market is  $N + 1$  and branches per neighborhood is one. When branching is allowed, the number of branches per market has an upper bound of  $\frac{\sum_{N_j=0}^N (N_j+1) \cdot N^*(N_j)}{N+1}$  and the number of branches per neighborhood has an upper bound of  $N + 1$ .

Second, market concentration should increase, while neighborhood concentration should decrease. In the no-branch case, the HHI measure for deposit shares in a market is  $\frac{1}{N+1}$  and the  $k$ -firm ratio is  $\frac{k}{N+1}$ . With branch networks, the corresponding measures for a market are strictly higher. In the no-branch case, the HHI measure and  $k$ -firm ratio for deposit shares in a neighborhood are both one. With branch networks the corresponding neighborhood measures are strictly lower. The intuition for the latter result is as follows. Within a small enough locality or neighborhood, vertical differentiation allows banks of different scope to co-exist without competing aggressively on price. Without such differentiation, fewer banks can coexist.

Third, the negative relationship between market concentration and market size should decrease in magnitude. In the model, market size is captured by  $N$ . Without bank branching, the market concentration as measured by the  $k$ -firm ratio where  $k = 1$  is  $\frac{1}{N+1}$ . This approaches 0 geometrically in  $N$ . With branching, this ratio is:

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<sup>5</sup>As a theoretical matter, this limiting result on the concentration of the deposit market might be seen as distinct from loan-side explanations of the bank size distribution. As I show in Appendix B, however, a model of loan heterogeneity can produce almost identical results in this limiting case.

$$s_{N+1} = 1 - \frac{r_N^{d*} - r_{N+1}^{d*}}{\frac{\gamma}{N}}$$

which, as we have seen, approaches .15 as  $N$  grows large. With branching, the slope of the concentration/market size relationship increases (and grows smaller in absolute value). Put differently, market concentration should increase more in larger markets than in smaller ones.

Fourth, with branching there should be a negative association between the size of a bank's branch network and its deposit rate. Consumers should be willing to accept a lower deposit rate from a bank with a large network. This negative association should increase in magnitude with the ability of banks to engage in branch network competition. Fifth, the variance in deposit interest rates should increase with branching. Greater product variety should lead to greater price dispersion.

#### IV Historical Background

In order to test these predictions, I exploit the history of bank regulation in the United States. I briefly outline the history of state bank-branching restrictions to explain their suitability for these tests. For most of the twentieth century, the United States consisted of at least 50 different banking markets. In the early part of the century, the charters of national banks were generally limited to individual states. After the Great Depression, a small number of bank holding companies formed across state lines. The Douglas Amendment to the Bank Holding Company Act of 1956, however, allowed states to restrict out-of-state bank holding companies from entering markets in their state.

Many states also severely restricted within-state branching. In so-called unit-branching states banks were allowed to have only one branch. This restricted both geographic competition for deposits and for loans. In some unit branching states, however, bank holding companies could own many individual, one-branch banks. Bank holding companies were free to organize their lending activity through an internal capital market. Individual banks within the holding company, however, still had to each comply with state and national banking regulations such as capital requirements. Depositors at one bank could not access accounts or services at another bank in the same holding company. As a result, a bank holding company was less able to compete for deposits than a multi-branch bank. I utilize this difference in institutional structure and in the timing of deregulation to measure the effects of increasing competition for deposits. I compare banking market structure in states that underwent regulatory changes to market structure free-branching states where no restrictions were placed on bank holding companies or banks. By doing so, I am able to separately consider the effect of allowing multi-branch bank holding companies to form

and of allowing holding companies to consolidate into individual banks.

## V Data and Empirical Analysis

I construct a panel dataset to test my predictions. The panel consists of MSA and non-MSA (county) markets from a subset of the 50 states from 1977-1997. I merge the Summary of Deposits dataset from the Federal Reserve, which contains branch level information on deposits, with the Call Report Data on bank balance sheets and bank holding company structure. I add to this market level and state level economic and demographic information from the Bureau of Economic Analysis.

I limit the sample to free-branching states and unit-branching states. A unit-branching state undergoes BHC deregulation when it allows holding companies to own multiple, single-branch banks. A unit-branching state undergoes bank-branching deregulation when it allows banks to own multiple branches by merger and acquisition. BHC deregulation precedes bank branching deregulation for every unit-branching state in the sample.

[To Be Completed]

## VI Conclusion

[To be completed]

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## VIII Appendix

### Appendix A - Model of Deposit Competition

The game is extensive form, but it will simplify exposition to denote all entry decision nodes as the “first stage” and to denote the (simultaneous move) pricing decision node as a “second stage.” In the first stage banks sequentially choose to enter a subset of  $N + 1$  distinct neighborhoods or localities. Sequential entry is assumed in order to guarantee the existence of an equilibrium in pure strategies. In the second stage banks set deposit interest rates in each neighborhood they enter. I assume there are  $(N + 1)^2$  potential entrants, so it is possible for every bank to enter every neighborhood. A strategy for bank  $l$  in the first stage, where  $l$  denotes the order of entry, consists of  $\{0, 1\}$  entry decisions for each of the  $N + 1$  neighborhoods *for any history of entry* by previous banks. Entry in each neighborhood is associated with a fixed cost  $F$ .

Each bank  $l$  thus chooses, for every history  $\mathbf{h}_{l-1}$ , a vector  $\mathbf{e}_l(\mathbf{h}_{l-1}) \in \mathbf{E} \subset \mathbb{Z}^{N+1}$ , where  $e_{ln}(\mathbf{h}_{l-1}) = 1$  if a bank enters neighborhood  $n$  and  $e_{ln}(\mathbf{h}_{l-1}) = 0$  otherwise. Each bank pays an entry cost  $c(\mathbf{e}_l(\mathbf{h}_{l-1})) = F \cdot (N_l + 1)$  where  $N_l + 1 = \sum_{n=1}^{N+1} e_{ln}(\mathbf{h}_{l-1})$  is the total number of neighborhoods that bank  $l$  enters given its strategy. I define a history  $\mathbf{h}_{l-1} \in \mathbf{H}_{l-1} \subset \mathbb{Z}^{(l-1) \times (N+1)}$  such that<sup>6</sup>:

$$h_{l-1} = \begin{bmatrix} \mathbf{e}_1(\mathbf{h}_1) \\ \mathbf{e}_2(\mathbf{h}_2) \\ \dots \\ \mathbf{e}_{l-1}(\mathbf{h}_{l-1}) \end{bmatrix}$$

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<sup>6</sup>For any bank  $l$ , the number of possible histories it faces at the time of entry is  $2^{(M+1) \cdot (l-1)}$ . Because the order of entry of banks 1 to  $l - 1$  is irrelevant for bank  $l$ 's entry decision, the number of strategically relevant histories is  $\frac{2^{(M+1) \cdot (l-1)}}{(l-1)!}$ . I therefore define the set  $\mathbf{H}_{l-1}$  to exclude matrices that are simply row permutations of one another. This implies that the extensive form of the entry game is a  $2^{M+1}$  order multinomial lattice.

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Subsequent to entry decisions by all  $(N+1)^2$  banks, each bank  $l$  simultaneously chooses deposit prices in every neighborhood for which  $e_{ln}(\mathbf{h}_{l-1}) = 1$ . A strategy in the second stage for bank  $l$  is a mapping from every entry history ( $\mathbf{h}_{(N+1)^2}$ ) to a vector of deposit interest rates. Each bank thus chooses  $\mathbf{r}^d(\mathbf{h}_{(N+1)^2}) \subset \mathbb{R}^{N_l+1} \forall \mathbf{h}_{(N+1)^2} \in \mathbf{H}_{(N+1)^2}$ <sup>7</sup>.

Consider a final-stage equilibrium in neighborhood  $n$  in which there exists some consumer who is indifferent between bank  $j$  and  $j-1 \forall j > 1$ . For any entry history of the game, a best response correspondence for each bank in neighborhood  $n$  is given by:

$$r_{nj}^d(\mathbf{r}_{n,/j}^d | N_{nj}, \mathbf{N}_{n,/j}) \in \arg \max_{r_{nj}^d} (r^l - r_{nj}^d) \cdot s_{nj}(\mathbf{r}_n^d, \mathbf{N}_n) \forall \mathbf{r}_{n,/j}^d, \mathbf{N}_n$$

where  $N_{nj} = \sum_{k=1, k \neq n}^{N+1} e_{jk}$  is the number of neighborhoods outside  $n$  in which bank  $j$  has a presence,  $N$  is the total number of outside neighborhoods. I refer to a bank's outside neighborhood presence ( $N_{nj}$ ) as its *outside* scope, so  $\mathbf{N}_n$  is a vector of outside scope for all banks in market  $n$ <sup>8</sup>. Within any neighborhood  $n$ , the actions of bank  $j$ 's competitors are summarized by  $\mathbf{r}_{n,/j}^d$ , a vector of prices in neighborhood  $n$ , and  $N_{n,/j}$ , a vector of outside scope<sup>9</sup>. Shaked and Sutton (1982) Proposition 1 shows that the profit function is continuous and quasi-concave in  $r_{nj}^d$ . Let the firm choose prices from a suitable, compact, convex set. The best response correspondence is then non-empty, convex, and has closed graph. By Kakutani's theorem a Nash equilibrium in prices exists for each neighborhood  $n$ .

For the first stage, each bank's best-response correspondence over entry must be optimal for any history of the game ( $\mathbf{h}_{l-1}$ ) and for any strategy profile of other banks [ $\mathbf{e}_{/l}(\mathbf{h}_{l-1}), \mathbf{r}^d(\mathbf{h}_{(N+1)^2})$ ]. I define this best-response correspondence  $e_l(\mathbf{e}_{/l}(\mathbf{h}_{l-1}), \mathbf{r}^d(\mathbf{h}_{(N+1)^2}) | \mathbf{h}_{l-1})$

<sup>7</sup>Recall that  $M_l$  is the total number of markets entered by bank  $l$  and is equal to the sum of the  $l$ th row of any entry history  $\mathbf{h}_{(M+1)^2}$ .

<sup>8</sup>Any bank  $j$  that operates in at least one market has scope  $M_{nj}$  equal to its total market presence minus one. When the identity of market  $n$  is irrelevant, I refer to this outside scope as  $N_j$ .

<sup>9</sup>All information on entry is contained in the first-stage history of the game ( $\mathbf{h}_{(N+1)^2}$ ). The entries in the vector  $N_n$ , for example, are obtained by (1) forming a vector of row sums for any row in  $\mathbf{h}_{(N+1)^2}$  in which the  $n$ th column is non-zero, (2) subtracting one from each entry, and (3) reordering this vector from smallest to largest. I can thus equivalently represent this correspondence as:

$$r_{nj}^d(\mathbf{r}_{n,/j}^d | \mathbf{h}_{(N+1)^2}) \in \arg \max_{r_{nj}^d} (r^l - r_{nj}^d) \cdot s_{nj}(\mathbf{r}_n^d, \mathbf{h}_{(N+1)^2}) \forall \mathbf{r}_{n,/j}^d, \forall \mathbf{h}_{(N+1)^2}$$

Note that a strategy for banks other than  $j$  in market  $n$  is a mapping from the history of the game to a vector of prices  $\mathbf{r}_{n,/j}^d(\mathbf{h}_{(N+1)^2})$ .

to be:

$$\begin{aligned} \arg \max_{\mathbf{e}_{ln} \in \{0,1\}} \sum_{n=1}^{n=N+1} e_{ln} \cdot \{\pi_l[\mathbf{r}_n^d(\mathbf{h}_{(N+1)^2}) | \mathbf{h}_{l-1}, \mathbf{e}_{/l}(\mathbf{h}_{/l-1})] - F\} \\ s.t. \pi_l[\mathbf{r}_n^d(\mathbf{h}_{(N+1)^2}) | \mathbf{h}_{l-1}, \mathbf{e}_{/l}(\mathbf{h}_{/l-1})] > 0, \forall e_{ln} = 1 \end{aligned}$$

While notationally cumbersome, the above program has a natural interpretation. For any given history  $\mathbf{h}_{l-1}$  at which bank  $l$  must make an entry decision, the bank chooses the best outcome along the path of the game induced by  $\mathbf{h}_{l-1}$  and other bank's entry ( $\mathbf{e}_{/l}(\mathbf{h}_{/l-1})$ ) and pricing ( $\mathbf{r}^d(\mathbf{h}_{(N+1)^2})$ ) strategies<sup>10</sup>.

Fix the strategies of other banks [ $\mathbf{e}_{/l}(\mathbf{h}_{/l-1}), \mathbf{r}^d(\mathbf{h}_{(N+1)^2})$ ] and consider the entry choice of bank  $l$  after some history of the game  $\mathbf{h}_l$ <sup>11</sup>. This is a discrete choice over  $2^{N+1}$  entry choices, where payoffs are an outcome of the second stage induced by other bank's strategies and bank  $l$ 's own pricing strategy. It follows that the set of best responses is nonempty for any history of the game and that a sub-game perfect equilibrium exists by Kuhn's Theorem. Because banks can be indifferent across entry decisions, the equilibrium need not be unique. As a result, the model does not permit an exact calculation of equilibrium neighborhood share for each bank.

It is still possible to obtain a lower bound for the neighborhood share, across all  $N + 1$  neighborhoods, of a bank with outside scope  $N_j$ . Such a bank would obtain its lowest neighborhood share in a neighborhood  $n$  where there exists a bank of every possible scope. A lower bound for its share across all  $N + 1$  neighborhoods is therefore given by  $(\frac{N_j+1}{N+1}) \cdot s_{nj}(\mathbf{r}_n^d, N_n)$  where  $N_n = \{0, 1, 2, \dots, N\}$ . I derive an expression for this lower bound.

The first order conditions for  $r_j^d$  can be expressed in matrix form as:

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & \dots & \dots & \dots \\ -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & \dots & \dots \\ 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} \\ \dots & \dots & \dots & 0 & -\frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} r_1^d \\ \dots \\ \dots \\ \dots \\ r_{N+1}^d \end{bmatrix} = \begin{bmatrix} \frac{1}{2}r^l \\ \dots \\ \dots \\ \dots \\ \frac{1}{2}r^l \\ \frac{1}{2}r^l - \frac{1}{2}\frac{\gamma}{N} \end{bmatrix}$$

The  $(N + 1) \times (N + 1)$  matrix of coefficients in this system is tridiagonal and almost-Toeplitz. I start with observation that for the  $(N + 1) \times (N + 1)$  tridiagonal, almost-Toeplitz

<sup>10</sup>The history  $h_{/l-1}$  must agree with  $h_{l-1}$  for all banks that enter before bank  $l$  and be determined by  $\mathbf{e}_{/l}(\mathbf{h}_{/l-1})$  subsequently. The history  $\mathbf{h}_{(M+1)^2}$  at which banks make their pricing decisions is thus completely specified by  $\mathbf{h}_{l-1}, \mathbf{e}_l(\mathbf{h}_{l-1}),$  and  $\mathbf{e}_{/l}(\mathbf{h}_{/l-1})$ . Together with  $\mathbf{r}(\mathbf{h}_{(M+1)^2})$ , these objects determine an outcome of the game.

<sup>11</sup>I am also fixing the pricing strategy of bank  $l$  after any history  $h_{(M+1)^2}$ .

matrix:

$$\begin{bmatrix} -\lambda & 1 & 0 & \dots & \dots & \dots \\ 1 & -2\lambda & 1 & 0 & \dots & \dots \\ 0 & 1 & -2\lambda & 1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & 1 & -2\lambda & 1 \\ \dots & \dots & \dots & 0 & 1 & -\lambda \end{bmatrix}$$

an exact solution for the (symmetric) inverse is known and given by:

$$a_{jk}^{-1} = \frac{1}{(1-\lambda^2)U_{N-1}(\lambda)} \cdot T_{j-1}(\lambda) \cdot T_{N+1-k}(\lambda), \quad 1 \leq j \leq k \leq N+1$$

where  $T$  and  $U$  are, respectively, Chebyshev polynomials of the first and second kind. To utilize this result, re-express the above system as:

$$\begin{bmatrix} -2 & 1 & 0 & \dots & \dots & \dots \\ 1 & -4 & 1 & 0 & \dots & \dots \\ 0 & 1 & -4 & 1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & 0 & 1 & -4 & 1 \\ \dots & \dots & \dots & 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} r_1^d \\ \dots \\ \dots \\ \dots \\ \dots \\ r_{N+1}^d \end{bmatrix} = \begin{bmatrix} -2r^l + r^l \\ -2r^l \\ \dots \\ \dots \\ -2r^l \\ -2r^l + r^l + \frac{1}{2} \frac{\gamma}{N} \end{bmatrix}$$

An exact expression for equilibrium prices is then:

$$r_j^d(r^l, N, \gamma) = r^l \cdot [-2 \cdot \sum_{k=1}^{N+1} a_{jk}^{-1} + a_{j1}^{-1} + a_{j,N+1}^{-1}] + a_{j,N+1}^{-1} \cdot \frac{2\gamma}{N} \quad (1)$$

$$= r^l + a_{j,N+1}^{-1} \cdot \frac{2\gamma}{N} \quad (2)$$

$$= r^l - \left(\frac{1}{2^2 - 1}\right) \cdot \frac{1}{U_{N-1}(2)} \cdot T_{j-1}(2) \cdot \frac{2\gamma}{N} \quad (3)$$

$$= r^l - \frac{\sqrt{3}}{3} \cdot \left[ \frac{(2 + \sqrt{3})^{j-1} + (2 - \sqrt{3})^{j-1}}{(2 + \sqrt{3})^N - (2 - \sqrt{3})^N} \right] \cdot \frac{2\gamma}{N} \quad (4)$$

where the lines 3 and 4 follow, respectively, from the matrix inverse given above and the non-recursive representation of the Chebyshev polynomials. The sum that is multiplied by  $r^l$  collapses to 1 in line 2 by the following argument.

$$-2 \cdot \sum_{k=1}^{N+1} a_{jk}^{-1} + a_{j1}^{-1} + a_{j,N+1}^{-1} = \quad (\text{Eq. A})$$



$$-2 \cdot \left[ \sum_{k=j}^{N+1} a_{jk}^{-1} + \sum_{k=1}^{j-1} a_{kj}^{-1} \right] + a_{j1}^{-1} + a_{j,N+1}^{-1} =$$

$$\frac{1}{(\lambda^2 - 1)U_{N-1}(\lambda)} \cdot \left[ 2 \sum_{k=j}^{N+1} T_{j-1}(\lambda) \cdot T_{N+1-k}(\lambda) + 2 \sum_{k=1}^{j-1} T_{k-1}(\lambda) \cdot T_{N+1-j}(\lambda) - T_{j-1}(\lambda) - T_{N+1-j}(\lambda) \right]$$

where the equation follows from the symmetry of the matrix inverse. Using standard rules for the algebra of Chebyshev polynomials, these sums can be simplified:

$$\sum_{k=j}^{N+1} T_{j-1}(\lambda) \cdot T_{N+1-k}(\lambda) = \frac{1}{2} \sum_{k=j}^{N+1} T_{N+j-k}(\lambda) + \frac{1}{2} \sum_{k=j}^{N+1} T_{|j-1-N-1+k|}(\lambda) \quad (*)$$

$$\sum_{k=1}^{j-1} T_{k-1}(\lambda) \cdot T_{N+1-j}(\lambda) = \frac{1}{2} \sum_{k=1}^{j-1} T_{N-j+k}(\lambda) + \frac{1}{2} \sum_{k=1}^{j-1} T_{|j-1-N-1+k|}(\lambda) \quad (**)$$

Combining (\*) and (\*\*):

$$\begin{aligned} & 2 \sum_{k=j}^{N+1} T_{j-1}(\lambda) \cdot T_{N+1-k}(\lambda) + 2 \sum_{k=1}^{j-1} T_{k-1}(\lambda) \cdot T_{N+1-j}(\lambda) = \\ & \sum_{k=j}^{N+1} T_{N+j-k}(\lambda) + \sum_{k=1}^{j-1} T_{N-j+k}(\lambda) + \sum_{k=1}^{N+1} T_{|j-1-N-1+k|}(\lambda) = \\ & \sum_{k=0}^{N-1} T_k(\lambda) + T_{N+1-j}(\lambda) + \sum_{k=0}^N T_k(\lambda) + T_{j-1}(\lambda) - 1 \end{aligned}$$

Equation A can then be simplified to yield:

$$\begin{aligned}
& -2 \cdot \sum_{k=1}^{N+1} a_{jk}^{-1} + a_{j1}^{-1} + a_{j,N+1}^{-1} = \\
& \frac{1}{(\lambda^2 - 1)U_{N-1}(\lambda)} \cdot \left[ \sum_{k=0}^{N-1} T_k(\lambda) + \sum_{k=0}^N T_k(\lambda) - 1 \right] = \\
& \frac{1}{U_{N-1}(\lambda)} \cdot \left\{ 1 + 2\lambda + \sum_{k=2}^{N-1} \frac{1}{2} [U_k(\lambda) - U_{k-2}(\lambda)] + \sum_{k=2}^N \frac{1}{2} [U_k(\lambda) - U_{k-2}(\lambda)] \right\} = \\
& \frac{1}{(\lambda^2 - 1)U_{N-1}(\lambda)} \left[ \frac{1}{2} U_N(\lambda) + U_{N-1}(\lambda) + \frac{1}{2} U_{N-2}(\lambda) \right] = \\
& \frac{1}{(\lambda^2 - 1)} \left[ 1 + \frac{1}{2} \cdot \frac{U_N(\lambda) + U_{N-2}(\lambda)}{U_{N-1}(\lambda)} \right] = \\
& \frac{1}{(\lambda^2 - 1)} [1 + \lambda]
\end{aligned}$$

When  $\lambda = 2$  this expression is equal to 1.

Using this closed form expression for  $r_j^d(r^l, N, \gamma)$  it is possible to obtain, for any given scope, a lower bound for the neighborhood share of a bank and to characterize this share as the number of neighborhoods  $N$  grows arbitrarily large. I focus on the case of a bank of scope  $N + 1$ , that is, a bank that enters every neighborhood. The existence of such a bank corresponds to the case where the fixed cost of entry is low. The neighborhood share for this bank is given by  $1 - F(\alpha_{N+1}) = 1 - \alpha_{N+1}$  in case of the uniform distribution, where  $\alpha_{N+1}$  represents a consumer who is indifferent between a bank of scope  $N + 1$  and  $N$ . It was shown above that, in equilibrium, the neighborhood share for the firm of greatest scope is given by:

$$\begin{aligned}
\lim_{N \rightarrow \infty} s_{N+1} &= 1 - \lim_{N \rightarrow \infty} \alpha_{N+1} \\
&= 1 - \lim_{N \rightarrow \infty} \frac{r_N^d(\cdot) - r_{N+1}^d(\cdot)}{\frac{\gamma}{N}} \\
&= 1 - \left( \frac{2}{2^2 - 1} \right) \lim_{N \rightarrow \infty} \frac{1}{U_{N-1}(2)} \cdot [T_N(2) - T_{N-1}(2)]
\end{aligned}$$

To characterize this share as  $N$  grows arbitrarily large, I refer to Nevai's (1979) results for the asymptotic ratios of orthogonal polynomials. For a Chebyshev polynomial of the second type:

$$\begin{aligned}\lambda U_N(\lambda) &= \frac{1}{2}U_{N+1}(\lambda) - \frac{1}{2}U_{N-1}(\lambda) \Rightarrow \\ \lim_{N \rightarrow \infty} \frac{U_{N-1}(\lambda)}{U_N(\lambda)} &= \lambda - \sqrt{\lambda^2 - 1}\end{aligned}$$

It follows that:

$$\begin{aligned}\lim_{N \rightarrow \infty} \frac{T_N(2)}{U_{N-1}(2)} &= \lim_{N \rightarrow \infty} \frac{\frac{1}{2}U_N(2) - \frac{1}{2}U_{N-2}(2)}{U_{N-1}(2)} \\ &= \lim_{N \rightarrow \infty} \frac{\frac{1}{2}(4U_{N-1}(2) - U_{N-2}) - \frac{1}{2}U_{N-2}(2)}{U_{N-1}(2)} \\ &= 2 - \lim_{N \rightarrow \infty} \frac{U_{N-2}(2)}{U_{N-1}(2)} \\ &= \sqrt{3}\end{aligned}$$

$$\begin{aligned}\lim_{N \rightarrow \infty} \frac{T_{N-1}(2)}{U_{N-1}(2)} &= \frac{\frac{1}{2}U_{N-1}(2) - \frac{1}{2}U_{N-3}(2)}{U_{N-1}(2)} \\ &= \frac{1}{2} - \frac{1}{2} \left[ \frac{4U_{N-2}(2) - U_{N-1}(2)}{U_{N-1}(2)} \right] \\ &= 2\sqrt{3} - 3\end{aligned}$$

It then follows that:

$$\begin{aligned}\lim_{N \rightarrow \infty} s_{N+1} &= 1 - \left( \frac{2}{2^2 - 1} \right) \lim_{N \rightarrow \infty} \frac{1}{U_{N-1}(2)} \cdot [T_N(2) - T_{N-1}(2)] \\ &= 1 - \frac{2}{3}(\sqrt{3} - (2\sqrt{3} - 3)) \\ &= \frac{2\sqrt{3}}{3} - 1 \\ &\approx .15\end{aligned}$$

The share of the largest bank is approximately .15 even as the number of neighborhoods– and banks–grows arbitrarily large.

## Appendix B - A Model of Loan Competition

I show that a model of lending can deliver an asymptotically equivalent result for the

share of the largest bank. The result illustrates the difficulty in empirically distinguishing the effects of loan and deposit side phenomenon in banking market structure. I present a simple equilibrium model of bank lending and deposit taking where the loan side determines market structure. In this model:

- (1) firm heterogeneity with respect to lending gives rise to differing loan market shares
- (2) loan market shares reflect differences in the underlying technology of lending or, alternatively, a bank's span of control
- (3) a right-skewed distribution of technology or organizational ability implies right-skewed market structures, and a small fraction of banks or a small number of banks can dominate lending even if the market is arbitrarily large, and
- (4) deposit market shares passively reflects loan market shares

I assume banks can acquire funds in a competitive borrowing market at rate  $r^b$  and lend in a competitive lending market with interest rate  $r^l$  where  $r^l > r^b$ . Each bank can make  $\$x$  worth of loans with expected return  $p(x, \theta) \cdot x$  where  $\theta$  is a firm-specific attribute distributed on  $[\underline{\theta}, \infty)$  according to  $F(\theta)$ . The probability of a loss  $(1 - p(x, \theta))$  increases with total lending, but this probability is smaller for firms with a high  $\theta$ . I assume that:

$$p(0, \theta) = 0, \frac{\partial p(0, \theta)}{\partial x} = 0 \quad \forall \theta$$

$$\frac{\partial p(x, \theta)}{\partial x} < 0, \frac{\partial p(x, \theta)}{\partial \theta} > 0, \frac{\partial^2 p(x, \theta)}{\partial x^2} < 0, \text{ and } \frac{\partial^2 p(x, \theta)}{\partial x \partial \theta} > 0 \quad \forall \theta, \forall x > 0$$

This simple framework is consistent with several empirical interpretations. Banks can differ in their lending technologies, so that different banks can “safely” handle different portfolio sizes. Banks could also differ in their span of control, or organizational capability to effectively monitor a large team of loan officers<sup>12</sup>.

Banks make an entry decision with associated fixed cost  $C$  before they realize their value of  $\theta$ . Conditional on entry, banks observe their value of  $\theta$  and choose a level of loans so as to:

$$\max_x r^l \cdot p(x, \theta) \cdot x - r^b \cdot x$$

The solution function  $x(r^l, r^b, \theta)$  and profit function  $\pi(r^l, r^b, C, \theta)$  are then strictly increasing in  $\theta$ :

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<sup>12</sup>Alternatively, banks can possess comparative advantage in different types of loans, indexed by  $\theta$ . A matching process in the loan market would then result in different loan types associating with different banks. In this setting, the size of a bank's optimal lending portfolio would follow from characteristics of the underlying borrowers as opposed to the bank's technology or organizational capability.

$$\begin{aligned}\frac{\partial x(r^l, r^b, \theta)}{\partial \theta} &= \frac{-[\frac{\partial^2 p(x(\cdot), \theta)}{\partial x \partial \theta} + \frac{\partial p(x(\cdot), \theta)}{\partial \theta}]}{\frac{\partial^2 p(x(\cdot), \theta)}{\partial x^2} + \frac{\partial p(x(\cdot), \theta)}{\partial \theta}} = \frac{(-) \cdot (+)}{(-)} = (+) \\ \frac{\partial \pi(r^l, r^b, C, \theta)}{\partial \theta} &= r^l \cdot \frac{\partial p(x(\cdot), \theta)}{\partial \theta} \cdot x(r^l, r^b, \theta) = (+)\end{aligned}$$

Banks lend more and make higher profits if they have a lower probability of loss for any given level of loans.

I next define an upward-sloping market supply curve for credit such that (1) the number of potential entrants can vary, and (2) a heterogeneous set of banks make loans. I modify a standard competitive model to obtain these two features. In a short-run equilibrium, an upward sloping supply curve is derived by *fixing* the number of potential entrants. In a long run equilibrium, only banks at minimum efficient scale make loans. With free entry the supply curve is perfectly elastic and the efficient loan amount for each bank—its minimum efficient scale—is given by the intersection of the average cost curve with the maximum over the set of average revenue curves. A long run supply curve can be defined in this way when the set of  $\theta$ 's has an upper bound with strictly positive mass. The set of banks that make loans in a long-run equilibrium are those that achieve this upper bound<sup>13</sup>. This conclusion does not follow when, as here, there is no such upper bound with strictly positive mass.

I first partially endogenize the set of potential entrants. For any pair of market prices  $\{r^l, r^b\}$ , I define the number of potential entrants be the largest number of firms that can each expect to make weakly positive profits:

$$\begin{aligned}N(r^l, r^b, C) &= \max\{N \in \mathbb{Z} \mid E_\theta[\min\{\pi_i(r^l, r^b, C, \theta)_{i=1}^N\}] \geq 0\} = \\ &\max\{N \in \mathbb{Z} \mid \int_\theta \pi(r^l, r^b, C, \theta) dF_{N,N}(\theta) \geq 0\}\end{aligned}$$

where  $F_{N,N}(\theta)$  is the distribution of the  $N$ th order statistic, the minimum of  $N$  independent draws from  $F(\theta)$ . I assume that potential entrants make an entry decision before realizing their values of  $\theta$ , so they will enter if they make weakly positive profits in expectation. The definition of potential entrants guarantees that all banks will enter<sup>14</sup>. If  $E_\theta[x(r^l, r^b, \theta)]$

<sup>13</sup>If the set of possible  $\theta$ 's is has a maximum at  $\bar{\theta}$ , then the maximum average revenue curve is well-defined ( $r_l \cdot p(x, \bar{\theta})$ ). The average cost curve is identical across banks. Define an equilibrium price  $r_l^*$  such that average revenue equals average cost at an optimum for a bank of type  $\bar{\theta}$ :

$$(r_l^* \cdot p(x(r_l^*, r^b, \bar{\theta}), \bar{\theta}) - r^b) \cdot x(r_l^*, r^b, \bar{\theta}) - C = 0$$

By the monotonicity of the profit function, profits must be strictly negative for all banks with  $\theta < \bar{\theta}$ .

<sup>14</sup>If the expected value of the minimum profit over  $N$  draws from  $F(\theta)$  is weakly positive then the expected

exists, the expected market supply function is given by:

$$X^s(r^l, r^b) = E_\theta[x(r^l, r^b, \theta)] \cdot N(r^l, r^b, C)$$

After entering, banks realize their value of  $\theta$  and set loan levels so as to maximize expected profits, conditional on  $\theta$ .

To obtain closed-form expressions, I assume:

$$p(x, \theta) = \begin{cases} 1 - \frac{x}{\theta} & x \leq \theta \\ 0 & x > \theta \end{cases}$$

It is easily verified that this functional form satisfies the restrictions given above. It follows that:

$$\begin{aligned} x(r^l, r^b, \theta) &= \frac{\theta}{2} \left(1 - \frac{r^b}{r^l}\right) \\ p(r^l, r^b, \theta) &= \frac{1}{2} \left(1 + \frac{r^b}{r^l}\right) \\ \pi(r^l, r^b, C, \theta) &= \frac{\theta}{4} (r^l - r^b) - C \end{aligned}$$

A convenience of the functional form chosen is that the equilibrium probability of loss does not depend on  $\theta$  so both loans and profits are affine functions of  $\theta$ . It follows that the expected market supply function as well as the distribution of loans and profits across banks can be derived from  $F(\theta)$ <sup>15</sup>.

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profit of any individual bank is also weakly positive.

<sup>15</sup>To illustrate, suppose  $\theta$  is distributed uniformly on the interval  $[\underline{\theta}, \bar{\theta}]$ . Then expected profits of the firm with the lowest draw of  $\theta$  is given by:

$$E_\alpha[\min\{\pi_i(r_l, r_b, C, \theta)_{i=1}^N\}] = \left(\frac{r_l - r_b}{4}\right) \cdot \left(\frac{\bar{\theta} + N\underline{\theta}}{N+1}\right) - F$$

The number of entrants is:

$$N(r_l, r_b) = \min\{N \in \mathbb{Z} \mid N \geq \frac{\frac{\bar{\theta}}{4}(r_l - r_b) - C}{C - \frac{\underline{\theta}}{4}(r_l - r_b)}\}$$

Because at least one entrant must achieve positive profits in expectation, the minimum  $r_l$  for which banks enter is given by:

$$r_l \geq r_b + \frac{4C}{\underline{\theta} + \underline{\theta}}$$

Similarly, if  $r_l$  is high enough, then even bank of type  $\underline{\theta}$  will earn weakly positive profits. A bank with any value  $\theta \geq \underline{\theta}$  will earn positive profits in expectation if:

$$r_l \geq r_b + \frac{4C}{\underline{\theta}}$$

With this definition of the expected market supply function, I can then examine the pattern of expected market shares across banks as the size of the market increases, that is, as the market demand function shifts out. It is easily seen that the number of entrants is unbounded and that the distribution of  $\theta$  governs the expected market shares across banks. The Lorenz curve provides a useful way of characterizing the concentration of bank lending<sup>16</sup>. I assume that  $\theta$  follows a Pareto distribution:

$$F(\theta) = \begin{cases} 1 - (\frac{\theta}{\underline{\theta}})^a & a > 1, \theta \geq \underline{\theta} \\ 0 & \theta < \underline{\theta} \end{cases}$$

The Lorenz curve takes the form:

$$L(F(\theta)) = 1 - (1 - F(\theta))^{1 - \frac{1}{a}}$$

so that  $1 - L(F(\theta')) = (1 - F(\theta'))^{1 - \frac{1}{a}}$  is the fraction of total loans made by firms with  $\theta \geq \theta'$ . As  $a \rightarrow 1^+$  the concentration of loans made becomes increasingly skewed toward the largest banks<sup>17</sup>. For any quantile of the largest banks  $(1 - F(\theta))$ , the fraction of loans made by this quantile becomes arbitrarily close to 1. This model of loans is thus consistent with extreme skewness in the size distribution of banks when size is measured by loan portfolios.

This model is also consistent with a high degree of skewness measured in the terms of number, as opposed to the quantile, of the largest banks. Market concentration is often measured by the  $k - firm$  ratio, here, the fraction of total loans made by the largest  $k$  banks. I focus on the case where  $k = 1$  and show that this model is also consistent with a non-zero market share for the largest bank as demand shifts out and the number banks in the market grows arbitrarily large. For any sequence of i.i.d. random variables, the ratio  $R_n = \frac{\max\{x_i\}_{i=1}^n}{\sum_{i=1}^n x_i}$  converges to 0 almost surely in  $n$  if  $E(x_i) < \infty$  (O'Brien 1980). It follows

that for Pareto-distributed random variables, a necessary condition for the market share of

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so that the number of potential entrants is unbounded (and the expected market supply function has infinite slope) as  $r_l$  approaches this value from below. It follows that the expected market supply function is given by:

$$\begin{aligned} X^s(r_l, r_b, C) &= E_\alpha[x(r_l, r_b, \theta)] \cdot N(r_l, r_b, C) \\ &= \begin{cases} 0 & r_l \in [0, r_b + \frac{4C}{\bar{\theta} + \underline{\theta}}) \\ N \cdot \frac{\bar{\theta} + \underline{\theta}}{4} (1 - \frac{r_b}{r_l}) & N \in \mathbb{Z}, r_l \in [r_b + \frac{4C}{\bar{\theta} + N\underline{\theta}}, r_b + \frac{4C}{\underline{\theta}}) \end{cases} \end{aligned}$$

<sup>16</sup>If, as was assumed for illustration,  $\theta$  has a uniform distribution, the largest 10% of banks account for 19% of lending.

<sup>17</sup>The Lorenz curve for the Pareto distribution is not defined if  $a \leq 1$  because the first moment no longer exists.

the largest firm to remain positive in the limit is that  $a \leq 1$ . For this distribution<sup>18</sup>, it can be shown (Bingham and Teugels, 1981) that as  $n \rightarrow \infty$ :

- (i)  $E[\frac{1}{R_n}] \rightarrow \frac{1}{1-a}, a \in (0, 1)$ <sup>19</sup>
- (ii)  $R_n \xrightarrow{d} y_a$ , where  $y_a$  has a non-degenerate distribution

Because  $R_n \in [0, 1]$  with probability one,  $E[R_n] > 0$  exists. It follows that when  $a < 1$ , the largest bank accounts for a non-zero proportion of total market lending in expectation even if the market grows arbitrarily large.

With this flexible apparatus for considering loan market shares, I consider a simple, spatial model of competition for deposits. I assume there are  $M + 1$  markets in which banks can locate<sup>20</sup>. In each market consumers inelastically supply  $\frac{1}{M+1}$  units of deposits to banks. As noted above, banks can access a market for debt at cost  $r^b$ . I assume Bertrand competition in deposit prices ( $r^d$ ) in each market. Banks pay a fixed cost  $F$  to enter each market. Banks simultaneously make an entry and price decision across each market conditional on their realization of  $\theta$ . By the usual logic of Bertrand competition, an equilibrium outcome for any particular market consists of one bank entering and charging a price  $r^{d*}$  so that its average cost for deposits equals that of its cost of outside funds ( $\frac{\frac{1}{M+1} \cdot r^{d*} + F}{\frac{1}{M+1}} = r^b$ ).

As there are many equilibria in this game, I impose a selection rule so that deposit shares across markets passively reflect loan shares. The intuition for this assumption is that banks with a greater need for funds are more likely to enter more markets. I assume the number of markets in which bank  $i$  enters in equilibrium and takes all the deposits is  $[s_i \cdot (M + 1)]$  where  $s_i$  is a bank's share of the loan market and  $[\cdot]$  represents the lowest integer part of the expression. This holds for all banks  $i$  except the largest bank. For the largest bank ( $s_N$ ) this number of markets is given by  $[s_N \cdot (M + 1)] + M + 1 - \sum_{i=1}^{N-1} [s_i \cdot (M + 1)]$ . If the number of deposit markets is large, then deposit market shares approximate loan market shares arbitrarily closely.

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<sup>18</sup>These results hold where  $x$  is an *iid* random variable with distribution function  $F(x)$  on  $\mathbb{R}_+$ ,  $F(0) = 0$ , and  $F(x) \in D(\alpha)$  for some  $\alpha \in (0, 1)$ , that is, a normalized sum of  $x$  converges in distribution to an  $\alpha$ -stable random variable.

<sup>19</sup>This result is straightforward to derive in the special case of the Pareto distribution (Zaliapin et al 2003).

<sup>20</sup>I have therefore assumed that firms can costlessly move across these  $M + 1$  markets in order to acquire funds. This would be a reasonable assumption for loan and deposit competition across a large metropolitan area made up of  $M + 1$  neighborhoods.



## VIII.1 Tables

**Table 1**

**Deregulation of Bank Branching by State**

State	Year of Deregulation		State	Year of Deregulation	
	Inter State	Intra State		Inter State	Intra State
Maine	1978	1975	Pennsylvania	1986	1982
Alaska	1982	<1970	South Carolina	1986	<1970
New York	1982	1976	Alabama	1987	1981
Connecticut	1983	1980*	California	1987	<1970
Massachusetts	1983	1984	Louisiana	1987	1988
Kentucky	1984	1990	New Hampshire	1987	1987
Rhode Island	1984	<1970	Oklahoma	1987	1988*
Utah	1984	1981	Texas	1987	1988*
Washington, DC	1985	<1970	Washington	1987	1985
Florida	1985	1988*	Wisconsin	1987	1990*
Georgia	1985	1983	Wyoming	1987	1988*
Idaho	1985	<1970	Colorado	1988	1991*
Maryland	1985	<1970	Delaware	1988	<1970
Nevada	1985	<1970	Mississippi	1988	1986
North Carolina	1985	<1970	South Dakota	1988	<1970
Ohio	1985	1979	Vermont	1988	1970
Tennessee	1985	1985	West Virginia	1988	1987*
Virginia	1985	1978	Arkansas	1989	1994*
Arizona	1986	<1970	New Mexico	1989	1991*
Illinois	1986	1988*	Nebraska	1990	1985*
Indiana	1986	1989	Iowa	1991	2001
Michigan	1986	1987	North Dakota	1991	1987*
Minnesota	1986	1993*	Kansas	1992	1987*
Missouri	1986	1990*	Montana	1993	1990*
New Jersey	1986	1977	Hawaii	1994	1986*
Oregon	1986	1985			

\* Indicates unit branching state

**Table 2**  
**Banking Market Characteristics**

		Average Market Characteristics				
		Pop	3-Firm	Herf	Banks	Markets
<b>MSA</b>						
1977	466,967	0.69 (0.17)	0.22 (0.11)	18.9 (28.2)	362	
1982	493,581	0.67 (0.16)	0.21 (0.10)	18.9 (27.6)	362	
1987	493,581	0.69 (0.15)	0.22 (0.10)	18.9 (25.5)	362	
1992	556,030	0.70 (0.15)	0.22 (0.10)	18.0 (21.8)	362	
1997	592,037	0.69 (0.13)	0.21 (0.10)	17.0 (18.0)	363	
<b>Non-MSA</b>						
1977	20,720	0.90 (0.14)	0.45 (0.25)	4.0 (2.9)	2,256	
1982	21,529	0.90 (0.14)	0.44 (0.24)	4.1 (2.8)	2,265	
1987	24,189	0.90 (0.14)	0.44 (0.24)	4.0 (2.8)	2,272	
1992	22,311	0.90 (0.13)	0.43 (0.23)	4.1 (2.7)	2,262	
1997	23,565	0.89 (0.13)	0.42 (0.23)	4.3 (2.67)	2,262	

**Notes:** Each column represents the market average for a given year with standard deviations in parentheses. "3 firm" is the fraction of deposits in a market held by the largest three banks or bank holding companies. "Herf" is the market Herfindahl index for deposits computed at the bank holding company level. "Banks" is the number of banks and bank holding companies.