Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 850 (2011) 1–30
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

# Discrete $R$ symmetries for the MSSM and its singlet extensions

Hyun Min Lee [a], Stuart Raby [b], Michael Ratz [c], Graham G. Ross [a,d], Roland Schieren [c], Kai Schmidt-Hoberg [d], Patrick K.S. Vaudrevange [e,*]

[a] *Theory Group, CERN, 1211 Geneva 23, Switzerland*
[b] *Department of Physics, The Ohio State University, 191 W. Woodruff Ave., Columbus, OH 43210, USA*
[c] *Physik-Department T30, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany*
[d] *Department of Physics, Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX 1 3NP, UK*
[e] *Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, 80333 München, Germany*

Received 24 February 2011; accepted 14 April 2011

Available online 19 April 2011

**Abstract**

We determine the anomaly free discrete $R$ symmetries, consistent with the MSSM, that commute with SU(5) and suppress the $\mu$ parameter and nucleon decay. We show that the order $M$ of such $\mathbb{Z}_M^R$ symmetries has to divide 24 and identify 5 viable symmetries. The simplest possibility is a $\mathbb{Z}_4^R$ symmetry which commutes with SO(10). We present a string-derived model with this $\mathbb{Z}_4^R$ symmetry and the exact MSSM spectrum below the GUT scale; in this model $\mathbb{Z}_4^R$ originates from the Lorentz symmetry of compactified dimensions. We extend the discussion to include the singlet extensions of the MSSM and find $\mathbb{Z}_4^R$ and $\mathbb{Z}_8^R$ are the only possible symmetries capable of solving the $\mu$ problem in the NMSSM. We also show that a singlet extension of the MSSM based on a $\mathbb{Z}_{24}^R$ symmetry can provide a simultaneous solution to the $\mu$ and strong CP problem with the axion coupling in the favoured window.


*Keywords:* MSSM; Discrete $R$ symmetries; Anomaly free symmetries; GUTs; Proton decay; $\mu$ problem; CP problem; NMSSM; String realization; Heterotic orbifold

* Corresponding author.
*E-mail address:* Patrick.Vaudrevange@physik.uni-muenchen.de (P.K.S. Vaudrevange).

0550-3213/$ – see front matter 
doi:10.1016/j.nuclphysb.2011.04.009

## 1. Introduction

Supersymmetric extensions of the standard model (SM), such as the minimal supersymmetric extension, the MSSM, promise to eliminate the hierarchy problem. However they also introduce serious potential problems and to be viable they must evade the $\mu$-problem and the problem associated with new baryon- and lepton-number violating terms. This suggests that there should be an additional underlying symmetry capable of controlling these terms. Dangerous dimension four operators can be forbidden by $R$- or matter parity [1–3], which is an anomaly free $\mathbb{Z}_2$ subgroup of the continuous baryon minus lepton symmetry $\mathrm{U}(1)_{B-L}$. Dimension five proton decay operators can be forbidden by 'baryon triality' [4], which combines with matter parity to 'proton hexality' [5,6]. The latter is the unique anomaly free discrete non-$R$ symmetry forbidding the dangerous operators while allowing the usual Yukawa couplings, the $\mu$-term and the effective neutrino mass operator. Anomaly freedom is believed to be a necessary property of discrete symmetries as otherwise quantum gravity effects may render them inefficient [4,7–9].

However, there are two unpleasant properties of these traditional discrete symmetries. First, they do not allow to address the $\mu$ problem. Second, they do not commute with the symmetries of the grand unified theories (GUTs) SU(5) or SO(10) [10]. In [11] a discrete $R$ symmetry was identified which can address the $\mu$ problem and commutes with SO(10). This $\mathbb{Z}_4^R$ symmetry is anomaly free through cancellation by the Green–Schwarz (GS) mechanism. In a recent paper [12] we have shown that this $\mathbb{Z}_4^R$ is the unique possibility which commutes with SO(10), and we have pointed out that it also solves the problem associated with dimension five proton decay operators. Furthermore it contains matter parity as a $\mathbb{Z}_2$ subgroup that is left unbroken after supersymmetry breaking.

In this paper we extend the discussion to consider the possible discrete symmetries of the MSSM which commute with SU(5). As we shall see, there are only five possibilities with the simplest one being the $\mathbb{Z}_4^R$. Our analysis applies to singlet extensions of the MSSM as well.

The paper is organized as follows. In Section 2 we prove that there are only five (generation-independent) discrete $\mathbb{Z}_M^R$ symmetries which (i) commute with SU(5), (ii) allow the usual Yukawa couplings and dimension five neutrino mass operator and (iii) address the $\mu$ and proton decay problems of the MSSM. Section 3 is dedicated to a more detailed discussion of the simplest such symmetry, $\mathbb{Z}_4^R$. We present a globally consistent string compactification with the exact MSSM spectrum below the compactification scale. The model exhibits the $\mathbb{Z}_4^R$ symmetry, which originates from the Lorentz group of compactified dimensions. In Section 4 we discuss discrete $R$ symmetries in singlet extensions of the MSSM. In a theory with the usual NMSSM couplings the discrete $R$ symmetries can, apart from suppressing the proton decay rate, provide us with a solution to the NMSSM hierarchy problem. In a different singlet extension, in which the singlet couples quadratically to the Higgs bilinear, we will identify a unique discrete $R$ symmetry capable of solving the $\mu$ and strong CP problems simultaneously. Finally, Section 5 contains our conclusions. In two appendices we present a re-derivation of discrete anomalies in the path integral approach and collect anomaly coefficients for discrete $R$ and non-$R$ symmetries.

## 2. Discrete symmetries of the MSSM

In this section we discuss discrete symmetries of the MSSM which commute with SU(5) and can solve the $\mu$ problem. As we shall see, the assumption that matter $\mathbb{Z}_M$ charges commute with SU(5) allows us to restrict possible $\mathbb{Z}_M$ symmetries of the MSSM, as well as singlet extensions, to only few possibilities. We start in Section 2.1 by showing that one cannot address the $\mu$

problem with non-$R$ symmetries. In Section 2.2 we then turn to the discussion of discrete $R$ symmetries, for which we prove that the order $M$ has to divide 24. Finally, in Section 2.2.2, we classify all possible charge assignments.

### 2.1. Non-$R$ discrete symmetries

We start by discussing non-$R$ symmetries. We show that such discrete symmetries that are consistent with SO(10) or SU(5) relations for matter, i.e. universal charges for quarks and leptons, cannot forbid the $\mu$ term (cf. the similar discussion in [13]).

Consider a $\mathbb{Z}_M$ symmetry under which the three generations of $Q$, $\overline{U}$ and $\overline{E}$ carry discrete charge $q_{\mathbf{10}}^g$ while $L$ and $\overline{D}$ carry $q_{\overline{\mathbf{5}}}^g$, where $g$ labels the generation index. Our conventions are given in Appendix B. If the $\mathbb{Z}_M$ charges obey the even stronger SO(10) relations (i.e. $q_{\mathbf{10}}^g = q_{\overline{\mathbf{5}}}^g$), the following discussion applies as well. The anomaly coefficients $A_3 := A_{\mathrm{SU}(3)_C\text{-}\mathrm{SU}(3)_C\text{-}\mathbb{Z}_M}$, $A_2 := A_{\mathrm{SU}(2)_\mathrm{L}\text{-}\mathrm{SU}(2)_\mathrm{L}\text{-}\mathbb{Z}_M}$, $A_1 := A_{\mathrm{U}(1)_Y\text{-}\mathrm{U}(1)_Y\text{-}\mathbb{Z}_M}$ and $A_0 := A_{\text{grav-grav-}\mathbb{Z}_M}$ are (cf. Eq. (B.17) in Appendix B)

$$A_3 = \frac{1}{2}\sum_{g=1}^{3}\left(3\cdot q_{\mathbf{10}}^g + q_{\overline{\mathbf{5}}}^g\right), \tag{2.1a}$$

$$A_2 = \frac{1}{2}\sum_{g=1}^{3}\left(3\cdot q_{\mathbf{10}}^g + q_{\overline{\mathbf{5}}}^g\right) + \frac{1}{2}(q_{H_u} + q_{H_d}), \tag{2.1b}$$

$$A_1 = \frac{1}{2}\sum_{g=1}^{3}\left(3\cdot q_{\mathbf{10}}^g + q_{\overline{\mathbf{5}}}^g\right) + \frac{3}{5}\cdot\frac{1}{2}\cdot(q_{H_u} + q_{H_d}), \tag{2.1c}$$

$$A_0 = \sum_{g=1}^{3}\left(10\cdot q_{\mathbf{10}}^g + 5\cdot q_{\overline{\mathbf{5}}}^g\right) + 2q_{H_u} + 2q_{H_d}, \tag{2.1d}$$

where the sum runs over the generation indices $g$ and $q_{H_u}$ and $q_{H_d}$ denote the $\mathbb{Z}_M$ charges of the up-type and down-type Higgs doublets, respectively. Anomaly freedom requires

$$(A_{1\leqslant i\leqslant 3} \bmod \eta) = \frac{1}{24}(A_0 \bmod \eta) = \rho \tag{2.2}$$

with $\rho \neq 0$ in the case of GS anomaly cancellation (cf. Eq. (A.23) in Appendix A.3). Here we define

$$\eta := \begin{cases} M & \text{for } M \text{ odd}, \\ M/2 & \text{for } M \text{ even}. \end{cases} \tag{2.3}$$

Condition (2.2) implies

$$A_2 - A_3 = 0 \bmod \eta \tag{2.4}$$

and hence, also in the case of generation-dependent $\mathbb{Z}_M$ charges,

$$\frac{1}{2}(q_{H_u} + q_{H_d}) = 0 \bmod \eta. \tag{2.5}$$

On the other hand, the condition that the $\mu$ term is allowed is

$$q_{H_u} + q_{H_d} = 0 \bmod M. \tag{2.6}$$

We therefore see that, if we demand SU(5) relations for matter charges, a non-$R$ $\mathbb{Z}_M$ symmetry cannot be used to address the $\mu$ problem, even if we allow for GS cancellation of anomalies.

### 2.2. *Discrete R-symmetries*

Having seen that non-$R$ symmetries cannot be used to address the $\mu$ problem, we turn to discuss discrete $R$ symmetries. In this subsection, we derive constraints on the order $M$ of $\mathbb{Z}_M^R$ symmetries that can solve the $\mu$ problem and accommodate the structure of the MSSM.

#### 2.2.1. *A constraint on the order M*

After adding the contribution of the gauginos and gravitino the anomaly coefficients are

$$A_3^R = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\bar{\mathbf{5}}}^g\right) - 3, \tag{2.7a}$$

$$A_2^R = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\bar{\mathbf{5}}}^g\right) + \frac{1}{2}(q_{H_u} + q_{H_d}) - 5, \tag{2.7b}$$

$$A_1^R = \frac{1}{2}\sum_{g=1}^{3}\left(3q_{\mathbf{10}}^g + q_{\bar{\mathbf{5}}}^g\right) + \frac{3}{5}\left[\frac{1}{2}(q_{H_u} + q_{H_d}) - 11\right], \tag{2.7c}$$

$$A_0^R = -21 + 8 + 3 + 1 + \sum_{g=1}^{3}\left[10\left(q_{\mathbf{10}}^g - 1\right) + 5\left(q_{\bar{\mathbf{5}}}^g - 1\right)\right] + 2(q_{H_u} + q_{H_d} - 2), \tag{2.7d}$$

where $q_{\mathbf{10}}$, $q_{\bar{\mathbf{5}}}$, $q_{H_u}$ and $q_{H_d}$ denote the $R$ charges of the matter and Higgs superfields, i.e. matter fermions and Higgsinos have charges $q-1$.

In the case $\rho \neq 0$, the GS mechanism requires the presence of an axion, such that $A_0^R$ is to be amended by the axino/dilatino contribution ($q_{\widetilde{a}} = -1$).

Subtracting the coefficients from each other leads to the universality conditions

$$A_2^R - A_3^R = 0 \bmod \eta \quad \curvearrowright \quad q_{H_u} + q_{H_d} = 4 \bmod 2\eta, \tag{2.8a}$$

$$A_1^R - A_3^R = 0 \bmod \eta \quad \curvearrowright \quad \frac{3}{5}\left[\frac{1}{2}(q_{H_u} + q_{H_d}) - 6\right] = 0 \bmod \eta. \tag{2.8b}$$

Eq. (2.8a) is equivalent to

$$\frac{1}{2}(q_{H_u} + q_{H_d}) = 2 + \eta\ell \tag{2.9}$$

with an integer $\ell$. Inserting this into Eq. (2.8b) yields

$$\frac{3}{5}[\ell\eta - 4] = k\eta \tag{2.10}$$

with another integer $k$. Altogether we find

$$[3\ell - 5k] = 12/\eta = \begin{cases} 24/M, & \text{for } M \text{ even}, \\ 12/M, & \text{for } M \text{ odd}. \end{cases} \tag{2.11}$$

Table 2.1
Phenomenologically attractive charge assignments. The charges $q^{\mathrm{sh}}_{H_u}$ and $q^{\mathrm{sh}}_{H_d}$ are Higgs charges shifted in such a way that the anomaly coefficients $A^R_i$ ($1 \leqslant i \leqslant 3$) are manifestly universal. $\rho$ is the universal value of the anomaly coefficients; $\rho \neq 0$ indicates GS cancellation of anomalies.

| $M$ | $q_{\mathbf{10}}$ | $q_{\bar{\mathbf{5}}}$ | $q_{H_u}$ | $q_{H_d}$ | $q^{\mathrm{sh}}_{H_u}$ | $q^{\mathrm{sh}}_{H_d}$ | $\rho$ | $A^R_0$ (MSSM) |
|---|---|---|---|---|---|---|---|---|
| 4 | 1 | 1 | 0 | 0 | 16 | 16 | 1 | 1 |
| 6 | 5 | 3 | 4 | 0 | 28 | 24 | 0 | 1 |
| 8 | 1 | 5 | 0 | 4 | 24 | 28 | 1 | 3 |
| 12 | 5 | 9 | 4 | 0 | 28 | 24 | 3 | 1 |
| 24 | 5 | 9 | 16 | 12 | 88 | 84 | 9 | 7 |

Table 2.2
Charge assignments which satisfy only the first two criteria. Both assignments have $\rho = 0$.

| $M$ | $q_{\mathbf{10}}$ | $q_{\bar{\mathbf{5}}}$ | $q_{H_u}$ | $q_{H_d}$ | $q^{\mathrm{sh}}_{H_u}$ | $q^{\mathrm{sh}}_{H_d}$ |
|---|---|---|---|---|---|---|
| 3 | 2 | 0 | 1 | 0 | 10 | 12 |
| 6 | 2 | 0 | 4 | 0 | 28 | 24 |

In both cases $24/M$ has to be integer, i.e. $M$ has to divide 24. Thus the possible values of $M$ are 3, 4, 6, 8, 12 and 24.[1] In what follows, we consider all these possibilities.

### 2.2.2. *Classification*

Given the constraints on the order $M$, it is straightforward to classify all phenomenologically attractive charge assignments. Here we assume that the charge assignments are family blind. Though not absolutely necessary it does ensure that the symmetry does not prevent mixing between families in the fermion mass matrix. The classification was done by a scan over all possible values of $M$. In addition to forbidding the $\mu$ term we require that

1. Mixed gauge–$\mathbb{Z}^R_M$ anomalies cancel, i.e. $A^R_{1 \leqslant i \leqslant 3} = \rho \bmod \eta$;
2. Yukawa couplings $\mathbf{10}\,\mathbf{10}\,H_u$ and $\mathbf{10}\,\bar{\mathbf{5}}\,H_d$ as well as the neutrino mass Weinberg operator $\bar{\mathbf{5}}\,H_u\bar{\mathbf{5}}\,H_u$ are allowed;
3. $R$-parity violating couplings are forbidden.

Under these constraints the allowed charge assignments are given in Table 2.1.

For completeness we note that there are only two more charge assignments that are allowed demanding just the first two conditions. They are given in Table 2.2.

One may ask whether there are additional discrete symmetries, such as $\mathbb{Z}^{(R)}_O \times \mathbb{Z}^{(R)}_P$, which cannot be written as single $\mathbb{Z}^{(R)}_M$ symmetries but also fulfill the three criteria above. The only candidates for such symmetries are based on the two patterns shown in Table 2.2. We find that by amending these assignments by the usual matter parity one arrives at the $\mathbb{Z}^R_6$ symmetry of Table 2.1. Hence our classification also comprises the $\mathbb{Z}^{(R)}_O \times \mathbb{Z}^{(R)}_P$ case. Of course, in extensions of the MSSM, extra states can enjoy additional symmetries.

[1] We exclude the case $M = 2$ since there are no meaningful order 2 discrete $R$ symmetries (cf. e.g. [14]).

*2.2.3. Dimension five nucleon decay operators*

Note that the third condition is sufficient to eliminate baryon and lepton number violation due to dimension four terms in the Lagrange density. However in the MSSM at dimension five there are problematic operators allowed that generate nucleon decay. To be consistent with the bounds on nucleon decay these must be suppressed by a mass scale more than eight orders above the Planck scale, a major problem. However in the case of the $\mathbb{Z}_M^R$ symmetries these operators are automatically absent. To see this note that the requirement that up- and down-type Yukawa couplings be allowed implies

$$3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} + q_{H_u} + q_{H_d} = 4 \bmod M. \tag{2.12}$$

Combining this with Eq. (2.8a) gives

$$3q_{\mathbf{10}} + q_{\bar{\mathbf{5}}} = 0 \bmod M, \tag{2.13}$$

showing that (for $M \neq 2$) the troublesome dimension five operators $\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}$ are automatically forbidden whenever the Yukawa couplings are allowed.

*2.2.4. The gravitational anomaly constraint*

For all charge assignments, the MSSM contribution to the gravitational anomaly is

$$A_0^R(\mathrm{MSSM}) = 7 \bmod \eta. \tag{2.14}$$

All cases except for $M = 6$ have $\rho \neq 0$ and hence require the presence of an axion $a$. Call the multiplet containing the axion $S$,

$$S|_{\theta=0} = s + \mathrm{i}a; \tag{2.15}$$

later we will identify $S$ with the dilaton. From the coupling to the gauge fields $\int \mathrm{d}^2\theta\, S W_\alpha W^\alpha$ one infers that the axino/dilatino has $R$ charge $-1$. Therefore, after adding the axino/dilatino contribution we obtain

$$A_0^R(\mathrm{MSSM} + \mathrm{axino/dilatino}) = 6 \bmod \eta. \tag{2.16}$$

The condition for anomaly freedom is

$$\frac{1}{24}\left(A_0^R \bmod \eta\right) = A_i^R \bmod \eta \tag{2.17}$$

for $1 \leqslant i \leqslant 3$. Now, since $A_i^R \in \mathbb{Z}$ and since the order $M$, and therefore $\eta$, divides 24, this condition is equivalent to

$$A_0^R = 0 \bmod \eta. \tag{2.18}$$

From Eq. (2.16) we see that the cases $M = 4$ and 12 are anomaly free. The case $M = 6$ is anomaly free with an axion that is singlet under $\mathbb{Z}_6^R$. All the other cases require additional states in order to cancel the gravitational anomaly.

However this does not necessarily require additional states in the low energy spectrum. This is because states contributing to the anomaly can acquire mass when the symmetry is spontaneously broken. Since the $R$ symmetry is broken in the hidden sector when supersymmetry is broken these states can acquire a mass of order the supersymmetry breaking scale in the hidden sector which can be as large as $10^{13}$ GeV. With this in mind we will not consider the gravitational anomaly any further.

Table 2.3
Field content of an $\mathrm{SO}(10)\times\widetilde{\mathbb{Z}}_M^R$ model which can produce the symmetries in Table 2.1.

(a) MSSM field content

| Label | SO(10) | $\widetilde{\mathbb{Z}}_M^R$ |
|---|---|---|
| $M_i$ | **16** | $r_{M_i}$ |
| $H$ | **10** | $r_H$ |

(b) Higgs sector

| Label | SO(10) | $\widetilde{\mathbb{Z}}_M^R$ |
|---|---|---|
| $\psi_H$ | **16** | $r_{\psi_H}$ |
| $\psi_{\overline{H}}$ | $\overline{\mathbf{16}}$ | $r_{\psi_{\overline{H}}}$ |

(c) $\widetilde{\mathbb{Z}}_M^R$-charges for different values of $M$, the resulting $\mathbb{Z}_M^R$-charge as a linear combination of the $\mathrm{U}(1)_X$ charge $Q_X$ and the $\widetilde{\mathbb{Z}}_M^R$ charge $r$. In the last column we list the value of the $\mathrm{SO}(10)$–$\mathrm{SO}(10)$–$\widetilde{\mathbb{Z}}_M^R$ anomaly $\rho$

| $M$ | $r_{M_i}$ | $r_H$ | $r_{\psi_H}$ | $r_{\psi_{\overline{H}}}$ | $\mathbb{Z}_M^R$ | $\rho$ |
|---|---|---|---|---|---|---|
| 6 | 3 | 2 | 2 | 4 | $5(-4Q_X+5r)$ | 2 |
| 8 | 4 | 2 | 7 | 1 | $3(Q_X-5r)$ | 3 |
| 12 | 9 | 8 | 8 | 4 | $5(-4Q_X+5r)$ | 5 |
| 24 | 12 | 2 | 23 | 1 | $7(Q_X-5r)$ | 11 |

### 2.2.5. *Compatibility with* SO(10)

By looking at the symmetries in Table 2.1 we observe that only the $\mathbb{Z}_4^R$ symmetry is compatible with a complete unification of quarks and leptons ($q_{\mathbf{10}}=q_{\bar{\mathbf{5}}}$). We will now show that also the other cases can potentially be in accordance with SO(10). The $\mathbb{Z}_M^R$ can be a mixture of the $\mathrm{U}(1)_X$ subgroup of SO(10) and an additional $\widetilde{\mathbb{Z}}_M^R$ symmetry which commutes with SO(10).

The crucial point is to realize that SO(10) has rank five, and SU(5) and the SM gauge group have rank four. Hence, there is an extra $\mathrm{U}(1)_X$ factor. We will denote the $\mathrm{U}(1)_X$ charge by $Q_X$. The branching rules are [15]

$$\mathrm{SO}(10)\supset \mathrm{SU}(5)\times \mathrm{U}(1)_X, \tag{2.19a}$$

$$\mathbf{10}\to \mathbf{5}_2+\bar{\mathbf{5}}_{-2}, \tag{2.19b}$$

$$\mathbf{16}\to \mathbf{10}_{-1}+\bar{\mathbf{5}}_3+\mathbf{1}_{-5}. \tag{2.19c}$$

Consider an SO(10) GUT with an additional $\widetilde{\mathbb{Z}}_M^R$ symmetry as given in Table 2.3. If the SU(5) singlets contained in the **16** and $\overline{\mathbf{16}}$ representations, $\psi_H$ and $\psi_{\overline{H}}$, attain VEVs, which have $\mathrm{U}(1)_X$ charge $Q_X=\pm 5$, we arrive at the following breaking pattern

$$\mathrm{SO}(10)\times\widetilde{\mathbb{Z}}_M^R\to \mathrm{SU}(5)\times\mathbb{Z}_M^R. \tag{2.20}$$

Note that there appears to be an additional $\mathbb{Z}_5$ symmetry, which does however not constrain any couplings since it is the non-trivial center of SU(5) (cf. [16,17]).

In summary, we can obtain our $\mathbb{Z}_M^R$ symmetries from an SO(10) GUT. However, the scenarios presented here are only toy models. First of all, further Higgs fields are needed to break SU(5) down to the SM. In addition, to obtain doublet–triplet splitting and get rid of dimension five operators larger Higgs representations are needed. Also, anomaly matching (cf. [16]) forces us to introduce extra representations because the value of $\rho$ does not equal the one given in Table 2.1.

These considerations also show that the SU(5) relations are mandatory. Since SU(5) and the standard model gauge group have the same rank, there is no U(1) with which our $\mathbb{Z}_M^R$s could mix upon breaking SU(5).

## 3. A simple $\mathbb{Z}_4^R$ symmetry in the MSSM

In Table 2.1 we survey all symmetries and charge assignments which commute with SU(5). The simplest one, the $\mathbb{Z}_4^R$, commutes also with SO(10). In what follows we will discuss this case in more detail.

### 3.1. Non-perturbative terms

The gauge invariant superpotential of the MSSM contains

$$\begin{aligned}\mathscr{W} = {} & \mu H_u H_d + \kappa_i L_i H_u \\ & + Y_e^{ij} H_d L_i \overline{E}_j + Y_d^{ij} H_d Q_i \overline{D}_j + Y_u^{ij} H_u Q_i \overline{U}_j \\ & + \lambda_{ijk}^{(0)} L_i L_j \overline{E}_k + \lambda_{ijk}^{(1)} L_i Q_j \overline{D}_k + \lambda_{ijk}^{(2)} \overline{U}_i \overline{D}_j \overline{D}_k \\ & + \kappa_{ij}^{(0)} H_u L_i H_u L_j + \kappa_{ijk\ell}^{(1)} Q_i Q_j Q_k L_\ell + \kappa_{ijk\ell}^{(2)} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell \\ & + \kappa_{ijk}^{(3)} Q_i Q_j Q_k H_d + \kappa_{ijk}^{(4)} Q_i \overline{U}_j \overline{E}_k H_d + \kappa_i^{(5)} L_i H_u H_u H_d. \end{aligned} \tag{3.1}$$

We see immediately that the coefficients $\mu$, $\kappa_i$, $\lambda_{ijk}^{(0)}$, $\lambda_{ijk}^{(1)}$, $\lambda_{ijk}^{(2)}$, $\kappa_{ijk\ell}^{(1)}$, $\kappa_{ijk\ell}^{(2)}$, $\kappa_{ijk}^{(3)}$, $\kappa_{ijk}^{(4)}$ and $\kappa_i^{(5)}$ are forbidden by $\mathbb{Z}_4^R$ perturbatively while $Y_{e,d,u}^{ij}$ and $\kappa_{ij}^{(0)}$ are allowed. In what follows we will show that at the non-perturbative level $\mu$ as well as $\kappa_{ijk\ell}^{(1)}$ and $\kappa_{ijk\ell}^{(2)}$ will be induced while the $R$ parity violating couplings $\kappa_i$ and $\lambda$ as well as the $\kappa^{(3-5)}$ remain zero. The reason is that the latter are forbidden by a $\mathbb{Z}_2$ subgroup of $\mathbb{Z}_4^R$ which is equivalent to matter parity. This subgroup is unbroken by the supersymmetry breaking sector and thus remains a symmetry of the full theory.

Let us spell out the argument in somewhat more detail. Call the $\mathbb{Z}_4^R$ transformation $\zeta$,

$$\begin{aligned} \zeta: \quad \text{matter superfield} &\to \mathrm{i}\cdot\text{matter superfield}, \\ \text{Higgs superfield} &\to \text{Higgs superfield}, \\ \theta &\to \mathrm{i}\cdot\theta, \\ \mathscr{W} &\to -\mathscr{W}. \end{aligned} \tag{3.2}$$

Now look at the transformation $\zeta^2$, under which matter superfields transform with a minus, Higgs superfields go into themselves and $\theta \to -\theta$. The transformation fermion $\to$ −fermion and $\theta \to -\theta$ is a symmetry of any SUSY theory, therefore $\zeta^2$ is equivalent to matter parity, and, in particular, anomaly free with $\rho = 0$. One can use the path integral (cf. Appendix A) to show that correlators that vanish due to a non-anomalous symmetry with $\rho = 0$ also vanish at the quantum level. Therefore, the matter parity subgroup contained in the $\mathbb{Z}_4^R$ will not be violated by quantum effects.

On the other hand, correlators which are only forbidden by $\mathbb{Z}_4^R$ but not by $\mathbb{Z}_2$, i.e. which are invariant under $\zeta^2$, can be non-trivial at the quantum level. A convenient way to parametrize effective couplings describing these effects involve the S field, which shifts under the $\mathbb{Z}_4^R$ symmetry as (cf. Eq. (A.22))

$$S \to S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}. \tag{3.3}$$

The discrete shift of $S$ is given by (cf. Eq. (A.23))

$$\Delta_{\mathrm{GS}} = \frac{1}{4\pi}(A_{G\text{-}G\text{-}\mathbb{Z}_4^R} \bmod 2) = \frac{1+2\nu}{4\pi} \tag{3.4}$$

with $\nu \in \mathbb{Z}$. This allows us to write down terms

$$\Delta\mathscr{W}_{\mathrm{np}} = \exp\left(-8\pi^2 \frac{1+2n}{1+2\nu} S\right)\left[B_0 + \overline{\mu} H_u H_d + \overline{\kappa}^{(1)}_{ijk\ell} Q_i Q_j Q_k L_\ell \right.$$
$$\left. + \overline{\kappa}^{(2)}_{ijk\ell} \overline{U}_i \overline{U}_j \overline{D}_k \overline{E}_\ell\right] \tag{3.5}$$

with some coefficients $B_0$, $\overline{\mu}$ and $\overline{\kappa}^{(1,2)}_{ijk\ell}$ and $n \in \mathbb{Z}$. Such superpotential terms are $\mathbb{Z}_4^R$ covariant, i.e. the exponential transforms with a minus under $\mathbb{Z}_4^R$ while the terms in the square brackets are invariant. Due to the fact that $S$ enters the gauge kinetic function, these terms are proportional to $\mathrm{e}^{-8\pi^2 \frac{1+2n}{1+2\nu} \frac{1}{g^2}}$. For $n = \nu = 0$ they can be interpreted as originating from t'Hooft instanton effects. The $8\pi^2$ in the exponential can also be obtained directly in a stringy computation [18]. The crucial property of the non-perturbative couplings (3.5) is that they are naturally suppressed.

The critical question concerns now the interpretation of the $\mathrm{e}^{-8\pi^2 \frac{1+2n}{1+2\nu} S}$ terms. So far we have shown that such terms are $\mathbb{Z}_4^R$ covariant. In the MSSM as a 'stand-alone' theory, $\mathrm{SU}(3)_C$ or $\mathrm{SU}(2)_\mathrm{L}$ instantons can generate such terms, but their magnitude turns out to be very small. Whether or not further terms, with given $n$ and $\nu$, appear depends on the model. Let us now make the very common assumption that there is a hidden sector that gets strong at some intermediate scale $\Lambda$. Then the non-perturbative terms related to the strong dynamics may well be the source of supersymmetry breakdown [19,20]. Given non-renormalizable interactions between the MSSM and the hidden sector, communicated by some messenger fields, $\Lambda$ sets the magnitude of the MSSM soft terms, $m_{\mathrm{soft}} \sim \Lambda^3/M_*^2$, with $M_*$ being the messenger scale. In such settings, holomorphic, i.e. superpotential, terms can also be induced by higher-dimensional operators. That is, the $\Delta\mathscr{W}_{\mathrm{np}}$ terms can appear with magnitude $m_{\mathrm{soft}} \sim \Lambda^3/M_*^2$, but in principle they may also be absent if there are no higher-dimensional operators connecting the MSSM sector with the hidden sector exhibiting strong dynamics. In other words, if the MSSM fields are singlets under the hidden sector gauge interactions, there is, a priori, no guarantee that the $\Delta\mathscr{W}_{\mathrm{np}}$ terms appear with reasonable size. If the scale of MSSM soft terms is related to some hidden sector strong dynamics, we expect the holomorphic terms also to appear, unless there are additional symmetries beyond $\mathbb{Z}_4^R$ that forbid such couplings. Assuming that the dominant non-perturbative scale is related to supersymmetry breakdown we expect that the $\Delta\mathscr{W}_{\mathrm{np}}$ terms are of the order of the soft supersymmetry breaking terms. We will mainly focus on gravity mediation, where $M_* = M_\mathrm{P}$ and these terms are of the order of the gravitino mass $m_{3/2}$ (in Planck units). Below in Section 3.5 we will present an explicit string theory example in which the non-perturbative $\mu$ term is directly connected to $m_{3/2}$.

At this point let us mention that for the case of discrete $R$ symmetries we disagree with statements made recently in [14], where it was claimed that, in the context of gravity mediation, $R$ symmetries will be broken at the Planck scale and be therefore ineffective. The claim relies on the observation that there are fields with Planck scale VEVs that break the $R$ symmetry. The derivation of this result relies on the inequality $|\langle\mathscr{W}\rangle| \leqslant \frac{1}{2} f_r |F|$ (cf. Eq. (9) in [14]) where $f_r$ is the $R$-axion decay constant. This was derived for the case of continuous $R$ symmetries by taking the limit of an infinitesimal transformation [14]. For the case of discrete $R$ symmetries the inequality is no longer true and there is no requirement that $R$-non-singlets acquire Planck scale VEVs. In this case the $R$ symmetry can be broken at a much lower scale. This is the case in the supergravity examples discussed here. In them the breaking of the $R$ symmetry occurs non-perturbatively at an intermediate scale in a hidden sector and it is the superpotential VEV $\langle \mathscr{W} \rangle$ rather than a field VEV that is the order parameter for $R$ symmetry breaking. Since the superpotential only appears at the non-perturbative level it is small. Also all other $R$ symmetry breaking terms are small. This applies also to other schemes such as the one discussed in [21], where a small $\langle \mathscr{W} \rangle$ is a consequence of an approximate $R$ symmetry. Here the $R$ symmetry is broken perturbatively, but again the order parameter, i.e. the superpotential VEV, is very small. In conclusion, $R$ symmetries are a useful tool also, or in particular, in gravity mediation, where the same parameter, the small superpotential VEV, both sets the scale of soft masses and cancels the vacuum energy. In what follows, we discuss how the connection between the $\Delta\mathscr{W}_{\mathrm{np}}$ terms and $m_{3/2}$ arises in the scheme of Kähler stabilization.

### *3.2. Dilaton stabilization and supersymmetry breaking*

At the present stage of the discussion, the $S$ field has no potential and supersymmetry is unbroken. An economical way to rectify this situation is to invoke the stringy scheme of Kähler stabilization [22–25] (see also Appendix C).[2] In this case the term of the form $\mathrm{e}^{-bS}$ represents a hidden sector gaugino condensate [20], which sets the scale for supersymmetry breakdown. According to the above discussion, in the presence of our $\mathbb{Z}_4^R$ symmetry

$$b = 8\pi^2 \frac{1+2n}{1+2\nu}. \tag{3.6}$$

Let us discuss what that means in the case of a hidden $\mathrm{SU}(N_c)$ theory with $N_f$ chiral superfields in the $\boldsymbol{N_c} + \overline{\boldsymbol{N_c}}$ representations. Here the coefficient $b$ is given by

$$b = \frac{3}{2\beta} = \frac{3 \cdot 8\pi^2}{3N_c - N_f}. \tag{3.7}$$

Therefore

$$\frac{3}{3N_c - N_f} = \frac{1+2n}{1+2\nu}. \tag{3.8}$$

In the scheme under consideration, supersymmetry is broken by a non-trivial VEV of $F_S$. This leads to gaugino and soft scalar masses, following the pattern of the so-called "dilaton dominated scenario" [26]. This scenario has a number of phenomenologically attractive features. In particular, due to flavour universality in the soft breaking sector, it avoids the SUSY FCNC problem. Also, most of the physical CP phases, e.g. $\arg(A^* M)$, vanish which ameliorates the SUSY CP problem. However in the dilaton dominated case the vacuum structure may favour an unacceptable colour breaking minimum [27]. Other phenomenological aspects have been discussed in [28].

Moreover, the (non-perturbative) superpotential acquires a non-trivial VEV as well,

$$\langle \mathscr{W} \rangle \sim \mathrm{e}^{-b\langle S\rangle} \neq 0. \tag{3.9}$$

All gauge invariant terms which have been forbidden because they have zero $R$ charge can now be obtained by multiplying them with $\langle \mathscr{W} \rangle$. $\langle \mathscr{W} \rangle$ will hence be the order parameter for $R$ symmetry breaking. Inserting this in Eq. (3.5) we find that there will be a $\mu$ term of the order of $\langle \mathscr{W} \rangle$,

[2] Alternatively, other stabilization schemes, such as racetrack mechanisms, may be applicable here.

i.e. of the order of the gravitino mass $m_{3/2}$, as well as $\kappa^{(1)}_{ijk\ell} \sim 10^{-15}/M_{\mathrm{P}}$. On the other hand, terms which have odd $\mathbb{Z}_4^R$ charge cannot be obtained by multiplying them by $\mathrm{e}^{-b\langle S\rangle}$; these are precisely the $R$ parity violating couplings $\kappa_i$, $\lambda^{(0)}$, $\lambda^{(1)}$ and $\lambda^{(2)}$ in Eq. (3.1), showing again that matter parity will not be broken.

### 3.3. Phenomenology

The suppression of the $\kappa^{(1)}$ term leads to a situation in which dimension five proton decay will be unobservably small. Therefore, proton decay will proceed through dimension six operators mediated by gauge boson exchange.

In settings with discrete $R$ symmetries one should worry about the cosmological domain wall problem [29]. The domain walls form at the stage of $R$ symmetry breaking, typically the scale of supersymmetry breaking. For the case of gravity mediation this is at an intermediate scale of $\mathcal{O}(10^{12})$ GeV. Provided the Hubble scale during inflation is below this scale, domain walls have sufficient time to form and then they will be inflated away. The requirement that no domain walls are created after inflation translates in an upper bound on the reheat temperature $T_R$, which, given the other bounds on $T_R$ in supersymmetric cosmology, appears rather mild.

A discrete $R$ symmetry may also be useful for inflationary scenarios. For example, in [30], it is argued that a $\mathbb{Z}_8^R$ symmetry, with inflaton field $\phi$ carrying $R$ charge 2, can be used to guarantee that the inflaton potential is flat near the origin and give enough inflation.[3]

In summary, for the case of gravity mediated supersymmetry breaking, non-perturbative effects naturally generate a $\mu$ parameter of the order of the gravitino mass. The symmetry ensures that the proton decay rate is well below the experimental limit and an exact matter parity is left that guarantees SUSY particles can only be pair produced and the lightest SUSY particle is stable. Thus one is left with the usual MSSM phenomenology with negligibly small corrections from higher dimension terms.

### 3.4. $\mathbb{Z}_4^R$ literature

A version of the $\mathbb{Z}_4^R$ symmetry has been prosed by Kurosawa et al. [31] where the traditional version of anomaly constraints was imposed, i.e. the possibility of GS anomaly cancellation has not been taken into account. This lead to a setting in which extra light charged states were required to cancel the anomaly.

The $\mathbb{Z}_4^R$ symmetry with GS anomaly cancellation has also been discussed by Babu et al. [11]. There are several aspects in which our analysis differs from or goes beyond [11]:

1. We discuss the uniqueness for the first time.
2. We point out that the $\mathbb{Z}_4^R$ symmetry also suppresses dimension five operators.
3. We present the first discussion of non-perturbative $\mathbb{Z}_4^R$ violating effects.
4. Related to the previous point, Babu et al. only discuss generation of the $\mu$ term by the Giudice–Masiero mechanism. However, there will also be a holomorphic non-perturbative (i.e. Kim–Nilles type) contribution.

[3] Note, their definition of the order of the discrete symmetry differs from ours. What they call $\mathbb{Z}_4^R$ we call $\mathbb{Z}_8^R$.

Table 3.1
Spectrum of the orbifold model from [32]. The representations w.r.t. $G_{\mathrm{SM}} \times [\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2)]_{\mathrm{hid}}$, their multiplicities (#) and labels are listed.

| # | Representation | Label | # | Representation | Label |
|---|---|---|---|---|---|
| 3 | $(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{6}}$ | $Q$ | 3 | $(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$ | $\overline{U}$ |
| 8 | $(\bar{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{3}}$ | $\overline{D}$ | 5 | $(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$ | $D$ |
| 7 | $(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{-\frac{1}{2}}$ | $L$ | 4 | $(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1},\mathbf{1})_{\frac{1}{2}}$ | $\overline{L}$ |
| 3 | $(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{1}$ | $\overline{E}$ | 33 | $(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})_{0}$ | $N$ |
| 5 | $(\mathbf{1},\mathbf{1};\mathbf{3},\mathbf{1},\mathbf{1})_{0}$ | $X$ | 5 | $(\mathbf{1},\mathbf{1};\bar{\mathbf{3}},\mathbf{1},\mathbf{1})_{0}$ | $\overline{X}$ |
| 6 | $(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{2})_{0}$ | $Y$ | 6 | $(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2},\mathbf{1})_{0}$ | $Z$ |

5. In [11] the $\mathbb{Z}_4^R$ is argued to originate from an 'anomalous' $\mathrm{U}(1)_R$. We are not aware of a model in which such an 'anomalous' $\mathrm{U}(1)_R$ appears in string models. However, we also cannot rule out this possibility.
6. We present a detailed discussion of the mixed hypercharge coefficient $A_1$.
7. Babu et al. do not discuss the gravitational anomalies.

### 3.5. *String theory realization*

In the above discussion we argued that, if some hidden sector strong dynamics was responsible for supersymmetry breakdown, also a $\mu$ term of the right size will be induced by this dynamics. In order to render our discussion more specific, we will now discuss an explicit, globally consistent string-derived model. Such models have the important property that they are complete, i.e. unlike bottom-up (or 'local') models they cannot be 'amended' by some extra states or sectors. This allows us to clarify whether or not a reasonable $\mu$ term will appear.

Making extensive use of the methods to determine the remnant symmetries described in [17], we were able to find examples realising the $\mathbb{Z}_4^R$ discussed in Section 3, based on the string model derived in [32] and similar models, with the exact MSSM spectrum, a large top Yukawa coupling, a non-trivial hidden sector etc. In what follows, we present an explicit example.

Consider the MSSM candidate model of [32]. It is obtained by the compactification of the $\mathrm{E}_8 \times \mathrm{E}_8$ heterotic string on a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with an additional freely acting $\mathbb{Z}_2$. At the orbifold point (where the VEVs of all fields are set to zero) the $\mathrm{E}_8 \times \mathrm{E}_8$ gauge group gets broken to

$$G_{\mathrm{SM}} \times \left[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2)\right]_{\mathrm{hid}} \tag{3.10}$$

times eight U(1) factors. One of them, denoted by $\mathrm{U}(1)_{\mathrm{anom}}$, appears anomalous, i.e. $\operatorname{tr} Q_{\mathrm{anom}} = 180 \neq 0$ using the normalization $|t_{\mathrm{anom}}|^2 = 15$. Hence, a one-loop Fayet–Iliopoulos $D$-term gets induced. Furthermore, the massless spectrum includes three generations of quarks and leptons and is summarized in Table 3.1. More details on the model can be found in [33].

Next, we choose a vacuum configuration in which the SM singlets

$$\begin{aligned}\{\phi_i\} = \{&\overline{X}_4, \overline{X}_5, X_3, X_4, X_5, Y_1, Y_2, Z_1, Z_2,\\ &N_1, N_2, N_4, N_7, N_{10}, N_{15}, N_{16}, N_{17},\\ &N_{20}, N_{21}, N_{25}, N_{27}, N_{28}, N_{30}, N_{32}, N_{33}\}\end{aligned} \tag{3.11}$$

attain VEVs. These fields are charged with respect to the hidden sector gauge group, the U(1) factors and several discrete symmetries (i.e. this orbifold compactification provides three $\mathbb{Z}_4^R$

Table 3.2
$\mathbb{Z}_4^R \times \mathbb{Z}_2$ charges of the fields with SM quantum numbers.

| Quarks and leptons | | | | | | Higgs and exotics | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $Q_1$ | 1 | 0 | $\overline{U}_1$ | 1 | 0 | $\overline{L}_1$ | 0 | 0 | $L_1$ | 0 | 0 |
| $Q_2$ | 1 | 1 | $\overline{U}_2$ | 1 | 1 | $\overline{L}_2$ | 0 | 0 | $L_3$ | 2 | 0 |
| $Q_3$ | 1 | 1 | $\overline{U}_3$ | 1 | 1 | $\overline{L}_3$ | 2 | 0 | $L_4$ | 0 | 0 |
| $\overline{D}_3$ | 1 | 1 | $L_2$ | 1 | 1 | $\overline{L}_4$ | 2 | 0 | $L_5$ | 0 | 0 |
| $\overline{D}_7$ | 1 | 0 | $L_6$ | 1 | 0 | $\overline{D}_1$ | 0 | 0 | $D_1$ | 0 | 0 |
| $\overline{D}_8$ | 1 | 0 | $L_7$ | 1 | 0 | $\overline{D}_2$ | 2 | 0 | $D_2$ | 2 | 0 |
| $\overline{E}_1$ | 1 | 0 | | | | $\overline{D}_4$ | 2 | 0 | $D_3$ | 0 | 0 |
| $\overline{E}_2$ | 1 | 1 | | | | $\overline{D}_5$ | 0 | 0 | $D_4$ | 2 | 0 |
| $\overline{E}_3$ | 1 | 1 | | | | $\overline{D}_6$ | 0 | 0 | $D_5$ | 2 | 0 |

symmetries reflecting the discrete rotational symmetry of the three $\mathbb{Z}_2$ orbifold planes and six $\mathbb{Z}_2$ factors coming from the space group selection rule, see Appendix B of [32]). Hence, the $\phi_i$ VEVs of Eq. (3.11) break these (gauge and discrete) symmetries and it turns out that

$$G_{\mathrm{SM}} \times \mathbb{Z}_4^R \times \mathbb{Z}_2 \tag{3.12}$$

remains unbroken, where $\mathbb{Z}_4^R$ is a mixture of an orbifold $\mathbb{Z}_4^R$ and other symmetries.[4] The $\mathbb{Z}_4^R \times \mathbb{Z}_2$ charges of the SM charged fields are listed in Table 3.2. From there we see that the $\mathbb{Z}_4^R$ factor gives a stringy realisation of the $\mathbb{Z}_4^R$ symmetry described in a bottom-up approach in Section 3. Furthermore, the $\phi_i$ VEVs also provide mass terms for the exotics, which are massless at the orbifold point, and allow us to cancel the Fayet–Iliopoulos $D$-term.

In detail, we find four Fayet–Iliopoulos monomials, i.e. monomials that are gauge invariant except for a total negative $\mathrm{U}(1)_{\mathrm{anom}}$ charge such that their VEVs can cancel the positive Fayet–Iliopoulos term in the $D$-term potential (cf. [34]). The monomials read

$$\left\{N_{28}^4 Y_1 Y_2,\ N_{28}^4 Z_1 Z_2,\ N_{33}^4 Y_1 Y_2,\ N_{33}^4 Z_1 Z_2\right\},$$

with $Q_{\mathrm{anom}}(\text{FI monomial}) = -15$. Further, we find monomials with zero or positive $\mathrm{U}(1)_{\mathrm{anom}}$ charge involving all $\phi_i$ fields from Eq. (3.11). Hence, this represents a $D$-flat configuration.

Matter fields are identified as fields with $\mathbb{Z}_4^R$ charge 1, see Table 3.2, and are given by $Q_1$, $Q_2$, $Q_3$, $\overline{U}_1$, $\overline{U}_2$, $\overline{U}_3$, $\overline{E}_1$, $\overline{E}_2$, $\overline{E}_3$, $\overline{D}_3$, $\overline{D}_7$, $\overline{D}_8$, $L_2$, $L_6$ and $L_7$. An inspection of the discrete charges of the Higgs candidates, i.e. the remaining $L$ and $\overline{L}$ fields, reveals that there is one massless Higgs pair at the perturbative level. Unfortunately, the additional $\mathbb{Z}_2$, which we cannot break, forbids some Yukawa couplings such that the charged lepton and $d$-type Yukawa couplings $Y_e$ and $Y_d$ have rank 2.

More explicitly, we have computed the couplings using the well-known string selection rules [35,36], which have been extended to the case of non-local GUT breaking [32]. For the Higgs mass matrix we find

$$\mathcal{M}_{\mathrm{Higgs}} = \begin{pmatrix} 0 & N_{15} & 0 & 0 \\ 0 & \phi^{13} & 0 & 0 \\ \phi^{11} & 0 & N_{10} & \phi^3 \\ \phi^{11} & 0 & \phi^3 & N_{10} \end{pmatrix}, \tag{3.13}$$

[4] Meanwhile a very similar string model exhibiting vacua without the extra $\mathbb{Z}_2$ has been found [18].

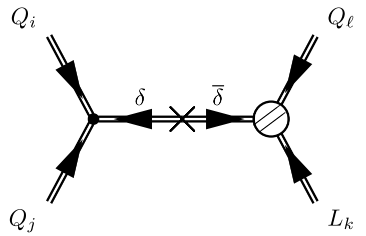


Fig. 1. Absence of induced $QQQL$ operators. Either one vertex or the mass term is forbidden by $\mathbb{Z}_4^R$.

where for example $\phi^3$ denotes a sum of known monomials in the fields of Eq. (3.11) starting at degree three. Therefore a linear combination of $\overline{L}_1$ and $\overline{L}_2$ as well as a linear combination of $L_1$, $L_4$ and $L_5$ remains massless. The mass matrix of the extra colour triplets

$$\{\delta_i\} = \{D_1, D_2, D_3, D_4, D_5\}, \tag{3.14a}$$

$$\{\bar{\delta}_i\} = \{\overline{D}_1, \overline{D}_2, \overline{D}_4, \overline{D}_5, \overline{D}_6\} \tag{3.14b}$$

is

$$\mathcal{M}_{\text{extra triplets}} = \begin{pmatrix} 0 & N_1 & \phi^9 & 0 & 0 \\ N_2 & 0 & 0 & N_{16} & N_{20} \\ 0 & \phi^3 & \phi^{13} & 0 & 0 \\ N_{28} & 0 & 0 & N_{10} & \phi^3 \\ N_{33} & 0 & 0 & \phi^3 & N_{10} \end{pmatrix}. \tag{3.15}$$

So we see that most exotics decouple at the linear level in the VEV fields $\phi_i$, one pair of exotic triplets gets masses at order nine in the $\phi_i$ fields and another one at order three. One may speculate that this leads to the presence of colour triplets somewhat below the GUT scale, which may account for the fact that, within the MSSM, the strong fine structure constant $\alpha_3 = g_3^2/(4\pi)$ turns out to be about 3% smaller than $\alpha_1$ and $\alpha_2$ at $M_{\text{GUT}}$. An important feature of the $\mathbb{Z}_4^R$ symmetry is that integrating out the triplets does not give rise to dimension 5 proton decay operators, as each triplet that couples to quarks and leptons pairs up with a triplet that does not (cf. the similar discussion in [37]). In other words, the mass partner $\delta$ of a triplet $\bar{\delta}$ that couples to $Q_\ell L_k$ (and therefore has $\mathbb{Z}_4^R$ charge 0) cannot couple to $Q_i\, Q_j$ (Fig. 1).

The Yukawa couplings are

$$Y_u = \overline{L}_1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & \phi^4 & \phi^4 \\ 0 & \phi^4 & \phi^4 \end{pmatrix} + \overline{L}_2 \begin{pmatrix} \phi^{12} & 0 & 0 \\ 0 & \phi^4 & \phi^4 \\ 0 & \phi^4 & \phi^4 \end{pmatrix}, \tag{3.16a}$$

$$Y_d = L_1 \begin{pmatrix} 0 & \phi^{22} & \phi^{22} \\ \phi^{22} & 0 & 0 \\ \phi^{22} & 0 & 0 \end{pmatrix} + L_4 \begin{pmatrix} 0 & 1 & \phi^{12} \\ 1 & 0 & 0 \\ \phi^{12} & 0 & 0 \end{pmatrix} \tag{3.16b}$$

$$+ L_5 \begin{pmatrix} 0 & \phi^{12} & 1 \\ \phi^{12} & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = Y_e^T. \tag{3.16c}$$

As already mentioned, $Y_d$ and $Y_e$ have rank 2.

A crucial property of the string embedding is that, unlike in the bottom-up approach, we have obtained an understanding of the origin of the $\mathbb{Z}_4^R$ symmetry. In stringy language discrete $R$ symmetries originate from what is called "$H$-momentum conservation" [35], which reflects discrete rotational symmetries of compact space–time dimensions. In our orbifold we have three $\mathbb{T}^2/\mathbb{Z}_2$ orbifold planes which can be rotated against each other by 180°. Each rotational symmetry manifests itself as a discrete order four $R$ symmetry in the effective field-theoretic description of the model. The remnant $\mathbb{Z}_4^R$ discussed above is a linear combination of such symmetries and other discrete symmetries, either coming from the space group selection rule or emerging from continuous U(1) symmetries through spontaneous breaking (see the general discussion in [17]). Note also that such discrete $R$ symmetries can already appear anomalous at the orbifold point [38]. That is, no mixing with the so-called anomalous U(1) is required to obtain a $\mathbb{Z}_M^R$ symmetry whose anomalies are canceled by the GS mechanism.[5]

Perhaps the most important property of the model is that there is a proportionality between the holomorphic mass term connecting $L_1$ and $\overline{L}_1$ and the superpotential VEV. This relation can be derived in an SU(6) orbifold GUT limit of the model, where it emerges due to gauge invariance in extra dimensions [39] (cf. also the field-theoretic discussion in [40,41]). Let us comment that the same SU(6) gauge symmetry also enforces the tree-level equality between the gauge and top–Yukawa couplings [42]. We hence see that, at least in this model, the superpotential VEV is both a measure for the gravitino mass, as usual, and the $\mu$ term.

## 4. Singlet extensions

In Section 2.2 we have shown that the requirement of universality for the mixed gauge anomalies constrains the order $M$ of a potential $\mathbb{Z}_M^R$ symmetry to be a divisor of 24. As we have seen, this analysis carries over in an obvious way to singlet extensions of the MSSM, since additional SM singlet fields cannot change the constraints coming from the mixed gauge anomalies. In such extensions the MSSM subsector still has to obey the criteria derived in Section 2.2. However, the extra (singlet) fields can be subject to additional symmetries.

In what follows we concentrate on two simple singlet extensions in which one or two singlet fields, respectively, are added. The first part of the discussion, Section 4.1, is on the so-called NMSSM, in which the singlet couples to the Higgs bilinear and there are cubic self-interactions. Section 4.2 is dedicated to a singlet extension of the MSSM which is capable of addressing the strong CP problem.

### 4.1. NMSSM

In the NMSSM, there is one additional singlet $N$ with superpotential

$$\mathscr{W} = \mathscr{W}_{\text{MSSM}}^{\mu=0} + \lambda N H_u H_d + \kappa N^3. \tag{4.1}$$

Let us now consider what this implies for the order $M$ of a $\mathbb{Z}_M^R$ symmetry.

#### 4.1.1. Constraints from NMSSM couplings

There are three different classes of $\mathbb{Z}_M^R$ symmetries for which the $N^3$-term of Eq. (4.1) implies different charges for the singlet $N$, i.e.

[5] The generality of the $\mathbb{Z}_4^R$ symmetry connected with a $\mathbb{Z}_2$ orbifold suggests that $\mathbb{Z}_4^R$ invariant models may also be obtained in other orbifold constructions such as $\mathbb{Z}_6$-II or $\mathbb{Z}_4 \times \mathbb{Z}_2$ orbifolds of $\mathbb{T}^6$.

Table 4.1
Charge assignments for the $\mathbb{Z}_4^R$ symmetry.

| $M$ | $q_{\mathbf{10}}$ | $q_{\bar{\mathbf{5}}}$ | $q_{H_u}$ | $q_{H_d}$ | $q_N$ |
|---|---|---|---|---|---|
| 4 | 1 | 1 | 0 | 0 | 2 |

Table 4.2
Charge assignments for the $\mathbb{Z}_8^R$ symmetry.

| $M$ | $q_{\mathbf{10}}$ | $q_{\bar{\mathbf{5}}}$ | $q_{H_u}$ | $q_{H_d}$ | $q_N$ |
|---|---|---|---|---|---|
| 8 | 1 | 5 | 0 | 4 | 6 |

$$M = 0 \bmod 3 \quad \Rightarrow \quad \text{no } N^3 \text{ term possible,} \tag{4.2}$$

$$M = 1 \bmod 3 \quad \Rightarrow \quad q_N = \frac{M+2}{3} \bmod M, \tag{4.3}$$

$$M = 2 \bmod 3 \quad \Rightarrow \quad q_N = \frac{2M+2}{3} \bmod M, \tag{4.4}$$

with $q_N$ the $\mathbb{Z}_M^R$ charge of $N$.

$M = 1 \bmod 3$. Let us first consider the case $M = 1 \bmod 3$. The term $\lambda N H_u H_d$ together with Eq. (2.8a) then implies

$$\begin{aligned} \left(\frac{M+2}{3} \bmod M\right) + (4 \bmod 2\eta) &= 2 \bmod M \\ \Rightarrow \quad \frac{M+8}{3} &= 0 \bmod M. \end{aligned} \tag{4.5}$$

This equation has only one non-trivial solution for integer $M$, namely $M = 4$. Note that in this case $q_N = 2 \bmod 4$ and a linear term in $N$ is also allowed in the superpotential. Strictly speaking this is not the NMSSM but it is viable if the linear term is very small. We will discuss later why this may be natural.

Following the analysis of Section 2.2 and using Eq. (4.3), the unique charge assignment compatible with the Weinberg operator is shown in Table 4.1.

This is exactly the $\mathbb{Z}_4^R$ symmetry which we discussed in Section 3. We have seen that the mixed gauge anomaly coefficients of this symmetry satisfy the Green–Schwarz condition. Of course the singlet does not change these coefficients, so the analysis still applies.

$M = 2 \bmod 3$. Let us now consider the case $M = 2 \bmod 3$. The term $\lambda N H_u H_d$ together with Eq. (2.8a) then implies

$$\frac{2M+8}{3} = 0 \bmod M. \tag{4.6}$$

The solutions to this equation are $M = 2, 8$. As we have noted earlier there are no meaningful $M = 2$ $R$ symmetries. The $M = 8$ case however is very interesting since, in this case, $q_N = 6 \bmod 8$ and the linear term in $N$ is forbidden. Following the analysis of Section 2.2 and using Eq. (4.4), the unique charge assignment compatible with the Weinberg operator is shown in Table 4.2.

As the singlet does not contribute to mixed gauge anomalies, we know already from Table 2.1 that the $\mathbb{Z}_8^R$ symmetry has $A^R_{1\leqslant i\leqslant 3}(\text{MSSM}) = \rho = 1$.

*4.1.2. The hierarchy problem*

Searching for possible $\mathbb{Z}_M^R$ symmetries in the context of the NMSSM we found that there are only two potential candidates: a $\mathbb{Z}_4^R$ and a $\mathbb{Z}_8^R$ symmetry. The $\mathbb{Z}_4^R$ symmetry is actually a subgroup of the $\mathbb{Z}_8^R$ symmetry, hence both symmetries are closely related. While the $\mathbb{Z}_4^R$ commutes with SO(10) the $\mathbb{Z}_8^R$ only commutes with SU(5). In both cases all dimension four and five baryon and lepton number violating operators are forbidden (except for the Weinberg operator), consistent with what we found in Section 2.2.

A potential problem with NMSSM models arises because SUSY breaking breaks the $R$ symmetry and in radiative order a linear term in $N$ is generated in the superpotential. If the coefficient of this linear term is larger than the square of the electroweak scale it will lead to a large VEV for the singlet $N$ and therefore to a destabilization of the SUSY solution to the gauge hierarchy problem. This has been studied in detail by Abel [43] who showed that the only dangerous operators that induce divergent tadpoles arise either from even terms in the superpotential or odd terms in the Kähler potential. He also showed that an $R$ symmetry can avoid such terms because of the different $R$ charges of the super- and Kähler-potential (cf. also [44]). From the charge assignments of Tables 4.1 and 4.2 for the singlet $N$ and the Higgs fields it is easy to show that the super- and Kähler-potentials actually do have exactly this structure in both the $\mathbb{Z}_4^R$ and the $\mathbb{Z}_8^R$ case and so in both cases radiative corrections do not destabilise the SUSY solution to the hierarchy problem.

The main difference between $\mathbb{Z}_4^R$ and $\mathbb{Z}_8^R$ is that the former allows a linear term even at tree level. Does this mean that it is necessary to have the full $\mathbb{Z}_8^R$ symmetry when building the NMSSM? In effective theories, such as those describing the massless degrees of freedom in string compactifications, the superpotential starts with cubic terms in the fields and the linear term only appears through the coupling of the singlet field to fields acquiring VEVs. If the only (non-moduli) fields, $\phi$, with VEVs above the electroweak scale are in the hidden sector the coupling will be suppressed by messenger field masses, $M_*$, which may be as large as the Planck scale. Allowing for trilinear couplings to messenger fields as well as trilinear couplings between messenger and hidden sector fields and assuming no additional symmetries, the leading term in the superpotential after integrating out the messenger fields is $N\phi^4/M_*^2$ with the messenger scale $M_*$. Taking Planck scale messengers, the constraint that this should not disturb the hierarchy is that $\langle\phi\rangle \leqslant \sqrt{M_W M_\mathrm{P}}$ which is satisfied if the dominant VEV comes from the SUSY breaking sector. In this case it is sufficient to impose just the $\mathbb{Z}_4^R$ symmetry when building the NMSSM.

The role of the SM singlets $\psi_2^{(i)}$ with $R$-charge 2 (such as $N$ for the case $\mathbb{Z}_4^R$) has recently been discussed in the context of singlet (moduli) stabilization [18]. There it was found that for a superpotential with generic coefficients the number of singlets with $R$-charge 2 should not exceed the number of fields $\phi_0^{(j)}$ with $R$-charge 0 since otherwise the $F$-term conditions would overconstrain the system. Moreover, the $\psi_2^{(i)}$ fields pair up with an equal number of $\phi_0^{(j)}$ fields. That is, for generic superpotential coefficients one might not expect to find vacua with an unbroken $\mathbb{Z}_4^R$ symmetry and a massless singlet with $R$-charge 2. However, it is quite conceivable that there are symmetries between the $F$-terms. In such a situation the $\psi_2^{(i)} - \phi_0^{(j)}$ mass matrix won't have full rank such that one is effectively left with one (or more) singlet(s) with $R$-charge 2. It will be interesting to see if this situation can be realized in string models in which there are additional symmetries, such as $D_4$ [45,46], relating the superpotential coefficients.

*4.1.3. Non-perturbative effects*

Non-perturbative effects may also be important in determining the low energy phenomenology. From Eq. (3.5) we see that the superpotential has a term of the form

$$\Delta\mathscr{W}_{\mathrm{np}} = B_0 \mathrm{e}^{-bS} \tag{4.7}$$

with a constant $b$. This parametrizes the non-perturbative effects discussed above, and may be interpreted as a hidden sector gaugino condensate. It provides the order parameter for local supersymmetry and generates the gravitino mass

$$\frac{\langle\Delta\mathscr{W}_{\mathrm{np}}\rangle}{M_{\mathrm{P}}^2} \sim \frac{\langle\lambda\lambda\rangle}{M_{\mathrm{P}}^2} \sim m_{3/2}. \tag{4.8}$$

$\Delta\mathscr{W}_{\mathrm{np}}$ has $R$-charge 2 (cf. the discussion in Section 3) and similar non-perturbative effects can contribute to further terms in the superpotential. The crucial property of the non-perturbative couplings is that they are naturally suppressed. To parametrize these effects we denote by a superfield $Y$ a non-perturbative term of the form given in Eq. (4.7) (scaled by the factor $M_{\mathrm{P}}^{-2}$) carrying $R$-charge 2 and we construct the superpotential involving $Y$ that is consistent with the relevant $R$ symmetry.

The lowest superpotential terms in $Y$ have the form

$$\begin{aligned}\Delta\mathscr{W}_{\mathbb{Z}_4^R} &= Y + Y^2 N + Y N^2 + Y H_u H_d \\ &\sim m_{3/2} M_{\mathrm{P}}^2 + m_{3/2}^2 N + m_{3/2} N^2 + m_{3/2} H_u H_d,\end{aligned} \tag{4.9}$$

$$\begin{aligned}\Delta\mathscr{W}_{\mathbb{Z}_8^R} &= Y + Y^2\big(N + Y N^2 + Y H_u H_d\big) \\ &\sim m_{3/2} M_{\mathrm{P}}^2 + m_{3/2}^2 N + \frac{m_{3/2}^3}{M_{\mathrm{P}}^2} N^2 + \frac{m_{3/2}^3}{M_{\mathrm{P}}^2} H_u H_d.\end{aligned} \tag{4.10}$$

All of these terms have magnitude determined by the gravitino mass scale. For gauge mediation this scale can be very small and these terms negligible. For gravity mediation however the gravitino mass scale is the scale of supersymmetry breaking in the visible sector and the unsuppressed terms cannot be neglected. In this case, the magnitude of the $\Delta\mathscr{W}_{\mathbb{Z}_4^R}$ terms is such as to reproduce the superpotential of the S-MSSM [47,48], where, apart from the usual NMSSM couplings also holomorphic mass terms for the singlets and the Higgs fields of the order $m_{3/2}$ are introduced. This extension of the SM has been shown to significantly reduce the fine tuning needed to accommodate the LEP Higgs mass bound [47,48]. Our analysis yields a justification for the small holomorphic terms, which have so far just been imposed by hand.

Interestingly the form of the non-perturbative effects is very sensitive to the underlying symmetry. For the case of $\mathbb{Z}_4^R$ there are additional unsuppressed linear and quadratic terms in $N$ as well as a non-perturbative contribution to the Higgsino mass. For the case of $\mathbb{Z}_8^R$ only the linear term in $N$ is unsuppressed. Because the magnitude of all these terms is determined by the gravitino mass they will not disturb the SUSY solution to the hierarchy problem. However, for the case of gravity mediation, the terms cannot be neglected and may be expected to significantly change the NMSSM phenomenology. Given the different non-perturbative terms appearing in the $\mathbb{Z}_4^R$ and $\mathbb{Z}_8^R$ we may expect these to have different phenomenological implications.

### 4.2. Discrete $R$ symmetries and the strong CP problem

The axion solution to the strong CP problem remains the most convincing to date. Since it is based on the existence of a Peccei–Quinn (PQ) symmetry that forbids the $\mu$ term it is of interest to ask whether a discrete $R$ symmetry can play the role of the PQ symmetry. Let us start by briefly discussing the role of axions in our setup with $\mathbb{Z}_M^R$ symmetries with particular focus on a possible solution of the strong CP problem. One potential candidate for such an axion is the universal Green–Schwarz axion, $a = \operatorname{Im} S$, cf. Eq. (2.15). As discussed in Appendix C, in the case of Kähler stabilisation we are — to leading order — left with a massless GS axion. However, as was e.g. shown in [49], the corresponding axion decay constant is of order the Planck scale if we demand the usual value for the unified gauge coupling. This is well outside the cosmologically allowed range $10^{10}$ GeV $< f_a < 3 \times 10^{11}$ GeV, assuming no fine tuning in the initial axion VEV [49].

Kim and Nilles [50] have proposed an interesting model that naturally gives an axion decay constant in the favoured range. They achieve this by requiring that the coupling of the MSSM singlet field that contains the axion to the Higgs supermultiplets be quadratic with a superpotential of the form

$$\mathscr{W} = \frac{\alpha}{M_{\mathrm{P}}} N^2 H_u H_d. \tag{4.11}$$

If the inverse mass scale, $M_{\mathrm{P}}$, associated with this operator (the mediator scale) is taken to be the Planck scale with $\alpha = \mathcal{O}(1)$ an electroweak scale $\mu$ term of $\mathcal{O}(m_{3/2})$ is generated if the singlet VEV ($\langle N\rangle \equiv f_a$) is in the desired range. This corresponds to $\langle N\rangle = \mathcal{O}(\sqrt{m_{3/2} M_{\mathrm{P}}})$ for the case of gravity mediated supersymmetry breaking with $m_{3/2}$ close to the electroweak breaking scale. Furthermore, the theory has a global (accidental) U(1) PQ symmetry under which $H_u H_d$ and the singlet $N$ transform non-trivially. Hence, the VEV of $N$ breaks the PQ symmetry and for the case of gravity mediated supersymmetry breaking the scale of breaking is of $\mathcal{O}(10^{10}\text{–}10^{11}\ \mathrm{GeV})$. So the associated axion coming from the singlet $N$ has the right properties to solve the strong CP-problem [50].

One has to ensure that higher-dimensional operators, which explicitly break the PQ symmetry, do not spoil the solution.[6] An additional complication in the construction of a viable model is that one needs an $|N|^6$ term in the scalar potential to get an intermediate scale VEV of the correct magnitude if the soft supersymmetry breaking mass of the $N$ field is $\mathcal{O}(m_{3/2})$. Since a superpotential term $N^4$ breaks the PQ symmetry too strongly to give a viable axion at least one additional singlet (called $X$) is needed to generate the $|N|^6$ term in the $F$-term potential. Let us consider whether the $\mathbb{Z}_M^R$ symmetries can give such a structure.

In order to construct a viable model with intermediate scale breaking we start with the superpotential

$$\mathscr{W} = \frac{\alpha}{M_{\mathrm{P}}} N^2 H_u H_d + \frac{\beta}{M_{\mathrm{P}}} X N^3, \tag{4.12}$$

which requires $\mathbb{Z}_M^R$ charges $q_N = -1$ and $q_X = 5$, cf. Table 2.1. Including soft SUSY breaking mass terms, this gives the potential

[6] A continuous $R$ symmetry can protect the PQ symmetry at higher orders if it is broken only by the superpotential VEV and the intermediate scale VEVs of singlets [51].

$$\mathscr{V} = \left| \frac{2\alpha}{M_{\mathrm{P}}} N H_u H_d + \frac{3\beta}{M_{\mathrm{P}}} X N^2 \right|^2 + \left| \frac{\beta}{M_{\mathrm{P}}} N^3 \right|^2 + m_N^2 |N|^2 + m_X^2 |X|^2 \tag{4.13}$$

that has the required $|N|^6$ stabilising term. Provided $m_X^2$ is negative, the field $X$ acquires a VEV

$$\langle X \rangle \sim \mathcal{O}\big(\langle N \rangle^3 / M_{\mathrm{P}}^2\big), \tag{4.14}$$

where we have used $\langle H_u \rangle \sim \langle H_d \rangle \sim (\langle N \rangle^2 / M_{\mathrm{P}}) = \mathcal{O}(m_{3/2})$. The superpotential in Eq. (4.12) has a global U(1) PQ symmetry under which $H_u H_d$, $N$ and $X$ transform non-trivially. However the superpotential is not the most general one allowed by an underlying discrete $R$ symmetry for this will allow additional terms of the form $N^p X^q$ for $p, q$ integer, where the values of $p, q$ are constrained by the choice of $\mathbb{Z}_M^R$. Such terms will break the PQ symmetry generating a mass for the would-be axion. If the axion solution to the strong CP problem is to be maintained this contribution to the mass should be five orders of magnitude smaller than the corresponding contribution from QCD, $\delta m_a \lesssim 10^{-5} m_a^{\mathrm{QCD}} \lesssim 10^{-9}$ eV [52,53]. This puts a strong constraint on the discrete symmetry for it must be large enough to forbid the additional terms to a high order.

Including the $N^p X^q$ term in Eq. (4.12) one sees that it is the interference between the last two terms in $|F_X|^2$ that gives the dominant contribution because the VEV of $X$ is smaller than the VEV of $N$. This term is of $\mathcal{O}(N^{p+3} X^{q-1} / M_{\mathrm{P}}^{p+q+1})$ in the potential and gives a contribution to the axion mass given by

$$\delta m_a^2 = \mathcal{O}\big(N^{p+1} X^{q-1} / M_{\mathrm{P}}^{p+q-2}\big) = \mathcal{O}\big(N^{p+3q-2} / M_{\mathrm{P}}^{p+3q-4}\big). \tag{4.15}$$

The leading terms for the candidate $\mathbb{Z}_M^R$ symmetries (for which $5q - p - 2 = 0 \bmod M$) are $N^2$, $N^4$, $X^2$, $N X^3$ and $N^8 X^2$ corresponding to $p + 3q - 2 = 0, 2, 4, 8$ and 12 for $M = 4, 6, 8, 12$ and 24 respectively. The constraint $\delta m_a^2 < 10^{-18}$ eV$^2$ is equivalent to $p + 3q - 2 > 8$ so we see that only $\mathbb{Z}_{24}^R$ is large enough to accommodate this method of simultaneously generating the $\mu$ term and solving the strong CP problem.

Note that the singlets also induce baryon and lepton number violation as well as a small amount of $R$ parity violation. In the case of the $\mathbb{Z}_{24}^R$ the leading order terms (cf. Table 2.1) are given by

$$\frac{X N^3}{M_{\mathrm{P}}^5} \mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}} \quad \text{and} \quad \frac{X N^2}{M_{\mathrm{P}}^3} \mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}. \tag{4.16}$$

With intermediate scale VEVs for $N$ and $\langle X \rangle = \mathcal{O}(\langle N \rangle^3 / M_{\mathrm{P}}^2)$ the contribution of these operators to nucleon decay is strongly suppressed lying below the irreducible dimension 6 operator contribution. Also the $R$ parity violation is negligible. In summary, a singlet extension of the MSSM with a $\mathbb{Z}_{24}^R$ symmetry provides us with a simultaneous solution to the $\mu$ and strong CP problems. The phenomenological implications for the Higgs structure may differ significantly from the NMSSM and remain to be analysed.

## 5. Conclusions

We have discussed possible discrete symmetries for the MSSM which commute with SU(5). We have seen that, in order to address the $\mu$ problem, these have to be $R$ symmetries. We have surveyed all possible discrete $\mathbb{Z}_M^R$ symmetries. Anomaly cancellation requires that the order $M$ be a divisor of 24. We identified 5 phenomenologically viable symmetries for the MSSM.

The simplest of the 5 MSSM symmetries is a $\mathbb{Z}_4^R$ which commutes with SO(10). This symmetry forbids all $R$-parity violating couplings, dimension five proton decay operators and the $\mu$ term at tree-level while allowing the usual Yukawa couplings and the neutrino mass operator. At the non-perturbative level the $\mu$ term and the dimension five proton decay operators are generated. We argued that in settings in which supersymmetry breaking is related to some non-perturbative dynamics the $\mu$ term will be of the order of the MSSM soft terms. In particular, in gravity mediation we will have $\mu \sim m_{3/2}$ and coefficients of the dimension five proton decay operators $\kappa_{ijk\ell}^{(1,2)} \sim m_{3/2}/M_{\mathrm{P}}^2$, i.e. sufficiently suppressed. Thus the $\mathbb{Z}_4^R$ symmetry provides us with a simultaneous solution to the arguably two most severe problems of the MSSM. We have discussed how to embed the $\mathbb{Z}_4^R$ into string theory. Specifically, we have constructed a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold with this $\mathbb{Z}_4^R$ and the exact MSSM spectrum below the compactification scale, in which the $\mathbb{Z}_4^R$ originates from the Lorentz symmetry of compactified dimensions. Due to the $\mathbb{Z}_4^R$ anomaly, the $\mu$ term is generated at the non-perturbative level. There is an exact matter parity and dimension five proton decay is well below experimental limits.

We have discussed the role of discrete symmetries in singlet extensions of the MSSM. There are two possible symmetries consistent with the structure of the NMSSM, $\mathbb{Z}_4^R$ and $\mathbb{Z}_8^R$, both of which are capable of solving the hierarchy problem. The $\mathbb{Z}_8^R$ allows the usual couplings while forbidding the linear term for the singlet at the perturbative level. In the $\mathbb{Z}_4^R$ case, one obtains holomorphic mass terms for the singlet and the Higgs at the non-perturbative level. We have argued that the size of such terms is of the order $m_{3/2}$, leading to an S-MSSM-like scheme in which the smallness of the explicit mass terms for the singlets and Higgs finds an explanation. As another application we have discussed how discrete $R$ symmetries can lead to approximate PQ U(1) symmetries capable of solving the strong CP problem. Given the upper bound on the order of $\mathbb{Z}_M^R$, $M \leqslant 24$, we identified $\mathbb{Z}_{24}^R$ as the unique possibility for solving the $\mu$- and strong CP problems simultaneously.


## Acknowledgements

We would like to thank Oleg Lebedev for useful discussions. This research was supported by the DFG cluster of excellence "Origin and Structure of the Universe", the SFB–Transregio 27 "Neutrinos and Beyond", LMUExcellent and the Graduiertenkolleg "Particle Physics at the Energy Frontier of New Phenomena" by Deutsche Forschungsgemeinschaft (DFG). S.R. acknowledges partial support from DOE grant DOE/ER/01545-892. The research presented here was partially supported by the EU ITN grant UNILHC 237920 (Unification in the LHC era) and by the UK Science and Technology Facilities Council Oxford particle theory rolling grant ST/G000492/1. H.M.L. is supported by the Korean–CERN Fellowship.


## Appendix A. Discrete anomalies in the path integral approach

In this appendix we re-derive Abelian discrete anomalies with the path integral method, following [38,54]. Among other things, we will describe how this allows us to understand the discrete version of the Green–Schwarz mechanism.

### A.1. Path integral derivation of anomalies

Consider a theory described by a Lagrange density $\mathscr{L}$ with a set of fermions $\Psi = [\psi^{(1)}, \ldots, \psi^{(M)}]$, where $\psi^{(m)}$ denotes a field transforming in the irreducible representation (irrep) $\boldsymbol{R}^{(m)}$ of all internal symmetries. A general transformation $\Psi \to U\Psi$ or, more explicitly,

$$\begin{bmatrix} \psi^{(1)} \\ \vdots \\ \psi^{(M)} \end{bmatrix} \to \begin{pmatrix} U^{(1)} & & 0 \\ & \ddots & \\ 0 & & U^{(M)} \end{pmatrix} \begin{bmatrix} \psi^{(1)} \\ \vdots \\ \psi^{(M)} \end{bmatrix}, \tag{A.1}$$

which leaves $\mathscr{L}$ invariant (up to a total derivative) denotes a classical symmetry.

A classical symmetry implies that certain correlators vanish at the classical level. To see this, consider the correlator

$$C_{n_1 \ldots n_M} = \big\langle \big(\psi^{(1)}\big)^{n_1} \cdots \big(\psi^{(M)}\big)^{n_M} \big\rangle. \tag{A.2}$$

Now, if the field combination $(\psi^{(1)})^{n_1} \cdots (\psi^{(M)})^{n_M}$ is not invariant under the symmetry transformation, we arrive at the (premature) conclusion that $C_{n_1 \ldots n_M} = 0$.

Classical chiral symmetries can be broken by quantum effects, i.e. have an anomaly. Specifically, consider a chiral transformation

$$\Psi(x) \to \Psi'(x) = \exp(\mathrm{i}\alpha P_\mathrm{L})\, \Psi(x), \tag{A.3}$$

where $\alpha = \alpha^{\mathrm{anom}} \mathsf{T}_{\mathrm{anom}}$ with $\mathsf{T}_{\mathrm{anom}}$ denoting the generator of the transformation and $\alpha^{\mathrm{anom}}$ being a parameter, and $P_\mathrm{L}$ is the left-chiral projector.

We wish now to show that this implies vanishing correlators at the classical level may appear at the quantum level. To this end, write the correlator as a path integral,

$$C_{n_1 \ldots n_M} = \int \mathcal{D}\Psi\, \mathcal{D}\overline{\Psi}\, \big(\psi^{(1)}\big)^{n_1} \cdots \big(\psi^{(M)}\big)^{n_M} \mathrm{e}^{\mathrm{i}\mathcal{S}}, \tag{A.4}$$

where $\mathcal{S}$ denotes the action, which is left invariant under (A.3). Now recall that under the transformation (A.1) the path integral measure undergoes a non-trivial change [55,56],

$$\mathcal{D}\Psi\, \mathcal{D}\overline{\Psi} \to J(\alpha)\, \mathcal{D}\Psi\, \mathcal{D}\overline{\Psi}, \tag{A.5}$$

where the Jacobian of the transformation is given by

$$J(\alpha) = \exp\left\{ \mathrm{i} \int \mathrm{d}^4 x\, \mathcal{A}(\alpha) \right\}. \tag{A.6}$$

The crucial observation is now that in the presence of a non-trivial Jacobian the full quantum correlator can be invariant. This is true regardless of whether the transformation (A.1) is continuous or discrete, or whether it is gauged or global.

The anomaly function $\mathcal{A}$ appearing in (A.6) decomposes into a gauge and a gravitational part [57–59],

$$\mathcal{A}(\alpha) = \mathcal{A}_{\mathrm{gauge}}(\alpha) + \mathcal{A}_{\mathrm{grav}}(\alpha), \tag{A.7}$$

with

$$\mathcal{A}_{\mathrm{gauge}}(\alpha) = \frac{1}{32\pi^2} \mathrm{Tr}[\alpha \mathcal{F} \widetilde{\mathcal{F}}], \tag{A.8}$$

$$\mathcal{A}_{\mathrm{grav}}(\alpha) = -\frac{1}{384\pi^2} \mathcal{R}\widetilde{\mathcal{R}}\, \mathrm{Tr}[\alpha]. \tag{A.9}$$

We have suppressed index contractions, i.e. $\mathcal{F}\widetilde{\mathcal{F}} = \mathcal{F}^{\mu\nu}\widetilde{\mathcal{F}}_{\mu\nu}$. Here $\mathcal{F}_{\mu\nu} = [D_\mu, D_\nu]$ is the field strength of the gauge symmetry, such that $\mathcal{F}_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ for a U(1) symmetry, $\mathcal{F}_{\mu\nu} = F^a_{\mu\nu}\mathsf{T}_a$ for non-Abelian gauge groups, and $\widetilde{\mathcal{F}}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\rho\sigma}$ denotes its dual. Similarly, $\mathcal{R}$ represents the Riemann curvature tensor and $\mathcal{R}\widetilde{\mathcal{R}} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{R}_{\mu\nu}{}^{\lambda\gamma}\mathcal{R}_{\rho\sigma\lambda\gamma}$. The trace 'Tr' runs over all internal indices.

Now we specialize to the case where $\alpha$ is a $\mathbb{Z}_M$ transformation. For the anomaly to be absent, i.e. $J(\alpha) = 1$, we arrive at the conditions [38,54]

$$G\text{–}G\text{–}\mathbb{Z}_M\colon \qquad \sum_{\boldsymbol{r}^{(f)}} \ell\big(\boldsymbol{r}^{(f)}\big) q^{(f)} = 0 \bmod \eta, \tag{A.10a}$$

$$\text{grav–grav–}\mathbb{Z}_M\colon \qquad \sum_{m} q^{(m)} = 0 \bmod \eta, \tag{A.10b}$$

where (cf. Eq. (2.3))

$$\eta = \begin{cases} M & \text{for } M \text{ odd}, \\ M/2 & \text{for } M \text{ even}, \end{cases}$$

and $q^{(m)}$ denotes the $\mathbb{Z}_M$ charge. The first sum runs over all irreducible representations $\boldsymbol{r}^{(f)}$ of $G$ with Dynkin index $\ell(\boldsymbol{r}^{(f)})$ while the second sum runs over all fermions. Our conventions are such that $\ell(\boldsymbol{N}) = 1/2$ for SU($N$) and $\ell(\boldsymbol{N}) = 1$ for SO($N$). (A.10) are the traditional discrete anomaly conditions [4,8] with the difference that the $\mathbb{Z}_M^3$ constraints do not appear; we will discuss $\mathbb{Z}_M^3$ anomalies separately in Section A.4.

### *A.2. Green–Schwarz mechanism and re-derivation of* $\delta_{\mathrm{GS}}$

Consider a theory with simple gauge group $G$ and an 'anomalous' Abelian gauge factor $\mathrm{U}(1)_{\mathrm{anom}}$. Under $\mathrm{U}(1)_{\mathrm{anom}}$ the dilaton superfield $S$ shifts according to

$$S \to S + \frac{\mathrm{i}}{2}\delta_{\mathrm{GS}}\Lambda(x) \tag{A.11}$$

with $\Lambda$ denoting the $\mathrm{U}(1)_{\mathrm{anom}}$ transformation, i.e. the chiral superfields follow the rule

$$\Phi^{(f)} \to \mathrm{e}^{-\mathrm{i}Q^{(f)}_{\mathrm{anom}}\Lambda}\Phi^{(f)}. \tag{A.12}$$

The corresponding transformation of the vector superfield $V_{\mathrm{anom}}$ is

$$V_{\mathrm{anom}} \to V_{\mathrm{anom}} + \frac{\mathrm{i}}{2}\big(\Lambda - \Lambda^\dagger\big) \tag{A.13}$$

with $\mathrm{Re}\,\Lambda|_{\theta=0} = \alpha$. In what follows, we derive the Green–Schwarz coefficient $\delta_{\mathrm{GS}}$ from the requirement of invariance of the full action.

The dilaton-dependent part of the Lagrange density is

$$\begin{aligned}\mathscr{L}_{\mathrm{dilaton}} = &- \int \mathrm{d}^4\theta \ln\big(S + S^\dagger - \delta_{\mathrm{GS}}V_{\mathrm{anom}}\big) \\ &+ \left[\int \mathrm{d}^2\theta \frac{S}{4}\,\mathrm{Tr}\,W_\alpha W^\alpha + \mathrm{h.c.}\right] \\ &+ \text{gravity terms}.\end{aligned} \tag{A.14}$$

The first line of this Lagrange density is already invariant under the combined transformations (A.11) and (A.13). The trace in the second line is supposed to run over all gauge factors, including $\mathrm{U}(1)_{\mathrm{anom}}$.

Decomposing the scalar component of the dilaton into a real and an imaginary (or axionic) part,

$$S|_{\theta=0} = s + \mathrm{i}a, \tag{A.15}$$

leads to the usual couplings of the axion $a$

$$\mathscr{L} \supset -\frac{a}{8} F_{\mathrm{anom}} \widetilde{F}_{\mathrm{anom}} - \frac{a}{8} F^a \widetilde{F}^a + \frac{a}{4} \mathcal{R}\widetilde{\mathcal{R}}, \tag{A.16}$$

where $F$ and $F_{\mathrm{anom}}$ denote the gauge field strength of $G$ and $\mathrm{U}(1)_{\mathrm{anom}}$ respectively.

Hence, under a $\mathrm{U}(1)_{\mathrm{anom}}$ transformation with parameter $\alpha$ the axionic Lagrange density shifts by

$$\Delta\mathscr{L}_{\mathrm{axion}} = -\frac{\alpha}{16} \delta_{\mathrm{GS}} F_{\mathrm{anom}} \widetilde{F}_{\mathrm{anom}} - \frac{\alpha}{16} \delta_{\mathrm{GS}} F^a \widetilde{F}^a + \frac{\alpha}{8} \delta_{\mathrm{GS}} \mathcal{R}\widetilde{\mathcal{R}}. \tag{A.17}$$

The Green–Schwarz term $\delta_{\mathrm{GS}}$ can now be inferred by demanding that the transformation of the axion $a$ cancels the anomalous variation of the path integral measure. The latter can be absorbed in a change of the Lagrange density

$$\begin{aligned}\Delta\mathscr{L}_{\mathrm{anomaly}} = {} & \frac{\alpha}{32\pi^2} F_{\mathrm{anom}} \widetilde{F}_{\mathrm{anom}} A_{\mathrm{U}(1)^3_{\mathrm{anom}}} \\ & + \frac{\alpha}{32\pi^2} F^a \widetilde{F}^a A_{G\text{-}G\text{-}\mathrm{U}(1)_{\mathrm{anom}}} \\ & - \frac{\alpha}{384\pi^2} \mathcal{R}\widetilde{\mathcal{R}} A_{\mathrm{grav}\text{-}\mathrm{grav}\text{-}\mathrm{U}(1)_{\mathrm{anom}}}.\end{aligned} \tag{A.18}$$

The coefficients $A$ are the anomaly coefficients given by

$$A_{\mathrm{U}(1)^3_{\mathrm{anom}}} = \frac{1}{3} \sum_m \big(Q^{(m)}_{\mathrm{anom}}\big)^3 = \frac{1}{3} \operatorname{tr} Q^3_{\mathrm{anom}}, \tag{A.19a}$$

$$A_{\mathrm{grav}\text{-}\mathrm{grav}\text{-}\mathrm{U}(1)_{\mathrm{anom}}} = \sum_m Q^{(m)}_{\mathrm{anom}} = \operatorname{tr} Q_{\mathrm{anom}}, \tag{A.19b}$$

$$A_{G\text{-}G\text{-}\mathrm{U}(1)_{\mathrm{anom}}} = \sum_{\boldsymbol{r}^{(f)}} \ell\big(\boldsymbol{r}^{(f)}\big) Q^{(f)}_{\mathrm{anom}}, \tag{A.19c}$$

where $Q^{(m)}_{\mathrm{anom}}$ denotes the $\mathrm{U}(1)_{\mathrm{anom}}$ charge. The first two sums run over all left-handed Weyl fermions while the last sum runs over all irreducible representations $\boldsymbol{r}^{(f)}$ of $G$ and $\ell(\boldsymbol{r}^{(f)})$ is the Dynkin index.

The axion shift allows us to cancel the grav–grav–$\mathrm{U}(1)_{\mathrm{anom}}$, $\mathrm{U}(1)^3_{\mathrm{anom}}$ and $G$–$G$–$\mathrm{U}(1)_{\mathrm{anom}}$ anomalies by demanding $\Delta\mathscr{L}_{\mathrm{anomaly}} + \Delta\mathscr{L}_{\mathrm{axion}} = 0$. This fixes the Green–Schwarz constant to be given by

$$2\pi^2 \delta_{\mathrm{GS}} = \frac{1}{24} \operatorname{tr} Q_{\mathrm{anom}} = \frac{1}{3} \operatorname{tr} Q^3_{\mathrm{anom}} = A_{G\text{-}G\text{-}\mathrm{U}(1)_{\mathrm{anom}}}, \tag{A.20}$$

which is in agreement with the result obtained in a string computation [60].

### A.3. Discrete Green–Schwarz mechanism

The Green–Schwarz mechanism also works if we replace $\mathrm{U}(1)_{\mathrm{anom}}$ by a discrete $\mathbb{Z}_M$. In this case the parameter $\alpha$ is no longer continuous but $\alpha = \frac{2\pi n}{M}$ with some integer $n$. Of course, there is no gauge field associated with the $\mathbb{Z}_M$, i.e. Eq. (A.13) does not apply here. The discussion then goes as in the previous subsection. The discrete Green–Schwarz constant is now defined in such a way that under the $\mathbb{Z}_M$ transformation of fields

$$\Phi^{(f)} \to \mathrm{e}^{-\mathrm{i}\frac{2\pi}{M}q^{(f)}}\Phi^{(f)} \tag{A.21}$$

the dilaton shifts according to

$$S \to S + \frac{\mathrm{i}}{2}\Delta_{\mathrm{GS}}, \tag{A.22}$$

where $\Delta_{\mathrm{GS}}$ is fixed only modulo $\eta$,

$$\pi M \Delta_{\mathrm{GS}} \equiv \frac{1}{24} A_{\text{grav-grav-}\mathbb{Z}_M} = A_{G\text{-}G\text{-}\mathbb{Z}_M} \bmod \eta. \tag{A.23}$$

The anomaly coefficients can be obtained from Eq. (A.19) by replacing the $\mathrm{U}(1)_{\mathrm{anom}}$ charges $Q^{(m)}_{\mathrm{anom}}$ by the $\mathbb{Z}_M$ charges $q^{(m)}$. Note that, unlike in the continuous case, the transformation of the axion is only fixed modulo $\eta$. In the main body of the paper we obtain constraints on possible discrete symmetries and charge assignments from the requirement that Eq. (A.23) possesses a solution, i.e. that the $A_{G\text{-}G\text{-}\mathbb{Z}_M}$ coefficients for different gauge factors $G$ coincide modulo $\eta$.

### A.4. A comment on $\mathbb{Z}_M^3$ anomalies

If the discrete symmetry is embedded in a continuous symmetry, the universality relations (A.20) also imply that

$$A_{\text{grav-grav-}\mathbb{Z}_M} - 8A_{\mathbb{Z}_M^3} = 0 \bmod \eta, \tag{A.24}$$

with

$$A_{\mathbb{Z}_M^3} = \mathcal{N}\sum_m \left(q^{(m)}\right)^3, \tag{A.25}$$

where $\mathcal{N}$ is a normalization factor compensating for the rescaling of the original U(1) charges $Q^{(m)}$ to integer $\mathbb{Z}_M$ discrete charges $q^{(m)}$. However, this relation is an embedding constraint rather than a true anomaly constraint. That is, if this relation is not satisfied, this does not necessarily imply a non-trivial variation of the path integral measure [38], and therefore it does not mean that classically forbidden correlators will appear at the quantum level.

The $\mathbb{Z}_M^3$ anomaly constraints have lead to some confusion in the literature. Banks and Dine [9] gave an argument for why there are no $\mathbb{Z}_M^3$ anomaly constraints. Following [8] they embedded the $\mathbb{Z}_M$ into a U(1) symmetry; however they broke this down to a $\mathbb{Z}_{P\cdot M}$ symmetry and were able to show that, while there is only a $\mathbb{Z}_M$ for the chiral states, the extra heavy states 'see' a $\mathbb{Z}_{P\cdot M}$ and can be chosen such that the anomaly conditions following from the U(1) constraints can be satisfied through their extra contributions.

From this one might conclude that, in order to satisfy the $\mathbb{Z}_M^3$ anomaly, the true symmetry has to be $\mathbb{Z}_{P\cdot M}$ rather than $\mathbb{Z}_M$. However, this is not necessarily the case as there can be discrete symmetries that cannot be obtained from continuous symmetries by *spontaneous* breaking in four dimensions. That is, the constraints from embedding $\mathbb{Z}_M$ in non-anomalous continuous symmetries are sufficient to ensure anomaly freedom but not necessary.

In order to be specific, let us look at the stringy origin of the $\mathbb{Z}_4^R$ symmetry discussed in Section 3 of the main body of the paper. The $\mathbb{Z}_4^R$ has a clear geometric interpretation in terms of remnants of the Lorentz group of compactified dimensions. However, some of the states transforming non-trivially under our $\mathbb{Z}_4^R$ are twisted states. These states are chiral massless states which appear only *after* orbifolding, i.e. after we have broken the continuous Lorentz symmetry down to a discrete subgroup. So it appears that there is no continuous interpolation between the continuous U(1) and the discrete $\mathbb{Z}_4^R$. Hence the derivation of discrete anomalies based on embedding discrete symmetries in continuous symmetries might not apply. On the other hand, the path integral method still works. This is consistent with the fact that in our orbifold construction there is no underlying $\mathbb{Z}_8^R$ while we believe that the theory is UV complete.

## Appendix B. $\mathbb{Z}_M$ and $\mathbb{Z}_M^R$ anomaly coefficients

We start by looking at the MSSM amended by ordinary, i.e. non-$R$, discrete symmetries, where the fermions have the same charges as the superfields $\Phi^{(f)}$ and turn then to the discussion of discrete $R$ symmetries.

### B.1. Anomaly coefficients for non-$R$ $\mathbb{Z}_M$ symmetries

The anomaly coefficients for discrete non-$R$ $\mathbb{Z}_M$ symmetries are well known [4,8,9],

$$A_{G\text{-}G\text{-}\mathbb{Z}_M} = \sum_{\boldsymbol{r}^{(f)}} \ell\bigl(\boldsymbol{r}^{(f)}\bigr) \cdot q^{(f)}, \tag{B.1a}$$

$$A_{\text{grav-grav-}\mathbb{Z}_M} = \sum_m q^{(m)}. \tag{B.1b}$$

These coefficients can be re-derived in the path integral approach [38] (cf. Appendix A). In Eq. (B.1a) we sum over all irreducible representations $\boldsymbol{r}^{(f)}$ of $G$ while in Eq. (B.1b) we sum over all fermions. $\ell(\boldsymbol{r}^{(f)})$ denotes the Dynkin index of the representation $\boldsymbol{r}^{(f)}$. The discrete charges $q$ are integers which are defined modulo $M$. Moreover, there might be mixed U(1) anomalies if the normalization of the U(1) factors is known. The coefficients are

$$A_{\mathrm{U}(1)\text{-}\mathrm{U}(1)\text{-}\mathbb{Z}_M} = \sum_m q^{(m)} \cdot \bigl(Q^{(m)}\bigr)^2 \tag{B.1c}$$

with $Q^{(m)}$ denoting the normalized U(1) charges. We will discuss this coefficient in more detail below.

Traditional anomaly freedom requires that for all anomaly coefficients

$$A = 0 \bmod \eta. \tag{B.2}$$

However, discrete anomalies can be canceled by the Green–Schwarz mechanism, in which case one has to demand

$$A_{G\text{-}G\text{-}\mathbb{Z}_M} = A_{\mathrm{U}(1)\text{-}\mathrm{U}(1)\text{-}\mathbb{Z}_M} = \frac{1}{24} A_{\text{grav-grav-}\mathbb{Z}_M} = \rho \bmod \eta. \tag{B.3}$$

An important comment concerns the mixed U(1)–$\mathbb{Z}_M$ anomaly coefficient (B.1c). Mixed U(1)–U(1)–$\mathbb{Z}_M$ anomalies are mostly ignored as they do not give meaningful constraints unless one knows the normalization of the charges [6,61]. Typically the sum in Eq. (B.1c) is not invariant under shifting some discrete charges by $M$. To see this, let us consider the example of hypercharge. We will denote the unnormalized $\mathrm{U}(1)_Y$ charge by $Q_Y^{(m)}$. The anomaly coefficient reads

$$A_1 = \sum_m \frac{3}{5}\big(Q_Y^{(m)}\big)^2 q^{(m)} = \rho \bmod \eta. \tag{B.4}$$

We have the freedom to shift the $\mathbb{Z}_M$ charges by integer multiples of $M$, i.e. we can define new $\mathbb{Z}_M$ charges $q'^{(m)} = q^{(m)} + k^{(m)}M$ with $k^{(m)} \in \mathbb{Z}$. With the new charges the condition for anomaly freedom is

$$\frac{3}{5}\sum_m \big(Q_Y^{(m)}\big)^2\big(q^{(m)} + k^{(m)}M\big) = \rho \bmod \eta \tag{B.5}$$

$$\Rightarrow \quad A_1 + \frac{3}{5}M \underbrace{\sum_m k^{(m)}\big(Q_Y^{(m)}\big)^2}_{=:n} = \rho \bmod \eta. \tag{B.6}$$

We can choose the $k^{(m)}$ such that $n$ is an arbitrary integer because, for example, $Q_Y(\overline{E}) = 1$. Hence, we arrive at

$$A_1 = \rho - \frac{3}{5}nM + m\eta, \quad m \in \mathbb{Z}. \tag{B.7}$$

This can be rewritten as

$$M \text{ odd:} \quad 5A_1 = 5\rho + (5m - 3n)M, \tag{B.8}$$

$$M \text{ even:} \quad 5A_1 = 5\rho + (5m - 6n)\frac{M}{2}. \tag{B.9}$$

Since $5m - 3n$ and $5m - 6n$ are arbitrary integers, we get

$$5A_1 = 5\rho \bmod \eta. \tag{B.10}$$

### B.2. Anomaly coefficients for $\mathbb{Z}_M^R$ symmetries

Now consider a $\mathbb{Z}_M^R$ symmetry, under which, by convention, the superpotential transforms as

$$\mathscr{W} \to \mathrm{e}^{2\pi \mathrm{i} q_{\mathscr{W}}/M}\mathscr{W} \tag{B.11}$$

with $q_{\mathscr{W}} = 2$. Accordingly, the superspace coordinates transform as

$$\theta \to \mathrm{e}^{2\pi \mathrm{i}/M}\theta, \tag{B.12}$$

such that $\mathrm{d}^2\theta$ transforms oppositely to $\mathscr{W}$. Superfields $\Phi^{(f)} = \phi^{(f)} + \sqrt{2}\theta\psi^{(f)} + \theta\theta F^{(f)}$ follow the law

$$\Phi^{(f)} \to \mathrm{e}^{2\pi \mathrm{i} q^{(f)}/M}\Phi^{(f)}. \tag{B.13}$$

Correspondingly, the fermions transform as

$$\psi^{(f)} = \mathrm{e}^{2\pi \mathrm{i}(q^{(f)}-1)/M}\psi^{(f)}. \tag{B.14}$$

For discrete $R$ symmetries, the anomaly coefficients read (cf. Appendix A)

$$A_{G\text{-}G\text{-}\mathbb{Z}_M^R} = \sum_{\boldsymbol{r}^{(f)}} \ell\big(\boldsymbol{r}^{(f)}\big) \cdot \big(q^{(f)} - 1\big) + \ell(\operatorname{adj} G), \tag{B.15a}$$

$$A_{\mathrm{U}(1)\text{-}\mathrm{U}(1)\text{-}\mathbb{Z}_M^R} = \sum_m \big(Q^{(m)}\big)^2 \cdot \big(q^{(m)} - 1\big), \tag{B.15b}$$

$$A_{\text{grav-grav-}\mathbb{Z}_M^R} = -21 + \sum_G \dim(\operatorname{adj} G) + \#\big(\mathrm{U}(1)\big) + \sum_m \big(q^{(m)} - 1\big). \tag{B.15c}$$

Here $q^{(f)}$ denote the $\mathbb{Z}_M^R$ charges of the superfields, the charges of the corresponding fermions are shifted by one unit, $q_{\psi^{(f)}} = q^{(f)} - 1$. In Eq. (B.15a) $\ell(\operatorname{adj} G) = C_2(G)$ represents the contribution from the gauginos, $\#(\mathrm{U}(1))$ denotes the number of U(1) gauginos. The first and second term on the right-hand side of Eq. (B.15c) represent the contributions from the gravitino and gauginos. A necessary condition for anomaly cancellation is the universality

$$\begin{aligned} A_{G\text{-}G\text{-}\mathbb{Z}_M^R} = A_{\mathrm{U}(1)\text{-}\mathrm{U}(1)\text{-}\mathbb{Z}_M^R} &= \frac{1}{24} A_{\text{grav-grav-}\mathbb{Z}_M^R} \\ &= \rho \bmod \eta. \end{aligned} \tag{B.16}$$

$\rho$ is a constant, which is related to the discrete shift (A.22) of the axion via $\rho = \pi M \Delta_{\mathrm{GS}}$.

### B.3. Summary of anomaly coefficients

The anomaly coefficients are given by

$$A_{G\text{-}G\text{-}\mathbb{Z}_M^{(R)}} = \sum_{\boldsymbol{r}^{(f)}} \ell\big(\boldsymbol{r}^{(f)}\big)\big(q^{(f)} - R\big) + \ell(\operatorname{adj} G) \cdot R, \tag{B.17a}$$

$$A_{\mathrm{U}(1)\text{-}\mathrm{U}(1)\text{-}\mathbb{Z}_M^{(R)}} = \sum_m \big(Q^{(m)}\big)^2\big(q^{(m)} - R\big), \tag{B.17b}$$

$$A_{\text{grav-grav-}\mathbb{Z}_M^{(R)}} = R \cdot \left[-21 + \sum_G \dim(\operatorname{adj} G) + \#\big(\mathrm{U}(1)\big)\right] + \sum_m \big(q^{(m)} - R\big), \tag{B.17c}$$

where we distinguish between discrete non-$R$ ($R = 0$) and $R$ ($R = 1$) symmetries. $\#(\mathrm{U}(1))$ denotes the number of U(1) gauginos. As discussed above, the mixed U(1)–U(1)–$\mathbb{Z}_M^{(R)}$ anomaly is only meaningful if one knows the normalization. In general, the coefficient $A_{\mathrm{U}(1)\text{–}\mathrm{U}(1)\text{–}\mathbb{Z}_M^{(R)}}$ is not invariant under shifts of the $\mathbb{Z}_M^{(R)}$ charges by integer multiples of $M$.

## Appendix C. A comment on Kähler stabilization

A possible way to stabilize the dilaton is through non-perturbative corrections to the Kähler potential [22,23]. Such corrections are expected to vanish in the limit of zero coupling and also to all orders in perturbation theory. The form of these corrections has been studied in the literature [24,25,62]. For a favourable choice of the parameters, this correction may allow one to stabilize the dilaton at a realistic value, $\operatorname{Re} S \simeq 2$, while breaking supersymmetry [24,25,62–64]. A common parametrization of the non-perturbative corrections reads

$$\mathrm{e}^K = \mathrm{e}^{K_0} + \mathrm{e}^{K_{\mathrm{np}}}, \tag{C.1}$$

$$\mathrm{e}^{K_{\mathrm{np}}} = c x^{p/2} \mathrm{e}^{-q\sqrt{x}}, \tag{C.2}$$

with $K_0 = -\ln(2x)$, $x = \operatorname{Re} S$, and parameters subject to $K'' > 0$ and $p, q > 0$. Supersymmetry is broken spontaneously by the $F$ term of the dilaton,

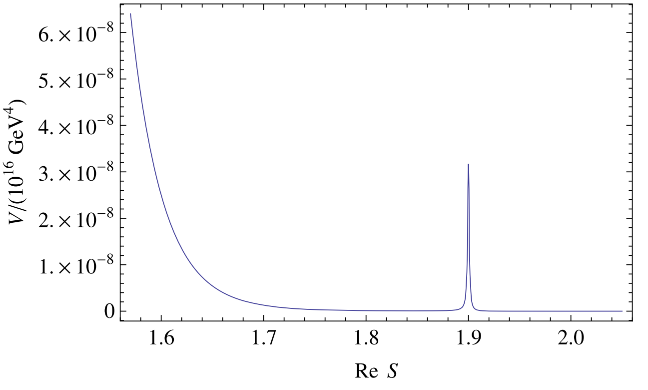


Fig. 2. Dilaton potential for $\{c_1, c_2, p_1, p_2, q\} = \{-28.8292, 22.6129, 2, 3, 4\}$.

$$F_S \sim \frac{\langle\lambda\lambda\rangle}{M_\mathrm{P}}. \tag{C.3}$$

For a single gaugino condensate, one has

$$\mathscr{W} = d\exp\left(-\frac{3S}{2\beta}\right), \tag{C.4}$$

where $3/(2\beta) = 8\pi^2/N$ and $d = -N/(32\pi^2\mathrm{e})$ for a condensing $\mathrm{SU}(N)$ group with no matter. Note that the scalar potential is independent of $\mathrm{Im}\,S$. That is, we are left with a GS axion.

The problem with this scheme is that the vacuum energy at the local minimum is typically positive (cf. [63]). Although Eq. (C.2) represents the 'standard' choice of the Kähler potential, there are no arguments that forbid additional terms of the same structure. That is, following Shenker's arguments [22] one may replace Eq. (C.2) by

$$\mathrm{e}^{K_\mathrm{np}} = \left(c_1 x^{p_1/2} + c_2 x^{p_2/2}\right)\mathrm{e}^{-q\sqrt{x}}. \tag{C.5}$$

In fact there seems to be no reason for not writing even more terms in the parentheses. One can then tune the vacuum energy in the local minimum to zero by carefully adjusting the coefficients (Fig. 2).

Of course there is still no reason for why the vacuum energy should vanish at the local minimum, but the above arguments may show that, in principle, the vacuum energy can be tuned to zero in this scheme.

## References

[1] G.R. Farrar, P. Fayet, Phys. Lett. B 76 (1978) 575.
[2] S. Dimopoulos, H. Georgi, Nucl. Phys. B 193 (1981) 150.
[3] S. Dimopoulos, S. Raby, F. Wilczek, Phys. Lett. B 112 (1982) 133.
[4] L.E. Ibáñez, G.G. Ross, Nucl. Phys. B 368 (1992) 3.
[5] K.S. Babu, I. Gogoladze, K. Wang, Phys. Lett. B 570 (2003) 32, hep-ph/0306003.
[6] H.K. Dreiner, C. Luhn, M. Thormeier, Phys. Rev. D 73 (2006) 075007, hep-ph/0512163.
[7] L.M. Krauss, F. Wilczek, Phys. Rev. Lett. 62 (1989) 1221.
[8] L.E. Ibáñez, G.G. Ross, Phys. Lett. B 260 (1991) 291.
[9] T. Banks, M. Dine, Phys. Rev. D 45 (1992) 1424, hep-th/9109045.
[10] S. Förste, H.P. Nilles, S. Ramos-Sánchez, P.K.S. Vaudrevange, arXiv:1007.3915 [hep-ph].
[11] K.S. Babu, I. Gogoladze, K. Wang, Nucl. Phys. B 660 (2003) 322, hep-ph/0212245.

[12] H.M. Lee, et al., Phys. Lett. B 694 (2011) 491, arXiv:1009.0905 [hep-ph].
[13] L.J. Hall, Y. Nomura, A. Pierce, Phys. Lett. B 538 (2002) 359, hep-ph/0204062.
[14] M. Dine, J. Kehayias, Phys. Rev. D 82 (2010) 055014, arXiv:0909.1615 [hep-ph].
[15] R. Slansky, Phys. Rept. 79 (1981) 1.
[16] C. Csáki, H. Murayama, Nucl. Phys. B 515 (1998) 114, hep-th/9710105.
[17] B. Petersen, M. Ratz, R. Schieren, JHEP 0908 (2009) 111, arXiv:0907.4049 [hep-ph].
[18] R. Kappl, et al., arXiv:1012.4574 [hep-th].
[19] E. Witten, Nucl. Phys. B 188 (1981) 513.
[20] H.P. Nilles, Phys. Lett. B 115 (1982) 193.
[21] R. Kappl, H.P. Nilles, S. Ramos-Sánchez, M. Ratz, K. Schmidt-Hoberg, P.K. Vaudrevange, Phys. Rev. Lett. 102 (2009) 121602, arXiv:0812.2120 [hep-th].
[22] S.H. Shenker, Presented at the Cargese workshop on random surfaces, Quantum Gravity and Strings, Cargese, France, May 28–June 1, 1990.
[23] T. Banks, M. Dine, Phys. Rev. D 50 (1994) 7454, hep-th/9406132.
[24] J.A. Casas, Phys. Lett. B 384 (1996) 103, hep-th/9605180.
[25] P. Binétruy, M.K. Gaillard, Y.-Y. Wu, Nucl. Phys. B 493 (1997) 27, hep-th/9611149.
[26] V.S. Kaplunovsky, J. Louis, Phys. Lett. B 306 (1993) 269, hep-th/9303040.
[27] J. Casas, A. Lleyda, C. Munoz, Phys. Lett. B 380 (1996) 59, arXiv:hep-ph/9601357.
[28] S.A. Abel, B.C. Allanach, F. Quevedo, L. Ibáñez, M. Klein, JHEP 0012 (2000) 026, hep-ph/0005260.
[29] S.A. Abel, S. Sarkar, P.L. White, Nucl. Phys. B 454 (1995) 663, hep-ph/9506359.
[30] K. Kumekawa, T. Moroi, T. Yanagida, Prog. Theor. Phys. 92 (1994) 437, arXiv:hep-ph/9405337.
[31] K. Kurosawa, N. Maru, T. Yanagida, Phys. Lett. B 512 (2001) 203, hep-ph/0105136.
[32] M. Blaszczyk, et al., Phys. Lett. B 683 (2010) 340, arXiv:0911.4905 [hep-th].
[33] H.M. Lee, S. Raby, M. Ratz, G.G. Ross, R. Schieren, K. Schmidt-Hoberg, P.K.S. Vaudrevange, Additional material for a $\mathbb{Z}_4^R$ string vacuum, http://einrichtungen.ph.tum.de/T30e/orbifolds/Z4Rvacuum1.pdf, 2011.
[34] F. Buccella, J.P. Derendinger, S. Ferrara, C.A. Savoy, Phys. Lett. B 115 (1982) 375.
[35] L.J. Dixon, D. Friedan, E.J. Martinec, S.H. Shenker, Nucl. Phys. B 282 (1987) 13.
[36] S. Hamidi, C. Vafa, Nucl. Phys. B 279 (1987) 465.
[37] K.S. Babu, S.M. Barr, Phys. Rev. D 48 (1993) 5354, hep-ph/9306242.
[38] T. Araki, et al., Nucl. Phys. B 805 (2008) 124, arXiv:0805.0207 [hep-th].
[39] F. Brümmer, R. Kappl, M. Ratz, K. Schmidt-Hoberg, JHEP 1004 (2010) 006, arXiv:1003.0084 [hep-ph].
[40] A. Hebecker, J. March-Russell, R. Ziegler, arXiv:0801.4101 [hep-ph].
[41] F. Brümmer, S. Fichet, A. Hebecker, S. Kraml, JHEP 0908 (2009) 011, arXiv:0906.2957 [hep-ph].
[42] P. Hosteins, R. Kappl, M. Ratz, K. Schmidt-Hoberg, JHEP 0907 (2009) 029, arXiv:0905.3323 [hep-ph].
[43] S.A. Abel, Nucl. Phys. B 480 (1996) 55, hep-ph/9609323.
[44] C. Panagiotakopoulos, K. Tamvakis, Phys. Lett. B 446 (1999) 224, hep-ph/9809475.
[45] T. Kobayashi, S. Raby, R.-J. Zhang, Nucl. Phys. B 704 (2005) 3, hep-ph/0409098.
[46] T. Kobayashi, H.P. Nilles, F. Plöger, S. Raby, M. Ratz, Nucl. Phys. B 768 (2007) 135, hep-ph/0611020.
[47] S. Cassel, D. Ghilencea, G. Ross, Nucl. Phys. B 825 (2010) 203, arXiv:0903.1115 [hep-ph].
[48] A. Delgado, C. Kolda, J.P. Olson, A. de la Puente, Phys. Rev. Lett. 105 (2010) 091802, arXiv:1005.1282 [hep-ph].
[49] P. Fox, A. Pierce, S.D. Thomas, hep-th/0409059.
[50] J.E. Kim, H.P. Nilles, Phys. Lett. B 138 (1984) 150.
[51] K.-Y. Choi, E.J. Chun, H.M. Lee, Phys. Rev. D 82 (2010) 105028, arXiv:1002.4791 [hep-ph].
[52] M. Kamionkowski, J. March-Russell, Phys. Lett. B 282 (1992) 137, hep-th/9202003.
[53] S.M. Barr, D. Seckel, Phys. Rev. D 46 (1992) 539.
[54] T. Araki, Prog. Theor. Phys. 117 (2007) 1119, hep-ph/0612306.
[55] K. Fujikawa, Phys. Rev. Lett. 42 (1979) 1195.
[56] K. Fujikawa, Phys. Rev. D 21 (1980) 2848.
[57] L. Álvarez-Gaumé, E. Witten, Nucl. Phys. B 234 (1984) 269.
[58] L. Álvarez-Gaumé, P.H. Ginsparg, Ann. Phys. 161 (1985) 423.
[59] K. Fujikawa, S. Ojima, S. Yajima, Phys. Rev. D 34 (1986) 3223.
[60] W. Lerche, B.E.W. Nilsson, A.N. Schellekens, Nucl. Phys. B 289 (1987) 609.
[61] L.E. Ibáñez, Nucl. Phys. B 398 (1993) 301, hep-ph/9210211.
[62] P. Binétruy, M.K. Gaillard, Y.-Y. Wu, Phys. Lett. B 412 (1997) 288, hep-th/9702105.
[63] T. Barreiro, B. de Carlos, E.J. Copeland, Phys. Rev. D 57 (1998) 7354, hep-ph/9712443.
[64] W. Buchmüller, K. Hamaguchi, O. Lebedev, M. Ratz, Nucl. Phys. B 699 (2004) 292, hep-th/0404168.