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Analysis of the Cost Structure of an Urban Bus Transit Property

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ABSTRACT

Past research on the cost structure of urban bus transportation shows conflicting results with respect to key economic issues such as economies of scale and other properties of the underlying technology. It is hypothesized that these results stem from three major problem areas: the form of the estimated cost model, definition of the output measure, and major characteristics of the data base. Utilizing longitudinal data from one bus property, this study estimates a general cost model which places very few a priori restrictions on the production structure. In addition, two different output measures are defined, and the cost model is estimated separately for each. Results of the study presented in this paper indicate that the general form cost model better represents bus transit technology than other more restrictive models, and that different output measures have a significant effect on the measurement of economies of scale. Results pertaining to factor substitution, separability, and homotheticity are also presented.

1. INTRODUCTION

The urban bus transit industry in the U.S. has undergone many changes over the past few decades. Following the shift from private to public ownership of all the major urban transit properties in the 1960's, federal, state and local governments launched a policy of large scale subsidization. Until 1975, the federal government's major contribution was the capital grant program, aimed at renewing the aging rolling stock and fixed facilities of the industry. In 1975, however, as transit operating costs and deficits increased at an ever increasing rate, the federal government joined state and local governments in contributing operating subsidies as well. By 1980, however, funding resources began to shrink. Federal operating subsidies are presently slated for abandonment within the next few years, while state and local subsidy resources are not expected to increase.

Against this backdrop of shifting revenue sources and rising costs, it is worthwhile to examine the economic structure of the bus transit industry. Such an examination is necessary to develop an understanding of the production conditions of bus transit services, and it can provide the information necessary to interpret the economic behavior of the transit firm in the context of public policy toward the transit industry.

Building upon a comprehensive review of previous research presented elsewhere (Berechman and Giuliano, 1982), this paper presents the results of an econometric analysis of bus transit which utilizes time series data from a major California bus transit operator. The paper begins with a short review of previous research which highlights some of the problems of these studies. It shows how this research differs from earlier efforts, and generates the set of hypotheses to be tested in this research. Afterwards, the econometric model utilized in the analysis is discussed, followed by a description of the data base. The paper then presents the results of the empirical estimation and analyzes these results in the context of transit firm production technology. The paper concludes with a summary of major findings and some suggestions on how this work can be extended to further examine the impact of public policy on transit firm behavior.

2. REVIEW OF PREVIOUS RESEARCH

Models utilized in the study of the cost structure of the industry fall into two categories. The vast majority of studies have estimated simple cost models which impose a number of restrictions on the cost structure and the underlying production technology; e.g., linearity in factor prices and output and thus constant marginal cost; or zero or unit factor elasticity of substitution. These restrictions have resulted in the major focus of these studies being the issue of scale economies. Some examples of studies which have estimated linear or log-linear cost models are Lee and Steedman, 1970; Nelson, 1972; Wabe and Coles, 1975; Fravel, 1978.

More recently, a few econometric studies of the bus transit industry have utilized models based on recent developments in production theory

(Williams and Dalal, 1981; Viton, 1981; Berechman, 1982). These studies estimated transcendental logarithmic cost models which place very few a priori restrictions on the economic characteristics of interest. More specifically, this type of model allows for nonunity factor elasticities of substitution, nonunity total cost elasticity, and a non-homothetic production structure. It is thus it possible to examine a broad range of economic characteristics in addition to scale economies such as factor substitution and demand, and the type of underlying production technology. These studies represent a significantly improved theoretical approach which is followed here as well. Homotheticity and separability of the production structure, factor substitution and cost elasticities, as well as economies of scale are examined in this research.

With the exception of Berechman (1982), all of these recent studies have utilized cross-sectional data, with the bus property as the unit of observation. Cross-sectional analysis implicitly assumes that observations are homogeneous and therefore that transit firms are comparable. However, there is a great deal of variation among transit firms; they not only operate in different markets facing quite varied demand environments, but they may also utilize different technologies to produce transit services. For example, the peak/base ratio is generally high in major metropolitan areas, while extra peak service is almost negligible in small and semi-rural urban areas. Since the costs of producing peak and base period services have been shown to differ significantly (Oram, 1979), this ratio may have an important effect on input factor demand and elasticity of factor substitution. In addition, service characteristics such as average speed, headways, and route lengths may affect service costs, and these characteristics are linked to the environment in which the firm operates.

The cross-section samples used in many previous studies consist of very heterogeneous properties. For example, the Lee and Steedman sample ranged in size from 28 vehicles to 1600 vehicles (1975), and the Viton sample consisted of firms which produced from 168,000 vehicle-miles to 88.5 million vehicle-miles per year (1981). The inclusion of bus properties of different sizes with different operating environments violates the theoretical requirement of uniform production conditions (Gold, 1981). In order to avoid the problem of a heterogenous sample, time series data from one bus property is utilized in this research. Problems of using longitudinal data are discussed in section 4.

The output measure selected for the analysis is another problem area of previous studies. The output of a transit firm is the aggregate of services provided which may be differentiated by such characteristics as route length, frequency, travel speed, and hours of operation. That is, result units of service are not homogeneous, yet some measure of aggregated output must be utilized, since the individual bus firm is the unit of observation. Moreover, the use of multiple output units is not likely to be a feasible solution to this problem because the costs of different types of service cannot be clearly differentiated from the data. Thus an aggregate measure which approximates service characteristics must be utilized.

Most of the previous studies have utilized service or capacity related measures such as bus-miles or bus-hours (e.g., Viton, 1981; Williams and Dalal, 1981). In this paper these are termed technical measures. In general such measures are highly correlated with major input cost factors such as labor and fuel, and they provide good common denominators of output levels of properties which otherwise might be quite different. However, the use of these measures in a cross section analysis tends to further obscure any differences in the production conditions among properties. For example, when measured in bus-miles, transit output in cities like New York and Los Angeles might be indistinguishable even though service characteristics and thus (presumably) production conditions are very different. Furthermore, technical measures do not reflect the economic motive for providing the services, the carrying of passengers.

A few of the previous studies have utilized demand-related measures such as passenger-trips, or passenger revenue (Williams and Hall, 1981; Berechman, 1982). These measures not only reflect differences in local production conditions, but are also directly related to actual market transactions. Thus the results from the cost analysis are readily amenable to economic interpretations.[1] In contrast with the technical measures, demand-related measures may not vary systematically with input

[1]Mohring (1972) utilizes a demand related output measure and includes the time that passengers contribute when traveling as a factor of production in the cost function.

items like labor and fuel. Clearly, every output measure has advantages and disadvantages.

These authors have argued elsewhere that the specification of the output measure might affect the outcomes of the analysis, particularly the measurement of scale economies (Berechman and Giuliano, 1982). Technical measures such as vehicle-miles represent output capacity, while demand-related measures such as passenger trips represent intensity of utilization of capacity. Thus when technical measures are used, economies of scale measures the change in total cost with respect to a change in capacity. When demand-related measures are used, economies of scale measures the change in total cost with respect to utilization of capacity. Thus, when using different output measures, economies of scale measure different aspects of the production structure, and it is therefore hypothesized that different cost elasticity estimates will result. In this research, the technical and demand-related output measures are, respectively, vehicle-miles and revenue passengers. The cost models are estimated separately for each output measure, and all production and cost characteristics are computed for both measures.

3. THE MODEL

Research in the duality theory of production has generated a number of functional forms for cost models which require very few a priori theoretical restrictions on the characteristics of the underlying production function (McFadden, 1978). In this study, a trascendental logarithmic function ("translog") was selected for estimation. The translog cost model is well documented in the literature (Christensen et al., 1973) and has been previously utilized in transportation research (Caves et al., 1980; Friedlander and Spady, 1981). The translog model meets the neoclassic theoretical conditions for a cost function, that is, it is continuous, nondecreasing concave, and homogeneous of degree one in input prices.[2] Cost minimization is assumed, and thus from Shephard's lemma, the first order partial derivatives of the cost function with respect to the input prices equal the cost minimizing factor quantities.[3] As illustrated below, this feature allows the estimation of factor demand equations.

In this study, the translog model is estimated with one output and four input factors: capital, labor, fuel, and maintenance. The specific cost function is,

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[2]The general form of the translog model is:

$$LnC = \alpha_0 + \sum_i^m \alpha_i \, LnY_i + \sum_i^n \beta_i \, LnP_i + \frac{1}{2} \sum_i^m \sum_j^m \delta_{ij} \, LnY_i \, LnY_j$$

$$+ \frac{1}{2} \sum_i^n \sum_j^n \gamma_{ij} \, LnP_i \, LnP_j + \sum_i^n \sum_j^m \rho_{ij} \, LnP_i \, LnY_j + u \quad .$$

with the following homogeneity and symmetry conditions imposed.

$$\sum_i^n \beta_i = 1; \sum_j^n \gamma_{ij} = 0 \ (i=1,\ldots,n); \sum_j^m \rho_{ij} = 0 \ (i=1,\ldots,m); \delta_{ij} = \delta_{ji}; \gamma_{ij} = \gamma_{ji}.$$

If $\rho_{ij} = 0$ for all ij, C is homothetic-homogeneous cost function.

[3]It is also assumed that there is no lagged behavior in the firm's response to factor price changes.

$$\begin{aligned}
LnC = {} & \alpha_0 + \alpha_Y \, LnY + \beta_K \, LnP_K + \beta_L \, LnP_L + \beta_F \, LnP_F + \beta_M \, LnP_M \\
& + \frac{1}{2} \delta_{YY}(LnY)^2 + \frac{1}{2} \gamma_{KK}(LnP_K)^2 + \frac{1}{2} \gamma_{LL}(LnP_L)^2 \\
& + \frac{1}{2} \gamma_{FF}(LnP_F)^2 + \frac{1}{2} \gamma_{MM}(LnP_M)^2 + \gamma_{KL} \, LnP_K \, LnP_L \\
& + \gamma_{KF} \, LnP_K \, LnP_F + \gamma_{KM} \, LnP_K \, LnP_M + \gamma_{LF} \, LnP_L \, LnP_F \\
& + \gamma_{LM} \, LnP_L \, LnP_M + \gamma_{FM} \, LnP_F \, LnP_M + \rho_{KY} \, LnP_K \, LnY \\
& + \rho_{LY} \, LnP_L \, LnY + \rho_{FY} \, LnP_F \, LnY + \rho_{MY} \, LnP_M \, LnY
\end{aligned} \tag{1}$$

where P_K, P_L, P_F and P_M are prices of capital, labor, fuel and maintenance, respectively, and Y = vehicles-miles in the first estimation and revenue passengers in the second. Homogeneity of degree one in prices is imposed; thus,

$$\begin{aligned}
\beta_K + \beta_L + \beta_F + \beta_M &= 1 \\
\gamma_{KK} + \gamma_{KL} + \gamma_{KF} + \gamma_{KM} &= 0 \\
\gamma_{LK} + \gamma_{LL} + \gamma_{LF} + \gamma_{LM} &= 0 \\
\gamma_{FK} + \gamma_{FL} + \gamma_{FF} + \gamma_{FM} &= 0 \\
\gamma_{MK} + \gamma_{ML} + \gamma_{MF} + \gamma_{MM} &= 0 \\
\rho_{KY} + \rho_{LY} + \rho_{FY} + \rho_{MY} &= 0
\end{aligned} \tag{2}$$

It is assumed that the transit firm faces competitive factor markets. Differentiating the cost function with respect to factor prices and using Shephard's lemma, the cost shares are obtained,

$$\frac{\partial LnC}{\partial LnP_i} = \frac{\partial C}{\partial P_i}\frac{P_i}{C} = \frac{X_iP_i}{C} = S_i \qquad i = 1,\ldots,4 \qquad (3)$$

where X_i is the cost minimizing factor quantity and S_i is the cost share of factor i in the total cost of producing output Y. Applying (3) to the cost function (1), the following factor demand functions are derived,

$$S_K = \frac{KP_K}{C} = \beta_K + \gamma_{KK}\, LnP_K + \gamma_{KL}\, LnP_L + \gamma_{KF}\, LnP_F + \gamma_{KM}\, LnP_M + \rho_{KY} LnY$$

$$S_L = \frac{LP_L}{C} = \beta_L + \gamma_{LL}\, LnP_L + \gamma_{KL}\, LnP_K + \gamma_{LF}\, LnP_F + \gamma_{LM}\, LnP_M + \rho_{LY} LnY$$

$$S_F = \frac{FP_F}{C} = \beta_F + \gamma_{FF}\, LnP_F + \gamma_{FK}\, LnP_K + \gamma_{FL}\, LnP_L + \gamma_{FM}\, LnP_M + \rho_{FY} LnY$$

$$S_M = \frac{MP_M}{C} = \beta_M + \gamma_{MM}\, LnP_M + \gamma_{KM}\, LnP_K + \gamma_{LM}\, LnP_L + \gamma_{FM}\, LnP_F + \rho_{MY} LnY \qquad (4)$$

where S_K, S_L, S_F and S_M are the shares of capital, labor, fuel and maintenance respectively. Since after imposing constraint (2) on the cost function (1) there are still 15 parameters to be estimated, and since the share equations do not add unknown parameters, it is useful to estimate (1) and (4) together. This approach increases the degrees of freedom without increasing the number of parameters to be estimated. More importantly, including (4) provides estimates of the demand for the input factors as functions of factor prices and the level of output.

Joint estimation of the cost and share equations requires that one of the share equations be deleted because a random error is associated with the cost and share functions; thus an additive disturbance is introduced to each of the five equations. Since the sum of the share functions equals unity at each observation, the sum of the error terms is zero (assuming they are normally distributed), and thus the covariance matrix of the cost share functions becomes singular and non-diagonal. This problem is avoided by deleting one of the share equations, in this case S_M.

In order to explore the production technology underlying the cost structure several elasticity measures are computed. First, the Allen partial elasticities of substitution between factors i and j, σ_{ij}, are computed from (1). Following Uzawa (1962),

$$\sigma_{ij} = C \left(\frac{\partial^2 C}{\partial P_i \partial P_j}\right) \bigg/ \left(\frac{\partial C}{\partial P_i} \cdot \frac{\partial C}{\partial P_j}\right) \tag{5}$$

which implies that $\sigma_{ij} = \sigma_{ji}$. For the translog model

$$\sigma_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2} \qquad i = 1,\ldots,4 \tag{6}$$

and

$$\sigma_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j} \qquad i,j = 1,\ldots,4 \tag{7}$$

where all variables are defined above. Notice that these partial elasticities of substitution are not constant (as for example in a Cobb-Douglas function), but vary according to the cost shares. By definition, if, for a given i and j $(i \neq j)$, $\sigma_{ij} > 0$, then factors i and j are substitutes, and if $\sigma_{ij} < 0$, factors i and j are complements.

Second, own and cross price elasticities of factor demand, ε_{ij} are computed. These elasticities are defined as

$$\varepsilon_{ij} = \frac{\partial LnX_i}{\partial LnP_j} , \qquad i,j = 1,\ldots,n \tag{8}$$

and Allen (1938) has shown that

$$\varepsilon_{ij} = \sigma_{ij} S_j , \qquad i,j = 1,\ldots,n \tag{9}$$

which implies that, in general, $\varepsilon_{ij} \neq \varepsilon_{ji}$.

The third elasticity measure of interest is the elasticity of total cost with respect to output, $\partial LnC/\partial LnY$. It is used to compute the degree of economies of scale in the production function and by extension the marginal cost function, as follows,

$$\frac{\partial LnC}{\partial LnY} = \alpha_Y + \frac{1}{2}\delta_{YY} LnY + \sum_{j=1}^{4} \rho_{jY} LnP_j \tag{10}$$

and

$$MC = \frac{\partial C}{\partial Y} = \frac{C}{Y}\left[\frac{\partial LnC}{\partial LnY}\right] . \tag{11}$$

Finally, it should be noted that in this study the cost function is assumed to be an exact representation of a minimum cost function for producing Y, given P_i $(i = 1,\ldots,4)$. An alternative approach is to consider the translog cost function as a second order approximation at a point (e.g., the mean) to an arbitrary twice differentiable cost function.[4] A major disadvantage of the latter approach with respect to this research is that test results regarding the underlying production technology, mainly factor separability, hold only at the point of approximation (see Denny and Fuss (1977), on this issue).

4. THE DATA BASE

The empirical analysis utilizes time series data from the Alameda-Contra Costa Transit District (ACTD). The ACTD is located in the San Francisco Bay Area and serves a 600 square mile, two county area. It has operated as a public enterprise since 1960. During the time period of this study (1972-1979), ACTD has remained a stable, medium size operation with a fleet of approximately 800 buses. Annual revenue passengers range from 50.5 million in fiscal year 1972 to 59.4 million in fiscal year 1979. It is assumed in the analysis that ACTD is in long-run equilibrium. ACTD provides a variety of transit services with local fixed route in the more densely populated areas making up the major portion. Commuter services within the East Bay Area, express service to San Francisco, and demand-responsive services in the suburban east county areas are also provided. Demand-responsive services underwent significant expansion during the 1970's, but continue to be only a small

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[4]Examples of empirical applications of the first approach are Berndt and Wood (1975) and Christensen and Green (1976). Friedlaender and Spady (1981) and Viton (1981) are examples [illegible] of the second approach.

proportion of total services provided. From this description of ACTD it can be seen that its operation is quite typical of many bus properties in the U.S. It is particularly representative of bus operations in urban areas where bus is the major mode of transit and where modest service expansion took place during the last two decades. It may not be representative, however, of transit operations in some northeastern cities where rail service is predominant, or where significant service reductions have taken place.

Data were collected from monthly reports and records of ACTD's accounting, maintenance, planning and bus operations departments. The monthly data were aggregated to quarterly data; thus the seven fiscal years yielded 28 observations. For computational purposes the data set was indexed, so that the first observation value of each variable was set to 100.

As stated earlier, two output measures and four input measures were selected. Total Vehicle Miles (VM) representing "technical" output is defined as the total number of miles logged by the bus fleet. This figure thus includes deadhead as well as in-service mileage. Revenue passengers, representing demand-related output, is defined as the number of fare paying passengers. The four input factor quantities were measured as follows: capital, total number of buses in fleet; labor, paid labor hours of driver, clerical and non-union employees; maintenance, paid labor hours of maintenance; fuel, gallons of diesel fuel. Table 1 presents the quantities of inputs and outputs in index form.

Table 1

Quantity Indices of Input and Output,

AC Transit 1972-1979 (Quarterly)

		Input Indices				Output Indices	
Year	Case	Capital	Labor	Maintenance	Fuel	Vehicle Miles	Revenue Passengers
1971	1	100.00	100.00	100.00	100.00	100.00	100.00
	2	99.96	104.21	100.33	101.93	101.93	108.58
1973	3	99.06	102.02	99.59	102.93	102.92	109.25
	4	97.74	104.55	100.51	105.39	105.39	113.68
	5	103.97	105.74	102.82	102.70	106.39	114.67
	6	104.19	109.57	103.84	107.39	107.39	115.66
1974	7	103.12	106.56	101.67	106.26	106.25	122.65
	8	103.03	109.49	107.40	111.45	111.45	126.69
	9	103.12	110.82	105.81	109.88	111.68	119.72
	10	103.97	116.24	108.38	111.93	111.92	112.76
1975	11	113.16	113.24	107.30	112.79	113.31	113.94
	12	112.94	115.75	109.85	117.28	117.39	118.80
	13	112.52	116.85	110.40	112.89	116.50	123.23
	14	111.75	117.89	113.78	117.30	115.70	127.66
1976	15	115.25	117.35	111.04	116.61	114.40	129.38
	16	114.87	117.80	113.34	119.11	116.29	124.76
	17	112.94	121.56	114.00	114.63	117.79	128.63
	18	112.82	122.98	111.56	121.14	119.29	132.51
1977	19	112.82	122.98	111.55	121.98	117.90	135.17
	20	112.26	123.08	116.30	123.25	119.90	133.58
	21	112.60	126.38	119.62	120.93	120.12	133.60
	22	110.85	124.53	118.38	124.81	120.34	133.63
1978	23	110.94	121.87	115.87	122.06	118.02	125.78
	24	108.54	122.83	117.69	124.49	120.21	132.39
	25	106.37	119.51	118.05	116.70	121.21	126.12
	26	106.37	122.68	125.20	123.31	119.73	121.46
1979	27	108.16	124.29	125.80	133.33	122.22	134.22
	28	107.86	120.53	125.31	127.00	118.17	136.19

In order to compute the per unit price of inputs, expenses were first allocated to the input factors and then (except for capital) divided by each input quantity. Labor expenses included total wages, fringe benefits and pension payments to drivers, clerical and non-union employees. Maintenance expenses included total wages, fringe benefits and pension payments to maintenance employees plus expenditures on parts and materials. Fuel expenses were total diesel fuel expenses.

Once the expenses were allocated, quarterly unit prices for labor, maintenance and fuel were obtained by dividing the expenses for each factor by the appropriate factor quantity. For example, the hourly price of labor was computed by dividing labor expenses by paid labor hours for each quarter.

A different approach was used to compute the cost of capital in order to come as close as possible to the economic cost of capital to the firm. A review of the literature indicated that many different methods have been used to calculate the cost of capital. However, only the Nelson formula (1972) takes the UMTA section 3 capital grant subsidy and age of the rolling stock into account. The formula used here is a modification of the Nelson approach, as follows[5]

$$P_{ki} = N_i(1 - .8)V_{0i}\exp(-\delta A)\delta \qquad (12)$$

[5]Nelson's formula is, $P = N(1 - .67s)V_0(z)\exp(-\delta A)(\delta + r)$, where s is the percent of buses paid by capital grant, r is the interest rate on municipal bonds, and $V_0(z)$ is the price of a new bus of type z. From the data it was impossible to establish $V_0(z)$ and $(1 - .67s)$. Furthermore, ACTD cannot legally float bonds to raise funds, and thus $r = 0$.

where P_{ki} = total cost of capital in quarter i

N_i = fleet size in quarter i

V_{0i} = price of a new bus in quarter i

δ = depreciation rate

A = average fleet age.

Per unit cost of capital in quarter i is thus P_{ki}/N_i

Total cost in nominal dollars, input price indices for each factor, and the cost shares are presented in Table 2. As explained earlier, the cost shares are defined as X_iP_i/C, where the X_i and P_i are the quantity and price of factor i respectively, and C, total cost, is computed as $\sum_{i=1}^{4} X_iP_i$.

Tables 1 and 2 also give an indication of some of the changes which took place during the period of study. Of all input factors, capital use varied the least, while labor, maintenance and fuel use changes ranged from about 20 to 25%. Vehicle miles increased approximately 20%, and revenue passengers went up about 35%. In contrast to these rather modest changes, total cost increased by almost 140%, in current prices, with all factor prices jumping by at least 100%, the largest price increases being for capital and fuel. At the same time, cost shares remained remarkably stable. It bears noting that during this period the fare recovery ratio (the ratio of fare revenue to total cost) declined from an average of 55% in FY 1972-73 to 33% in FY 1978-79, and operating subsidies correspondingly increased. Thus the period of study was one in which relatively modest increases in output were accompanied by major increases in cost and subsidy revenues.

Table 2

Total Cost, Cost Shares and Prices of Inputs (Indices)

AC Transit 1972-1979 (quarterly)

			Cost Shares[a]				Input Price Indices			
Year	Case	Total Cost	Capital	Labor	Maint.	Fuel	Capital	Labor	Maint.	Fuel
1972	1	6,518,153	.0805	.7174	.1208	.0361	100.00	100.00	100.00	100.00
	2	6,723,042	.0876	.7204	.1186	.0356	112.21	99.39	100.90	100.00
1973	3	6,832,306	.0849	.7140	.1202	.0354	111.57	102.27	104.74	100.00
	4	7,306,604	.1066	.6857	.1161	.0340	151.84	102.50	107.14	100.00
	5	7,169,995	.0868	.7480	.1215	.0338	114.07	108.47	107.61	100.00
	6	7,446,516	.1084	.7448	.1211	.0354	146.63	108.25	100.28	105.55
1974	7	7,625,807	.0806	.7298	.1212	.0488	113.63	111.70	115.45	148.89
	8	8,536,145	.0888	.7146	.1171	.0466	140.26	119.14	118.17	148.89
	9	9,563,471	.0769	.6600	.1092	.0485	135.90	121.82	125.28	177.77
	10	10,528,462	.0783	.6913	.1090	.0393	151.02	133.91	134.52	155.55
1975	11	10,503,309	.0848	.7003	.1097	.0439	149.93	138.93	136.37	172.22
	12	11,163,621	.0900	.6756	.1086	.0457	169.57	139.36	140.10	183.33
	13	11,515,716	.0911	.6916	.1121	.0446	177.62	145.77	145.10	194.44
	14	11,150,771	.0724	.7284	.1166	.0478	111.40	147.34	145.10	194.44
1976	15	11,807,200	.0712	.7091	.1144	.0466	138.98	152.58	154.44	200.56
	16	11,926,538	.0668	.7000	.1184	.0471	132.19	151.56	158.22	200.56
	17	12,672,198	.0658	.7135	.1164	.0427	140.75	159.06	164.30	200.56
	18	13,061,945	.0653	.7129	.1128	.0432	143.42	160.72	160.47	205.55
1977	19	13,382,212	.0662	.7127	.1088	.0463	149.73	165.87	165.72	216.66
	20	14,026,435	.0753	.6871	.1063	.0455	179.18	167.47	162.85	222.22
	21	14,333,280	.0746	.6978	.1124	.0441	181.02	169.26	171.00	222.22
	22	14,107,292	.0803	.7262	.1183	.0466	194.64	175.95	178.95	222.22
1978	23	13,934,726	.0748	.7221	.1182	.0461	179.00	176.57	180.53	222.22
	24	16,282,691	.0760	.6779	.1184	.0403	217.24	192.19	208.14	222.22
	25	15,822,000	.0722	.6669	.1141	.0388	204.53	188.80	194.30	222.22
	26	16,516,000	.0713	.6653	.1037	.0398	210.95	191.55	173.62	227.77
1979	27	16,866,000	.0715	.6713	.1115	.0432	212.40	194.80	189.89	233.33
	28	16,258,000	.0871	.7077	.1276	.0469	250.21	204.16	210.26	255.00
Average			.0800	.7100	.1151	.0426				

[a]Cost shares may not sum up to unity because of changes in ACTD accounting procedures which affected the definition of certain cost items.

4. RESULTS AND ANALYSIS

The four equation system consisting of the cost function (1) and the share equations for capital, labor and fuel (4) presented in Section 3 was estimated with the nonlinear iterative Zellner method (Zellner, 1962). This method yields parameter estimates which are asymptotically equivalent to maximum likelihood estimates. The estimates are also invariant with respect to the particular share equation deleted from the system (Christensen and Green, 1976; Caves et al., 1980).

Table 3 presents the results of the cost model system estimation. As discussed earlier, the model is estimated twice, once with vehicle miles and once with revenue passengers as the output measure. Theoretical requirements for a well-behaved cost function are that it is concave in input prices, and that the factor demand (share) functions are strictly positive. The first condition is satisfied if the Hessian Matrix $[\partial C/\partial p_i \partial p_j]$ based on the above estimates is negative semidefinite. It was found to be so for each mean year observation. To satisfy the second condition, the input cost share equations were fitted with the price data, using the above estimates, and found to be positive at each observation.[6]

Utilizing time series data made it necessary to consider two statistical problems, namely linear dependency among the explanatory variables and autocorrelated disturbances. Linear dependency can yield

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[6]These tests do not guarantee that the cost function meets the above requirements globally, as the test results apply only within a neighborhood of the observed prices.

Table 3

Parameter Estimate of Cost Model, AC Transit

(1972-1979)

Parameter	Vehicle Miles Estimate	Vehicle Miles Standard Error	Revenue Passengers Estimate	Revenue Passengers Standard Error
A_0	12.656	19.264	0.740	16.255
α_Y	-6.723	8.178	-1.068	6.793
δ_{YY}	1.724	1.736	0.392	1.420
β_K	0.396	0.060	0.260	0.055
β_L	1.541	0.312	0.933	0.259
β_F	0.090	0.029	0.0367	0.022
β_M	-1.027		-0.2294	
γ_{KK}	0.0663	0.003	0.0671	0.0036
γ_{LL}	0.191	0.065	0.209	0.0645
γ_{FF}	0.0406	0.002	0.0396	0.0021
γ_{MM}	0.0907		0.0964	
γ_{KL}	-0.0636	0.011	-0.0607	0.012
γ_{KF}	-0.0006	0.0016	0.0003	0.0015
γ_{KM}	-0.0020		-0.0067	
γ_{LF}	-0.0293	0.007	-0.0291	0.0065
γ_{LM}	-0.0981		-0.119	
γ_{FM}	-0.0105		-0.0107	
ρ_{KY}	-0.0677	0.013	-0.0383	0.0113
ρ_{LY}	-0.175	0.066	-0.0454	0.054
ρ_{FY}	-0.0116	0.0062	-0.0003	0.0045
ρ_{MY}	0.254		0.0840	
R^2(adj.)	.877		.453	
Durbin-Watson	1.38		.79	

inefficient estimates. An inspection of the partial correlation coefficients matrix showed that the output variables LnY and LnY^2 in the cost function are highly correlated. Indeed, the deletion of LnY^2 from the model substantially lowered the standard error of α_y, with virtually no change in all other estimated parameters.[7] Recognizing the unavoidable impact on the efficiency of the estimated parameters of LnY and LnY^2, the original specification of the model was nonetheless maintained in order to derive the cost elasticity factors as a function of output.

The Durbin-Watson statistic was used to test the hypothesis that the disturbances are positively autocorrelated, and the value of this statistic is also given in Table 3. Since all of the values fall within the lower and upper test critical values, the test results were inconclusive at the 5% level of significance.

Table 3 also gives the adjusted R^2 for each equation. As anticipated, R^2 for the vehicle-miles equation is higher than that of the revenue passengers equation, indicating the closer relationship of the technical output measure to the factor inputs.

Elasticities

Turning now to the measurement of factor and price elasticities, Table 4 presents estimate of the Allen partial elasticities of substitution (σ_{ij}), and Table 5 gives own and cross price

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[7] When the model was run with $\delta_{YY} = 0$, the following results were obtained: for vehicle miles, $\alpha_y = 1.398$ with s.e. = 0.0923; for revenue passengers, $\alpha_y = .810$ with s.e. = .139.

Table 4

Allen Partial Elasticities of Substitution Estimates

for Selected Mean Fiscal Years

	1972-3		1975-6		1978-9	
Parameter	VM	RP	VM	RP	VM	RP
σ_{KK}	-1.86	-1.77	-.60	-.46	-.54	-.40
σ_{LL}	-.03	-.005	-.03	-.003	-.06	-.02
σ_{FF}	-1.21	-1.70	-1.72	-2.20	-2.25	-2.67
σ_{MM}	-.99	-.59	-.85	-.42	-.80	-.36
σ_{KL}	-.19	-.24	-.19	-.42	-.25	-.19
σ_{KF}	.85	1.07	.82	1.08	.81	1.09
σ_{KM}	.81	.36	.76	.22	.76	.22
σ_{LF}	.08	.09	.11	.11	-.03	-.03
σ_{LM}	-.16	-.41	-.20	-.46	-.26	-.54
σ_{FM}	-.96	-.99	-.96	-.99	-1.18	-1.22

Table 5

Estimated Own and Cross Price Elasticities

of Input Demand for Selected Mean Fiscal Years

	1972-3		1975-6		1978-9	
Parameter	VM	RP	VM	RP	VM	RP
ε_{KK}	-.16	-.15	-.06	-.03	-.04	-.03
ε_{LL}	-.02	-.003	-.02	-.002	-.04	-.01
ε_{FF}	-.05	-.07	-.07	-.10	-.10	-.12
ε_{MM}	-.18	-.07	-.09	-.04	-.10	-.04
ε_{KL}	-.06	-.03	-.02	-.04	-.02	-.02
ε_{LK}	-.006	-.003	-.01	-.03	-.02	-.009
ε_{KF}	.04	.05	.04	.05	.04	.05
ε_{FK}	.07	.09	.06	.08	.06	.08
ε_{MK}	.07	.03	.05	.02	.06	.01
ε_{KM}	.09	.04	.09	.03	.09	.03
ε_{LF}	.003	.004	.005	.005	-.001	-.001
ε_{FL}	.06	.06	.08	.08	-.02	-.02
ε_{LM}	-.02	-.05	-.02	-.05	-.03	-.06
ε_{ML}	-.11	-.29	-.14	-.32	-.17	-.36
ε_{FM}	-.11	-.12	-.11	-.11	-.13	-.14
ε_{MF}	-.04	-.04	-.04	-.04	-.05	-.06

elasticities of demand (ε_{ij}) for the four input factors. Results are presented for the selected mean fiscal years 1972-73, 1975-76, and 1978-79.

The elasticity estimates suggest several important conclusions, some of which stand in direct contrast to those of other studies. First, all the own price elasticities have the correct negative sign, indicating that the demand for each factor is responsive to change in its own price. However, the estimated elasticities are rather small, paticularly for labor demand.[8] Differences between the vehicle mile and revenue passenger equation system estimates should also be noted. While in no case are there conflicting results (e.g., no two factors are substitutes with RP but complements with VM), there are some sizeable numerical differences. In particular, the labor-maintenance and capital-maintenance cross partial elasticities, as well as the labor own price elasticity show noteworthy differences. Clearly, the output measure used does affect the estimated results.

The partial elasticity parameters presented in Table 4 indicate that labor and capital inputs are slightly complementary, while labor and maintenance and fuel and maintenance inputs display stronger complementarity. On the other hand, capital and fuel, and capital and maintenance, are substitutable. These results do not support those of

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[8]Viton (1981) is the only study which reports the own price elasticity of labor demand. The vehicle miles estimate here roughly corresponds to the Viton estimate for large bus systems of -0.039.

Williams and Hall (1981), who concluded from their analysis that all inputs (capital, labor and fuel) are substitutes for each other.

The technical substitution which exists between capital and fuel and capital and maintenance, and the complementarity which exists between fuel and maintenance can be explained as follows. Capital, being relatively cheap owing to the capital subsidy, is used to offset maintenance costs; that is, new capital requires less maintenance, and the capital stock is turned over more rapidly because of its reduced price. This explanation conforms to previous research on the impact of capital subsidies on the transit industry (Charles River Associates, 1977). Moreover, the newer buses are less fuel efficient because of air conditioning and the additional weight of extra equipment such as handicapped lifts. In fact, the average fuel efficiency for ACTD declined from 4.93 mpg in 1972-73 to 4.76 mpg in 1978-79. Thus maintenance and fuel tend to be complementary. The reason for complementarity between labor and maintenance is less clear. Williams and Dalal (1980) who report similar findings argue that bus labor (due to price increases) entails less use of rolling stock which in turn requires less maintenance.

The weak complementarity between labor and capital observed here contradicts other studies (e.g., Williams and Hall, 1981; Williams and Dalal, 1980). However, this complementarity seems reasonable in the context of the current one bus, one driver technology which characterizes most bus service (Berechman, 1982). The opportunities for labor-capital substitutions are quite limited; bus size can be adjusted, and routes might possibly be scheduled so as to minimize the driver's unproductive time. By and large, however, the one bus, one driver fixed proportions technology is the dominating factor. In addition, it should be recalled that the labor input factor used here also includes the "fixed" clerical and administrative labor input. It is therefore possible that complementarity observed here reflects the relatively fixed nature of both capital and labor.

The Structure of the Cost Function

The general structure of the cost function as given by (1) assumes non-homotheticity, non-unitary elasticity of factor substitution and non-separability of factor prices. Technically, these assumptions are introduced by having $\rho_{iY} \neq 0$ for some i (non-homoteticity), and $\gamma_{ij} \neq 0$ for some i,j (non-unitary elasticity of substitution and non-separability). Since the estimation procedure yields maximum likelihood estimates, the value of the likelihood function at convergence can be used to test for the validity of these assumptions. Specifically, some of the parameters are restricted to equal zero, and the ratio, R, of the maximum value of the restricted likelihood function to that of the unrestricted function is computed. The resulting test statistic (-2LnR) is asymptotically distributed as $\chi^2(n)$, where n equals the number of restrictions being imposed.

Three such tests were carried out, and the results are reported in Table 6. First, homotheticity was tested by setting all ρ_{iY} $(\rho_{KY}, \rho_{LY}, \rho_{FY})$, to zero and computing the likelihood ratio test statistics

Table 6

Test Results of Homotheticity, Separability and Cobb-Douglas Production Technologies

Test	Test Statistic* VM	RP	$\chi^2(n)$ at 0.01 level
1. Homotheticity[a]	21.7	18.26	n=3 $\chi^2(3)=11.3$
2. Linear Separability (M,F),(K,L)[b]	1.42	2.12	n=4 $\chi^2(7)=18.5$
3. Cobb Douglas[c]	74.1	72.5	n=10 $\chi^2(10)=23.2$

a) $\rho_{KY} = \rho_{LY} = \rho_{FY} = 0$

b) $\gamma_{MK} = \gamma_{ML} = \gamma_{FK} = \gamma_{FL} = 0$

c) $\gamma_{ij} = 0$ for all ij; $\delta_{YY} = 0$; $\rho_{iY} = 0$ for all i.

for VM and RP. The critical value of $\chi^2(3)$ at the 0.01 level of significance is less than the two test statistics, implying a rejection of the null hypothesis in favor of the alternative of a non-homothetic production structure.

Second, the results on factor substitution give rise to the hypothesis that maintenance and fuel inputs are separable from capital and labor inputs. A necessary and sufficient condition for linear separability is that $S_M\gamma_{FL} - S_F\gamma_{ML} = 0$, and $S_M\gamma_{FK} - S_F\gamma_{MK} = 0$. Since $S_M, S_F > 0$, the condition is met if $\gamma_{FL} = \gamma_{ML} = \gamma_{FK} = \gamma_{MK} = 0$. The results of the test indicate that the null hypothesis of linear separability cannot be rejected at 0.01 level.[9]

Finally, the hypothesis that the technology which underlies the cost function is of the Cobb-Douglas type was tested by setting $\delta_{YY} = 0$, $\gamma_{ij} = 0$ and $\rho_{iy} = 0$ for all i. Again, the null hypothesis is rejected in favor of the alternative of non Cobb-Douglas type technology. The results of these three tests thus imply a non-homothetic cost function with linear separability between K,L and M,F. The results of previous studies which also estimated a translog cost function indicated a homothetic function (Williams and Dalal, 1980; Williams and Hall, 1981; Berechman, 1982).

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[9]Notice that the elasticity of substitution between labor and maintenance, σ_{LM}, is negative, thus implying complementarity. Thus, the above result may perhaps be explained by the fact that the marginal rate of substitution between M and F is not independent of L. See Berndt and Christensen (1973) on the separability conditions.

This result is a particularly important because a homothetic cost function implies that given a proportional increase in inputs, scale economies are a function only of the level of output. Because of the political constraints it faces as a public enterprise, the bus firm has much less control over the level of output than factor utilization. This characteristic of the cost function may thus have significant policy consequences. For example, a future decline in the demand for factors may result when capital and operational subsidies are reduced.

Economies of Scale

Until recently, the consensus among transportation analysts was that constant returns exist for all but the largest bus firms which are characterized by decreasing returns (McGillivary et al., 1980).[10] Recent studies utilizing similar cost models have found both increasing and decreasing returns (e.g., Williams and Dalal, 1980; Viton, 1981; Berechman, 1982).

Economies of scale can be measured by computing the elasticity of total cost with respect to output and subtracting it from unity. Cost elasticities, marginal cost, average cost, and the scale economies measures are presented for mean fiscal years 1972-73 through 1978-79 in Table 7. It was hypothesized above that scale economies depend on the output measure. Cost elasticities are greater than unity for VM and less than unity for RP. Correspondingly, diseconomies of scale are observed

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[10]Largest firms are those with 2000 or more vehicles.

Table 7

Cost Elasticities,[a] Marginal Costs,[b] Average Cost and

Scale Economies for Mean Fiscal years 1972/3-1978/9

	Cost Elasticities[a]		Marginal Cost[b]		Average Cost		Scale Economies[c]	
Year	VM	RP	VM	RP	VM	RP	VM	RP
1972/3	1.251	0.742	1.289	0.400	1.031	0.539	-.25	.26
1973/4	1.342	0.804	1.478	0.438	1.102	0.546	-.34	.20
1974/5	1.425	0.823	2.022	0.628	1.420	0.763	-.42	.17
1975/6	1.471	0.847	2.278	0.661	1.549	0.781	-.47	.15
1976/7	1.513	0.846	2.615	0.721	1.729	0.853	-.51	.15
1977/8	1.525	0.840	2.888	0.797	1.894	0.949	-.52	.16
1978/9	1.521	0.831	3.19	0.889	2.102	1.070	-.52	.17
Average	1.436	0.819					-.43	.18

Note: Average and marginal costs are in nominal dollars.

a) $\partial LnC/\partial LnY = \alpha_Y + \delta_{YY} LnQ + \sum_i \rho_{iY} Lnp_i$

b) $MC = \partial C/\partial Y$

c) $1 - [\partial LnC/\partial LnY]$.

for VM, and economies of scale are observed for RP. As expected, these results illustrate the distinction between economies of scale with respect to changes in capacity and with respect to changes in capacity utilization discussed in Section 3. In this case, changes in the scale of operation are associated with increasing unit costs, while changes in utilization are associated with decreasing unit costs. Thus part of the controversy regarding economies of scale in bus transit can be attributed to the selection of the output measure.

The results in Table 7 indicate that in terms of vehicle miles and with respect to its own cost function, the bus firm is operating beyond the optimal level of output. Moreover, since the mean output level increased each year, these results imply that the firm has moved to increasingly inefficient output levels over the the period of study. On the other hand, it might be argued that diseconomies with respect to vehicle miles are irrelevant; since passengers pay for the service, economies of scale with respect to passengers is the important factor. Under economies of scale, the output level should be maintained by providing subsidies. However, given the prevailing fares, the subsidy required to support these increases in output has grown drastically: from $12.3 million in 1972-73 to $44.1 million in 1978-79.

6. CONCLUSIONS

This research has shown that the nature of the data sample and the output measure selected have a significant effect on the results of an econometric analysis of bus transit. By utilizing a time-series data set in order to minimize environmental effects, the structure of the resulting cost function was found to differ in important ways from the structure implied by previous cross-section studies. The use of two output measures in this study showed that results are affected by the type of output measure used. Factor substitution and price elasticities differed numerically for VM and RP, but were not contradictory. Similarly, the structure of the cost function was found to be the same for both output measures. However, scale economies were found to be in striking contrast: diseconomies of scale with respect to vehicle miles, and economies of scale with respect to revenue passengers.

The characteristics of the bus transit supply process described in this research yield some interesting observations on the behavior of the transit firm. First, the rigidity in factor proportions (the small factor and price elasticities) implies that the firm is inflexible in adjusting to changing factor prices and output levels. In addition to the constraints imposed by the technology itself (one man, one bus), transit union contract work rules further limit factor utilization options.

Second, the findings on economies of scale merit discussion. An obvious question that these results generate is which scale measure is relevant for policy decisions? At this time, the answer is unclear. On the one hand, it can be argued that transit firms have not had to pay much attention to passengers. Subsidies have been allocated on the basis of service area populations (a proxy for service provided), and until 1980 have been sufficient to cover the revenue shortfall, indicating that both transit subsidizers and providers have not been much concerned with transit ridership. Under these circumstances, the vehicle-miles measure is more appropriate. On the other hand, there is no economic justification for providing vehicle-miles; the purpose of transit service is to generate trips. Thus even though service decisions may have not been based on passenger demand, from an economic point of view they should be, and revenue passenger measures are more correct. Indeed, the reduction in the availability of subsidies seems to indicate that public policy towards transit service is changing, that passenger demand is becoming a much more important factor.

As subsidies are reduced, the present levels of output and fares cannot be maintained, and transit firms will be forced to make adjustments. Based on the results presented here, and assuming cost minimization behavior, the transit firm should reduce vehicle miles but maintain or increase passengers. For an urban transit firm, this might be accomplished by reducing suburban service, which generates many miles and (relatively) few passengers, and increasing central city service, which generates (relatively) few miles but many passengers.

The findings on economies of scale also indicate that even if the existing level of output in vehicle-miles were optimal in terms of passengers, then perhaps the output should be supplied by more than one firm in order to reduce unit costs. Since transit firms are usually spatial monopolies, an increase in the number of transit suppliers in an area would require radical institutional change. However, the current interest in transit service contracting is aimed at achieving just such a change.

In conclusion, some suggestions for further research are in order. First, the issue of transit output remains to be resolved. It would be desirable to use more than one output measure in the cost function, either a technical and a demand related measure, or a set of more differentiated measures which reflect the various types of services provided (e.g., express, local, and demand-responsive). Unfortunately, small sample size precluded this approach here.

Second, this research has presented the results of an econometric analysis of the cost structure of a transit firm. However, this analysis pertains to only one side of the transit service "picture." In order to make this picture complete, the demand for transit should also be examined, so that the transit firm's response to different subsidization schemes can be examined, and optimal output levels under these schemes be determined.

Finally, a caveat is in order. This research is based on the behavior of only one firm, AC Transit. While it is typical of a great number of transit firms throughout the US, it may not be typical of the entire industry. Consequently, additional analyses of other types of transit firms are necessary in order to gain a greater understanding of the economic structure of the bus transit industry.

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