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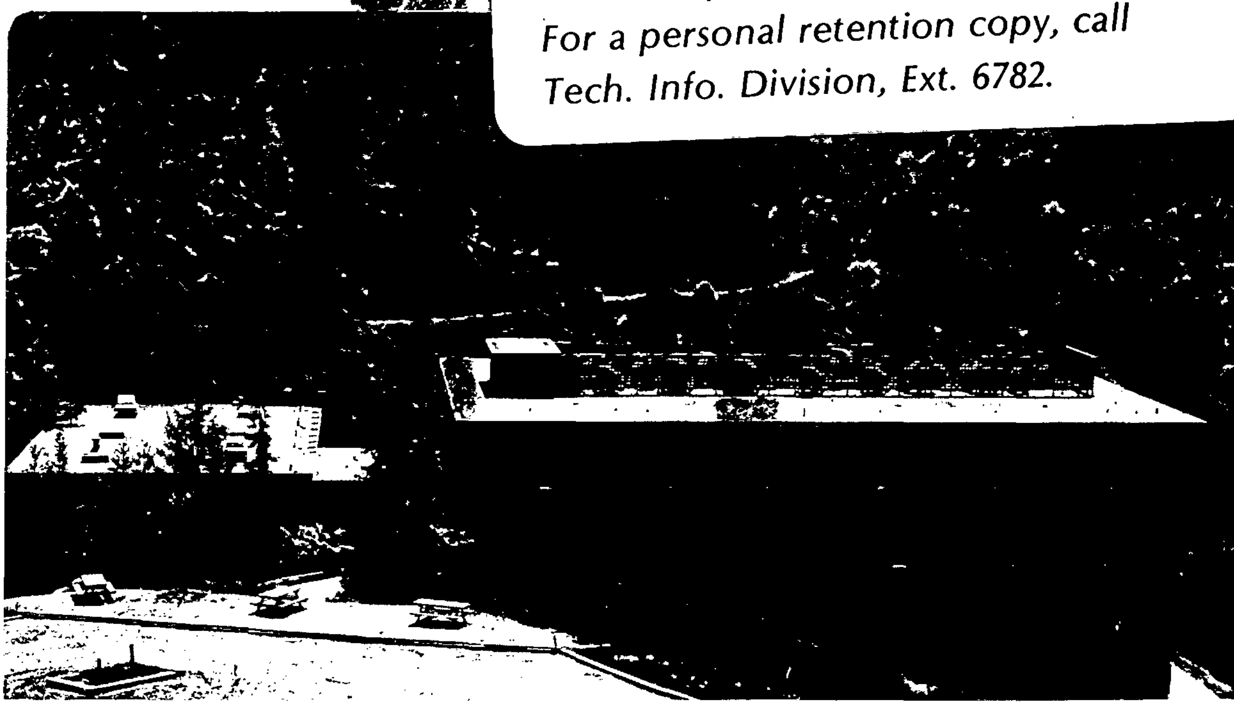

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We report the direct observation of a tangent bifurcation at the period-five window of a driven nonlinear semiconductor oscillator, by observing the fifth iterate of the dynamical variable of the system in real time. The tangent bifurcation is accompanied by an intermittency as predicted by Manneville and Pomeau.

For nonlinear systems described by an iterated map of the form $x_{n+1} = f(\lambda,x_n)$ of the unit interval onto itself, a tangent bifurcation [1] occurs at a value of the control parameter $\lambda = \lambda_c$ such that a higher iterate of the map $x_{n+m} = f^m(x_n)$ just becomes tangent to the line $x_{n+m} = x_n$. This is illustrated in fig. 1 where we plot the fifth iterate $f^5(x)$ versus x for the logistic map $x_{n+1} = \lambda x_n(1-x_n)$ at $\lambda = \lambda_c = 3.73775$. For $\lambda \gtrsim \lambda_c$, $f^5(x)$ intersects the $x_{n+5} = x_n$ line at ten fixed points defined by $f^5(x_i) = x_i$, of which only five are stable, with slope less than +1. These give rise to the period-five window; in general, all periodic windows in the chaotic regime are the result of a tangent bifurcation. As λ is increased, the five stable points become unstable as the slope becomes less than -1, and ten new stable fixed points arise for the tenth iterate, $f^{10}(x)$. This is the period doubling pitchfork bifurcation route to chaos [1,2].

Manneville and Pomeau have pointed out [3] that the region $\lambda \lesssim \lambda_c$ is also quite interesting and leads to an intermittency route to chaos. This route is characterized by periodic phases separated by aperiodic bursts. The geometric picture proposed by Manneville and Pomeau to explain intermittency is as follows. Consider, once again, the logistic map. For $\lambda \lesssim \lambda_c$ the five points which were tangent at $\lambda = \lambda_c$ are now slightly above or below the 45° line. Figure 2 shows an expansion near one of the tangent points. There is a channel formed by the 45° line and one side of $f^5(x)$ through which a trajectory must traverse from A to B. The dots and arrows represent the sequence of five-fold iterations through the channel corresponding to a quasi-periodic region, or laminar phase, whose length is proportional to the number of iterations, i.e.

the time of passage. When the iterates exit at B, they move erratically producing an aperiodic burst, until they reenter one of the five channels at A. Although intermittency has been experimentally observed in thermal convection experiments [4,5], chemical reactions [6], and other systems, no experiment has linked intermittency conclusively with the phenomena of tangent bifurcation.

In this paper we report direct observation of a tangent bifurcation and intermittency in a driven nonlinear semiconductor oscillator which has been previously shown to display the universal character of the period doubling route to chaos [7]. The system is a series connected inductor, resistor, and nonlinear varactor diode driven at resonance by a voltage $V(t) = V_0 \sin(2\pi t/T)$, where $T = 12.5$ μsec. We assume the following correspondences between the logistic map and experimental quantities: $\lambda \leftrightarrow V_0$, $x_n \leftrightarrow I(t)$, and $x_{n+m} \leftrightarrow I(t + mT)$, where $I(t)$ is a peak value of the current in the circuit.

We chose the period-five window because it is noise free and has no hysteresis. The fifth iterate of the current is determined in the following way. A sample-and-hold circuit samples $I(t)$ at a peak and holds that value until $t \gtrsim 5T$, repeating the procedure every 10T. The resulting waveform (fig. 3a) is fed to the horizontal input of an oscilloscope. The current waveform (fig. 3b) is fed to the vertical input. The oscilloscope is intensity-strobed at points corresponding to 5T after the initiation of the sample-and-hold. Both the hold pulse and intensity-strobe pulse are derived from the driving voltage. The result is a real-time display on the oscilloscope of $I(t + 5T)$ versus $I(t)$.

Figures 4a-d show a series of oscilloscope photographs taken at four different values of V_o near the value V_{oc} for the period-five window. The line $I(t + 5T) = I(t)$ is hand drawn. In fig. 4a the system is fully chaotic before the period-five window, $V_o < V_{oc}$. The full map is visited fairly uniformly by the iterates. Although the photo trace appears to touch at tangent points the $I(t + 5T) = I(t)$ line, this is only because of the finite line width of the trace. Careful measurement with a window comparator shows that these points lie off the $I(t + 5T) = I(t)$ line by an amount too small to be resolved in the photograph. The discontinuities and double-valuedness are attributed to higher dimensional effects. Otherwise, the photograph has a good correspondence with the fifth iterate of the logistic map, fig. 1. In fig. 4b the system is intermittent, $V_o \gtrsim V_{oc}$. The channels are visited slowly which makes them brighter on the oscilloscope than the rest of the map, which is too faint to show up in the photograph. This photo offers direct evidence for Manneville and Pomeau's geometric interpretation of intermittency, fig. 2. Figures 5a-c show the intermittency as it appears in the current signal. Figure 5a is an oscilloscope photograph of $I(t)$ at the period-five window taken at 15 milliseconds per sweep. The relatively slow sweep makes the signal appear smooth or laminar with dark horizontal lines corresponding to the peaks and valleys of the current (fig. 3b). Figures 5b and c show the laminar phases separated by aperiodic bursts during intermittency. In fig. 4c the system is periodic near the start of the period-five window $V_o = V_{oc}$. Only one stable fixed point of the fifth iterate is visited whenever the system is periodic. We photographed all five by reducing V_o slightly to take the system in and out of chaos and taking a multiple exposure. In fig. 4d the system is chaotic after the period-five window has bifurcated into chaos, $V_o > V_{oc}$. The map has grown across the $I(t + 5T) = I(t)$ line as expected.

To summarize, we have presented direct experimental evidence for a tangent bifurcation in a real physical system and have shown that it gives rise to intermittency according to the geometrical picture of Manneville and Pomeau. We have also studied the intermittency in more quantitative detail by other methods [8].

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Figure Captions

Fig. 1. The fifth iterate of the logistic map at $\lambda = \lambda_c = 3.73775$ showing the five fixed points x_i at the period-five window.

Fig. 2. Expanded view of fig. 1 near one of the fixed points for $\lambda \gtrsim \lambda_c$, showing a bottleneck through which successive iterates pass from point A to point B.

Fig. 3. (a) Oscilloscope photo of waveform of the sample-and-hold circuit for $V_o = V_{oc}$ for the period-five window. The value of I(t) at point 1 is held for slightly longer than 5T = 62.5 μsec, where T is the period of the driving voltage. The oscilloscope is intensity-strobed at point 2, etc. (b) The waveform of current I(t) at the period-five window. The periodicity is 5T.

Fig. 4. Oscilloscope photo of I(t + 5T) versus I(t) for the current in driven nonlinear oscillator, showing directly the fifth iterate map and a tangent bifurcation; the diagonal line is hand drawn. (a) For driving voltage $V_o < V_{oc}$, in the fully developed chaotic regime. (b) For $V_o \lesssim V_{oc}$, showing intermittency. (c) For $V_o = V_{oc}$, showing the period-five window. (d) For $V_o > V_{oc}$, in the chaotic region following the period-five window.

Fig. 5. Oscilloscope photo of I(t) versus time at 15 milliseconds per sweep. (a) For the driving voltage $V_o = V_{oc}$ at the period-five window. (b) and (c) For $V_o \gtrsim V_{oc}$, showing intermittency.

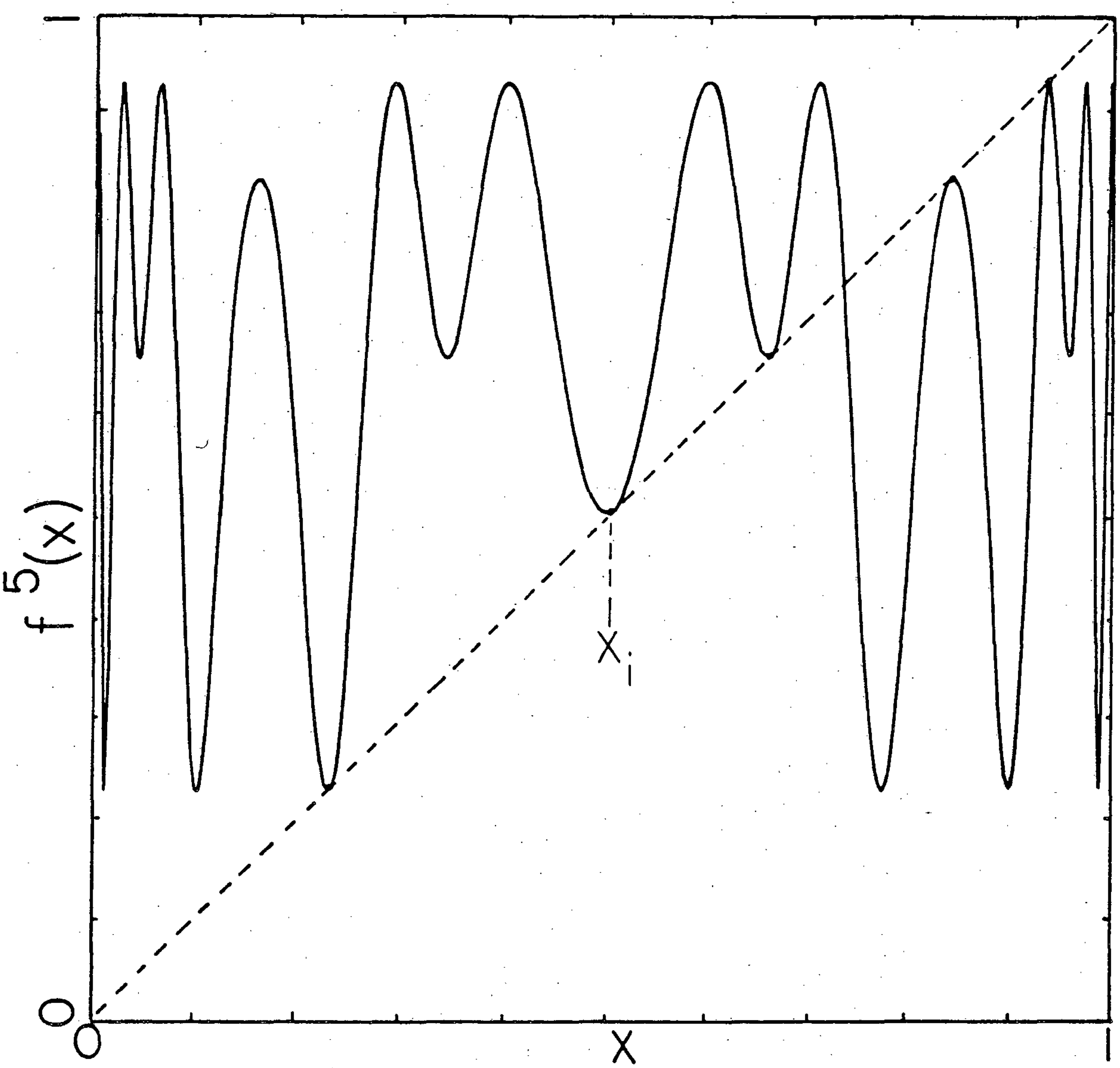

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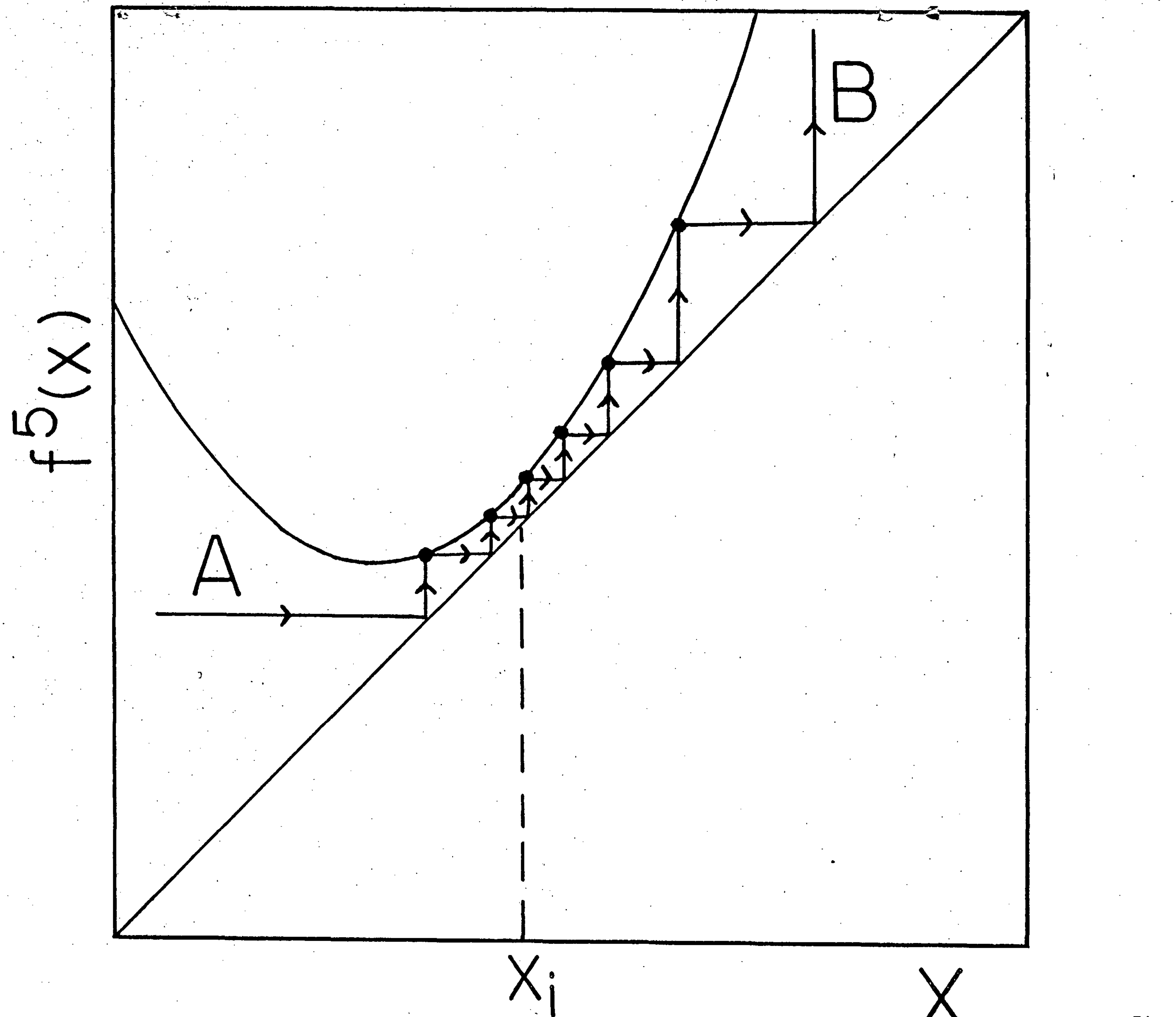
Fig. 1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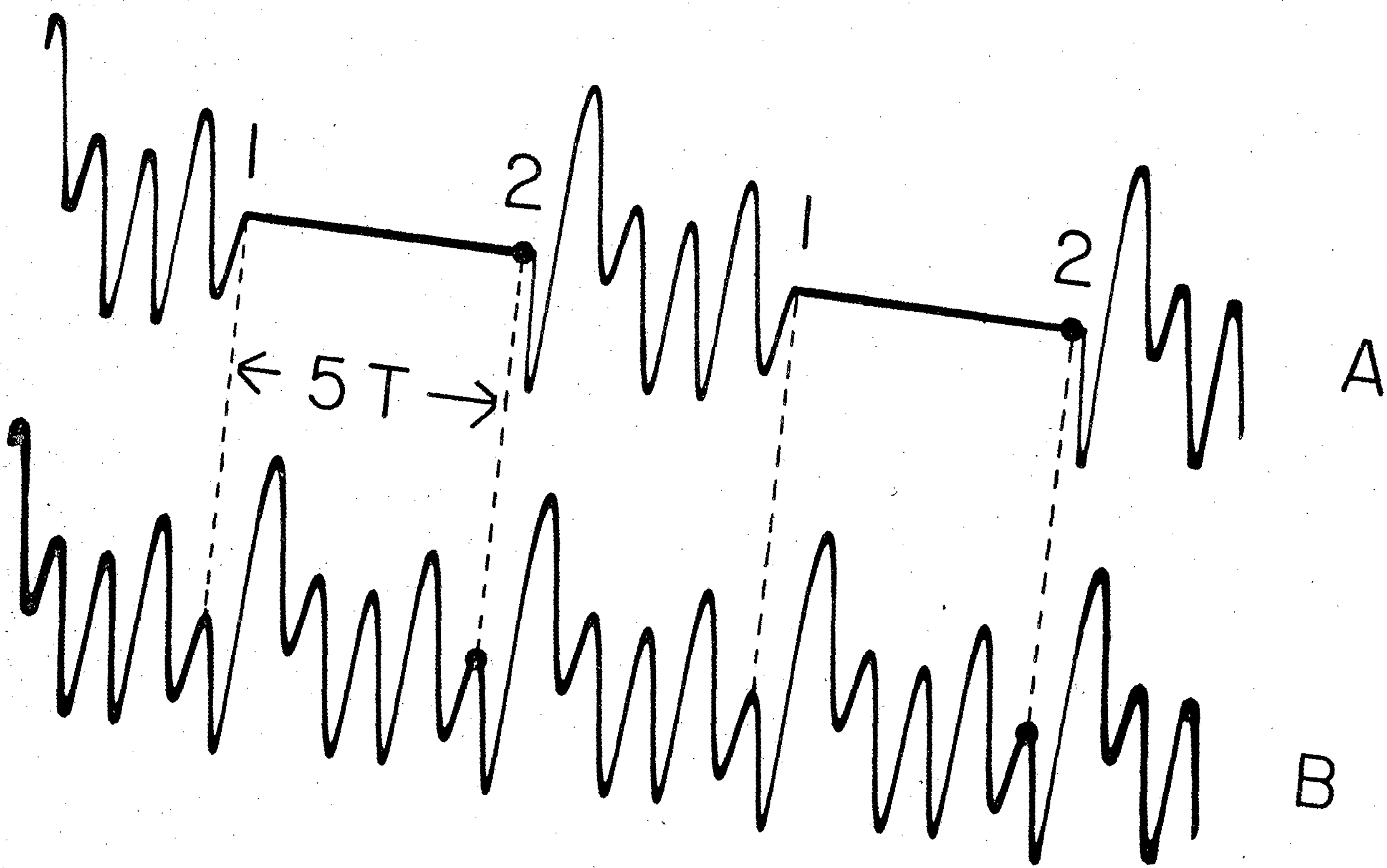

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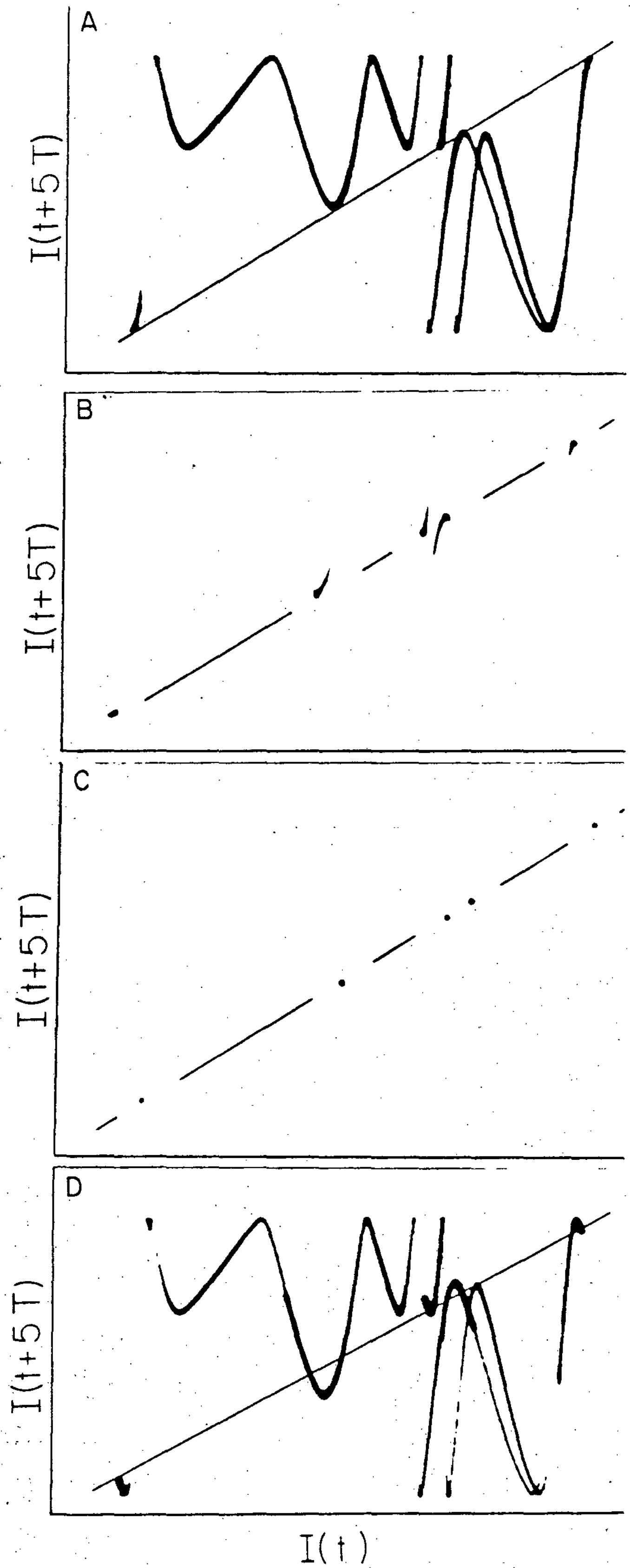

Fig. 4

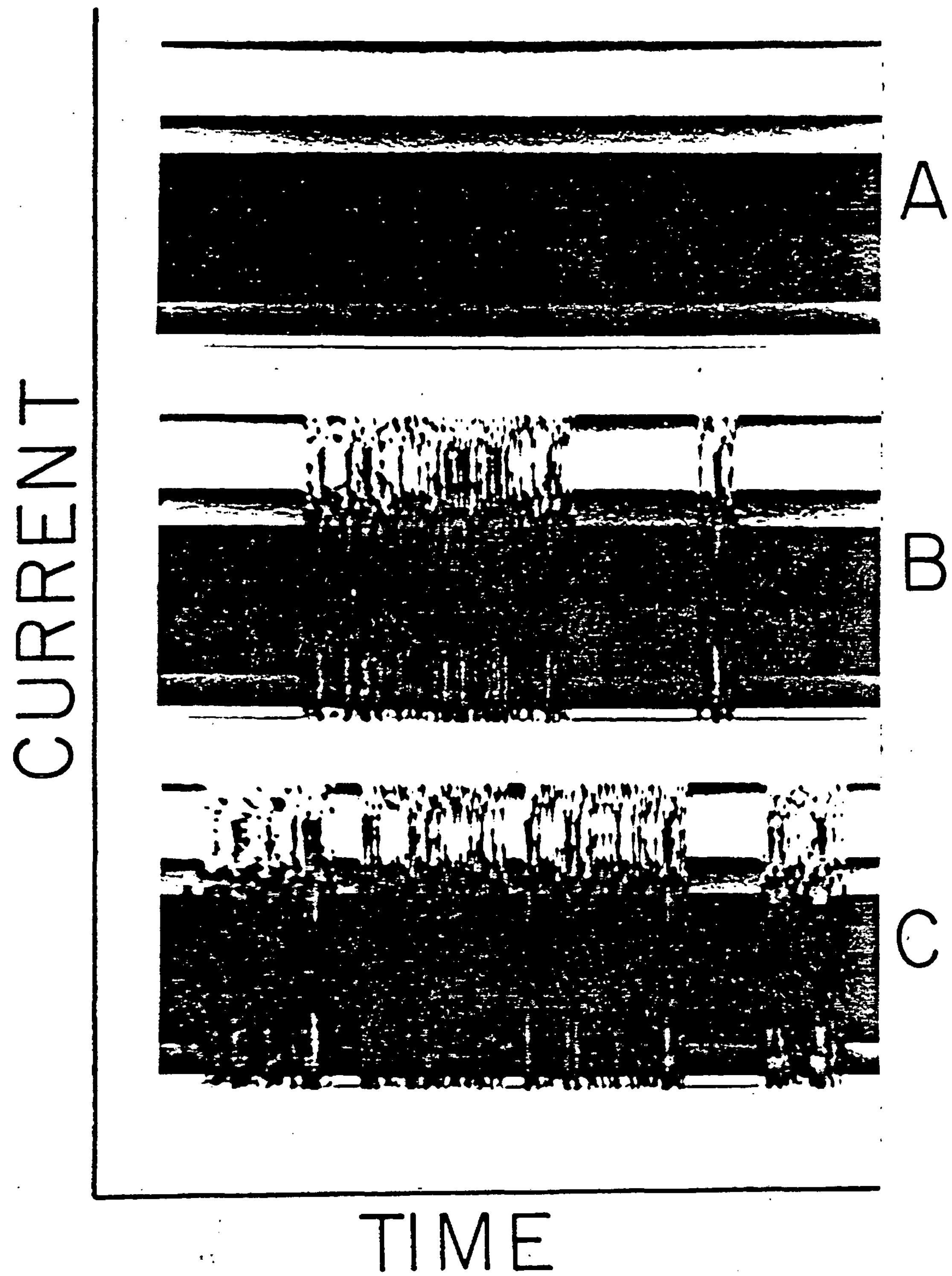

Fig. 5

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