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POTENTIAL SCATTERING AS OPPOSED TO SCATTERING ASSOCIATED WITH INDEPENDENT PARTICLES IN THE S-MATRIX THEORY OF STRONG INTERACTIONS.

~~THE ROLE OF THE POTENTIAL AND UNSTABLE IN THE S-MATRIX THEORY OF STRONG INTERACTIONS~~

Geoffrey F. Chew and Steven C. Frautschi

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THE ROLE OF THE POTENTIAL AND UNSTABLE ELEMENTARY PARTICLES IN THE S-MATRIX THEORY OF STRONG INTERACTIONS*

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We should like to propose in terms of the analytic continuation of the S matrix a relativistic definition of a generalized two-body "potential" that appears to have three useful properties: (A) Its role in the dynamics is analogous to that of an ordinary potential, and in the nonrelativistic limit its relation to the potential defined by Charap and Fubini[1] may be established. (B) Its long-range and medium-range parts may be evaluated for arbitrary energies in terms of one- and two-body S-matrix elements. (C) It allows a precise and physically reasonable characterization of resonances associated with "unstable elementary particles" as opposed to "dynamical" resonances.

Our definition is made within the Mandelstam framework, which describes the scattering amplitude for three different two-body reactions by a single analytic function.[2] Suppose the two particles whose mutual interaction is of interest are called a and b. Then we label the three Mandelstam channels as follows:

I.	$a + b \to a + b$	(barycentric energy squared = s),
II.	$a + \bar{a} \to b + \bar{b}$	(" " " = t),
III.	$a + \bar{b} \to a + \bar{b}$	(" " " = u).

Roughly speaking, channel II provides the "direct" forces for channel I, while channel III provides "exchange" forces. More precisely, we define

the generalized direct potential $V_I^{II}(t, s)$ as the channel II absorptive part, A_2, minus the contribution from $\rho_{st}^{I(el)}$, the elastic double-spectral function for channel I. That is,

$$V_I^{II}(t,s) = A_2(t,s) - \frac{1}{\pi} \iint ds' \frac{\rho_{st}^{I(el)}(s',t)}{s' - s} . \qquad (1)$$

The generalized exchange potential has a corresponding definition in terms of channel III and $\rho_{su}^{I(el)}$.

The elastic double-spectral function for channel I is given by the Cutkosky graphs of Fig. 1.[3] If we diagonalize the S matrix in channel I with respect to all internal quantum numbers (isotopic spin, strangeness, etc.), then the Cutkosky recipe[3] gives us (if spin complications are ignored)

$$\rho_{st}^{I(el)}(s,t) = \frac{1}{\pi} \frac{1}{q_s \sqrt{s}} \iint dt' dt'' \frac{A_2^*(t',s) A_2(t'',s)}{K^{1/2}(q_s^2; t, t', t'')}$$

$$+ \frac{1}{\pi} \frac{1}{q_s \sqrt{s}} \iint du' du'' \frac{A_3^*(u', s) A_3(u'',s)}{K^{1/2}(q_s^2; t, u', u'')} , \qquad (2)$$

where $K(q^2, t, t', t'') = t^2 + t'^2 + t''^2 - 2(tt' + tt'' + t't'') - \frac{tt't''}{q^2}$,

with a corresponding formula for $\rho_{su}^{I(el)}(s,u)$ in which t is replaced by u and the bilinear combinations $A_2^* A_3$ and $A_3^* A_2$ appear. The range of integration in (2) is restricted to the region in which K is positive. These formulas were first derived by Mandelstam from the elastic unitarity condition.[2]

Now, by definition,

$$A_2(t,s) = V_I^{II}(t,s) + \frac{1}{\pi}\int ds' \frac{\rho_{ct}^{I(es)}(s',t)}{s'-s} ,$$

$$A_3(u,s) = V_I^{III}(u,s) + \frac{1}{\pi}\int ds' \frac{\rho_{su}^{I(es)}(s',u)}{s'-s} , \qquad (3)$$

so if the generalized potentials V_I^{II} and V_I^{III} are given one may compute the elastic double-spectral functions by iteration of formula (2), as originally emphasized by Mandelstam.[2] All statements to this point have been formal and, correspondingly, exact. We now consider, in order, the three aspects of our generalized potential that were listed in the first paragraph.

(A) If one considers nonrelativistic scattering by a superposition of Yukawa direct and exchange potentials;[4]

$$V_{dir}(r) = -\int dt\ g_{dir}(t)\ \frac{e^{-\sqrt{t}\,r}}{r} ,$$

$$V_{ex}(r) = -\int du\ g_{ex}(u)\ \frac{e^{-\sqrt{u}\,r}}{r} , \qquad (4)$$

then, except for trivial questions of normalization, if $V_I^{II}(t,s)$ is replaced by $g_{dir}(t)$ and $V_I^{III}(u,s)$ by $g_{ex}(u)$, the equations determining the double-spectral functions differ from Eqs. (2) and (3) above only in the replacement of the factor $\sqrt{s}$ by $M_a + M_b$. Thus our generalized potentials determine the dynamics to the same extent and in much the same way as an ordinary potential. In fact it is easy to show that although the "potential" defined by Eq. (1) is in general

energy-dependent and complex, it becomes real for s below the inelastic threshold, and when $M_a^2, M_b^2 \gg m_\pi^2$, as in nucleon-nucleon scattering, the dependence on s in the elastic region is weak (especially for small t). It has been pointed out to us by J. Charap that in such a case, if one wishes to use the potential in a Schrödinger equation, some modification is required because even when q_s^2 is small there may be substantial contributions from large values of s' in the integral on the right-hand side of Eq. (3). Such contributions behave like an extra term in a nonrelativistic potential, and can be calculated in a simple way from a knowledge of the generalized potential.[5] Once this is done one obtains the nonrelativistic potential already defined by Charap and Fubini.[1] Of course if one works directly with the integral equations (2) and (3), as one must in π-π and π-N scattering, this modification is unnecessary.

(B) The Cutkosky recipe[3] tells us how to compute the generalized potentials in terms of analytic continuations of S-matrix elements. It is convenient to classify contributions according to "range," that is, in terms of the masses of the various intermediate states in channels II and III. The very-long-range one-particle contributions are trivial, as usual, and require no special comment. The medium-range two-particle contributions to V_I^{II} are associated with the graphs of Fig. 2, where the omission of the box diagrams should be noted. The latter are contained in $\rho_{st}^{I(el)}$ and must be eliminated according to Formula (1). Formulas for the graphs of Fig. 2 will be similar to formula (2) above, with s and t interchanged and a K function that depends on the masses of the exchanged particles. Thus, a sufficient knowledge of the absorptive parts for the appropriate two-body reactions will allow a calculation of the part of

the direct potential associated with two-particle transfer; the exchange potential can be handled in a similar way. Keeping only the one- and two-particle contributions to the generalized potential is equivalent to the "strip approximation" described in an earlier letter.[6]

We may comment here that the approximation of neglecting or treating phenomenologically the exchange of more than two particles does not depend for its validity on the energy. The approximation appears just as plausible at high as at low energies, so, as explained previously,[6] the "peripheral" approach is by no means restricted to the elastic region. Our "potential" automatically develops an imaginary part equal to $\rho_{st}^{I(in)}$ for s above the inelastic threshold.

(C) We finally discuss a matter of principle, assuming that somehow V_I^{II} and V_I^{III} are completely known. Our comments here are a synthesis of remarks made at various times by others but never, to our knowledge, collected in one place. It has been shown above that a knowledge of the generalized potential for channel I is equivalent to a complete knowledge of the double-spectral functions as well as a knowledge of the single-spectral functions for channels II and III. In order to define the amplitude completely, however, we further require, in addition to any over-all subtraction constants, the single-spectral function for channel I (i.e., the elastic absorptive parts of a finite number of low partial waves in channel I). Here there arises the famous CDD ambiguity,[7] which is now recognized as equivalent to the possibility of unstable elementary particles with the quantum numbers of channel I. One way to characterize the ambiguity is in terms of the N/D technique of partial-wave calculation introduced by Chew and Mandelstam.[8] Let us briefly review the essential features of this method.

One starts with a knowledge of the complete discontinuity across the unphysical cuts of the partial-wave amplitude and of the inelastic discontinuity on the physical cut.[9] All this information can be obtained once our generalized potentials are given. The denominator function may then be defined by

$$D_\ell(s) = \exp\left(-\frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \, \frac{\delta_\ell(s')}{(s' - s_0)(s' - s)} \right) , \qquad (5)$$

where δ_ℓ is the real part of the phase shift ($\delta_\ell(s_0) = 0$), and separate dispersion relations may be written down for numerator and denominator functions. These dispersion relations, once established, lead to a linear integral equation[10] whose solution, if it exists at all, is believed to be unique for the given discontinuities,[8] but there is an ambiguity with respect to the number of subtractions. Unitarity restricts the asymptotic behavior of the quotient N_ℓ/D_ℓ, but an arbitrary number n of subtractions in D_ℓ is possible, provided it is matched by a corresponding number in N_ℓ. It is possible to associate the $2n$ subtraction constants with the positions and residues of n poles on the unphysical sheet.[11] These we wish to call "unstable elementary particles." Poles on the unphysical sheet that occur even when no "extra" subtractions are made we wish to call "dynamical resonances."

For nonrelativistic scattering an unambiguous distinction can be made.[4] If one wants the N/D solution that corresponds to "pure" potential scattering one write the dispersion relation for D_ℓ with no arbitrary constants:[12]

$$D_\ell(s) = 1 - \frac{s - s_0}{\pi} \int_0^\infty ds' \, \frac{\operatorname{Im} D_\ell(s')}{(s' - s_0)(s' - s)} \, . \qquad (6)$$

We propose that this oft-discussed prescription be extended to the relativistic case as a <u>definition</u> of pure potential scattering. Such a solution (if it exists) is completely determined once the generalized potential is given, and corresponds to the condition $\delta_\ell \xrightarrow[s \to \infty]{} \delta_\ell(\infty)$, where $\delta_\ell(\infty) < \pi$, as may be seen from Eq. (5):[13]

$$\lim_{s \to \infty} D_\ell(s) = \exp\left[\frac{\delta_\ell(\infty)}{\pi} \ln s + \text{constant} \right] \propto s^{\delta_\ell(\infty)/\pi} \, . \qquad (7)$$

Evidently, making n arbitrary subtractions in D_ℓ corresponds to $n\pi \leq \delta_\ell(\infty) < (n+1)\pi$.

We are aware that when unitarity in channels II and III is included in the discussion (as well as crossing symmetry, if one or both of these channels corresponds to the same reaction as channel I) then the inclusion or exclusion of unstable elementary particles in channel I is not completely arbitrary. For example, Froissart has recently shown that over-all unitarity (in all three channels) uniquely determines all but the S and P waves once the complete double-spectral function is given.[14] (Even the P wave is determined if one accepts the Pomeranchuk high-energy relations.) Nevertheless it seems to us helpful to have a clean definition of pure potential scattering that can be applied in all situations. The prescription proposed here has often been privately discussed by workers in the field,[15] but never with relation to a precise definition to the "potential."

REFERENCES

* This work was performed under the auspices of the U.S. Atomic Energy Commission.

1. J. Charap and S. Fubini, Nuovo cimento 14, 540 (1959).

2. S. Mandelstam, Phys. Rev. 112, 1344 (1958).

3. R. E. Cutkosky, J. Math. Phys. 1, 429 (1960); Phys. Rev. Letters 4, 624 (1960).

4. R. Blankenbecler, M. L. Goldberger, N. N. Khuri, and S. B. Treiman, Annals of Physics 10, 62 (1960).

5. The correction may be calculated up to any finite value of t by a finite number of iterations of Eq. (3), taking the difference between relativistic and nonrelativistic values of the integral as the correction to the potential. In practice, in the N-N problem, if one wishes to calculate only the one- and two-pion parts of the potential ($t < 9\, m_\pi^2$) a single iteration suffices. We are extremely grateful to M. Froissart for assistance in clarifying this question.

6. G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 5, 580 (1960).

7. L. Castillejo, R. Dalitz, and F. Dyson, Phys. Rev. 101, 453 (1956).

8. G. F. Chew and S. Mandelstam, Phys. Rev. 119, 467 (1960).

9. For simplicity we assume no real poles in channel I, i.e., no bound states or stable elementary particles. Such poles, if present, give rise to no essential difficulty.

10. The possibility of generalizing the N/D method so as to achieve linear equations even when the inelastic discontinuity is included was pointed out to us in a private communication from James Ball and William Frazer (Physics Department, University of California--San Diego).

11. The $2n$ constants have some restrictions on their range of values, corresponding to the requirement that the poles not appear on the physical sheet.

12. This is the type of solution chosen in the work of Chew and Mandelstam, Ref. 8, on the π-π interaction. It also is the basis for the Chew-Low effective-range formula for the π-N (3/2, 3/2) resonance.

13. This asymptotic behavior was pointed out to us in a private conversation by M. Froissart (Physics Department, University of California, Berkeley).

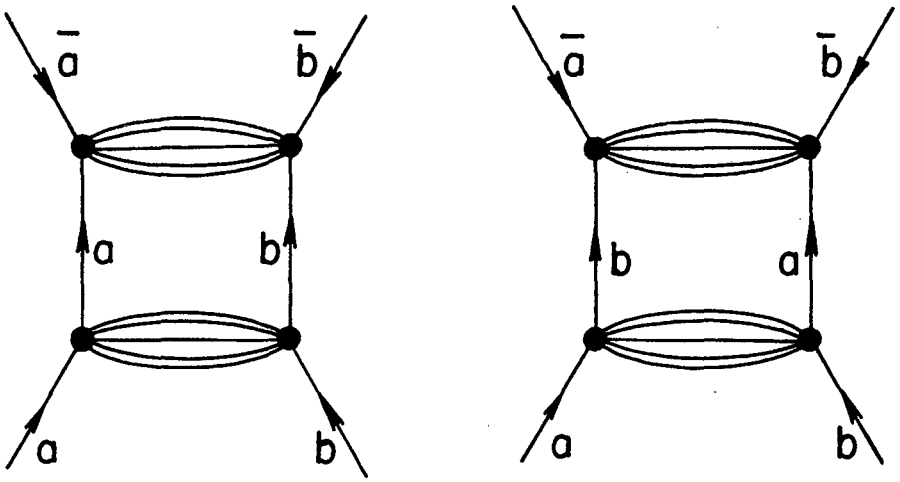
14. M. Froissart, "Asymptotic Behavior and Subtractions in the Mandelstam Representation," University of California Physics Department Preprint, Berkeley, California, 1960.

15. For example, by M. Gell-Mann and S. Mandelstam.

FIGURE CAPTIONS

Fig. 1. Cutkosky graphs for $\rho_{ct}^{I(e\ell)}$. To obtain $\rho_{su}^{I(e\ell)}$ the lines for $\bar{a}$ and $\bar{b}$ should be interchanged.

Fig. 2. Cutkosky graphs providing double-spectral functions for the two-body "direct potential." A subtraction term corresponding to $J = 0$ in channel II must also be included.



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