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Capacity Planning for Process Industries under Competition and Resource Scarcity

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Abstract

We consider a competitive version of the traditional capacity planning model of production with capacity constraints. In the general case, multiple products are produced with constrained resources and capacities. Production quantities are allocated across markets, and competitors also produce for these markets. Prices and profits depend on production and allocation decisions. We first consider a single-tier version of this problem without interactions with the raw material supply tier and utilize a Cournot framework. We next extend our model using a successive “Bertrand-Cournot” framework to include interactions with the raw material supplying sector whose supplies are limited, and where prices reflect these limitations. Such situations have recently occurred in several process industry settings including the petro-refining and metal processing sectors, such as steel and copper.

1. INTRODUCTION

Traditional capacity planning models are usually cast in terms of allocating scarce resources and capacity to competing products. They are most commonly formulated as linear programs that minimize costs for a single producer, with products, demand levels and prices given. However, in many process industries, changes in demand due to the emergence of new markets, competition, capacity constraints and interactions with the raw material supply tier often cause significant market effects across one or more product segments including changes in availability of products, price and profit. These situations occur frequently in sectors that produce high volume, commodity products for which capacity is expensive and not easily expanded. Examples can be found across a variety of process industry sectors such as petro-

refining, petrochemicals, basic chemicals, cement, fertilizers, pharmaceuticals, rubber, paper, food processing and metals.

The petro-refining industry appears to be a particularly egregious example of jockeying by crude oil producers and refiners to control prices and profits with results that are painful to consumers. In the first quarter of 2004, gasoline prices were rising sharply in the United States. Certainly, the limited supply of oil, exacerbated by uncertainties surrounding international oil supplies due to the Iraq war, was a major reason for the rise in the price of crude oil. But, an article in *Business Week* (Coy, 2004) noted that capacity constraints at refineries were also a factor. The *Financial Times* (McNulty, 2004) suggested that the lack of capacity in U.S. refineries contributed significantly to these record prices. The article went on to say that there had been no refineries built in the United States for 28 years, that the number of refineries had actually halved since 1981 and that there was little incentive for producers to add capacity given the average rates of return on capacity investments.

Fuel prices in 2005 continued to rise, and the finger pointing also continued. The Organization of the Petrol Exporting Countries (OPEC) has historically been blamed for keeping oil scarce to raise prices. But, in 2005, OPEC offered to make two million more barrels of oil a day available if only it could be processed (Catan and Morrison, 2005). The capacity bottleneck in the refining tier was becoming obvious, to the point that President Bush at a White House press conference in October of 2005 called for more refining capacity (*CNN/Money*, 2005). The refiners in turn complained that their returns on investment in refining capacity were extremely low and risky due to the cyclical nature of the industry and the uncertainty over whether additional crude oil supplies would continue over a longer term to utilize this new refining capacity (Catan and Morrison, 2005).

Raw material shortages have become an issue not only for oil, but for many other commodities as well. The rapid growth of China created significant shortages in steel and copper at the start of 2004. For example, the price of hot-rolled steel coil rose more than 20% and that of steel wire and rod rose by 50% over the first two months of 2004. As a result, downstream producers of parts and components using steel

were also forced to raise prices. By mid 2005, copper prices reached record highs as demand outstripped supply, and stockpiles were used up (Morrison, 2005).

There are many situations in the process industry for which high capacity costs coupled with long planning horizons, raw material shortages and modest rates of return on investments leave producers with little incentive to add capacity or to carry excess capacity. In these situations, capacity constraints effectively lead to markets being “on allocation.” In some cases, such as the oil refining and steel industries, there are interactions between capacity constraints, shifts in demand curves and raw material availability (e.g., crude in oil-refining and scrap metal in steel).

Traditional models of capacity planning with constrained resources formulate the problem of allocation of scarce resources and capacity to competing products as a linear programming model of cost minimization with known demand levels (Sanderson, 1978). These models are applicable in a stable economic environment, with low rates of change. Consequently, factories can operate in a relatively stable manner, with capacity and mix changes being made infrequently. However, in the last decade, we have seen rapid changes in many process industry sectors usually brought about by economic growth in emerging economies, with the consequent increases in demand. In such situations, raw material availability and processing capacity can both be significant constraints. Here, the traditional capacity planning formulations are not able to capture the competitive nature of this situation, nor the interactions between production quantities, capacity constraints, raw material availability and price. Consequently, it is doubtful whether product mix and market allocation decisions made using a single-producer cost minimization model could correctly reflect the interactions between decisions of competitors and the response of markets and prices to production quantities. Indeed, we will show that concepts such as Lagrange multipliers, reflecting the marginal value of capacity, can be subject to misinterpretation to the point of being misleading.

In this paper, we formulate a competitive version of the traditional capacity planning model of production with capacity constraints. We first analyze the single-tier version of this problem without interactions with the raw material supply tier and utilize a Cournot framework. In this context, we were

able to extract closed form solutions that provide insight and structure to our subsequent analysis. In addition to production quantity decisions, we are also able to find Lagrange multipliers for capacity constraints in the competitive, profit maximization setting. The interpretation of these Lagrange multipliers is fundamentally different from those obtained from traditional planning models.

We next extend our model using a successive “Bertrand-Cournot” framework to the two-tier case to include interactions with the raw material supplying sector where supplies are limited, and where prices reflect these limitations. Constraints on production capacity can effectively reduce the market power of the raw material supplier. However, the intensity of competition in each sector is crucially important in determining market power. In this case, we use small examples to provide explicit solutions that provide insight; larger problems require numerical solution methods. We show that it is possible to have a situation in which capacity is fully utilized, but for which the multiplier on capacity is zero and, yet, the value of a marginal expansion of capacity can still be positive. Here, Lagrange multipliers can have a very different interpretation than the traditional one of marginal value for increases in a constrained resource. We develop a computational method to solve larger problems and test this method on an illustrative example. This analysis also provides insight into the impact of production efficiencies (i.e., supplier’s and producers’ unit production costs) and market parameters (i.e., market size and customer price sensitivity) on prices, production quantities and profits for both the supplier and the producers’ tiers.

This paper is organized as follows. In the next section, we review the relevant literature. In Section 3, we formulate the single-tier capacity planning problem under competition without interactions with the raw material supply tier. In Section 4, we extend our model formulation and analysis to consider a two-tier model that includes interactions with the raw material supply tier. We also examine the special case of a single product, single raw material with a monopoly supplier. In this context, we consider both homogenous and heterogeneous producers. In Section 5, we develop a computational method to solve general versions of this problem with multiple producers, products and raw materials. We test this method on an illustrative example. In the concluding section, we summarize the major results of the paper and suggest future research directions.

2. LITERATURE REVIEW

There is substantial literature on capacity planning or aggregate planning decisions that address production mix, capacity allocation, seasonal inventory planning and distribution planning with annual planning horizons. Early formulations of aggregate capacity planning decisions employed linear programming (Hansmann and Hess, 1960) and quadratic cost models (Holt, Modigliani et al., 1960) to represent the problem. Extensions of the linear programming model included its use in hierarchical control schemes, sometimes through column generation techniques (Lasdon and Terjung, 1971). Such models correspond to annual budgeting and sales planning cycles, and usually serve the purpose of inter-functional coordination. Surveys of aggregate planning models are presented by Hax (1978) and Nam and Logendran (1992). However, none of this work captures the relationship between production quantities and prices, and the interactions between capacity constraints, shifts in demand curves and the raw material supply tier.

Models of Cournot competition have a long history going back to the original paper by Cournot (1838). The literature on capacity constrained competition is, however, quite limited. Moreover, managerial issues are rarely addressed in this literature. Haskell and Martin (1994) examine the behavior of capacity constrained producers, and conclude that they exhibit Cournot-like behavior. Herk (1993) formulates a two-stage duopoly model with capacity choice followed by price competition and shows that capacity choices exhibit Cournot behavior. Of the papers that are similar to the setting of our work are Karmarkar and Pitbladdo (1993, 1994) who study a two-stage, single-tier formulation with entry at the first stage and multi-product Cournot competition with capacity constraints at the second stage. Our work differs from theirs in that we focus on the tactical (second stage) problem with fixed (given) capacity, permit heterogeneity of producers and consider multiple products, multiple capacity constraints and interactions with the raw material supply tier.

Previous work that is closest to our model is that of Zappe and Horowitz (1993) who examine a multi-product, multi-market model and study the effect of capacity on competitive response under quasi-

Cournot conjectures. In addition, they embed a capacity decision into the Cournot quantity decision. As a consequence, they limit their analysis to a problem with two identical producers, one product, two capacity levels and two markets, and restrict their investigation to the symmetric case. Further, they do not consider the two-tier case that includes the raw material supplier. In contrast, our emphasis is on developing methods to analyze planning decisions in a multi-tier setting consisting of producers and the raw material supplier. To this end, we focus primarily on the capacity allocation decision at the producers tier. We explicitly consider both the effect of short term capacity constraints on prices and production quantities under competition, as well as examining the impact of interactions with the raw material supply tier. As we show, the effect of the latter is quite significant and leads to results that are quite different from traditional planning models.

3. THE SINGLE-TIER MODEL

Consider m producers indexed by $i \in (1, 2, \dots, m)$ producing n commodity products indexed by $j \in (1, 2, \dots, n)$. Let variable q_{ij} represent the production quantity for product j at producer i and $Q_j = \sum_{i=1}^m q_{ij}$

denote the total amount of product j available in the market. For example, in the context of the petrochemical industry, these producers can be considered as the refiners of crude oil (e.g. Shell, Exxon Mobil, Chevron, etc.), while products could be the different grades of gasoline (e.g. Regular, Plus and Supreme). We assume that demand for product j is characterized by an affine inverse demand function $p_j = a_j - b_j Q_j$, where p_j is the price for product j , while a_j and b_j are parameters. Here, a_j/b_j can be considered as the market size and $1/b_j$ can be regarded as the customer price sensitivity. This can also be viewed as an affine approximation of the actual demand function and has been commonly used in the literature (Karmarkar and Pitbladdo, 1993; Corbett and Karmarkar, 2001). We are given:

v_{ij} : production cost per unit of product j at producer i (\$/unit),

c_{ij} : capacity required per unit of product j at producer i (capacity units/unit), and

d_i : total capacity available at producer i .

We assume quantity competition to model capacity planning decisions under competition, as price competition with more than one producer will lead to marginal cost pricing. We utilize a Cournot framework in which producers make decisions on production quantities by allocating capacity across products and by choosing how to allocate production across markets. Given a demand curve, this establishes product prices.

The Cournot framework is the standard mechanism used to study competitive interactions with a small number of producers (i.e., oligopoly). This framework assumes that one producer observes or forecasts the production of the other producer and is a “one shot game”, the profits of producer i is its payoff and the strategy space of a producer i is the possible production quantities that can be produced. At equilibrium, each producer produces a profit maximizing output, given its knowledge of the other producer’s production quantity.

The Cournot framework is particularly appropriate for our process industry setting for several reasons. First, due to high fixed costs of entry, there are usually only a small number of producers producing any particular type of commodity product. Second, environmental regulations typically require full public disclosure of production quantities and capacities (For instance, see Energy Information Administration, 2006). Finally, short term capacity is very expensive to change so that the interactions between producers can be regarded as a “one shot game”. In the Cournot framework, producer i solves the following problem:

$$(P1) \quad \Pi_i = \text{Max} \sum_{j=1}^n \{(p_j - v_{ij})q_{ij}\} \quad (1)$$

Subject to:

$$\sum_{j=1}^n c_{ij}q_{ij} \leq d_i, \quad (2)$$

$$q_{ij} \geq 0, \forall j. \quad (3)$$

Objective function (1) maximizes profits at producer i by choosing the appropriate production quantity, q_{ij} , given the production quantities at the other producers (i.e., q_{pj} , $\forall p \neq i$). Constraint (2) ensures capacity limits at producer i , while non-negativity constraints are enforced by (3).

Next, we substitute $p_j = (a_j - b_j \sum_{i=1}^m q_{ij})$ in (1). Note that (P1) is a concave optimization problem.

We then relax constraint (2) by introducing Lagrange multiplier λ_i to get the following dual problem for the i^{th} producer:

$$\Pi_i^D = \text{Max}_{q_{ij} \geq 0, \lambda_i \geq 0} \sum_{j=1}^n \{(a_j - b_j \sum_{i=1}^m q_{ij} - v_{ij} - \lambda_i c_{ij})q_{ij}\} + \lambda_i d_i \quad (4)$$

From (4), the sufficient first order conditions with respect to q_{ij} can be found by setting $\frac{\partial \Pi_i^D}{\partial q_{ij}} = 0$, so

that:

$$2q_{ij} + \sum_{t \neq i} q_{tj} = \frac{a_j - v_j - \lambda_i c_{ij}}{b_j}, \forall i, j. \quad (5)$$

Proposition 1: There exists a unique vector of equilibrium order quantities $q_{ij}^*, \forall i, j$.

Proof. Observe from (5) that $\sum_j \sum_i \sum_{t \neq i} \frac{\delta q_{ij}}{\delta q_{tj}} \leq 1$. By Friedman (1986), page 84, Theorem 3.4, there exists a

unique equilibrium. ■

In light of Proposition 1, we can find the equilibrium production quantities $q_{ij}^*, \forall i, j$, by solving the system of equations defined in (5) to get:

$$q_{ij}^* = \frac{\{a_j - m(v_{ij} + \lambda_i c_{ij}) + \sum_{t \neq i} (v_{tj} + \lambda_t c_{tj})\}}{(m+1)b_j} \quad \forall i, j. \quad (6)$$

The total quantity q_j^* of product j in the market is given by:

$$q_j^* = \sum_i q_{ij}^* = \frac{ma_j}{(m+1)b_j} + \frac{\sum_{i \neq t} \{ \sum (v_{tj} + \lambda_t c_{tj}) - m(v_{ij} + \lambda_i c_{ij}) \}}{(m+1)b_j}. \quad (7)$$

As expected, it can be observed from (6) and (7) that for a given j , q_{ij}^* and q_j^* increase when market size increases, and when production costs, capacity requirements and costs of capacity expansion increase for competing producers (i.e. $\forall t, t \neq i$). On the other hand, q_{ij}^* and q_j^* decrease when the number of competing producers and customer price sensitivity increase, and when capacity requirements, costs of capacity expansion and production costs increase at producer i . The equilibrium price p_j^* for product j is given by:

$$p_j^* = a_j - b_j \sum_i q_{ij}^* = \frac{a_j - \sum_{i \neq t} \{ \sum (v_{tj} + \lambda_t c_{tj}) - m(v_{ij} + \lambda_i c_{ij}) \}}{(m+1)}. \quad \forall j \quad (8)$$

It can be seen from (8) that the equilibrium price increases as market size increases, but decreases as the number of producers increases. To find the marginal value of capacity expansion under the equilibrium production quantities, we can use (5) to find λ_i , the Lagrange multiplier corresponding to producer i as:

$$\lambda_i = \frac{a_j - v_{ij} - b_j(2q_{ij}^* + \sum_{t \neq i} q_{tj}^*)}{c_{ij}}. \quad (9)$$

This shows that for a given producer, the Lagrange multiplier representing the marginal value of capacity expansion is increasing with market size for the product, but decreasing with production costs and production quantities at this producer. In addition, observe that the marginal value of capacity expansion at any producer is reduced by the total equilibrium production quantities produced by the competitors. This effect is not captured by the traditional, non-competitive Lagrange multiplier.

Next, we compare the marginal value of capacity expansion and the equilibrium production quantities and prices with the Non Competitive (NC) or monopolistic case. To perform this analysis, we set $i = (NC)$, $m = 1$ in (6) and (8) to get the production quantities and prices in this case as:

$$q_{(NC)j}^* = \frac{a_j - v_{(NC)j} - \lambda_{(NC)}c_{(NC)j}}{2b_j} \quad (10)$$

$$p_{(NC)j}^* = \frac{a_j + v_{(NC)j} + \lambda_{(NC)}c_{(NC)j}}{2} \quad (11)$$

Proposition 2. When the capacity constraint is not binding and when production costs are equal across all producers, the monopolist produces less of the product at a higher price.

Proof. Let $v_j = v_{ij} = v_{(NC)j}, \forall i$ represent the production cost for product j , which is invariant across m producers. When the capacity constraint is not binding, $\lambda_{ij} = \lambda_{(NC)} = 0, \forall i, j$. Under these assumptions

and using (7) and (10), we get $q_j^* = \frac{m}{(m+1)} \left(\frac{a_j - v_j}{b_j} \right) > q_{(NC)j}^*, \forall m \geq 2$. It then follows that

$$p_j^* = a_j - b_j q_j^* < a_j - b_j q_{(NC)j}^* = p_{(NC)j}^*, \forall m \geq 2. \blacksquare$$

4. THE TWO-TIER MODEL

In this section, we extend the single tier model to incorporate interactions with the raw material supplying sector, where supplies are limited, and where raw material prices reflect these limitations. This situation occurs in oil production, where the supply of crude is often tightly controlled by a group of countries (e.g., OPEC). It also occurs in a number of other industries, for which the item in limited supply could be a raw material commodity like copper, a rare material like uncut diamonds or a manufactured product like DRAM chips. To model this case, we consider a *monopolist* price setting supplier and consider r raw materials indexed by $k \in (1, 2, \dots, r)$. Further, let p^k represent the price of these raw materials, v^k represent production costs per unit of the raw material and r_{ijk} be a scale factor representing the number of

units of raw material required for the production of a unit of product j at producer i . In this case, producer i considers the following problem:

$$(P2) \quad \Pi_i = \text{Max} \sum_{j=1}^m \{(p_j - (v_{ij} + \sum_{k=1}^r p^k r_{ijk}))q_{ij}\}$$

Subject to:

$$\sum_{j=1}^n c_{ij} q_{ij} \leq d_i, \quad (12)$$

$$q_{ij} \geq 0, \forall j.$$

Note that (P2) is identical in structure to (P1) with v_{ij} replaced by $\bar{v}_{ij} = (v_{ij} + \sum_{k=1}^r p^k r_{ijk})$. Therefore, the

first order conditions for (P2) can be obtained by replacing v_{ij} with \bar{v}_{ij} in (5). Similar to Proposition 1, we

can show that equilibrium exists and is unique. If $p^k, \forall k$ is given, then the equilibrium production quantities and prices can be found by replacing v_j with \bar{v}_j in (6) through (8). However, finding p^k is

intricate and requires constructing the demand function $Q^k(p^k)$ for each raw material k , and then solving the supplier's problem:

$$(P3) \quad \Pi^o = \text{Max} \sum_{k=1}^r \{(p^k - v^k)Q^k(p^k)\} \quad (13)$$

Subject to:

$$\sum_{k=1}^r c^k Q^k(p^k) \leq d^o, \quad (14)$$

$$p^k \geq 0.$$

Problems (P3) and (P2) can be considered as a successive ‘‘Bertrand-Cournot’’ framework. In (P3), the supplier sets raw material prices to maximize profits subject to capacity constraints on raw material production. At this price, the raw material production quantity equals the total requirements across all producers. Given these raw material prices, the producers solve the Cournot game (P2) to determine the optimal production quantities by allocating capacity across a set of products. Given a demand curve for

the product, this establishes prices for the end customer. The successive ‘‘Bertrand-Cournot’’ framework is similar in concept to the successive Cournot framework commonly used in the literature (Machlup and Taber, 1960; Greenhut and Ohta, 1979; Abiru, 1988; Corbett and Karmarkar, 2001). However the successive Cournot framework cannot be always employed in our context, as in general, the raw material demand function $Q^k(p^k)$ may not be invertible.

To solve (P3), we first need to construct $Q^k(p^k)$, $\forall k$. To develop this function, let $Q^k = \sum_{i=1}^m \sum_{j=1}^n r_{ijk} q_{ij}$ represent the total amount of raw material k required across all producers. We can replace v_j with \bar{v}_j in (6), use the resulting expression in $Q^k = \sum_{i=1}^m \sum_{j=1}^n r_{ijk} q_{ij}$ to construct $Q^k(p^k)$. We can then substitute function $Q^k(p^k)$ into (13) and (14) to solve (P3) to find the values of p^k . This can then be used to calculate \bar{v}_j . Finally, the equilibrium producer’s quantities and prices can be calculated by

employing (6) through (8) with this calculated value of \bar{v}_j and the condition $\sum_{j=1}^n c_{ij} q_{ij}^* = d_i$, $\forall i$. In

Section 5, we develop an efficient method to execute this procedure for large real problems. However, this procedure does not provide insight into how prices, production quantities and capacity constraints interact across producers and with supplier prices, production quantities and capacity constraints. To develop this understanding, we next consider smaller versions of this problem.

4.1 THE SINGLE RAW MATERIAL, SINGLE PRODUCT PROBLEM

There are some very significant strategic interactions that occur between the raw material supplier tier and the product processing tier. Some of these interactions are best revealed by examining simplified versions of the problem. As a start, we consider the problem in which there are multiple producers making a single product with a capacity constraint, and in which there is a single raw material supplied by a monopolist

supplier. This setting allows us to derive insights into the relationship between capacity constraints and supply decisions and into the interpretation of Lagrange multipliers in a multi-tier competitive setting.

In the context of this simplified problem, we are able to demonstrate that, when all producers are homogenous with respect to capacity limits, there are systematic situations for which the Lagrange multipliers for the capacity constraints are zero, but a marginal increase in capacity can result in positive benefits for the producers. In short, the usual interpretation of Lagrange multipliers is not valid. We further show that when producers are heterogeneous and have varying capacity constraints, the Lagrange multipliers are generally not zero, but, again, their interpretation requires care. We show that in the latter case there is a critical marginal producer, whose capacity constraint has a significant effect on the entire production tier.

4.1.1 Homogenous Capacity Across Producers

Consider the scenario in which there are m producers in the production tier, a single product is manufactured and all producers have equal variable manufacturing costs and equal capacity so that $v_i = v$ and $d_i = d$, $\forall i = 1, 2, \dots, m$. In addition, there is a single raw material required (i.e., $k = 1$), and we can assume without loss of generality that one unit of the raw material is required for one unit of the product (i.e., $r = 1$). Further, let p^0 represent the price of this raw material. The problem for producer i is:

$$(P4) \quad \tilde{\Pi}_i = \text{Max}\{(a - b \sum_i q_i - (v + p^0))q_i\}$$

Subject to: $q_i \leq d$ and $q_i \geq 0$.

The supplier considers the following problem:

$$(P5) \quad \tilde{\Pi}^o = \text{Max}\{(p^0 - v^o)\tilde{Q}(p^0)\}$$

Subject to: $\tilde{Q}(p^0) \leq d^0$, and $p^0 \geq 0$.

Proposition 3. The only possible solutions for (P4) are $q_i^* = d \quad \forall i$, or $q_i^* = q^* < d \quad \forall i$.

Proof. Suppose not, $q_i^* = d \quad \forall i \neq j$ and $q_j^* < d$, or $q_i^* > q_j^*$. Then,

$$\left. \frac{\partial \Pi_i}{\partial q_i} \right|_{q_i=q_i^*, q_j=q_j^*} = a - 2bq_i^* - bq_j^* - b \sum_{p \neq i, j} q_p - (v + p^0) \geq 0,$$

$$\left. \frac{\partial \Pi_i}{\partial q_j} \right|_{q_j=q_j^*, q_i=q_i^*} = a - 2bq_j^* - bq_i^* - b \sum_{p \neq i, j} q_p - (v + p^0) = 0.$$

But, this is a contradiction, if $q_i^* > q_j^*$. Therefore, the result holds. ■

Proposition 3 implies that if the supplier has capacity $d^0 > md$, then they would sell at price $p^0 = p^*$, so that at this price, each producer would order d and the total supplier production quantity corresponds to md . When $d^0 \leq md$, then the supplier would sell at price p^0 , so that $p^* \leq p^0 \leq a - v$. At this price, each producer orders q^* , where $q^* \leq d$ and the total raw material production quantity is mq^* . Figure 1 shows the demand function faced by the monopolist supplier.

INSERT FIGURE 1 ABOUT HERE

To determine q^* and p^* , we consider the problem for producer i and note that due to Proposition 3, this now reduces to $\Pi_i = (a - bmq - v - p^0)q$. The optimal production quantity q^* can be obtained by

setting $\frac{\partial \Pi_i}{\partial q} = 0$, so that $q^* = \frac{(a - v - p^0)}{b(m + 1)}$. The total quantity of product in the market is

$Q = mq^* = \frac{m}{(m + 1)} \frac{(a - v - p^0)}{b}$. From Figure 1, note that $p^0 = p^*$ is chosen so that

$Q = \frac{m}{(m+1)} \frac{(a-v-p^*)}{b} = md$. This implies that $p^* = a - v - b(m+1)d$. We next analyze the marginal value of capacity expansion in this context.

Proposition 4. The producers' Lagrange multipliers are identically zero, i.e., $\lambda_i = 0, \forall i$.

Proof. Observe from Figure 1, that the maximum production d for each producer is achieved when $p^0 = p^* = a - v - b(m+1)d$. Thus, when $p^0 > p^*$, $q^* = \frac{(a-v-p^0)}{b(m+1)} < d$. This implies that the producer's capacity is not exceeded and by Proposition 3, this is true for all producers. Therefore, each producer's Lagrange multiplier, λ_i , which measures the marginal value of capacity expansion will always be zero for all producers. ■

Next, consider a situation in which we increase each producer's capacity by ϵ , $\epsilon \rightarrow 0$ so that equilibrium is maintained and so that there are no fixed costs of capacity expansion. The following proposition shows that even a small increase in capacity will result in increased profits for the producer, if there is sufficient supply to use this capacity.

Proposition 5. There is a positive change in the i^{th} producer's profit when its capacity is increased by ϵ , $\epsilon \rightarrow 0$.

Proof. Let $\tilde{\Pi}(d)$ represent the producer's profits when capacity is d and $\tilde{\Pi}(d+\epsilon)$ represent the profit when capacity is increased to $d+\epsilon$. Then:

$$\tilde{\Pi}(d) = (a - bmd - v - p^*)d = d^2b \text{ and } \tilde{\Pi}(d+\epsilon) = (a - bm(d+\epsilon) - v - p^*)(d+\epsilon) = (d+\epsilon)^2b,$$

$$\text{so that } \lim_{\epsilon \rightarrow 0} \frac{\tilde{\Pi}(d+\epsilon) - \tilde{\Pi}(d)}{\epsilon} = 2bd > 0. \blacksquare$$

If a producer solves (P4) in a myopic single-tier manner, as expected by Proposition 4, the Lagrange multiplier would be zero and they could conclude that the marginal value of capacity expansion is zero. However, as shown by Proposition 5, this conclusion is not valid if we consider multi-tier effects. Propositions 4 and 5 together imply that the usual interpretation of Lagrange multipliers is not valid in this competitive, multi-tier context.

We now consider the supplier. The supplier's initial profit is $\Pi^o(md) = (p^* - v^o)md = (a - v - b(m+1)d - v^o)md$. When the supplier changes capacity to meet the increased supply requirement of ϵ by each of the m producers, their new capacity is $m(d + \epsilon)$. The resulting profit is now $\Pi^o(m(d + \epsilon)) = (a - v - b(m+1)(d + \epsilon) - v^o)m(d + \epsilon)$. The change in profit due to this capacity change is $\lim_{\epsilon \rightarrow 0} \frac{\Pi^o(m(d + \epsilon)) - \Pi^o(md)}{\epsilon} = p^*m - bm(m+1)d$, where

$p^* = a - v - b(m+1)d$. Therefore, $\lim_{\epsilon \rightarrow 0} \frac{\Pi^o(m(d + \epsilon)) - \Pi^o(md)}{\epsilon} > 0$, if $p^* > db(m+1)$. This will

hold when the optimal supplier's price is sufficiently high, which would entice the supplier to marginally expand capacity and sell more product or when the producer's capacity d is sufficiently low providing the basis for capacity expansion. This analysis shows that it could be profitable for both producers and the supplier to expand capacity by a small amount. However, were the producers to only consider the myopic problem (P4), by Proposition 3, they might erroneously conclude that the marginal value of capacity expansion is zero. This could lead to the scenario when both the supplier and producers may not even marginally increase their capacity, when this could be profitable for both of them.

4.1.2 Heterogeneous Capacity Across Producers

Now consider a scenario in which capacity differs across the producers, but without loss of generality the producers still have identical variable manufacturing costs. The problem for producer i is now given by

$$(P6) \quad \hat{\Pi}_i = \text{Max}\{(a - b\sum_i q_i - (v + p))q_i\}$$

Subject to: $q_i \leq d_i$ and $q_i \geq 0$.

The supplier considers the following problem:

$$(P7) \quad \hat{\Pi}^o = \text{Max}\{(p^o - v^o)\hat{Q}(p^o)\}$$

$$\text{Subject to: } \hat{Q}(p^o) \leq d^0 \text{ and } p^o \geq 0.$$

Note that (P7) is similar in structure to (P5), but the imputed demand function $\hat{Q}(p^o)$ is different. Figure 2 shows the demand function for this case.

INSERT FIGURE 2 ABOUT HERE

We next explain how Figure 2 is constructed by considering what happens as raw material price p^0 changes. Without loss of generality, assume that producers are indexed in order of increasing capacity; i.e. $d_i \leq d_j$ if $i < j$. When $p^0 > a - v$, no producer will produce a positive quantity and $Q = 0$. As the price p^0 is lowered below $a - v$, by Proposition 3 and the subsequent analysis, all producers begin to

produce and they will make the same production decision $q_i = \frac{(a - v - p^0)}{b(m + 1)}$, so that the demand

function for the raw material is $Q_1(p^0) = mq_i = \frac{m}{(m + 1)} \frac{(a - v - p^0)}{b}$. The corresponding inverse

demand function is $p_1(Q) = a - v - b \frac{(m + 1)}{m} Q$. The demand function will hold until producer $i = l$

reaches its capacity limit first. That limit is reached at p_1^0 such that $q_1 = \frac{(a - v - p_1^0)}{b(m + 1)} = d_1$ or when

$p_1^0 = a - v - b(m + 1)d_1$. At this price, all producers make the same quantity decision, so that the total

raw material quantity is $Q_l = md_l$. This implies that the demand function for the raw material takes the

form of the linear line segment for price p^0 , such that $p_1^0 \leq p^0 \leq a - v$.

As price p^0 is reduced below p_1^0 , producer $i = 1$ remains at capacity so only the remaining $(m - 1)$ producers continue to increase production. So the demand function for the raw material now becomes

$$Q_2(p^0) = \frac{(m-1)(a-v-bd_1-p^0)}{b} \quad \text{and the corresponding inverse demand function is}$$

$$p_2(Q) = a - v - md_1 - b \frac{m}{(m-1)} Q. \quad \text{The demand function will hold until producer } i = 2 \text{ reaches its}$$

capacity limit, reached at p_2^0 such that $q_2 = \frac{(a-v-bd_1-p_2^0)}{bm} = d_2$ or when

$$p_2^0 = a - v - bd_1 - bmd_2. \quad \text{At this price, the total raw material quantity is } Q_2 = d_1 + (m-1)d_2. \quad \text{Here}$$

again, the demand function for the raw material takes the form of the linear line segment, for prices p^0

such that $p_2^0 \leq p^0 \leq p_1^0$.

Using the same reasoning, we can show that:

$$Q_u(p^0) = \left(\frac{m-u+1}{m-u+2} \right) \left(\frac{1}{b} \right) \left(a - v - b \sum_{i=1}^{u-1} d_i - p^0 \right) \quad (15)$$

and each line segment is defined in the range $Q_u < Q < Q_{(u-1)}$, with break points

$$Q_u = \sum_{i=1}^{u-1} d_i + (m-u+1)d_u \quad \text{for } u = 1 \text{ to } m \text{ and } Q_0 = 0. \quad \text{For notational compactness, we write}$$

$$Q_u(p^0) = C_u - D_u p^0, \quad \text{where } C_u = \left(\frac{m-u+1}{m-u+2} \right) \left(\frac{a-v-b \sum_{i=1}^{u-1} d_i}{b} \right) \quad \text{and } D_u = \left(\frac{m-u+1}{m-u+2} \right) \left(\frac{1}{b} \right).$$

The inverse demand function for the raw material is:

$$p_u(Q) = \left(a - v - b \sum_{i=1}^{u-1} d_i \right) - bQ \frac{(m-u+2)}{(m-u+1)}, \quad u = 1 \text{ to } m. \quad (16)$$

For notational convenience, let $A_u = (a - v - b \sum_{i=1}^{u-1} d_i)$ and $B_u = \frac{b(m-u+2)}{(m-u+1)}$, so that

$p_u(Q) = A_u - B_u Q$. Note that each line segment $p_u(Q)$ is defined in the range $p_u^0 < p^0 < p_{(u-1)}^0$,

where break points $p_u^0 = a - v - b \sum_{i=1}^{u-1} d_i - b(m+2-u)d_u$ for $u = 1$ to m and $p_0^0 = a - v$.

Note that we will have t breakpoints, so that $Q_t \geq d^0$ and $Q_{(t-1)} < d^0$, and that we will need to consider t line segments of the demand function $Q_u(p^0)$. The demand function $\hat{Q}(p^0) = \min_{u=1 \text{ to } t} \{Q_u(p^0)\}$.

Proposition 6. The supplier's problem with heterogeneous capacity across producers is a concave optimization problem.

Proof. Consider the supplier's problem with heterogeneous capacity across producers represented by (P7). In this problem, it is sufficient to show that $p^0 \hat{Q}(p^0)$ is a concave function of p^0 . Since

$$\hat{Q}(p^0) = \min_u \{Q_u(p^0)\}, \quad \text{where} \quad Q_u(p^0) = C_u - D_u p^0, \quad \text{we} \quad \text{get}$$

$$p^0 \hat{Q}(p^0) = p^0 \min_u \{Q_u(p^0)\} = \min_u \{p^0 Q_u(p^0)\} = \min_u \{C_u p^0 - D_u (p^0)^2\}.$$

As $C_u p^0 - D_u (p^0)^2$ is concave in p^0 , $\forall u$, we get that $p^0 \hat{Q}(p^0)$ is a concave function of p^0 . ■

In light of Proposition 6, we can find the optimal value of (P7) using the following procedure. First,

consider break points (p_u^0, Q_u) , evaluate $\hat{\Pi}(p_u^0, Q_u) = (p_u^0 - v^0)Q_u$ and find

$\hat{\Pi}^1 = \text{Max}_u \{\hat{\Pi}(p_u^0, Q_u)\}$ and let $p_s^0 = \text{Arg max}_u \{\hat{\Pi}(p_u^0, Q_u)\}$. Next, consider

$Q_s(p^0) = C_s - D_s p^0$, the line segment representing the demand function bounded by the break points

$(Q_{(s-1)}, Q_s)$ Also, consider $Q_{(s+1)}(p^0) = C_{(s+1)} - D_{(s+1)}p^0$, the line segment bounded by break

points $(Q_s, Q_{(s+1)})$. Then, compute $\hat{\Pi}^2 = \text{Max}_{p^0 \geq 0} \{C_s p^0 - D_s(p^0)^2\}$,

$\hat{\Pi}^3 = \text{Max}_{p^0 \geq 0} \{C_{(s+1)} p^0 - D_{(s+1)}(p^0)^2\}$, $\hat{\Pi} = \text{Max} \{ \hat{\Pi}^1, \hat{\Pi}^2, \hat{\Pi}^3 \}$ and the optimal raw

material price $\hat{p}^0 = \text{Arg max} \{ \hat{\Pi}^1, \hat{\Pi}^2, \hat{\Pi}^3 \}$.

The above analysis shows that the equilibrium solution can either lie at a break point on the demand curve for the supplier at which some producer w just reaches capacity, or between break points $(w-1)$ and w for some w at which producers 1 to $(w-1)$ are at capacity, while producers w to m are below capacity. We denote the w^{th} producer as the marginal producer and next analyze the marginal value of capacity expansion at the two cases of the equilibrium solution.

Case 1: Equilibrium Solution Between Break Points

When the equilibrium solution lies between the $(w-1)^{\text{th}}$ and w^{th} break point, first consider producers $i = w$ to m . As these producers are not yet at capacity, $\lambda_i = 0, \forall i = w$ to m . For producers $i = 1$ to $w-1$, since all the producers are at capacity d_i , a single-tier myopic analysis of (P6) will imply that the marginal value of capacity expansion $\lambda_i = p^* - v - \hat{p}^0$, where $p^* = a - b\hat{Q}$ is the optimal product price. However, we show in the analysis below that this interpretation would be misleading, as it does not consider the multi-tier effects due to interaction between the supplier and the producers.

When the equilibrium solution is between break points, the supplier problem (P7) reduces to $\hat{\Pi}^o = \text{Max}_{p^0 \geq 0} \{(p^0 - v^0)(C_w - D_w p^0)\}$. Since this is concave in p^0 , we can easily find p^0 by setting

$$\frac{d\{(p^0 - v^0)(C_w - D_w p^0)\}}{dp^0} = 0 \text{ so that } \hat{p}^0 = \frac{(C_w - v_o D_w)}{2D_w}, \hat{Q} = C_w - D_w \hat{p}^0 \text{ and } p^* = a - b\hat{Q}.$$

First, consider producer $i \leq w-1$. Initially, the optimal profit for this producer is $\hat{\Pi}_i(d_i) = (p^* - v - \hat{p}^0)d_i$. Suppose, we increase the producer capacity by ϵ , where $\epsilon \rightarrow 0$, so that equilibrium is maintained and there are no fixed costs of capacity expansion. Then, if the supplier makes the additional raw material available to the producer, the optimal profit for this producer is now $\hat{\Pi}_i(d_i + \epsilon) = (p^* - v - \hat{p}^0 + B_w \epsilon)(d_i + \epsilon)$.

Therefore, the marginal value of capacity expansion is $\lim_{\epsilon \rightarrow 0} \frac{\hat{\Pi}_i(d_i + \epsilon) - \hat{\Pi}_i(d_i)}{\epsilon} = (p^* - v - \hat{p}^0) + b \frac{(m - w + 2)}{(m - w + 1)}$. Observe that this is similar to λ_i with an additional factor $b \frac{(m - w + 2)}{(m - w + 1)}$, which represents the additional value of capacity expansion to this producer due to the competitive interactions between producers for the raw material. Note that this additional value increases as the number of producers who are at capacity increase (i.e., as $(w-1)$ increases), so that this producer uses this additional capacity to strengthen their market position. The multiplier λ_i does not capture this effect and thus caution must be used when we interpret this parameter in this multi-tier, competitive context.

Now consider the supplier. Initially, the optimal profit for the supplier is $\hat{\Pi}^o = (\hat{p}^0 - v_o)\hat{Q}$. If the supplier increases the supply of raw material to accommodate producer i , then the supplier's profit is

$$\hat{\Pi}_{new}^o = (\hat{p}^0 - v_o - (b + B_w)\epsilon)(\hat{Q} + \epsilon). \quad \text{Therefore,} \quad \lim_{\epsilon \rightarrow 0} \frac{\hat{\Pi}^o - \hat{\Pi}_{new}^o}{\epsilon} = (\hat{p}^0 - v_o) -$$

$$(A_w + v_o) \frac{(m - w + \frac{3}{2})}{(m - w + 2)}. \quad \text{This implies that} \quad \lim_{\epsilon \rightarrow 0} \frac{\hat{\Pi}^o - \hat{\Pi}_{new}^o}{\epsilon} > 0, \quad \text{if}$$

$$(\hat{p}^0 - v_o) > (A_w + v_o) \frac{(m - w + \frac{3}{2})}{(m - w + 2)}. \quad \text{This inequality shows that it is profitable for the supplier to}$$

increase the raw material supply to accommodate producer i when the existing profit margin is greater

than a threshold. Note that as w or the number of producers who are at capacity *increases*, this threshold *decreases*. This is because the price reduction due to increase in supply affects only producers who are not at capacity and now this would impact fewer high volume producers who are not yet at capacity. This analysis shows that while it is possible that both the producer and the supplier can make higher profits, solving the myopic, single-tier problem underestimates the marginal value of capacity expansion and the producers, if myopic, may not choose to increase capacity.

Case 2: Equilibrium Solution at the Break Points

If we are at the w^{th} breakpoint, then $\lambda_i = 0, \forall i = w + 1 \text{ to } m$. For producers $i = 1 \text{ to } w$, since all the producers are at capacity d_i , a single-tier myopic analysis of (P6) implies that the marginal value of capacity expansion would be $\lambda_i = p^* - v - \hat{p}^0$, where $p^* = a - b\hat{Q}$ is the optimal price of the producer. Here again, we show in the analysis below that this interpretation would be misleading, as it does not consider the multi-tier effects due to interactions between the supplier and the producers.

First, consider producer w . Initially, the optimal profit for this producer is $\hat{\Pi}_w(d_w) = (p^* - v - p_w^o)d_w$, where $p^* = a - bQ_w$. Suppose, we increase the producer capacity by $\epsilon, \epsilon \rightarrow 0$ so that equilibrium is maintained and there are no fixed costs of capacity expansion. Then, if the supplier makes additional raw material available to the producer, the optimal profit for this producer is $\hat{\Pi}_w(d_w + \epsilon) = (p^* - v - p_w^o + b\epsilon)(d_w + \epsilon)$.

Therefore, the marginal value of capacity expansion is $\lim_{\epsilon \rightarrow 0} \frac{\hat{\Pi}_w(d_w + \epsilon) - \hat{\Pi}_w(d_w)}{\epsilon} = (p^* - v - p_w^o) + bd_w$. Observe that this is similar to λ_i with an additional factor bd_w that represents the additional value of capacity expansion to this producer due to the competitive interactions between producers for the raw material. The multiplier λ_i will not capture this

effect and thus caution must be used when we interpret this parameter in this multi-tier, competitive context.

Now consider the supplier. Initially, the optimal profit for the supplier is $\hat{\Pi}^o = (p_w^0 - v_o)Q_w$. If the supplier increases the supply of raw material to accommodate producer w , then the supplier's profit is

now $\hat{\Pi}_{new}^o = (p_w^0 - v_o - b \in (m - w + 2))(Q_w + (m - w + 1) \in)$. Therefore, $\lim_{\in \rightarrow 0} \frac{\hat{\Pi}^o - \hat{\Pi}_{new}^o}{\in} =$

$(p_w^0 - v_o) - b \frac{(m - w + 2)}{(m - w + 1)} Q_w$. This implies that $\lim_{\in \rightarrow 0} \frac{\hat{\Pi}^o - \hat{\Pi}_{new}^o}{\in} > 0$, if

$(p_w^0 - v_o) > b \frac{(m - w + 2)}{(m - w + 1)} Q_w$. This inequality shows that it is profitable for the supplier to

accommodate producer w only when the current profit margin is greater than a threshold. Note that, unlike Case 1, as w or the number of producers who are at capacity *increases*, this threshold also *increases*. This is because there would be fewer producers to sell the additional supply, while price reduction due to this additional supply is felt across all producers. This analysis again shows that while it is possible that both the producer and supplier can increase their profits, the producer may not increase capacity even by a small amount, perhaps because solving the myopic, single-tier problem underestimates the marginal value of capacity expansion and because they are also unsure if the supplier would provide additional raw material.

In both cases, predicting whether the supplier would provide additional raw material is complicated, as it requires knowledge of the raw material supplier's margin and demand function, and understanding whether the raw material price is located at a break point or at a line segment and knowing how many producers are below and above this region. This, coupled with the underestimation of the marginal value of expansion at the producers tier, may explain in part why both producers and the supplier could be reluctant to increase capacity even by small amounts. Such reluctance is particularly evident in the petro-refining industry.

5. COMPUTATIONAL METHOD

We next develop a method to solve the general versions of the producer's problem (i.e. (P2)) and the supplier's problem (i.e. (P3)). If we are given raw material prices p^k , $k = 1$ to r , then (P2) reduces to (P1)

with v_{ij} replaced by $\bar{v}_{ij} = (v_{ij} + \sum_{k=1}^r p^k r_{ijk})$. Then, using (6) the equilibrium production quantities are:

$$q_{ij}^* = \frac{\{a_j - m(v_{ij} + \sum_{k=1}^r p^k r_{ijk} + \lambda_i c_{ij}) + \sum_{t \neq i} (v_{tj} + \sum_{k=1}^r p^k r_{tjk} + \lambda_t c_{tj})\}}{(m+1)b_j}, \quad \forall i, j. \quad (17)$$

Let s_i be the slack variable associated with capacity constraint (12). The complimentary slackness conditions associated with these constraints at the equilibrium production quantities are:

$$\sum_{j=1}^n c_{ij} q_{ij}^* + s_i = d_i, \quad \forall i = 1 \text{ to } m, \text{ and} \quad (18)$$

$$\lambda_i s_i = 0 \quad \forall i = 1 \text{ to } m. \quad (19)$$

Rather than explicitly construct the inverse demand function, $Q^k(p^k)$, required to solve (P3), we

implicitly represent this function by defining $Q^k = \sum_{i=1}^m \sum_{j=1}^n r_{ijk} q_{ij}^*$ and introduce this as constraints along

with (17) through (19) in (P3) to get:

$$(P8) \quad \Pi^o = \text{Max} \sum_{k=1}^r \{(p^k - v^k) Q^k\}$$

Subject to (17), (18), (19), and:

$$\sum_{k=1}^r c^k Q^k \leq d^o, \quad (20)$$

$$Q^k = \sum_{i=1}^m \sum_{j=1}^n r_{ijk} q_{ij}^*, \quad \forall k, \quad (21)$$

$$p^k, \lambda_i, s_i \geq 0, \quad \forall i, k.$$

Proposition 7. Solving (P8) is equivalent to solving (P2) and (P3).

Proof. Observe that (P8) consists of (P3) and definitional constraints (17) through (19) that represent the optimal solution of (P2) that is obtained for any value of p^k , including its optimal value. Therefore solving (P8) is equivalent to solving (P2) and (P3). ■

The following steps outline the procedure to solve (P8).

Step 1: Initialization: Set $\alpha=0$, $\lambda_i^0=0$ and $\lambda_i = \lambda_i^{(\alpha)} = 0$.

Step 2: Solve:

$$(P5) \quad \Pi^o = \text{Max} \sum_{k=1}^r \{(p^k - v^k)Q^k\}$$

Subject to: (17), (20), (21) and $p^k \geq 0, \forall k$.

Note that in (P5), Q^k can be substituted in the objective function and in (20) using (17) and (21). This reduces (P5) to a standard concave quadratic programming problem in p^k , which can be solved using commercially available software such as Matlab (MathWorks Inc. 1998). Let $\hat{p}^k \forall k$ represent the optimal solution to this problem.

Step 3: Use \hat{p}^k in (17) to find \hat{q}_{ij}^* , $\forall i,j$. and use \hat{q}_{ij}^* in (21) to find \hat{Q}^k .

Step 4: Use \hat{q}_{ij}^* in (18) to compute $\hat{s}_i \forall i$. Let $P = \{p \mid \hat{s}_p < 0\}$. If $P = \{ \}$, set $\lambda_i = 0, \forall i$ and stop.

Otherwise, set $\lambda_i = 0, \forall i \notin P$ and $\lambda_i^{(\alpha+1)} = \lambda_i^{(\alpha)} + \delta, \delta \in R^+, \forall i \in P$. Finally, set $\lambda_i^{(\alpha+1)} = \lambda_i$,

α to $\alpha + 1$ and go to Step 2.

Proposition 8. Steps 1 through 4 provide an optimal solution to (P8).

Proof. Observe that for any value of raw material prices $p^k, \forall k$, the producer's problem is a concave optimization problem. Therefore, the Kuhn-Tucker conditions are necessary and sufficient for this problem. Since this procedure enforces the Kuhn-Tucker conditions for any value of raw material prices, including the optimal raw material prices $\hat{p}^k, \forall k$, this procedure provides an optimal solution to (P8). ■

In light of Propositions 7 and 8, we can use this procedure to find the optimal solution to problems (P2) and (P3). We use this procedure in the following illustrative example.

5.1 AN ILLUSTRATIVE EXAMPLE

To better understand how production efficiencies (i.e., supplier's and producers' unit production costs) and market parameters (i.e., market size and customer price sensitivity) affect price, production quantities and profits at both the supplier and producers tiers, we considered a ten-producer, three-product, one-raw material problem. This problem size was chosen to correspond to the ten major petrochemical refining companies in the United States (Platts, 2006), each of whom produce three grades of gasoline (i.e., regular, plus and supreme) refined from crude oil (i.e., raw material).

For this example, we first solved (P8) using Steps 1 through 4 of this procedure, which was programmed in Matlab. We then changed the supplier's unit production cost from the base level in increments of 10% from -50% to 50%. As expected, the price of the raw material increases with an increase in raw material unit production costs. But this, in turn, leads to less demand for the raw material and, consequently, lower profits for the supplier. For the producer, an increase in raw material price increases the price offered to end customers. This lowers end customer demand and hence their production quantities for the products. However, the impact on producer's profit depends on the conversion factor (i.e., r_{ijk}) representing the rate at which the raw material is consumed by the producer to provide the end product. In particular, profits go up for producers with lower conversion factors, while they go down for producers with higher conversion factors. The implications in the petro-refining industry are that producers could benefit from technologies that decrease conversion factors (i.e., improve yields) particularly when raw material (i.e., crude prices) increase.

To understand the impact of a producer's unit production costs on price, quantity and profit at the supplier and producers, we varied the value of this parameter across the three products and ten producers from the base level in increments of 10% from -50% to 50%. With an increase in unit production costs at the producer, these costs are passed on to customers and end product prices increase. This reduces end

product demand and consequently the producer's production quantity. This in turn leads to a decline in producer's profits. As the total production quantities decrease, demand for raw material decrease, which causes raw material production quantities and prices to drop. This contributes to a decline in supplier profits. Thus, a decrease in production efficiencies at the producers causes profits for both the supplier and producers to decline.

Finally, we wanted to analyze how market size and customer price sensitivity affect price, production quantity and profit at the supplier and producers. Recall that the market size and customer price sensitivity for product j are a_j/b_j , and $1/b_j$, respectively. To vary market size for a product, we varied a_j across the three products from the base level in increments of 10% from -50% to 50%. This analysis showed that as market size increased, as expected, prices, production quantities and profits increased at both the supplier and the producers, and the converse also holds. This seems consistent with trends in the petrochemical industry. Here, an increase in market size due to emerging markets like China and due to gas guzzling vehicles such as SUV's (U.S. Census Bureau News, 2004), has been observed to increase prices, production quantities and profits for refiners and the crude oil supplier (Energy Information Administration, 2006). We also analyzed the impact of customer price sensitivity by varying b_j across the three products from the base level in increments of 10% from -50% to 50%. Here, we also changed a_j as required to ensure that market size remained unchanged. This analysis showed that as customer price sensitivity decreased, prices, production quantities and profits increased at both the supplier and the producers, and the converse also holds. Again, this seems consistent with trends in the petrochemical industry, in which customers seem more insensitive to gas prices due to life style choices (Victorian Transport Policy Institute, 2005) and this has led to an increase in prices, production quantities and profits for refiners and the crude oil supplier (Energy Information Administration, 2006).

6. CONCLUSIONS

We have considered a competitive version of the traditional capacity planning model of production with capacity constraints. We first analyzed the single-tier version of this problem without interactions with the

raw material supplier tier and utilized a Cournot framework. Here, producers make decisions on production quantities by allocating capacity across products and can also choose how to allocate production across markets. Given a demand curve, this establishes product prices. In this case, we were able to establish that a unique equilibrium of production quantities exists and were able to extract closed form solutions that provide insight, and structure for our subsequent analysis. In addition to production quantity decisions, we were also able to find Lagrange multipliers for capacity constraints in the competitive, profit maximization setting. The interpretation of these Lagrange multipliers representing the marginal value of capacity expansion is fundamentally different from those obtained from traditional planning models. In particular, we found that marginal value of capacity expansion at any producer is reduced by the total equilibrium production quantities produced by the competitors. This effect is not captured by the traditional non-competitive Lagrange multiplier. We also find that when capacity constraints are not tight and production costs are equal across producers, the total production quantities are higher and prices are lower in comparison to the non-competitive case.

We next extend our model to include interactions with the raw material supplying sector for which supplies are limited and prices reflect these limitations. Here, we use a successive “Bertrand-Cournot” framework in which a monopolist supplier sets raw material prices to maximize profits so that the raw material production quantity equals the total requirements across all producers. Given these raw material prices, the producers solve the Cournot game to determine the optimal production quantities by allocating capacity across a set of products.

To understand the strategic interactions between the raw material supplier and the product processing tier, we examine simplified versions of this problem. In particular, we consider the single-raw-material, single-product problem with homogenous and heterogeneous capacity across producers. When producers are homogenous with respect to capacity limits, there are systematic situations for which Lagrange multipliers for capacity constraints are zero, but a marginal increase in capacity can result in positive benefits for the producers. Therefore, the usual interpretation of Lagrange multipliers is not valid. We also show that it is profitable for the supplier to increase capacity marginally and supply to the producers when

the optimal price set by the supplier is high or the producer's capacity is sufficiently low. While such marginal increases in capacity could be profitable to both the supplier and producers, this is not apparent if the producer does not consider multi-tier effects in their decision making process.

We further show that when producers are heterogeneous with varying capacity constraints, Lagrange multipliers are not generally zero, but again, their interpretation requires care. In particular, we show that the multiplier does not capture the additional value of capacity for the producer due to competitive interactions between producers for the raw material used. We also show that it is profitable for the supplier to marginally increase capacity when the profit margin on the raw material is greater than a threshold; and this threshold changes with the number of producers at capacity. Here again, we find that while capacity expansion could be profitable for both the supplier and producers, this is not evident if the producer does not consider multi-tier effects in their decision making process.

We also present a computational method to solve the general problem. We use this method on an illustrative example to better understand how production efficiencies at the supplier and producers affect production quantities and prices of the raw material and the product, and profits at the supplier and producer's tiers. We also consider the impact of market size and customer price sensitivity on these aspects.

This paper presents several avenues for future research. First, this problem could be extended to incorporate multi-period effects using inventory constraints. Second, we could consider the impact of yield uncertainty at both the supplier and producer tiers. Both these extensions would require significant modifications to the computational method to solve the general problem. Finally, another direction could be to extend this problem to multiple suppliers and producers. In this case, the entire structure of the supplier problem has to be changed, as price competition with more than one supplier would lead to marginal cost pricing. One plausible approach could be to change the supplier problem to quantity rather than price competition. However, the optimal solution for such a problem may not be always computable, as in general, the raw material demand function may not be invertible. We hope this paper provides the stimulus and building blocks to examine these new, exciting and challenging avenues for future research.

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FIGURE 1. Demand Function for the Supplier With Homogenous Producers

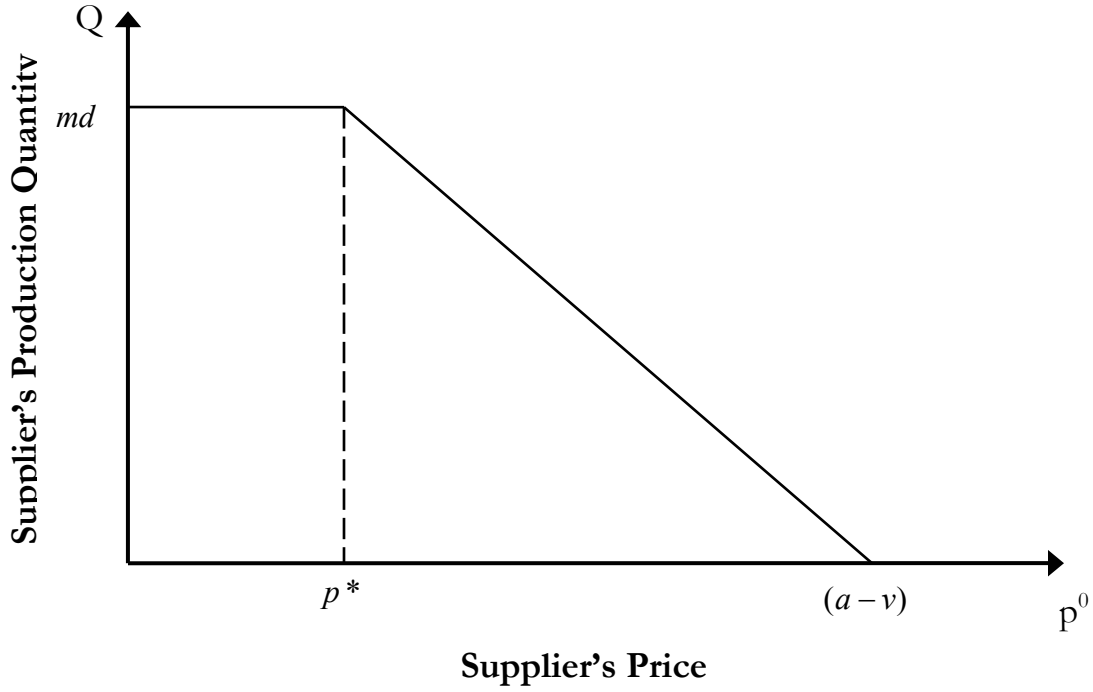


FIGURE 2. Demand Function for the Supplier With Heterogeneous Producers

