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Authors
Goudreau, Gerald
Nickell, Robert
Dunham, Robert

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by

G. L. GOUDREAU
R. E. NICKELL
and
R. S. DUNHAM

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and
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by

Gerald L. Goudreau
Graduate Student in Civil Engineering
University of California, Berkeley

and

Robert E. Nickell
Research Engineer, Rohm and Haas Co.
Huntsville, Alabama

and

Robert S. Dunham
Graduate Student in Civil Engineering
University of California, Berkeley

Faculty Investigators

Professor Karl S. Pister
Professor Jerome L. Sackman
Professor Robert L. Taylor

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ABSTRACT

A general computer program for the linear elastostatic analysis of two dimensional locally orthotropic solids is described in this report. Solids of revolution bonded to a shell of revolution, as well as states of plane strain or plane stress in solids bonded to a cylindrical case may be analyzed. The finite element method, with constant-strain solid elements, and linear-moment shell elements, is employed. The local elastic axes of orthotropy for each element may vary with respect to the global coordinate system. A sample problem is presented.

A complete user's manual and a listing of the program source deck are included as appendices.

TABLE OF CONTENTS

INTRODUCTION

Many problems have stimulated the separate development of theories and solution techniques for general bounded continua and for thin shells. The composite problem of a solid confined by a much stiffer thin case requires the application of both theories.

In the past decade the development of "finite element" methods has provided approximate solutions to a wide class of boundary value problems, for arbitrary loading, geometry, and inhomogeneity. This technique discretizes the field problem by suitably expanding the primary dependent variables over conveniently chosen subdomains (or finite elements) in terms of a finite number of nodal values.

The basic constant-strain (or linear-displacement) triangular element was presented in 1956 for the problem of plane stress by Turner, Clough, Martin, and Topp [1]*. It was programmed for large systems and applied by Wilson [2]. Rashid [3] introduced the linear-displacement, triangular toroidal element to analyze solids of revolution with axisymmetric loading. Wilson [4] incorporated the two solutions into a general computer program, which also permitted the assemblage of four triangular elements into a general quadrilateral element.

A finite element solution for shells of revolution was developed by Grafton and Strome [5], using polynomial expansions for the displacements. Becker and Brisbane incorporated this element in their solid of revolution isotropic elasticity program [6]. However, the shell's cubic displacement expansion is incompatible with the linear

*Numbers in brackets refer to references.

displacement expansion of the solid element. To overcome this difficulty Herrmann, Taylor, and Green [7] formulated a shell solution which employed a linear expansion for both components of the displacement field and the primary moment, thus providing an element compatible with the linear displacement expansion used in the triangular continuum element.

This report describes a computer program, called PALØS (Plane and Axisymmetric Locally Orthotropic Solids with Shell). A linear displacement plane stress and axisymmetric triangular element continuum program is combined with a linear displacement cylindrical shell and shell of revolution programs. The problem of plane strain differs from that of plane stress only in the material properties, and is thus included as a third option. Orthotropic material properties are defined for each element with respect to its individual elastic axes. Mixed force and displacement boundary conditions are permitted, including the possibility of mixed conditions along boundary segments sloping with respect to the global coordinate system.

The orthotropic properties are assumed to be known. They may be determined experimentally, or as in the case of composites materials, analytically predicted from the isotropic properties of the constituents. This has been done in the case of wire reinforced propellant grains by Herrmann, Mason, and Chan [8], and has been the principal motivation in the development of this program.

AXISYMMETRIC AND PLANE SOLIDS

The linear elastic analyses of plane stress or strain solids and axisymmetrically loaded solids of revolution are both linear boundary value problems in two independent variables, and may be regarded as similar problems in the calculus of variations. The finite element method is conveniently derived from the latter formulation. The Theorem of Minimum Potential Energy [9] for linear elastic bodies takes the form (in matrix notation)

$$V = \int_V \left(W - [u]^T \, [f] \right) dv - \int_S [u]^T \, [p] \, ds = \min \qquad (1)$$

where the integration is carried out over the volume of a plane section of unit thickness or one radian of arc. The surface integration is performed over the part of the surface on which stresses are prescribed.

$$W = \frac{1}{2} \, [\tau]^T \, [\epsilon - \epsilon_t]$$

$$[\tau]^T = \langle \tau_{rr}, \tau_{\theta\theta}, \tau_{zz}, \tau_{rz} \rangle \quad \text{or} \quad \langle \tau_{xx}, \tau_{zz}, \tau_{yy}, \tau_{xy} \rangle$$

$$[\epsilon]^T = \langle \epsilon_{rr}, \epsilon_{\theta\theta}, \epsilon_{zz}, 2\epsilon_{rz} \rangle \quad \text{or} \quad \langle \epsilon_{zz}, \epsilon_{zz}, \epsilon_{yy}, 2\epsilon_{xy} \rangle$$

$$[\epsilon_t]^T = T\langle \alpha_r, \alpha_\theta, \alpha_z, 0 \rangle \quad \text{or} \quad T\langle \alpha_x, \alpha_z, \alpha_y, 0 \rangle \qquad (2)$$

$[\alpha]$ = orthotropic coefficients of linear expansion

T = difference in temperature from stress free state

$[u]$ = displacement vector field

$[f]$ = body force vector field

$[p]$ = prescribed surface tractions on S.

The potential energy V will be a minimum when its first variation vanishes

$$\delta V = 0 \tag{3}$$

For a linear elastic solid the constitutive equations are

$$[\tau] = [C][\epsilon - \epsilon_t] = [C][\epsilon] \tag{4}$$

where $[C]$ and $[\beta]$ are arrays of anisotropic constants of generalized Hooke's law to be discussed later.

The general strain displacement equations for infinitesimal deformations are

$$\epsilon_{ij} = \frac{1}{2}\left(u_i\big|_j + u_j\big|_i\right) \tag{5}$$

where a $|$ denotes covariant differentiation. These specialize to

$$\epsilon_{11}: \quad \epsilon_{rr} = \frac{\partial u_r}{\partial r} \quad \text{or} \quad \epsilon_{xx} = \frac{\partial u_x}{\partial x}$$

$$\epsilon_{22}: \quad \epsilon_{\theta\theta} = \frac{u_r}{r} \quad \text{or} \quad \epsilon_{zz} = 0$$

$$\epsilon_{33}: \quad \epsilon_{zz} = \frac{\partial u_z}{\partial z} \quad \text{or} \quad \epsilon_{yy} = \frac{\partial u_y}{\partial y}$$

$$\epsilon_{13}: \quad \epsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right) \quad \text{or} \quad \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

under the respective assumptions of either axisymmetric or plane deformation.

Substitution of (5) into (4) and then into (1) yields a quadratic functional of the displacement field. The theorem states that of all admissible displacement fields, that which minimizes the potential energy will satisfy the equilibrium of the system. Admissibility conditions are that [u] must be continuous with piecewise continuous second partial derivatives over the domain, and satisfy the displacement boundary conditions.

Setting the first variation of this quadratic functional equal to zero yields the linear Euler differential equations associated with the variational problem. They are the displacement equations of equilibrium. The advantage of the variational approach is that the direct methods of the calculus of variations may be applied to seek an approximate solution to problems of arbitrary geometry and inhomogeneity not amenable to closed form solution.

The Ritz method reduces the problem to one of an ordinary minimum by expressing the displacement field as a linear combination of a finite set of suitable functions. Then, integrations may be carried out, reducing the potential energy functional to an ordinary quadratic function of the unknown coefficients of the combination. The vanishing of the variations with respect to each of the coefficients yields a set of linear algebraic equations.

The finite element method differs from the usual application of the Ritz method in that instead of assuming functions defined over the entire domain, the domain is subdivided into a set of subdomains, or "finite elements" and functions defined separately over each element. The theorem requires, however, that the assumed displacement fields $[u^m]$ be continuous across element boundaries, which is accomplished by a suitable choice of generalized coordinates. The total potential energy is then the sum of those of the separate elements.

$$V = \sum_{m} V^m \tag{6}$$

Derivations of the stiffness matrices for the assumed linear displacement, plane and axisymmetric triangular elements are presented in [2-4], but will be given here to include a locally orthotropic solid. The linear displacement field assures continuity between elements under deformation, and is simply given in terms of six generalized coordinates.

$$\begin{aligned} u_1(x_1,x_3) &= b_1 + b_2x_1 + b_3x_3 \\ u_3(x_1,x_3) &= b_4 + b_5x_1 + b_6x_3 \end{aligned} \tag{7}$$

or for each element

$$[u^m] = [\phi^m][b^m] \quad . \tag{8}$$

Evaluating the displacements at the three vertices, i, j, k,

$$[u_o^m] = [\phi_o^m][b^m] \tag{9}$$

where

$$[\phi_o^m] = \begin{vmatrix} \bar{\phi}_o^m & 0 \\ 0 & \bar{\phi}_o^m \end{vmatrix} \tag{10}$$

and

$$[\bar{\phi}_o^m] = \begin{vmatrix} 1 & x_1^i & x_3^i \\ 1 & x_1^j & x_3^j \\ 1 & x_1^k & x_3^k \end{vmatrix} \tag{11}$$

Solving the set (9) for the generalized coordinates in terms of the nodal point displacements,

$$[b^m] = [\phi_o^m]^{-1} \; [u_o^m] = [h^m][u_o] \tag{12}$$

where

$$[\bar{\phi}_o^m]^{-1} = \frac{1}{2A} \begin{vmatrix} \left(x_1^j x_3^k - x_1^k x_3^j\right) & \left(x_1^k x_3^i - x_1^i x_3^k\right) & \left(x_1^i x_3^j - x_1^j x_3^i\right) \\ \left(x_3^j - x_3^k\right) & \left(x_3^k - x_3^i\right) & \left(x_3^i - x_3^j\right) \\ \left(x_1^k - x_1^j\right) & \left(x_1^i - x_1^k\right) & \left(x_1^j - x_1^i\right) \end{vmatrix} \tag{13}$$

and

$$2A = x_1^j \left(x_3^k - x_3^i\right) + x_1^i \left(x_3^j - x_3^k\right) + x_1^k \left(x_3^i - x_3^j\right) \tag{14}$$

For the plane problem $x_1 = x$, $x_3 = y$. For the axisymmetric problem $x_1 = r$, $x_3 = z$.

Applying the strain-displacement Eq. (5) to (8)

$$[\varepsilon^m] = [g^m][b^m] \quad . \tag{15}$$

For plane strain,

$$\varepsilon_x = b_2, \; \varepsilon_z = 0, \; \varepsilon_y = b_6, \; \gamma_{xy} = b_3 + b_5$$

and

$$[g^m(x,y)] = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{vmatrix} \tag{16}$$

For the axisymmetric case,

$$\varepsilon_r = b_2, \quad \varepsilon_\theta = \frac{1}{r} b_1 + b_2 + \frac{z}{r} b_3, \quad \varepsilon_z = b_6, \quad \gamma_{rz} = b_3 + b_5$$

and

$$[g^m(r,z)] = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{1}{r} & 1 & \frac{z}{r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{vmatrix} \tag{17}$$

Substituting (12) into (15)

$$[\varepsilon^m] = [g^m][h^m][u_o] \tag{18}$$

Substituting (18) into the stress-strain law (4) and (12) into (8), and then into (1) and (6), the total potential energy becomes

$$V = \sum_m \frac{1}{2} [u_o]^T [k^m][u_o] - \sum_m [u_o]^T [Q^m] \tag{19}$$

where

$$[k^m] = [h^m]^T \int_V [g^m]^T [C^m][g^m] dv \; [h^m] \tag{20}$$

and

$$[Q^m] = [h^m]^T \int_V \left([g^m]^T [\beta^m] T + [\phi^m]^T [f^m]\right) dv$$

$$+ [h^m]^T \int_A [\phi^m]^T [p^m] \, da \tag{21}$$

Setting the first variation equal to zero yields the governing set of linear algebraic equations

$$[K][u_o] = [Q] \tag{22}$$

where

$$[k] = \sum_m [k^m] \tag{23}$$

is a symmetric stiffness matrix, and

$$[Q] = \sum_m [Q^m] \tag{24}$$

is the set of generalized forces associated with the nodal point displacements $[u_o]$. If sufficient displacements are specified to prevent rigid body motion, $[K]$ will be positive definite and a unique solution will exist.

For a locally orthotropic solid referred to global coordinate system, the integrand of the stiffness integral for the axisymmetric body

$$[g]^T [C][g] = \tag{25}$$

$$\begin{bmatrix} C_{22}\frac{1}{r^2} & (C_{12}+C_{22})\frac{1}{r} & \left(C_{22}\frac{z}{r^2} + C_{24}\frac{1}{r}\right) & 0 & C_{24}\frac{1}{r} & C_{23}\frac{1}{r} \\ & (C_{11}+2C_{12}+C_{22}) & \left[(C_{12}+C_{22})\frac{z}{r} + (C_{14}+C_{24})\right] & 0 & (C_{14}+C_{24}) & (C_{13}+C_{23}) \\ & & \left(C_{22}\frac{z^2}{r^2} + 2C_{24}\frac{z}{r} + C_{44}\right) & 0 & \left(C_{24}\frac{z}{r} + C_{44}\right) & \left(C_{23}\frac{z}{r} + C_{34}\right) \\ & & & 0 & 0 & 0 \\ & & & & C_{44} & C_{34} \\ \text{symmetric} & & & & & C_{33} \end{bmatrix}$$

and that for the generalized forces

$$[\phi]^T [f] + [g]^T [\beta] T = \begin{bmatrix} F_1 + \frac{1}{r} \beta_2 T \\ rF_1 + (\beta_1 + \beta_2) T \\ zF_1 + \left(\frac{z}{r} \beta_2 + \beta_4\right) T \\ F_3 \\ rF_3 \\ zF_3 + (\beta_3 + \beta_4) T \end{bmatrix} \tag{26}$$

where $F_1 = \rho a_1$, $F_3 = \rho a_3$, $a_r = r\omega^2$, and a_i are accelerations. The integrand for the plane problem can be obtained by setting all coefficients with subscript (2) equal to zero. The resulting array is then constant.

Integration of (25) and (26) is performed numerically by using a quadratic interpolation formula. For a triangular region, [10]

$$\int_A f \, da = \frac{1}{3} (f_a + f_b + f_c) A \tag{27}$$

where f is assumed to be a quadratic function over the triangle and f_a, f_b, f_c are the integrand evaluated at the midpoints of the sides. For most problems this scheme is sufficiently accurate (capturing several integrals exactly,) and more efficient than exact integrals involving the evaluation of logarithms.

Four triangles may be combined to form a general quadrilateral. Since the middle node is coupled only to the corner nodes of its own quadrilateral, the middle node displacements can be eliminated at the element level [4].

The stiffness matrix [K] is symmetric and by proper nodal sequencing is banded. These features are utilized to efficiently solve the set of Eqs. (22) by Gaussian elimination. In the computer program the banded stiffness matrix is generated in core a block at a time. The block is modified for displacement boundary conditions by inserting the specified displacements into the equations. For mixed boundary conditions along a sloping boundary, the two nodal point degrees of freedom are first transformed from the global to the sloping coordinate directions, and then the displacement degree of freedom is modified. The block is then reduced to triangular form and stored on tape. The process is repeated until all equations are generated and reduced. Finally the displacements are determined by back substitution. Average strains and stresses are determined for each quadrilateral and triangular element at its centroid.

LOCALLY ORTHOTROPIC MATERIALS

The constitutive law for linear elastic orthotropic materials was given in the previous section as

$$[\tau] = [C][\epsilon] - [\beta]T \tag{3}$$

where with respect to its elastic axes

$$[C] = [\bar{C}] = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 \\ & \bar{C}_{22} & \bar{C}_{23} & 0 \\ & & \bar{C}_{33} & 0 \\ \text{sym} & & & \bar{C}_{44} \end{bmatrix} , \quad [\beta] = [\bar{\beta}] = \begin{bmatrix} \bar{\beta}_1 \\ \bar{B}_2 \\ \bar{B}_3 \\ 0 \end{bmatrix} \tag{28}$$

and shear is confined to the 1-3 plane. The moduli in (28) are not conveniently expressible in terms of the technical or engineering constants. However, the coefficients of the strain stress law are.

$$[\bar{\epsilon}] = [A][\bar{\tau}] + [\bar{\epsilon}_t]$$

where (29)

$$[A] = [\bar{C}]^{-1}$$

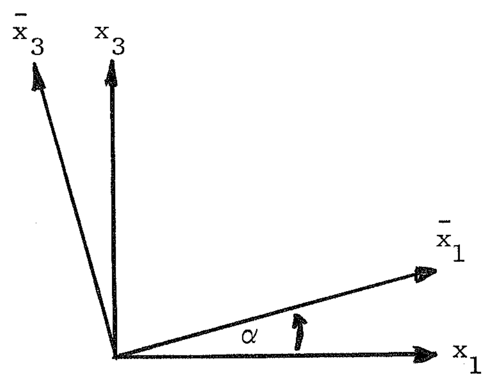
Then, [11],

$$A_{11} = \frac{1}{E_{11}} , \quad A_{22} = \frac{1}{E_{22}} , \quad A_{33} = \frac{1}{E_{33}}$$

$$A_{12} = -\frac{\nu_{21}}{E_{11}} = -\frac{\nu_{12}}{E_{22}} , \quad A_{23} = -\frac{\nu_{32}}{E_{22}} = -\frac{\nu_{23}}{E_{33}} \tag{30}$$

$$A_{13} = -\frac{\nu_{13}}{E_{33}} = -\frac{\nu_{31}}{E_{11}} , \quad A_{44} = \frac{1}{G_{13}}$$

If the elastic axes in the 1-3 plane are at some angle and from the global axes, then it is convenient to transform the material law to the global coordinate system.



(31)

A full array of material constants results.

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ & C_{22} & C_{23} & C_{24} \\ & & C_{33} & C_{34} \\ \text{sym} & & & C_{44} \end{bmatrix} , \quad [\beta] = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} \tag{32}$$

The transformation between the coordinate systems is

$$\bar{x}_i = a_{ij}\, x_j \qquad (i,j = 1,2,3) \tag{33}$$

where

$$a_{ij} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}$$

and where summation is implied over repeated indices in the same term of an expression.

Since (33) is an orthogonal transformation,

$$a_{ki}\, a_{kj} = \delta_{ij} \tag{34}$$

Thus,

$$x_k = \delta_{ki}\, x_i = a_{jk}\, a_{ji}\, x_i = a_{jk}\, \bar{x}_j \tag{35}$$

and

$$\frac{\partial x_k}{\partial \bar{x}_j} = a_{jk} \tag{36}$$

Displacements transform according to (33)

$$\bar{u}_i = a_{ik}\, u_k \quad . \tag{37}$$

Derivatives transform according to a second rank tensor law

$$\frac{\partial \bar{u}_i}{\partial \bar{x}_j} = a_{ik} \frac{\partial u_k}{\partial x_\ell} \frac{\partial x_\ell}{\partial \bar{x}_j} = a_{ik}\, a_{j\ell} \frac{\partial u_k}{\partial x_\ell} \tag{38}$$

Thus, the strains transform, using (5) and (38),

$$\bar{\varepsilon}_{ij} = \frac{1}{2} \left[a_{ik}\, a_{j\ell} \frac{\partial u_k}{\partial x_\ell} + a_{j\ell}\, a_{ik} \frac{\partial u_\ell}{\partial x_k} \right] \tag{39}$$

$$\bar{\varepsilon}_{ij} = a_{ik}\, a_{j\ell}\, \varepsilon_{k\ell} \tag{40}$$

which is also a second rank tensor law.

Expanding (40) and representing the transformation of the independent components as a matrix transformation,

$$[\bar{\varepsilon}] = [R][\varepsilon] \tag{41}$$

where

$$[R] = \begin{bmatrix} c^2 & 0 & s^2 & cs \\ 0 & 1 & 0 & 0 \\ s^2 & 0 & c^2 & -cs \\ -2cs & 0 & 2cs & (c^2-s^2) \end{bmatrix}$$

and $[\epsilon]$ are the strains defined in (2). Since the strain energy is invariant with respect to coordinate transformation,

$$[\bar{\tau}]^T \, [\bar{\epsilon}] = [\tau]^T \, [\epsilon] \tag{42}$$

$$[\bar{\tau}]^T \, [R][\epsilon] = [\tau]^T \, [\epsilon] \tag{43}$$

Thus

$$[\tau] = [R]^T \, [\bar{\tau}] \tag{44}$$

Assuming the material to be orthotropic in the local system

$$[\bar{\tau}] = [\bar{C}][\bar{\epsilon}] - [\bar{\beta}] \, T \tag{45}$$

$$[\tau] = [R]^T \, [\bar{C}][\bar{\epsilon}] - [R]^T \, [\bar{\beta}] \, T$$

$$[\tau] = [R]^T \, [\bar{C}][R][\epsilon] - [R]^T \, [\bar{\beta}] \, T \tag{46}$$

Thus, $$[\tau] = [C][\epsilon] - [\beta] \, T \tag{3}$$

where

$$[C] = [R]^T \, [\bar{C}][R] \tag{47}$$

and

$$[\beta] = [R]^T \, [\bar{\beta}] \tag{48}$$

Expanding (47) and (48), we get the coefficients in the global system

$$
\begin{aligned}
C_{11} &= \bar{C}_{11}\, c^4 + \bar{C}_{33}\, s^4 + 2(\bar{C}_{13} + 2\,\bar{C}_{44})\, c^2 s^2 \\
C_{12} &= \bar{C}_{12}\, c^2 + \bar{C}_{23}\, s^2 \\
C_{13} &= \bar{C}_{13}\,(c^4 + s^4) + (\bar{C}_{11} + \bar{C}_{33} - 4\bar{C}_{44})\, c^2 s^2 \\
C_{14} &= \left[\bar{C}_{11}\, c^2 - \bar{C}_{33}\, s^2 - (\bar{C}_{13} + 2\,\bar{C}_{44})\,(c^2 - s^2)\right] cs \\
C_{22} &= \bar{C}_{22} \\
C_{23} &= \bar{C}_{23}\, c^2 + \bar{C}_{12}\, s^2 \\
C_{24} &= (\bar{C}_{12} - \bar{C}_{23})\, cs \\
C_{33} &= \bar{C}_{33}\, c^4 + \bar{C}_{11}\, s^4 + 2(\bar{C}_{13} + 2\,\bar{C}_{44})\, c^2 s^2 \\
C_{34} &= \left[-\,\bar{C}_{33}\, c^2 + \bar{C}_{11}\, s^2 + (\bar{C}_{13} + 2\,\bar{C}_{44})\,(c^2 - s^2)\right] cs \\
C_{44} &= \bar{C}_{44}\,(c^2 - s^2) + (\bar{C}_{11} - 2\,\bar{C}_{13} + \bar{C}_{33})\, c^2 s^2 \\
C_{ij} &= C_{ji} \quad (i > j) \\
\beta_1 &= \bar{\beta}_1\, c^2 + \bar{\beta}_3 s^2 \\
\beta_2 &= \bar{\beta}_2 \\
\beta_3 &= \bar{\beta}_1\, s^2 + \bar{\beta}_3 c^2 \\
\beta_4 &= (\bar{\beta}_1 - \bar{\beta}_3)\, cs
\end{aligned}
\tag{49}
$$

AXISYMMETRIC AND CYLINDRICAL SHELLS

In order to construct a finite element solution for shells of revolution and cylindrical shells in a state of plane strain which satisfies all the continuity requirements at element interfaces and uses only displacements as primary variables, it is necessary to use at least a linear expansion for the in-plane displacement u and at least a cubic expansion for the transverse displacement w. Grafton and Strome [5] use that expansion. Such a displacement field is incompatible with the linear displacement triangular solid element if the two are used together. To restrict both components of shell displacement to linear form, it is necessary to retain a third primary dependent variable. The finite element solution presented by Herrmann, Taylor, and Green [7] retains the moment along the generator as the third dependent variable.

The curved shell geometry is approximated by straight meridional segments (i.e., conical frustrum or flat plate elements). The contribution to the functional V for both such elements is

$$V = \int_B \left\{ \frac{1}{2} \left(A_{11}\epsilon_{11}{}^2 + A_{22}\epsilon_{22}{}^2 + \bar{D}_{22}\chi_{22}{}^2 - \frac{1}{D_{11}} M_{11}{}^2 \right) \right.$$

$$+ A_{12}\epsilon_{11}\epsilon_{22} + \frac{\partial M_{11}}{\partial s}\theta + \left(1 - \frac{D_{12}}{D_{11}}\right) M_{11}\chi_{22} + \frac{\sin\phi}{r}\Big[(M_{11}$$

$$\left. + D_{12}\chi_{22})\,\epsilon_{11} + D_{22}\chi_{22}\epsilon_{22}\Big] - q_n w - q_1 u \right\} dv$$

$$- \int_C \left(N_{11}{}^a u + Q_1{}^a w + \theta^a M_{11} \right) dc$$

where r is the radius of the shell, ϕ the angle the conical frustrum makes with the r axis, and θ is the change in slope of the shell along the generator. For the cylindrical shell $r \to \infty$. The surface integral is to be evaluated over the reference surface of the shell and the line integral over the boundaries where $N_{11}{}^{a}$, $Q_{1}{}^{a}$, and θ^{a} are the specified values of generator in plane stress resultant, transverse shear, and slope respectively.

$$\begin{bmatrix} N_{11} \\ N_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \end{bmatrix}$$

$$\begin{bmatrix} M_{11} \\ M_{22} \end{bmatrix} = - \begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \chi_{11} \\ \chi_{22} \end{bmatrix} \tag{51}$$

$$\bar{D}_{22} = D_{22} - \frac{D_{12}^{\ 2}}{D_{11}}$$

$$A_{\alpha\beta} = h\ \bar{C}_{\alpha\beta} \qquad\qquad (\alpha, \beta = 1, 2) \tag{52}$$

$$D_{\alpha\beta} = h^2/12\ A_{\alpha\beta}$$

where h is the average shell thickness of the element, and

$$\bar{C}_{\alpha\beta} = C_{\alpha\beta} - \frac{C_{\alpha n} C_{\beta n}}{C_{nn}} \tag{53}$$

are the orthotropic elastic moduli of the three dimensional stress strain law with respect to the local shell coordinate system, modified for the assumption of plane stress through the shell thickness. This modification is not to be confused with the modification for plane stress that also is made if the cylindrical shell in the plane problem is assumed to be in a state of plane stress in the longitudinal direction. In that case the cylindrical shell becomes a beam.

The strain (or curvature) - displacement - temperature relations for the axisymmetric shell are

$$\epsilon_{11} = \frac{\partial u}{\partial s} + \alpha_{11} T$$

$$\epsilon_{22} = \frac{1}{r} [u \cos \varphi + w \sin \varphi] + \alpha_{22} T \tag{54}$$

$$\chi_{22} = \frac{1}{r} \cos \varphi \frac{\partial w}{\partial s}$$

where $\alpha_{\gamma\gamma}$ are the orthotropic coefficients of linear expansion (also modified for the assumption of plane stress through the thickness). For the cylindrical shell

$$\epsilon_{11} = \frac{\partial u}{\partial s} + \alpha_{11} T$$

$$\epsilon_{22} = + \alpha_{22} T \tag{55}$$

$$\chi_{22} = 0$$

The validity of the variational form (50) is verified by determining the Euler equations and finding them to be the inplane and transverse force equilibrium equations, the constitutive equation for M_{11}, and the natural boundary conditions, all expressed in terms of the three primary variables u, w, and M_{11}.

The three primary variables may be expressed in terms of nodal point values by linear interpolation formuli

$$u(s) = \left(1 - \frac{s}{L}\right) u_i + \frac{s}{L} u_j$$

$$w(s) = \left(1 - \frac{s}{L}\right) w_i + \frac{s}{L} w_j \tag{56}$$

$$M_{11}(s) = \left(1 - \frac{s}{L}\right) M_{11i} + \frac{s}{L} M_{11j}$$

where L is the length of the element. The strain-curvature displacement relations (54) or (55) may be applied, and then the integration in (50) performed. A quadratic function of the nodal point values of the three primary dependent variables results. The vanishing of the variations with respect to these quantities yields the governing set of linear algebraic equations.

Once the primary variables u, w, M_{11} have been obtained, the stress resultants can be obtained from (51) after eliminating χ_{11} .

EXAMPLE - ORTHOTROPIC SPHERE

To evaluate the numerical accuracy of PALØS in the solution of locally orthotropic problems, an orthotropic sphere under external pressure was considered (Fig. 1). A closed form solution is obtained from Lekhnitskii [11]. The material is assumed to be orthotropic with respect to a spherical coordinate system, and governed by the stress strain law

$$\begin{bmatrix} \sigma_\rho \\ \sigma_\varphi \\ \sigma_\theta \end{bmatrix} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{12} \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} \\ \bar{C}_{12} & \bar{C}_{23} & \bar{C}_{22} \end{bmatrix} \begin{bmatrix} \epsilon_\rho \\ \epsilon_\varphi \\ \epsilon_\theta \end{bmatrix} \qquad (57)$$

Although the material used is actually isotropic in the tangent plane, the stress-strain matrix when transformed to the cylindrical coordinate system utilized in PALØS (32), becomes full.

Taking the closed form solution from [11],

$$u(\rho) = A\rho^{-\frac{1}{2}+n} + B\rho^{-\frac{1}{2}-n} \qquad (58)$$

where

$$n = \sqrt{\frac{1}{4} + \frac{2(\bar{C}_{22}+\bar{C}_{23}-\bar{C}_{12})}{\bar{C}_{11}}} \; . \qquad (59)$$

For an external pressure q, inner radius R_o, outer radius R,

$$A = -\frac{q\ R^{\frac{3}{2}+n}}{R^{2n} - R_o^{\ 2n}} \cdot \frac{1}{\bar{C}_{11}\left(-\frac{1}{2} + n\right) + 2\bar{C}_{12}}$$

$$B = \frac{q\ R^{\frac{3}{2}-n}}{R^{2n} - R_o^{\ 2n}} \cdot \frac{1}{-\bar{C}_{11}\left(\frac{1}{2} + n\right) + 2\bar{C}_{12}} \tag{60}$$

The strains are determined by the relations

$$\epsilon_\rho = \frac{du}{d\rho} = A\left(-\frac{1}{2} + n\right)\rho^{\left(-\frac{3}{2} + n\right)} + B\left(-\frac{1}{2} - n\right)\rho^{\left(-\frac{3}{2} - n\right)}$$

$$\epsilon_\varphi = \epsilon_\theta = \frac{u}{\rho} \tag{61}$$

and the stresses determined from the strains by (57).

The finite element solution takes advantage of the spherical symmetry of the problem by modeling only a thin wedge in the r-z plane as a solid of revolution, and requiring the sides of the wedge to be surfaces of symmetry (i.e. no tangential shear forces or normal displacements). This is represented by rollers along those surfaces (Fig. 2). Comparing the results for wedges at two different positions in the quadrant (thus requiring different material property transformations), shows the accuracy in capturing the local orthotropic character. Solutions are compared in Fig 3 for the following set of data

$\bar{C}_{11} = 200.$ $\quad R_o = 5.$

$\bar{C}_{12} = 100.$ $\quad R = 10.$

$\bar{C}_{22} = 300.$ $\quad q = 100.$

$\bar{C}_{23} = 150.$

The solution for both wedges are the same to two significant figures, and fall right on the exact curves. This demonstrates the accuracy of the material transformation.

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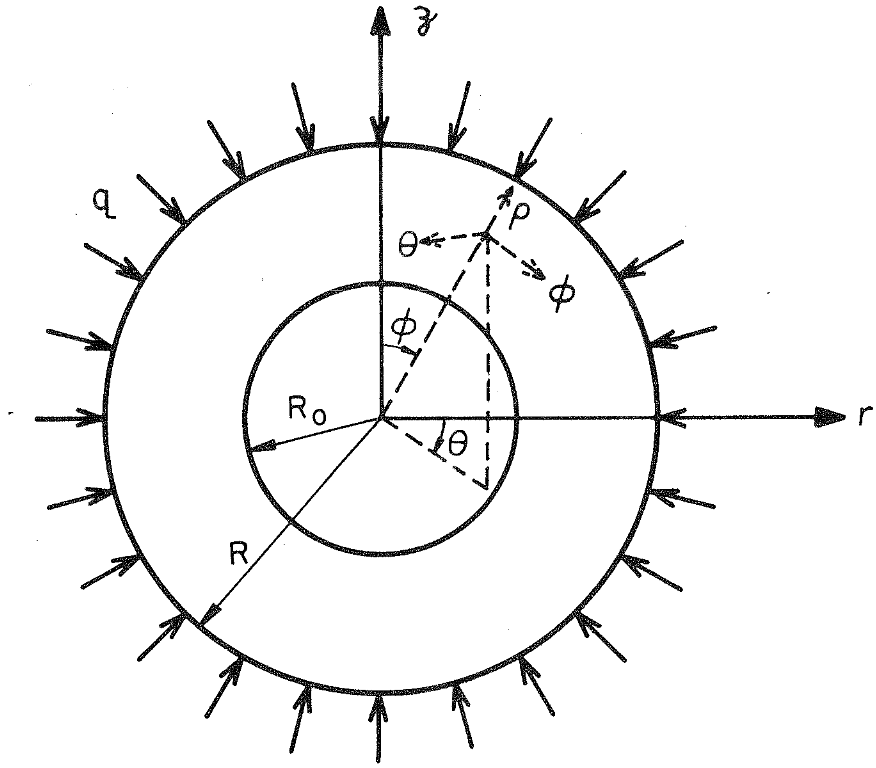


FIG. 1

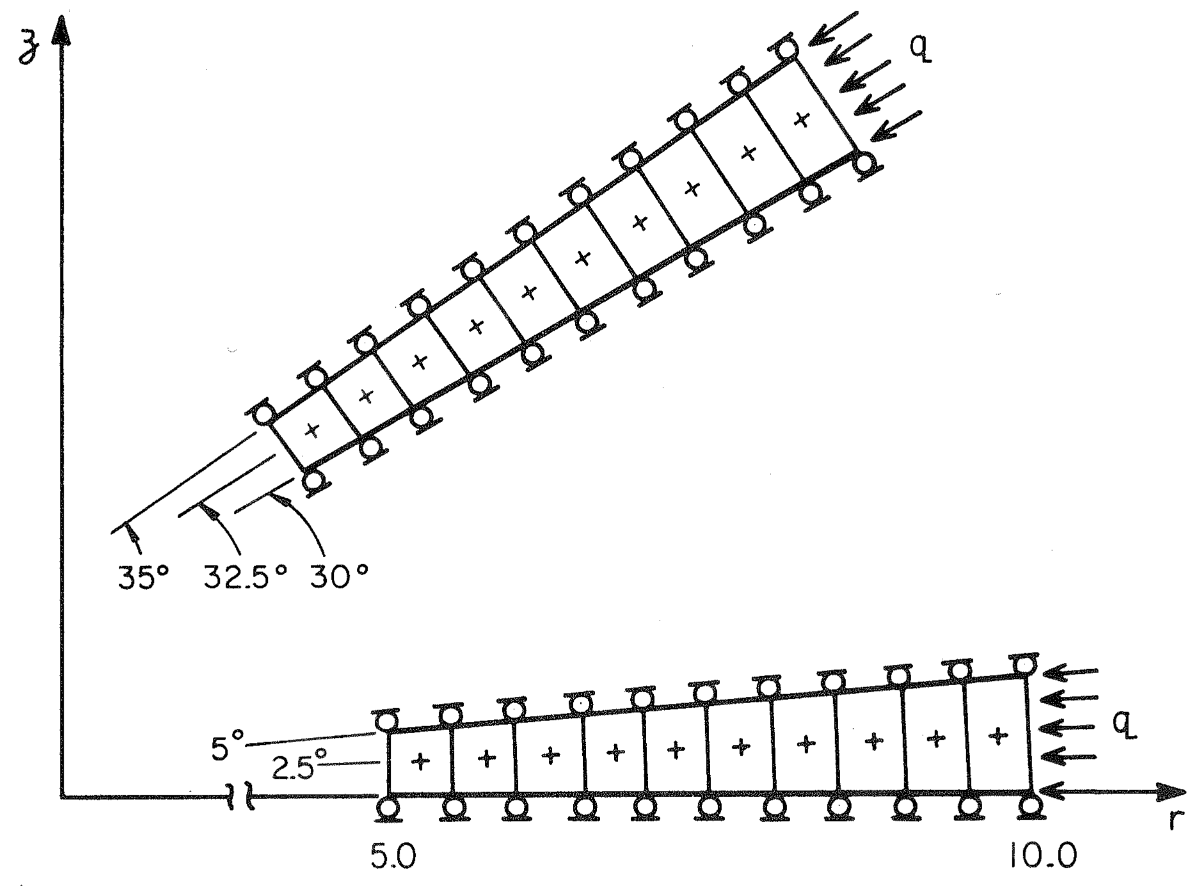


FIG. 2

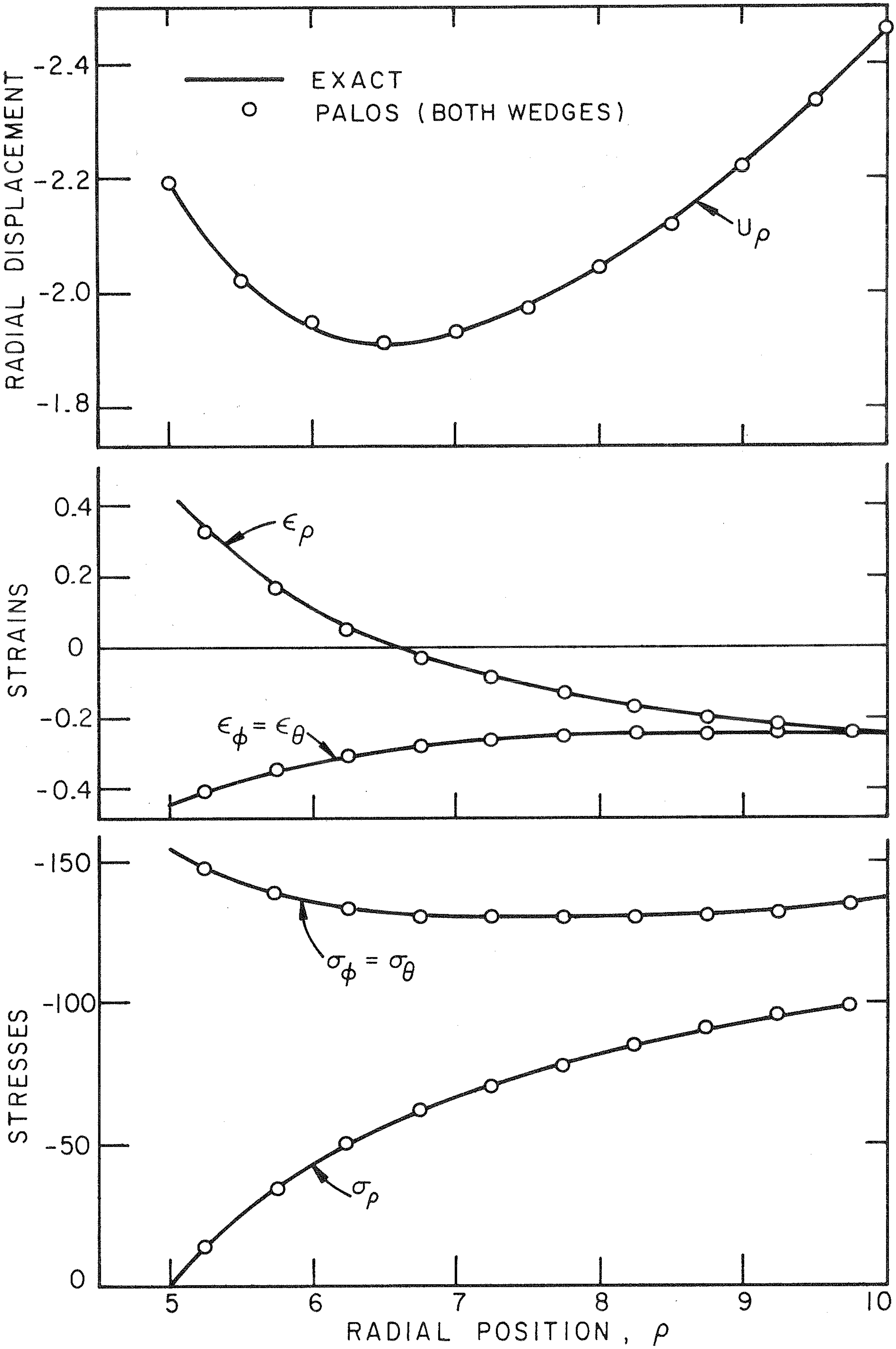


FIG. 3 ORTHOTROPIC SPHERE
EXTERNAL PRESSURE

```
PALOS  LEKHNITSKII PRESSURIZED SPHERE LOCAL ORTHOTROPIC CHECK CASE
   22   10    1    1
    1    1                   SOLID
 0.0         200.        100.        100.        300.        150.        300.        50.
 0.0         0.0         0.0
1    1   10 5.
1   11   10 10.
1   12      5.          5.                                                5.
1   22      10.         5.                                                5.
    1    1    2   13   12    1 2.5
   10   10   11   22   21    1 2.5
   11   22 100.
END
```

APPENDIX A - USER'S MANUAL

I. IDEALIZATION

The two dimensional region defining a body in plane stress or plane strain, or the meridional plane of an axisymmetrically loaded body of revolution, must be discretized by a set of nodal points described by x-y or r-z coordinates, respectively. The triangular or quadrilateral subregions bounded by the lines interconnecting the nodes form the "finite elements" of the structure. The layout of this "mesh" is the critical part of the analysis and requires judgment and experience of the stress analyst. The local and global fineness of the mesh is governed by the regions of principal interest, degree of accuracy sought, as well as the capacity of the program.

One limiting consideration is the band width of the problem. The largest difference between node numbers of any element may not exceed 26 in the given code. Thus, node numbers should be assigned in a regular pattern. For maximum efficiency, nodes should be numbered across the narrower width of the region.

The maximum number of nodes, elements, materials, sets of temperature properties, boundary cards, and band width listed here correspond to the dimension statements in the included listing. They may be changed as the capacity of the computer permits. The corresponding DATA statements should also be changed.

This program is dimensioned for a 32K IBM 7040-7094 DCS system. Overlay is required, and the appropriate levels are defined by the ORIGIN cards in the listing. It has been converted to the CDC 6400 where with a 32K core and 55000 octal cells available, 600 nodes and

500 elements can be accomodated without overlay. Only the dimensions of CODE, X, Y, UX, UY, T, IX, ANGLE need be changed, as well as the corresponding data statements MAXNP and MAXEL.

II. INPUT DATA

A. Identification Card (A6,66H)

This card must start with the program title PALØS, followed by a space. Columns 7-72 of this card may contain title information for the problem which is printed at the top of each page of output.

B. Control Card (6I5,10X,4F10.0)*

Columns	1-5	Number of nodal points (1100 maximum)
	6-10	Number of elements (1000 maximum)
	11-15	Number of different materials (25 maximum)
	16-20	Number of boundary pressure cards (200 maximum)
	25	Program option 0 axisymmetric analysis 1 plane strain analysis 2 plane stress analysis
	30	If this column contains a 1, the program will only generate and output the input data, and then go to the next problem. If 0 or blank, it will proceed with the solution of the problem.

*All fixed point variables (indicated by In formats) must be right adjusted in the field of width n columns.

41-50 Reference temperature (stress free temperature)

51-60 Acceleration in the x direction

61-70 Acceleration in the y direction (or the z direction in the axisymmetrics case)

71-80 Angular velocity

C. Material Property Information

The following group of cards must be supplied for each different material:

Material Control Card - (2I5,F10.0,5X,A5,48H)

Columns

1-5 Material identification (number from 1 to 25)

6-10 Number of different temperatures for which properties are given (up to 4)

11-20 Mass density of the material

26-30 Enter the word SHELL for a shell element or SØLID for an elasticity element. If blank, an elasticity element will be inferred.

31-78 Title information identifying the material to be printed with input.

80 One if yield criteria to be specified.

Following cards - two for each temperature (three if yield criterion specified).

First Card (8F10.0)

Columns

1-10 Temperature

Columns	Elasticity Element				Shell Element	Isotropic Value
11-20	C_{11}:	C_{rr}	or	C_{xx}	C_{ss}	$\lambda + 2\mu$
21-30	C_{12}:	$C_{r\theta}$	or	C_{xz}	$C_{s\theta}$	λ
31-40	C_{13}:	C_{rz}	or	C_{xy}	C_{xn}	λ
41-50	C_{22}:	$C_{\theta\theta}$	or	C_{zz}	$C_{\theta\theta}$	$\lambda + 2\mu$
51-60	C_{23}:	$C_{\theta z}$	or	C_{zy}	$C_{\theta n}$	λ
61-70	C_{33}:	C_{zz}	or	C_{yy}	C_{nn}	$\lambda + 2\mu$
71-80	C_{44}:	G_{rz}	or	G_{xy}	$G_{s\theta}$	μ

Second Card (3F10.0)

1-10	β_1:	β_r	or	β_x	β_s	$(3\lambda + 2\mu)\alpha$
11-20	β_2:	β_θ	or	β_z	β_θ	$(3\lambda + 2\mu)\alpha$
21-30	β_3:	β_z	or	β_y	β_n	$(3\lambda + 2\mu)\alpha$

All orthotropic properties are assumed to be with respect to the local axes of orthotropy. The $[\beta]$ vector is the set of coefficients of T in the stress-strain law. It is the matrix product of the (3×3) moduli matrix $[C]$ times the vector $[\alpha]$ of orthotropic coefficients of linear expansion. The Lame constants λ, μ for the isotropic case are related to the engineering constants E, ν by

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

$$\mu = \frac{E}{2(1+\nu)}$$

For the case of plane stress ($\tau_{zz} = 0$), the properties will be modified accordingly unless C_{zz} in columns 41-50 is zero, indicating that they have already been modified. In the case of a shell element the properties are modified for the assumed state of plane stress through the thickness, unless C_{nn} in columns 61-70 are zero indicating they have already been modified.

Third Card (7F10.0) - only if yield criterion to be specified

Columns	1-10	Z_{11}:	Z_{rr-rr}	or	Z_{xx-xx}
	11-20	Z_{12}:	$Z_{rr-\theta\theta}$	or	Z_{xx-zz}
	21-30	Z_{13}:	Z_{rr-zz}	or	Z_{xx-yy}
	31-40	Z_{22}:	$Z_{\theta\theta-\theta\theta}$	or	Z_{zz-zz}
	41-50	Z_{23}:	$Z_{\theta\theta-zz}$	or	Z_{zz-yy}
	51-60	Z_{33}:	Z_{zz-zz}	or	Z_{yy-yy}
	61-70	Z_{44}:	Z_{rz-rz}	or	Z_{xy-xy}

where for any solid material, yield will occur when

$$[\tau]^T [Z][\tau] = 1$$

and the $[\tau]$ are the three normal and shear stresses defined earlier by (2).

D. Nodal Point Cards (I1,I4,3X,2I1,6F10.0)

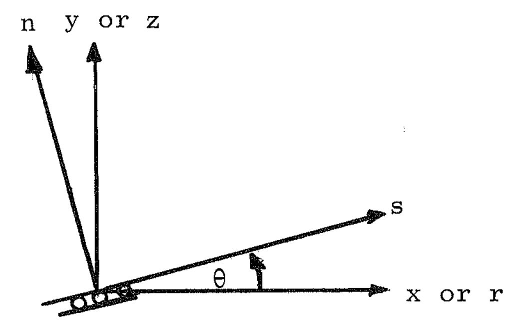
Columns	1	A 1 specifies that the coordinates to follow are polar (ρ,θ), Cartesian if blank
	2-5	Nodal point number
	9	Put 1 if z or y displacement to be specified
	10	Put 1 if r or x displacement to be specified

If zero or blank, then corresponding force will be specified. The nodal point force is the total force per unit of thickness or per radian of circumference.

If the node is a "dummy" shell node (see next section), then put 1 in column 10 if shell bending moment will be specified. If zero or blank, rotation will be specified.

Columns	11-20	r or x ordinate, or polar ρ
	21-30	z or y ordinate, or polar θ (degrees)
	31-40	r or x force or displacement (for "dummy" shell node rotation or moment)
	41-50	z or y force or displacement
	51-60	temperature (shell thickness if "dummy" shell node)
	61-70	boundary angle (degrees from r or x axis)

If a boundary point is constrained to move in a direction s, input the angle (in degrees) from r or x axis positive as shown.



$0 \leq \theta < 360^{\circ}$

In this case leave columns 9 and 10 blank. Columns 31-40 are then the s force and columns 41-50 are the n displacement.

Nodal point cards must be in numerical sequence. If cards are omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. However, if the end point of the interval is in polar coordinates, the generated points are set at equal increments of angle by an interpolation formula linear in θ . Nodal temperatures are also interpolated linearly. If the end point boundary codes (9 and 10) are the same at each end of the interval, that code will be assigned to the generated nodes. If they are different, the codes will be assumed zero. The generated nodal point forces or displacements will be zero.

E. <u>Element Cards</u> (6I5,F10.0)

One card for each element

Columns 1-5 Element number

6-10 Nodal point I

11-15 Nodal point J

16-20 Nodal point K

21-25 Nodal point L

29-30 Material number

31-40 Local angle of orthotropy (degrees from x or r axis)

1. Solid Element

An element may be triangular or quadrilateral, depending on whether it is defined by three or four nodes respectively. If the global axes transform from r to z or x to y by a counter-clockwise rotation, then the nodal sequence I-J-K-L must be a counterclockwise one around the element. If the element is a triangle, set L = K.

2. Shell Element

To facilitate internal coding, shell elements must also be defined by four nodes. Two nodes must be coincident at each end of the element. It is recommended that these nodes be defined at the shell solid interface. One pair of "real" nodes contains translational boundary conditions of the shell nodes and the nodal point temperatures. The other "dummy" nodes contain the rotational boundary conditions and the shell thicknesses. On the element card, the nodes must be permuted such that

I = real node

J = dummy node at I

K = dummy node at L

L = real node

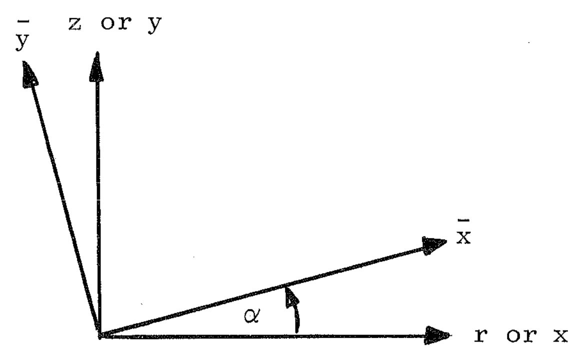
The shell element is assumed to be of constant thickness, (the average of the thicknesses specified at the two ends of the element).

3. Band Width

The maximum difference between any pair of I, J, K, L may not exceed 26.

4. Angle of Local Orthotropy

If the axes of orthotropy of an element are different from the global axes, that angle (in degrees) from the r or x axis is input positive as shown.

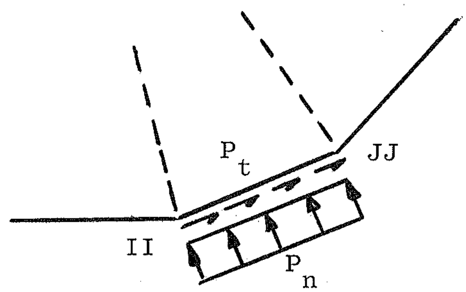


Element cards must be in numerical sequence. If cards are omitted, elements are generated by incrementing I, J, K and L of the previous element by one. The material identification code and the local angle of orthotropy are set equal to that of the previous card. The last element of the problem must always be input.

F. Pressure Cards (2I5,2F10.0)

One card for each boundary segment which is subjected to a normal or tangential pressure.

Columns	1-5	Nodal point II
	6-10	Nodal point JJ
	11-20	Normal pressure
	21-30	Tangential pressure



The boundary element must be on the left as one progresses from II to JJ. Surface tensile force is input as a negative pressure. For pressure on a shell element II corresponds to node L and JJ to node I of the element card.

III. OUTPUT INFORMATION

The following information is output by the computer.

A. Printout of the input data: Polar coordinate data is converted to Cartesian form. Plane stress and shell material properties are printed both for the input and modified form.

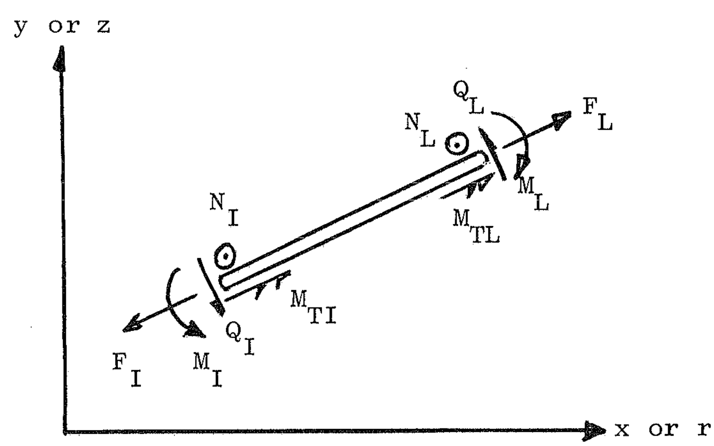
B. Printout of the computed nodal point displacements.

C. Printout for an elasticity element at its centroid of the stresses and strains in the following coordinate systems:

1. global
2. principal (with separate angle for stresses and strains)
3. normal and tangential to J-K face,

plus the strain energy density and the yield criterion $[\tau]^T [Z][\tau]$.

D. Printout for each end of a shell element of stress resultants:



where hoop forces N_I, N_L, and hoop moments M_{TI} and M_{TL} exist only for the axisymmetric shell.

E. Printout of the total work done by the external forces per unit thickness or per radian of arc.

APPENDIX B - PROGRAM LISTINGS

DECKS

Main Level

1. PALØS
2. MØDU

Level 1

3. MESH

Level 2

4. BLØK
5. MØDI
6. QUDF
7. TRIS
8. INTE
9. SHERM

Level 3

10. BCSB
11. STRS
12. SSTR

```
$IBFTC PALOS   DECK,LIST
C
C*****     AXISYMMETRIC, PLANE STRAIN, OR PLANE STRESS LOCALLY
C*****     ORTHOTROPIC WITH SHELL
C
      COMMON
     1 A(54,108), B(108), NB, ND, ND2, MBAND, NUMBLK
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON /YIELD/
     1 IZD(25),ZD(7,25)
      DATA
     1 NTEMP/4/,SOLID/6H SOLID/,SHELL/6H SHELL/
C
C*****     A PROGRAM BY WILSON, TAYLOR, NICKELL, DUNHAM AND GOUDREAU
C
      NB=27
      ND=54
      ND2=108
   10 CONTINUE
      CALL LOCATE
      IF (ISTOP.NE.0) GO TO 10
      CALL BLOCKS
      IF (ISTOP.NE.0) GO TO 10
      ENERG=0.0
      CALL BACSUB
      CALL STRESS
      IF (ISTOP.NE.0) GO TO 10
      CALL SHLSTR
      ENERG=0.5*ENERG
      WRITE (6,2000) ENERG
      GO TO 10
C
 2000 FORMAT (36H1WORK OF FORCE BOUNDARY CONDITIONS =E15.5)
      END
```

```
$IBFTC MODU     DECK,LIST
      SUBROUTINE MODULI (N,I,J,K,L)
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON/QADTRI/
     1 XXX(5), YYY(5), EE(13), C(4,4), TT(4)
      EQUIVALENCE
     1 (C11,EE(1)),(C12,EE(2)),(C13,EE(3)),(C22,EE(4)),(C23,EE(5)),
     2 (C33,EE(6)),(C44,EE(7))
      EQUIVALENCE
     1 (EE(1),CSS,A11),(EE(2),CST,A12),(EE(3),A22),(EE(4),CTT,D11),
     2 (EE(5),D12),(EE(6),D22),(EE(7),D22B),(EE(8),BETS),(EE(9),BETT)
C
C*****    INITIALIZATION
C
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPE=IABS(IX(N,5))
      IX(N,5)=-IX(N,5)
      IF (TYPE(MTYPE).EQ.SHELL) GO TO 10
      IF (K.EQ.L) TEMP=(T(I)+T(J)+T(K))/3.0
      IF (K.NE.L) TEMP=0.25*(T(I)+T(J)+T(K)+T(L))
      GO TO 25
   10 TA=0.5*(T(J)+T(K))
      IF (TA.GT.0.0) GO TO 20
      WRITE (6,3000) N
      ISTOP=1
      RETURN
   20 TEMP=0.5*(T(I)+T(L))
   25 CONTINUE
C
C**** FORM STRESS STRAIN RELATIONS
C
      DO 30 M=2,NTEMP
      IF(E(M,1,MTYPE).GE.TEMP) GO TO 40
   30 CONTINUE
   40 RATIO=0.0
      DEN=E(M,1,MTYPE)-E(M-1,1,MTYPE)
      IF(DEN.EQ.0.0) GO TO 50
      RATIO=(TEMP-E(M-1,1,MTYPE))/DEN
   50 DO 60 KK=1,10
   60 EE(KK)=E(M-1,KK+1,MTYPE)+RATIO*(E(M,KK+1,MTYPE)-E(M-1,KK+1,MTYPE))
      TEMP=TEMP-TR
      IF (TYPE(MTYPE).EQ.SHELL) GO TO 100
C
C**** QUADRILATERAL ELEMENT
C     EE(1-10)=CRR,CRT,CRZ,CTT,CTZ,CZZ,GRZ,BRR,BTT,BZZ
C     EE(1-10)=CXX,CXZ,CXY,CZZ,CZY,CYY,GXY,BXX,BZZ,BYY
C
C*****    ROTATE TO GLOBAL STIFFNESSES
```

```
C
      CS=COS(ANGLE(N))
      SN=SIN(ANGLE(N))
      CS2=CS**2
      SN2=SN**2
      CS4=CS2**2
      SN4=SN2**2
      TT(1)=EE(8)*CS2+EE(10)*SN2
      TT(2)=EE(9)
      TT(3)=EE(8)*SN2+EE(10)*CS2
      TT(4)=(EE(8)-EE(10))*CS*SN
      DO 70 M=1,4
   70 TT(M)=TT(M)*TEMP
      TEMP=2.*(C13+2.*C44)*CS2*SN2
      C(1,1)=C11*CS4+C33*SN4+TEMP
      C(3,3)=C33*CS4+C11*SN4+TEMP
      C(1,3)=C13*(CS4+SN4)+(C11+C33-4.*C44)*CS2*SN2
      TEMP=(C13+2.*C44)*(CS2-SN2)
      C(1,4)=( C11*CS2-C33*SN2-TEMP)*CS*SN
      C(3,4)=(-C33*CS2+C11*SN2+TEMP)*CS*SN
      C(4,4)=C44*(CS2-SN2)**2+(C11-2.*C13+C33)*CS2*SN2
      C(2,2)=C22
      C(1,2)=C12*CS2+C23*SN2
      C(2,3)=C23*CS2+C12*SN2
      C(2,4)=(C12-C23)*CS*SN
      C(2,1)=C(1,2)
      C(3,1)=C(1,3)
      C(3,2)=C(2,3)
      C(4,1)=C(1,4)
      C(4,2)=C(2,4)
      C(4,3)=C(3,4)
      RETURN
C
C**** SHELL ELEMENT
C
  100 A11=TA*CSS
      A12=TA*CST
      A22=TA*CTT
      DEN=TA*TA/12.
      D11=DEN*A11
      D12=DEN*A12
      D22=DEN*A22
      D22B=D22-D12**2/D11
      BETS=BETS*TA*TEMP
      BETT=BETT*TA*TEMP
      RETURN
C
 3000 FORMAT (31H3ZERO SHELL THICKNESS, ELEMENT=I3)
      END
```

```
$ORIGIN        LEVEL1
$IBFTC MESH    DECK,LIST
      SUBROUTINE LOCATE
C
C*****     FORMATION OF MESH LAYOUT
C
      COMMON
     1 A(54,108), B(108), NB, ND, ND2, MBAND, NUMBLK
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      DATA MAXNP/1100/, MAXEL/1000/
      COMMON /YIELD/
     1 IZD(25),ZD(7,25)
      DIMENSION
     1 OPTION(3,3),IBC(200),JBC(200),PN(200),PT(200),XTYPE(8),XC(2),
     2 FX(2),FY(2),IN(2),ANG(1)
      EQUIVALENCE
     1 (IBC,A),(JBC,A(1,5)),(PN,A(1,9)),(PT,A(1,13)),(ANG,A(1,17))
      DATA
     1 HED(15)/6H STRUC/,HED(16)/6HTURE 1/,HED(17)/6H-2-3 =/,
     2OPTION(1,1)/6HAXISYM/,OPTION(2,1)/6HMETRIC/,OPTION(3,1)/6H R-T-Z/,
     3OPTION(1,2)/6HPLANE /,OPTION(2,2)/6HSTRAIN/,OPTION(3,2)/6H X-Z-Y/,
     4OPTION(1,3)/6HPLANE /,OPTION(2,3)/6HSTRESS/,OPTION(3,3)/6H X-Z-Y/,
     5 PI/0.017453293/,BLANK/6H      /,PALOS/6HPALOS /,END/6HEND   /
      INTEGER TAG
C
C**** READ AND PRINT CONTROL INFORMATION
C
    5 READ (5,1000) (HED(I),I=1,12)
      IF (HED(1).EQ.END) STOP
      IF (HED(1).NE.PALOS) GO TO 5
      READ (5,1001) NUMNP,NUMEL,NUMMAT,NUMPC,NPP,ISTOP,TR,ACEL1,ACEL3,
     1 ANGFQ
      HED(13)=OPTION(1,NPP+1)
      HED(14)=OPTION(2,NPP+1)
      HED(18)=OPTION(3,NPP+1)
      WRITE (6,2000) HED,NUMNP,NUMEL,NUMMAT,NUMPC,TR,ACEL1,ACEL3,ANGFQ
      IF (NUMNP.LE.MAXNP) GO TO 6
      WRITE (6,3000) MAXNP
      ISTOP=1
    6 IF (NUMEL.LE.MAXEL) GO TO 8
      WRITE (6,3001) MAXEL
      ISTOP=1
      RETURN
    8 CONTINUE
C
C**** READ AND PRINT MATERIAL PROPERTIES
C
      MPRINT=0
      DO 20 M=1,NUMMAT
      READ (5,1002) MTYPE,NUMTC,RO(MTYPE),TYPE(MTYPE),XTYPE,IZD(MTYPE)
      IF (NUMTC.LE.0) NUMTC=1
```

```
      IF (TYPE(MTYPE).EQ.BLANK) TYPE(MTYPE)=SOLID
      MPRINT=MPRINT-3*(1+NUMTC+IZD(MTYPE))
      IF (NPP.EQ.2.OR.TYPE(MTYPE).EQ.SHELL) MPRINT=MPRINT-3*NUMTC
      IF(MPRINT.GT.0) GO TO 10
      MPRINT=50
      WRITE (6,2001) HED
   10 WRITE (6,2002) MTYPE,NUMTC,RO(MTYPE),TYPE(MTYPE),XTYPE
      DO 16 I=1,NUMTC
      READ  (5,1003) (E(I,J,MTYPE),J=1,11)
      WRITE (6,2003) (E(I,J,MTYPE),J=1,11)
C
C**** MODIFY FOR PLANE STRESS
C
      IF (NPP.NE.2) GO TO 12
      IF(E(I,5,MTYPE).EQ.0.0) GO TO 12
      EA=1./E(I,5,MTYPE)
      E(I,2,MTYPE)=E(I,2,MTYPE)-E(I,3,MTYPE)**2*EA
      E(I,4,MTYPE)=E(I,4,MTYPE)-E(I,3,MTYPE)*E(I,6,MTYPE)*EA
      E(I,7,MTYPE)=E(I,7,MTYPE)-E(I,6,MTYPE)**2*EA
      E(I,9,MTYPE)=E(I,9,MTYPE)-E(I,3,MTYPE)*E(I,10,MTYPE)*EA
      E(I,11,MTYPE)=E(I,11,MTYPE)-E(I,6,MTYPE)*F(I,10,MTYPE)*EA
      E(I,3,MTYPE)=0.0
      E(I,5,MTYPE)=0.0
      E(I,6,MTYPE)=0.0
      E(I,10,MTYPE)=0.0
   12 CONTINUE
C
      IF (TYPE(MTYPE).NE.SHELL) GO TO 14
      IF (E(I,7,MTYPE).EQ.0.0) GO TO 14
      EA=1./E(I,7,MTYPE)
      E(I,2,MTYPE)=E(I,2,MTYPE)-E(I,4,MTYPE)**2*EA
      E(I,3,MTYPE)=E(I,3,MTYPE)-E(I,4,MTYPE)*E(I,6,MTYPE)*EA
      E(I,5,MTYPE)=E(I,5,MTYPE)-E(I,6,MTYPE)**2*EA
      E(I,9,MTYPE)=E(I,9,MTYPE)-E(I,4,MTYPE)*E(I,11,MTYPE)*EA
      E(I,10,MTYPE)=E(I,10,MTYPE)-E(I,6,MTYPE)*E(I,11,MTYPE)*EA
      E(I,4,MTYPE)=0.0
      E(I,6,MTYPE)=0.0
      E(I,7,MTYPE)=0.0
      E(I,11,MTYPE)=0.0
   14 CONTINUE
C
      IF (NPP.EQ.2.OR.TYPE(MTYPE).EQ.SHELL)
     1 WRITE (6,2003) (E(I,J,MTYPE),J=1,11)
C
   16 CONTINUE
C
      DO 18 I=NUMTC,NTEMP
      DO 18 J=1,11
   18 E(I,J,MTYPE)=E(NUMTC,J,MTYPE)
C
C**** READ YIELD CRITERIA
C
      IF(IZD(MTYPE).EQ.0) GO TO 20
      READ  (5,1003) (ZD(I,MTYPE),I=1,7)
      WRITE (6,2004) (ZD(I,MTYPE),I=1,7)
```

```
   20 CONTINUE
C
C**** READ AND PRINT NODAL POINT DATA
C
      N=0
      MPRINT=0
      DO 95 L=1,NUMNP
      IF(L-N) 70,75,55
   55 READ(5,1006) TAG,N,CODE(N),X(N),Y(N),UX(2*N-1),UX(2*N),T(N),ANG(N)
      IF (TAG.GT.0) Y(N)=PI*Y(N)
      CODE(N)=ABS(CODE(N))
      DV=X(N)
      DU=Y(N)
      IF (L-N) 60,80,100
   60 DZ=N-L+1
      DT=(T(N)-T(L-1))/DZ
      IF (TAG.GT.0) GO TO 65
      DX=(X(N)-X(L-1))/DZ
      DY=(Y(N)-Y(L-1))/DZ
      GO TO 70
   65 DV=SQRT(X(L-1)**2+Y(L-1)**2)
      DU=ATAN2(Y(L-1),X(L-1))
      DX=(X(N)-DV)/DZ
      DY=(Y(N)-DU)/DZ
   70 CODE(L)=0.0
      IF(CODE(N).EQ.CODE(L-1)) CODE(L)=CODE(L-1)
      IF (ANG(N).EQ.ANG(L-1)) ANG(L)=ANG(L-1)
      UX(2*L-1)=0.0
      UX(2*L)=0.0
      T(L)=T(L-1)+DT
      IF (TAG.GT.0) GO TO 75
      X(L)=X(L-1)+DX
      Y(L)=Y(L-1)+DY
      GO TO 85
   75 DV=DV+DX
      DU=DU+DY
   80 IF (TAG.EQ.0) GO TO 85
      X(L)=DV*COS(DU)
      Y(L)=DV*SIN(DU)
   85 IF (MPRINT.GT.0) GO TO 90
      MPRINT=50
      WRITE (6,2005) HED
   90 WRITE (6,2006) L,CODE(L),X(L),Y(L),UX(2*L-1),UX(2*L),T(L),ANG(L)
      IF (ANG(L).NE.0.0) CODE(L)=-ANG(L)*PI
   95 MPRINT=MPRINT-1
      GO TO 110
  100 WRITE (6,3002) N
      ISTOP=1
  110 CONTINUE
C
C**** READ AND PRINT ELEMENT PROPERTIES
C
      N=0
      MPRINT=0
  130 READ (5,1008) M,(IX(M,I),I=1,5),ANGLE(M)
```

```
  140 N=N+1
      IF(M.LE.N) GO TO 160
      IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
      ANGLE(N)=ANGLE(N-1)
  160 IF(MPRINT.GT.0) GO TO 170
      MPRINT=50
      WRITE (6,2007) HED
  170 WRITE (6,2008) N,(IX(N,I),I=1,5),ANGLE(N)
      MPRINT=MPRINT-1
      IF (M-N) 185,180,140
  180 IF (NUMEL-N) 185,190,130
  185 WRITE (6,3003) N
      ISTOP=1
  190 CONTINUE
C
C**** READ AND PRINT PRESSURE BOUNDARY CONDITIONS
C
      IF (NUMPC.EQ.0) GO TO 310
      MPRINT=0
      DO 300 L=1,NUMPC
      READ  (5,1010) IBC(L),JBC(L),PN(L),PT(L)
      IF(MPRINT.GT.0) GO TO 290
      MPRINT=50
      WRITE (6,2009) HED
  290 WRITE (6,2010) IBC(L),JBC(L),PN(L),PT(L)
  300 MPRINT=MPRINT-1
  310 CONTINUE
C
C**** DETERMINE BAND WIDTH AND COMPUTE SURFACE INTEGRALS
C
      MB=0
      DO 340 N=1,NUMEL
      ANGLE(N)=PI*ANGLE(N)
      DO 340 I=1,4
      IF (NUMPC.LE.0) GO TO 240
      J=I+1
      IF (I.EQ.4) J=1
      K=IX(N,I)
      L=IX(N,J)
      DO 230 MM=1,NUMPC
      IF (K.NE.IBC(MM).OR.L.NE.JBC(MM)) GO TO 230
      IBC(MM)=0
      XC(1)=CODE(K)
      XC(2)=CODE(L)
      IN(1)=K
      IN(2)=L
      DX=X(L)-X(K)
      DY=Y(L)-Y(K)
      XX=DX/6.0
      YY=0.5*X(K)+XX
      XX=1.0+XX/YY
```

```
      IF(NPP.NE.0) XX=1.0
      IF(NPP.NE.0) YY=0.5
      FX(1)=YY*(PT(MM)*DX-PN(MM)*DY)
      FX(2)=XX*FX(1)
      FY(1)=YY*(PT(MM)*DY+PN(MM)*DX)
      FY(2)=XX*FY(1)
      DO 225 K=1,2
      L=IN(K)
      C=XC(K)
      IF(C.GE.0.0) GO TO 205
      UX(2*L-1)=UX(2*L-1)+FX(K)*COS(C)-FY(K)*SIN(C)
      GO TO 225
  205 IF(C.EQ.1.0.OR.C.EQ.11.0) GO TO 210
      UX(2*L-1)=UX(2*L-1)+FX(K)
  210 IF(C.EQ.10.0.OR.C.EQ.11.0) GO TO 225
      UX(2*L)=UX(2*L)+FY(K)
  225 CONTINUE
  230 CONTINUE
  240 DO 325 J=1,4
      K=IABS(IX(N,I)-IX(N,J))
      IF (K.GT.MB) MB=K
  325 CONTINUE
  340 CONTINUE
      IF(NUMPC.EQ.0) GO TO 355
      DO 350 MM=1,NUMPC
      IF(IBC(MM).EQ.0) GO TO 350
      WRITE (6,3004) MM
      ISTOP=1
  350 CONTINUE
  355 MBAND=2*MB+2
      IF(MBAND.LE.ND) GO TO 360
      WRITE (6,3005) MBAND
      ISTOP=1
  360 RETURN
C
 1000 FORMAT (12A6)
 1001 FORMAT (6I5,10X,4F10.0)
 1002 FORMAT (2I5,F10.0,4X,9A6,I2)
 1003 FORMAT (8F10.0)
 1006 FORMAT (I1,I4,F5.0,6F10.0)
 1008 FORMAT (6I5,F10.0)
 1010 FORMAT (2I5,2F10.0)
 2000 FORMAT (1H1 12A6,10X 6A6/
     1    29H0NUMBER OF NODAL POINTS------ I3/
     2    29H0NUMBER OF ELEMENTS---------- I3/
     3    29H0NUMBER OF DIFF. MATERIALS--- I3/
     4    29H0NUMBER OF PRES./SHEAR CARDS- I3/
     5    29H0REFERENCE TEMPERATURE------- F12.2/
     6    29H0X1 ACCELERATION------------- E12.4/
     7    29H0X3 ACCELERATION------------- E12.4/
     8    29H0ANGULAR VELOCITY------------ E12.4/)
 2001 FORMAT (1H1 12A6,15X 6A6)
 2002 FORMAT (17H0MATERIAL NUMBER=I3,23H, NUMBER OF TEMP CARDS=I3,
    1 15H, MASS DENSITY=1PE11.3,4X 9A6/
    2 15H0   TEMPERATURE 11X4HC-11 11X4HC-12 11X4HC-13 11X4HC-22
```

```
     3 11X4HC-23 11X4HC-33 11X4HG-13 / 26X4HB-11 11X4HB-22 11X4HB-33 )
2003 FORMAT (1H0 F14.2,7E15.5/15X,3E15.5)
2004 FORMAT (19H0YIELD COEFFICIENTS 7X4HZ-11 11X4HZ-12 11X4HZ-13 11X
     1 4HZ-22 11X4HZ-23 11X4HZ-33 10X5HZB-13 /1H0 14X,7E15.5)
2005 FORMAT (1H1 12A6,10X 6A6/
     1        124H0NODAL POINT        CODE X1-ORDINATE X3-ORDINATE X1-LOA
     2D OR DISPLACEMENT X3-LOAD OR DISPLACEMENT   TEMPERATURE          AN
     3GLE/1H0)
2006 FORMAT(I12,F12.0,2F12.4,2E24.7,2F14.4)
2007 FORMAT (1H1 12A6,10X 6A6/
     1        62H0     ELEMENT      I      J      K      L      MATERIAL
     2 ANGLE/1H0)
2008 FORMAT (1I13, 4I6, 1I13,E20.7)
2009 FORMAT (1H1 12A6,10X 6A6/
     1 42H0PRESSURE AND/OR SHEAR BOUNDARY CONDITIONS /1H0 4X1HI 4X1HJ
     2 7X8HPRESSURE 10X5HSHEAR /1H0)
2010 FORMAT (2I5,2F15.5)
3000 FORMAT (24H0NUMBER OF NODES EXCEEDS I5)
3001 FORMAT (27H0NUMBER OF ELEMENTS EXCEEDS I5)
3002 FORMAT (26H0NODAL POINT CARD ERROR N= I5)
3003 FORMAT (23H0ELEMENT CARD ERROR, N=I5)
3004 FORMAT (24H0PRESSURE CARD ERROR, N= I5)
3005 FORMAT (27H0BANDWIDTH EXCEEDED, MBAND=I5)
      END
```

```
$ORIGIN         LEVEL1
$IBFTC BLOK     DECK,LIST
      SUBROUTINE BLOCKS
      COMMON
     1 A(54,108), B(108), NB, ND, ND2, MBAND, NUMBLK
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON /QUADST/
     1 S(10,10), P(10)
      DIMENSION LM(4)
C
C*****    INITIALIZATION
C
      NUMBLK=0
      DO 50 N=1,ND2
      B(N)=0.0
      DO 50 M=1,ND
   50 A(M,N)=0.0
C
C*****    FORM STIFFNESS MATRIX IN BLOCKS
C
   60 NUMBLK=NUMBLK+1
      NM=NB*NUMBLK
      NL=NM-NB+1
      KSHIFT=2*NL-2
      IF (NM.GT.NUMNP) NM=NUMNP
C
C*****    SELECT ELEMENT IN BLOCK
C
      DO 210 N=1, NUMEL
      MTYPE=IX(N,5)
      IF(MTYPE.LE.0) GO TO 210
      DO 80 I=1,4
      IF (IX(N,I).LT.NL) GO TO 80
      IF (IX(N,I).LE.NM) GO TO 90
   80 CONTINUE
      GO TO 210
   90 DO 5 I=1,10
      P(I)=0.0
      DO 5 J=1,10
    5 S(I,J)=0.0
      IF (TYPE(MTYPE).EQ.SOLID) CALL QUAD(N)
      IF (TYPE(MTYPE).EQ.SHELL) CALL SHERM(N)
      IF (ISTOP.NE.0) GO TO 210
C
C*****    MODIFY FOR SLOPING BOUNDARY CONDITIONS
C
      NNN=4
      IF(TYPE(MTYPE).EQ.SHELL) NNN=2
      DO 400 NN=1,NNN
      MM=IX(N,NN)
      IF(CODE(MM).GE.0.0) GO TO 400
      CC=-CODE(MM)
```

```
      DX=COS(CC)
      DY=SIN(CC)
      II=2*NN-1
      DEN=P(II)*DX+P(II+1)*DY
      P(II+1)=P(II+1)*DX-P(II)*DY
      P(II)=DEN
      DO 420 J=1,8,2
      IF(J.EQ.II) GO TO 420
      DEN=S(II,J)*DX+S(II+1,J)*DY
      S(II+1,J)=S(II+1,J)*DX-S(II,J)*DY
      S(II,J)=DEN
      S(J,II+1)=S(II+1,J)
      S(J,II)=S(II,J)
      DEN=S(II,J+1)*DX+S(II+1,J+1)*DY
      S(II+1,J+1)=S(II+1,J+1)*DX-S(II,J+1)*DY
      S(II,J+1)=DEN
      S(J+1,II+1)=S(II+1,J+1)
      S(J+1,II)=S(II,J+1)
  420 CONTINUE
      DEN=S(II,II)*DX*DX+2.0*S(II+1,II)*DX*DY+S(II+1,II+1)*DY*DY
      S(II,II+1)=DX*DY*(S(II+1,II+1)-S(II,II))+S(II+1,II)*(DX*DX-DY*DY)
      S(II+1,II+1)=S(II,II)*DY*DY-2.*S(II+1,II)*DX*DY+S(II+1,II+1)*DX*DX
      S(II,II)=DEN
      S(II+1,II)=S(II,II+1)
  400 CONTINUE
C
C*****   ADD STIFFNESS AND FORCE VECTOR
C
      DO 166 I=1,4
  166 LM(I)=2*IX(N,I)-2
      DO 200 I=1,4
      DO 200 K=1,2
      II=LM(I)+K-KSHIFT
      KK=2*I-2+K
      B(II)=B(II)+P(KK)
      DO 200 J=1,4
      DO 200 L=1,2
      JJ=LM(J)+L-II+1-KSHIFT
      LL=2*J-2+L
      IF(JJ) 200,200,195
  195 A(JJ,II)=A(JJ,II)+S(KK,LL)
  200 CONTINUE
  210 CONTINUE
      IF (ISTOP.NE.0) GO TO 560
C
C*****    ADD CONCENTRATED FORCES, MODIFY FOR DISPLACEMENT B. C.
C
      DO 250 N=NL,NM
      K=2*N-KSHIFT-1
      C=CODE(N)
      IF (C.EQ.0.0) GO TO 240
      IF (C.EQ.1.0.OR.C.EQ.11.0) CALL MODIFY (K,UX(2*N-1))
      IF (C.LT.0.0.OR.C.EQ.10.0.OR.C.EQ.11.0) CALL MODIFY (K+1,UX(2*N))
  240 B(K)=B(K)+UX(2*N-1)
      B(K+1)=B(K+1)+UX(2*N)
```

```
  250 CONTINUE
C
C*****    REDUCE BLOCK OF EQUATIONS
C
      DO 550 N=1,ND
      IF(A(1,N).EQ.0.) GO TO 550
      B(N)=B(N)/A(1,N)
      DO 540 L=2,MBAND
      IF(A(L,N).EQ.0.) GO TO 540
      C=A(L,N)/A(1,N)
      I=N+L-1
      J=0
      DO 530 K=L,MBAND
      J=J+1
  530 A(J,I)=A(J,I)-C*A(K,N)
      B(I)=B(I)-A(L,N)*B(N)
      A(L,N)=C
  540 CONTINUE
  550 CONTINUE
  560 IF (NM.EQ.NUMNP) RETURN
      IF (ISTOP.NE.0) GO TO 60
C
C*****    WRITE BLOCK OF REDUCED EQUATIONS ON TAPE
C
      WRITE (8) (B(J),(A(I,J),I=1,MBAND),J=1,ND)
C
C*****    SHIFT BLOCK OF EQUATIONS UP FOR NEXT BLOCK
C
      DO 575 N=1,ND
      MM=ND+N
      B(N)=B(MM)
      B(MM)=0.
      DO 575 M=1,MBAND
      A(M,N)=A(M,MM)
  575 A(M,MM)=0.0
      GO TO 60
      END
```

```
$IBFTC MODI    DECK,LIST
      SUBROUTINE MODIFY (N,X)
      COMMON
     1 A(54,108), B(108), NB, ND, ND2, MBAND, NUMBLK
      DO 250 M=2,MBAND
      K=N-M+1
      IF (K.LE.0) GO TO 235
      B(K)=B(K)-A(M,K)*X
      A(M,K)=0.0
  235 K=N+M-1
      IF(K.GT.ND2) GO TO 250
      B(K)=B(K)-A(M,N)*X
      A(M,N)=0.0
  250 CONTINUE
      A(1,N)=0.0
      B(N)=0.0
      RETURN
      END
```

```
$IBFTC QUDF     DECK,LIST
      SUBROUTINE QUAD(N)
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON /QUADST/
     1 S(10,10), P(10)
      COMMON/QADTRI/
     1 XXX(5), YYY(5), EE(13), C(4,4), TT(4)
      COMMON/INTEG/XI(10),XX(6),YY(6),COMM
C
      IF (ISTOP.NE.0) GO TO 90
C
C*****      FORM STRESS-STRAIN RELATIONSHIP
C
      CALL MODULI (N,I,J,K,L)
      MTYPE=IABS(IX(N,5))
      EE(11)=RO(MTYPE)*ACEL1
      EE(12)=RO(MTYPE)*ACEL3
      EE(13)=RO(MTYPE)*ANGFQ*ANGFQ
C
C*****    ZERO OUT C(M,2) AND C(2,M) FOR PLANE STRAIN OR STRESS
C
      IF(NPP.EQ.0) GO TO 90
      TT(2)=0.0
      DO 80 M=1,4
      C(M,2)=0.
   80 C(2,M)=0.
   90 CONTINUE
C
C*****      FORM QUADRILATERAL STIFFNESS MATRIX
C
      DO 100 M=1,4
      MM=IX(N,M)
      XXX(M)=X(MM)
  100 YYY(M)=Y(MM)
      IF (K.NE.L) GO TO 110
      CALL TRISTF(1,2,3,NPP,ISTOP)
      IF (COMM.LE.0.0) GO TO 200
      GO TO 120
  110 XXX(5)=0.25*(XXX(1)+XXX(2)+XXX(3)+XXX(4))
      YYY(5)=0.25*(YYY(1)+YYY(2)+YYY(3)+YYY(4))
      CALL TRISTF(1,2,5,NPP,ISTOP)
      IF (COMM.LE.0.0) GO TO 200
      CALL TRISTF(2,3,5,NPP,ISTOP)
      IF (COMM.LE.0.0) GO TO 200
      CALL TRISTF(3,4,5,NPP,ISTOP)
      IF (COMM.LE.0.0) GO TO 200
      CALL TRISTF(4,1,5,NPP,ISTOP)
      IF (COMM.LE.0.0) GO TO 200
  120 IF (ISTOP.NE.0) RETURN
C
C*****    SOLVE FOR MIDDLE NODAL POINT DISPLACEMENTS
```

```
C
      IF (K.EQ.L) GO TO 170
      DO 150 II=1,9
      CC=S(II,10)/S(10,10)
      P(II)=P(II)-CC*P(10)
      DO 150 JJ=1,9
  150 S(II,JJ)=S(II,JJ)-CC*S(10,JJ)
      DO 160 II=1,8
      CC=S(II,9)/S(9,9)
      P(II)=P(II)-CC*P(9)
      DO 160 JJ=1,8
  160 S(II,JJ)=S(II,JJ)-CC*S(9,JJ)
  170 CONTINUE
      RETURN
  200 WRITE (6,3000) N
      ISTOP=1
      RETURN
C
 3000 FORMAT (33H0ZERO OR NEGATIVE AREA, ELEMENT = I4)
      END
```

```
$IBFTC TRIS     DECK,LIST
      SUBROUTINE TRISTF(II,JJ,KK,NPP,ISTOP)
      COMMON /QUADST/
     1 S(10,10), P(10)
      COMMON/QADTRI/
     1 XXX(5), YYY(5), EE(13), C(4,4), TT(4)
      COMMON/INTEG/XI(10),XX(6),YY(6),COMM
      DIMENSION F(6,10),H(6,10),DD(3,3),LM(3),TP(6),D(6,6)
      DATA D/36*0.0/
C
C*****    INITIALIZATION
C
      LM(1)=II
      LM(2)=JJ
      LM(3)=KK
      XX(1)=XXX(II)
      XX(2)=XXX(JJ)
      XX(3)=XXX(KK)
      YY(1)=YYY(II)
      YY(2)=YYY(JJ)
      YY(3)=YYY(KK)
      COMM=XX(2)*(YY(3)-YY(1))+XX(1)*(YY(2)-YY(3))+XX(3)*(YY(1)-YY(2))
      IF (COMM.LE.0.0.OR.ISTOP.NE.0) RETURN
      DO 100 I=1,6
      DO 100 J=1,10
      F(I,J)=0.0
  100 H(I,J)=0.0
C
C*****    FORM INTEGRAL (G)T*(C)*(G)
C
      CALL INTER(NPP)
      D(2,6)=XI(1)*(C(1,3)+C(2,3))
      D(3,5)=XI(1)*C(4,4)+C(2,4)*XI(4)
      D(5,5)=XI(1)*C(4,4)
      D(5,6)=XI(1)*C(3,4)
      D(6,6)=XI(1)*C(3,3)
      D(3,6)=XI(1)*C(3,4)+XI(4)*C(2,3)
      D(2,5)=XI(1)*(C(1,4)+C(2,4))
      D(1,5)=XI(2)*C(2,4)
      D(1,6)=XI(2)*C(2,3)
      D(1,1)=XI(3)*C(2,2)
      D(1,2)=XI(2)*(C(1,2)+C(2,2))
      D(1,3)=XI(5)*C(2,2)+XI(2)*C(2,4)
      D(2,2)=XI(1)*(C(1,1)+2.0*C(1,2)+C(2,2))
      D(2,3)=XI(1)*(C(1,4)+C(2,4))+XI(4)*(C(1,2)+C(2,2))
      D(3,3)=XI(1)*C(4,4)+2.0*XI(4)*C(2,4)+XI(6)*C(2,2)
      DO 110 I=2,6
      DO 110 J=1,I
  110 D(I,J)=D(J,I)
C
C*****    4. FORM COEFFICIENT-DISPLACEMENT TRANSFORMATION MATRIX
C
      DD(1,1)=(XX(2)*YY(3)-XX(3)*YY(2))/COMM
      DD(1,2)=(XX(3)*YY(1)-XX(1)*YY(3))/COMM
```

```
      DD(1,3)=(XX(1)*YY(2)-XX(2)*YY(1))/COMM
      DD(2,1)=(YY(2)-YY(3))/COMM
      DD(2,2)=(YY(3)-YY(1))/COMM
      DD(2,3)=(YY(1)-YY(2))/COMM
      DD(3,1)=(XX(3)-XX(2))/COMM
      DD(3,2)=(XX(1)-XX(3))/COMM
      DD(3,3)=(XX(2)-XX(1))/COMM
      DO 120 I=1,3
      J=2*LM(I)-1
      H(1,J)=DD(1,I)
      H(2,J)=DD(2,I)
      H(3,J)=DD(3,I)
      H(4,J+1)=DD(1,I)
      H(5,J+1)=DD(2,I)
  120 H(6,J+1)=DD(3,I)
C
C*****     5. FORM ELEMENT STIFFNESS MATRIX (H)T*(D)*(H)
C
      DO 130 J=1,10
      DO 130 K=1,6
      IF (H(K,J).EQ.0.0) GO TO 130
      DO 129 I=1,6
  129 F(I,J)=F(I,J)+D(I,K)*H(K,J)
  130 CONTINUE
      DO 140 I=1,10
      DO 140 K=1,6
      IF (H(K,I).EQ.0.0) GO TO 140
      DO 139 J=1,10
  139 S(I,J)=S(I,J)+H(K,I)*F(K,J)
  140 CONTINUE
C
C*****     FORM THERMAL AND BODY FORCE VECTOR
C
      TP(1)=EE(11)*XI(1)+EE(13)*XI(7)+TT(2)*XI(2)
      TP(2)=EE(11)*XI(7)+EE(13)*XI(9)+(TT(1)+TT(2))*XI(1)
      TP(3)=EE(11)*XI(8)+EE(13)*XI(10)+TT(2)*XI(4)+TT(4)*XI(1)
      TP(4)=EE(12)*XI(1)
      TP(5)=EE(12)*XI(7)
      TP(6)=EE(12)*XI(8)+(TT(3)+TT(4))*XI(1)
      DO 160 I=1,10
      DO 160 K=1,6
      P(I)=P(I)+H(K,I)*TP(K)
  160 CONTINUE
      RETURN
      END
```

```
$IBFTC INTE    DECK,LIST
      SUBROUTINE INTER(NPP)
      COMMON/INTEG/XI(10),XX(6),YY(6),COMM
      DIMENSION X(6),Y(6),XM(6)
      EQUIVALENCE (X(1),XX(1)),(Y(1),YY(1))
      X(4)=(X(1)+X(2))/2.
      X(5)=(X(2)+X(3))/2.
      X(6)=(X(3)+X(1))/2.
      Y(4)=(Y(1)+Y(2))/2.
      Y(5)=(Y(2)+Y(3))/2.
      Y(6)=(Y(3)+Y(1))/2.
      IF (NPP) 10,30,10
   10 DO 20 I=4,6
   20 XM(I)=1.
      GO TO 40
   30 DO 35 I=4,6
   35 XM(I)=X(I)
   40 DO  50 I=1,10
   50 XI(I)=0.0
      DO 100 I=4,6
      XI(1)=XI(1)+XM(I)
      XI(7)=XI(7)+XM(I)*X(I)
      XI(8)=XI(8)+XM(I)*Y(I)
      IF(NPP.NE.0) GO TO 100
      IF(X(I).EQ.0.0) GO TO 100
      XI(2)=XI(2)+XM(I)/X(I)
      XI(3)=XI(3)+XM(I)/(X(I)**2)
      XI(4)=XI(4)+XM(I)*Y(I)/X(I)
      XI(5)=XI(5)+XM(I)*Y(I)/(X(I)**2)
      XI(6)=XI(6)+XM(I)*Y(I)**2/(X(I)**2)
      XI(9)=XI(9)+XM(I)*X(I)**2
      XI(10)=XI(10)+XM(I)*X(I)*Y(I)
  100 CONTINUE
      DO 150 I=1,10
  150 XI(I)=XI(I)*COMM/6.
      RETURN
      END
```

```
$IBFTC SHEL    DECK,LIST
      SUBROUTINE SHERM(N)
C
C*****    SHELL STIFFNESS SUBROUTINE USING HERRMANN FORMULATION
C
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON /QUADST/
     1 S(10,10), P(10)
      DIMENSION
     1 R(1),Z(1)
      COMMON/QADTRI/
     1 XXX(5), YYY(5), EE(13), C(4,4), TT(4)
      EQUIVALENCE
     1 (EE(1),A11),(EE(2),A12),(EE(3),A22),(EE(4),D11),(EE(5),D12),
     2 (EE(6),D22),(EE(7),D22B),(EE(8),BETS),(EE(9),BETT),(R,X),(Z,Y)
      REAL L
C
C*****    INTERPOLATE FOR TEMPERATURE-DEPENDENT PROPERTIES
C
      CALL MODULI (N,I,II,JJ,J)
      IF (ISTOP.NE.0) RETURN
      R(II)=-1.0
      R(JJ)=-1.0
      A=Z(I)-Z(J)
      B=R(J)-R(I)
      L=SQRT(A**2+B**2)
      IF (NPP.NE.0) GO TO 200
C
C*****    EVALUATION OF INTEGRALS FOR AXISYMMETRIC SHELL
C
      IF (R(I).EQ.0.0) R(I)=0.001*R(J)
      IF (R(J).EQ.0.0) R(J)=0.001*R(I)
      XI1=0.5*(R(J)+R(I))/L
      XI2=1.0/L
      XI3=0.5/L
      IF (B.EQ.0.0) GO TO 100
      XI4=ALOG(R(J)/R(I))/(B*L)
      XI5=(XI2-R(I)*XI4)/B
      XI6=(XI1-2.*R(I)*XI2+R(I)**2*XI4)/B**2
      GO TO 110
  100 XI4=XI2/R(I)
      XI5=XI3/R(I)
      XI6=1.0/(3.*R(I)*L)
  110 XI7=(R(I)+2.*R(J))/(6.0*L)
      XI8=(R(I)+3.*R(J))/(12.0*L)
C
C*****    FORM ELEMENT STIFFNESS MATRIX OF SHELL
C
      TEMP=XI4*(B/L)**2*D22B
      S11=A11*XI1-2.*A12*B*(XI2-XI3)+A22*B**2*(XI4-2.*XI5+XI6)
      S12=-A11*XI1-A12*B*(2.*XI3-XI2)+A22*B**2*(XI5-XI6)
```

```
      S13=-A12*A*(XI2-XI3)+A22*A*B*(XI4-2.*XI5+XI6)
      S14=-A12*A*XI3+A22*A*B*(XI5-XI6)
      S22=A11*XI1+2.*A12*B*XI3+A22*B**2*XI6
      S23=A12*A*(XI2-XI3)+A22*A*B*(XI5-XI6)
      S24=A12*A*XI3+A22*A*B*XI6
      S33=A22*A**2*(XI4-2.*XI5+XI6)+TEMP
      S34=A22*A**2*(XI5-XI6)-TEMP
      S35=XI1-(XI2-XI3)*B*(1.-D12/D11)
      S36=-XI1-XI3*B*(1.-D12/D11)
      S44=A22*A**2*XI6+TEMP
      S45=-S35
      S46=-S36
      S55=-(XI1-2.*XI7+XI8)*L*L/D11
      S56=-(XI7-XI8)*L*L/D11
      S66=-XI8*L*L/D11
C
C*****    FORM ELEMENT FORCE VECTOR
C
      FF1=-BETS*XI1*L+BETT*B*0.5
      FF2=BETT*A*0.5
      FF3=BETS*XI1*L+BETT*B*0.5
      GO TO 300
C
C*****    FORM ELEMENT STIFFNESS MATRIX OF BEAM SHELL
C
  200 S11=A11/L
      S12=-S11
      S13=0.0
      S14=0.0
      S22=S11
      S23=0.0
      S24=0.0
      S33=0.0
      S34=0.0
      S35=1./L
      S36=-S35
      S44=0.0
      S45=S36
      S46=S35
      S55=-L/(3.*D11)
      S56=0.5*S55
      S66=S55
C
C*****    FORM ELEMENT FORCE VECTOR
C
      FF1=-BETS
      FF2=0.0
      FF3=-FF1
C
C*****    TRANSFORM ELEMENT STIFFNESS TO GLOBAL SYSTEM
C
  300 A=A/L
      B=B/L
      S(1,1)=S11*B**2+2.*S13*A*B+S33*A**2
      S(1,2)=S13*(B**2-A**2)+A*B*(S33-S11)
```

```
      S(1,3)=S35*A
      S(1,5)=S36*A
      S(1,7)=S12*B**2+S34*A**2+A*B*(S14+S23)
      S(1,8)=S14*B**2-S23*A**2+A*B*(S34-S12)
      S(2,2)=S11*A**2-2.*S13*A*B+S33*B**2
      S(2,3)=S35*B
      S(2,5)=S36*B
      S(2,7)=S23*B**2-S14*A**2+A*B*(S34-S12)
      S(2,8)=S12*A**2+S34*B**2-A*B*(S23+S14)
      S(3,3)=S55
      S(3,5)=S56
      S(3,7)=S45*A
      S(3,8)=S45*B
      S(5,5)=S66
      S(5,7)=S46*A
      S(5,8)=S46*B
      S(7,7)=S22*B**2+S24*A*B*2.+S44*A**2
      S(7,8)=S24*(B**2-A**2)+A*B*(S44-S22)
      S(8,8)=S22*A**2+S44*B**2-2.*A*B*S24
      DO 120 KK=1,8
      DO 120 LL=1,KK
  120 S(KK,LL)=S(LL,KK)
C
C*****     TRANSFORM LOAD TO GLOBAL SYSTEM
C
      P(1)=FF1*B+FF2*A
      P(2)=FF2*B-FF1*A
      P(7)=FF3*B+FF2*A
      P(8)=FF2*B-FF3*A
      RETURN
      END
```

```
$ORIGIN          LEVEL1
$IBFTC BCSB      DECK,LIST
       SUBROUTINE BACSUB
       COMMON
     1 A(54,108), B(108), NB, ND, ND2, MBAND, NUMBLK
       COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
       NU=ND*NUMBLK+1
   400 DO 450 M=1,ND
       N=ND+1-M
       C=B(N)
       DO 425 K=2,MBAND
       L=N+K-1
   425 C=C-A(K,N)*B(L)
       B(N)=C
       NM=N+ND
       B(NM)=C
       NU=NU-1
       ENERG=ENERG+C*UX(NU)
   450 UX(NU)=C
       NUMBLK=NUMBLK-1
       IF (NUMBLK.LE.0) RETURN
       BACKSPACE 8
       READ  (8) (B(J),(A(I,J),I=1,MBAND),J=1,ND)
       BACKSPACE 8
       GO TO 400
       END
```

```
$IBFTC STRS    DECK,LIST
      SUBROUTINE STRESS
C
C*****     THIS SUBROUTINE PRINTS THE DISPLACEMENTS AND CALCULATES THE
C*****     STRESS FIELD AT THE ELEMENT CENTROID.
C
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON /YIELD/
     1 IZD(25),ZD(7,25)
      DIMENSION
     1 EPS(9), SIG(9)
      COMMON/QADTRI/
     1 XXX(5), YYY(5), EE(13), C(4,4), TT(4)
      DATA PII /28.648/
C
C*****     WRITE DISPLACEMENTS OF ELASTICITY NODES
C
      MPRINT=0
      DO 200 N=1,NUMNP
      IF (X(N).LT.0.0) GO TO 200
      CC=-CODE(N)
      IF (CC.LE.0.0) GO TO 230
      DX=COS(CC)
      DY=SIN(CC)
      TEMP=UX(2*N-1)*DX-UX(2*N)*DY
      UX(2*N)=UX(2*N-1)*DY+UX(2*N)*DX
      UX(2*N-1)=TEMP
  230 IF (MPRINT.GT.0) GO TO 220
      MPRINT=50
      WRITE (6,2000) HED
  220 WRITE (6,2001) N,X(N),Y(N),UX(2*N-1),UX(2*N)
      MPRINT=MPRINT-1
  200 CONTINUE
C
C*****     COMPUTE AND OUTPUT ELASTICITY ELEMENT STRESSES
C
      MPRINT=0
      DO 300 N=1,NUMEL
      MTYPE=IABS(IX(N,5))
      IF(TYPE(MTYPE).NE.SOLID) GO TO 300
C
C*****      FORM STRESS-STRAIN RELATIONSHIP
C
      CALL MODULI (N,I,J,K,L)
C
C*****     CALCULATION OF STRAINS AT CENTER NODAL POINT
C
      IF (K.EQ.L) GO TO 10
      F=0.25
      NM=4
      UK=F*(UX(2*I-1)+UX(2*J-1)+UX(2*K-1)+UX(2*L-1))
```

```
      WK=F*(UX(2*I)+UX(2*J)+UX(2*K)+UX(2*L))
      XK=F*(X(I)+X(J)+X(K)+X(L))
      YK=F*(Y(I)+Y(J)+Y(K)+Y(L))
      GO TO 20
   10 XK=X(K)
      YK=Y(K)
      F=1.0
      NM=1
      K=2*K
      UK=UX(K-1)
      WK=UX(K)
   20 CONTINUE
      DO 80 NN=1,4
   80 EPS(NN)=0.0
      DO 90 NN=1,NM
      M=NN+1
      IF(NN.EQ.4) M=1
      I=IX(N,NN)
      J=IX(N,M)
      AJ=X(J)-X(I)
      AK=XK-X(I)
      A=AJ-AK
      BJ=Y(J)-Y(I)
      BK=YK-Y(I)
      B=BJ-BK
      D=AJ*BK-AK*BJ
      D=F/D
      I=2*I
      J=2*J
      EPS(1)=EPS(1)+D*(B*UX(I-1)+BK*UX(J-1)-BJ*UK)
      EPS(3)=EPS(3)+D*(-A*UX(I)-AK*UX(J)+AJ*WK)
      EPS(4)=EPS(4)+D*(-A*UX(I-1)+B*UX(I)-AK*UX(J-1)+BK*UX(J)+
     1 AJ*UK-BJ*WK)
   90 CONTINUE
      IF (NPP.EQ.0) EPS(2)=UK/XK
C
C*****    CALCULATION OF STRESSES
C
      DO 120 L=1,4
      SIG(L)=-TT(L)
      DO 120 LL=1,4
  120 SIG(L)=SIG(L)+C(L,LL)*EPS(LL)
C
C*****    CALCULATE PRINCIPAL STRESSES AND STRAINS
C
      DD=(EPS(1)+EPS(3))*0.5
      DX=EPS(1)-EPS(3)
      DR=SQRT(DX**2+EPS(4)**2)*0.5
      EPS(5)=DD+DR
      EPS(6)=DD-DR
      IF(EPS(4).EQ.0.0.AND.DX.EQ.0.0) GO TO 400
      EPS(7)=PII*ATAN2(EPS(4),DX)
      CC=(SIG(1)+SIG(3))*0.5
      CX=0.5*(SIG(1)-SIG(3))
      CR=SQRT(CX*CX+SIG(4)**2)
```

```
      SIG(5)=CC+CR
      SIG(6)=CC-CR
      SIG(7)=PII*ATAN2(SIG(4),CX)
C
C*****    STRESSES PARALLEL TO LINE J-K
C
      I=IX(N,2)
      J=IX(N,3)
      ANG=2.*ATAN2(Y(J)-Y(I),X(J)-X(I))
      COS2A=COS(ANG)
      SIN2A=SIN(ANG)
      SIG(8)=-CX*COS2A-SIG(4)*SIN2A+CC
      EPS(8) = CX*SIN2A-SIG(4)*COS2A
C
C*****    ENERGY
C
      SIG(9)=0.0
      DO 100 L=1,4
  100 SIG(9)=SIG(9)+EPS(L)*SIG(L)
      SIG(9)=0.5*SIG(9)
C
C**** CHECK YIELD STRESS CRITERION
C
      EPS(9)=0.0
      IF(IZD(MTYPE).NE.0) EPS(9)=ZD(1,MTYPE)*SIG(1)**2+2.0*ZD(2,MTYPE)*
     1 SIG(1)*SIG(2)+2.0*ZD(3,MTYPE)*SIG(1)*SIG(3)+ZD(4,MTYPE)*SIG(2)**2
     2 +2.0*ZD(5,MTYPE)*SIG(2)*SIG(3)+ZD(6,MTYPE)*SIG(3)**2+ZD(7,MTYPE)*
     3 SIG(4)**2
C
      IF(MPRINT.GT.0) GO TO 290
      MPRINT=16
      WRITE (6,2002) HED
  290 WRITE (6,2003) N,XK,YK,(SIG(I),I=1,9 ),MTYPE,(EPS(I),I=1,9)
  300 MPRINT=MPRINT-1
      RETURN
  400 ISTOP=1
      RETURN
C
 2000 FORMAT (1H1 12A6,10X 6A6/5H0  NP 11X2HX1 11X2HX3 15X2HU1 15X2HU3//
     1 )
 2001 FORMAT (I5,2F13.2,2E17.5)
 2002 FORMAT (1H1 12A6,10X 6A6/
     1                  5H0 ELM  4X 2HX1 6X 2HX3  6X 8H1-STRESS 4X 8H2-S
     2TRESS 4X 8H3-STRESS 3X 9H13-STRESS 2X 10HMAX-STRESS 2X 10HMIN-STRE
     3SS 2X 5HANGLE 3X 9HJK-STRESS 6X 6HENERGY/10X 8HMATERIAL
     4  7X 8H1-STRAIN 4X 8H2-STRAIN 4X 8H3-STRAIN 3X 9H13-STRAIN
     5 2X 10HMAX-STRAIN 2X 10HMIN-STRAIN 2X 5HANGLE 4X 8HJK-SHEAR
     6 7X5HYIELD)
 2003 FORMAT (1H0 I4,2F8.2,1P6E12.3,0P1F7.2,1P2E12.3 / I15,6X,1P6E12.3,
     1 0P1F7.2,1P2E12.3)
      END
```

```
$IBFTC SSTR     DECK,LIST
      SUBROUTINE SHLSTR
      COMMON /NPELD/
     1 HED(18),NUMNP,NUMEL,NPP,TR,ACEL1,ACEL3,ANGFQ,E(4,11,25),RO(25),
     2 TYPE(25),CODE(1100),X(1100),Y(1100),UX(1100),UY(1100),T(1100),
     3 IX(1000,5),ANGLE(1000),SOLID,SHELL,NTEMP,ISTOP,ENERG
      COMMON/QADTRI/
     1 XXX(5), YYY(5), EE(13), C(4,4), TT(4)
      EQUIVALENCE
     1 (EE(1),A11),(EE(2),A12),(EE(3),A22),(EE(4),D11),(EE(5),D12),
     2 (EE(6),D22),(EE(7),D22B),(EE(8),BETS),(EE(9),BETT),(R,X),(Z,Y)
      REAL L,MI,MJ,MTI,MTJ
C
C*****    INITIALIZATION
C
      MPRINT=0
      DO 100 N=1,NUMEL
      MTYPE=IABS(IX(N,5))
      IF(TYPE(MTYPE).NE.SHELL) GO TO 100
C
C*****    INTERPOLATE FOR TEMPERATURE-DEPENDENT PROPERTIES
C
      CALL MODULI (N,I,II,JJ,J)
      A=Y(I)-Y(J)
      B=X(J)-X(I)
      L=SQRT(A**2+B**2)
      A=A/L
      B=B/L
      UI=UX(2*I-1)*B-UX(2*I)*A
      WI=UX(2*I-1)*A+UX(2*I)*B
      UJ=UX(2*J-1)*B-UX(2*J)*A
      WJ=UX(2*J-1)*A+UX(2*J)*B
      ESS=(UJ-UI)/L
      MI=UX(2*II-1)
      MJ=UX(2*JJ-1)
      IF(NPP.NE.0) GO TO 200
C
C*****    FORMATION OF STRESS RESULTANTS FOR CONICAL SHELL
C
      ETSI=UX(2*I-1)/X(I)
      ETSJ=UX(2*J-1)/X(J)
      XTTI=B*(WJ-WI)/L/X(I)
      XTTJ=B*(WJ-WI)/L/X(J)
      FSI=A11*ESS+A12*ETSI-BETS
      FSJ=A11*ESS+A12*ETSJ-BETS
      FTI=A22*ETSI+A12*ESS-BETT
      FTJ=A22*ETSJ+A12*ESS-BETT
      MTI=MI*D12/D11-D22B*XTTI
      MTJ=MJ*D12/D11-D22B*XTTJ
      QI=(MJ-MI)/L+(MI-MTI)*B/X(I)
      QJ=(MJ-MI)/L+(MJ-MTJ)*B/X(J)
      IF(MPRINT.GT.0) GO TO 110
      MPRINT=15
      WRITE (6,2000) HED
```

```
  110 WRITE (6,2001) N,I,X(I),Y(I),FSI,FTI,QI,MI,MTI,J,X(J),Y(J),
     1 FSJ,FTJ,QJ,MJ,MTJ
      GO TO 90
C
C*****     FORMATION OF STRESS RESULTANTS FOR BEAM SHELL
C
  200 QI=(MJ-MI)/L
      QJ=QI
      FSI=A11*ESS-BETS
      FSJ=FSI
      IF(MPRINT.GT.0) GO TO 210
      MPRINT=15
      WRITE (6,2002) HED
  210 WRITE (6,2003) N,I,X(I),Y(I),FSI,QI,MI,J,X(J),Y(J),FSJ,QJ,MJ
   90 MPRINT=MPRINT-1
  100 CONTINUE
      RETURN
C
 2000 FORMAT (1H1 12A6,10X 6A6/24H0SHELL STRESS RESULTANTS/
     1 8H0ELEMENT 6X 1HI 4X 4HR(I) 4X 4HZ(I) 10X 3HFSI 10X 3HFTI 11X
     2 2HQI 11X 2HMI 10X 3HMTI/
     3 7H NUMBER  7X 1HJ 4X 4HR(J) 4X 4HZ(J) 10X 3HFSJ 10X 3HFTJ 11X 2HQ
     4J 11X 2HMJ 10X 3HMTJ )
 2001 FORMAT (1H0I5,I9,0P2F8.3,1P5E13.4/I15,0P2F8.3,1P5E13.4)
 2002 FORMAT (1H1 12A6,10X 6A6/24H0SHELL STRESS RESULTANTS/
     1 8H0ELEMENT 6X 1HI 4X 4HX(I) 4X 4HY(I) 10X 3HFSI 11X 2HQI 11X 2HMI
     2/7H NUMBER  7X 1HJ 4X 4HX(J) 4X 4HY(J) 10X 3HFSJ 11X
     3 2HQJ 11X 2HMJ)
 2003 FORMAT (1H0I5,I9,0P2F8.3,1P3E13.4/I15,0P2F8.3,1P3E13.4)
      END
$ENTRY          PALOS
$DATA
```