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The Influence of the Home Learning Environment on Preschool Children's Informal Mathematical Development: Variation by Age and Socioeconomic Status

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The Influence of the Home Learning Environment on Preschool Children's Informal Mathematical Development: Variation by Age and Socioeconomic Status

by

Lydia Laurene DeFlorio

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ABSTRACT

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In the United States, children from families of lower socioeconomic status (SES) generally enter kindergarten with significantly less mathematical knowledge than children from families of middle SES. Research reveals that this discrepancy is present, although to a lesser degree, at age three years, the age many children enter preschool for the first time (Starkey & Klein, 2008). This mixed-methods correlational study explores relationships between elements of the home learning environment and the mathematical knowledge of three- and four-year-old children from lower and middle SES families in order to better understand the potential reasons for this discrepancy. First, I compare responses from 179 parents, balanced for child's age and family's SES, on a questionnaire designed to capture aspects of the quantity and quality of mathematical support children receive in the home. The questionnaire contained items about the frequency and range of activities children engage in to support the development of informal mathematics, parent practices supporting their children's mathematical development, and parents' beliefs and knowledge about typical early mathematical development in the United States. Next, I analyze videos obtained for a subset of the sample (n = 26) of parents and children engaging in activities at home, believed by parents to support early mathematical development. Specifically, I examine differences in the amount, type, and complexity of math content included in each activity, as well as differences in parents' teaching behavior as they assist their children with mathematical tasks. Correlational relationships between both the questionnaire and video data are and children's scores on a comprehensive mathematical assessment are explored.

Results from the questionnaire component indicate very few differences in the amount and types of mathematical support three- and four-year-old children in both SES groups receive in the home, but very clear SES differences in parents' beliefs about early mathematical development and how it is best supported, as well as SES differences in parents' knowledge about early mathematical development. Compared to lower SES parents, middle SES parents of children at both ages hold higher expectations in terms of skills they expect children to possess by age 5, as well as a more accurate understanding of which mathematical skills are within the developmental range of most children by age 5. Both of these constructs have predictive value, as evidenced by multiple regression models, for children's mathematical knowledge as measured by the math assessment. Middle SES parents are also more likely to provide support for the development of these skills by embedding math in the home routine, encouraging made-up games involving math, and by reading books with mathematical content to their children.

The video analyses suggest that parents of children in both age and SES groups focus on similar mathematical concepts when engaging in math activities with their children, but there are qualitative differences in parents' teaching behavior that are predictive of children's performance on the math assessment when the child's age and family SES is held constant. Specifically, middle SES parents tend to structure the demands of activities in such a way that the children successfully respond to approximately 80% (three-year-olds) or 90% (four-year-olds) of the math included independently, compared to lower SES parents, who tend to structure the demands of activities such that children are only responding to approximately 50% (three-year-olds) or 70% (four-year-olds) of the math independently. Furthermore, the type of assistance provided for the problems or tasks that children do not respond to independently, appears to both vary by age and SES, and predicts unique variance in children's mathematical knowledge.

This dissertation is dedicated to my grandparents,
Rod and Lydia Rodriguez
for a lifetime of unconditional love, support, and inspiration;
and for instilling in all of us,
a love for children,
a belief in fairness and equality,
and the responsibility to make this world a better place.

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CHAPTER ONE: INTRODUCTION

Rationale

There is a growing consensus among researchers and practitioners in child development and early childhood education that children's experiences during the first five years of life influence many aspects of development throughout childhood, adolescence, and even adulthood. The skills and abilities children bring with them to kindergarten predict early academic achievement (Alexander & Entwisle, 1988; Duncan, et. al., 2007), which in turn predicts overall educational attainment (Entwisle, Alexander, & Olson, 2005). However, children's experiences prior to kindergarten differ tremendously both between and within families and communities. In the United States, socioeconomic status (SES) has been associated with differences in early childhood outcomes spanning a wide range of developmental domains, including physical, socioemotional, and cognitive. Although the precise mechanisms linking SES with specific child outcomes are not yet well understood, research in child development and early childhood education consistently demonstrates that upon kindergarten entry, children from lower SES families fare poorer than children from middle and higher SES families in health (Brooks-Gunn & Duncan, 1997; Case, Lubotsky, & Paxson, 2002; Chen, Martin, & Matthews, 2006), behavioral or emotional self-regulation (NICHD, 2005; Yeung, Linver, & Brooks-Gunn, 2002), language skills (Hoff, 2006; Hoff & Tian, 2005), and cognitive abilities (Laosa, 1984; Magnuson & Duncan, 2006; NICHD, 2005; West, Denton, & Reaney, 2000; Yeung, Linver, & Brooks-Gunn, 2002).

One important aspect of cognitive development that is strongly influenced by SES is early, or informal, mathematical knowledge. Informal mathematical knowledge differs from formal mathematical knowledge in that informal mathematics does not involve the manipulation or use of symbols, but instead entails problem-solving with concrete objects or mental representations of objects. Throughout this dissertation, I use the terms early/informal mathematical knowledge/development/abilities interchangeably. In addition to providing the foundation for later mathematical understandings, children's early mathematical abilities have powerful implications for their later success in school. For example, Duncan and colleagues (2007) conducted a meta-analysis of longitudinal data collected on more than 20,000 children in three countries to identify any links between later school success and literacy, mathematical, attention, and socioemotional skills upon kindergarten entry. Of all skills assessed, early mathematical competence was the most powerful predictor of overall academic achievement throughout the elementary school years.

Unfortunately, all children do not begin school with the same level of mathematical competence. Prior to kindergarten entry, group differences favoring children of middle and upper SES have been found consistently across several mathematical domains including number and arithmetic (Ginsburg et. al., 1998; Ginsburg & Russell, 1981; Griffin & Case, 1997; Jordan, Huttenlocher, & Levine, 1992; Kirk, Hunt, & Volkmar, 1975; Saxe, Guberman, & Gearhart, 1987; Starkey, Klein, & Wakeley, 2004), spatial and geometric reasoning (Starkey & Klein, 2007; Starkey, Klein, &Wakeley, 2004), measurement (Starkey & Klein, 2007; Starkey, Klein, & Wakeley, 2004), and pre-algebraic pattern knowledge (Starkey & Klein, 2007; Starkey, Klein, & Wakeley, 2004). On average, children from lower SES families are entering kindergarten approximately one year developmentally behind in mathematics than children from higher SES families (Hughes, 1986; Klein, et. al., 2002). These differences are present at age three years (Starkey & Klein, 2007), and persist regardless of children's participation in school readiness programs such as Head Start (Administration for Children and Families, 2005; Pigott & Israel, 2005; Starkey, Klein, & Wakeley, 2004).

Addressing this discrepancy in early mathematical knowledge and subsequent achievement gap in K-12 mathematics has become a national priority. The No Child Left Behind Act (2002) mandates nationwide efforts to improve children's school readiness skills, including early mathematical knowledge, as well as the development and implementation of strategies to reduce the achievement gap in K-12 mathematics. More recently, the Obama administration has proposed an overhaul of many key tenets of No Child Left Behind, but closing the achievement gap in mathematics remains a primary objective in the reform initiative (U.S. Department of Education, 2010). In order to develop interventions that are both effective and efficient, we need to develop a better understanding of the relationship between SES and early mathematical development. We need to know why SES matters, how SES matters, and which mediating variables along the potential causal pathway lend themselves best to large-scale intervention.

Examining children's home learning environments is of particular importance in attempting to understand and explain the relationship between SES and early mathematical development. While it also makes sense to examine the classroom learning environment, the SES discrepancy in early mathematical knowledge is present by age three (Starkey & Klein, 2007), the age many children enter preschool or childcare for the first time (National Goals Panel, 1997). Thus, it is likely that some element, or elements, in the home environment contributes to the origin of the achievement gap observed in preschool and beyond.

A small body of work investigating children's home learning environments at age four reveals variation by SES in ways that may affect children's early mathematical development. For example, four-year-old children from middle SES families are more likely than children from lower SES families to have mathematics activities embedded into their home routines (Stipek, Milburn, Clements, and Daniels, 1992), to engage in number activities that span a greater range of complexity than those of lower SES children (Saxe, Guberman, & Gearhart, 1987), and tend to have parents with higher expectations for early mathematical development than do lower SES children (Starkey, et. al., 1999).

The purpose of this study is to further investigate the level of support young children receive in the home for early mathematical development, with the primary goal being the identification of potential contributors to the SES gap in early mathematical knowledge. This study expands on previous work in this area in at least two important ways, including (1) the analysis of video data depicting parents and children engaging in activities reported by parents as mathematically supportive and typically done in their home, and (2) contrasting the mathematical development and support children receive across two years of preschool.

First, previous studies have relied primarily on parent self-reports to capture various aspects of children's home learning environments. Although informative on some level, data obtained by parent report on its own is typically not sensitive enough to capture potential qualitative differences in children's home activities. Questionnaire data tells us that most parents of four-year-old children report using store bought games, for example, to support their children's mathematical development (Saxe, Guberman, & Gearhart, 1987; DeFlorio & Starkey, 2007). However, even when examples of game titles are provided, we are forced to treat that data with the assumption that all children's experiences of playing store-bought games are the same when clearly that is unlikely. Some games are inherently more difficult and/or mathematically relevant than others. They also vary in terms of objectives, game pieces, graphics, and rules. Even when multiple parents report playing the exact same game with their children, there is plenty of room for variation in the mathematical support afforded to children during that experience. In the popular children's game Chutes and Ladders, for example, children use a spinner to determine how many spaces they get to move from squares numbered one to one hundred. If a player lands on a space with a ladder, he or she "climbs" the ladder to a space closer to one hundred. If a player lands on a space with a chute, he or she "slides" down the chute to a space closer to one. Imagine one parent-child dyad playing this game. In this dyad, the parent expects the child to recognize the numerals on the spinner, count the appropriate number of spaces, and move her game token accordingly. Prior to each turn, the parent asks the child questions regarding the numeral she would like the spinner to stop on so that her game token will land on a ladder, or conversely which numeral she would like to avoid to escape a chute. In this scenario, the child is getting support for a range of mathematical concepts, including numeral recognition, counting, and arithmetic reasoning. Compare this to a different scenario where a parent might tell the child which numeral the spinner stopped on, and then tells the child which square to move her game piece to. In this scenario, mathematics is essentially removed from the experience. The inability to capture these types of qualitative differences in children's activities is a serious limitation of using only parent self-report data to examine children's home learning environments.

While this study utilizes parent questionnaire data, it also includes analyses of video data captured on a subset of the sample, and child outcome data in the form of scores on a comprehensive child math assessment. The videos show parents and children engaging in some of the activities they report doing at home to support early mathematical development. The activities were coded and analyzed for the purpose of capturing any SES differences in mathematical content and complexity, as well as parental teaching practices. Results from both the questionnaire and video analyses are then compared to children's scores on a comprehensive child math assessment, providing deeper insight into the relationship between SES, the home learning environment, and children's developing mathematical knowledge.

The second way this study expands upon previous work is that it seeks to identify meaningful differences in the home learning environments of children over two years of preschool. Previous studies investigating potential SES differences in the mathematical support children receive in the home have primarily focused on prekindergarten, or four-year-old, children (Stipek, et. al., 1992; Starkey, et. al., 1999). Although Starkey and Klein (2007) collected home environment data on children of both ages, those results were not reported by age group. Instead, the home environment data was collapsed across the age groups and general differences between all lower and middle SES homes were reported. There is some evidence to suggest that the three-year-old year is an important one in regards to emerging SES differences in children's mathematical knowledge. Saxe, et. al. (1987) administered a series of mathematical tasks to both middle- and working-class two- and four-year old children. While there were SES differences in the performance of four-year-old children, there were no SES differences for two-year-olds. This suggests that the SES differences first become apparent at age three.

Research Questions

1. Is there a correlational relationship between the type or frequency of specific activities children do in the home and their early mathematical knowledge? Which aspects of these activities vary by age (three years vs. four years) and SES?
2. Is there a correlational relationship between parents' beliefs of and/or knowledge about early mathematical development and children's early mathematical knowledge? Which aspects of parents' knowledge and/or beliefs vary by age (three years vs. four years) and SES?
3. Are there qualitative differences by age (three years vs. four years) and/or SES in the activities done in the home to support early mathematical development?

These questions are addressed by analyzing children's scores on a child math assessment, parent responses from a parent questionnaire, and video data collected in a subset of the children's homes.

Theoretical Framework

This investigation was guided primarily by theory and research on early mathematical cognition and development. Research on early mathematical cognition and development has been approached in the literature from several theoretical perspectives, each providing its own unique contribution to this area of study (for a review, see Case, Griffin & Kelly, 2001; Ginsburg, et. al., 1998). This particular study was influenced by theory and research from three different theoretical perspectives within that greater literature base—nativism, constructivism, and sociocultural theory—that together provide a picture of the development of informal mathematics in early childhood.

From the nativism perspective, human infants possess some basic numerical competence, including a sensitivity to number. Research using habituation-dishabituation paradigms, for example, indicate that very young infants are able to discriminate between small sets of objects (Antell & Keating, 1983; Canfield & Smith, 1996; Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1983, 1990; Strauss & Curtis, 1981; van Loosbroek & Smitsman, 1990), and slightly older infants can determine which of two sets is more numerous (Cooper, 1984). These abilities are present independent of environmental inputs, and serve as the foundation for children to develop a system of informal mathematics, as children are inherently able to attend to quantity in their environment.

Although these primitive structures are important, it is through environmental inputs that informal mathematics develops. Research and theory from the constructivist perspective highlight the importance of the physical environment. Children's conceptual understanding of number, in part, develops in the context of play, exploration, and reflection, as they manipulate various objects around them (Ginsburg, et. al., 1998). Children use objects to create sets, which are then subjected to any number of arithmetic transformations. In the process, children learn that adding to a set makes it larger, taking away from or dividing a set makes it smaller, and so on. By age two, children in the United States typically understand that addition produces more, subtraction produces less, and can solve simple non-verbal addition and subtraction problems (Simon, Hespos, & Rochat, 1995; Sophian & Adams, 1987; Starkey, 1992; Wynn, 1992). During the preschool years, children's informal numerical knowledge expands rapidly, again, in part due to their experiences with physical, or mental representations of, objects. However, the characterization of the growth depends upon their social environment as well.

Although the core mathematical properties or principles of informal mathematics (i.e., a basic understanding of quantity) are believed to be universal, the rate at which these are acquired and how they manifest in everyday life are culturally influenced (Guberman, 1999). Sociocultural theory is the third theoretical perspective guiding this study. Culturally desired informal mathematical skills and abilities are largely taught to children via their social environment, both directly and indirectly, via social interaction. Vygotsky (1978) contends that much of children's learning occurs in the social context, through imitation, scaffolding, and direct instruction. While at any given moment children's knowledge, or "actual developmental level," in any number of domains can be observed, measured, or assessed, it is within their "zone of proximal development" that learning and development best occurs. Vygotsky defines the zone of proximal development as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (pg. 86)." From this theoretical perspective, children receiving scaffolding within their individual zones of proximal development from adults or more competent peers will advance through higher levels of cognitive functioning within the targeted domain more rapidly than those who do not receive such guidance. Jordan, Huttenlocher, and Levine (1992) presented low and middle-income kindergartners with four types of calculation tasks. Three of the tasks presented the calculation problems verbally (e.g., story problems, word problems, and number fact problems), and one of the tasks presented the calculation problems non-verbally. The nonverbal procedure involved constructing an initial set, hiding that set, performing an arithmetic transformation so that the child could see the number of objects being added or subtracted, and then asking the child to replicate the final set. Middle-income children outperformed the low income children on all three of the verbal tasks, but there were no differences between groups in the nonverbal task. This suggests that the low income children understood the basic principles of addition and subtraction as well as the middle-income children, but to some degree, lacked the culturally transmitted tools (i.e., math-specific language, symbols, and strategies) possessed by the middle-income children.

Identifying potential SES-related differences in children's social interactions where informal mathematical knowledge is likely to be transmitted may provide valuable insight into the causes of the SES-related discrepancy in early mathematical knowledge upon kindergarten entry. This dissertation study examines the types of activities lower and middle SES children do at home to support the development of informal mathematics, as well as characteristics of the social interaction when the activities are done with a parent.

Background of This Study

The data used in this study was obtained from a subset of families who participated in a larger research project at the University of California, Berkeley, directed by Prentice Starkey and Alice Klein. The larger study was conducted to examine the development of mathematical knowledge from the beginning of preschool through the end of the kindergarten year, and included children of middle and lower SES in the United States, China, and Japan. There were two cohorts of children in each country, with approximately 100 children per cohort (approximately 50 from lower SES families, and 50 from middle SES families). Children in the first cohort were three-years-old at the beginning of the study, and children in the second cohort were four-years-old. Both cohorts were followed for two years, resulting in a rich dataset on three-, four-, and five-year-old children's mathematical development over that time. General results from this study reveal that in all three countries, children of middle SES families have more extensive mathematical knowledge at age three than children of lower SES families have at age three (Starkey & Klein, 2007). In the United States, this discrepancy in knowledge widens throughout preschool and kindergarten, but in China and Japan, this discrepancy narrows over the same time period. The widening or narrowing of the gap can be at least partially explained by elements of the preschool classroom learning environment, including the amount of time teachers spend on math and the curriculum used, but it is the home learning environment that is likely responsible for the uniform existence of the SES gap found in early mathematical knowledge. This dissertation study involves a deeper analysis of the home learning environments of the American sample.

CHAPTER TWO: LITERATURE REVIEW

SES, Children's Home Environments, and Early Mathematical Knowledge

In the United States, children's informal mathematical knowledge varies as a function of SES by the time children are three years old (Starkey & Klein, 2007). The reasons for this discrepancy are not well understood, although it is likely to stem at least in part, from differences in children's home learning environments. Still in the embryonic phase, research identifying which elements of the home environment influence young children's mathematical development has not been widely reported in the literature. However, studies examining relationships and pathways between SES, home learning environments, and more general measures of cognitive ability such as intelligence and general achievement tests are more widespread, and may prove valuable in guiding, organizing, and interpreting research on the cognitive domain of informal mathematical knowledge.

I begin this chapter by reviewing selected research focusing on relationships and potential causal pathways from SES to children's cognitive or intellectual development, with the goal of identifying potentially important mediators from SES to early mathematical development. Although an increasingly rich body of literature exists exploring the pathways between SES and children's cognitive or academic outcomes in middle childhood and adolescence, here I focus only on children prior to the beginning of compulsory education (i.e., kindergarten and younger). It is likely that SES, or components and correlates of SES, act upon development in different ways or to different degrees throughout childhood, thus I limit this literature review to studies of children in the age group of interest.

Next, I turn to the literature focusing specifically on young children's early mathematical development, the goal being to reconcile what we know about the effects of SES on cognitive development with what we know about SES and mathematical development.

From SES to Early Cognitive Development

Identifying causal pathways from SES to children's cognitive development is particularly challenging, as SES is not something that is easy to define or measure. There is some consensus among researchers that SES is a complex, multidimensional construct that somehow mediates and/or moderates an individual's access to financial, human, and social capital, but there is little consensus on how to best capture SES (Bradley & Corwyn, 2002; Hoff, Laursen, & Tardif, 2002). Traditional indicators of SES have included parents' income, educational attainment, and/or occupation (Entwisle & Astone, 1994; Ensminger & Fothergill, 2003; White, 1982), but whether any indicator contributes uniquely to children's cognitive development has not been systematically evaluated. Instead, it appears as though researchers with similar interests or theoretical orientations tend to use the same indicator as a proxy of SES. As such, it made sense to organize this section of the literature review by indicator of SES.

Parents' Income

Studies exploring the effects of income on young children's cognitive outcomes are typically approached from at least one of two theoretical perspectives (Mayer, 1997). The first, frequently termed the investment, or financial capital, perspective, posits that income affects children's development both directly and indirectly by allowing parents to purchase goods, services, and experiences for their children to enhance development. Examples include things like nutritious food, medical care, quality child care, homes in safe neighborhoods, cognitively stimulating materials, organized recreational activities, and trips to cognitively stimulating places, such as museums (Bradley, et. al., 2001, Haveman & Wolfe, 1995). The second perspective, often referred to as the family stress, or family process, perspective, maintains parental stress is the mediating variable between income and children's developmental outcomes. Parents enduring economic hardship suffer from more stress than parents who are not, and this stress affects parent characteristics that matter to children. Examples of such characteristics include parental responsiveness or warmth, parental monitoring or supervision, and parental disciplinary practices (Duncan, Brooks-Gunn, & Klebanov, 1994; Hashima & Amato, 1994; McLoyd, 1990).

It is becoming increasingly clear that constructs from the two competing perspectives are not mutually exclusive, but that they may act upon development in different ways. Guo & Harris (2000) were among the first researchers to examine causal pathways from income to cognitive development incorporating constructs from both perspectives. To assess the influence of a family's ability to provide material resources on children's early literacy, language, and mathematical abilities they used measures of children's cognitive stimulation in the home, the physical home environment, and quality of child care. To assess the influence of nonmaterial resources, they used a measure of parenting warmth. Of each of these measures, cognitive stimulation in the home was the most powerful variable mediating between income and children's cognitive ability. Parenting warmth and the physical home environment were also significant mediators, but to a much lesser degree. Although informative, one limitation of this study was the narrow range of items used to assess cognitive stimulation in the home. The cognitive stimulation measure was comprised only of the frequency the mother reads to her child, the number of books the child owns, whether or not the child owns a record or tape player, the number of times the child visited a museum in the past year, and the number of magazines the family subscribed to. However, subsequent studies examining pathways from income to general intelligence (Linver, Brooks-Gunn, & Kohen, 2002) and early literacy and mathematical achievement (Yeung, Linver, & Brooks-Gunn, 2002) using a slightly wider range of items have confirmed the mediating effects of cognitive stimulation in the home. In contrast to Guo & Harris, however, these studies suggest that constructs from the family stress perspective have little influence on cognitive development, and are more likely mediators between income and children's behavioral outcomes.

Although the effects of income on children's cognitive development appear to operate through the amount of cognitive stimulation in children's home, it is important to point out that income effects in general are relatively very small for most children when compared to effects of other parent characteristics, including race, gender, and educational attainment, (Blau, 1999). First and foremost, the relationship between income and children's developmental outcomes is nonlinear, with the largest effects being at the lowest income levels. Increases in a family's income make little or no difference to the majority of children in terms of developmental gains, unless income is so low that basic needs, including food, clothing, shelter, and medical care, are scarcely being met (Blau, 1999; Mayer, 1997). For families living in such impoverished conditions, it is still not a given that income will negatively affect children's long-term development. Factors including the age of the child during poverty, the duration of poverty, and the depth of poverty all contribute to the child's eventual outcome (Brooks-Gunn & Duncan, 1997; McLoyd, 1998).

Poverty appears to have the strongest influence on children's development during preschool and early elementary school, particularly when it is persistent and deep (Miller &

Davis, 1997; Yeung, Linver, & Brooks-Gunn, 2002). Persistent poverty is associated with less responsive parenting practices, less cognitive stimulation in the home, and lower scores on measures of school readiness when compared to both children who have never been poor and children who were only poor for part of their lives (NICHD, 2005). Children who spend at least four years in persistent poverty score almost 10 points lower on IQ tests administered at age 5 than children who are not poor during that same time period (Duncan, Brooks-Gunn, & Klebanov, 1994). Even still, the income effects for these children decrease significantly when parent and family characteristics are considered as covariates (Blau, 1999).

Parents' Educational Attainment

Parents' level of educational attainment is perhaps the most widely-used indicator, or proxy, for SES in the early childhood development and education literature (Ensminger & Fothergill, 2003). Although either parents' level of educational attainment can be utilized as an indicator of SES (Ganzach, 2000), remarkably little is known about the influence of the fathers' level of educational attainment on early cognitive development. However, the positive relationship between maternal level of education and measures of intelligence, achievement, and school readiness prior to compulsory education has been well established in the literature (Bee, et.al., 1982; Christian, Morrison, & Bryant, 1998; Haveman & Wolfe, 1995; Murnane, Maynard, & Ohis, 1981; Rosenzweig & Wolpin, 1994; West, Denton, & Reaney, 2000). The more education a mother possesses, the higher her child(ren) tends to score on such measures at kindergarten entry. Additionally, children born to mothers with less than a high school education have significantly higher retention rates in kindergarten and first grade (Byrd & Weitzman, 1994), and are nearly twice as likely to be placed in special education by the age of ten (Holloman, Dobbins, & Scott, 1998).

In an effort to better understand the relationship between maternal education and children's early cognitive development, parenting differences among highly and less educated mothers have received considerable empirical attention. For instance, research shows that the quality of the home environment, including safety, cleanliness, and cognitive stimulation is positively correlated with levels of maternal education (Klebanov, Brooks-Gunn, & Duncan, 1994; Menaghan & Parcel, 1991). Hill & Stafford (1980) examined time diaries kept by mothers who were college educated, high school graduates, and non-high school graduates to identify potential group differences in parenting practices. They found that mothers with at least some college spent more time per week engaged in direct care of their young children (ages 0-5) than mothers with less than a high school education, regardless of the number of hours worked per week outside the home. In addition to spending more time with their children in general, the mothers with higher education provided a wider variety of care to their children, spending significantly more time playing with them, teaching them, and taking them to developmentally stimulating places outside of the home than mothers with lower levels of education.

Not only do mothers with higher levels of educational attainment spend more time interacting with their children, differences in the quality of those interactions have also been identified. One type of mother-child interaction that has been studied extensively is the mother-as-teacher interaction. Numerous studies have been conducted where mothers with varying levels of educational attainment are instructed to teach their young children a specific task or set of tasks, and then those interactions are subjected to qualitative analyses (e.g., Borduin & Henggeler, 1981; Brophy, 1970; Harris, Terrel, & Allen, 1999; Hess & Shipman, 1965; Laosa, 1980). The results of these studies consistently demonstrate that mothers with higher levels of education engage in a wider range of teaching behaviors with varying degrees of complexity.

They ask their children more questions during the interactions, teach their children explicit strategies to use, and are generally more organized in terms of the overall presentation of the task(s). Mothers with lower levels of educational attainment, on the other hand, are most likely to rely on the use of direct commands or control (physical and verbal) during teaching interactions with their children.

This difference in maternal teaching styles appears to matter to children in important ways. Hess & McDevitt (1984) found that children whose mothers who relied on direct control or commands as their primary teaching strategy performed significantly lower on measures of school readiness prior to kindergarten entry than did children whose mothers utilized more collaborative teaching strategies. They suggest that overreliance on direct control or commands is not conducive to children's cognitive development as it changes the focus of the interaction from problem-solving to obedience and conformity while simultaneously having a negative effect on children's motivation, locus of control, and self-confidence.

Parents' Occupational Status

Parents' occupational status and vocational choices appear to affect children's developmental outcomes indirectly via parent values and beliefs (Kohn, 1963; 1986; Pearlin & Kohn, 1966; Wright & Wright, 1976). The type of work done by a head of household influences not only social class, but also life experiences. Life experiences in turn, influence parents' values and beliefs. In traditional "blue collar," or manual labor jobs, employee characteristics that are rewarded often include conformity to rules, ability to follow instructions, and working collaboratively as a team to accomplish objectives. In traditional "white collar" jobs, characteristics such as initiative, self-direction, and ability to work independently are more likely to result in occupational advancement. These experiences influence the values parents hold for their children, as well as the beliefs they hold for what is best for their children, both of which influence parents' behavior towards their children.

Although the majority of work investigating the links between occupational status, parenting, and cognitive outcomes focuses on older children and adolescents, research conducted by Luster, Rhoades, and Hass (1989) offers one demonstration of how this causal chain may operate in early childhood. The findings from their study revealed that occupational status is significantly correlated to parental values of self-direction and conformity. Mothers with higher occupational prestige (either from their own occupation or the occupation of their spouse) reported valuing child characteristics such as the ability to think independently and to be curious about many things, while mothers with lower occupational prestige valued child characteristics such as keeping to oneself and being polite to adults. These values were then manifested in mothers' childrearing beliefs. Mothers who valued self-direction tended to believe that maternal responsiveness, reading and talking to children, and allowing young children to explore their environments freely and safely were important components of child-rearing. Conversely, mothers who valued conformity were more inclined to stress their role of disciplinarian as the most important component of child-rearing. These mothers often held the belief that maternal responsiveness should be deliberately limited, so not to create a spoiled or demanding child. In the final stage of their analysis, it becomes evident that these child-rearing beliefs influence mothers' behavior towards and regarding their young children, as these beliefs are reflected in measures of maternal involvement, maternal warmth, disciplinary practices, and mothers' activities with their children.

Limitations of the Cognitive Development Research

Although insightful on some levels, the studies just described represent an incomplete picture, at best, of some of the variables and processes mediating between SES and children's cognitive development. It is clear, for example, that a family's income influences children's access to cognitively stimulating materials and activities, but one could easily argue that possession of such materials or participation in such activities means little without considering the context in which they are used or occur. What benefits are derived from owning 20 books if a child has no one who reads to him or her? Will this child fare better or worse than the child who only owns three books, but is read those three books every night before bed? Furthermore, does the specific act of reading to children influence their cognitive development directly, or is it that parents who frequently read to their children differ from parents who do not in other ways that are important for development? Research exploring causal pathways from indicators of SES to young children's cognitive outcomes does little to answer these kinds of questions.

What research does provide are starting points for further exploration. Studies exploring causal pathways from income to children's cognitive and intellectual development have largely relied on subscales, or adaptations of subscales, from the Home Observation for Measurement of the Environment (HOME) Inventories (Caldwell, & Bradley, 1984) to capture SES differences in the home environment. The full-length version of the HOME inventory for early childhood, ages 3 years to 5 years, 11 months, consists of 55 items clustered into eight subscales, including learning materials, language stimulation, physical environment, parental responsivity, learning stimulation, modeling of social maturity, variety of experience, and acceptance of the child. Some items rely on observer ratings of the environment, and others rely on parent's self reports. Although useful as a general measure of the quality of children's home environments, the narrow range and number of items, even in the rare occasions where the full scale is actually used, make it difficult to identify which specific materials, activities, practices, and home characteristics are most beneficial for children's cognitive development, and impossible to identify which are most beneficial for children's early mathematical development. Items from the HOME inventory most frequently used to assess cognitive stimulation in the home, for example, include the number of books a child owns, how often a child is read to, whether or not a child owns a CD/tape/record player and at least five children's CDs/tapes/records, how often a child goes on outings with other family members, how often a child visits a museum, how many magazine subscriptions a family has, and whether children own any items used for learning numbers, letters, colors, and shapes or sizes. There appears to be an underlying assumption that possession of these particular materials and engagement in these specific activities reflects the amount of cognitive stimulation in the home, despite any differences in context that may exist (e.g., proximity to museums, the content of the magazines subscribed to, the quantity, quality, and variety of support in the home for learning numbers). While research does suggest an association between the items on the HOME inventory and children's scores on both intellectual (Linver, Brooks-Gunn, & Kohen, 2002) and achievement (Yeung, Linver, & Brooks-Gunn, 2002) assessments, even when only a few items are used (Guo & Harris, 2000), its practical value is minimal in terms of informing intervention efforts to reduce the SES-related gap in early mathematics. If income influences children's cognitive development through access to cognitively stimulating materials and experiences, then it is possible that income also influences children's mathematical development through access to materials and experiences that support early mathematical development. If true, it makes sense on an intuitive level to provide lower-income children with the materials and experiences that their families cannot afford. However, in order to do this we need to know which specific materials and experiences, if any, actually matter to children's early mathematical development, and of those, which ones lower-SES children lack access to.

In addition to identifying SES differences in the specific materials children possess and the activities they engage in at home to support early mathematical development, the literature on the effects of maternal education offers support for exploring SES differences in the quantity and quality of the stimulation the materials and activities provide (Hill & Stafford, 1980; Klebanov, Brooks-Gunn, & Duncan, 1994; Menaghan & Parcel, 1991). From a quantitative perspective, the frequency in which a child engages in a specific activity at all may influence the amount of cognitive gain derived from that activity, but from a qualitative perspective, the frequency in which a child engages in that activity with an adult utilizing teaching methods that are conducive to learning will likely have a greater impact on cognitive development (Vygotsky, 1978). Since mothers' teaching interactions in general problem-solving tasks vary as a function of SES, it is possible that their teaching interactions vary in math-specific tasks as well. Examining the quantitative and qualitative differences in the ways parents support early mathematical development in the home is necessary in order to develop effective and efficient interventions.

Finally, research exploring links between parental occupation and children's outcomes highlights the importance of parents' values and beliefs as determinants of parenting behavior. While occupational status has been shown to influence parents' values and beliefs about child-rearing (Kohn, 1963; Luster, Rhoades, & Hass, 1989; Pearlin & Kohn, 1966; Wright & Wright, 1976), occupational status is not the sole contributor. Values and beliefs about child-rearing can be influenced by many things, including, culture, religion, education, family, friends, political affiliation, mass media, mental health, life experiences, and so on. Values and beliefs about child-rearing can encompass an infinite number of things, from discipline, to education, to what constitutes a healthy breakfast. In regards to identifying mediators from SES to children's cognitive development, and subsequently to children's early mathematical development, it is important to identify and understand potential group differences in parents' values and beliefs about how children learn, what they should be learning, and whose responsibility it is to teach them.

From SES to Early Mathematical Development

Up to this point, I have focused on identifying how SES matters for children's general cognitive development, in an attempt to identify variables that may mediate between SES and early mathematical development. In the next section, I review what is known about the relationship between SES, select variables identified in the preceding sections, and early mathematical development. More specifically, I focus on known SES differences in the types of materials children use and activities children do at home to support early mathematical development and the frequency in which children use or engage in them, differences in the quality of support children receive when engaged in mathematical activities, and differences in parenting values and beliefs about early mathematical development that may influence parents' behavior and children's outcomes.

Materials and Activities Supporting Early Mathematical Development

Saxe, Guberman, and Gearhart (1987) were among the first researchers to examine SES differences in the types of materials used and activities engaged in by children to support early mathematical development in the home. They interviewed parents of two- and four-year-old children regarding their use of specific activities used to learn number in the home. Activities included store-bought games specifically designed to teach number, store-bought games that use number but were not designed for the specific purpose of teaching number, made-up games and activities using number, commercial books for children containing number activities, and watching educational television shows that teach number. Mothers were asked to indicate the frequency in which their child engaged in each type of activity, and to provide specific examples and descriptions of each. They were then asked to rank each activity from the activity engaged in most frequently to least frequently, and then again from the activity they believed to be most important for teaching and learning about number to least important for teaching and learning about number. Results indicated there were no significant differences between the two SES groups in terms of the range and frequency of activities children engaged in. However, further analyses revealed some interesting SES differences between four-year-olds in the complexity of the activities. Recoding of the data demonstrated that children in the middle SES group not only engaged in activities with more complex goal structures than lower SES children, but also the activities of middle SES children spanned a greater range of complexity than the activities of lower SES children.

Starkey, et. al., (1999) also examined children's home activities for SES differences. Parent questionnaires were administered to parents of prekindergarten children, with approximately half the sample consisting of lower SES families and half of the sample from middle SES families. Parents were presented with seven types of activities typically used to support early mathematical development and asked to indicate the frequency their child engaged in each. Results indicated that middle SES children were more likely to use math in the home routine, engage in made-up games involving math, read number/shape books, and use a computer with math software than lower SES children. Lower SES children, on the other hand, watched more television shows with math content than middle SES children.

Stipek, Milburn, Clements, and Daniels (1992) offer further support that parents in middle SES homes provide a wider range of mathematical support in the home. Using maternal education as a proxy for SES, their study showed that four and five-year old children in middle SES families were more likely to have mathematics embedded into their daily routines than were children in lower SES families. However, the two groups did not differ in terms of the frequency of didactic activities specifically used for the purpose of promoting early numeracy.

The studies just described highlight the importance of using more comprehensive and specific measures of children's activities than those included in the HOME inventory (Caldwell & Bradley, 1984), as they provide a very different picture of the importance of children's access to material resources and experiences in the home. Based on the research on children's activities as they pertain to mathematical development, it may not be prudent, for example, to supply lower SES families with store-bought games for the purpose of narrowing the achievement gap, as parents in both SES groups report that their children engage in this type of activity at the same frequency already (Saxe, et. al., 1987; Starkey et. al., 1999; Stipek, et. al., 1992). However, working with parents to increase the amount of math embedded into children's daily routines, for example, may prove more fruitful if it turns out there is a causal relationship between math embedded into children's home routines and their mathematical knowledge.

Parental Support of Children's Early Mathematical Activities

The finding by Saxe, et. al., (1987) regarding differences in the complexity of activities done by lower and middle SES children underscores the importance of looking beyond simply what children have in terms of material resources to how those resources are used. For example, all parents in their sample reported using made-up games to teach their children about number at least once a week, thus if only frequency had been reported, the two SES groups would have appeared identical on this particular measure. However, by examining the complexity of parents' made-up games, very clear SES differences emerged. Saxe and colleagues identified four levels of complexity to describe the activities parents provide their children to support early numerical development. The most basic level, "Level 1," included activities using number words and recognizing numerals. Examples of activities at this level include number songs and playing with magnetic numbers. "Level 2" activities were slightly more complex in that they required children to count single sets of objects, such as fingers, toes, coins, or toys. "Level 3" consisted of activities involving numerical comparisons of two or more sets, and at the highest level, "Level 4," were activities requiring children to perform actual arithmetic operations with objects. By age four, 90% of middle SES children's mathematical activities involved Level 3 and Level 4 goal structures, compared to only 65% of lower SES children's activities, providing correlational support for the position that the complexity of children's home activities contribute to the SES gap in early mathematical development.

In addition to the complexity of an activity, it is likely that aspects of the parents' involvement in children's activities vary between SES groups as well. Since the amount of time mothers spend teaching and playing with their children correlates positively with the mother's level of educational attainment (Hill & Stafford, 1980), and mothers' educational attainment is an indicator of family SES, we can infer that middle SES mothers spend more time teaching and playing with their children than lower SES parents.

In addition to probable differences in the amount of time mothers spend teaching and playing with their children, there are likely differences between SES groups in terms of the quality of these interactions. It has been widely established in the literature that middle SES mothers talk to their young children more frequently and use more complex patterns of speech (Hoff, 2003), ask more questions (Borduin & Henggeler, 1981; Brody, 1968; Hoff-Ginsberg, 1991), encourage independent problem-solving (Luster, Rhoades, & Haas, 1989), and provide more scaffolding (Laosa, 1980) during interactions with their children than lower SES parents. Lower SES parents, on the other hand, are more controlling during parent-child interactions (Hoff, Laursen, & Tardif, 2002), and are more likely to utilize direct commands to teach new skills or obtain children's compliance (Borduin & Henggeler, 1981; Brody, 1968). What is not clear is whether these differences are related to young children's mathematical knowledge.

Finally, SES differences in parent-child teaching and playing interactions might also be influenced by the types of activities children most frequently engage in. It is possible that some types of activities more readily lend themselves to scaffolded learning opportunities as they require assistance of an adult or older child, thus changing the quality of the experience. For example, it may be that made-up games involving math, reading math-related books, and using math in the home routine are all activities likely done with an adult or older child, whereas watching television, playing with blocks, construction toys, and store-bought games may be something children tend to do alone or with same-age peers. If true, then middle SES children have more opportunities for adult-guided learning than lower SES children do based upon the types of activities they engage in most frequently (Starkey, et. al., 1999).

Values and Beliefs Supporting Early Mathematical Development

Disentangling parenting values from parenting beliefs is particularly difficult, as measurements of each likely tap into the other. Whether parent beliefs are influenced more heavily by their values, or their values are influenced more heavily by their beliefs is a matter of philosophical debate, and not particularly relevant to this discussion. For the purpose of this study, I make the assumption that values and beliefs are not mutually exclusive in that parents place greater value on the things they believe to be most important for their children's development, and that their beliefs are stronger for those values that matter to them most. From this perspective, values and beliefs can be viewed as part of the same parenting construct.

In his review of the literature on parenting beliefs about cognitive development, Miller (1988) identified four themes constraining or guiding research in this area. The first theme involves capturing what parents believe about children's cognitive development, both in regards to general processes and specific abilities. Studies organized around this theme seek to explain parents' beliefs about what children learn, when they learn it, how they learn best, and why parents believe this is so. The second theme involves understanding where and how parents get their ideas about children's learning and development. These studies attempt to identify origins of parents' ideas by either making comparisons between groups with assumed differences in experiences, such as gender (mothers vs. fathers) or caregiving roles (parents vs. teachers), or by asking parents direct questions about where their ideas about children come from. The third theme addresses the relationship between what parents think and what parents do. More specifically, research from this perspective examines the degrees to which parenting behavior is influenced by their own values and beliefs about children. Finally, the fourth theme involves linking the accuracy of parents' conceptions about child development with actual child outcomes. Although these themes were identified from the general cognitive development literature, research investigating parenting beliefs about early mathematical development can be characterized the same way. However, as I will demonstrate, some themes have been explored more widely than others.

Parent Beliefs about Early Mathematical Development. Three specific types of parent beliefs relating to early mathematical development have been reported in the literature. These include beliefs about the influence of the home and preschool environments on early mathematical development, beliefs about how children best learn foundational mathematical concepts, and beliefs about which mathematical skills and abilities typically emerge during the preschool years.

Research examining parents' beliefs about the relative influence of the home and preschool environments on children's early mathematical development suggest SES-related differences in parents' beliefs. Starkey, et. al., (1999) found SES differences in parents' beliefs about the influence of the home and preschool environments on early mathematical development, with more lower SES than middle SES parents believing that the preschool contributes more than the home environment in preparing their children for math in kindergarten, and more middle SES than lower SES parents believing that the home contributes more.

In regards to group differences in beliefs about how children best learn foundational mathematical concepts, Stipek, et. al., (1992) found that mothers with higher levels of education placed higher emphasis on child-centered, embedded, and informal activities to teach their children early mathematical skills, but mothers with lower levels of education believed performance-oriented, didactic, adult-led activities were more important for teaching young children mathematical skills.

Finally, in terms of the skills and abilities parents believe emerge during the preschool years, Starkey, et. al., (1999) presented parents with 13 mathematical skills or abilities and asked them to indicate which they believed typically developed by the end of preschool. Eight of the skills or abilities were within the developmental range of most five year old children, and five skills and abilities were beyond the developmental range of most five year old children. The expectations of middle SES parents were higher than those of the lower SES parents for all abilities that were within the developmental range.

Origin of Parents' Beliefs about Mathematical Development. It is not clear where parents' beliefs about early mathematical development come from, as this is not an area that has been explored in the literature, with one small exception. In their analysis of a subset of the four-year-old children that will be used in this dissertation study, DeFlorio & Starkey (2007) reported on parents' familiarity with the kindergarten mathematics curriculum used at the school their child would be attending. Almost half, 46%, of middle SES parents reported knowing what would be taught in the curriculum compared to only 16% of lower SES parents. Of those that knew the curriculum, the overwhelming majority, 86% of middle SES parents and 71% of lower SES parents, reported that they learned about it from an older child's experience in the same school or classroom. Less common resources included visits to the kindergarten classroom and information from other parents. Although interesting, this particular finding does little to shed light on the origins of parents' beliefs about early mathematical development, and more research is clearly needed.

Parents' Beliefs and Parents' Behavior. The relationship between parents' beliefs about early mathematical development and parenting practices facilitating early mathematical development has not been explored directly. However, there is some evidence that providing parents with training on how to support children's mathematical development, hence likely altering their beliefs at least to some degree, leads to changes in how parents teach mathematical skills to their preschool children. Starkey & Klein (2000) held a series of "family math workshops" for two groups of lower SES families with the specific purpose of teaching parents how to best support their children's mathematical development. The workshops were held bi-weekly over the course of four months, and were conducted by two teachers of the same ethnicity as the families (one group was African-American, the other was Latino). Each meeting focused on a specific mathematical domain (e.g., number, arithmetic, patterns), and consisted of an introduction to the topic, and a demonstration and actual practice doing activities to support development in that domain. The teachers emphasized the importance of limiting distractions and using specific scaffolding techniques to the parents through both modeling and direct instruction of desired parental behavior. Parents were given access to a lending library of math activities, but were not required to take materials home or engage in any specific activities with their children outside of the workshops. During the four month workshop, all parents utilized the lending library, and all reported doing the activities at home with their children. The resulting changes in parents' beliefs and practices were evident in children's pre- and post-test scores on a children's math assessment, as they made significantly more progress in mathematical knowledge over the course of the school year compared to control children attending the same preschool programs but whose families did not participate in the workshops.

Accuracy of Parents' Beliefs about Children's Mathematical Development. The relationship between the accuracy of parents' conceptions about early mathematical development and children's mathematical knowledge has not been directly explored in the literature.

However, preliminary analyses on a subset of the data used in this dissertation suggests there are possible SES differences, at least in terms of the accuracy of parents' knowledge about the mathematical abilities within and beyond the developmental range of typical children at age five (DeFlorio & Starkey, 2007). Parents of four-year-old children were presented with 23 mathematical skills and asked to indicate whether or not they believed most children acquire each skill by their 5th birthday. Similar to the procedure used by Starkey and colleagues (1999), an overall mean expectation score was computed with higher scores reflecting higher parent expectations. The data were then subjected to additional scoring for accuracy. Affirmative responses to skills generally accepted by educators (California State Department of Education, 2009; National Council of Teachers of Mathematics, 2000) to be within the developmental range of children at the end of prekindergarten (e.g., rote counting to 10), as well as negative responses to skills generally accepted to be beyond the developmental range of most children at the end of prekindergarten (e.g., rote counting to 100) were scored as correct. All other responses were scored as incorrect. These preliminary analyses showed that the middle SES parents not only held significantly higher expectations for 5-year-olds' mathematical abilities compared to lower SES parents, but they were also more accurate when it came to judging which skills and abilities children were capable of, particularly when the skill or ability was within children's developmental range. In regards to incorrect responses, middle SES parents were more likely to overestimate children's potential, while lower SES parents were more likely to underestimate their potential. Further analysis with the full dataset is warranted to determine if these SES differences are correlated with children's mathematical knowledge in preschool.

Conclusions and Implications

In order to address the SES achievement gap in early mathematical ability upon school entry both effectively and efficiently, it is necessary to understand the relationship between SES and early mathematical development. We need to know where along the causal pathway differences exist, as well as why they exist, in order to identify the most promising points for intervention. No Child Left Behind (2002) mandates that the achievement gap in mathematics be eliminated by the year 2014. Given how little is currently known about the early contributors to this gap, I decided to utilize the greater cognitive development literature as guidance. I looked at the effects of parents' income, educational attainment, and occupational status on young children's cognitive development in hopes of identifying important variables that mediate the effects of SES on children's early mathematical development. I found that cognitive stimulation in the home environment, the quality of support children receive from their parents during cognitively stimulating activities, parents' beliefs and values, and characteristics of the physical home environment all potentially mediate the effects of SES on children's cognitive development. I then reviewed the research in early mathematical development in search of empirical evidence suggesting that the same variables mediate between SES and children's early mathematical development as cognitive development. Given the scarcity of empirical work describing the relationships between the mediators of interest and early mathematical development, definitive conclusions are hard to draw.

Current empirical work provides little evidence to suggest that financial resources play an important part in the SES discrepancies found in children's early mathematical development. Although the possession of cognitively stimulating materials and participation in cognitively stimulating activities appears to mediate the effects of income on general cognitive development (Guo & Harris, 2000; Linver, Brooks-Gunn, & Kohen, 2002; Yeung, Linver, & Brooks-Gunn, 2002), both lower and middle SES children appear to have somewhat equal access to material goods supporting math, including store-bought games, books containing number or shapes, and didactic materials, such as workbooks and flashcards (Saxe, et. al., 1987; Starkey et. al., 1991; Stipek et. al, 1992). In terms of the activities they engage in to support early mathematical development, middle SES children are more likely to have math embedded into their home routines and play made-up games involving math than are lower SES children. Interestingly, both are activities that presumably do not cost money, thus participation does not have to be a function of income. However, it would be premature to suggest that income does not influence early mathematical development at all based solely on these findings for at least two reasons. First, income may very well influence children's early mathematical development in ways yet to be identified through research. It is possible that there are important SES differences in children's access to materials and activities that support early mathematical development, but that only certain materials or activities matter at all, certain materials or activities matter more than others, or certain materials or activities matter only when used in combination with others. These are important questions yet to be addressed in the literature. Second, it also possible that income influences young children's early mathematical development in additional ways that have nothing to do with children's access to materials or activities that support math. Clearly, more research is needed before conclusions are made and interventions are developed.

Perhaps more important than the types of math-related materials and activities used in the home is the quality of support children receive when engaged in mathematical activities. The literature exploring relationships between maternal education and children's cognitive development suggests that the characteristics of mother-child teaching interactions differ significantly by SES (e.g., Borduin & Henggeler, 1981; Brophy, 1970; Harris, Terrel, & Allen, 1999; Hess & Shipman, 1965; Laosa, 1980). In comparison to lower SES mothers, middle SES mothers ask more questions, provide more positive feedback and motivation, and teach their children explicit strategies to use in problem-solving tasks. Lower SES mothers give their children more direct commands. The teaching behaviors utilized more often by the middle SES mothers appear to be more conducive to children's learning, and are positively correlated with measures of cognitive development and school readiness (Hess & McDevitt, 1984). Examining mothers' teaching behaviors while engaged in mathematical activities with their children may provide important insight into the SES-related discrepancies in early mathematical development, particularly if the findings mimic those in the general cognitive literature. Although there is no evidence in the general cognitive development literature to suggest maternal teaching behaviors differ as a function of what they are attempting to teach, it is not known if characteristics of the mother-child interaction remain constant regardless of the task they are teaching. In other words, do mothers utilize the same teaching behaviors across all of their teaching interactions, or does their behavior change depending on the context (e.g., familiarity with activity, knowledge of subject matter, perceived importance of the child mastering the content, complexity of the task)? Exploring the determinants of maternal teaching behavior is a potentially important step in the process of improving the quality of parent-child teaching interactions during mathematics activities in the home.

Although very few studies investigating the relationship between parents' beliefs and children's early mathematical development have been reported in the literature, it appears that parents' beliefs about some aspects of early mathematical development differ between SES groups. One important difference involves parents' beliefs about children's mathematical capabilities prior to kindergarten entry (DeFlorio & Starkey, 2007). Compared to lower SES parents, middle SES parents of four-year-olds possess a more accurate understanding of which skills and abilities are within the developmental range of most prekindergarten children. Middle SES parents also tend to have higher expectations in terms of the skills and abilities they believe children should master prior to entering kindergarten. This difference in expectations is potentially important, as it appears such expectations have predictive value for children's performance on prekindergarten math assessments.

Future research exploring parents' beliefs about early mathematical development should include measures to identify potential group differences in the origins of their beliefs as well. Knowing where parents get their information about children's mathematical development may be valuable for designing interventions, particularly if their trusted resources differ by SES. A wide-scale internet campaign, for example, might be an appropriate means of intervention for middle SES parents who use the internet daily as a form of self-education, but may be of little value to lower SES parents without a computer or internet access. The family math intervention implemented by Starkey & Klein (2000) indicates that lower SES parents are willing to change their parenting practices to optimize their children's development when they are given the training and tools to do so, but we need research to identify the most effective and efficient means of making this happen.

As evident throughout this literature review, research exploring the relationship between SES and early mathematical development is clearly limited. However, the process of identifying mediators from SES to children's general cognitive development proved useful for framing the research on SES differences in early mathematical development that does exist. Perhaps even more importantly, the areas in which more research is needed became more salient. This dissertation work represents a contribution to this base of literature.

CHAPTER THREE: METHODS

Design and Participants

Design of the Study

In this correlational study, I employed a mixed methods approach to examine the ways in which aspects of the home environment may contribute to the SES gap present in early mathematics over two years of preschool: the preschool entry year (pre-prekindergarten, or PPK) when children are generally three-years-old, and the prekindergarten year (PK) when children are generally four-years-old. Throughout the remainder of this dissertation I use the term age to refer to these two years of preschool, as preschools do not generally refer to the PPK and PK years as grades. The participants included two cohorts of children. The PPK cohort consisted of children in the preschool entry year (i.e., the year before prekindergarten), and the PK cohort consisted of children in the prekindergarten year (i.e., the year before kindergarten). Within each cohort, approximately half of the participants were from lower SES families and half were from middle SES families. Data on the children's home environments were obtained by administering a comprehensive questionnaire to all parents in the second half of the school year (February through May), and results were compared to the children's performance on a child math assessment administered in the spring (April through May) of the same academic year. A subset of families were observed in the home as well. The purpose of the home observations was to capture potential qualitative differences by age and SES in children's home activities perhaps not captured by the parent questionnaire.

Participants

The sample included a total of 178 children attending 10 preschools in the San Francisco Bay Area. The overall sample was balanced for gender, with 88 girls and 90 boys. The PPK cohort consisted of 90 children, 44 of which were characterized as lower SES, as determined by qualification for the state and federally subsidized preschools they attended which served exclusively low income families, and 46 of which were characterized as middle SES, also determined by tuition rates and population served by the children's preschools. At the beginning of the school year, the mean age of the children in both SES groups was 3 years, 5 months (range: 2 years, 11 months to 3 years 10 months). The PK cohort consisted of 88 children, 40 of which were lower SES and 48 of which were middle SES determined by the same criteria. The mean age at the beginning of the school year for participants in the PK cohort was 4 years, 4 months (range: 3 years, 11 months to 4 years, 9 months) for the lower SES group and 4 years, 5 months (range 3 years, 11 months to 4 years, 9 months) for the middle SES group. Additional demographic information by age and SES is provided in Table 1.

Within each cohort and SES group, a subset of children were selected to participate in the home observation component of the study. It is important to note that the original purpose for collecting the video data was to capture footage for use in presentations and a possible documentary, as opposed to systematic analyses. Thus, random selection was not utilized. Teachers were asked which parents and children in their classrooms might be willing to let a researcher into their home to videotape the parent and child doing math activities together. In the event teachers suggested more than one child, children's personality during assessments and parent participation in the project were considered. It was important to film children who would not be upset or otherwise resistant to being observed, and it was rationalized that parents who had higher rates of participation (e.g., completed the parent questionnaire and other parent measures) would be more receptive to being filmed as well. In the event that more than one child per classroom met these criteria, the eligible children's names were drawn at random. Parents were contacted, and one child from each participating classroom was selected. Parents were paid $100.00 for their participation in the video component of the study.

Table 1
Additional Demographic Information by Age and Socioeconomic Status

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
	n = 44	*n* = 46	*n* = 40	*n* = 48
	%	%	%	%
Ethnicity				
African American	54.5	10.9	50.0	27.1
Hispanic	25.0	2.2	22.5	6.3
Caucasian	13.6	73.9	15.0	45.8
Asian/Pacific Islander	4.5	13.0	7.5	14.6
Other	2.3	--	5.0	6.3
Mother's Level of Educational Attainment				
Less than High School	15.9	2.2	7.5	--
High School	27.3	--	27.5	2.1
Some College or Vocational School	34.1	10.9	50.0	27.1
College	4.5	41.3	7.5	29.2
Some Graduate/Professional School	6.8	10.9	5.0	10.4
Graduate/Professional School	9.1	34.8	2.5	29.2
Declined to State	2.3	--	--	2.1

Note. Mother's level of education was not available for two children in the sample, both of whom lived with their fathers only. In these two cases, the father's level of educational attainment was used.

During the school year, a total of 20 parent-child dyads participated in the observation component of the study. Two of those observations were completely unusable due to camera malfunction, resulting in 18 observations for year 1. Ten children participated from the PPK cohort, five of which were lower SES, and five of which were middle SES. The five lower SES children were African-American, and of the five middle SES children, one was African-American and four were Caucasian. A total of 8 children participated from the PK cohort, four of which were lower SES, and four of which were middle SES. Of the four lower SES children, two were African-American, one was Caucasian, and one was Hispanic. Of the four middle SES children, one was African-American, two were Caucasian, and one was reported to be of "other" ethnicity.

The design of the larger study was such that data were collected on children in both cohorts over a two-year period. As such, home video observations were also conducted in the spring of the second year when the initial PPK children were in PK, and the PK children were in kindergarten. To increase the number of home observations on children in the prekindergarten year, observations from an additional eight children from the PPK cohort who were observed the following year, during prekindergarten, were included in this video analysis. These eight participants were selected using the same criteria as in year one, and include observations on three lower SES children and five middle SES children. Of the three lower SES children, one was African-American, one was Hispanic, and one was Caucasian. All five of the middle SES children were Caucasian. Results from a series of ANOVAs revealed no significant differences in mathematical knowledge at the end of prekindergarten between the PPK and PK cohorts for either SES group, thus it was appropriate to include the additional videos. The videos from the children in kindergarten were not included in this analysis, as this study focuses on children in the preschool years.

The video sample was not randomly selected from the main sample, thus the generalizability of the video data analysis is limited. The mean scores on the children's math assessments from each group within the video sample do not differ significantly from those of the main sample. However, given the selection criteria for participation in the video study, it may be that the activities captured on video are not representative of the home activities of all children in the main sample. The means and standard deviations for children's scores on the child math assessment for both the main and video sample are listed in Table 2. SES differences within cohorts mirror those in the main sample, with middle SES children in the video sample significantly outperforming lower SES children in the video sample in both cohorts, F (1, 22) = 23.736, $p < .001$, thus indicating that even if the video sample is not completely representative of the main sample, the video analysis may be used to shed light on differences in the home learning environment that contribute to the SES gap in early mathematical knowledge.

Table 2

Mean Composite Scores on the Child Math Assessment by Age and SES for the Main Sample and Video Subsample

	Main Sample		Video Subsample	
	Lower SES	Middle SES	Lower SES	Middle SES
	M (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Participants				
PPK Cohort Preschool Entry Year	.33 (.15) (n = 44)	.55 (.17) (n = 46)	.33 (.13) (n = 5)	.69 (.15) (n = 5)
PK Cohort Prekindergarten Year	.48 (.15) (n = 40)	.73 (.12) (n = 48)	.40 (.12) (n = 4)	.65 (.10) (n = 4)

Measures

Child Math Assessment

Children were administered the Child Math Assessment (CMA), developed by Klein & Starkey (2000), and modified specifically for this study, in the spring of the preschool academic year. The CMA consisted of 17 tasks covering a range of mathematical domains including number sense, arithmetic, geometric reasoning, measurement, and patterns. The tasks were designed in accordance with the standards set by the National Council of Teachers in Mathematics (NCTM, 2000), and each task consisted of one to six problems. Number sense tasks included object counting, construction of equivalent sets, quantitative set comparison, knowledge of number order, and ordinal number. Arithmetic tasks included one-set addition and subtraction with concrete objects present, one-set addition and subtraction with concrete objects hidden, addition and subtraction without concrete objects, and two-set addition and subtraction with objects hidden. Geometric reasoning included a shape recognition task, a shape naming task, and a reasoning about shape transformation task. In the measurement domain, the three tasks included direct measurement, creation with a nonstandard unit, and measurement with a nonstandard unit. Finally, pattern knowledge was assessed using a pattern duplication and a pattern extension task.

The CMA was administered individually to all participants in two twenty-minute sessions, with each session administered on separate days by a female graduate student in a private space at the child's preschool. Children with limited English proficiency were assessed in their native language by a bilingual graduate student. All sessions were videotaped, coded, and later scored by a graduate student researcher. Children earned a score of "1" for each correct response on the CMA. Mean scores for each of the 17 tasks were calculated individually, and a single composite score was produced by calculating the mean scores of all 17 tasks. Finally, subcomposite scores for the five mathematical domains (i.e., number sense, arithmetic, geometric reasoning, pattern knowledge, and measurement) were generated by dividing the mean score for each task within the domain by the total number of tasks in the domain.

Parent Questionnaire

Parents were asked to complete a parent questionnaire during the second half of the school year. The questionnaire was designed to obtain information regarding the ways parents support early mathematical development in the home and the types of activities their children engage in at home, parents' beliefs about early mathematical development and how it is best supported, and parents' knowledge about children's early mathematical development. The questionnaire was also used to collect demographic information about each family. A copy of the questionnaire is located in Appendix A.

To find out how parents were supporting early mathematical development in the home, parents were asked to report on the frequency of their children's use of 12 specific types of activities with mathematical content plus any others that were not listed. These activities included using store-bought games involving math, playing with math-related toys, playing with blocks or other construction toys, practicing origami (paper folding) or kitagami (paper cutting), doing art activities involving patterns or symmetry, made-up games involving math, using a computer with math software, reading books such as counting or shape books, using math workbooks, singing/listening to songs that use math, watching television shows that teach math, and using math in the home routine. Parents were asked to select a response of "daily," “weekly,” “monthly,” or “rarely/never” to indicate the frequency their child engaged in each activity and were asked to provide examples of each.

Parents were also presented with three statements reflecting different pedagogical approaches towards early learning and asked to identify which of the approaches they used on a regular basis at home to help their children develop mathematical knowledge and skills. Parents were asked to place a check next to the approaches they used, and then rank them in order of their beliefs about which approach is most important, less important, and least important when more than one was selected. The three statements were as follows: (1) “I give my child math-related tasks or ask math-related questions during ongoing domestic routines (e.g., We use measuring cups or spoons while preparing food),” (2) “I set aside time to be with my child on a regular basis to help him/her develop cognitive skills (e.g., We look at a number book, play a board game, or use math software together),” and (3) “I enrich my child’s play time (alone or with other children) by providing math-related toys and materials (e.g., My child spontaneously plays with cards or shape puzzles, or watches Sesame Street alone).”

Parents’ beliefs about early mathematical development and the ways which they felt it is best supported were assessed three ways. First, parents were asked to consider the relative contribution of the home environment versus the preschool environment in preparing children for math in kindergarten, and to indicate in percentages how much they believe each contributed. Responses were then scored into three categories based upon the percentages given. These categories included the home environment contributes more, 2) both the home and the school contribute equally, and 3) the preschool contributes more. Next, parents were asked to consider the relative contribution of children’s spontaneous play (alone or with peers) and adult- (parent- and teacher-) directed activities in preparing children for math in kindergarten. Again they were asked to indicate the percentage each contributes, and responses were categorized as 1) spontaneous play contributes more, 2) spontaneous play and adult-directed activities contribute equally, or 3) adult-directed activities contributes more. Finally, parents were presented with 23 mathematical abilities or skills that in the United States, typically emerge between ages 3 and 8 years and asked whether they believed typical children have developed each by their 5^{th} birthday. Affirmative responses were scored as “1,” and negative responses were scored as “0.” A complete list of abilities and skills is listed in Appendix A.

Finally, parents’ knowledge about early mathematical development was assessed by recoding their responses to the 23 mathematical abilities just described. While eight of the 23 skills or abilities presented to parents were not skills that most children in the United States develop before their fifth birthday, the remaining 15 were skills that typically emerge between ages 3 and 5 years. Affirmative responses corresponding with skills or abilities that are typically present by age 5 were scored as “1,” as were negative responses to skills and abilities that are not typically present until after age 5. All other responses were scored as “0.”

Home Observations/Videos

Home observations were conducted with approximately 15% of the sample over a two-year period. Parents were asked to demonstrate two activities they do with their child on a regular basis to support their child’s early mathematical development. They were instructed that one activity should be something they do with their child with the specific objective of supporting math, and one activity should be something they do as part of their home routine that includes math, but where math is not the primary focus. Parents who stated they did not use math with their child in their home routine were asked to select a second activity using the same criteria as the first activity.

The decision to let the parents select their own activities to demonstrate is not a common practice in early childhood research, but was made by the PIs of the larger study after careful consideration of the study's sample, aims, and intended use of the video data. The larger study included lower and middle SES children in the United States, China, and Japan. The study was largely exploratory in nature, with the overarching goal of identifying differences in early mathematical support received by children in the three countries. Restricting parents' activity choices would have been counterproductive to that goal. It was acknowledged from the onset of the study that the video data might not be representative of what parents actually do with their children in the home, but agreed that at the very least we would have a snapshot of what parents think they should be doing with their children to support developing mathematical knowledge. We expected to parents to put their best foot forward, such that the demonstrated activities may not truly mirror similar activities done without an observer present. However, we rationalized that if we were able to identify country and/or SES differences between children and activities in these "polished" demonstrations, it was likely the differences would be equally or more pronounced in everyday, non-observed settings.

A second, unexpected, consequence of allowing parents to select their own activities was the variation among parents' beliefs about what constitutes an activity. Among the five parents who chose reading books as one activity to demonstrate, for example, three parents read one book each, one parent read three books, and one parent read four books. A decision had to be made whether each book counted as an individual activity or if the activity was the act of reading regardless of the number of books read. There were also several instances of secondary activities being embedded within primary activities. For example, one child was doing addition and subtraction problems with toy cars. She and her mother took turns constructing sets, performing arithmetic transformations and asking the other to count the final set. Upon completion of the final problem, the mother complimented the child on her counting skills and prompted her to count as high as she could. The child then proceeded to count to 66. In instances like these, it was difficult to tell if the parent was ignoring the two activity guideline and demonstrating a third activity, or if the parent's intent was to show a range of loosely related things they do with their child within a single activity.

To foster comparison across data sources, I ultimately chose to separate activities using the same 13 categories (e.g., store-bought games, made-up games, reading books with math content) presented in the first question of the parent questionnaire, irrespective of how many examples of that activity were demonstrated. The parents who read one book during the observation were credited with one activity in the reading books with math content category, as was the parent who read four books. In the event that an activity could be classified as more than one activity type, I selected the category most appropriate given the context and perceived intent of the parent. The specific examples of activities observed in each category are presented in Table 3. Using this criterion to separate the observations into specific activities typically resulted in two activities per observation. However, three observations of PPK children (one lower SES, two middle SES) ended up with three activities. Each of these observations contained at least one activity that was relatively short (four minutes or less), thus it is possible the parents felt the need to extend the length of the observation by adding an additional activity. Then again, it could be a difference of perception, in that what I considered to be two separate activities, the parent believed to be two parts of the same activity. With no way to know for certain, the data from all three activities, when applicable, were included in the analysis.

All home observations were videotaped and converted to digital files. The digital files were then partitioned into individual files, with each file consisting of one activity, using Windows Movie Maker. Each file was reviewed carefully for the following criteria: 1) the activity contained at least one occurrence of math, and 2) the activity contained both parent/guardian and focal child participation. Three activities were excluded under the first criteria, and one activity was excluded under the second. An additional three activities were also excluded for severe technical failure (e.g., no audio or video). The final sample of observations included 26 children engaging in a total of 56 activities (see Table 4).

Table 3
Specific Activities Observed for Each of the 13 General Activity Types

Store-Bought Games
Addition War
Checkers
Chutes and Ladders
Don’t Wake Daddy
Franklin Goes to School
Hi-Ho Cherry Oh
Monopoly Jr.
Pokémon Sorry
Pokémon Yahtzee
Skip-Bo
Trouble
Wizard of Oz Board Game

Math-Related Toys
N/A

Blocks/Construction Toys
Building a robot with Legos

Origami/Kitagami
N/A

Art Activities
Making a bracelet with beads

Made-Up Games
Addition/Subtraction with objects (e.g., candles, cheerios, crackers, pennies, toys)
Constructing shapes with pencils
Counting objects (e.g., pennies, steps, toys)
Creating a number line with Uno cards
Matching sets of macaroni to numerals
Playing store

Songs
N/A

Reading Books
Bambi
Color Farm
Four (appeared to be part of a curriculum series)
Five (appeared to be part of a curriculum series)
Guess How Much I Love You
Little Rabbit’s First Time Book
Richard Scarry’s Pop-Up Numbers
Ship Shape
Ten Little Witches
Turn and Learn: Learn to Spell, Learn to Count

Workbooks/Worksheets/Flashcards
Addition/subtraction flashcards
Matching sets to numerals
Shape matching/naming/tracing
Tracing/copying/writing numerals
Written addition/subtraction problems

Computer with Math Software
Arthur’s Computer Adventure
Reader Rabbit (Math Activities)

Home Routine
Baking (e.g., cookies, cake)
Calendar (e.g., counting days to a specific event)
Checking the mail
Dialing the telephone
Folding Laundry
Getting cotton swabs to clean ears
Preparing food (e.g., fruit skewers, lunch)
Matching socks
Picking lemons to make lemonade
Setting the table

TV Shows
N/A

Table 4
Observations and Activities by Age and SES Group

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
	n	*n*	*n*	*n*
Observations	5	5	7	9
Activities	13	13	13	17

There was no way to ascertain whether all of the video observations were representative of the activities parents actually do with their children to support mathematical development. However, during the coding process, details about the child's perceived familiarity with the activity, the level of explanation of rules or objectives provided by the parent to the child at the beginning of each activity, and verbal references made by either the parent or child to previous engagement in the same or similar (e.g., baking cookies instead of a cake) activity were documented. In the lower SES PPK group, four of the 13 activities were clearly typical for those children, and one activity had clearly never been done before with that child, as evidenced by the mother explicitly stating this was a new idea and the child never catching on to the desired response. The remaining eight activities done by children in this group may or may not have been typical. In the middle SES PPK group, it appeared as though 12 of the 13 activities were typical for the individual children in that group. The children did not need explanations for rules or procedures, made references to at least one other time they engaged in the same activity (e.g., "remember last time when I won?"), and appeared to know what was expected from them in terms of responses. There was one activity in this group that may or may not have been typical for that child. The father suggested they make shapes with pencils, and the child agreed that was a good idea and they proceeded to do so. Whether the child knew what to do because it was within his developmental range, or because he had prior experience making shapes with pencils was unclear. In the lower SES PK group, nine of the 13 activities appeared typical of the parent-child dyads, three may or may not have been typical, and one was likely not typical. In the case of the non-typical activity, the parent produced two brand new board games with the pieces still wrapped in cellophane. The child appeared not to have prior experience with either game, as he really struggled to learn the rules and procedures. Finally, in the middle SES PK group, 15 of the 17 activities appeared typical to those children, and two may or may not have been typical.

In the end I decided to include all activities in this analysis, regardless of whether I believed the activity was typical for the family. The rationale for this decision was twofold. First, it is possible that while the activity selected may have been novel to the child, perhaps engaging in novel activities in general is how developing mathematical knowledge is supported in some homes. Second, it was also possible that parents introduced new activities during the observation out of necessity, as perhaps those particular parents really did not have two activities they do with their children on a regular basis. If true, then it is likely those children are receiving the least amount of parental support for early mathematical development, thus it was important to keep them in the analysis.

Each observation was subjected to three iterations of coding, with each activity viewed in its entirety a minimum of twice at each step. The first iteration involved recording the length of the observation from beginning to end, as well as the number of minutes within the activity that contained at least one occurrence of math. An occurrence of math was defined as the use of any mathematical language (e.g., number words, shape names, arithmetic questions) or engagement in mathematical behavior (e.g., counting, constructing a set or pattern) by the focal child. Mathematical activity performed by another participant (e.g., parent, sibling) was considered a math occurrence only when the mathematical behavior was accompanied by mathematical language and it was clear that the focal child observed it. For example, in board games each participant takes individual turns rolling the dice and moving the appropriate number of spaces. For the focal child, this sequence of events almost always resulted in two math occurrences. The first occurrence involved counting the dots on the dice, and the second involved moving the game token the appropriate number of spaces. Although the other participant(s) typically engaged in the same sequence during their turns, older children and adults did not always verbally count or enumerate the dots on the dice or count out loud while moving their game token. When either of these acts were done so that the focal child could see and hear the mathematical activity occurring, the act counted as an occurrence of math.

All math occurrences were coded to differentiate between occurrences in which the child was an active participant, and the occurrences in which the child's role was that of passive observer. Regardless of whether the math was performed or observed by the focal child, each minute of the total activity received a score of "1," if math occurred at any point during the interval. If no math occurred, that interval was scored as "0." Typically, the final interval did not last exactly one minute. Final intervals lasting less than 30 seconds were generally comprised of the parent talking to the observer, and any math that occurred happened in the first second or two of the interval as the activity was ending. Final intervals lasting longer than 30 seconds, however, generally included at least 20 seconds where the parent and child were still engaged in the activity. Thus the final interval was only coded if it exceeded 30 seconds in length. The sum of the scores were then computed, producing the number of minutes that included math in each activity. Finally, the number of math occurrences was then divided by the length of the activity to produce the mean number of math occurrences per minute.

The second iteration of coding involved coding for specific math content, and consisted of four steps for occurrences in the number and arithmetic domains, and three steps for math occurrences in the other domains. The mathematical content of each activity was recorded at levels of specificity using the guidelines laid out by Starkey (2004). These guidelines are included in Appendix B. Each occurrence of math within an activity was coded individually. On the broadest level, each occurrence was coded for the mathematical domain, or domains, being supported, including number and arithmetic, geometry and space, measurement, graphing, and data collection, and patterns. There were a few mathematical occurrences that did not fall into any of these domains, thus a domain of "other" was also included.

The next step of coding within this iteration involved documenting the mathematical process observed within the domain for each occurrence. Mathematical processes can be described as specific mathematical tasks, such as counting a set, or solving an addition problem with concrete objects. Starkey (2004) developed a comprehensive list of mathematical processes commonly observed in preschool classrooms (see Appendix B). This list served as the guideline for coding mathematical processes, although not every process on his list was observed in the home activities, and not every process observed was on his list. A complete list of processes observed, along with descriptions and examples is presented in Appendix C.

Math occurrences just in the domains of number or arithmetic were subjected to an additional step in the coding process using the four goal levels identified by Saxe, et. al., (1987). Each goal level represents a differential level of numerical complexity. Level one goals were those that involved the denotation of number in a nominal sense only. Math processes at the preschool level at this goal level may include things like numeral naming or recognition, or rote counting. Level two goals were more advanced, in that they entailed the numerical representation of single sets. Examples of common math processes in this category include counting a set of objects and constructing a set. Level three goals are those that involve the comparison of two or more sets, such as deciding which of two groups of objects is more numerous. Finally, level four goals are those that entail actual arithmetic operations. A common example includes solving small addition and subtraction problems using concrete objects.

It should be noted that my use of this particular coding scheme differs somewhat from its original purpose. Whereas Saxe and colleagues developed these codes to capture the overarching goal of specific types of activities, I used them to code individual number and arithmetic processes occurring with the activities regardless of the activity type, thus results from the two studies may not be comparable. Coding for the overarching goal in the current study was not appropriate, as number and arithmetic were not the primary mathematical focus in several of the activities. Math occurrences in domains other than number and arithmetic were not coded for goal level.

The final and most specific level of coding in the second iteration involved recording the problem details for each mathematical process or type of knowledge being supported. Examples of problem details include set sizes, the actual numerals or number of objects used in arithmetic problems, the shape names being used, etc.. Thus, each math occurrence within an activity had a code for mathematical domain, mathematical process or knowledge being supported, goal level (number and arithmetic only) and problem details.

Identifying occurrences of math within each activity was surprisingly straightforward, as was coding for domain, problem details, and goal level. However, it would be ideal to get interrater reliability on the coding for mathematical processes, as that was more challenging. This was not possible to do given the timeline for this project, and the specialized training in early mathematical development that a reliability coder needs to code for math processes with accuracy. In lieu of a second coder for this iteration of coding, I reviewed each activity at least three times and consulted one of my committee members for all questionable processes to ensure the coding was as complete and accurate as possible.

The third and final iteration of coding involved recording who was involved in each math occurrence, whether the child's response was correct or incorrect, if help was provided, and if so, the teaching strategy used by the parent to help the child and the reason help was given. To distinguish occurrences where the child was an active participant in the math occurrence from those occurrences where the child observed someone else doing math, each occurrence was coded as "child," "parent," or "other participant" to indicate which person in the activity was the active participant. In the event the child and parent shared participation in the occurrence, the occurrence was coded as "child with help."

The teaching strategies used by parents were coded using a scheme loosely adapted from two coding schemes found in the literature, one originally used to record maternal teaching behaviors during structured tasks and free play (Kermani & Brenner, 2001), and another used to record the scaffolding behaviors of preschool teachers during large group activities (Pentimonti & Justice, 2010). These coding schemes were selected after extensive review and categorization of the teaching strategies observed in the home video data, as each contained categories of teaching behavior comparable to those observed in this dataset. Both coding schemes are alike in that they contain teaching strategies or scaffolding behaviors that are considered high control or support, as well as scaffolding behaviors that are considered low control or support, which was important to capture in this study. However, neither scheme captured the full range of teaching strategies observed in these data and both had contained strategies not observed in these data, thus further adaptation was necessary. The final coding scheme included seven teaching strategies used by parents, each representing a differential level of support for the child. In instances where more than one strategy was used for one math occurrence, the strategy providing the highest amount of support was recorded. Definitions and examples of each teaching strategy in the coding scheme are provided in Table 5.

On the low support end of the continuum was non-instructive feedback, which involved the parent indicating to the child that his/her response was incorrect without providing any additional cues or information to help the child solve the problem. Examples of non-instructive feedback might include comments made by parents such as, "no, that's not right," "try again," or repeating the initial instructions.

The next level on the continuum of teaching strategies included prompts given by the parent to assist the child. To be credited as a prompt, the teaching strategy must have included some degree of scaffolding, but the child must have completed at least 51% of the task independently. There were three types of prompts given by parents to assist the children, including general prompts, strategy prompts, and specific prompts. General prompts provided the least amount of support of the three, and involved parents providing a non-specific hint to the child. For example, if a child incorrectly counts a set of eight objects and says there are five, a general prompt might be "I think there are more than five." In this example, the "hint" that differentiates this from non-instructive feedback is "more than five." Not only is the parent saying the child is incorrect, but the parent is also prompting the child to count higher than five. Strategy prompts provide more support for children than general prompts, in that the child is given a specific strategy to use to correctly complete the task at hand. Using the same example, a strategy prompt might include the parent instructing the child to "count slower," or to "touch each [object] as you count it." Within the prompt category, specific prompts provide the child with the highest level of support, in that the parent provides specific information, beyond suggesting a strategy, to assist the child in completing the task. Imagine a child attempting to count the set of eight who arrives at the response of five because he/she is using an incorrect counting sequence such as "1, 2, 3, 4, 7, 8, 5." A specific prompt might involve the parent having the child start over, but interrupting the child at four, saying "5, 6," and letting the child resume the counting sequence from six.

On the higher-support end of the continuum, are the strategies I have termed as participation. Participation differs from prompts, in that the parent takes responsibility for completing at least 50% of the task for the child. There are two levels of participation. The first, co-participation, occurred when the parent and child did the task together, each contributing equally to a successful outcome. For example, the parent and child count the eight objects together, or perhaps take turns saying each number, one through eight, in the counting string. The second level, parent-led participation, occurred when the parent provided direct help to the child, either physically or verbally, completing more than 51% of the task for the child but without providing the end correct response. The parent may tell the child, "I'll point to the objects and we will count together," and then proceed to touch each object in the count while saying all but the last number word in the sequence for the child. Thus, the child gets to the correct response of eight, but only because the parent took him/her more than half of the way there. On the highest-support end of the teaching strategy curriculum, parents provide children with the correct response. The parent says, "No, there aren't five, there are eight."

Perhaps as important as the strategy used to help children is the reason help was given at all. The coding scheme used to record the reasons for help was developed specifically from this data. During the multiple viewings of the video, a running list of all observed reasons for help was created, resulting in six categories that captured all math occurrences in this dataset. Unlike the teaching strategy scheme, there is no inherent hierarchy within the codes for reasons parents chose to help their children. The six codes include (1) the child's response was incorrect, (2) the child said or indicated (e.g., shrugging shoulders) he/she did not know the answer to a question or did not know how to do something, (3) the child hesitated or failed to provide a response at all when a response was clearly expected, (4) the child became distracted and lost focus on the task at hand, (5) the child explicitly requested help from the parent, and (6) no observable reason, as the child was never given the opportunity to complete the task independently.

Children's familiarity with the activities demonstrated appeared to influence the amount of assistance provided by parents. Although parents were instructed to demonstrate activities typically done in the home, as discussed previously, not all children appeared familiar with all activities. To examine whether a relationship between children's perceived familiarity with an activity and the amount of help parents provide during that activity might exist, I compared the mean proportion of activities consisting of assisted math occurrences for three groups of perceived typicality: typical (n = 40), may or may not be typical (n = 14), and probably not typical (n = 2). Activities perceived as typical had the lowest proportion of assisted occurrences (M = .19; SD = .21). Activities perceived as may or may not be typical consisted of a mean proportion of .35 (SD = .29) assisted occurrences, and activities perceived as not typical had a mean proportion of .79 (SD = .14) assisted occurrences. Thus, it appears as though help was more prominent in activities less familiar to children. Again, I made the decision to include these activities in the analyses for reasons already addressed in this chapter.

Interrater Reliability. Interrater reliability was established on 20% of the activities, stratified for preschool year and SES, for both the teaching strategy and reasons for help coding schemes. For each coding scheme, both percentage of rater agreement and a Cohen's kappa value were calculated. Cohen's kappa was selected as it takes into account the number of inter-rater agreements occurring by chance. Percent agreement was 92% for teaching strategies and 91% for reasons for help. Kappa was .896 for teaching strategies and .824 for help, which is considered excellent. Discrepancies were resolved by having both coders review the video together and discussing the issue until a consensus was reached.

Table 5
Parental Teaching Strategies

	General Code	Specific Code	Definition/Description	Examples
Low Support	Feedback	Non-Instructive Feedback	The parent provides feedback that child is incorrect, asks/prompts/implies child to try again without further help or instruction.	--“No, that’s not right.” --“Count them again.” --repeats initial instruction.
	Prompts	General Prompt	The parent provides a general prompt or hint to assist the child in responding correctly.	--“I think there are more than 7.” --“How many more do you need?” --“What comes after 12?”
		Strategy Prompt	The parent suggests a strategy for the child to use.	--“Use your fingers.” --“Touch each one as you count it.” --“Count them altogether.”
		Specific Prompt	The parent provides specific information (beyond a strategy) to help the child, but the child still completes more than 50% of the task independently.	--Parent provides a skipped or unknown numeral in a number series. --“You have 3, so you need 2 more” (constructing a set of 5).
High Support	Participation	Co-participation	The parent and child do the task together, with each contributing to the outcome equally.	--Parent and child count simultaneously. --Parent and child take turns saying the numeral in a counting string. --Parent touches each object while child counts alone.
		Parent-led participation	The parent provides direct help in completing the task, either physically or verbally, completing more than 50% of the task for the child.	--“I’ll touch each one, and we’ll count together.” --For written arithmetic problems, parent holds up the correct number of fingers and asks child to count them.
	Provides Response	Parent Provides Correct Response	The parent provides the child with the correct response or completes the task for the child.	--child incorrectly states a set has five objects, parent says, “no, there are six.”

CHAPTER FOUR: THE CHILD MATH ASSEMSSMENT RESULTS AND DISCUSSION

Children's Mathematical Knowledge

This first set of analyses was conducted for the purpose of confirming expected age and SES differences in children's mathematical knowledge, as measured by the Child Math Assessment (CMA; Klein & Starkey, 2000), in this sample of children. Results from the larger study indicated that in the United States, PK children had more extensive mathematical knowledge as measured by the CMA than PPK children, as did middle SES children compared to lower SES children (Klein, et. al., 2002; Starkey & Klein, 2007). However, because only a subset of the children from the larger study was utilized for this study, it was necessary to reanalyze the CMA data to ensure the pattern of findings remained the same with the smaller dataset. Children's mean composite scores on the CMA, as well as mean subcomposite scores in the number, arithmetic, geometric reasoning, measurement, and pattern domains are compared.

Mean Composite Scores

A 2 x 2 ANOVA was conducted to evaluate the effects of age and SES on children's composite scores on the CMA. As depicted in Figure 1, there were significant main effects for age, F (1, 174) = 60.825, $p < .001$, and SES, F (1, 174) = 115.616, $p < .001$. Within each age group, PK children outperformed PPK children, and within each SES group, middle SES children outperformed lower SES children at a level exceeding chance. Effect sizes were calculated using Cohen's d (Cohen, 1988), with results indicating very large effects for both age (d = .96) and SES (d = 1.43).

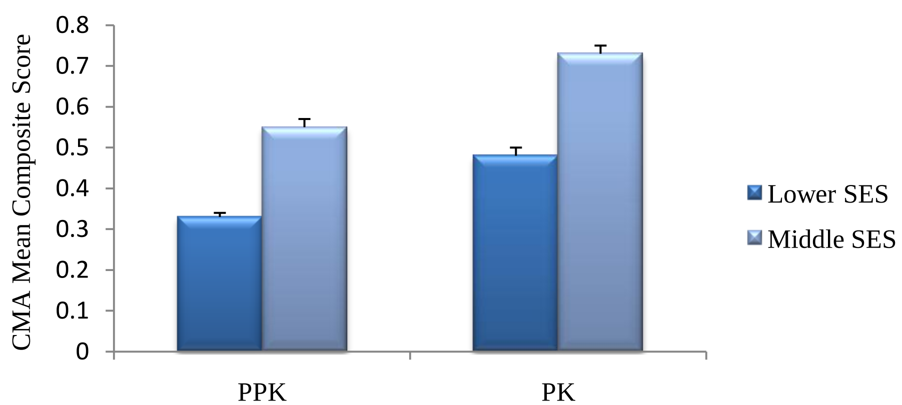

Figure 1. Mean composite scores on the CMA by age and SES. Error bars represent standard errors.

Mean Subcomposite Scores

The effects of age and SES on the children's five subcomposite scores were also examined using a series of 2 x 2 ANOVAs. Results followed the same trend as for the composite score, with significant main effects for age and SES on each of the subcomposite scores, with no significant interaction between age and SES. As with the composite scores, PK children also scored higher than PPK children on the number sense subcomposite, F (1, 174) = 49.478, $p < .001$, $d = .86$, arithmetic subcomposite, F (1, 174) = 36.412, $p < .001$, $d = .83$, geometric and spatial reasoning subcomposite, F (1, 174) = 32.039, $p < .001$, $d = .75$, measurement subcomposite, F (1, 174) = 9.869, $p < .01$, $d = .46$, and pattern knowledge subcomposite, F (1, 174) = 26.791, $p < .001$, $d = .77$. Middle SES children scored higher than lower SES children on the number sense subcomposite, F (1, 174) = 125.814, $p < .001$, $d = 1.52$, arithmetic composite, F (1, 174) = 59.131, $p < .001$, $d = 1.08$, geometric and spatial reasoning subcomposite, F (1, 174) = 60.251, $p < .001$, $d = 1.10$, measurement subcomposite, F (1, 174) = 23.235, $p < .001$, $d = .72$, and pattern knowledge subcomposite, F (1, 174) = 16.906, $p < .001$, $d = .59$. The means and standard deviations for each subcomposite by age and SES group are reported in Table 6.

Table 6
Children's Composite and Subcomposite Scores on the CMA

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
	M (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Composite Score	.33 (.15)	.55 (.17)	.48 (.15)	.73 (.12)
Number Sense	.31 (.19)	.59 (.24)	.47 (.20)	.84 (.13)
Arithmetic	.39 (.16)	.57 (.17)	.57 (.19)	.75 (.17)
Geometric Reasoning	.32 (.20)	.54 (.21)	.60 (.20)	.71 (.17)
Measurement	.44 (.27)	.59 (.29)	.66 (.23)	.75 (.25)
Pattern Knowledge	.07 (.19)	.19 (.30)	.15 (.20)	.41 (.24)

Discussion

As expected, children's mathematical knowledge varied by age and SES. Children in the PK year of preschool held more extensive mathematical knowledge than children in the PPK year of preschool, and middle SES children held more extensive mathematical knowledge compared to lower SES children. This was also true for the overall mathematical knowledge, as evidenced by children's CMA composite scores, as well as for knowledge within the specific domains of number, arithmetic, geometric reasoning, measurement, and patterns.

Consistent with previous findings (Hughes, 1986), the mathematical knowledge of lower SES PK children was very similar to that of middle SES PPK children. Thus, just prior to kindergarten entry, the lower SES children in this sample were in fact about a year developmentally behind middle SES children in terms of their mathematical knowledge and skills. As such, it follows that these lower SES children were less prepared for mathematics in kindergarten, despite their participation in a prekindergarten program. The question is why?

Observational data reported by Starkey and Klein (2007) on the preschool classrooms attended by the children in this sample showed that the teachers of the middle SES children spent more time during the school day on mathematics compared to the teachers of lower SES children. However, as Starkey and Klein also report, the SES differences in mathematical knowledge observed in this sample were present among the PPK cohort during the first few weeks of preschool, thus differences in teacher practices and/or support for mathematics in the classroom cannot be the sole contributor to the SES differences reported here. Since children are entering preschool with differential levels of mathematical knowledge, it is likely middle and lower SES children are receiving differential levels of support for mathematics at home as well.

CHAPTER FIVE: THE PARENT QUESTIONNAIRE RESULTS AND DISCUSSION

Parent Questionnaire

The parent questionnaire was used to investigate potential group differences in the practices, beliefs, and knowledge about informal mathematics held by parents of both low and middle SES PPK and PK children in attempt to generate possible explanations for the group differences revealed by the CMA analyses. Analyses of the parent questionnaire included contrasting the types of activities children engage in at home to support early mathematical development, parents' beliefs about early mathematical development and how it is best supported, parents' knowledge about children's mathematical development, and finally, exploring correlations between these each of these variables and mathematical knowledge as demonstrated by performance on the CMA.

Home Support for the Development of Early Mathematical Knowledge

Range of Home Activities. Parents were asked to report on the frequency of their children's use of 12 specific types of activities with mathematical content, plus any others that were not listed (i.e., a 13th category of "other"). Although the instructions stated parents should indicate whether each activity was done on a daily, weekly, monthly, or rarely/never basis, preliminary analyses of these data resulted in the collapsing of responses across daily and weekly, and then again across monthly and rarely/never categories (DeFlorio & Starkey, 2007). The distribution of the preliminary results suggested that most parents had the tendency to select only daily or weekly for all activities done more than once a month, as opposed to selecting daily for some activities and weekly for others. The distribution followed a similar pattern for monthly vs. rarely/never responses as well. It is likely that children's engagement in specific activities fell between the frequency choices (e.g., twice a week), so some parents consistently selected the higher frequency option while others consistently selected the lower frequency option. Thus, we decided that reporting activities done at least once a week or less than once a week was more meaningful than reporting the four categories separately.

For each of the 13 activities, parents' responses of "daily" and "weekly" were scored as "1," and parents' responses of "monthly" and "rarely/never" were scored as "0." These scores were summed across the 13 activities to produce a range score, indicating the number of different types of activities (e.g., store bought games, plays with math-related toys) done in the home to support early mathematical development at least once a week. The mean range scores for the PPK cohort were 5.50 (SD = 2.98) for lower SES children, and 6.80 (SD = 2.68) for middle SES children. For the PK cohort, the mean range scores were 5.83 (SD = 2.84) for lower SES children, and 6.58 (SD = 3.07) for middle SES children. A 2 x 2 ANOVA revealed a main effect for SES, F (1,174) = 5.601, $p < .05$, $d = .36$, thus middle SES children appear to engage in a greater range of activities to support mathematical development at home. There were no main effects for age, thus there do not appear to be differences in the range of activities done by older and younger preschool age children.

Participation in Specific Activities. Parents' responses for individual activities done at least once per week were compared as well. A two-way MANOVA was conducted to examine potential age and SES differences in the frequency children engage in each of the 13 activity types. Results revealed a significant multivariate effect for SES only, Wilks' $\Lambda = .689$, F (26, 324) = 2.548, $p < .001$. Follow up univariate analyses on the individual activity types indicated that middle SES children were more likely than lower SES children to play made-up games involving math, F (2, 174) = 9.526, $p < .001$, read books with math content, F (2, 174) = 6.888, $p = .001$, use a computer with math software, F (2, 174) = 3.056, $p = .05$, and have math embedded into their home routine, F (2, 174) = 12.972, $p < .001$. The mean proportions of children engaging in each activity at least once a week by age and SES are presented in Table 7.

Table 7
Mean Proportion of Children Participating in Each of the 13 Activity Types at Least Once Per Week by Age and SES

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
Activity Type	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Store-Bought Games	.50 (.51)	.39 (.49)	.48 (.51)	.42 (.50)
Math-Related Toys	.59 (.50)	.48 (.51)	.60 (.50)	.46 (.50)
Blocks or Construction Toys	.80 (.41)	.83 (.38)	.68 (.47)	.71 (.46)
Origami/Kitagami	.43 (.50)	.35 (.48)	.40 (.50)	.33 (.48)
Art Activities	.34 (.48)	.41 (.50)	.45 (.50)	.48 (.50)
Made-Up Games	.50 (.51)	.85 (.36)	.60 (.50)	.81 (.39)
Books with Math Content	.55 (.50)	.89 (.31)	.63 (.49)	.69 (.47)
Math Workbooks	.30 (.46)	.17 (.38)	.38 (.49)	.50 (.50)
Computer with Math Software	.20 (.41)	.35 (.48)	.30 (.46)	.50 (.50)
Singing Math Songs	.41 (.50)	.57 (.50)	.33 (.47)	.40 (.49)
TV Shows with Math Content	.46 (.50)	.50 (.51)	.50 (.51)	.46 (.50)
Math in the Home Routine	.25 (.44)	.67 (.47)	.25 (.44)	.52 (.50)
Other Math Activities	.18 (.39)	.35 (.48)	.25 (.44)	.29 (.46)

Pedagogical Approaches Utilized by Parents. To explore whether or not parents' preferred methods of supporting children's mathematical development differ by age or SES, the mean number of parents in each group endorsing each of the three pedagogical approaches as the most important were calculated and compared using a 2 X 2 ANOVA. The mean number of parents endorsing each approach by age and SES is presented in Table 8. A main effect for age was found for the third pedagogical approach, enriching children's playtime with math-related toys and materials, with more PK parents endorsing this approach than PPK parents, F (1, 174) =

5.280, $p < .05$. A main effect for SES was found for two of the three of the pedagogical approaches, with middle SES parents more likely then lower SES parents to endorse embedding math in the children's home routine, $F(1, 174) = 5.064$, $p < .05$. and lower SES parents more likely to endorse one-on-one time with their children for the specific purpose of developing cognitive skills, $F(1, 174)$, 3.895, $p = .05$.

Table 8

Mean Proportion of Parents Endorsing Each Pedagogical Approach as the Most Important Used in Their Home to Help Children Develop Mathematical Knowledge

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
Pedagogical Approach	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
I give my child math-related tasks or ask math-related questions during ongoing domestic routines.	.25 (.44)	.43 (.50)	.23 (.42)	.35 (.48)
I set aside time to be with my child on a regular basis to help him/her develop cognitive skills.	.68 (.47)	.52 (.51)	.55 (.50)	.42 (.50)
I enrich my child's play time (alone or with other children) by providing math-related toys and materials.	.43 (.50)	.20 (.40)	.50 (.51)	.46 (.50)

Parent Beliefs About Mathematical Development

Home vs. School Influences on Early Mathematical Development. To capture parent beliefs about the relative contribution of the home and preschool environments to preparing children for math in kindergarten, the mean number of parents reporting that the home contributes more, the home and preschool contribute equally, and the preschool contributes more were compared using a series of 2 X 2 ANOVAs. Results indicated that middle SES parents were more likely to endorse the belief that the home contributes more than lower SES parents, $F(1, 174) = 6{,}483$, $p < .05$. As illustrated in Figure 2, there was an interaction effect, $F(1, 174) = 4.880$, $p < .05$, for the belief that preschool contributes more. For both lower SES cohorts, preschool was generally thought to contribute more than the home, while for middle SES cohorts, the parents of PPK children rarely believed preschool contributes more, and parents of PK children were divided in their beliefs. There were no main effects for SES or cohort for the belief that the home and preschool contribute equally.

The Influence of Spontaneous Play vs. Adult-Directed Activities on Early Mathematical Development. To capture parent beliefs about the relative contribution of spontaneous play and adult-directed activities on preparing for math in kindergarten, the mean number of parents reporting that the spontaneous play contributes more, spontaneous play and adult-directed activities contribute equally, and adult-directed activities contribute more were computed for each group. A series of 2 X 2 ANOVAs revealed a main effect for the belief that play and adult-directed activities contribute equally, with lower SES parents more likely to endorse this belief, F (1, 174) = 4.998, $p < .05$ (see Figure 3).

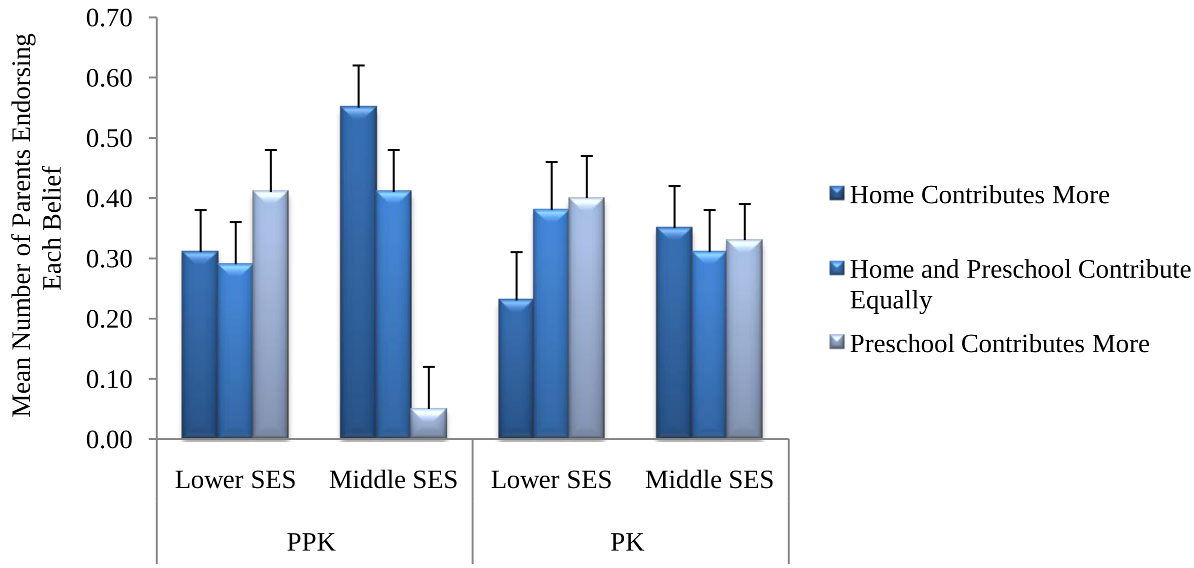

Figure 2. Parent beliefs about the relative contributions of the home and preschool to preparing their children for mathematics in kindergarten. Error bars represent standard errors.

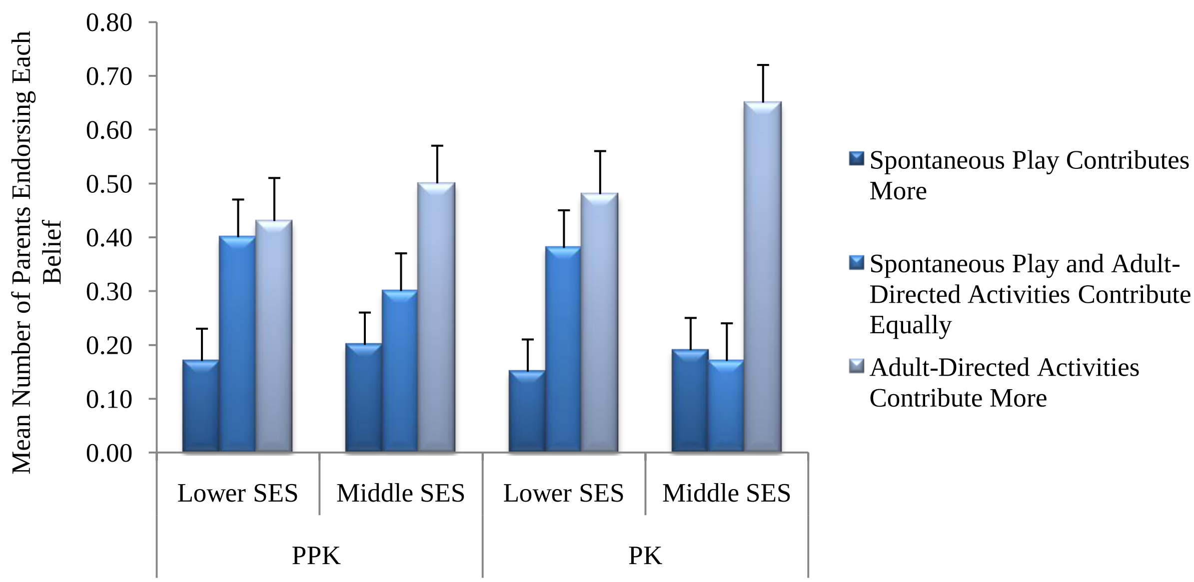

Figure 3. Parent beliefs about the relative contributions of spontaneous play and adult-directed activities to preparing their children for mathematics in kindergarten. Error bars represent standard errors.

Parent Math Expectations. Parental beliefs, or expectations, about children's mathematical abilities were assessed by presenting parents with 23 mathematical skills and abilities and asking them to endorse those they believe are typically present by children's 5th birthdays. Means, presented in Table 9, were computed and compared using a 2 X 2 ANOVA to see if expectations differ between the age and SES groups. Parent's expectations did not differ by age, but there was a main effect for SES, F (1, 174) = 16.94, $p < .001$, d = .62. Middle SES parents held higher expectations (M = .66, SD = .14) for their children than lower SES parents (M = .56, SD = .19).

Table 9

Parental Expectations About Specific Mathematical Skills and Abilities Within and Beyond 5-Year-Old Children's Developmental Range by SES Group

	Lower SES		Middle SES	
Mathematical Skill or Ability	*M*	*SD*	*M*	*SD*
Within Developmental Range				
Count a row of 10 objects*	.93	(.26)	.99	(.10)
Construct an equivalent set of 7 objects*	.75	(.44)	.88	(.32)
Show second ordinal position in a row of 5*	.77	(.42)	.89	(.31)
Solve addition/subtraction problems with objects*	.61	(.49)	.78	(.42)
Arrange 5 sticks in order of increasing length***	.62	(.49)	.86	(.35)
Sort a set of objects into 2 groups by color**	.82	(.39)	.97	(..17)
Shape naming: Circle, square, triangle**	.83	(.37)	.97	(.18)
Understand spatial words (e.g., on, under, behind)*	.80	(.40)	.93	(.26)
Measure a pencil using string***	.35	(.48)	.69	(.46)
Create an A-B-A-B pattern with colored beads**	.75	(.44)	.94	(.25)
Use a computer with math software***	.57	(.50)	.84	(.37)
Beyond Developmental Range				
Read the numerals from 1 to 10***	.67	(.47)	.91	(.28)
Solve single digit addition/subtraction problems presented on flashcards*	.37	(.49)	.21	(.41)
Read basic arithmetic symbols*	.33	(.47)	.50	(.50)

*$p < .05$, ** $p < .01$, *** $p < .001$

Parent Knowledge about Early Mathematical Development

Accuracy of Parent Math Expectations. In addition to looking at the nature of parent beliefs and expectations, it is possible there are group differences in the accuracy of these beliefs. To explore this further, responses to the parent expectation questions were recoded to reflect correct and incorrect beliefs. Mean accuracy scores were computed for each of the four groups and compared using a 2 X 2 ANOVA. Middle SES parents demonstrated a more accurate understanding ($M = .78$, $SD = .10$) of which skills and abilities are within a typical five-year-old child' developmental range compared to lower SES parents ($M = .70$, $SD = .13$), F (1, 174) = 19.569, $p < .001$, $d = .66$. There were no significant differences by age.

Further exploration of the SES differences found in parents' correct and incorrect beliefs was conducted by comparing means by SES group for each of the 23 skills or abilities using the original coding scheme. A two-way MANOVA revealed a significant multivariate effect for SES only, Wilks' $\Lambda = .633$, F (23, 152) = 3.824, $p < .001$. Follow up univariate analyses on the individual skills or abilities revealed that parents in the two SES groups differed significantly in their beliefs about 14 of the abilities, 11 of which were with children's developmental range, and three of which were not. For the 11 skills with children's developmental range, lower SES parents were more likely to underestimate children's abilities, whereas for the three skills beyond most five-year-old children's developmental range, middle SES parents were more likely to overestimate children's ability to read numerals and arithmetic symbols, but lower SES parents were more likely to overestimate children's ability to solve addition and subtraction problems presented on flashcards.

Correlations Between the Parent Questionnaire and Children's CMA Scores.

Although interesting, group differences on their own aren't necessarily useful unless such differences matter for young children's developing mathematical knowledge. The next step in these analyses was to see which items on the parent questionnaire actually correlate with children's scores on the CMA. Individual Pearson correlation coefficients were computed between children's CMA scores and each score calculated in the activity, beliefs, and knowledge sections of the parent questionnaire described above. A total of nine scores from the parent questionnaire were significantly correlated with children's CMA scores. From the activity section, significant correlations with the CMA included the score indicating the range of math activity types done at least once per week ($r = .16$, $p = .05$), and the individual scores for art activities ($r = .15$, $p = .05$), made-up games ($r = .22$, $p = .01$), using a computer with math software ($r = .25$, $p = .01$), and math in the home routine ($r = .27$, $p = .01$) at least once per week. The pedagogical approach, "I set aside time to be with my child on a regular basis to help him/her develop cognitive skills," was negatively correlated to children's math scores ($r = -.26$, $p = .01$). Parent beliefs about the relative contribution of home vs. preschool and spontaneous play vs. adult-directed activities were not significantly correlated with children's mathematical scores, but parent beliefs, or expectations, about the skills and abilities of typical 5-year-old children were significantly correlated ($r = .39$, $p = .01$). Finally, in terms of parent knowledge, correlations were significant for both the accuracy of parents' expectations about the skills and abilities of typical 5-year-old children ($r = .39$, $p = .01$). It is important to note that main effects for SES were previously reported for each of these parent questionnaire scores except art activities done at least once per week, as well as a main effect for age for using a computer with math software at least once a week, thus further analysis is necessary to eliminate potential confounds with SES and age.

Hierarchical multiple regression was used to determine the unique contribution of parent questionnaire variables and children's scores on the CMA. Variables were entered into the model in two blocks. The first block was entered using the forward method, and consisted of SES and child's age at spring testing. Age was used instead of preschool year, as it is really what preschool year represents (i.e., children in the PK cohort are approximately one year older than children in the PK cohort). The second block consisted of each variable determined by the previous correlational analyses to be significantly correlated with children's CMA scores. This block was entered and analyzed using the stepwise method.

The regression analysis yielded a total of 3 significant models. The first model showed that children's SES and age at testing accounted for a significant amount of variability in posttest scores on the CMA, $R^2 = .54$, $F\ (2, 175) = 103.017$, $p < .001$. In addition to SES and age at testing, the second model included parents' expectations of mathematical knowledge at age five years, $R^2 = .58$, $F\ (3, 174) = 81.081$, $p < .001$. The final model included SES, age at testing, parents' expectations of mathematical knowledge at age five years, and parents' knowledge about mathematical development at age five (i.e., the accuracy score), $R^2 = .60$, $F\ (4, 173) = 64.111$, $p < .001$, indicating that both parents' expectations and parents' knowledge about early mathematical development predict unique variance in children's scores on the CMA above and beyond SES and children's age. All other explanatory variables were excluded from the model via the stepwise procedure, indicating no unique contribution to the variance in CMA scores. The unstandardized beta, standard error, and standardized Beta for the three models are reported in Table 10.

Table 10
Parent Responses from the Parent Questionnaire as Predictors of CMA Scores

	b	*SE b*	*β*
Model 1			
Constant	-.631	.097	
SES	.233	.021	.565
Age at Testing	.174	.020	.449
Model 2			
Constant	-.731	.095	
SES	.206	.021	.501
Age at Testing	.171	.019	.440
Parent Expectations	.254	.061	.216
Model 3			
Constant	-.845	.105	
SES	.195	.021	.475
Age at Testing	.171	.019	.439
Parent Expectations	.182	.066	.154
Parent Accuracy	.237	.096	.140

Summary and Discussion

The primary objective of the questionnaire component of this study was to identify and explore possible factors in the home environment contributing to known differences in mathematical development among children in the PPK and PK years of preschool from lower and middle SES families. Parent responses to questions about the range and type of activities done to support mathematical development, the pedagogical approach deemed most important by parents for supporting children's mathematical development, parents' beliefs about early mathematical knowledge, and parents' knowledge about early mathematical development were compared with children's scores on a child math assessment.

There were no age differences in the range of activities parents reported their children engaging in on a weekly basis, and for the most part very few differences by age in regards to the types of activities PPK and PK children typically do. Children in the younger group were more likely than children in the older group to play with blocks and construction toys, and children in the older group were more likely to use math workbooks and sing math songs, but children's engagement in the other 10 (nine plus one "other") activity types does not appear to vary much over the preschool years. There also does not appear to be wide variation in terms of the context endorsed by parents of PPK and PK children as most important for those activities to occur. Although more parents of PK children endorsed enriching the child's playtime with cognitively stimulating materials as the most important approach for supporting mathematical development, many parents also indicated that spending one-on-one time helping their children develop cognitive skills was equally important.

The SES differences in the direct mathematical support provided in the home were more pronounced. Middle SES children engaged in a greater range of activities on a weekly basis compared to lower SES children. This is discrepant with the previous findings reported by Saxe et. al., (1987) that the range of home activities do not differ by SES. However, it should be noted that the current questionnaire inquired about a wider range of activity types than used in the Saxe et. al. study, and only four activity types were common to both studies.

Consistent with previous findings (Starkey, et. al., 1999), middle SES children engaged in made-up games involving math, used math in the home routine, read math-related books, and used computers with math software more frequently than lower SES children. There were no SES differences for the remaining nine activity types. Middle SES parents were more likely to endorse the pedagogical approach of embedding mathematics into the home routine, while lower SES parents showed a preference for setting aside one-on-one time with their children to work on cognitive skills. This is also consistent with previous findings (Stipek, et. al., 1992).

Parents' beliefs and knowledge about early mathematical development varied little as a function of children's age, but more so as a function of SES. Similar to Starkey, et. al., (1999), middle SES parents in general were more likely to endorse the belief that the home environment bears more responsibility for preparing children for math in kindergarten than the preschool environment, but it is likely that the PPK parents are carrying most of this effect. Middle SES parents of the younger preschoolers rarely endorsed the belief that the preschool has a greater contribution than the home, but by the end of prekindergarten, there was more variation in their beliefs. Lower SES parents of children in both age groups were more likely to endorse the belief that preschool contributes more, with less than one-quarter of all lower SES parents endorsing the belief that the home contributes more.

Parents' beliefs about the contribution of children's spontaneous play versus adult-directed activities to preparing children for math in kindergarten are interesting in that very few parents across age and SES groups believed that spontaneous play contributed more. This was true even for middle SES parents of PK children who as a group, endorsed enriching their children's playtime by providing math-related toys and materials as the most important pedagogical approach used in the home at the greatest frequency. This suggests that the majority of parents recognize the value of adult-child teaching interactions for children's acquisition of mathematical knowledge, even if for lower SES parents they may believe the primary responsibility for those interactions falls on preschool staff.

There were no differences between age groups in parents' expectations of young children's mathematical abilities at age five, but parents' expectations in the two SES groups differed significantly. Middle SES parents held higher expectations than lower SES parents. This is consistent with the findings of Starkey, et. al.. (1999). In addition to having higher expectations of their children's mathematical abilities, middle SES parents' expectations were also more accurate. Compared to lower SES parents, middle SES parents were more knowledgeable about which skills or abilities were within the developmental range of most typical five-year-old children. It is interesting, and perhaps sheds additional light on the extent of parents' knowledge about early mathematical development, that middle SES parents were more likely to overestimate children's abilities to read numerals and arithmetic symbols, but lower SES parents were more likely to overestimate children's ability to solve addition and subtraction problems presented on flashcards. Presumably, solving addition and subtraction problems presented on flashcards requires the ability to read numerals and some arithmetic symbols, yet the percentage of lower SES parents responding affirmatively to each suggests at least some parents in this group lack understanding of the sequential nature of mathematics.

Despite the many differences between SES groups that emerged from the parent questionnaire, only parental expectations of children's mathematical ability at age five and the accuracy of those expectations contributed unique variance to children's mathematical knowledge on the CMA above and beyond age and SES. Children's activities in the home as measured by this questionnaire do not appear to be associated with children's mathematical ability at any level beyond chance. However, this null result should not be perceived as conclusive evidence that activities and supports in the home are not important. Instead, it is possible that it is not the types of activities children do at home, but the qualitative differences in the mathematical experiences embedded within those activities that matter for children's developing mathematical knowledge.

One qualitative difference in home activities that was not completely captured, but was supported by the parent questionnaire involves the content of adult-child (or older peer-child) interaction when engaging in activities believed to support early mathematical development. Because middle SES parents have higher expectations about their children's mathematical abilities, it is possible they encourage or attempt to teach their children more sophisticated or complex mathematical concepts than lower SES parents. Furthermore, because middle SES parents have a more accurate understanding of skills within children's typical developmental range, it may be that they are more likely to engage in activities within the zone of proximal development, as opposed to lower SES parents who showed a greater tendency to underestimate children's potential for mathematical ability. If true, this could at least partially explain the high association found between parents' expectations and knowledge about mathematical development and child math assessment scores, as well as explain the finding by Saxe et. al., (1987) that middle SES children engage in more home activities with higher goal structures than lower SES children.

A second, potential qualitative difference not captured by the parent questionnaire but possibly influencing middle and lower SES children's early mathematical development may involve characteristics of the parent-child teaching interaction beyond just the complexity of the math being done. As discussed in the literature review chapter, SES differences in parents' scaffolding behavior during problem-solving and other teaching tasks are well documented (e.g., Borduin & Henggeler, 1981; Hoff-Ginsberg, 1991; Laosa, 1980), and these differences are correlated with children's outcomes on general measures of cognitive ability during the preschool years (Hess & McDevitt, 1984). Parents' scaffolding behavior during authentic, parent-initiated activity, or mathematical activity in general, has not been systematically examined and reported in the literature, thus it is not known if the SES differences in teaching strategies utilized by parents and described in the literature review generalize to these situations as well. These potential qualitative differences are examined in the next chapter.

CHAPTER 6: THE HOME OBSERVATIONS RESULTS AND DISCUSSION PART III

Home Observations/Videos

The purpose of the home observation component of this study was to examine potential qualitative age and SES differences present in children's home activities not captured by the questionnaire. Specifically, I focus on the mathematical content and complexity, children's participatory role, and the teaching strategies used by parents to help children produce correct responses to mathematical questions or tasks. When applicable, I also compare results from the video analyses to the children's scores on the CMA. Given the variation in the number of activities demonstrated by each parent-child dyad, unless otherwise specified, the unit of analysis was the observation (i.e., the full collection of activities recorded for each parent and child). Although I had hoped to examine the characteristics of specific activity types, the small sample size proved to be a limitation. Separating the data into groups beyond age and SES had the tendency to dilute the data to a point where analysis was no longer meaningful. There simply was not enough data for each of the age or SES groups to systematically look at between group variation in math content present in one specific type of activity versus another type of activity. In the end, the mathematical content was analyzed by age and SES only, collapsed across activity type.

General Characteristics of the Home Observations

A series of 2 X 2 ANOVAs revealed no significant differences between groups for mean length of observation, mean number of minutes including math, mean number of math occurrences per activity, or mean number of math occurrences per minute. The means, standard deviations, and ranges for each are reported in Table 11. Overall, observations ranged from 11 to 47 minutes in length (M = 24.76; SD = 10.99). The number of minutes including math ranged from 10 to 47 minutes (M = 21.36; SD = 9.22), and the number of math occurrences ranged from 23 to 232 per observation (M = 98.12; SD = 56.39). Finally, the mean number of math occurrences per minute ranged from 1.26 to 9.28 (M =4.17; SD = 2.08).

Activity Types Observed. At the most general level, activity type was coded using the same categories from the parent questionnaire. Activities observed in the home included store-bought games (n = 12, or 21% of activities), art activities involving math (n = 1, or 2%), math-related toys (n = 1, or 2%), construction toys (n = 1, or 2%), made-up games involving math (n = 12, or 21%), reading books with math content (n = 5, or 9%), using workbooks, worksheets, or flashcards (n = 6, or 11%), using a computer with math software (n = 2, or 4%), and using math in the home routine (n = 16, or 29%). The frequency and percentages of each activity type by age and SES are reported in Table 12.

Table 11.

General Characteristics of the Home Observations, Including the Length of the Observation, the Number of Minutes that Included Math, the Number of Occurrences of Math, and the Number of Math Occurrences Per Minute

	M	*SD*	Minimum	Maximum
<u>Length of Observation</u>				
PPK				
Lower SES	23.20	8.73	11.00	35.00
Middle SES	25.20	13.63	15.00	45.00
PK				
Lower SES	29.33	13.53	13.00	47.00
Middle SES	22.33	9.66	13.00	44.00
<u>Minutes with Math</u>				
PPK				
Lower SES	18.00	5.52	10.00	25.00
Middle SES	19.80	8.81	10.00	31.00
PK				
Lower SES	26.33	11.83	13.00	47.00
Middle SES	20.78	9.28	12.00	42.00
<u>Occurrences of Math</u>				
PPK				
Lower SES	65.60	25.48	43.00	102.00
Middle SES	86.80	60.00	23.00	179.00
PK				
Lower SES	130.17	51.41	61.00	209.00
Middle SES	101.11	65.13	50.00	232.00
<u>Math Occurrences per Minute</u>				
PPK				
Lower SES	3.08	1.12	1.26	4.08
Middle SES	3.49	1.82	1.44	5.53
PK				
Lower SES	4.98	2.35	2.67	8.46
Middle SES	4.62	2.34	2.08	9.28

Table 12
Activity Types Observed by Age and SES

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
Activity Type	% (*n*)	% (*n*)	% (*n*)	% (*n*)
Store-Bought Game	15.4 (2)	7.7 (1)	23.1 (3)	35.3 (6)
Art Activity	7.7 (1)	--	--	--
Construction Toy	--	7.7 (1)	--	--
Math-Related Toy	--	7.7 (1)	--	--
Made-Up Game	23.1 (3)	30.8 (4)	23.1 (3)	11.8 (2)
Reading Books	7.7 (1)	23.1 (3)	--	5.9 (1)
Workbook/Worksheet/Flashcards	7.7 (1)	--	30.8 (4)	5.9 (1)
Computer with Math Software	--	7.7 (1)	7.7 (1)	--
Home Routine	38.5 (5)	15.4 (2)	15.4 (2)	41.2 (7)
Total	100 (13)	100 (13)	100 (13)	100 (17)

Mathematical Content and Complexity

The first step in analyzing the mathematical content in children's home activities was to capture the mathematical domain, or domains, being supported within each observation. The number/arithmetic domain was the most widely supported domain across both age and SES groups, appearing in at least one activity in all observations. Across age and SES groups, number/arithmetic was the sole domain supported in 46% of the observations. There were no differences by age or SES in the mean range of domains supported per observation (M = 1.65, SD = .80).

Support in the domains of geometry and space, measurement, patterns, and "other," was seemingly dependent on the activities parents chose to demonstrate. The percentage of observations including support for the domains of number/arithmetic, geometry and space, measurement, pattern knowledge, and "other," are reported in Table 13. Although there were no significant differences by age or SES, it must be noted that these are percentages based on very small groups of children, and across entire observations.

Table 13
Percentage of Observations Including Support in Each Mathematical Domain

	PPK		PK	
	Lower SES	Middle SES	PK Lower SES	PK Middle SES
Domain	%	%	%	%
Number /Arithmetic	100	100	100	100
Geometry/Space	0.00	40.0	28.6	11.1
Measurement	80.0	20.0	14.0	22.2
Patterns	20.0	0.00	0.00	11.1
Other	0.00	40.0	0.00	0.00

When considering the 56 individual activities observed, this picture changes somewhat. The majority of individual activities, 73%, provide support for number/arithmetic only. In regards to the remaining activities, 11% were essentially non-number activities with some number concept or concepts infused into the activity by the parent (e.g., the main activity was matching socks based on length, but at some point the parent asks the child to count the number of pairs), and 8% were essentially number activities with non-number mathematical content infused into the activity by the parent (e.g., the main activity is to set a table with four place settings, parent asks child to identify the shape of the napkins and bowls). The remaining 7% of activities were split equally between number activities with mathematical concepts outside of the number domain necessarily introduced for the purpose of completing the primary activity (e.g., the instructions for a worksheet are to draw a circle around all sets with *n* objects), and activities that involved a shift in focus from number to a second mathematical domain (e.g., parent reads a number book to the child, followed by a shape book). There was also a pattern activity done by one middle SES PK child that did not include any support for number.

The second step in analyzing the mathematical content of the activities involved examining the specific mathematical processes within each domain for the purpose of identifying group similarities and differences in the specific mathematical skills utilized in the activities. A total of 38 distinct mathematical processes were observed, with just five of those accounting for over 80% of all processes. The five most common mathematical processes included enumerating or counting a set (33%), numeral recognition or naming (25%), counting objects to a specified number (14%), set construction (5%), and addition/subtraction with concrete objects (4%). Detailed descriptions of each process, along with examples are presented in Appendix C. The general findings are reported here first by number and arithmetic, and then subsequently for the geometry/space, measurement, patterns, and "other" domains.

Number and arithmetic. Math occurrences in the domains of number and arithmetic accounted for 92% of all math occurrences observed, and spanned the full range of the four complexity levels (Saxe, et. al., 1987; see Table 14). A total of 29% of occurrences were coded

as level 1, 55% as level 2, 3% as level 3, and 5% as level 4. There were no significant differences by age or SES in the proportion of occurrences within observations that contained mathematical processes in each of the four goal levels, although this might be attributed to the small sample size. A potential interaction effect was detected for level four processes, F (1, 22) = 3.187, p = .088, indicating no SES difference in the proportion of observations containing level four processes in the PPK group, but the proportion of observations containing level four processes being higher for middle SES compared to lower SES children in the PK group.
To further examine this trend with a larger sample size, the mean proportion of activities containing processes at each of the four levels was computed. Using proportions of occurrences within activities (n = 56) rather than observations (n = 26), the interaction trend for level four processes disappeared, but a main effect for age emerged in its place, F (1, 52) = 4.684, p < .05. Children in the prekindergarten group were more likely than children in the preschool entry group to engage in arithmetic. Between group differences remained non-significant for activities containing level one, two, and three processes. The percentage of observations and activities containing mathematical processes in each level by age and SES are reported in Table 14.

Table 14
Percentage of Observations and Activities Containing Mathematical Processes in Each of Four Goal Levels by Age and SES

	PPK				PK			
	Lower SES		Middle SES		Lower SES		Middle SES	
Complexity Level	Obs %	Act %	Obs %	Act %	Obs %	Act %	Obs %	Act %
1 (no sets, denotation only)	100	54	100	62	86	77	89	59
2 (single sets)	100	100	100	92	100	92	100	94
3 (comparison of multiple sets)	20	8	40	15	29	15	50	29
4 (arithmetic with multiple sets)	80	31	60	23	43	38	89	71

Obs = Percentage of Observations
Act = Percentage of Activities

Level one processes. There were nine level one mathematical processes observed across age and SES groups, including numeral recognition/naming, knowledge of number order, rote counting, backward rote counting, ordinal position, knowledge of odd/even, fraction recognition/naming, copying/tracing numerals, and writing numerals. The mean proportion of observations containing each level one mathematical process by age and SES is reported in Table 15. Numeral recognition/naming was the most common level one process for children across age and SES groups, with no significant group differences in the mean proportion of observations including that process. Knowledge of number order was the second most commonly observed level one process, although it was more common for lower SES children than middle SES children in both age groups, F (1, 22) = 5.409, p < .05.

Upon deeper investigation, I found that with the exception of one occurrence in the middle SES PPK group where a number order question was posed as part of a game, all other math occurrences involving the knowledge of number order process occurred only in the context of counting or numeral identification. Parents posed number order questions or explicitly taught their children about number order when the children either made a mistake counting, or failed to correctly name a numeral (e.g., the child counted "1, 2, 3, 5," and the parent responded by asking "what number comes after three?").

The remaining goal one processes were not subjected to statistical comparison due to the infrequency of occurrence, but are described here nonetheless. Traditional rote counting forward appeared only in lower SES PK observations, whereas rote counting backward appeared in both middle SES PPK and PK observations. Ordinal number only appeared in one observation of a lower SES PPK child, and knowledge of even and odd properties of number only appeared in one observation of a middle SES PPK child. Fraction recognition/naming was present in one observation of one child in each of the age and SES groups except lower SES PK. Finally, only lower SES children, in both PPK and PK engaged in copying or tracing numerals, and only lower SES PK children engaged in writing numerals from memory.

Level two processes. All observations consisted of level two mathematical processes (see Table 15). Across observations, four specific processes were common among children in each age and SES group. These included enumerating/counting a set, counting objects to a specified number, set construction, and counting a subset. There were no significant differences by age of SES for any of these processes. Less frequent level two math processes observed included matching sets to numerals, estimation of quantity, and set decomposition. Matching sets to numerals was observed in one observation of a lower SES PK child and one observation of a middle SES PK child. Estimation of quantity and set decomposition were observed in single observations of middle SES PK children only.

Level three processes. Level three mathematical processes were extremely rare throughout all of the observations (see Table 15). Typically when two or more sets were involved, some type of arithmetic computation was also involved, thus changing the process to level four. There were only two observed math processes, one-to-one correspondence and quantitative comparison of two or more sets, which qualified as level three. All instances of one-to-one correspondence occurred during home routine activities when children were setting the table. Children typically set the table for four or five people, placing one napkin, glass, plate, etc., at each setting. Quantitative comparison of two or more sets occurred even less frequently than one-to-one correspondence, and typically involved the parent asking the child which of two sets was more/less numerous. There were no significant group differences for either of the level three processes.

Level four processes. There were a total of four level four mathematical processes observed, the most common being addition and subtraction with concrete objects (see Table 15). Surprisingly, there were no significant differences by age or SES in terms of the mean proportion of observations including this process. In the lower PPK group, one parent demonstrated an activity where addition with concrete objects was the primary activity, but three of the five parents in this same group incorporated one addition with concrete objects occurrence into activities that otherwise were comprised of goal 1 and 2 processes. This inclusion of the single concrete objects problem by those three parents resulted in 80% of observations on children in this group, or four out of five, coded as including this math process.

Table 15
Mean Proportion of Observations Containing Mathematical Processes in Each of Four Goal Levels by Age and SES

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
	M	*M*	*M*	*M*
Level 1				
Rote Counting, Forward	--	--	.43	--
Rote Counting, Backward	--	.20	--	.11
Knowledge of Number Order	.60	.20	.57	.11
Ordinal Position	.20	--	--	--
Knowledge of Even/Odd	--	.20	--	--
Numeral Recognition/Naming, Integers	.60	.60	.71	.89
Numeral Recognition/Naming, Fractions	.20	.20	--	.11
Copying/Tracing Numerals	.20	--	.29	--
Writing Numerals	--	--	.43	--
Level 2				
Estimating Quantity of a Set	--	--	--	.11
Enumerating/Counting a Set	1.00	1.00	1.00	1.00
Enumerating/Counting a Subset	.40	.20	.14	.22
Counting Objects to a Specified Number	.60	.40	.71	.67
Set Construction	1.00	.80	.86	.67
Set Decomposition	--	--	--	.11
Matching Sets to Numerals	--	--	.14	.11
Level 3				
One-to-One Correspondence	.20	--	.14	.38
Quantitative Comparison of Two or More Sets	--	.40	.29	.33
Level 4				
Addition/Subtraction with Concrete Objects	.80	.40	.29	.78
Addition/Subtraction with Hidden/Imaginary Objects	--	--	.14	--
Formal Addition/Subtraction	--	--	.14	.33
Informal Division	--	--	.14	--

That the lower SES PPK group had the highest percentage of observations with addition/subtraction with concrete objects processes seemed contrary to what I actually observed in the videos, so I separated the observations by individual activities and ran the analysis again. Conducting the analysis with individual activities instead of entire observations by child yielded a significant interaction effect for age and SES, $F\ (1, 52) = 5.480$, $p < .05$. Lower SES children's activities were more likely to include this process ($M = .31$; $SD = .48$) compared to middle SES children in PPK ($M = .15$; $SD = .38$), but middle SES children's activities were more likely to include this process ($M = .65$, $SD = .49$) compared to lower SES children ($M = .23$, $SD = .44$) in PK. The additional math processes included addition and subtraction with hidden/imaginary objects, formal addition/subtraction, and informal division. Addition and subtraction with hidden/imaginary objects was observed in one observation of a lower SES PK child, whereas formal addition was only seen in lower and middle SES PK observations. Informal division accounted for only one occurrence in one activity done by a lower SES PK child.

To summarize, there were no significant differences by age or SES in the proportion of observations containing levels one, two, three, and four mathematical processes. When analyzed by individual activities as opposed to full observations, PK children had a higher proportion of activities containing level four processes compared to PPK children.

Set sizes. To further explore potential age and SES differences in mathematical content, the set sizes or problem details were compared between groups for the five most common mathematical processes: numeral recognition/naming, enumerating/counting a set, counting objects to a specified number, set construction, and addition/subtraction with concrete objects. For the four number processes, this involved calculating the mean, median, and range of numerals or set sizes for each occurrence by age and SES (see Table 16). For the arithmetic process, each addition or subtraction occurrence was separated into the augend/minuend, addend/subtrahend, and sum/difference, and then the mean, median, and range of each were calculated by age and SES.

A series of two-way ANOVAs revealed main effects for age in two of the four number processes and SES for one number process. Compared to children in PPK, PK children were engaging in math processes with larger set sizes for enumerating/counting a set, $F\ (1, 813) = 15.590$, $p < .001$, and set construction, $F\ (1, 128) = 6.349$, $p = .01$. There was also a main effect for SES in the counting objects to a specified number process, with middle SES children counting to larger numbers than lower SES children, $F\ (1, 348) = 3.923$, $p < .05$. There were no group differences for numeral recognition/naming. For the arithmetic process, there was a main effect for age for the augend/minuend, F (1, 99) = 6.845, p = .01, and the addend/subtrahend, F (1, 99) = 4.495, p < .05, with PK children engaging in problems with larger numbers than PK children. There were no significant differences between groups for the sum/difference.

Table 16
Set Sizes for Most Common Math Processes by Age and SES

	PPK		PK	
Math Process	Lower SES	Middle SES	Lower SES	Middle SES
Numeral Recognition/Naming				
Mean	4.22	5.56	4.22	57.95
SD	2.80	4.71	2.48	306.85
Median	3.00	4.00	4.00	4.00
Minimum	.00	.00	1.00	.00
Maximum	10.00	30.00	12.00	2804.00
Enumerating/Counting a Set				
Mean	4.39	4.73	5.96	5.80
SD	3.07	5.38	5.38	4.12
Median	4.00	4.00	4.00	5.00
Minimum	.00	1.00	1.00	.00
Maximum	25.00	60.00	24.00	29.00
Counting Objects to a Specified Number				
Mean	4.67	8.00	4.17	4.97
SD	2.67	21.80	2.44	3.29
Median	4.00	4.00	4.00	4.00
Minimum	1.00	1.00	1.00	1.00
Maximum	10.00	100.00	15.00	17.00
Set Construction				
Mean	3.34	3.50	5.06	4.92
SD	2.81	1.61	3.61	4.39
Median	2.00	3.00	5.00	4.00
Minimum	1.00	1.00	1.00	1.00
Maximum	10.00	8.00	15.00	17.00
Addition/Subtraction with Concrete Objects				
Augend/Minuend				
Mean	2.81	5.05	6.25	5.36
SD	1.38	2.18	2.44	4.15
Median	3.00	5.00	6.00	4.00
Minimum	1.00	1.00	2.00	1.00
Maximum	6.00	9.00	10.00	17.00
Addend/Subtrahend				
Mean	2.81	2.05	3.63	3.10
SD	1.76	1.28	2.39	2.18
Median	2.50	2.00	3.00	2.50
Minimum	.00	1.00	1.00	1.00
Maximum	6.00	5.00	10.00	9.00
Sum/Difference				
Mean	5.63	5.95	6.63	6.86
SD	2.42	2.58	2.92	2.54
Median	4.50	6.00	7.00	7.00
Minimum	2.00	2.00	.00	3.00
Maximum	10.00	11.00	11.00	13.00

Geometry/Space, Measurement, Patterns, and "Other." Math processes in the geometry and space, measurement, patterns, and the "other" domain were so infrequent across observations that comparison by age and SES was not meaningful. Table 17 contains the mean proportion of observations containing each of the processes observed in the geometry/space, measurement, pattern, and other/time domains. Only five of the 26 observations included processes in the geometry/space domain, with the majority of processes occurring in only two of those observations. Even if group differences did exist, it is likely there was not enough data to capture them. Math processes in the geometry and space domain consisted of shape matching, shape naming/recognition, tracing shapes, drawing/constructing shapes, shape analysis, decomposition of shapes, and using spatial terms to denote location.

Table 17

Mean Proportion of Observations Containing Specific Mathematical Processes in the Geometry/Space, Measurement, Patterns, and Other Domains by Age and SES

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
	M	*M*	*M*	*M*
Geometry/Space				
Shape Matching	--	.20	.15	--
Shape Naming/Recognition	--	.40	.14	.11
Tracing Shapes	--	--	.14	--
Drawing/Constructing Shapes	--	.20	.15	--
Shape Analysis	--	.20	.15	--
Shape Decomposition	--	.20	--	--
Using Spatial Terms to Describe Location	--	.20	--	.11
Measurement				
Direct Measurement	.40	.20	--	.11
Measurement with a Non-Standard Unit	.20	--	--	--
Denotation of Formal Units of Measurement	--	--	.14	.11
Graphing	.20	--	--	--
Patterns				
Pattern Identification/Creation	.20	--	--	.11
Pattern Extension	.20	--	--	.11
Other (Time)				
Reasoning about Time	--	.20	--	--
Denotation of Formal Units of Time	--	.20	--	--

Math processes in the measurement domain included direct measurement, measurement with a nonstandard unit, and denotation of formal units of measurement (e.g., teaspoon, tablespoon, cup). Although it appears the lower SES PPK children may be receiving more support in the measurement domain then children in the other groups, this is another instance where the proportions used may be misleading. There were only five children in the lower SES PPK video group. Two parents in this group demonstrated matching socks by length for their home routine activity, one parent used a piece of string to measure the child's wrist when making a bracelet, and one parent chose to play a store-bought game that used a grid with a graphing component to keep score. Although 80% of the 5 lower SES PPK children received support for measurement, in each case, there were no more than five math occurrences within the measurement domain, and those occurrences occurred only within one activity of each observation. Thus the children received some support for measurement concepts, but perhaps not as much as appeared to be implied by the percentages.

Only two observations contained processes in the pattern domain, those processes including creating a pattern unit, identifying a pattern, and pattern extension. In both instances, creating and extending patterns were the primary activity being done.

Finally, there were two mathematical processes observed that did not fall into any of these domains. Both processes involved time, but were distinctly different from each other. The first process involved reasoning about time, in that the discussion focused around different activities and what time those are typically done (e.g., what time do you wake up/go to school/eat dinner?) The second process involved the denotation of units of time (e.g., 23 seconds, 3 minutes). These processes were seen in one observation each.

Children's Participation During the Home Observation

Active vs. Passive Participation. The analyses presented thus far have included all math occurrences observed in the home activities, regardless of whether the child's participation was active (i.e., the child performed at least part of the occurrence) or passive (i.e., the child observed the parent or other participant performing the entire occurrence). The mean proportion of occurrences children actively participated in compared to the mean proportion of occurrences observed did not vary significantly by age or SES. The mean number of occurrences actively participated in was $M = .73$ ($SD = .12$) for lower PPK children, $M = .59$ ($SD = .26$) for middle PPK children, $M = .70$ ($SD = .19$) for lower SES PK children, and $M = .71$ ($SD = .22$) for middle SES PK children.

Math Performed Independently vs. Performed with Assistance from an Adult. The next step in this analysis was to examine group differences in characteristics of the occurrences in which children were active participants. I began by specifically examining whether children 1) provided the correct response independently (i.e., without help), 2) provided the correct response with help, 3) provided an incorrect response without help, and 4) provided an incorrect response despite help. It should be noted that it was the child's final response for each math occurrence that was coded, as opposed to every response given within a single math occurrence. The mean proportions of children's correct responses produced independently and with assistance are depicted in Figure 4. A series of 2 X 2 ANOVAs revealed SES differences in the mean proportion of independent correct responses, $F(1, 22) = 9.278$, $p < .01$, and the mean proportion of correct responses with scaffolding, $F(1, 22) = 9.022$, $p < .01$. The mean proportion of independent correct responses was higher for middle SES children than lower SES children, and the mean proportion of correct responses with help was higher for lower SES children compared to middle SES children. Group differences by age were not significant.

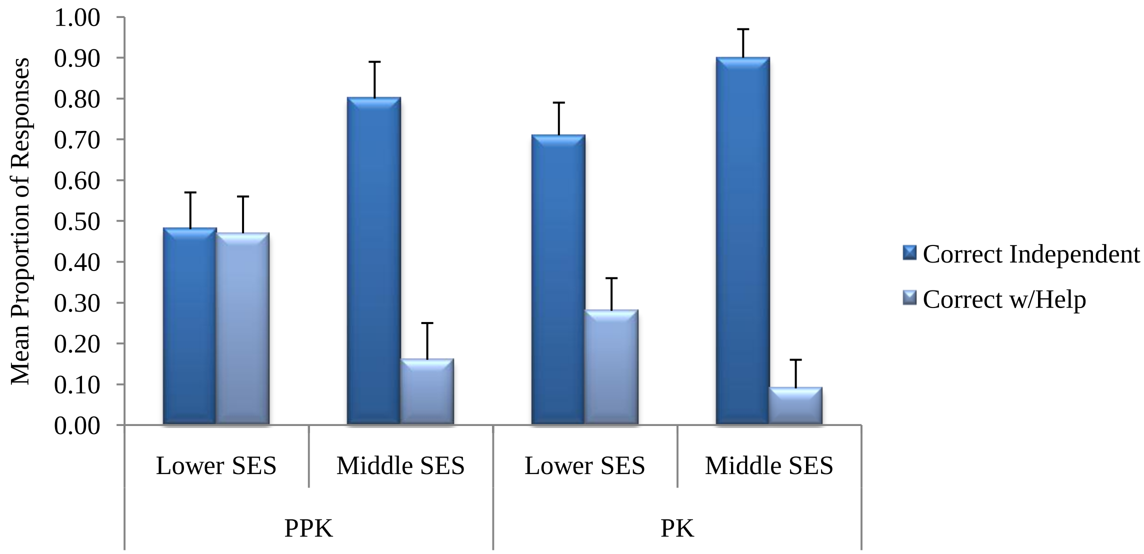

Figure 4. Mean proportions of children's correct responses with and without assistance by age and SES. Error bars represent the standard errors.

Math occurrences with incorrect final responses, either with or without assistance, were rare. This is not to say that children did not make mistakes, as they clearly did, but typically parents responded by providing some type of help or scaffolding until the child produced the correct response. The mean proportion of children's responses that were incorrect without help was $M = .04$ ($SD = .03$) for lower PPK children, $M = .04$ ($SD = .05$) for middle PPK children, $M = .01$ ($SD = .01$) for lower SES PK children, and $M = .01$ ($SD = .03$) for middle SES PK children. A 2 X 2 ANOVA revealed a main effect for age, F (1, 22) = 5.772, $p < .05$, suggesting that parents of PPK children are somewhat more likely than parents of PK parents to ignore their children's incorrect responses. Results were similar for the mean proportion of children's responses that were incorrect despite help. A 2 X 2 ANOVA revealed a main effect for age, F (1, 22) = 6.254, $p < .05$, with the mean proportion for PPK children being $M = .01$, $SD = .02$, and the mean proportion for PK children being $M = .00$, $SD = .00$. There were no differences by SES.

Teaching Strategies Utilized by Parents

The first step in analyzing parents' teaching strategies was to collapse the occurrences of children's correct and incorrect responses without assistance, and then the occurrences of children's correct and incorrect responses with assistance. The rationale for combining children's correct and incorrect responses was that for these analyses, it was the parents' teaching behavior that is of interest. The mean proportion of total responses with and without help from parents closely mirrors the mean proportion of responses correct with and without help (see Figure 5). Again, the mean proportion of responses without help was higher for middle SES children compared to lower SES children, F (1, 22) = 8.875, $p < .01$.

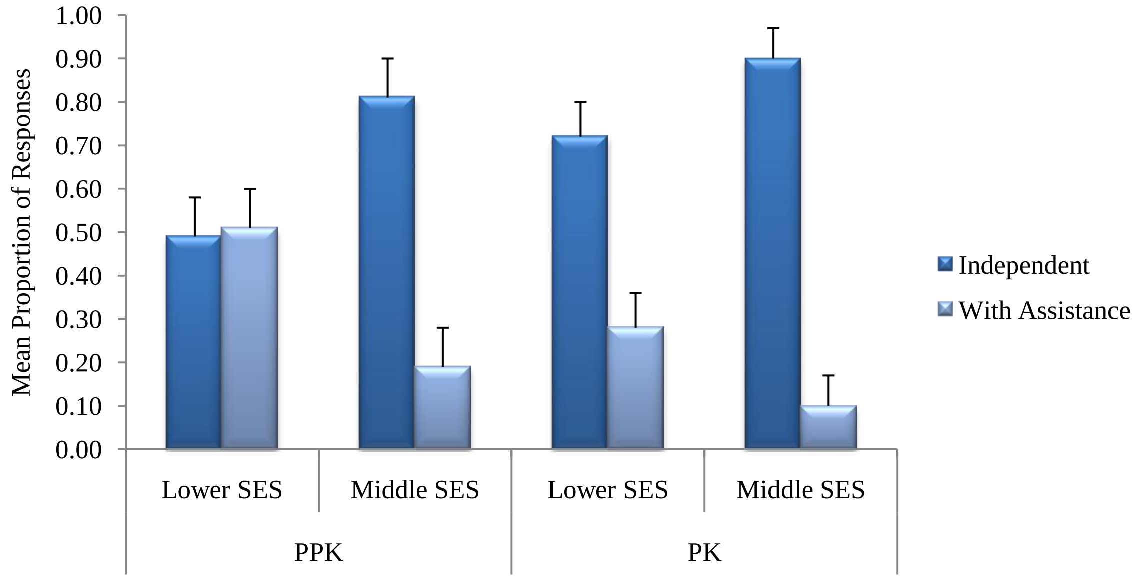

Figure 5. Mean proportions of children's total responses with and without assistance by age and SES. The error bars represent the standard errors.

The SES difference in the amount of assistance provided by parents presented a challenge for determining how to most appropriately analyze and report the data. There were two primary sets of analyses of interest: the type of teaching strategies or help used and the reason help was provided. On the one hand, computing the frequencies or proportions of assisted occurrences by observation and then comparing those means by group ensured that each observation carries equal weight within the age and SES groups. However, this may not be the best approach for capturing differences between the groups. Because the lower SES children had so many more occurrences where help was provided, using frequency data by child resulted in lower SES children having higher mean counts for nearly every type of teaching strategy and reason for help category. Using proportions instead of frequency alleviated this issue, in that a proportion was computed for each child based upon the individual number of assisted occurrences observed, but resulted in mean proportions that did not take into account the variation in the frequency of assisted occurrences by age and SES. If child A's observation contained two occurrences where help was provided, and child B's observation contained 30 occurrences of where help was provided, the type of help/reason for help codes for a single occurrence inevitably carried more weight for Child A than for Child B.

An alternative method of analysis was to create proportion scores for type of help and reason for help variables by age and SES group, as opposed to by individual children. More specifically, proportion scores for each category were computed by dividing the number of occurrences in that category by the total number of assisted occurrences across children in each age and SES group. The limitation of this approach is that the more occurrences with assistance within an observation, the more influence that one observation, or parent, has upon the mean score for the group.

Preliminary analyses showed that the method used to analyze the type of help provided influenced the results, whereas analyses of the reasons for help remained generally the same regardless of the method used. After carefully considering the limitations of each approach, as well as ruling out other options (e.g., creating a score to capture both frequency and proportion), I decided to analyze and report the type of help findings by both proportions by observation and proportions by group. As I will demonstrate below, the means proportions followed similar trends either way the data was analyzed, with the primary difference between the two methods being the significance levels of group differences. However, given the limitations inherent in each approach, the findings of each should be interpreted cautiously. The results showing reasons for help are presented using proportions by observation only, as it is the more conservative of the two approaches and significance levels were not affected.

Types of Help. Parents were observed using seven teaching strategies, or types of help, across the math occurrences where children were active participants. Teaching strategies that provided the child with enough support to complete at least 50% of the mathematical task independently were considered low support, and those strategies that resulted in the parent completing at least 50% of the mathematical task were considered high support. Low support strategies included non-instructive feedback (e.g., “no, that’s not right”), and then three types of prompts, each providing slightly different levels of support: general prompts (e.g., “I think you might have skipped a number”), strategy prompts (e.g., “Try counting again more slowly, and touch each one as you count it”), and specific prompts (e.g., child counts, “1, 2, 3, 6, 7, 8,” and parent interrupts saying, “3, 4,” and child resumes counting, “5, 6, 7, 8, 9, 10.”). High support strategies included two categories of parent participation and one category where the parent simply provides the correct response. Parent participation included co-participation (e.g., parent and child count all objects in a set together) and parent-led participation (e.g. parent touches objects and counts out loud while child says some number words in the sequence with the parent).

Mean Proportions by Observation. There were no main or interaction effects by age or SES group in the proportion of low or high support strategies used by parents (see Figure 6). Nor were there differences by age or SES in the proportion of specific teaching strategies used over the course of each observation. The lack of between group variation is likely due, at least in part, to the small sample size as many of the means appear to differ by age and/or SES (see Table 18), and those differences generally mirror the differences (discussed below) that are significant when the scaffolding occurrences are collapsed across observations in each age and SES groups and the proportion of scaffolding types are compared.

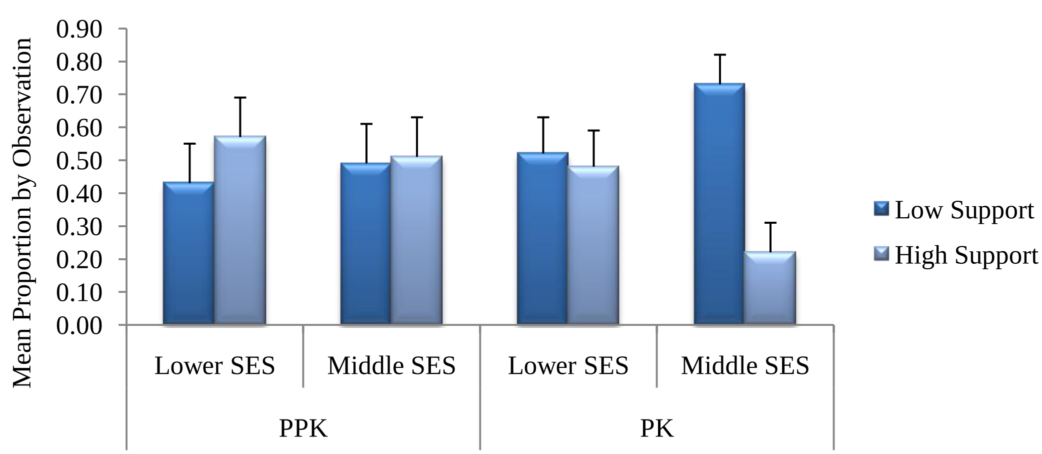

Figure 6. Mean proportion of occurrences with low and high support by age and SES (Proportions calculated by observation). The error bars represent the standard errors.

Table 18
Parents' Utilization of Teaching Strategies by Observation

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
Teaching Strategy	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Feedback	.16 (.18)	.13 (.15)	.14 (.13)	.25 (.15)
Prompts	.27 (.20)	.37 (.36)	.38 (.19)	.49 (.20)
General	.04 (.05)	.02 (.05)	.10 (.14)	.12 (.22)
Strategy	.07 (.07)	.20 (.21)	.07 (.13)	.19 (.27)
Specific	.16 (.14)	.15 (.15)	.21 (.14)	.17 (.16)
Participation	.22 (.15)	.21 (.21)	.21 (.11)	.05 (.12)
Co-Participation	.05 (.10	.04 (.09)	.03 (.04)	.03 (.08)
Parent-Led Participation	.17 (.17)	.16 (.15)	.18 (.10)	.02 (.04)
Provides Response	.35 (.25)	.30 (.28)	.27 (.27)	.22 (.25)

Proportions across Age and SES Group. These proportions were computed by computing the total number of occurrences including each type of help by the total number of overall occurrences with help for each age and SES group. A two-way ANOVA revealed main effects for age, F (1, 386) = 7.500, $p < .01$, and SES, F (1, 386) = 35.349, $p < .001$, for the mean proportion of high and low support scaffolding observed across the observations. As shown in Figure 7, parents of PK children demonstrated a higher proportion of low support teaching strategies compared to parents of PPK children, as did middle SES parents compared to lower SES parents. Conversely, parents of PPK children demonstrated a higher proportion of high support teaching strategies compared to parents of PK children, as did lower SES parents compared to middle SES parents.

Within the low support category, there were no significant differences between age and SES groups for the proportion non-instructive feedback provided by parents, but there was a main effect for SES for the proportion of prompts given by parents, F (1, 386) = 27.433, $p <$ .001. Teaching strategies used by middle SES parents included a higher proportion of prompts compared to lower SES parents. The means and standard deviations for both low and high support categories are reported in Table 19. There were no significant differences by age and SES in the proportion of general and specific prompts provided by parents, but there was a main effect for SES for the proportion of strategy prompts provided by parents, F (1, 386) = 43.672, p

< .001. Middle SES parents utilized a higher proportion of strategy prompts compared to lower SES parents.

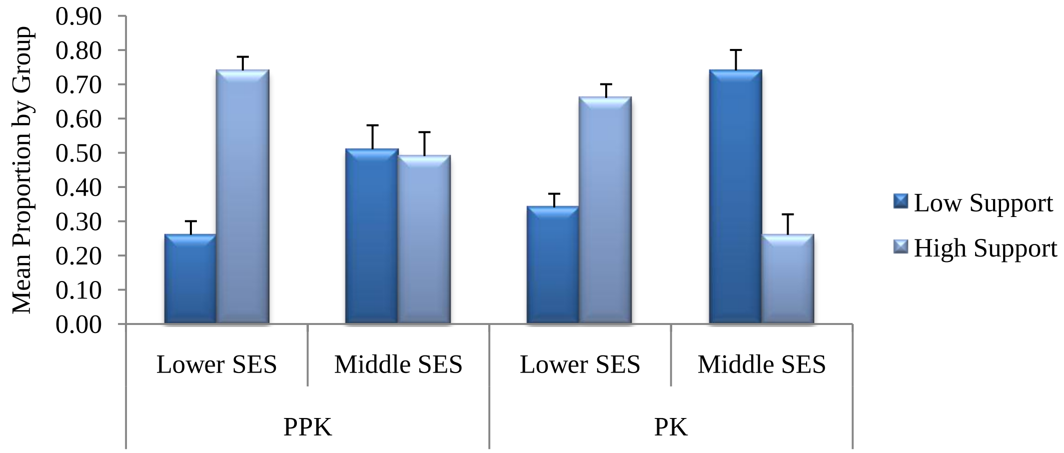

Figure 7. Mean proportion of occurrences with low and high support by age and SES (Proportions calculated by collapsing across age and SES groups).

Within the high support category, parents of PPK children utilized a higher proportion of parent participation compared to parents of PK children, F (1, 386) = 9.907, $p < .005$. This effect was mirrored in the co-participation category, F (1, 386) = 5.845, $p < .05$, but there was an interaction effect for the parent-led participation category, F (1, 386) = 5.169, $p < .05$. Teaching strategies utilized by middle SES parents of PPK children included a higher proportion of parent-led participation compared to lower SES parents of PPK children, but the opposite was true of middle and lower SES parents of PK children. Finally, there was a main effect of SES for the proportion of providing child with the correct response, F (1, 386) = 24.386, $p < .001$. Lower SES parents of children in both age groups utilized this strategy at a higher proportion than middle SES parents.

Reasons for Help. There were no significant differences between age and SES groups for the reasons parents chose to provide help (see Table 20). The most common reason (62% across age and SES groups) parents provided help was to assist children in producing a correct response when their initial response was incorrect. There was no observable reason for help provided in a full 25% of occurrences where help was provided. Far less common reasons included children hesitating or failing to provide a response at all (6%), children requesting help from their parents (2%), children responding by saying “I don’t know,” or otherwise indicating they didn’t know (2%), and children losing focus on the task at hand (1%).

Table 19
Parents' Utilization of Teaching Strategies Collapsed Across Age and SES Groups

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
Teaching Strategy	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Feedback	.09 (.29)	.08 (.27)	.09 (.29)	.21 (.41)
Prompts	.17 (.38)	.44 (.50)	.25 (.43)	.52 (.50)
General	.04 (.19)	.03 (.16)	.06 (.23)	.10 (.30)
Strategy	.04 (.19)	.23 (.43)	.04 (.21)	.30 (.46)
Specific	.10 (.30)	.18 (.39)	.15 (.35)	.13 (.34)
Participation	.26 (.44)	.31 (.47)	.20 (.40)	.07 (.25)
Co-Participation	.11 (.32)	.10 (.31)	.04 (.19)	.03 (.18)
Parent-Led Participation	.15 (.36)	.21 (.41)	.17 (.37)	.03 (.18)
Provides Response	.47 (.50)	.18 (.39)	.46 (.50)	.20 (.40)

Table 20.
Reasons Parents Provided Help by Age and SES

	PPK		PK	
	Lower SES	Middle SES	Lower SES	Middle SES
Reason for Help	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)	*M* (*SD*)
Child's Response Incorrect	.67 (.30)	.60 (.27)	.53 (.23)	.68 (.28)
No Observable Reason	.28 (.28)	.19 (.32)	.31 (.26)	.23 (.22)
Hesitated/No Response	.02 (.03)	.10 (.22)	.07 (.09)	.05 (.06)
Requested by Child	.02 (.04)	.04 (.10)	.01 (.02)	.00 (.00)
Lost Focus	.01 (.01)	.00 (.00)	.02 (.04)	.00 (.00)
Child Responds "I Don't Know"	.00 (.00)	.00 (.00)	.02 (.04)	.04 (.11)

Correlations Between the Home Observations and CMA Scores

There were no significant correlations between the general characteristics of the activities demonstrated, including activity type, length, minutes including math, total number of math occurrences, and mean number of math occurrences per minute and children's CMA scores. There were also no significant correlations between the mathematical content or complexity observed and children's CMA scores. Given the variation in activities demonstrated by parents and children, this was not surprising. From the parent questionnaires, we know that parents report their children generally engage in approximately five to seven types of mathematically supportive activities per week, and these observations provide only a snapshot of one or two examples of specific activities falling into each of those activity types. It is likely that characteristics of children's activities such as length, mathematical content, and mathematical complexity are at least somewhat dependent on the demands of the activity itself, thus a more congruent sample of activities from each parent-child dyad would likely be needed to capture a potential correlations between these variables and children's mathematical knowledge.

Although also likely influenced to some degree by the demands of the activity, children's degree of active participation and parent's teaching strategies may be more stable across activities. Individual Pearson correlation coefficients were computed between children's composite scores on the CMA and the proportions of math occurrences actively participated in by children, the proportions of children's responses generated independently, the proportion of assisted math occurrences with low support, and the proportion of assisted math occurrences with high support. There was no significant correlation between the proportion of occurrences participated in actively by the child, but there were significant correlations between CMA scores and the proportion of children's independent responses ($r = .82$, $p < .001$), between children's CMA scores and the proportion of occurrences with low support help ($r = .54$, $p = .005$), and between children's CMA scores and the proportion of occurrences with high support help ($r = .79$, $p < .001$).

Hierarchical multiple regression was used to determine the unique contribution, if any, of the proportion of children's independent responses and the proportion of responses including low and high support teaching strategies scores on the CMA. Similar to the procedure used for the parent questionnaire, variables were entered into the model in two blocks. The first block consisted of SES and child's age at spring testing during the preschool year in which the video observation occurred. Age was again used instead of cohort. The second block consisted of the proportion of occurrences where children's responded independently, the proportion of occurrences where low support help was provided, and the proportion of occurrences where high support help was provided.

Ideally, variables from the parent questionnaire, particularly the parents' expectations and knowledge scores for mathematical knowledge at age five, would be included in this analysis. However, comparison of the mean expectation and accuracy scores for the larger sample and the video sample were not comparable. Lower SES parents of PPK children in the video sample held much higher expectations ($M = .78$, $SD = .13$) of mathematical ability at age five compared to lower SES parents of PPK children in the main sample ($M = .52$, $SD = .19$), $F(1, 39) = 8.670$, $p = .005$, and middle SES parents of PK children in the video sample demonstrated a more accurate understanding of mathematical ability age five ($M = .86$, $SD = .06$) compared to middle SES parents of PK children in the main sample ($M = .78$, $SD = .09$), $F(1, 51) = 6.047$, $p < .05$. Although for each measure the remaining age/SES groups were more or less comparable, the expectation and accuracy scores were not correlated with CMA scores for the small video sample only, and thus were excluded from the analysis.

The regression analysis yielded a total of two significant models. The first model showed that children's SES and age at testing accounted for a significant amount of variability in posttest scores on the CMA, $R^2 = .67$, $F\ (2, 23) = 23.086$, $p < .001$. In addition to SES and age at testing, the second model included parents' use of both low and high support teaching strategies, $R^2 = .84$, $F\ (4, 21) = 27.688$, $p < .001$, suggesting that parents' use of low and high support scaffolding predicts unique variance in children's scores on the CMA above and beyond SES and children's age. The proportion of children's independent responses in the observed activities was excluded from the model, indicating no unique contribution to the variance in CMA scores. The unstandardized beta, standard error, and standardized Beta for the two models are reported in Table 21.

Table 21
Parents' Use of No, Low, and High Support Teaching Strategies as Predictors of Children's Scores on the CMA

	b	*SE b*	*β*
Model 1			
Constant	-.481	.244	
SES	.302	.049	.735
Age at Testing	.131	.049	.320
Model 2			
Constant	.154	.234	
SES	.198	.042	.482
Age at Testing	.054	.040	.132
Low Support	-.423	.351	-.124
High Support	-.462	-.462	-.462

Although the use of low and high support teaching strategies both predicted unique variance in children's CMA scores, it should be noted that those relationships were negative. Using these models alone, it would appear as though providing any type of support is negatively associated with children's mathematical knowledge. However, as discussed previously, parents in the lower SES group tended to provide their children with more assistance overall. So much so, that when the teaching strategies were analyzed by frequency, parent's in the lower SES group appeared to utilize nearly each type of strategy more frequently than did middle SES parents. Middle SES children, on the other hand, had a greater frequency and proportion of independent responses. To account for this, a second set of regression analyses were conducted.

Variables were again entered in two blocks. The first block included SES and age at testing. The second block however, included the proportion of occurrences where more than 50% of the task was done independently by the child (e.g., no support + low support). It was not necessary to include the proportion of occurrences where less than 50% of the task was done independently by the child (e.g., high support) as the sum of these two proportions always equaled 1.00, and the use of either proportion would yield the same result.

Again, this analyses resulted in two significant models. The first model remained the same as Model 1 from the previous analysis in that children's SES and age at testing accounted for a significant amount of variability in posttest scores on the CMA, $R^2 = .67$, $F(2, 23) = 23.086$, $p < .001$. The second model showed that in addition to SES and age at testing, the children's ability to respond independently or with low support assistance positively predicted their scores on the CMA, $R^2 = .83$, $F(3, 22) = 35.697$, $p < .001$. The unstandardized beta, standard error, and standardized Beta for the two models are reported in Table 22.

Table 22
The Proportion of Math Occurrences Completed at Least 50% Independently as a Predictor of Children's Scores on the CMA.

	b	*SE b*	*β*
Model 1			
Constant	-.481	.244	
SES	.302	.049	.735
Age at Testing	.131	.049	.320
Model 2			
Constant	-.454	.179	
SES	.210	.042	.510
Age at Testing	.070	.039	.170
≥ 50% Independently	.480	.105	.489

Summary and Discussion

The purpose of including the home observations in this study was to further examine the qualitative nature of the support children receive for early mathematical development in the home. Specifically, I looked at the mathematical content and complexity, children's participation and responses, and the strategies parents use to teach mathematical concepts and facts to their children. When appropriate, I compared the results with children's performance on the CMA. Given the small size of the home observation sample, the non-random selection used to select the sample, and the variation in activities being compared, these findings should be interpreted with caution as they may not be generalizable to larger populations of lower and middle SES preschool children. More extensive research is clearly needed.

Perhaps the most unexpected finding from these analyses was the similarity between groups in regards to the mathematical content observed in the home activities. Across age and SES groups, children's activities primarily consisted of the same five core mathematical processes: numeral recognition/naming, enumerating/counting a set, counting to a specified number, set construction, and addition/subtraction with concrete objects. Even when the mathematical focus of the activity was in another non-number mathematical domain (e.g., geometry), parents typically found a way to embed at least one of these core processes into the activity. This indicates that lower and middle SES parents of three- and four-year-old children in the United States share a common understanding of the mathematical skills their children should be learning. While there were some differences by age and SES in terms of set sizes and problem details for these processes, these differences were likely appropriate as scores from the CMA suggest differential levels of mathematical knowledge between the groups. The activities selected by parents did very little to shed light on group similarities or differences in mathematical domains other than number and arithmetic.

There was also little variance by age and SES in the complexity levels of the math observed. The lack of statistically significant variation was not anticipated even after all of the videos had been viewed multiple times, and may be due at least in part to the small sample size and limited number of activities observed. That there were no differences by age or SES in the proportion of activities containing levels one, two, and three processes was not surprising as Saxe and colleagues (1987) did not find age or SES differences at levels one or two, and the level three occurrences were so rare. However, the minimal between group variation in the proportion of observations containing level four processes was not consistent with the findings from that same study.

As mentioned previously, lower SES children in the PPK group had the highest proportion of observations containing the mathematical process of addition/subtraction with concrete objects. This was perplexing as the performance of this group on the CMA indicates that children in this group are far below children in the other three groups in their ability to count and construct even very small sets, so for parents to ask them to add and subtract multiple sets seems beyond their developmental range. This may at least partially explain why PPK children in the lower SES group had such a high proportion of assisted math occurrences relative to children in the other three groups.

Children's level of active participation did not differ by age or SES, but the proportion of their independent responses compared to the proportion of their assisted responses was largely a function of SES. Middle SES children in the PPK year were able to provide the correct response without assistance in approximately 80% of all attempts. For middle SES children in the PK year, this increases to approximately 90%. The percentage of correct responses provided without assistance was considerably less in the lower SES groups, with approximately 70% for children in the PK group, and just under 50% for children in the PPK group.

It is important to point out that not all instances of help occurred only when children needed it. In fact, for approximately one-quarter of occurrences where assistance was provided the reason a parent chose to provide help could not be determined from the video alone. In many cases it appeared as though the parents perceived that the demands of the task were too difficult and were attempting to provide scaffolding to lessen those demands. However, there were also instances where parents may have been attempting to increase the difficulty of a task by posing questions or encouraging children to use more complex strategies in their attempts to complete it. Without asking the parents about their intent, it is not possible to know for sure.

While there are no specific guidelines for establishing the optimal proportion of an activity that should be within or just beyond a child's actual developmental level for learning to occur, it may have been that the parents of lower SES children either 1) believed that the optimal proportion of an activity that should be beyond their children's developmental range was higher than middle SES parents did, and as such intentionally gave their children a higher proportion of mathematical tasks or problems that they believed their children could not solve without assistance, 2) were less in tune with their own child's actual developmental level and unintentionally provided a higher proportion of mathematical tasks or problems that were too difficult for their child to solve without help, or 3) held a belief different from middle SES parents about their role as the adult in adult-child teaching interactions which manifested in lower SES parents providing more direct instruction to their children compared to middle SES parents.

We did not ask parents about their intent, rationale, or thoughts behind any of their choices or behavior, but anecdotally speaking, there were probably some lower SES parents that fell into each of these categories. Within the lower SES PPK group, for example, there was one parent who chose addition with concrete objects as one of her activities. The parent appeared to know from the onset that the activity was too difficult for her child, as she insisted on providing high support assistance for nearly every single problem despite the child's continued requests for the mother to let her try on her own. Towards the end of the activity the mother gave in briefly, and it became clear that the task at hand was far beyond the developmental range of that particular child as the child failed miserably. Similarly, there were three other parents in this same group that presented their children with activities clearly beyond their developmental range but appeared genuinely surprised, and later frustrated, with the amount of support they needed to provide in order to get their child to the correct response. The fifth and final parent in this group appeared very much like the middle SES parents in that she selected activities that were mostly within the developmental range of her child. Given the small sample size it is not possible to know whether these parent profiles are isolated examples or representative of subgroups within the lower SES population.

Whether or not the teaching strategies used by parents when assisting their children differed significantly by age or SES depended upon the method of analysis used. However, it is likely that the utilization of no support/low support versus high support strategies is important, as it was the only variable from these analyses correlated with children's scores on the CMA. Furthermore, it appears that the use of no/low support strategies or high support strategies have predictive value over and above age and SES on children's scores on the CMA, even when the more conservative method was used in the models. Parents' use of no/low support teaching strategies was associated with higher scores on the CMA, and parents' use of high support strategies were associated with lower scores on the CMA.

Regardless of whether mean proportions were computed by observation or by age/SES group, the most common teaching strategy used among middle SES parents of children in both age groups were prompts, and within the prompt category strategy prompts were most common. Strategy prompts differ from the other teaching strategies observed in this study, as it is the only teaching strategy that explicitly shows children how to solve problems while still allowing for the child to complete the majority of the task independently. Conversely, the use of strategy prompts was rare among lower SES parents in both age groups. These findings are consistent with research from the general cognitive development literature showing that middle SES parents are more likely than lower SES parents to teach their children explicit strategies during problem-solving tasks (Brophy, 1970; Hess & Shipman, 1965; Laosa, 1980).

The most common teaching strategy used by lower SES parents of PPK children was to provide the child with the correct response, and for lower SES parents of PK children the most common was either prompts, with specific prompts being the most frequent, or providing the child with the correct response depending on whether mean proportions of observation or mean proportions by group are used. This is interesting, as both specific prompts and providing the correct response both entail the parent giving the child at least part of the desired answer.

It is possible that parents' decisions to use low or high support teaching strategies are child-driven, in that children with lower mathematical skills may need more support or higher-support overall. If true, it may be the case that the level of support observed in this study is at least partially confounded with some other variable. Although it was not possible to evaluate this in the current study, it may be that the parents' ability to provide their children with activities that provide support for the mathematical concepts both within and just beyond the child's range has implications for children's developing mathematical knowledge.

Finally, there were no significant differences by age or SES observed in this study in regards to the apparent reasons parents chose to provide their children with help on various math occurrences. Across age and SES groups, the most common reason help was provided was that the child's initial response was incorrect. Less common reasons included no response from the child, the child lost focus, help was requested by the child, and the child indicating he or she did not know the correct answer. However, for approximately 25% of assisted math occurrences, the reason for help could not be reliably determined. From an anecdotal perspective, it appeared that in some cases the parents were attempting to extend the mathematical content or complexity by embedding cues or questions into an ongoing task, while in others there were no clues as to the parents' intent, thus it is possible that the reasons parents provide help to their children during mathematical tasks do vary by age, SES, or some other undetermined characteristic but that those differences were not captured in this particular study.

CHAPTER 7: CONCLUSIONS AND IMPLICATIONS

Research Questions and Conclusions

This study was guided by three central research questions:

1. Is there a correlational relationship between the type or frequency of specific activities children do in the home and their early mathematical knowledge? Which aspects of these activities vary by age (three years vs. four years) and SES?
2. Is there a correlational relationship between parents' beliefs of and/or knowledge about early mathematical development and children's early mathematical knowledge? Which aspects of parents' knowledge and/or beliefs vary by age (three years vs. four years) and SES?
3. Are there qualitative differences by age (three years vs. four years) and/or SES in the activities done in the home to support early mathematical development?

Research Question #1

In addressing the SES gap in early mathematical knowledge, it would be helpful to know if there are specific activities done in the home that are actually correlated with, and perhaps predictive of, children's mathematical knowledge prior to kindergarten entry. If such an activity or range of activities exists, then ensuring lower SES children have access to the activity or activities may prove beneficial towards reducing the knowledge gap. The results from this study show a positive correlational relationship between children's mathematical knowledge and a few specific types of activities, including art activities involving symmetry or patterns, made-up games involving math, using a computer with math software, and using math in the home routine. The three former activities are all activities more likely to be done by middle SES children than lower SES children, and older children were more likely than younger children to use a computer with math software. However, none of these activities on their own had predictive value on children's CMA scores, so working with lower SES parents to increase the frequency of these activities in their home may not do much in the way of reducing the achievement gap in early mathematics.

Research Question #2

Perhaps the most interesting and potentially useful information collected from the parent questionnaire was the expectation and knowledge data captured by asking parents which skills and abilities they think typical children are capable of at age five years. Both parents' beliefs about the mathematical knowledge of five-year-old children and the accuracy of those beliefs were not only positively correlated with children's CMA scores, but both predicted unique variance over and above age and SES in children's CMA scores despite significant differences in mean scores by SES. That the parents of children in the two age groups did not differ in their beliefs or knowledge of skills at age five suggests this knowledge has little or nothing to do with the proximity to age five of their own child, but instead a more general discrepancy by SES in terms of parents' knowledge about early mathematical development. One practical implication of these findings might be to focus intervention efforts on providing lower SES parents with more knowledge about early mathematical development.

Research Question #3

The small sample size of the home observation component of this study combined with the considerable variation in the activities makes it difficult to draw definitive conclusions about qualitative differences in children's home activities, but there are certainly differences worth examining further with a larger sample. For example, there were very few age or SES differences in the mathematical content and complexity in the home activities observed. On the one hand this can be construed as parents sharing a common set of beliefs about which skills or processes should be emphasized during the preschool years, but on the other, given the differential levels of mathematical knowledge between the groups, some children might not have been developmentally ready for specific skills to be introduced. For reasons not yet understood, lower SES parents in general tended to incorporate a higher proportion of mathematical skills that were too difficult for children to master without a significant amount of assistance. This mismatch between children's developmental level and the skills parents choose to work on may not be conducive to children's learning.

There also appears to be age and SES differences in the type of help parents provide their children when engaged in mathematical activities. In the middle SES group, the parents of children in the younger group utilized a combination of low and high support teaching strategies with their children, and the parents of children in the older group demonstrated a strong preference for low support strategies, regardless of the approach used to analyze the data. Things were a little less clear for the lower SES group, as the findings were not consistent between the two methods used to analyze the data. The more conservative of the two analytic approaches resulted in a near equal balance of low and high support in the activities of both age groups, and the less conservative method resulted in higher proportions of high support strategies in the activities of both age groups. This suggests that at minimum, the teaching strategies used by middle and lower parents of prekindergarten children differ considerably from each other just prior to kindergarten entry, which is important because the proportion of no/low (or high) support teaching strategies utilized in the observation was a significant predictor of children's mathematical knowledge as measured by the CMA.

General Conclusions and Implications for Future Research

The findings from this study indicate that rather than focusing on the types of activities children do in the home, efforts might be better spent working to increase lower SES parents' knowledge about early mathematical development. It may also be prudent to teach parents about the predictive value of low support teaching strategies on children's mathematical knowledge, as well as providing them with strategies to use. Future research examining the qualitative nature of children's home activities, including parent-child teaching interactions is warranted.

The home observations included in this study were not captured with the intention of subjecting them to systematic analysis. However, the resulting dataset was so rich it made sense to include them in this study. Given the findings, and potential implications of those findings, the next step toward understanding the SES differences in early mathematical knowledge is to expand upon this piece of the study with a larger, randomly selected sample and more rigorous design. Future studies should include more stringent guidelines in terms of the types of activities demonstrated and possibly even the specific activities demonstrated to ensure that comparable data are obtained from each group. Viewing video of the observation with the parent immediately after the observation commences would be of extreme value, as inquiries could be made as to parents' intentions, rationale, and beliefs guiding their decisions throughout the observation, thus providing an even greater contribution to understanding how children's home experiences impact their mathematical knowledge.

Finally, future research should focus on the potential value of educating lower SES parents about early mathematical development and how to provide optimal support for their own children's mathematical knowledge. Starkey and Klein (2000) did this successfully with their family math project. The nature of that intervention was such parents attended workshops where they learned how to support their children's mathematical development using very specific activities with their children. Many of the activities were similar to what one might expect to see a preschool teacher doing with children in the classroom, and parents were given the resources to facilitate the activities much like a preschool teacher might in terms of materials for the activity, minimizing other distractions, providing optimal levels of scaffolding, and monitoring children's progress. At the end of the intervention, children whose parents participated in the intervention showed greater gains in mathematical knowledge from pretest to posttest compared to children whose parents did not participate. However, even with this robust intervention, it is still unlikely that the lower SES children in this intervention received as much support for early mathematics in the home, as their middle SES peers. Results from the parent questionnaire used in this study suggest that mathematics is infused into their daily lives via made-up games, home routines, and having access to math-related materials during periods of play. It would be interesting to see if an intervention with an embedded math focus coupled with specific focal activities, such as those used by Starkey and Klein, results in even greater gains in children's mathematical knowledge among lower SES preschool children, thus reducing the SES gap in early mathematical knowledge.

Limitations of This Study

One limitation of this study involves the use of the parent questionnaire data for capturing the amount of mathematics support children receive in their home environments. Use of the questionnaire relies exclusively on self-report measures, which may or may not reflect what is actually done in the home. Questions are subject to the respondents' interpretations, and what may constitute a "math-related" to one parent might not be considered so by another. Self-report instruments always come with some risk of respondents over-or-under-reporting, perhaps reporting what they believe is expected of them rather than what they really believe or practice. This specific questionnaire, although appropriate for an exploratory study such as this one, may have been too general for accurately capturing the amount of mathematical support children receive in the home.

Many aspects of parent beliefs and knowledge about early mathematical development and how it is best supported are also difficult to capture using a questionnaire. The questionnaire was useful for examining parents beliefs and knowledge about specific mathematical skills, but provided little in the way of understanding parents' more general beliefs about teaching and learning mathematics during early childhood. Perhaps a more informative method for obtaining this information would entail the use of semi-structured interviews with parents, thus allowing for open-ended discussion about what parents believe and know about early mathematical devlopment, the origin of those beliefs and knowledge, and the relationship between what parents believe or know about early mathematical development and their reported practices for supporting it.

. There are also multiple limitations in the home observation component of this study, many of which I have already addressed. The small size of this sample combined with the variation in activities selected and demonstrated by parents made systematic comparisons of mathematical content and complexity difficult to conduct and interpret meaningfully. Additionally, possibly because the participants in this study were not randomly selected or due to the small number of children represented from each age and SES group, the video sample may not have been a completely accurate representation of the larger sample from which is was drawn. However, these data were collected for exploratory purposes and will prove valuable for informing future research.

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APPENDIX A

Parent Questionnaire

(Questions 3, 8, 9, and 12 were not used for this study)

1. We would like to know about the activities and materials your child uses at home that support her or his mathematical development. From the list below, please check which activities and materials your child uses at home, indicate how often they are used, and **give some examples** in the space provided.

Activities and Materials Used at Home	Daily	Weekly	Monthly	Rarely/Never
Store-bought games involving math (e.g., games with dice) Example(s): ______________________________	_____	_____	_____	_____
Playing with math-related toys (e.g., play money) Example(s): ______________________________	_____	_____	_____	_____
Playing with blocks or other construction toys Example(s): ______________________________	_____	_____	_____	_____
Origami (paper folding) or Kitigami (paper cutting) Example(s): ______________________________	_____	_____	_____	_____
Art activities involving patterns or symmetry Example(s): ______________________________	_____	_____	_____	_____
Made-up games involving math such as counting steps Example(s): ______________________________	_____	_____	_____	_____
Reading books such as counting or shape books Example(s): ______________________________	_____	_____	_____	_____
Using workbooks for math such as number coloring books Example(s): ______________________________	_____	_____	_____	_____
Using a computer with math software Example(s): ______________________________	_____	_____	_____	_____
Singing/listening to songs that use math (e.g., number songs) Example(s): ______________________________	_____	_____	_____	_____
Watching TV shows that teach math Example(s): ______________________________	_____	_____	_____	_____
Using math in a home routine (e.g., child uses measuring cup while you prepare food) Example(s): ______________________________	_____	_____	_____	_____
Other math activities (Please list): ______________________________	_____	_____	_____	_____

2. Which of the following approaches do you use at home on a regular basis to help your child develop mathematical knowledge and skills? Check the approach(es) you use and rank them from most important (1), to less important (2), to least important (3) in your home.

_______ I give my child math-related tasks or ask math-related questions during ongoing domestic routines (e.g., We use measuring cups or spoons while preparing food).

_______ I set aside time to be with my child on a regular basis to help him/her develop cognitive skills (e.g., We look at a number book, play a board game, or use math software together).

_______ I enrich my child's play time (alone or with other children) by providing math-related toys and materials (e.g., My child spontaneously plays with cards or shape puzzles, or watches Sesame Street alone).

3. We would like to know about the technology your child uses at home that support her or his development. From the list below, please check which types of technology your child uses at home, indicate how often they are used, and **give some examples** in the space provided.

Technology Used with Child at Home	Daily	Weekly	Monthly	Rarely/Never
Television programs for children Example(s): ______________________________	_____	_____	_____	_____
Computer and children's software: Example(s): ______________________________	_____	_____	_____	_____
Electronic games: Example(s): ______________________________	_____	_____	_____	_____
Videotape player and monitor Example(s): ______________________________	_____	_____	_____	_____
Audiotape player or other educational technologies Example(s): ______________________________	_____	_____	_____	_____

4. INSTRUCTIONS: These following questions concern children's mathematical development during the preschool years. Which of the following abilities or skills do you believe *typical children* have developed before their 5th birthday? **(Please circle "yes" or "no".)**

a. Recite numbers from 1 to 10	Yes	No
b. Recite numbers from 1 to 100	Yes	No
c. Count a row of 10 objects	Yes	No
d. Use the numerals from 1 to 10 to make a number pattern, such as "odd - even -odd - even"	Yes	No
e. Read the numerals from 1 to 10	Yes	No
f. Align a row of 7 miniature umbrellas with a row of 7 dolls	Yes	No
g. In a row of 5 dolls, show which is second in the row	Yes	No
h. Solve small addition or subtraction problems presented with objects, such as 3 blocks and 2 blocks = ________ blocks	Yes	No
i. Use pattern blocks to construct a larger geometric shape, such as using 2 right triangles to construct a square	Yes	No
j. Use a calculator to solve single-digit addition or subtraction problems, such as 8 - 2 =	Yes	No
k. Share 12 crackers equally among 3 friends	Yes	No
l. Solve single-digit addition or subtraction problems presented on flashcards, such as 5 + 3 =	Yes	No
m. Read basic arithmetic symbols, such as "+", "-"	Yes	No
n. Arrange 5 sticks in order of increasing length	Yes	No
o. Sort a set of objects into 2 groups based on color such as red and blue	Yes	No
p. Measure the width of a sheet of paper using a ruler	Yes	No
q. Name the following shapes: circle, triangle, and square	Yes	No
r. Determine that a triangle matches another identical triangle which is in a different spatial orientation such as upside down	Yes	No
s. Understand spatial words such as "under", "on", "next to", and "behind"	Yes	No
t. Measure the length of a pencil using string	Yes	No
u. Measure the angles of a triangle	Yes	No
v. Use colored beads to make a simple pattern, such as "red-blue-red-blue"	Yes	No
w. Use a computer with age-appropriate software to learn math concepts	Yes	No

5. To what extent do your child's preschool teachers provide opportunities for learning math in their classroom? (Please circle your choice.)

Not at all   Slightly   Moderately   A Great Deal   Don't Know

Comment:

6. Consider the relative contribution of the home environment and the preschool environment in preparing children for math in kindergarten. Please indicate in percentages how much you think each contributes. (Please total to 100%.)

Home	_______%
Preschool	_______%
Total	100%

7. Consider the relative contribution of children's spontaneous play and adult-directed activities in preparing children for math in kindergarten. Please indicate in percentages how much you think each contributes. (Please total to 100%.)

Spontaneous play (alone or with peers)	_______%
Adult- (parent- and teacher-) directed activities	_______%
Total	100%

8. Do you know what is taught in the kindergarten math curriculum at the elementary school that your child will be attending?

_______ Yes _______ No

If **yes**, please list what, to your knowledge, is taught in this curriculum, and then go to Question 9.
If **no**, skip Question 9 and go to Question 10.

9. How did you learn about the kindergarten math curriculum? (Please check all that apply.)

_______ Don't know the kindergarten math curriculum

_______ Visit to kindergarten

_______ Older sibling's experience in kindergarten

_______ Parents in your preschool

_______ Other: (Please specify) ____

Please provide the following information about your family background:

10. Name of person completing this questionnaire: ____

11. What is your relationship to the child? (Please check one.)

_______ Mother
_______ Father
_______ Guardian
_______ Other:____________________________

12. Please provide some information about your child's birth:

Child's date of birth: _________

Child's weight at birth: _______

Length of pregnancy with child (in weeks): __________

13. Do you have other children? YES __ NO____

If yes, please indicate the age and gender of each child:

_________________________________ ____

_________________________________ ____

14. What is the primary language spoken in your home? __________________________

15. Education of child's parents or guardians (Please check the category that applies.):

Mother or guardian:

Highest level of education completed:

_______ Some High School

_______ High School

_______ Vocational/Technical School

_______ Some College

_______ College

_______ Some Graduate/Professional School

_______ Graduate/Professional School

Father or guardian:

Highest level of education completed:

_______ Some High School

_______ High School

_______ Vocational/Technical School

_______ Some College

_______ College

_______ Some Graduate/Professional School

_______ Graduate/Professional School

APPENDIX B

Math Content List (Starkey, 2004)

Number

Mathematical Process or Knowledge	Problem Details
Subitizing	Range of set sizes
Rote Counting	Particular number words
Counting Objects	Range of set sizes
Numerical Estimation	Range of set sizes
One-to-One Correspondence (without counting)	Range of set sizes
Set Construction	Provoked or unprovoked; set relations
Numerical Composition of Sets	Set sizes
Reciting Ordinal Number Words	Set relations (e.g., more, less, same)
Using Ordinal Number Words for Position	Particular number words
Numeral Recognition or Naming	Particular number words
Relating Sets of Objects to Numerals	Particular numerals
Quantifying Money	Values of coins
Other (specify):	

Arithmetic

Mathematical Process or Knowledge	Problem Details
Addition with Visible Objects	Particular problems (e.g., augend, addend)
Subtraction with Visible Objects	Particular problems (e.g., minuend, subtrahend)
Addition with Hidden or Imaginary Objects	Particular problems; Strategies (e.g., fingers)
Subtraction with Hidden or Imaginary Objects	Particular problems; Strategies (e.g., fingers)
Division with Visible Objects	Particular problems; Strategies (e.g., fingers)
Division with Hidden or Imaginary Objects	Particular problems; Strategies (e.g., fingers)
Informal Fraction Knowledge and Fraction Terms	Particular fractions and terms (e.g., half)
Numerical Decomposition of Sets	Sizes of to-be-decomposed sets
Two-Set Addition or Subtraction	Initial relations and transformations
Other (specify):	

Patterns

Mathematical Process or Knowledge	Problem Details
Identification or Description of Patterns	Pattern units and elements used
Pattern Unit Duplication	Description of ordered set
Pattern Duplication	Pattern units; Elements Used; # of Repetitions
Pattern Extension	Pattern units; Elements Used; # of Repetitions
Other (specify):	

Measurement, Graphing, and Data Collection

Mathematical Process or Knowledge	Problem Details
Comparing and Ordering Using Direct Comparison of Length	Describe or sketch comparisons
Comparing and Ordering Using Direct Comparison of Weight	Describe or sketch comparisons
Comparing and Ordering Using Direct Comparison of Capacity	Describe or sketch comparisons
Nonstandard Measurement of Length	Repeating unit vs. multiple copies of unit
Nonstandard Measurement of Weight	Type of unit and how measured
Nonstandard Measurement of Capacity	Type of unit and how measured
Standard Measurement of Length	Type of unit; elements being measured
Standard Measurement of Weight	Type of unit; elements being measured
Standard Measurement of Capacity	Type of unit; elements being measured
Other (specify):	

Space/Geometry

Mathematical Process or Knowledge	Problem Details
Identify/Name 2-Dimensional Shapes	Particular shapes and names used
Identify/Name 3-Dimensional Shapes	Particular shapes and names used
Matching Figures	Particular figures
Combining Shapes	Shapes used: What was produced
Shape Analysis: Sides and Corners/Angles	Shapes used: Features were located or counted?
Shape Analysis: Faces, Edges, and Vertices	Shapes used: Features were located or counted?
Constructing or Drawing Figures	Figures produced
Physical Transformation of Figures	Describe or sketch transformation
Mental Rotation of Figures	Describe or sketch transformation
Classifying Figures	Type of class; members and distracters
Location in Small-Scale Space	Describe space and location
Location in Large-Scale Space	Describe space and location

Logical Relations, Calendar, or Other

Mathematical Process or Knowledge	Problem Details
Inferences About Quantitative Relations	State premises
Seriation	Number and type of elements in the series
Sorting and Classifying	Composition of classes; 1-level or hierarchical
Time	Temporal units used
Temporal Sequence	Event sequence
Other (specify):	

APPENDIX C

Math Processes Observed

Math Process	Description	Example
<u>Number</u>		
Rote Counting, Forward	Reciting numerals in sequence from small to large.	"1, 2, 3, 4, 5, 6, 7, 8, 9, 10"
Rote Counting, Backward	Reciting numerals in backward sequence from large to small	"10, 9, 8, 7, 6, 5, 4, 3, 2, 1"
Knowledge of Number Order	Knowledge of the location of numerals on the number line (or within the counting sequence) relative to another numeral	Parent: "What comes after four?" Child: "Five."
Ordinal Position	Denotation of the ordinal position of objects in a set	"This one is first, this one is second, and this one is third."
Knowledge of Even/Odd	Knowledge that a number is either even or odd	Parent: "Is two an even number or an odd number?" Child: "Odd, I mean even."
Numeral Recognition/Naming, Integers	Verbal denotation of a whole numeral	Parent: "What number did your spinner land on?" Child: "Four."
Numeral Recognition/Naming, Fractions	Verbal denotation of a fraction	Parent: "Now we need to add one-half of a cup of oil."
Copying/Tracing Numerals	Tracing numerals, typically written in dots, or copying a numeral written by someone else	Child uses a pencil to trace the numerals "1, 2, 3, 4, 5"
Writing Numerals	Writing numerals from memory	Child writes a "6" after solving the problem 4 + 2.
Estimating Quantity of a Set	Estimating the quantity of a set	Parent: "How many lemons do you think we need to make a pitcher of lemonade?" Child: "Maybe seven?"

Math Process	**Description**	**Example**
Enumerating/Counting a Set	Enumerating or counting objects with the intention of producing the cardinal value of the set	Parent: "How many did you roll?" Child (counts dots on die): "1, 2, 3."
Enumerating/Counting a Subset	Enumerating or counting a subset of objects with the intention of producing the cardinal value of the subset	Parent: "You have seven animals here, but how many of them have four legs?" Child: "1, 2, 3, 4"
Counting Objects to a Specified Number	Counting objects only to a specified number (slightly different than set construction in that the counting stops when a specific number is reached as opposed to when the set is complete)	Child moves game token five spaces after drawing a card with the numeral five.
Set Construction	Constructing a set with a specific number of objects	After identifying that she needs four bowls to set the table, child goes to kitchen and retrieves four bowls.
Set Decomposition	Separating objects in a single set to make two or more distinct sets	Parent: "You can break up the 7 to be 3 and 4, 2 and 5, or 6 and 1"
Matching Sets to Numerals	Matching sets of objects to a numeral	Parent: "What number is this?" Child: "Six" Parent: "Which groups have six in them? Circle all of the groups of six."
One-to-One Correspondence	Matching two or more sets of objects using one-to-one correspondence	Child places one plate, one fork, and one glass at each of four place settings
Quantitative Comparison of Two or More Sets	Judging which of two sets is more/less numerous	Child: "I have six and you have three." Parent: "Who has more?" Child: "I do."

Math Process	**Description**	**Example**
Addition/Subtraction with Concrete Objects	Solving verbal addition and/or subtraction problems with objects	Parent: "If you have seven cars and you add three, how many cars do you have?" Child (creates a set of seven, a set of three, and counts them all): "Ten."
Addition/Subtraction with Hidden/Imaginary Objects	Solving verbal or non-verbal addition and/or subtraction problems where sets of objects are initially visible but hidden before the sum/difference is produced.	Child is playing a computer game. The character on the screen shows child 2 objects, then hides them behind a curtain. The character shows 1 more object, and then hides that behind the screen. Child is asked to reproduce the total number of objects hidden behind the screen.
Formal Addition/Subtraction	Solving printed addition and/or subtraction problems presented with numerals and arithmetic symbols (answer may be verbal or written)	In response to the problem 13 – 2 = __ presented on a flashcard, the child counts silently and says "eleven."
Informal Division	Dividing a set of objects into equal sets	Parent: "Share all of the pennies so that we both have the same."
<u>Geometry/Space</u>		
Shape Matching	Matching two or more of the same shape	Child is presented a worksheet with a column of shapes on both ends of the paper. Parent asks child to draw a line between all of the shapes that are the same.
Shape Naming/Recognition	Naming or identifying shapes	Child showed a row containing three circles and one square. Parent asks child to point to all of the circles in the row.
Tracing Shapes	Tracing shapes, typically drawn with dots	Child traces a square made of dotted lines.

Math Process	Description	Example
Drawing/Constructing Shapes	Drawing or constructing shapes from memory, without an example to trace or copy	Parent: "Can you make a rectangle with these pencils?"
Shape Analysis	Verbalization or identification of shape attributes, including sides and angles	Parent: "How many lines does a square have?"
Shape Decomposition	Decomposing a single shape into two different shapes	Parent: "Look, the square has two triangles in it. When you put the two triangles together, you make a square."
Using Spatial Terms to Describe Location	Use of spatial terms such as "above," "below," and "middle" to describe location	Parent: "Can you put the candle in the middle of the table?"
<u>Measurement</u>		
Direct Measurement	Using direct comparison of physical attributes to compare two or more objects on the basis of length, width, or capacity	Child holds up two socks side-by-side to decide if they are the same length.
Measurement with a Non-Standard Unit	Using a non-standard unit to measure an object	Parent "measures" the child's wrist with a piece of string.
Denotation of Formal Units of Measurement	The use of formal units of measurement in speech	Parent: "We need to add a tablespoon of vanilla."
Graphing	Using simple x and y coordinates to identify a specific location or plot data on a grid.	A store-bought game used a grid to keep score. The grid had the numerals 1-5 across the top and pictures of various Pokémon characters down the side. The child placed a token in the square corresponding to the number of dice he rolled with that particular character facing upwards (picture dice).
<u>Patterns</u>		
Pattern Identification/Creation	Verbally identifying or describing a repeating pattern	Child: "I am going to make my pattern berry-pineapple-marshmallow, berry-pineapple-marshmallow."

Math Process	**Description**	**Example**
Pattern Unit Creation	Construction of a pattern unit for use in a repeating pattern	Child created two separate rows of colored beads, each following the same A-A-B-B-C pattern unit.
Pattern Extension	Constructing a repeating pattern	Child made her berry-pineapple-marshmallow pattern and with three repeating units.

<u>Other (Time)</u>

Reasoning about Time	Reasoning about time	Parent: “What time do you normally go to bed?” Child: 8:00 Parent: “That’s right, you usually go to bed at 8:00, but not always, right?” Child: “No, sometimes I stay up later and listen to Sissy play the piano. Then I go to bed at 9:00.”
Denotation of Formal Units of Time	The use of formal units of time in speech	Parent: “Let’s see how fast you can take the napkins to the dining room. I’ll time you.” Child runs to the dining room and back. Parent: “Twenty-nine seconds. That was really fast.”