AUCTION EQUILIBRIUM WITH COSTLY INFORMATION ACQUISITION

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ABSTRACT

This paper presents a simple model of auction equilibrium. The distinctive feature of the model is that each bidder may discover the value that the item represents for herself, provided she spends some amount in order to be well informed. For each agent, the decision of whether or not to acquire information depends on a private cost of information acquisition and on her conjectures regarding the behavior of other bidders. A rational expectations equilibrium is characterized.

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I. INTRODUCTION

This paper presents a simple model of auction equilibrium. The distinctive feature of the model is that agents can purchase information regarding their valuations of the object for which they are bidding. More specifically, although the value of the object is unknown to the bidders, each bidder might discover her private value, provided she spends some private amount in order to be well informed.

For each agent, the decision of whether or not to acquire information depends on the cost of information acquisition and on his or her conjectures regarding the behavior of other bidders. Then, a rational expectations equilibrium --defined as a situation when those conjectures are fulfilled-- is characterized.

The idea of *costly* information acquisition within an auction seems to be very natural. For example, consider the process of soliciting bids to perform some task, such as constructing a house or building a new military jet. The bidders know their costs imperfectly although with some effort can narrow their uncertainty. How much the bidders expend in narrowing uncertainty depends on the expected benefit from being well informed. This in turn is coupled to the strategy for forming a bid. The higher the bid, the higher the likelihood of a positive surplus but the lower the likelihood of winning.

In spite of its relevance, however, the issue of endogeneous information acquisition has been scarcely analyzed. The main reason for this is that models with costly information acquisition tend to become readily intractable. Hence, few specific results have been derived.

An important merit of the present paper is that it introduces endogenous information acquisition in a tractable way. That is, the structure of our model is about as simple as possible, while still capturing many important aspects of the problem. This tractability allows us to answer interesting questions that have not been directly approached before. In that sense, this paper distinguishes itself from all previous models of which we are aware.

II. BACKGROUND

In this section we review the literature related to our topic, before describing our model and presenting our results. Our review will only cover those models with costly information acquisition as an endogenous element of the bidding process. There are only a few papers satisfying this condition since most of auction theory focuses on models in which bidders have some exogenous level of information.[1]

One of the earliest papers explicitly combining information acquisition with bidding strategy is by Schweizer and Ungern-Sternberg (1983). Their model presents a common-value auction in which each bidder draws an estimate of the value from an interval centered on the true value. The length of the interval around the true value can be narrowed, but only at some cost. Since the authors

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[1] For further discussion of this point and reviews of the auction literature in general, see Wilson (1992), Milgrom (1985), McAfee and McMillan (1987), and Engelbrecht-Wiggans (1980).

are unable to compute a closed form solution for the model, the analysis resorts to simulations for the case of two bidders. The paper represents an interesting example of the role of information, but no definitive conclusions are derived.

Another model with costly information acquisition is incidentally examined in the well-known paper by Milgrom (1981). This paper studies an auction in which *k* identical items are offered for sale. Sealed bids are tendered, and the *k* highest bidders each receive one object. In an example, the author briefly discusses a two-stage model whereby in the first stage, a bidder can decide, at a cost, to become perfectly informed about the common value of an object being auctioned.

Lee (1984), in turn, develops a two-agent model with incomplete information in the sense that bidders do not know with certainty whether their opponents are informed. The item being auctioned can take only two values, say V_h and V_l, with probabilities p and $1- p$, respectively. The true value of the item can be completely revealed at some cost. Lee characterizes a symmetric equilibrium in which each bidder buys information with a positive probability and, if so, then uses a randomized bid strategy whose distribution depends on the value he discovers. In Lee (1985), the model is extended to the case with larger number of bidders.

Matthews (1984) provides a broad treatment of information acquisition. In his model, each bidder shares a common, but unknown, value. Further, each bidder can buy a signal correlated with the true value. The author discusses a symmetric equilibrium. An unpleasant feature of the model is that it turns out to be cumbersome: even in a simple example, Matthews is unable to give a precise description of the equilibrium bidding function. Furthermore, since all bidders have identical information costs, all of them end with the same level of information. Intuitively, it would seem more likely that bidders are heterogeneous; with different bidders having different information levels. Thus, we feel that Matthews' model ignores an important and realistic issue. Hausch and Li (1993) develop a common-value model closer in spirit to Lee's. The good can take two values, 1 and 0, with probabilities p and 1- p, respectively. Any potential bidder who wishes to bid needs to pay a fixed cost and, once in the auction, can buy a private signal giving information about the true value. The accuracy of the signal is positively related to the amount spent on it. The authors characterize a symmetric equilibrium in which bidders enter the auction with a fixed probability q and, if so, they bid according to a randomized bid strategy which depends on the signal they have received. The analysis is extended to the private-value case in Hausch and Li (1993a).

In the present paper, we allow a continuum of feasible values (rather than a finite number, as in Lee's and Hausch and Li's models), and we endow each bidder with a private cost (rather than a common cost, as in Matthews' model). Under those circumstances, to examine symmetric equilibria seems to be unnatural. We prefer, instead, to characterize an asymmetric equilibrium in which different bidders will endogenously find it convenient to acquire different information levels.

III. THE MODEL

A. The Environment

Consider a large number of risk neutral bidders, $i = 1, 2,...,N$, who are trying to buy an object in a first-price sealed-bid auction.[2] Let us assume that the object has some particular value, V_i, for each particular bidder, i, but the value V_i is unknown, even to the agent i himself. In fact, the valuations of the object are random variables with identical density functions:

$$V_i \sim f(.) \qquad i = 1, 2,..., N \tag{1}$$

where f is a continuous density function with support in the interval $[V_l, V_u]$ and with expected value $\underline{V}$.

An agent can follow one of two alternate strategies. First, he might spend an amount c_i in order to discover the true value that the object represents for himself. Then, based on his findings, he might form an optimal bid so that his expected profit is maximized conditioned to the new information he has received. On the other hand, this agent might decide to save c_i and just use the expectation of V_i as an estimate of his (unknown) true valuation. In this case, he would form an optimal bid conditioned to his beliefs and prior information.

The costs of information are also random variables with a common distribution. Each agent knows his particular cost but is unaware of the costs of other bidders. Specifically,

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[2] In a first-price sealed-bid auction, each bidder submits his bid in a private way, without observing the bids offered by his competitors. The auction is won by the bidder who presents the highest bid.

$$c_i \sim g(.) \qquad i = 1, 2, \ldots, N \tag{2}$$

where g is a continuous density function in the interval $[C_l, C_u]$. This distribution is also common knowledge.

In these circumstances, an agent should evaluate all the probable outcomes he could face in case he decides to be informed. Those outcomes depend, of course, on the behavior of his competitors and, in particular, on the information that other bidders have with respect to *their* valuations of the object.

A conventional way of determining the bid for each agent is to compute the Bayesian Nash equilibrium. Each player shares knowledge of a common distribution on (c,V) and knows her own c. Should the agent buy information, she will also know her own V. In any case, she will only have a distribution on the costs and values for the other bidders. Unfortunately, computing a Bayesian Nash equilibirum is extremely difficult, particularly with a continuum of possible costs and values and more than two bidders. Motivated by Grossman and Stiglitz (1980), we take a somewhat more tractable view of each agent's perception of the actions of other agents.

We start with each bidder assuming that a proportion p of bidders chooses not to be informed about their valuations and consequently are just using the common expectation, $\underline{V}$, as a bid.[3] Therefore, if a bidder pays c_i and then he

[3] Strictly speaking, it is not necessary to assume that all uninformed bidders are offering $\underline{V}$. All the results of the model would go through provided at least two uninformed bidders are offering $\underline{V}$. This latter assumption, in turn, seems to be reasonable because: (a) if only one bidder is offering $\underline{V}$, he would find it convenient to offer slightly less than that; and (b) if none bidder is offering $\underline{V}$, some bidders would have an incentive to increase their bids. The assumption in the text is convenient, but not strictly necessary.

discovers that his valuation is V_i, his optimal bidding strategy would have the form $b_i = B(p, V_i)$, and his expected gross profit -- given that particular valuation -- would be $\pi = \pi(p, V_i)$. Since *ex ante* V_i is a random variable, $\pi(p, V_i)$ is also random, and the value of the information is given by its expected value. Define the expected profit as $\Pi(p) = E(\pi(p,V))$, where the expectation is over V and is based on the density function f. This is the expected profit <u>before</u> information is acquired.

On the other hand, if the agent refuses to buy information, he will find it optimal to bid just $\underline{V}$, given his conjectures that $pN - 1 > 0$ other bidders are behaving in identical way. If he offers less than $\underline{V}$, that could not increase his expected profit, because the probability of winning the auction would be zero anyway; if he offers more than $\underline{V}$, he would increase his chances of "winning" the auction, but in unfavorable conditions.[4] In short, he will offer $\underline{V}$, and his expected profit would be zero.[5]

Obviously, the decision of whether or not to acquire information depends simply on the relation between the value of the information --i.e., the positive

[4] Note that a symmetric equilibrium where all uninformed bidders offer less than $\underline{V}$ is not possible. Suppose, to the contrary, that all uninformed bidders offer $\underline{V} - \delta$, where $\underline{V} \geq \delta > 0$. In that case, any single bidder would find it profitable to offer $\underline{V}-(\delta/2)$ since then he will have a positive expected profit, provided all other bidders maintain their previous strategies. Therefore, the original set of strategies could not be an equilibrium. This is the same argument that in a Bertrand equilibrium, price will equal marginal cost.

[5] We will always assume that p is bounded away from the extreme cases 0 and 1. If the bidder conjectures that he is the only uninformed bidder, it is not certain anymore that someone else has a valuation $\underline{V}$. Then, the bidder might find it optimal to offer less than $\underline{V}$. A common-value, two-bidder auction in which only one is uninformed has been analyzed in Milgrom and Weber (1982b) and Engelbrecht-Wiggans et al. (1981). Those papers derive a mixed-strategy equilibrium and show that the expected profit of the uninformed bidder is zero.

amount of expected gross profit that information provides-- and the cost of information acquisition. For a particular p, the relation between $\Pi(p)$ and c_i only depends on c_i: if c_i is too high, the agent *i* will refuse to be informed. A Rational Expectations Equilibrium is given by a value p^e such that the actual proportion of bidders for which $\Pi(p^e) < c_i$ is (nearly) equal to p^e itself.[6]

B. The Equilibrium

Let us first consider the economic problem of a bidder, say *i*, in case he decides to buy information and then discovers a value V_i. This agent conjectures that $n < N$ other bidders are refusing to pay for information and are bidding just the expected value of V, while other $[(N-n)-1]$ bidders have already discovered their respective valuations. If V_i is less than $\underline{V}$, this agent perceives that his probability of winning the auction is zero, because n other agents will be offering $\underline{V}$. If V_i is greater than $\underline{V}$ we shall proceed by assuming that the optimal bid has the form $b_i = B(p,V_i)$; i.e., it is a function of the observed V_i and conjectured p, where B is a continuous, strictly increasing function in V_i, which in equilibrium should be common to all agents.[7] Therefore, the maximum expected profit of the bidder is

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[6] While a Bayesian Nash equilibrium might be more common in the literature on games of incomplete information, computational complexity is a real issue in analyzing auctions and bidding problems, particularly when there are multiple agents and information acquisition. The Rational Expectations equilibrium adopted here is more tractable; furthermore, it is commonly used in other contexts, such as Grossman and Stiglitz's (1980) model of information acquisition in asset markets with uncertain returns.

[7] The assumption that B is common to all agents follows from the symmetry of the situations faced by all bidders. The differences in information costs are relevant to determine whether or not to acquire

$$\pi(p, V_i) = (V_i - b_i)\ [F(B^{-1}(b_i))]^{[(1-p)N-1]} \qquad (3)$$

given by the expression:

where p (the proportion of uninformed bidders) is defined as (*n*/N), B^{-1} is the inverse of B with respect to V and, of course, $(1-p)N-1 \geq 0$.

In the right-hand side of expression (3), the second factor represents the probability of winning the auction, i.e., the probability that each of the other [(1-p)N-1] bidders who are getting information discovers a valuation smaller than V_i.

To derive the function B, we use the envelope theorem which ensures that

$$\frac{d\pi(p, V_i)}{dV_i} = [F(B^{-1}(b_i))]^{[(1-p)N-1]} \qquad (4)$$

$d\pi/dV_i = \partial\pi/\partial Vi$. Therefore:

$$\frac{d\pi(p, V_i)}{dV_i} = [F(Vsubi)]^{[(1-p)N-1]} \qquad (5)$$

But, since $b_i = B(V_i)$, we end with:

Now, this differential equation can be solved by simple integration, and by imposing the boundary condition that $\lim_{V \to E(V)} \pi(p,V) = 0$. Hence, we obtain:

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information, but are totally irrelevant at the moment of forming an optimal bid after being informed. Once the bidder has acquired information, bygones are bygones, and the observed value, V_i, is all that matters.

$$\pi(p, V_i) = \int_{\underline{V}}^{Vsubi} (F(\Theta))^{[(1-p)N-1]} \, d\Theta \qquad (6)$$

where $\pi(p,V_i)$ is a function of the random variable V_i.

Finally, substituting (6) into (3), and solving for b_i, we can get the following

$$B(p, V_i) = V_i - \frac{\int_{\underline{V}}^{V_i} F(\Theta)^{[(1-p)N-1]} \, d\Theta}{F(V_i)^{[(1-p)N-1]}} \qquad (7)$$

expression for the optimal bid:

It is trivial to check that $\partial B(p,V_i)/\partial V_i > 0$, so that the original assumption is satisfied. Standard arguments (see Wilson (1977)) ensure that equation (7) is the unique symmetric equilibrium bidding strategy, among the class of differentiable, strictly increasing functions on $[\underline{V}, V_u]$. Note that, given the presence of uninformed bids, the auction can be thought as a conventional auction with $\underline{V}$ as the reservation price of the seller. With this interpretation, the equivalence between first-price and second price auctions holds, and $B(p, V_i)$ can be thought as the expectation of the second highest valuation among a fictitious seller and the informed bidders, conditioning on V_i being the highest valuation.

The value of the information is given by the expectation of $\pi(p, V_i)$ over the

$$\Pi(p) = \int_{\underline{V}}^{V_u} \left[\int_{\underline{V}}^{\Phi} (F(\Theta))^{[(1-p)N-1]} \, d\Theta \right] f(\Phi) \, d\Phi \qquad (8)$$

relevant interval $[\underline{V}, V_u]$. Then, where $\Pi(p)$ is the (*ex ante*) expected gross profit of a bidder, should he decide to buy information, conditioned on his beliefs that the other [(1-p)N-1] bidders are also becoming informed about their respective valuations. Those beliefs about the behavior of the competitors are summarized in p. Note that $d\Pi/dp > 0$ for all relevant p.

Consequently, we obtain the decision rule:

$$\begin{array}{l} \text{Buy information if } \Pi(p) \geq c_i \\ \text{Do not buy information otherwise} \end{array} \qquad (9)$$

Finally, we impose the equilibrium condition that the expectations of the bidders are rational. Hence, the value of p conjectured by all bidders when taking their decisions should be consistent with the true p[8]. Let G be the distribution function of information costs, and define

$$\Omega(x) \equiv 1 - G(x) \qquad (10)$$

Thus, $\Omega(x)$ is the proportion of bidders with information costs greater than x. Obviously, $d\Omega/dx \leq 0$.

Since p can only take discrete values, a rational expectations equilibrium, p^e, will be defined by the condition:

[8] This idea depends on a somehow heuristic application of the Law of Large Numbers. Strictly speaking, this law would only apply if the number of bidders goes to infinity, so our reasoning should be seen as an approximation for a large N.

$$\Omega(\Pi(p^e)) \geq p^e \;; \quad p^e + \frac{1}{N} \geq \Omega(\Pi(p^e + \frac{1}{N})) \qquad (11)$$

$$p^e \; \varepsilon \{ \frac{i}{N} : i = 2,\ldots,N-2 \}$$

where

The last condition defines the feasible range of p consistent with the previous discussion. Recall that the model is not interesting for the case p = 0 (i.e., all bidders are informed). That situation corresponds to the standard model where everybody knows his true valuation, and this has been extensively studied in auction literature. On the other hand, if p = 1 (no one buys information), everybody would use the same strategy and their expected profits would be zero. This would be the case, if the minimum information cost, C_l, is very high with respect to the maximum valuation V_u. Finally, the extreme cases with only one informed or uninformed bidder has been analyzed elsewhere by other authors.[9]

The following lemma results directly from the fact that Ω declines as p increases.

<u>Lemma 1</u>: If Ω(Π(1/N)) > (1/N) and Ω(Π(1-(1/N)) < 1 - (1/N), then there exists a unique rational expectations proportion of uninformed bidders, p^e, in the range (1/N, 1-(1/N)).

This is illustrated in Figure 1. Three different distribution of information

[9] See footnote 1.

cost are shown. In panels (a) and (c), costs are either too high or too low for an interior p^e exists.

Finally, we derive an expression for the expectation of the winning bid. That

$$E_N(b) = \int_{\underline{V}}^{Vsubu} \left[z - \int_{\underline{V}}^{z} \left(\frac{F(\Theta)}{F(z)}\right)^{[(1-p)N-1]} d\Theta \right] \gamma_{N^*}(z)dz + \overline{V}\,\Gamma_{N^*}(\overline{V}) \quad (12)$$

is,

where b represents the winning bid, γ_{N*} is the density function of the maximum in a sample of [(1-p)N] draws on V, and Γ_{N*} represents the corresponding distribution function. This expression will be useful later.

IV. RESULTS

A. <u>The Effects of "Hypotheticalness"</u>

We begin with an extension of the model that is formally trivial, but seems to have empirical relevance. Consider now the case when the bidders attribute some positive probability to the event in which (ex post) the transaction is not really consummated by the auctioneer. Specifically, each bidder believes that, with probability α, the auction is only "cheap talk" and that the auctioneer is not really willing to sell the object to the winner of the auction.

In an obvious way, the decision rule given by (9) can now be restated as:

Buy information if $(1-\alpha)\Pi(p) \geq c_i$

Do not buy information otherwise (9')

The effect of uncertainty regarding the validity of the auction is just to reduce the value of information and the incentive of each agent to be well informed. Consequently, it also increases the equilibrium proportion of uninformed bidders, p^e. That is, the greater the "hypotheticalness" of the auction, the greater the proportion of bidders who decide not to buy information and the less informative the bids will be.

For particular forms of the density functions f and g, expression (9') allows us to compute the "critical level of hypotheticalness," defined as a value α^* such that $(1-\alpha^*)\ \Pi(1) \approx c_I$ or, more precisely, $\Omega[(1-\alpha)\Pi(1-(1/N))] > 1-(1/N)$ for any $\alpha \geq \alpha^*$. That is, for any level of confidence smaller than that corresponding to $(1-\alpha^*)$, there is no solution to (11) in the relevant range, since no one is willing to spend sure resources to increase the low chances of winning limited profits. Thus, the auction does not have to be purely hypothetical for everyone to remain uninformed. This behavior is illustrated in Figure 2. The figure shows how Ω depends on p for various α. Ω moves outward with decreases in α. For α^*, no interior p^e exists.

From the point of view of the seller, we can think of the efficiency of an auction as its ability to induce revelation of the true values of the bidders. In this sense, the more hypothetical is the auction, the less efficient it is. For levels of hypotheticalness above α^*, the auction delivers no relevant information. An illustrative example of this point is given by the case of a firm going public. In a common procedure, the firm asks selected potential investors to state the price that they would be willing to pay for the offering. After their statements arrive, the firm decides whether or not to go ahead with the sale. Our previous discussion suggests that, if investors need to investigate the true value of the firm and if they think it is highly probable that the sale will not be consummated, the pre-issue gathering of information will be barely informative for the firm. Most potential investors will find it inconvenient to spend resources to learn the exact value of the firm.

B. <u>Comparative Statics</u>

In the following discussion, we examine the effects of the number of bidders, and specific changes in the distributions of information costs and valuations.

1. <u>The Number of Bidders</u>. For any fixed p, a greater number of bidders implies a smaller value of information; that is, $\partial\Pi(p,N)/\partial N < 0$. This follows immediately from the fact that, given a particular p, a greater N implies a smaller value for the integral in brackets in expression (8), above. Therefore, an increase in the number of bidders represents an upward shift in the graph of the function $\Omega(\Pi(p, N))$ --as defined by expression (11)-- and, in turn, this ensures a higher equilibrium value for p. This has been illustrated in Figure 3.

More formally, consider the equilibrium condition (11) as an implicit function in p and N, and --just for the sake of exposition-- assume that the first inequality holds as equality. Then, the equilibrium condition is:

$$H(p, N) = 0 \tag{13}$$

$$H(p, N) \equiv \Omega(\Pi(p, N)) - p \tag{14}$$

where the function H is defined as[10]

$$\frac{dp}{dN} = - \frac{\partial H / \partial N}{\partial H / \partial p}$$
$$= - \frac{(\partial\Omega/\partial\Pi)(\partial\Pi/\partial N)}{(\partial\Omega/\partial\Pi)(\partial\Pi/\partial p) - 1} \geq 0 \tag{15}$$

Therefore,

which validates the previous informal argument.

Now, since the equilibrium p, p^e, increases with N (provided Ω is not "flat"), it follows from the expressions (13)-(14) and the monotonicity of Ω that the equilibrium value of $\Pi(p^e, N)$ should be smaller the greater the value of N. Since the density function f has not changed, this is only possible through an increase of the exponent [(1-p)N-1] in expression (8). These arguments lead to the following:

Proposition 1. A greater number of bidders implies no larger *proportion* of bidders buying information, but a greater *number* of well-informed bidders.

It is well known that, in the standard model of auction, an increase of the

[10] Here we have made explicit the presence of N as an argument of Π.

number of bidders increases the expectation of the winning bid. In our case, the effect of the increase of the total number of bidders on the expectation of the winning bid can be analyzed from expression (12) above. Heuristically, an increase in N causes both an increase in the expression in brackets, for any given z, and a shift of the mass of $\gamma_{N^*}(z)$ to the right. Since B(p,z) is increasing in *z*, both effects act in the same direction. Then, the next proposition follows.

Proposition 2. An increase of the number of bidders implies an increase of the expected value of the winning bid.

A formal proof of this statement is provided in the appendix.[11]

2. <u>Variations in the distribution of information costs</u>. For this case, it is convenient to rewrite the equilibrium condition (13) as:

$$p^e = 1 - G(\Pi(p^e)) \tag{13'}$$

where G is the cumulative function corresponding to the density g. Next, let us begin by considering an arbitrary change in g. Since $\Pi(p^e)$ is not affected by the changes in g, it follows that changes in the initial equilibrium p^e depend only on changes of $G(\Pi(p))$ when evaluated at that point. Formally, consider two different distribution functions, G_0 and G_1, with p_0 and p_1 as the corresponding equilibrium proportions. Clearly

[11] An alternate proof can be based on the fact that the number of informed bidders increases with the total number of bidders. Thus, in accordance with a previous remark, the situation is equivalent to an increase of the number of bidders in a standard auction with reservation price $\underline{V}$.

$$\Omega_1(\Pi(p_0)) \;\geq\; \Omega_0(\Pi(p_0)) \;\; iff$$

$$G_0(\Pi(p_0)) \;\geq\; G_1(\Pi(p_0))$$

where the subscript {0,1} identifies the pre and post-change functions. Therefore, $p_1 \geq p_0$ *iff* $G_1(\Pi(p_0)) \leq G_0(\Pi(p_0))$, as illustrated in Figure 4.

The intuition behind is obvious. For any value of p, including the equilibrium p^e, the value of information is independent of G. Thus, the original equilibrium can only be changed with changes in the proportion of bidders with information costs below or above the corresponding value of information.

To advance a further step, consider the case where changes in g induce an increase in p^e and, consequently, a decrease in [(1-p)N-1]. The latter effect acts in the same way as an exogenous decrease of N. Hence, it causes a decrease of the integral in the expression for the expected winning bid, E(b), above, and a simultaneous shift of mass of the density γ_{N^*} to the left. Of course, to have a *strict* change of p, the change in G must be big enough to overcome the discrete nature of p. This discussion can be summarized as follows:

Proposition 3. The effect of a change in the distribution of information costs depends on the change of G, evaluated at the initial equilibrium value of information. If $G(\Pi(p^e))$ increases, then the proportion of informed bidders as well as the expectation of the winning bid will (weakly) increase; if $G(\Pi(p^e))$ decreases, then both variables will (weakly) decrease.

3. <u>Variations in the Distribution of Valuations</u>. We will continue to use the subscripts {0, 1} to refer to pre and post-change functions, respectively. Our discussion will be focused on some specific cases, and we do not presume to be exhaustive.

Let us consider first a mean-increasing/shape-preserving shift of f; i.e., for some $s > 0$,

$$f_1(z) = f_0(z - s) \qquad z \in [V_l + s, V_u + s] \tag{16}$$

It is not difficult to see that, in this case, the function Π(p) does not change.

$$\Pi_1(p) = \int_{\bar{V}+s}^{Vsubu + s} \left[\int_{\bar{V}+s}^{\Phi} (F_1(\Theta))^{[(1-p)N-1]} \, d\Theta \right] f_1(\Phi)\, d\Phi$$

$$= \int_{\bar{V}}^{V_u} \left[\int_{\bar{V}}^{\Phi} F_1(\Theta + s))^{[(1-p)N-1]} \, d\Theta \right] f_1(\Phi + s) d\Phi \tag{17}$$

$$= \int_{\bar{V}}^{V_u} \left[\int_{\bar{V}}^{\Phi} F_0(\Theta))^{[(1-p)N-1]} \, d\Theta \right] f_0(\Phi) d\Phi = \Pi_0(p)$$

In effect, for any arbitrary p, we have:

where the notation is self-evident. Since Ω has not changed, the equilibrium value of p remains unchanged.[12]

However, the change on f increases the expectation of the winning bid by an amount equal to *s*. To see this, notice that:

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12 This result is intuitively clear. Holding p fixed, the shift on the distribution of valuations increases the probability of higher valuations, but it also asks for higher bids. The net effect on the value of information is null.

$$E_1(b) = \int_{\overline{V}+s}^{Vsubu\ +\ s} \left[z - \int_{\overline{V}+s}^{z} \left\{ \frac{F_1(\Theta)}{F_1(z)} \right\}^{[(1\ -\ p)N\ -\ 1]} d\Theta \right] \gamma_1(z)dz + (\overline{V} + s)\Gamma_1(\overline{V} + s) \quad (18)$$

$$= \int_{\overline{V}}^{V_u} \left[z - \int_{\overline{V}}^{z} \left\{ \frac{F_1(\Theta + s)}{F_1(z + s)} \right\}^{[(1\ -\ p)N\ -\ 1]} d\Theta \right] \gamma_0(z)dz + \overline{V}\ \Gamma_0(\overline{V}) + s\ \Gamma_0(\overline{V})$$

$= s + E_0(b)$.

Thus, we are able to conclude that:

Proposition 4. A mean-increasing shape-preserving shift of the density function f implies an increase of the expectation of the winning bid by the amount of the mean shift, while the proportion of bidders who buy information remains unchanged.

The implicit message here can be rephrased as follows: all that matters is the magnitude of information costs compared with potential gains, and not the magnitude of information costs compared with valuations.

Finally, we would like to analyze the case of a symmetric mean preserving spread of f that we define as a change of f such that, for some function s(x):

$$f_1(z) = f_0(z) + s(z) \qquad z \in [V_l, V_u] \qquad (19)$$

and the following conditions are satisfied:

$$(i) \quad \int_{V_l}^{Vsubu} s(z)\, dz = 0$$

$$(ii) \quad \int_{V_l}^{V_u} s(z)\, z\, dz = 0$$

$$(iii) \quad F_0(z) \leq F_1(z) \qquad z \leq \overline{V}$$

$$(iv) \quad F_0(z) \geq F_1(z) \qquad z \geq \overline{V}$$

The conditions (*i*)-(*iv*) seem to be a natural description of many typical situations. Of course, (*i*) and (*ii*) are self-explanatory. In turn, (*iii*) and (*iv*) guarantee that the new distribution gives more mass to the tails of the support $[V_l,V_u]$ but preserving a sort of symmetry in the shift. Figure 5 gives an example where those conditions hold, showing the density, f, the distribution function, F, and the analogous functions for s.

We want to determine the effect of this change on p^e. From Figure 1 and the arguments there, we know that p^e will increase (decrease) if the shift of f produces an upward (downward) shift of Ω viewed as a function of p. In turn, for any given p^e, Ω will move up (down) if the changes of f induce a negative (positive) change on $\Pi(p^e)$. It is intuitive to think that, if the distribution of the values concentrates around the mean, the benefit of acquiring private information should decrease; then, the proportion of uninformed bidders should go up. However, that conjecture is not necessarily true. The following result states that the direction of the change of $\Pi(p^e)$ cannot be determined under the general conditions of a symmetric mean preserving shift.

Proposition 5. Consider an equilibrium proportion of uninformed bidders, p^e, and then suppose a symmetric mean preserving spread as defined by (*i*)-(*iv*) above. The effect of that change on p^e and $\Pi(p^e)$ is indeterminant without further restrictions.

The proof of the result is given in the appendix. The rationale behind the result is that, as the distribution of the values shrinks around the mean, the gain from acquiring information is reduced by the need of even greater precision. In turn, the need for even greater precision comes from the fact that the competitors already have relatively accurate information. Although the sign of the movement of p^e is undetermined under the general conditions of a symmetric mean preserving shift, it is easily determined if, in addition to (*i*)-(*iv*), the following constraints are

(v) f is symmetric; and

$$(vi)\quad f_1(\overline{V}+z) = \lambda\ f_0(\overline{V}+\lambda z), \qquad z\, \varepsilon\, [\overline{V} - \frac{\overline{V} - V_l}{\lambda},\ \overline{V} + \frac{V_u - \overline{V}}{\lambda}],\ \lambda > 0$$

imposed:

If $\lambda > 1$, the additional conditions imply that f_1 is obtained just by "compressing" the original function f_0 horizontally and "dilating" it vertically in a proper way. This is the case illustrated in Figure 6. Of course, the opposite is true when $\lambda < 1$. It is easy to check that f_1 is a well-defined density function.

$$F_1(x) \equiv Fsub1(\overline{V} + (x - \overline{V}))$$

$$= \int_{-\infty}^{overlineV + (x - \overline{V})} \lambda \; f_0(\overline{V} + \lambda(y - \overline{V}))\, dy \qquad (20)$$

For concreteness, let's consider $\lambda > 1$. Thus,

$$F_1(x) = \int_{-\infty}^{overlineV + (x - \overline{V})} f_0(z)\, dz = F_0(\lambda x + (1-\lambda)\overline{V}) \qquad (21)$$

Then, letting $z = \underline{V} + \lambda(y-\underline{V})$, we get:

$$\Pi_1(p^e) \equiv \int_{\underline{V}}^{\overline{V} - \frac{\overline{V} - V_l}{\lambda}} \psi(F_1(x))\, dx = \int_{\underline{V}}^{\overline{V} - \frac{\overline{V} - V_l}{\lambda}} \psi(F_0(\lambda x + (1-\lambda)\overline{V}))dx \qquad (22)$$

Now define $\psi(X) \equiv X^{[(1-p)N-1]} - X^{[(1-p)N]}$. Then, as is shown in the appendix:

$$\Pi_1(p^e) = \frac{1}{\lambda} \int_{\underline{V}}^{V_u} \psi(F_0(w))\, dw = \frac{\Pi_0(p^e)}{\lambda} \qquad (23)$$

Finally, define $w \equiv \lambda x + (1-\lambda)\underline{V}$, obtaining:

Therefore, if $\lambda > 1$, $\Pi_1(p^e) \le \Pi_0(p^e)$, and it follows that the equilibrium p should increase as the mass of the density f is concentrated around $\underline{V}$. In this case, the intuitive conjecture holds. Therefore,

Proposition 6. Consider a symmetric, mean preserving change of the density f, so that its mass becomes more concentrated around the mean valuation in accordance with (*v*)-(*vi*)above. Then, at the new equilibrium, the incentive to acquire information and the proportion of informed bidders decrease.

V. GENERALIZED INFORMATION

For completeness, we will now suggest a natural way in which the simplest model might be extended to cover a finite number of feasible information levels, rather than just the extreme cases of null or perfect information. A formal treatment of this extension would be technically cumbersome. The following is purely descriptive and informal.

Let us say that, as before, the object has some particular value, V_i, for each particular bidder, *i*, but the values V_i are unknown. The valuations are drawn from the common knowledge distribution f on $[V_l, V_u]$.

Now, assume that each agent can buy one among *k* different levels of information, say $I_1,...,I_k$, regarding her private valuation. By acquiring I_j $(1 \leq j \leq k)$, the bidder can observe a signal s with conditional density $h_j(s \mid V_i)$. Let $\Lambda_j(V_i)$ be the set of all values that the signal can take when V_i is the true valuation of the observer, given the information level I_j. Further, let h_j, j = 1,...,k, be the (unconditional) density function of the signals given the information level I_j. Allowing some loss of generality, say that, for a bidder *i,* the information level I_j has a cost $c_i j$, where c_i is a private parameter for each bidder.

Following Matthews (1984), we will order the information levels saying that, for $1 \leq j < k$, I_{j+1} is more *informative* than I_j in the sense that, for any given set of available actions, the supremum of the expected profit of a bidder having I_{j+1} is greater than the supremum of the expected profit of a bidder with I_j. Since the true valuations are not observable, the optimal bid of each bidder should only depend on the observable signal. Thus, we will consider that a bidder with information level I_j uses a bidding functions with the form $B_j(s)$, where s represents the signal she observes. Let us assume that, for all j, the function B_j is differentiable and strictly increasing in the relevant range $\cup_{v\varepsilon[Vl\ ,Vu]} \Lambda_j(V)$; then, it has an inverse, say σ_j, which is also strictly increasing.

Let us consider the situation of a bidder i who is deciding whether to buy the information level I_k. If, after acquiring that information, she observes a signal s

$$\Pi_k = \int_v^V \int_{\Lambda subk(V)} (V - B_k(s))\, Q(B_k(s))\, h(s|v)\, f(v) ds\, dv \qquad (24)$$

and she presents a bid $B_k(s)$, her expected profit would be:

The factor $Q(B_k(s))$ represents the probability of winning the auction by bidding an amount $B_k(s)$. To give content to $Q(B_k(s))$, assume that the bidder conjectures that proportions $p_1,...p_{k-1}$ are acquiring the information levels $I_1,...I_{k-1}$, respectively.

$$Q(B_k(s)) = H_1(\sigma_1(B_k(s)))^{p_1 N} \ldots H_k(\sigma_k(B_k(s)))^{(1-\Sigma_{j=1}^{k-1} p_j)N-1} \qquad (25)$$

Thus, $Q(B_k(s))$ can be written as:

where H_j represents the distribution function corresponding to the density h_j.

Introducing (25) into (24), differentiating with respect to B_k, setting it to 0 and, finally, using the fact that at equilibrium $\sigma_k(B_k(s)) = s$, we obtain a system of differential equation where the optimal bid depends on the competitors' strategies, B_j $(j < k)$, and on the assumed values $p_1,...p_{k-1}$. The solution of that system of equations, provided it exists, yields the equilibrium bidding functions for a given set $p_1,...,p_{k-1}$. Suppose that the set $B_1,...B_k$ solves that system of first order conditions. Then, from (24), we obtain an expression for the expected profit associated with each information level, as a function of the conjectured p's.

Given a set of conjectures $p_1,...p_{k-1}$, a bidder with information cost c will position herself in the informational level where the net value of information is maximized. For a bidder with information cost c, we define the net value of information level I_j by the expression:

$$\Pi_j - cj \qquad j = 1,...k \qquad (26)$$

Clearly, if $c_m > c_n$, then the bidder with c_m will necessarily choose a less informative information level than a bidder with cost c_n: otherwise, one of those bidders would not be taking the best action. Then, in accordance with the simplest model, a natural definition of equilibrium is that the proportion of bidders in each information level I_j is (nearly) equal to p_j itself. This completes the description of the environment.

To establish sufficient conditions for the existence of this equilibrium, as well as its characterization at this level of generality, is beyond our present objective. In this paper, we will not make further attempt in that direction. That represents a room for further developments.

VI. CONCLUSIONS

This paper has examined an extremely common situation in auctions and bidding, yet one usually neglected in the auction literature. Specifically, we examined the case of a set of bidders formulating bids based on incomplete information on value with the option of purchasing additional information. This situation applies to many bidding contexts, including competitive bidding by suppliers with unknown and somehow different production costs. Not only did we solve for the closed form expressions for the bidding function and the expected winning bid, but we also developed comparative statics results on the effects of the number of bidders and changes in the distribution of valuations and information costs.

In addition to its applications within the realm of the auction literature, we argued that our model might be useful to evaluate the informational content of some procedures commonly used to assess the value that an object represents for a group of individuals. In particular, we offered an immediate implications of costly information acquisition for the case of firms going public. The idea of costly information acquisition is clearly relevant in other contexts, as for example in principal-agent models. Although the principal is unable to observe the action of the agent, he can certainly become better informed by spending resources. The explicit consideration of this fact, perhaps along the lines suggested by this paper, might provide useful insights into those problems. Indeed, costly information acquisition seems to be a natural companion for the costly state verification setup commonly used in the literature.

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APPENDIX

A. <u>*Proof of Proposition 2*</u>. Let us define:

$$N^* \equiv (1\text{-}p)N \qquad \text{(A-1)}$$

and

$$B(z,\ N^*) \ = \ z \ - \ \int_{\overline{V}} supz\ \{\frac{F(\Theta)}{F(z)}\}^{[N^* - 1]} \ d\Theta \quad if\, z \ \geq \ \overline{V} \qquad (A\text{-}2)$$

$$= \ \overline{V} \qquad\qquad if \ \ z \ \leq \ \overline{V}$$

Clearly,

$$(i) \qquad \frac{\partial B(z,\ N^*)}{\partial z} \ \geq \ 0 \qquad (A\text{-}3a)$$

$$(ii) \qquad \frac{\partial B(z,\ N^*)}{\partial N^*} \ \geq \ 0 \qquad (A\text{-}3b)$$

$$(iii) \qquad E(b) \ = \ \int_{V_l}^{Vsubu} B(z,\ N^*)\ \gamma_{N^*}(z)dz \qquad (A\text{-}3c)$$

and that the expectation of the winner bid can be written simply as:

$$\frac{dE(b)}{dN} \ = \ \frac{dE(b)}{d\,N^*}.\frac{d\,N^*}{dN} \qquad (A\text{-}4)$$

Now,

From proposition 1 in the text, we already know that the second factor is positive. Thus, it suffices to prove that the first is non-negative. Differentiating (A-3c), we can say that:

$$\frac{dE(b)}{dN^*} = \int_{V_l}^{V_u} \frac{\partial B(z, N^*)}{\partial N^*} \gamma(z)\, dz + \int_{V_l}^{V_u} B(z, N^*) \frac{\partial \gamma(z)}{\partial N^*} dz \qquad (A-5)$$

where, for notation convenience, the subscript of the density function γ_{N^*} has been dropped. It follows from (A-3b) that the first term in this expression is nonnegative. In turn, the second term can be written as:

$$\int_{0}^{V_u} B(z, N^*) \frac{\partial \gamma(z)}{\partial N^*} dz \equiv \int_{V_l}^{V_u} B(z, N^*) \frac{\partial}{\partial N^*} (\frac{\partial \Gamma(z)}{\partial z})\, dz$$

$$\equiv \int_{V_l}^{V_u} B(z, N^*) \frac{\partial}{\partial z} (\frac{\partial \Gamma(z)}{\partial N^*})\, dz \qquad (A-6)$$

which, by straightforward integration by part, yields:

$$\equiv B(z,N^*) \frac{\partial \Gamma(Z)}{\partial N^*} \Big|_{V_l}^{V_u} - \int_{V_l}^{V_u} \frac{\partial B(z,N^*)}{\partial z} \frac{\partial \Gamma(z)}{\partial N^*} dz \qquad (A-7)$$

Finally, we use the fact that, for all N^*, $\Gamma(v) = 0$ and $\Gamma(V) = 1$, so that its derivative at those points is zero. Therefore, we end with:

$$\int_{V_l}^{V_u} B(z, N^*) \frac{\partial \gamma(z)}{\partial N^*} = - \int_{V_l}^{V_u} \underset{(+)}{\frac{\partial B(z, N^*)}{\partial z}} \underset{(-)}{\frac{\partial}{\partial N^*}(F(z)^{N^*})} dz \geq 0 \qquad (A-8)$$

and the result follows immediately.

B. _Proof of Proposition 5_. Let 0 and 1 identify the pre and post-change functions, respectively. Also, let p^e denote the equilibrium value of p before the change of the density f. Thus, from expression (8) in the text,

$$\Pi_i(p^e) \equiv \int_{\underline{V}}^{V_u} M_i(\Phi) F_i'(\Phi) d\Phi, \qquad (A\text{-}9)$$

$$M_i(\Phi) \equiv \int_{\underline{V}}^{\Phi} Fsubi(\theta)^{N^*} \, d\theta \qquad (A\text{-}10)$$

where $i = \{0, 1\}$,
and $N^* \equiv [(1-p^e)N-1]$.

$$\Pi_i(p^e) = M_i(\Phi) Fsubi(\Phi) \Big|_{\underline{V}}^{V_u} - \int_{\underline{V}}^{V_u} M_i'(\Phi) F_i(\Phi) d\Phi$$

$$= M_i(V_u) - \int_{\underline{V}}^{V_u} F_i(\Phi)^{N^*} F_i(\Phi) d\Phi$$

$$= \int_{\underline{V}}^{V_u} [F_i(x)^{N^*} - F_i(x)^{N^*+1})] dx \qquad (A\text{-}11)$$

Integrating by parts and noting that $F(V_u)=1$ and $M(\underline{V}))=0$,
Now, let us define $\psi(X) \equiv X^{N^*} - X^{N^*+1}$, to have:

$$\Pi_i(p^e) = \int_{\underline{V}}^{Vsubu} \psi(F_i(x)) \, dx \qquad (A\text{-}12)$$

We seek to show that the expression

$$\Pi_0(p^e) - \Pi_1(p^e) = \int_{\underline{V}}^{V_u} [\psi(F_0(x)) - \psi(F_1(x))] \, dx \qquad (A\text{-}13)$$

can take any sign, even after imposing the conditions in (i)-(iv) of the text.

Note that the function $\psi(x)$ has a maximum at $x = N^*/(N^*+1) < 1$, and an

$$\Psi(F_i(Vsubu)) = \Psi(1) = 0 \qquad (A\text{-}14)$$

inflection point at $x = (N^*-1)/(N^*+1)$. Also,
while

$$\Psi(F_i(\overline{V})) = F_i(overlineV)^{N^*} - F_i(\overline{V})^{N^*+1} > 0 \qquad (A\text{-}15)$$

Finally, define

$$v^* \equiv F_0^{-1}\left(\frac{N^*}{N^*+1}\right) \qquad (A\text{-}16)$$

Now consider a case in which

$$\begin{aligned} F_0(x) &= F_1(x) && \underline{V} \leq x \leq v^* \\ &\geq F_1(x) && v^* < x \leq V_u \end{aligned}$$

Since $F_0 \geq F_1$ while ψ is decreasing in $[v^*, V_u]$, it follows that $\psi(F_0) \leq \psi(F_1)$ in that interval. Thus,

$$\Pi_0(p^e) - \Pi_1(p^e) = \int_{v^*}^{V_u} [\psi(F_0(x)) - \psi(F_1(x))]\, dx \leq 0 \qquad (A\text{-}17)$$

and then $\Omega_0(p^e) \geq \Omega_1(p^e)$.

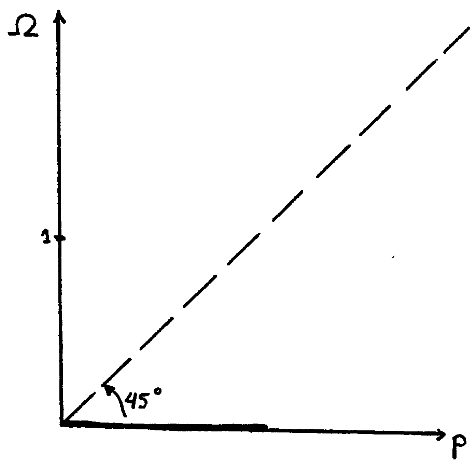
In other words, we have seen that, after the change in f, the function Ω has a downward jump at the original equilibrium value of p, p^e. From Figure 1, it is clear that the new equilibrium p should be smaller than p^e.

On the other hand, consider now a case where

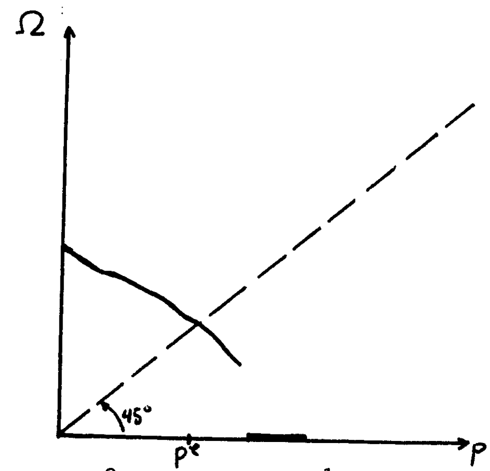
$$\begin{aligned} F_0(x) &= F_1(x) && \underline{V} \leq x \leq v^* \\ &\geq F_1(x) && v^* < x \leq V_u \end{aligned}$$

A similar argument shows that, in this case, $\Omega_0(p^e) \leq \Omega_1(p^e)$. Then, the new equilibrium value of p should be greater than the original one.

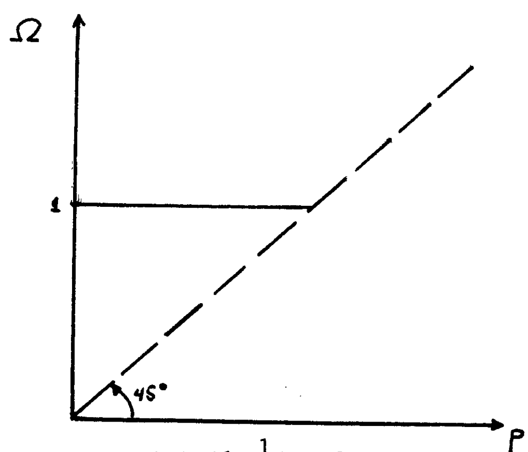
Proposition 5 follows.



(*a*) $\Pi\left(\frac{2}{N}\right) \geq C_u$;



(*b*) $\Pi\left(\frac{2}{N}\right) < C_u$; $\Pi\left(1-\frac{1}{N}\right) > C_l$



(*c*) $\left(1-\frac{1}{N}\right) \leq C_l$

Figure 1: Determination of the equilibrium value of p

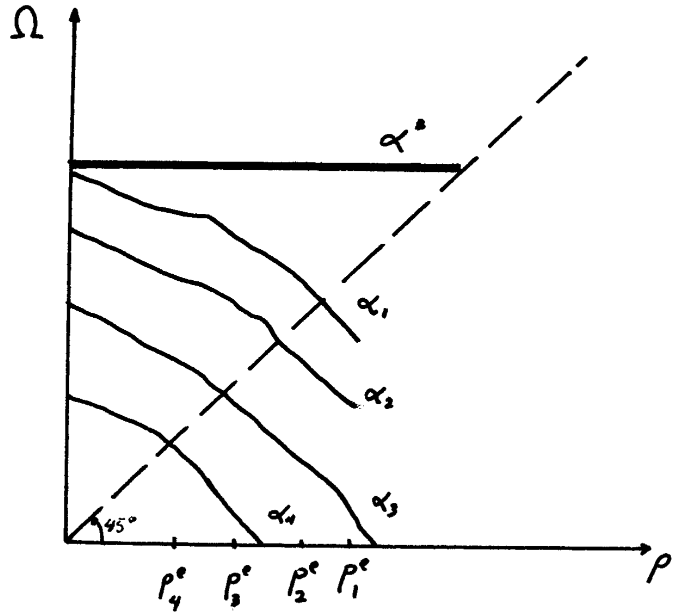


Figure 2: Critical level of "Hypotheticalness"

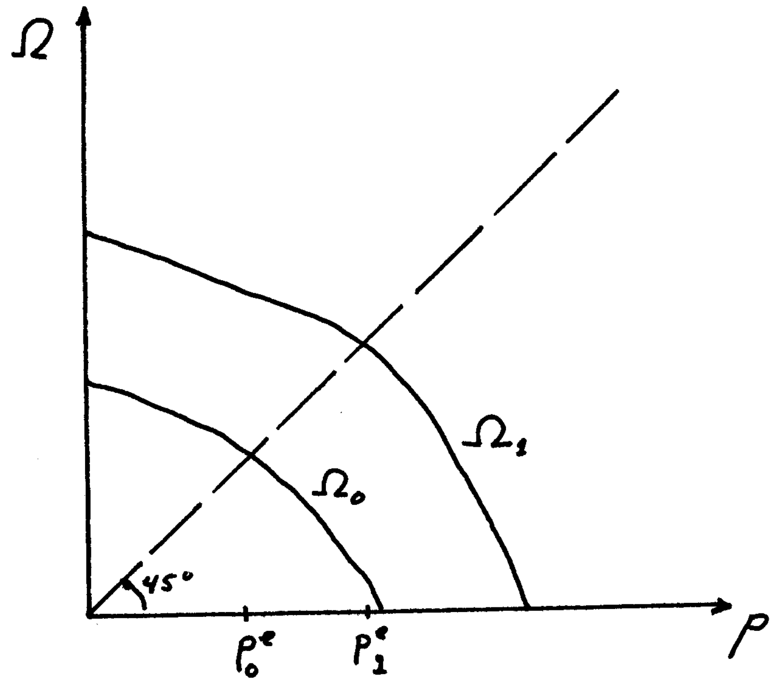


Figure 3: Effect of the number of bidders on $\Omega(\Pi(p))$, $N_1 > N_0$

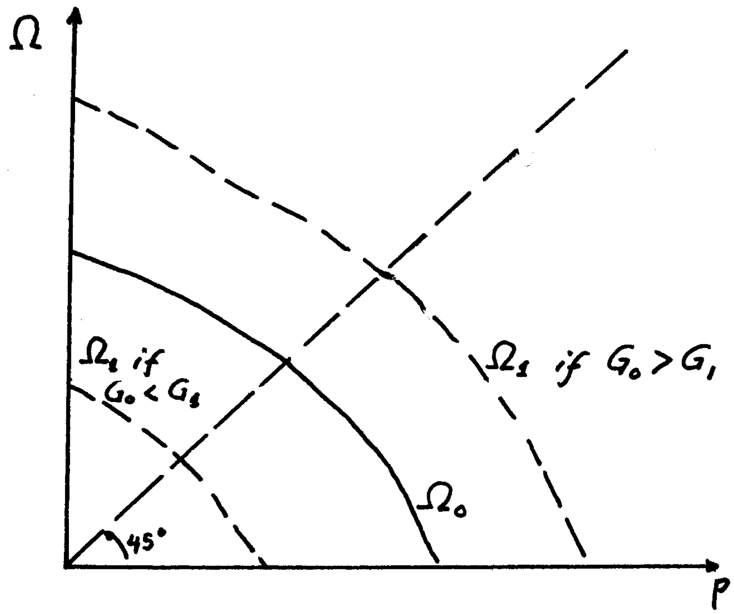


Figure 4: Shifts on $\Omega(\Pi(p))$ given changes of g

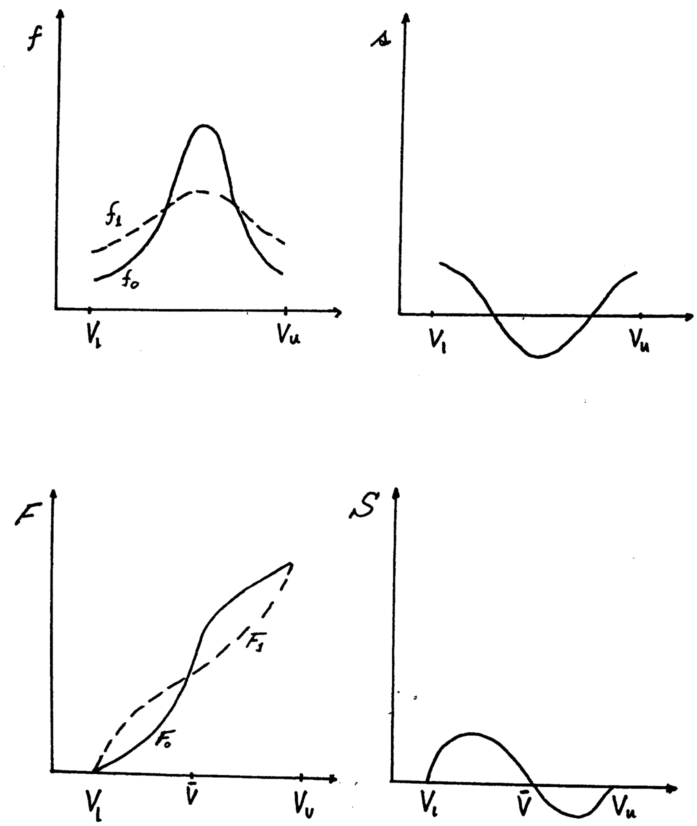


Figure 5: Symmetric Mean-Preserving shift of f

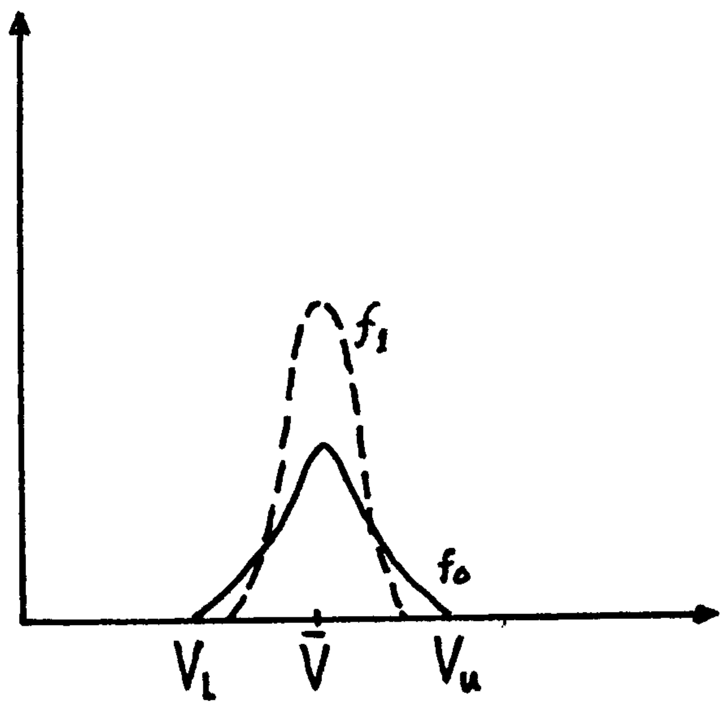


Figure 6: Special case of Symmetric Mean-Preserving shift of f