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Authors
Silverman, Dennis
Ting, Peter D
Yesian, Harry J

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Diffractive Pomeranchukon Bootstrap and Inclusive Experiments*

Dennis Silverman,† Peter D. Ting, and Harry J. Yesian‡
Department of Physics, University of California, San Diego, La Jolla, California 92037
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A bootstrap of a diffractive Pomeranchuk singularity which gives rise to constant total cross sections is carried out in the framework of a multiperipheral model for particle production in which the Pomeranchuk singularity is exchanged at most once. The results for multiplicity distributions and inclusive experiments are presented and contrasted with those of multi-Pomeranchukon-exchange models in order to provide tests for the nature of the Pomeranchuk singularity. In this diffractive model the single-particle spectrum has a limiting distribution at high energies, but does not have pionization. As a consequence, the average multiplicity, $\bar{n}$, is asymptotically constant. A set of coupled nonlinear integral equations for the Pomeranchukon residue functions are derived and solutions are presented which explicitly exhibit factorization. The model predicts that the elastic contribution saturates unitarity at $t \approx -0.6$ GeV^2, where the P' Regge trajectory passes through zero.

I. INTRODUCTION

There are currently two major points of view concerning the Pomeranchuk phenomenon. On the one hand, the Pomeranchukon is considered to be the highest-lying moving Regge pole in the complex J plane similar to the other Regge poles.[1] The other point of view is that the Pomeranchukon is to be treated as a unique diffraction phenomenon,[2,3] distinct from the ordinary, lower-ranking Regge singularities. This dichotomy of opinion has as its basis a wide range of empirical and theoretical ideas.[4]

Experimentally, the nature of the Pomeranchukon has been investigated in the behavior of the total and elastic cross sections. The predictions for inclusive experiments such as the single-particle spectrum and average multiplicity have been made by using multiperipheral models[5,6] in which the Pomeranchukon is an ordinary Regge trajectory.[7] In order to use these experiments to learn about the nature of the Pomeranchukon, it would be valuable to have the predictions of a diffractive Pomeranchukon model for comparison. In this paper, we develop such a model, in order to find contrasting theoretical predictions and thereby to help unravel the nature of the Pomeranchuk singularity.

In this model, we unite the appealing physical ideas of constant cross sections at high energies as given by a diffractive Pomeranchukon with the multiperipheral models for particle production and the determination of Regge singularities by a bootstrap. To do this in a multiperipheral production amplitude, we consider the Pomeranchukon as a unique diffractive phenomenon or Regge singularity at $J=1$ which is assumed to occur at most once in the chain. The multiperipheral amplitudes containing the Pomeranchukon exchange will then generate an output Pomeranchukon singularity through unitarity which will be self-consistent if the Pomeranchukon is a fixed pole at $J=1$.

Those production amplitudes not containing the Pomeranchukon exchange are considered to generate and possibly bootstrap the nonleading Regge singularities $(P', \rho, \omega, \ldots)$. The occurrence of a single Pomeranchukon in the chain can be interpreted as a diffractive scattering with resulting multiperipheral fragmentation of the beam and the target into particles on the left and the right of the Pomeranchukon, respectively. The coupling of a specific production model with the diffractive scattering enables us to use the results of multiperipheral dynamics to make definite predictions on production cross sections, multiplicity distributions, and inclusive experiments in a diffraction model. The resulting bootstrap equations allow us to predict the behavior of the residue function of the Pomeranchukon and the other lower-lying Regge singularities.

In the development of this particular model, we were strongly motivated to find a solution giving rise to a constant total cross section which would still be consistent with the multiperipheral bootstrap philosophy.[8] Presently existing bootstrap models can only accommodate cross sections which vanish as the incident energy becomes asymptotically large.[9,10] Nonvanishing asymptotic cross sections are compelling from models based on a classical picture of hadrons (as extended objects with many internal degrees of freedom). Further appeal can be drawn from the principle that the strong interactions be as strong as possible, consistent with unitarity.[11]

In this model, we consider a general multiperipheral production amplitude in which the Pomeranchuk singularity is assumed to be exchanged at most once,[2,3] unlike other Regge singularities. The motivation for this choice resulted from the desire to bootstrap a Pomeranchukon of intercept unity. Multiperipheral ladders with only meson trajectories exchanged yield output singularities dependent on coupling strengths and do not contain a regulating mechanism which would generate a Pomeranchukon with an intercept of exactly unity. Therefore, the Pomeranchukon must occur in the multiperipheral production amplitude, and for a bootstrap it must have an input intercept $\alpha_{\text{in}}(0)=1$. Now, it is well known that the exchange of an arbitrary number of Pomeranchukons of intercept unity in a multiperipheral production amplitude gives rise to a cross section which violates the Froissart bound.[12] Furthermore, the exchange of more than one Pomeranchukon gives rise to extra $\ln s$ factors in the total cross section which preclude the possibility of a bootstrap. Therefore, we limit ourselves to having only one Pomeranchukon exchanged in the production amplitude.

If the Pomeranchukon is considered a moving trajectory, the output of a single exchange is a total cross section $\sigma \propto (\ln s)^{-1}$, which prevents a bootstrap solution based on the input assumption of a constant total cross section $[\alpha_{\text{in}}(0)=1]$. We will show below that a Pomeranchukon which is a fixed pole at $J=1$ does bootstrap itself, and this solution is taken as the basis of our model. Thus, at asymptotic energies, the elastic scattering amplitude has the form $T(s,t)=is\beta(t)$.

Since the multiperipheral bootstrap model is an attempt to incorporate multiparticle intermediate states into s-channel unitarity, we are not constrained by the fact that a fixed pole at $J=1$ violates t-channel elastic unitarity if continued to the first t-channel threshold.[13] In fact, fixed poles are allowed if "protective moving cuts" are present.[14,15]

In the usual multiperipheral models, a principle of factorization along the chain is used which demands that the Pomeranchukon be allowed to appear many times just as the other singularities. This multiple occurrence of the Pomeranchukon is motivated by field-theory concepts of allowing indefinitely repeated substructures. The exchange of a Pomeranchukon only once would be a very nonlocal interaction in field theory. It may be consistent with a pure S-matrix philosophy since one need only specify a particular amplitude for particle production. It is the bootstrap requirements that determine whether one has made the correct choice. Through the use of s-channel unitarity, we find that this is a bootstrap solution.

In this model, the lower-ranking Regge singularities are considered to generate themselves in a consistent bootstrap fashion. They do *not* build up the Pomeranchukon component. Only those terms containing the Pomeranchukon exchange build up the Pomeranchukon component. Analysis of multiperipheral models of the Amati, Fubini, and Stanghellini[5] (AFS) type indicate that the kernel of the resulting integral equation has insufficient strength to generate a high-lying singularity near $J=1$. In these models, the major components of the kernel are the low-energy resonances to the π-π channel and the resulting eigenvalue equations indicate that a singularity near $J\cong 0.33$ is produced.[16] Such a singularity can be interpreted as one of the lower-ranking meson Regge poles (P'). Furthermore, investigations of multi-Regge models have shown that meson exchanges are capable of bootstrapping themselves while still giving reasonable values for the parameters of the model.[17]

Asymptotically, then, the unitarity equation for the leading vacuum singularity is represented pictorially in Figs. 1(a) and 1(b). In this scheme, the multiperipheral fragmentation adjacent to the Pomeranchuk exchange build up the Regge behavior corresponding to the P' exchange. Hence, in this model, there is no triple-Pomeranchuk coupling.[18,19] In Fig. 1(c), we illustrate how non-Pomeranchukon exchanges generate the nonleading Regge behavior.

The absorptive part of the elastic scattering amplitude has a resulting two-component form. These two components are the contributions from those terms having only mesons exchanged and those which have one Pomeranchukon exchanged in the multiperipheral chain. In our model, there is a one-to-one correspondence between these two components and the meson and Pomeranchukon exchanges generated by unitarity in the elastic

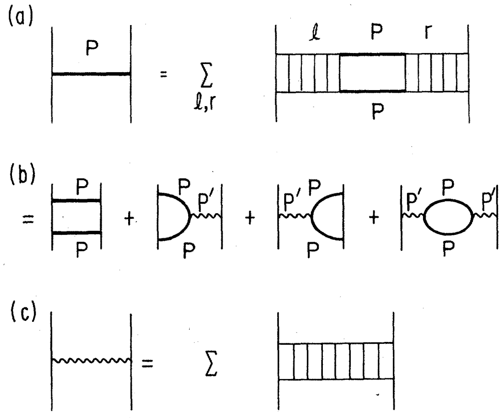

FIG. 1. (a) and (b) Asymptotic unitarity equations for the diffractive Pomeranchuk pole; (c) Unitarity equation for ordinary Regge behavior.

absorptive part. This one-to-one correspondence is consistent with the hypothesis of duality as formulated by Harari.[20]

Many of the results of the fixed-pole Pomeranchukon model differ markedly from those of the usual multiperipheral models. The results dependent on the nonleading singularities, however, [Fig. 1(c)] are the same as those in the usual multiperipheral models. The asymptotic results are that all cross sections approach constants and that the elastic differential cross sections are functions only of t, the square of the invariant momentum transfer. The single-particle spectrum obeys the hypothesis of limiting fragmentation[21] or scaling,[22] but does not have the pionization component. The average multiplicity approaches a constant at high energies, and in a specialized model the distribution of multiplicities is close to being geometric. The usual multiperipheral models with multi-Pomeranchukon exchange (MPE), as well as the field-theory-based parton models, all predict pionization and that the average multiplicities increase logarithmically with energy. This contrast with the properties of the single-Pomeranchukon-exchange model presented here gives increased importance to the experiments on single-particle distributions and multiplicity distributions, since they can now be used to determine the nature of the Pomeranchuk singularity.[23]

The momentum-transfer aspects of our bootstrap are summarized by a nonlinear integral equation for the Pomeranchukon residue function $\beta(t)$. This integral equation can be solved to exhibit the exponential behavior of $\beta(t)$ for small values of t, and to relate the slope of the diffraction peak to the vanishing of the absorptive part of the P'-trajectory residue function at the ghost-eliminating point $\alpha_{P'}(t_0)=0$.

We first demonstrate the diffractive Pomeranchukon bootstrap and derive the results which follow from single-Pomeranchukon exchange and the general properties of the multiperipheral model. The fixed-pole Pomeranchukon bootstrap is given in Sec. II. The results for the cross sections are derived in Sec. III, while the properties of scaling, pionization, and average multiplicity are presented in Sec. IV. After this general discussion, we make some more detailed assumptions and approximations in order to derive more specific results. In Sec. V we derive the distribution of multiplicities under the strong-ordering multiperipheral assumption, while in Sec. VI we present and discuss the nonlinear bootstrap equation for the Pomeranchukon residue function. In the concluding Sec. VII we contrast the diffractive Pomeranchukon model with the results of the MPE models and discuss the important experiments for deciding between these models.

II. POMERANCHUKON BOOTSTRAP

Within the context of a multiperipheral model for particle production, it has been recognized that if a Regge trajectory of intercept unity is allowed to be exchanged an arbitrary number of times, then the total cross section will violate the unitarity bound.[12,24] In addition, if a moving Pomeranchukon with $\alpha_P(0)=1$ is exchanged n times in the multiperipheral chain, then the resulting cross section behaves asymptotically as

$$\sigma \sim [\ln(\ln s)]^{n-1}/\ln s\,, \tag{2.1}$$

indicative of a complicated cut at $J=1$, so that even if we restrict the Pomeranchukon to be exchanged a finite number of times, the behavior of the cross section is incompatible with the implicit input assumption of a constant total cross section, or with generating an output pole with intercept unity. For these reasons, multiperipheral bootstrap models which employ the unitarity condition as a basic ingredient have not succeeded in generating a self-consistent Pomeranchuk singularity of intercept unity.[9,10]

We choose to avoid these difficulties and accomplish a self-consistent bootstrap of the Pomeranchuk singularity as a Regge pole of intercept unity, by considering the Pomeranchukon as a diffractive phenomenon which can only be exchanged once in the multiperipheral chain. However, even with this restriction, we find that if the Pomeranchukon is a moving pole with $\alpha_P(0)=1$ the total cross section behaves asymptotically as $(\ln s)^{-1}$. This behavior is just that of the well-known AFS cut[5] at $\alpha_c(0)=2\alpha_P(0)-1$, and reflects the lack of consistency between the input and output solutions. When we impose bootstrap consistency, we find that the Pomeranchukon is simply a fixed pole at $J=1$.

In order to demonstrate that a fixed-pole diffractive Pomeranchukon bootstraps itself, let us now consider specific contributions to the absorptive part of the elastic scattering amplitude, $A(s,t)$, at high energies. The unitarity condition tells us that the n-particle-production contribution is given by

$$A_n(s,t)=\tfrac{1}{2}\int d\phi_n T_{in}T^*_{nf}\delta^4(P_a+P_b-P'_a-P'_b)\,, \qquad (2.2)$$

where T_{in} is the matrix element for the process $a+b\to n$, and where the differential volume element of phase space is

$$d\phi_n=(2\pi)^{4-3n}\delta^4\Big(\sum_i k_i-P_a-P_b\Big)\prod_{i=1}^{n}d^4k_i\,\delta^+(k_i^{\,2}-m_i^{\,2})\,. \qquad (2.3)$$

Here P_a, P_b, P'_a, P'_b are the four-momenta of the initial and final hadrons a and b, while the k_i refer to the four-momenta of the intermediate-state particles. The invariant energy squared and momentum transfer squared are given by

$$s=(P_a+P_b)^2,\quad t=(P'_a-P_a)^2\,. \qquad (2.4)$$

As a dynamical assumption, we take a multiperipheral model for our production amplitudes with simple factorization properties along the chain and damping in the momentum-transfer variables. In a multi-Regge model, we would distinguish two major components of the multiperipheral chain. The first is some average meson Regge trajectory $(P',\rho,\omega,\ldots)$ and represents the bulk of the multiperipheral chain. The second component is the Pomeranchuk singularity, which we specify to appear at most once in the chain, and is a fixed pole at $J=1$. In an AFS-type multiperipheral model, the first component would be replaced by the elementary-pion exchanges forming the links of the chain, whereas the rungs of the ladder would be represented by the low-energy resonance component of π-π scattering.

Our production amplitudes are of two types: those which contain the Pomeranchukon exchange and those which do not. We hypothesize that those multiperipheral amplitudes which contain *only* ordinary Regge exchanges (or low-energy π-π resonances in the case of an AFS model) build up through unitarity only the ordinary Regge exchanges in the elastic absorptive part. The sum of all the multiperipheral graphs with only ordinary Regge exchanges gives, therefore, the following contribution to the absorptive part for $t\le 0$:

$$A^M(s,t)=\beta_M(t)s^{\alpha_M(t)},\quad \alpha_M(t)<1 \qquad (2.5)$$

while the n-particle production contribution is also typically power-behaved in s, up to logarithmic factors. Since $\alpha_M(t)<1$, these contributions represent only finite-energy corrections to the dominant Pomeranchukon component.

On the other hand, those multiperipheral amplitudes in which the Pomeranchukon is exchanged give the dominant contributions to the absorptive part at high energies, and we shall proceed now to analyze them.[25] We distinguish those terms where the Pomeranchukon is adjacent to an end particle from those terms where the Pomeranchukon is exchanged in the central portion of the multiperipheral chain. Consider, first, the central diagrams. Let l particles be produced to the left of the Pomeranchukon exchange and $r=n-l$ particles to the right (see Fig. 2). Its contribution to $A_n(s,t)$ is given by

$$A_n^{l,r}(s,t)=\frac{(2\pi)^{4-3n}}{2}\int\prod_{i=1}^{n}d^4k_i\,\delta^+(k_i^{\,2}-m_i^{\,2})\,\delta^4\Big(\sum_{i=1}^{n}k_i-P_a-P_b\Big)T_l(P_a;k_1,\ldots,k_l)T_l^*(P'_a;k_1,\ldots,k_l)\beta(t_+)\beta(t_-)\Sigma^2$$
$$\times T_r(P_b;k_{l+1},\ldots,k_n)T_r^*(P'_b;k_{l+1},\ldots,k_n)\,\delta^4(P'_a+P'_b-P_a-P_b)\,, \qquad (2.6)$$

where

$$t_+=\Big(P'_a-\sum_{i=1}^{l}k_i\Big)^2,\quad t_-=\Big(P_a-\sum_{i=1}^{l}k_i\Big)^2,$$

and where we have explicity displayed the factorization property of our multiperipheral amplitude in its dependence on the momenta of the left- and right-hand particles. $\beta(t_\pm)$ are the Pomeranchukon residue functions and Σ^2 is the square of the Regge propagator for the fixed Pomeranchukon pole, where $\Sigma=(k_l+k_{l+1})^2$.

By introducing

$$\int d^4K\,d^4K'\,\delta^4\Big(K-\sum_{i=1}^{l}k_i\Big)\delta^4\Big(K'-\sum_{i=l+1}^{n}k_i\Big) \qquad (2.7)$$

we can write

$$A_n^{l,r}(s,t)=\frac{2}{(2\pi)^4}\int d^4K\,d^4K'\,\delta^4(K+K'-P_a-P_b)\beta(t_+)\beta(t_-)s_0{}^4A_l^{P'}(s_l,t)\left(\frac{s}{s_ls_r}\right)^2A_r^{P'}(s_r,t)\,, \tag{2.8}$$

where we have defined

$$A_l^{P'}(s_l,t)=\frac{(2\pi)^{4-3l}}{2}\int\prod_{i=1}^{l}d^4k_i\,\delta^+(k_i{}^2-m_i{}^2)\delta^4\Big(K-\sum_{i=1}^{l}k_i\Big)\,T_l(P_a;k_1,\ldots,k_l)T_l^*(P_a';k_1,\ldots,k_l)\,, \tag{2.9}$$

with a similar expression for $A_r^{P'}(s_r,t)$. $s_l=K^2$ and $s_r=K^2$ and $s_r=K'^2$ are the invariant squared masses of the particles produced to the left and right of the Pomeranchukon exchange, respectively, and we have taken the result of multiperipheral kinematics to replace Σ by[26] $s_0{}^2s/s_ls_r$ as the Pomeranchukon propagator. The superscript P' on $A_l^{P'}(s_l,t)$ denotes the fact that vacuum quantum numbers are being projected in the t channel. The functions $A_l^{P'}$ and $A_r^{P'}$ also have, in general, an off-mass-shell dependence in t_+ and t_-. The Pomeranchukon residue functions $\beta(t_\pm)$ are damped in t_+ and t_- and hence limit the important regions of these variables to be small.

To calculate the contributions of these diagrams to the total absorptive part $A(s,t)$, we sum over l and r,

$$A^{\text{central}}(s,t)=\sum_{n=4}^{\infty}A_n(s,t)=\sum_{n=4}^{\infty}\sum_{l=2}^{n-2}A_n^{l,n-l}(s,t) \tag{2.10}$$

or

$$A^{\text{central}}(s,t)=\sum_{l=2}^{\infty}\sum_{r=2}^{\infty}A_{l+r}^{l,r}(s,t)\,. \tag{2.11}$$

After transforming the integrations to invariant variables, we have

$$A^{\text{central}}(s,t)=\frac{s}{(2\pi)^4}\int\frac{dt_+dt_-\beta(t_+)\beta(t_-)}{[-\Delta(t,t_+,t_-)]^{1/2}}\int\frac{ds_lds_r}{s_l{}^2s_r{}^2}\,s_0{}^4A^{P'}(s_l,t)A^{P'}(s_r,t)\,, \tag{2.12}$$

where

$$A^{P'}(s_l,t)=\sum_{l'=2}^{\infty}A_{l'}^{P'}(s_l,t)\,, \tag{2.13}$$

with

$$\Delta(t,t_+,t_-)=t^2+t_+{}^2+t_-{}^2-2t(t_++t_-)-2t_+t_- \tag{2.14}$$

defining the boundary of integration to be $\Delta(t,t_+,t_-)\leq 0$. The factors $\beta(t_+)$ and $\beta(t_-)$ provide sufficiently rapid damping in t_+ and t_- so that the integrations over these variables may be performed, yielding a finite result. Any s dependence of the remaining integrals lies implicit in the upper limits of integration on s_l and s_r, which grows with s, roughly as $s_ls_r<ss_0$. To investigate this possibility, we can choose the asymptotic form of $A^{P'}(s_{l,r},t)$, which by hypothesis had Regge behavior in the energy variable [Eq. (2.5)]. So as far as the energy dependence is concerned, we have

$$A^{\text{central}}(s,t)\sim s\int\frac{ds_l}{s_l{}^{2-\alpha_{P'}(t)}}\;\frac{ds_r}{s_r{}^{2-\alpha_{P'}(t)}}f(t)\,. \tag{2.15}$$

Now since $\alpha_{P'}(t)<1$, the integrand is sufficiently damped in these variables to render the result independent of the upper limit of integration,

$$A^{\text{central}}(s,t)\sim s\tilde{f}(t)\,, \tag{2.16}$$

so that for all values of t the energy dependence of this contribution to the absorptive part is just that of a fixed pole at $J=1$, consistent with our input assumption.

Other contributions to $A(s,t)$ exist as well. These are the contributions of the end diagrams and the elastic scattering term (Pomeranchukon adjacent to both end particles). We can calculate a typical end-diagram contribution by merely replacing one of the absorptive parts $A^{P'}(s_l,t)$ in Eq. (2.12) by $\pi\delta(s_l-m_a{}^2)$. A similar analysis follows and we find the energy dependence of these

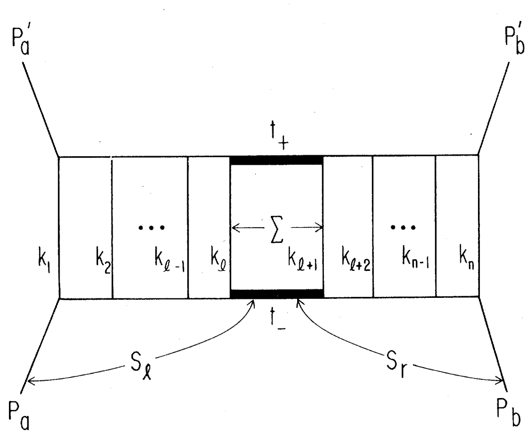

FIG. 2. Kinematical variables for the central Pomeranchukon contribution to unitarity.

contributions is simply

$$A^{\text{end}}(s,t) \sim s\int \frac{ds_r f'(t)}{s_r^{\,2-\alpha_{P'}(t)}} \sim s f''(t)\,, \tag{2.17}$$

which is also that of a fixed pole at $J=1$. The elastic scattering contribution is obtained by replacing both absorptive parts in Eq. (2.12) by $\pi\delta(s_{l,r}-m_{a,b}^{\,2})$ and trivially has the energy dependence of a fixed pole,

$$A^{\text{elastic}}(s,t) \sim s\tilde{f}(t)\,. \tag{2.18}$$

This completes the proof of the fixed-pole Pomeranchukon bootstrap. In the following section, we consider the behavior of the various cross sections in this model.

III. BEHAVIOR OF CROSS SECTIONS

Let us now consider the contribution of the fragmentation processes discussed in Sec. II to various partial cross sections.

The optical theorem gives us the relation between the absorptive part of the elastic scattering amplitude, at $t=0$, and the total cross section. In our normalization, and at high energies, it becomes

$$\sigma_T(s) = \sum_n \sigma_n(s) = \frac{1}{s}\sum_n A_n(s,0)\,. \tag{3.1}$$

The elastic differential cross section is given by

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{1}{16\pi s^2}\,|T(s,t)|^2\,, \tag{3.2}$$

where $T(s,t)$ is the elastic scattering amplitude. The fixed Pomeranchukon pole gives the form

$$T(s,t) = iA(s,t) = is\beta(t) \tag{3.3}$$

at high energy, and yields an energy-independent elastic differential cross section whose integral is finite,

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{1}{16\pi}\,\beta^2(t)\,. \tag{3.4}$$

From Eq. (2.12), we determine the differential cross section for double diffraction dissociation to be given by

$$\frac{d\sigma}{ds_l\,ds_r\,dt} = s_0^{\,4}\,\frac{\beta^2(t)}{16\pi^3}\,\frac{A^{P'}(s_l,0;t)}{s_l^{\,2}}\,\frac{A^{P'}(s_r,0;t)}{s_r^{\,2}}\,, \tag{3.5}$$

where, for the sake of completeness, we have explicitly denoted the off-mass-shell dependence of the fragmentation absorptive parts $A^{P'}$ in $t_+=t_-=t$. Note that it is a limiting distribution, independent of the incident energy. The corresponding differential cross section for single dissociation is simply

$$\frac{d\sigma}{ds_r\,dt} = s_0^{\,2}\,\frac{\beta^2(t)}{16\pi^2}\,\frac{A^{P'}(s_r,0;t)}{s_r^{\,2}} \tag{3.6}$$

and is in principle a simply measurable experimental quantity. Note that because of the simple factorization properties of the Pomeranchuk singularity, the double-dissociation cross section, Eq. (3.5), can be obtained from the single-dissociation cross section, Eq. (3.6), for the case of identical incident hadrons.

On the basis of our previous analysis in Sec. II, we find that the fragmentation of the incident hadrons builds up a total inelastic cross section which is constant at high energies,

$$\sigma_{\text{inel}}(s) = \frac{1}{s}\sum_{n\geq 3} A_n(s,0) = \text{const}\,. \tag{3.7}$$

The crucial points in obtaining these results were

(a) that there is sufficient momentum-transfer damping as provided by the Pomeranchukon residue functions $\beta(t)$ so that the integration over this variable gives a finite result, and

(b) that the large-subenergy behavior of the fragmentation absorptive parts, $A^{P'}(s,0;t)$, is such as to make the integrals over these variables in Eqs. (2.12) and (2.17) independent of the upper limits and hence finite.

Now from Eq. (2.9) we see that each term in the sum for $A^{P'}(s,0)$ in Eq. (2.13) is a positive definite quantity. Therefore each term $A_l^{P'}(s,0)$ in the sum must share the above property (b). Consequently, when we calculate a typical cross section for n-particle production from Eq. (2.8), we find that it too gives a finite result. In this way, we find that all partial cross sections are constant at high energies,

$$\sigma_n(s) = \text{const}\,, \tag{3.8}$$

and they sum to give a constant total cross section. In this model, the cross section was composed of the diffractive fragmentation of the initial hadrons. Later, we shall explore in greater detail the predictions which follow from a more specific model for this fragmentation.

IV. BEHAVIOR OF INCLUSIVE EXPERIMENTS AND AVERAGE MULTIPLICITY

A. Multiperipheral Fragmentation

One of the main purposes of presenting this model is to show that all multiperipheral models do not have the same properties for inclusive experiments and that these experiments can be important in distinguishing between models. In this section, we discuss the results of the diffractive Pomeranchukon model for the single-particle spectrum in regard to its scaling behavior[21,22] and also show that there is no pionization component at high energies. As a consequence, the average multiplicity is asymptotically finite.

B. Proof of Limiting Fragmentation

We will now prove that the single-particle spectrum has a limiting distribution in this model. The proof for any other inclusive cross section will be a simple generalization of this. The essence of the proof lies in the dynamic decoupling of the target fragments from the projectile fragments provided by the factorization across the Pomeranchuk link, and in the convergence of integrals whose s-dependent limits become unimportant at large s. The resulting cross sections become independent of s and form a limiting distribution.

We begin by describing the general Pomeranchukon-exchange contribution as a factorized form of the left-hand (target) fragments k_i, $i=1,\ldots,l$; the Pomeranchukon exchange; and the right-hand (projectile) fragments k_j, $j=l+1,\ldots,n$, where $n=l+r$ (see Fig. 3). With phase space $d\phi_i=d^3k_i/2k_{i0}$, the single-particle spectrum for observing a particle of momentum $\vec{k}$ can be written as a unitarity summation over all additionally produced particles on the chain and is given by

$$d\sigma\bigg/\left(\frac{d^3k}{2k_0}\right)=\frac{1}{s}\sum_{l=3}^{\infty}\sum_{m=2}^{l-1}\int\prod_{i=1}^{m-1}d\phi_i\prod_{i=m+1}^{l}d\phi_i\,|T_l(k_1,\ldots,k,\ldots,k_l)|^2\Sigma^2\beta^2(Q^2)\times\sum_{r=2}^{\infty}\int\prod_{j=l+1}^{l+r}d\phi_j\,|T_r(k_{l+1},\ldots,k_n)|^2\delta^4\left(\sum_i k_i-P_a-P_b\right). \tag{4.1}$$

We add an integration over the Pomeranchukon-exchange loop,

$$\int d^4Q\,\delta^4\left(\sum_{j=l+1}^{n}k_j-P_b+Q\right), \tag{4.2}$$

and convert the integration to invariant variables corresponding to the squared fragmentation subenergies $s_l=(P_a+Q)^2$, $s_r=(P_b-Q)^2$, the squared momentum transfer Q^2, and the invariant $\mathbf{k}\cdot Q$ with Jacobian $s^{-1}J$. Furthermore, since the dynamics of our model give $s_l, s_r \ll s$, a detailed analysis of multiperipheral kinematics gives the following factorized form for Σ:

$$\Sigma=\frac{s}{s_l s_r}f(k_i)f(k_j), \tag{4.3}$$

where k_i and k_j refer, respectively, to momenta of target and projectile fragments. Integrating over the $d\phi$'s and summing over all particles then gives the functional form

$$d\sigma\bigg/\left(\frac{d^3k}{2k_0}\right)=\int\frac{ds_l\,ds_r\,d(k\cdot Q)\,dQ^2}{s_l^2 s_r^2}J\beta^2(Q^2)\times F_a(s_l,P_a\cdot k,k\cdot Q,Q^2)A_b^{P'}(s_r,0;Q^2), \tag{4.4}$$

where $A_b^{P'}(s_r,0;Q^2)\sim\gamma_b^{P'}s_r^{\alpha_{P'}(0)}$ is the fragmentation absorptive part from the unitarity sum on the right of the Pomeranchukon exchange. On the left-hand side, the sum $F_a(s_l,\ldots)$ is known, from the previous analysis of Silverman and Tan,[27] to behave as $s_l^{\alpha_{P'}(0)}$. The subscripts a and b denote that these functions depend in general on the nature of the target and projectile particles.

We now investigate the question of limiting fragmentation by showing that the asymptotic single-particle spectrum depends only on the momenta $k_\parallel$ and $k_\perp^2$ measured in the lab (target-particle a) frame, and is independent of the total energy squared s. Since $P_a\cdot\mathbf{k}=m_a k_0$, s is not present in the functions in Eq. (4.4). Any remaining s dependence for the single-particle spectrum lies implicitly in the definition of the upper limits of integration. However, since the integrals over s_l and s_r converge, they will be dominated by finite values at large s, in which case the Jacobian becomes

$$J=\frac{\Theta(H)}{2\sqrt{H}}, \tag{4.5}$$

$$H=-\left[\left(\frac{k_0-k_\parallel}{m_a}\right)(s_l-Q^2-m_a^2)-2\mathbf{k}\cdot Q\right]^2-4Q^2k_\perp^2.$$

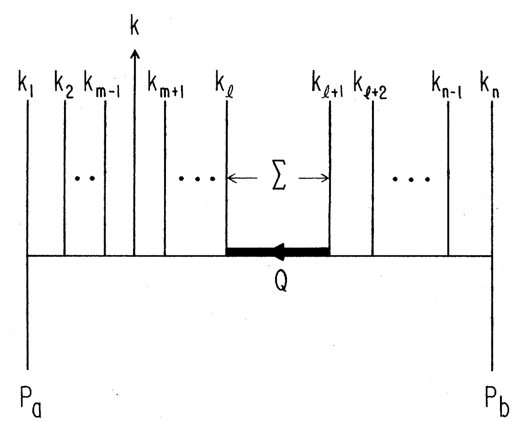

FIG. 3. Diagram for the calculation of the single-particle spectrum for target fragments.

This Jacobian shows that the $k \cdot Q$ integration in Eq. (4.4) is also limited, independent of s. After integrating over Q^2, $k \cdot Q$, s_l, and s_r, we find that the single-particle spectrum will depend only on $k_\parallel$ and $k_\perp^2$ and will be independent of s. It is therefor a limiting distribution.

C. Absence of Pionization

We can easily calculate in this model the behavior of the single-particle spectrum for increasing longitudinal momentum, $k_\parallel$, in the lab system. This is effectively determined by the lower limit of the s_l integration in Eq. (4.4), which is a function of $k_\parallel$. To see this, we use the kinematical relation

$$s_l = (P_a + Q)^2 = -m_a^2 + Q^2 + 2m_a E \,, \tag{4.6}$$

where E is the total energy of the target fragments in the lab frame. Since it is a sum of individual particle energies, which are positive definite quantities, we have the inequality

$$E \geq k_0 \,. \tag{4.7}$$

Consequently, for large $k_\parallel$ in the lab frame, the effective lower limit on the s_l integration becomes

$$s_l \geq 2m_a k_\parallel \,. \tag{4.8}$$

Hence

$$d\sigma \bigg/ \left(\frac{d^3k}{2k_0}\right) \propto \int_{2m_a k_\parallel} \frac{ds_l}{s_l^{\,2-\alpha_{P'}(0)}} \propto k_\parallel^{\,\alpha_{P'}(0)-1} \,. \tag{4.9}$$

In the lab frame, the occurrence of pionization as predicted by the usual multiperipheral models[5,27-30] is such that the limiting distribution of the single-particle spectrum becomes a constant for large k_0 or $k_\parallel$. Instead, the diffractive Pomeranchukon model has a limiting distribution which vanishes for large $k_\parallel$, and therefore does not have pionization.

Another way to see this is the following. An equivalent definition of pionization is that the single-particle spectrum as measured in the center-of-mass system as a function of the scaled variable $x = 2k_\parallel^{\text{c.m.}} s^{-1/2}$ and $k_\perp^2$ has a finite, nonzero limit as $x \to 0$. That is,

$$\lim_{x\to 0} \sigma_T^{-1}\, d\sigma \bigg/ \left(\frac{d^3k}{2k_0}\right) = \lim_{x\to 0} f(x, k_\perp^2) = f(k_\perp^2) \,. \tag{4.10}$$

Previous studies[5,27-31] of the single-particle spectrum have been made of the diagram in Fig. 4, where A_l and A_r are forward absorptive parts. They have shown that if we look near $k_\parallel^{\text{c.m.}} \approx O(1)$, so that $x = 0$, then $f(k_\perp^2)$ will be finite and nonzero only if a Pomeranchukon is present in *both* absorptive parts [Fig. 5(a)]. However, in this model we exchange a fixed-pole Pomeranchukon only once. Therefore, only one of the absorptive parts (A_l or A_r) can be Pomeranchukon-dominated, and as a result there is no pionization.

At present accelerator energies, there will be contributions at $x = 0$ (slow particles in the c.m. system) from the diagrams which have Pomeranchukon exchange in one absorptive part and $P'(f_0)$ in the other [see Fig. 5(b)], which die out as

$$d\sigma \bigg/ \left(\frac{d^3k}{2k_0}\right) \underset{x=0}{\sim} \frac{(s^{1/2})^{\alpha_{P'}(0)}(s^{1/2})}{s} \sim (s^{1/2})^{\alpha_{P'}(0)-1} \,, \tag{4.11}$$

since at $x = 0$ both absorptive parts receive a subenergy $\propto s^{1/2}$ (see Refs. 27, 28, and 31 for details). This corresponds to the tail of the target (projectile) fragmentation at $k_\parallel \sim O(s^{1/2})$ as measured in the target (projectile) frame given by Eq. (4.9). The diffractive Pomeranchukon model

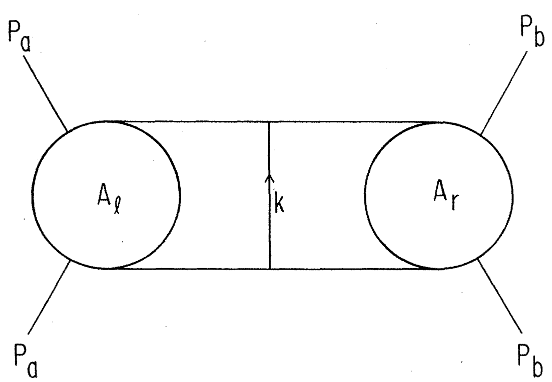

FIG. 4. Diagram for the single-particle spectrum in terms of forward absorptive parts.

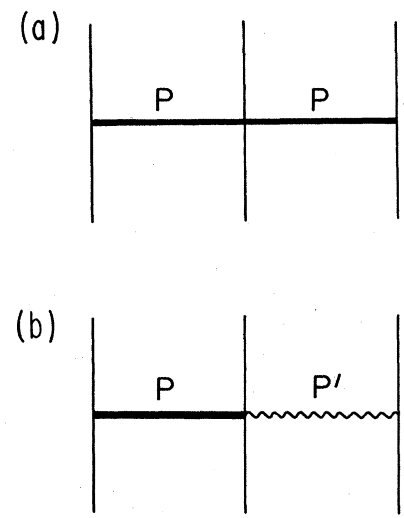

FIG. 5. (a) Diagram for pionization in the MPE model; (b) Diagram for single-particle spectrum for $k_\parallel \sim O(s^{1/2})$ in the diffractive Pomeranchukon model.

predicts that the current observation of these slow particles in the c.m. system ($x=0$) will fall off[32] slowly as $\sim s^{-1/4}$.

The asymptotic relation between the lab momentum, $k_{\parallel}$, and the scaled variable x, for $x \geq O(s^{-1/2})$, is given by

$$k_{\parallel}=\frac{-m_a|x|}{2}+\frac{k_\perp{}^2+\mu^2}{2m_a|x|}, \tag{4.12}$$

where m_a is the mass of the target particle and μ is the mass of the observed particle. From Eq. (4.9), we then calculate the asymptotic behavior of the single-particle spectrum $f(x, k_\perp{}^2)$ near $x \approx 0$ to be

$$\left(\frac{-m_a|x|}{2}+\frac{k_\perp{}^2+\mu^2}{2m_a|x|}\right)^{\alpha_{P'}(0)-1} \sim |x|^{1-\alpha_{P'}(0)}. \tag{4.13}$$

D. Average Multiplicity

The average multiplicity may be computed from the integral over the single-particle spectrum as

$$\bar{n}\sigma_{\text{inel}}=\int d\sigma\Big/\left(\frac{d^3k}{2k_0}\right)\frac{d^3k}{2k_0}. \tag{4.14}$$

Treating the target and projectile fragmentations independently, they each contribute at large $k_{\parallel}$ [see Eq. (4.9)]

$$\bar{n} \propto \int_{O(1)}^{s/2m} k_{\parallel}{}^{\alpha_{P'}(0)-1}\frac{dk_{\parallel}}{k_{\parallel}}\int f(k_\perp{}^2)dk_\perp{}^2+\text{const}. \tag{4.15}$$

The integration over $k_\perp{}^2$ converges because of the rapid damping in this variable. The integral over $k_{\parallel}$ also converges and is independent of s as $s\to\infty$, so that the average multiplicity becomes a constant. If, on the other hand, pionization were present, the result would have been a logarithmically increasing average multiplicity, since

$$\bar{n} \propto \int^{s/2m}\frac{dk_{\parallel}}{k_{\parallel}} \sim \ln s. \tag{4.16}$$

V. MULTI-REGGE MODEL OF FRAGMENTATION

The distribution of multiplicities (or the production cross sections σ_n) can be determined from the diffractive Pomeranchukon model once the dynamical distribution of the fragmentation absorptive parts $A_n^{P'}$ is specified. As an illustration to compare with MPE results, we will use the simple Chew-Pignotti multi-Regge model[8] to compute the fragmentation $A_n^{P'}$.

Again, we decompose the cross sections into three main pieces: the elastic scattering part, and, for the inelastic cross sections, the end-diagram and central-diagram contribution, respectively. We shall treat each case in turn. For the elastic cross section, we have from Eq. (3.4) [taking the form $T_{ab}(s,t)=is\beta_{ab}(t)$]

$$\begin{aligned}\sigma_{\text{el},ab}(s)&=\frac{1}{16\pi s^2}\int dt\, s^2|\beta_{ab}(t)|^2\\&=\frac{1}{16\pi}\int dt\,|\beta_{ab}(t)|^2\\&\equiv C_{ab}=\text{const},\end{aligned} \tag{5.1}$$

where the subscripts a, b refer to the initial hadrons participating in the reaction.

To calculate the end-diagram contribution to the cross section to produce n particles, σ_n, we take for $A_{n-1,b}^{P'}(s_r)$ the form given by the Chew-Pignotti model of multi-Regge exchange of an effective meson trajectory M, which in our normalization is given by

$$A_{n-1,b}^{P'}(s_r)=\frac{\pi}{s_0}\left(\frac{s_r}{s_0}\right)^{2\bar{\alpha}_M-1}\frac{[g^2\ln(s_r/s_0)]^{n-1}}{(n-1)!}G_{Mb}{}^2, \tag{5.2}$$

where $\bar{\alpha}_M$ is the averaged or effective meson Regge pole and b identifies the participating hadron. g is the coupling strength of the two-Reggeon-particle vertex, and G_{Mb} refers to the coupling of the meson Regge exchange to the external hadron b. Then the contribution of both end diagrams to σ_n at large s is

$$\begin{aligned}\sigma_{n,ab}^{\text{end}}(s)=&\frac{1}{16\pi s^2}\int_{-\infty}^{0}dt\int_{s_{\text{th}}}^{s}ds_r|\beta_{aM}(t)|^2\Sigma^2\\&\times\frac{1}{s_0}\left(\frac{s_r}{s_0}\right)^{2\bar{\alpha}_M-1}\frac{[g^2\ln(s_r/s_0)]^{n-1}}{(n-1)!}G_{Mb}{}^2\\&+(a\to b),\end{aligned} \tag{5.3}$$

where Σ is the fixed-pole Pomeranchukon Regge behavior and $\beta_{aM}(t)$ is the Pomeranchukon residue for coupling to the effective meson trajectory M.

For simplicity, we let $s_0=s_{\text{th}}\approx 1\ \text{GeV}^2$. Taking the strong-ordering limit $\Sigma\approx(s/s_r)s_0$, substituting $u=\ln(s_r/s_{\text{th}})$, and integrating over t, we obtain

$$\sigma_{n,ab}^{\text{end}}=C_{aM}\frac{(g^2)^{n-1}}{(n-1)!}G_{Mb}{}^2\int_0^{\ln s}u^{n-1}e^{-2(1-\bar{\alpha}_M)u}du+(a\to b). \tag{5.4}$$

As s, the total energy squared, approaches infinity, this becomes a constant,

$$\sigma_{n,ab}^{\text{end}}=C_{aM}\frac{G_{Mb}{}^2}{g^2}r^n+(a\to b), \tag{5.5}$$

with

$$r=\frac{g^2}{(2-2\bar{\alpha}_M)}=\frac{g^2}{[1+g^2-\alpha_{P'}(0)]}<1. \tag{5.6}$$

Here we have used the Chew-Pignotti bootstrap equation

$$\alpha_{P'}(0)=2\bar{\alpha}_M-1+g^2\,, \tag{5.7}$$

so that in the strong-ordering limit we predict that the contribution of the end diagram will form a "geometric series."[33] This series can be summed to yield the contribution of the end diagrams to the total inelastic cross section,

$$\sigma^{\text{end}}_{\text{inel},ab}=\sum_{n=1}^{\infty}C_{aM}\frac{G_{Mb}^{\;2}}{g^2}r^n+(a\to b)$$

$$=C_{aM}\frac{G_{Mb}^{\;2}}{g^2}\left(\frac{r}{1-r}\right)+(a\to b)\,. \tag{5.8}$$

Now let us evaluate the central-diagram contributions. In this case the Pomeranchukon couples only in the internal portion of the multiperipheral chain, so that it gives contributions to σ_n only for $n\geq 2$. As before, we employ the Chew-Pignotti model to obtain

$$\sigma^{\text{central}}_{n,ab}(s)=\frac{1}{16\pi s^2}\sum_{m=1}^{n-1}\int_{-\infty}^{0}dt\int_{s_{\text{th}}}^{s}\frac{ds_r}{s_0}\int_{s_{\text{th}}}^{s/s_r}\frac{ds_l}{s_0}\,G_{aM}^{\;2}\left(\frac{s_r}{s_0}\right)^{2\bar{\alpha}_M-1}\frac{[g^2\ln(s_r/s_0)]^{m-1}}{(m-1)!}\,|\beta_{MM}(t)|^2\Sigma^2$$
$$\times\left(\frac{s_l}{s_0}\right)^{2\bar{\alpha}_M-1}\frac{[g^2\ln(s_l/s_0)]^{n-m-1}}{(n-m-1)!}G_{Mb}^{\;2}\,, \tag{5.9}$$

where in the strong-ordering limit $\Sigma\sim(s/s_rs_l)s_0^{\,2}$. If we set $s_0=s_{\text{th}}\sim 1\text{ GeV}^2$, perform the integration over t, and substitute $u_r=\ln(s_r/s_0)$, $u_l=\ln(s_l/s_0)$, we get

$$\sigma^{\text{central}}_{n,ab}(s)=\frac{G_{aM}^{\;2}C_{MM}(g^2)^{n-2}G_{Mb}^{\;2}}{(n-2)!}\int_0^{\ln s}du_r\int_0^{\ln s-u_r}du_l(u_r+u_l)^{n-2}e^{-2(1-\bar{\alpha}_M)(u_r+u_l)}\,. \tag{5.10}$$

As s increases, the contributions approach constants,

$$\sigma^{\text{central}}_{n,ab}(s)=C_{MM}\frac{G_{aM}^{\;2}}{g^2}\,\frac{G_{Mb}^{\;2}}{g^2}(n-1)r^n\,, \tag{5.11}$$

where r is defined by Eq. (5.6). Furthermore, we can sum this series to obtain the contribution of the central diagrams to the total inelastic cross section,

$$\sigma^{\text{central}}_{\text{inel},ab}=\sum_{n=2}^{\infty}\sigma^{\text{central}}_{n,ab}=C_{MM}\frac{G_{aM}^{\;2}}{g^2}\,\frac{G_{Mb}^{\;2}}{g^2}\left(\frac{r}{1-r}\right)^2. \tag{5.12}$$

As an application, we will study the multiplicity distribution in pp collisions by assuming the produced particles to be ρ mesons which subsequently decay into pion pairs. We further make the simplification that $G_{pM}^{\;2}=g^2$ and $C_{MM}=C_{Mp}=C_{pp}$. Summing the central- and end-diagram contributions [Eqs. (5.5) and (5.11)] for producing n ρ's, we obtain

$$\sigma_{n,pp}=C_{pp}(n+1)r^n\,, \tag{5.13}$$

an almost geometric multiplicity distribution.[33] In Fig. 6, we have fitted the multiplicity-distribution data from the Echo Lake experiment[34,35] by using Eq. (5.13). It should be noted, however, that the average multiplicity computed from these data gives evidence for $\bar{n}\sim\ln s$, in disagreement with this single-Pomeranchukon-exchange model.[23] The total inelastic and total cross sections are

$$\sigma_{\text{inel},pp}=\sum_{n=1}^{\infty}\sigma_{n,pp}=C_{pp}\frac{r(2-r)}{(1-r)^2}\,, \tag{5.14}$$

$$\sigma_{\text{T},pp}=\sigma_{\text{el},pp}+\sigma_{\text{inel},pp}=C_{pp}(1-r)^{-2}\,. \tag{5.15}$$

The average multiplicity of produced ρ's is then the constant

$$\bar{n}_\rho=(\sigma_{\text{inel},pp})^{-1}\sum_{n=1}^{\infty}n\sigma_{n,pp}=\frac{2}{(1-r)(2-r)}\,. \tag{5.16}$$

If we assume that two-thirds of the final pions are charged, then the average charged multiplicity will be

$$\bar{n}_{\text{ch}}=2+\tfrac{2}{3}\times 2\times\bar{n}_\rho=2+\frac{8}{3(1-r)(2-r)}\,. \tag{5.17}$$

Reasonable bootstrap parameters of $\bar{\alpha}_M=0.4$ and $g^2=0.7$, which generate $\alpha_{P'}(0)=0.5$ via Eq. (5.7), give the ratio $r=0.58$. The resulting multiplicity, $\bar{n}_{\text{ch}}=6.5$, is consistent with current cosmic-ray experimental data.[34] We can also calculate the ratio of elastic to total cross section, which is

$$\frac{\sigma_{\text{el},pp}}{\sigma_{\text{T},pp}}=(1-r)^2\approx 0.17\,, \tag{5.18}$$

while the present experimental value is $\approx$0.24.

VI. BOOTSTRAP OF THE POMERANCHUKON RESIDUES

In the preceding sections, we have discussed the bootstrap in terms of the s dependence of the absorptive part $A(s,t)$. We now examine the t dependence of the bootstrap, which is simplified by the separation of the s and t dependence of the asymptotic fixed-pole amplitude $A(s,t)=s\beta(t)$, where $\beta(t)$ is the Pomeranchukon residue function.

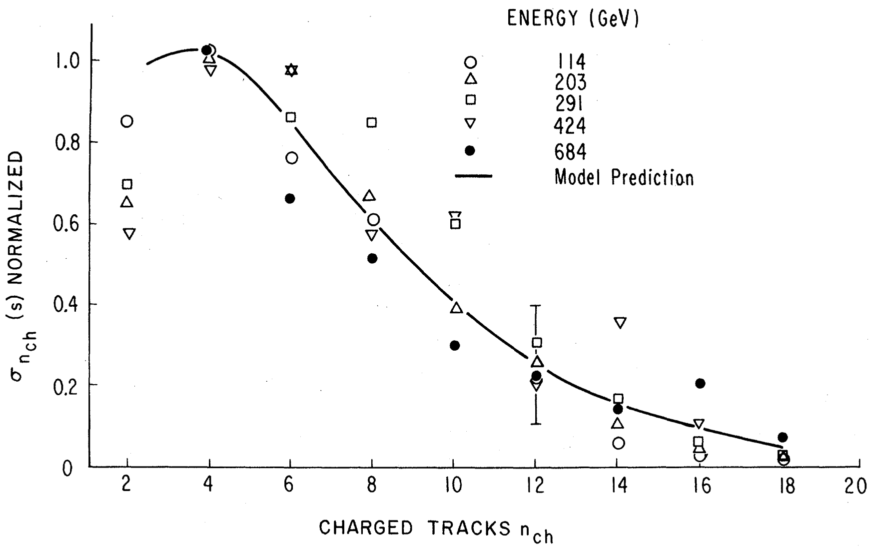

FIG. 6. Comparison with cosmic-ray data of asymptotic multiplicity distribution. Cross sections for various energies in Ref. 34 are normalized to unity at the point $n_{ch}=4$. The fitted curve is $\sigma_{n_{ch}}=c(N+1)r^N$, with $N=\frac{1}{2}(n_{ch}-2)$ and $c=1$, $r=0.53$.

The bootstrap equations for the Pomeranchukon residues will be nonlinear integral equations, as can be seen from Fig. 1(b). If we assume that pion exchange dominates in coupling to internal Pomeranchukons,[5] then the amplitude for the Pomeranchukon exchange is proportional to the product of two Pomeranchukon residues, such as $\beta_{\pi\pi}(t_+)\beta_{\pi\pi}(t_-)$ for the central diagram, and this is used to give an output residue $\beta_{ab}(t)$.

The bootstrap equations are derived from the unitarity relation, Eq. (2.12), by doing the integrals over s_l and s_r. We define residues $\gamma_a(t;t_+,t_-)$ of the fragmentation absorptive parts by

$$A_a^{P'}(s_l,t;t_+,t_-)=\frac{\pi}{s_0}\left(\frac{s_l}{s_0}\right)^{\alpha_{P'}(t)}\gamma_a(t;t_+,t_-)[1-\alpha_{P'}(t)]\,. \tag{6.1}$$

Then, using the multiperipheral integration limits $s_l s_r \leqslant s s_0$, we find for the contribution of the central Pomeranchukon-exchange term (Fig. 7)

$$A_{ab}(s,t)=\frac{s}{16\pi^2}\int\frac{dt_+dt_-\beta_{\pi\pi}(t_+)\beta_{\pi\pi}(t_-)}{[-\Delta(t,t_+,t_-)]^{1/2}}\times\gamma_a(t;t_+,t_-)\gamma_b(t;t_+,t_-)\,. \tag{6.2}$$

Similarly, contributions from the end diagrams and the elastic diagram are obtained by using $A_a^{P'}=\pi\delta(s_l-m_a^2)$, and likewise for hadron b. The sum of all these terms gives the following integral equation for the Pomeranchukon residue function:

$$\beta_{ab}(t)=\frac{1}{16\pi^2}\int\frac{dt_+dt_-}{[-\Delta(t,t_+,t_-)]^{1/2}}[\beta_{ab}(t_+)\beta_{ab}(t_-)+\beta_{a\pi}(t_+)\beta_{a\pi}(t_-)\gamma_b+\beta_{\pi b}(t_+)\beta_{\pi b}(t_-)\gamma_a+\beta_{\pi\pi}(t_+)\beta_{\pi\pi}(t_-)\gamma_a\gamma_b]\,. \tag{6.3}$$

Since the total cross section $\sigma_{ab}=\beta_{ab}(0)$, we evaluate the equation at $t=0$, where the boundaries require $t_+=t_-=t'$, and in this limit

$$\lim_{t\to 0}\int\frac{dt_+dt_-}{[-\Delta(t,t_+,t_-)]^{1/2}}=\pi\int_{-\infty}^{0}dt'\,. \tag{6.4}$$

Then the left-hand side of Eq. (6.3) is the total cross section, while the right-hand side gives the contributions of elastic, single-diffractive, and double-diffractive scattering,

$$\beta_{ab}(0)=\frac{1}{16\pi}\int dt'[\beta_{ab}{}^2(t')+\beta_{a\pi}{}^2(t')\gamma_b(0;t')+\beta_{\pi b}{}^2(t')\gamma_a(0;t')+\beta_{\pi\pi}{}^2(t')\gamma_a(0;t')\gamma_b(0;t')]\,, \tag{6.5}$$

or

$$\sigma_T = \sigma_{el} + \sigma_{dissoc\,(b)} + \sigma_{dissoc\,(a)} + \sigma_{dissoc\,(a+b)} \;. \tag{6.6}$$

For purposes of illustrating the nature of these bootstrap equations, we shall exhibit some solutions obtained by assuming a simplified behavior of the input functions γ which arise from purely meson-exchange multiperipheral graphs. The coupled set of nonlinear equations for $\pi\pi$, πN, and NN scattering are

$$\beta_{\pi\pi}(t) = \frac{1}{16\pi^2}\int \frac{dt_+dt_-\beta_{\pi\pi}(t_+)\beta_{\pi\pi}(t_-)}{[-\Delta(t,t_+,t_-)]^{1/2}}(1+\gamma_\pi)^2\,, \tag{6.7a}$$

$$\beta_{\pi N}(t) = \frac{1}{16\pi^2}\int \frac{dt_+dt_-}{[-\Delta(t,t_+,t_-)]^{1/2}}[\beta_{\pi N}(t_+)\beta_{\pi N}(t_-)(1+\gamma_\pi)+\beta_{\pi\pi}(t_+)\beta_{\pi\pi}(t_-)\gamma_N(1+\gamma_\pi)]\,, \tag{6.7b}$$

$$\beta_{NN}(t) = \frac{1}{16\pi^2}\int \frac{dt_+dt_-}{[-\Delta(t,t_+,t_-)]^{1/2}}[\beta_{NN}(t_+)\beta_{NN}(t_-)+2\beta_{\pi N}(t_+)\beta_{\pi N}(t_-)\gamma_N+\beta_{\pi\pi}(t_+)\beta_{\pi\pi}(t_-)\gamma_N{}^2]\,. \tag{6.7c}$$

Since the dependence of $\gamma_a(t;t_+,t_-)$ on t_+ and t_- arises from integrations, but not from any form factors, it may well be approximately independent of these variables, and we will henceforth consider it only as a function of t.

Empirically, we know more about the residue functions $\beta_{ab}(t)$ than about the functions $\gamma(t)$. Hence, we shall assume solutions for $\beta_{ab}(t)$ and find the γ's necessary to generate them. From experiments, we know that $\beta_{\pi N}(t)$ and $\beta_{NN}(t)$ have almost the same form,[36] and we may expect $\beta_{\pi\pi}(t)$ to have the same form also. Therefore, we investigate the class of solutions for which

$$\beta_{\pi\pi}(t) = \sigma_{\pi\pi} f(t)\,, \tag{6.8a}$$

$$\beta_{\pi N}(t) = \sigma_{\pi N} f(t)\,, \tag{6.8b}$$

$$\beta_{NN}(t) = \sigma_{NN} f(t)\,, \tag{6.8c}$$

where $f(0)=1$. From $f(t)$, we define

$$h(t) = \frac{1}{16\pi^2}\int \frac{dt_+dt_-}{[-\Delta(t,t_+,t_-)]^{1/2}} f(t_+)f(t_-)\,, \tag{6.9}$$

where specific choices for $f(t)$ will be considered later. With the above assumptions, we can algebraically solve Eqs. (6.8a)–(6.8c) for $\sigma_{\pi\pi}$, $\sigma_{\pi N}$, and σ_{NN} in terms of $f(t)/h(t)$, $\gamma_\pi(t)$, and $\gamma_N(t)$. We find that these solutions factorize,

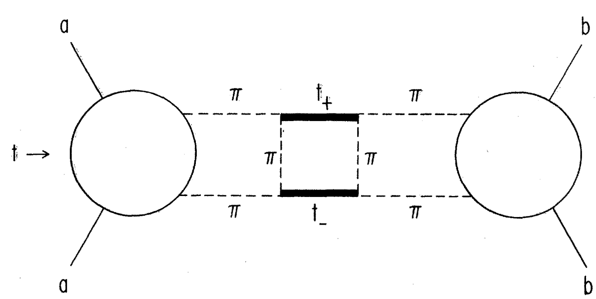

FIG. 7. Central Pomeranchukon contribution with AFS multiperipheral model for fragmentation.

$$\sigma_{\pi\pi}\sigma_{NN} = \sigma_{\pi N}{}^2\,. \tag{6.10}$$

In addition, they give a restriction between $\gamma_\pi(t)$ and $\gamma_N(t)$,

$$\gamma_N(t) = K[1+\gamma_\pi(t)] - K^2, \tag{6.11}$$

where K is an arbitrary positive constant. The solutions are

$$\sigma_{\pi\pi} = [f(t)/h(t)][1+\gamma_\pi(t)]^{-2}, \tag{6.12a}$$

$$\sigma_{\pi N} = K\sigma_{\pi\pi}, \tag{6.12b}$$

$$\sigma_{NN} = K\sigma_{\pi N}, \tag{6.12c}$$

where the elastic cross sections are just

$$\sigma_{el,ab} = \sigma_{ab}{}^2 h(0). \tag{6.13}$$

For a given $f(t)$, we can find the appropriate $\gamma_\pi(t)$ from Eq. (6.12).

We shall examine two cases for $\gamma_\pi(t)$. The first is the one which gives an exponential solution for the residues $f(t)=e^{at/2}$. This gives, from Eq. (6.9), $h(t)=(16\pi a)^{-1}e^{at/4}$, and therefore requires a form for $\gamma_\pi(t)$ and $\gamma_N(t)$ from Eqs. (6.12a)–(6.12c) and (6.11):

$$\gamma_\pi(t) = Ae^{at/8} - 1, \tag{6.14a}$$

$$\gamma_N(t) = KAe^{at/8} - K^2, \tag{6.14b}$$

where $A^2 = 16\pi a/\sigma_{\pi\pi}$. If we reverse the reasoning and consider the forms (6.14a) and (6.14b) as arising from the meson-exchange multiperipheral graphs, then the solutions to the bootstrap equations are

$$\beta_{\pi\pi}(t) = \sigma_{\pi\pi}e^{at/2}, \tag{6.15a}$$

$$\beta_{\pi N}(t) = K\sigma_{\pi\pi}e^{at/2}, \tag{6.15b}$$

$$\beta_{NN}(t) = K^2\sigma_{\pi\pi}e^{at/2}, \tag{6.15c}$$

and

$$\sigma_{el,ab} = \sigma_{ab}{}^2(16\pi a)^{-1}.$$

Another set of interesting solutions may be generated by assuming that $\gamma_\pi(t)$ and $\gamma_N(t)$ are approximately independent of t or constants. This will lead to solutions in terms of Bessel functions which are commonly associated with diffraction models. If we again assume that the residue functions have the same form for $\pi\pi$, πN, and NN scattering [Eq. (6.8)], then we must satisfy Eq. (6.12) with $\sigma_{\pi\pi}(1+\gamma_\pi)^2$ a constant,

$$f(t)=\frac{\sigma_{\pi\pi}(1+\gamma_\pi)^2}{16\pi^2}\int\frac{dt_+dt_-}{[-\Delta(t,t_+,t_-)]^{1/2}}f(t_+)f(t_-). \tag{6.16}$$

To solve this we introduce the Fourier-Bessel transform,

$$f(t)=\int_0^\infty b\,db\,J_0(b(-t)^{1/2})g(b), \tag{6.17a}$$

$$g(b)=\int_0^\infty (-t)^{1/2}d(-t)^{1/2}J_0(b(-t)^{1/2})f(t), \tag{6.17b}$$

and make use of the identity

$$\frac{2\Theta(-\Delta)}{[-\Delta(t,t_+,t_-)]^{1/2}}=\pi\int_0^\infty b\,db\,J_0(b(-t)^{1/2}) \times J_0(b(-t_+)^{1/2})J_0(b(-t_-)^{1/2}) \tag{6.18}$$

to transform the integral equation for $\beta(t)$ into an algebraic equation for $g(b)$. This gives

$$g(b)=(8\pi)^{-1}\sigma_{\pi\pi}(1+\gamma_\pi)^2g^2(b). \tag{6.19}$$

Thus, at any given value of b, either $g(b)=0$ or $g(b)=8\pi[\sigma_{\pi\pi}(1+\gamma_\pi)^2]^{-1}$. One of the simplest non-trivial solutions corresponds to scattering by an absorbing disk of radius R,

$$g(b)=8\pi[\sigma_{\pi\pi}(1+\gamma_\pi)^2]^{-1}\Theta(R-b). \tag{6.20}$$

The resulting residue function is given by

$$f(t)=\frac{4\pi R^2}{\sigma_{\pi\pi}(1+\gamma_\pi)^2}\left(\frac{2J_1(R(-t)^{1/2})}{R(-t)^{1/2}}\right), \tag{6.21}$$

where at $t=0$, the quantity in large round parentheses is unity. From the given constants γ_π and γ_N, we can solve Eq. (6.11) for K:

$$K=\tfrac{1}{2}(1+\gamma_\pi)\{1\pm[1-4\gamma_N/(1+\gamma_\pi)^2]^{1/2}\}. \tag{6.22}$$

We can then write the solutions to these equations as

$$\begin{aligned}&\sigma_{\pi\pi}=4\pi R^2/(1+\gamma_\pi)^2,\\&\sigma_{\pi N}=K\sigma_{\pi\pi},\\&\sigma_{NN}=K^2\sigma_{\pi\pi},\\&\beta_{ab}(t)=\sigma_{ab}\left(\frac{2J_1(R(-t)^{1/2})}{R(-t)^{1/2}}\right),\\&\sigma_{\mathrm{el},ab}=\sigma_{ab}{}^2/(4\pi R^2).\end{aligned} \tag{6.23}$$

The shape of $f(t)$ is roughly exponential near $t=0$ and may be compared to an exponential fit by the expansion

$$\frac{2J_1(R(-t)^{1/2})}{R(-t)^{1/2}}=1+\tfrac{1}{8}R^2t+O(t^2)\cong e^{at/2}=1+\tfrac{1}{2}at+O(t^2), \tag{6.24}$$

where $4\pi R^2=16\pi a$.

The solutions examined above all had the same t dependence for the residues $\beta_{\pi\pi}$, $\beta_{\pi N}$, and β_{NN}. We also examined the possibility of a different exponential t dependence for each of these residues, i.e.,

$$\beta_{\pi\pi}(t)=\sigma_{\pi\pi}e^{a_{\pi\pi}t/2},$$

$$\beta_{\pi N}(t)=\sigma_{\pi N}e^{a_{\pi N}t/2},$$

and

$$\beta_{NN}(t)=\sigma_{NN}e^{a_{NN}t/2}.$$

The bootstrap equations gave not only the factorization restriction $a_{\pi\pi}+a_{NN}=2a_{\pi N}$, but also the condition $a_{\pi N}{}^2=a_{\pi\pi}a_{NN}$, which together imply

$$a_{\pi\pi}=a_{\pi N}=a_{NN}. \tag{6.25}$$

Hence, this attempted solution reduced to the previous case of identical t dependence for the three residue functions. So under the assumption of exponential Pomeranchukon residue functions, our model gives not only factorization, but predicts that the t dependence for πN scattering will be the same as for NN scattering. Experimentally, this is very well satisfied.[36]

We can ask specifically what our bootstrap equations imply for the behavior of the absorptive part $A(s,t)$ away from $t=0$. From Fig. 1(b) and Eqs. (6.1), (6.7) we see that the inelastic contributions to unitarity, $A_{\mathrm{inel}}(s,t)=A(s,t)-A_{\mathrm{el}}(s,t)$, are proportional to the absorptive part of the P' residue function. We expect that the ghost-eliminating mechanism in Regge theory would give $\gamma(t_0)=0$, when $\alpha_{P'}(t_0)=0$. In fact, Eqs. (6.14), which are only expected to be valid for small t, actually imply a zero for $\gamma(t)$. Experimentally, $t_0\approx-0.6$ GeV^2. At $t=t_0$ then, the elastic contribution in Eq. (6.7) should completely saturate unitarity. In Fig. 8, we plot $A(s,t)$ and $A_{\mathrm{el}}(s,t)$ derived from an exponential parametrization of p-p elastic scattering at 21 GeV/c. The difference

$$A_{\mathrm{inel}}(s,t)=s\left(\sigma_{pp}e^{at/2}-\frac{\sigma_{pp}{}^2}{16\pi a}e^{at/4}\right) \tag{6.26}$$

vanishes when

$$t=\frac{4}{a}\ln\frac{\sigma_{pp}}{16\pi a}. \tag{6.27}$$

For the fitted parameters, $\sigma_{pp}=39$ mb and $a=10$

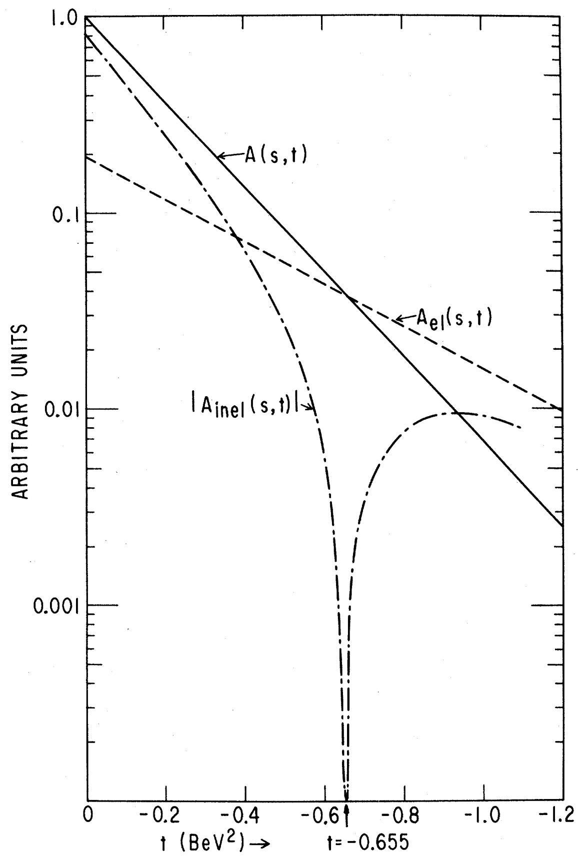

FIG. 8. $A(s,t) = s\sigma_T e^{at}$, $A_{el}(s,t)$, and $A_{inel}(s,t)$ from p-p elastic scattering at 21 GeV/c.

GeV^{-2}, this gives $t = -0.65\ \text{GeV}^2$, in excellent agreement with the above value for t_0.

VII. CONCLUSIONS

We have developed a multiparticle production model in contrast with the usual multi-Pomeranchukon-exchange models in order to give a dynamical alternative against which we can test the multi-Pomeranchukon-exchange model in inclusive experiments. The alternative was achieved by requiring constant total cross sections for hadronic collisions within the framework of a multiperipheral model for particle production. A consistent bootstrap solution for the leading asymptotic behavior of the elastic scattering amplitude was found to be a Pomeranchuk singularity which appears only once in the multiperipheral chain, and which is a fixed pole at $J = 1$. The leading behavior is built up through unitarity only by those multiperipheral chains in which one Pomeranchuk is exchanged and the resulting physical picture is that of a diffractive model. The fragmentation of the incident hadrons is described by the multiperipheral chains adjacent to the Pomeranchukon exchange.

In contrast with other multiperipheral bootstrap models we find that all partial cross sections are asymptotically finite, including the elastic cross section. In particular, the width of the diffraction peak will not shrink for elastic scattering. As a consequence of the energy-independent cross sections, the average multiplicity becomes asymptotically constant. In a particular model for the hadronic fragmentation, the distribution of final-particle multiplicities is close to geometric, whereas the usual multiperipheral models in the weak-coupling limit[8,29] predict that this distribution will be a Poisson distribution with an average multiplicity which grows like the logarithm of the incident energy.

The mechanism of single-Pomeranchukon exchange leads to a dynamics that is quite different from MPE models. Most of the available energy is deposited into the relative motion of the beam fragments from the target fragments, which are scattered by Pomeranchukon exchange, rather than into particle production. This, in turn, will manifest itself as a gap in the longitudinal-momentum spectrum of the produced secondary particles. Thus at asymptotic energies there will be no pionization of the kind produced in MPE models. At present energies the gap between the fragments does not appear, due to the slow falloff of diffractive fragmentation with longitudinal momentum. In addition, there are multiperipheral contributions without Pomeranchukon exchange which produce a uniform spectrum in $dk_\parallel/k_0$ and which also fall off slowly in s. This gap in the spectrum of secondaries will lead to an asymptotically constant average multiplicity. Despite the lack of pionization, we have shown that the single-particle distribution scales, i.e., the fragments asymptotically form a limiting distribution in the lab or projectile frame.

Because of the simple nature of the fixed-pole Pomeranchukon, the s and t dependence of the bootstrap decouples. As a result, some simple models of the behavior of the residue were obtained which explicitly showed the factorization of the fixed pole in a coupled-channel problem. This diffractive model also allows us to associate the vanishing residue of the P' trajectory with the vanishing of the inelastic contribution to the absorptive part at $t \simeq -0.6\ \text{GeV}^2$.

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†Present address: Department of Physics, University of California, Irvine, California 92664.

‡Present address: Department of Physics, University of California, Berkeley, California 94720.

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