Lawrence Berkeley National Laboratory
Recent Work

Title
HIGHER TRANSITIONS IN n-MESIC ATOMS

Permalink
https://escholarship.org/uc/item/4348d5n3

Authors
Jenkins, David A.
Kunselman, Raymond.

Publication Date
1966-10-18

eScholarship.org Powered by the California Digital Library
University of California

UCRL-17211

University of California

Ernest O. Lawrence Radiation Laboratory

HIGHER TRANSITIONS IN π-MESIC ATOMS

TWO-WEEK LOAN COPY

This is a Library Circulating Copy which may be borrowed for two weeks. For a personal retention copy, call Tech. Info. Division, Ext. 5545

Berkeley, California

UCRL-17211 c.2

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Physical Review Letters

UCRL-17211
Preprint

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

AEC Contract No. W-7405-eng-48

HIGHER TRANSITIONS IN π-MESIC ATOMS

David A. Jenkins and Raymond Kunselman

October 18, 1966

Higher Transitions in π-Mesic Atoms*

David A. Jenkins and Raymond Kunselman

Lawrence Radiation Laboratory
University of California
Berkeley, California

October 18, 1966

ABSTRACT

We have measured the $3d \rightarrow 2p$, $4f \rightarrow 3d$, and $5g \rightarrow 4f$ π-mesic x-ray energies and widths for a selection of isotopes from $Z = 16$ (sulfur) to $Z = 94$ (plutonium). Our objective was to measure shifts and widths of the pion energy levels caused by the strong-interaction force of the nucleus. Earlier work reported a general survey of π-mesic x-ray energies[1] and a detailed investigation of the $2p \rightarrow 1s$ x-ray energies and widths.[2] The higher transitions reported in this paper are needed to determine the pion-nucleon interaction parameters. We have combined the present data with earlier data to find such parameters, and find fair agreement with values predicted from pion-nucleon scattering and pion production.

Pi-mesic x rays were observed with a lithium-drifted germanium detector which had a resolution of about 4 keV for the Na^{22} 511-keV line and which measured 2 cm^2 by 1 cm deep. Pulses from the detector passed through a preamplifier which had a cooled field-effect transistor, and a 4096-channel analog-to-digital converter (Nuclear Data model ND 161F) was used with a computer (Digital Equipment model PDP-5) so that broad energy windows could be examined while still efficiently using the high detector resolution.[3] The energy scale and instrumental resolution were calibrated with a mercury pulser and with sources of known energy, and digital stabilizers were used during the measurements to stabilize the bias and gain of the amplifier on peaks in the spectrum produced by radioactive sources (Am^{241} and Na^{22}). Our energy scale was chosen such that one channel in the pulse-height spectrum corresponded to 0.3 keV, and we use this as our minimum error; in some cases this error had to be increased because of the low number of counts in the x-ray peaks. A computer program is now being used to more accurately measure the centroids and widths of the peaks, and these results will be presented later.

We observe those transitions that were expected to show a strong nuclear perturbation in isotopes that were readily available. For example, the 2p level is perturbed by the nucleus in the Z = 25 region, and therefore we observed the 3d → 2p transition from Z = 16 (sulfur) to Z = 27 (cobalt) in elements that were at least 84% isotopically pure. We could not observe the 3d → 2p transition in higher-Z elements because of the low x-ray yield caused by nuclear capture from the 3d state. With some isotopes we observed an energy-level splitting caused by the magnetic-dipole and electric-quadrupole moments of the nucleus. Since these splittings are not well resolved by our detector, they confuse the measurement of line widths, and we have restricted our analysis of level broadening to those isotopes with splittings that are calculated to be less than 0.3 keV, except bismuth which has a splitting of 0.6 keV.

The π-mesic x-ray energies and widths which we measured are presented in Table I. We list the principle quantum number n for the lower level of the x-ray transition, the measured x-ray energy, the energy computed from the Klein-Gordon equation with a correction for reduced mass, the vacuum-polarization correction computed according to Mickelwait,[4] and the finite Coulomb-size correction computed according to Pustavolov with a nuclear radius of 1.2 $A^{1/3}$ fermi (where A is the mass number).[5] The vacuum-polarization correction was calculated only to first order; higher-order corrections have been discussed by Wichmann and Kroll[6] and are expected to be small even for high-Z nuclei. The Coulomb correction is small because the pion is far from the nucleus, and for the 4f → 3d and 5g → 4f x rays this correction is negligible. The sum of the Klein-Gordon value, the vacuum-polarization shift, and the Coulomb shift give a total calculated energy, E_c. The difference between this energy and the measured energy (ΔE) is interpreted as the nuclear shift for the lower level of the x-ray transition. The calculated value of the energy shift is described below, and represents a best fit to the data for an optical model of the nucleus. The measured value of the width (w_n) is the full width at half-maximum of the peak after the instrumental width has been removed. The measured value is derived from the data by a method described in reference 2, and the calculated value (Γ) is described below.

In Table II we list other mesic x-ray data that were used in determining the interaction parameters. The data for oxygen have been revised from that presented in reference 2 because of an error in the background-subtraction analysis, and the errors on the widths of the fluorine and sodium lines have been increased because of uncertainties introduced by the background analysis. The Al^{27} data is from reference 7.

To compute the shift of the x-ray energies, we follow the formalism developed by the Ericsons in which the nucleus is represented by an optical-model potential of the form

$$V(r) = 2\pi\left[(b_0 + \frac{T\cdot\tau}{A} b_1)\rho(r) - (c_0 + \frac{T\cdot\tau}{A} c_1)\nabla\cdot\rho(r)\nabla\right] \qquad (1)$$

where $\rho(r)$ is the nuclear density, ∇ is the gradient operator, T is the isotopic spin of the pion, τ is the isotopic spin of the nucleus, and we take $T\cdot\tau = N - Z$. The constants b_i and c_i are simply related to the pion-nucleon scattering lengths:

$$\begin{aligned} b_0 &= (a_1 + 2a_3)/3 & c_0 &= (4a_{33} + 2a_{31} + 2a_{13} + a_{11})/3 \\ b_1 &= (-a_1 + a_3)/3 & c_1 &= (2a_{33} + a_{31} - 2a_{13} - a_{11})/3, \end{aligned} \qquad (2)$$

where $a_{2t,\,2j}$ is written for the channel with isotopic spin t and spin j. For $\rho(r)$ we used a Saxon Woods potential

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}}, \qquad (3)$$

where ρ_0 is a normalization constant determined from $\int\rho(r)dV = A$, and c and a are constants which we have taken from Elton[8] and both depend on A. The energy shift is computed from perturbation theory

$$\Delta E = \int \psi^* V(r)\psi \, dV, \tag{4}$$

where ψ is the meson wave function, and we have not included the Lorentz-Lorenz effect or the normalization for the gradient interaction discussed by the Ericsons.[9, 10] These effects tend to cancel, and should not influence our results by more than 10%. We also neglect corrections due to the Fermi motion of the nucleons, two-nucleon absorption, and nuclear correlations.

The constants b_i and c_i in Eq. (1) are determined by a least-squares fit to the data of Tables I and II with a method described by Mandel.[11] However, we did not include the isotopes with $Z > 83$ in this analysis because their shape is distorted and Eq. (3) is not expected to be a good description of their nuclear density. The best fit for the energy shifts was obtained with (units $\hbar = c = m_\pi = 1$)

$$\begin{aligned} b_0 &= -0.0197 \pm 0.0004 & c_0 &= 0.131 \pm 0.011 \\ b_1 &= -0.064 \pm 0.013 & c_1 &= -0.018 \pm 0.090, \end{aligned} \quad \text{(measured)} \tag{5}$$

which can be compared to the scattering lengths[12] of Eq. (2)

$$\begin{aligned} b_0 &= -0.012 \pm 0.004 & c_0 &= 0.21 \pm 0.01 \\ b_1 &= -0.097 \pm 0.007 & c_1 &= 0.18 \pm 0.01. \end{aligned} \quad \text{(calculated with no nuclear corrections)} \tag{6}$$

Including nuclear corrections with the data in Eq. (6), one gets[13]

$$\begin{aligned} b_0 &= -0.028 \pm 0.006 & c_0 &= 0.19 \pm 0.02 \\ b_1 &= -0.10 \pm 0.01 & c_1 &= 0.16 \pm 0.01. \end{aligned} \quad \text{(calculated with nuclear corrections)} \tag{7}$$

There is qualitative agreement between the measured and calculated values of the constants; and the nuclear corrections, as discussed by the Ericsons, tend to improve the agreement between the measured and calculated values. The measured values are compared to the values calculated with parameters from Eq. (5) in Tables I and II and Fig. 1.

The widths of the x-ray lines are more difficult to calculate because the pion must be absorbed by two nucleons to conserve momentum. In our analysis we use an optical model of the form[14]

$$\Gamma = A_1 \int \rho(r)^2 |\psi|^2 \, dV + A_2 \int \rho(r)^2 |\nabla\psi|^2 \, dV, \qquad (8)$$

where $\rho(r)$ and ψ are defined above, and A_i are constants determined by a least-squares fit to the width data of Tables I and II. We find (excluding the fluorine, sodium, and $Z > 83$ points):

$$A_1 = 0.220 \pm 0.017 \qquad A_2 = 1.97 \pm 0.26 \qquad \text{(measured)}. \qquad (9)$$

These parameters have been calculated by M. Ericson from the reaction $N + N \rightarrow N + N + \pi$ to give[14]

$$A_1 = 0.299 \pm 0.036 \qquad A_2 = 2.15 \pm 0.51, \qquad \text{(calculated)} \qquad (10)$$

in agreement with experiment. The measured widths, and widths calculated from Eqs. (8) and (9), are shown in Tables I and II and Fig. 2.

The fluorine and sodium width data are not explained by this calculation. The reason may be that fluorine and sodium have nucleons in the d shell, whereas the lower-Z nuclei have only s and p nucleons. If we assume that the capturing nucleons are in a relative s state, nucleons in the d shell cannot participate in the capture process, and they dilute the effect of the nucleons that can capture a pion. Since the probability of finding a particular nucleon in the nucleus is $1/A$ and the probability of capture is proportional to the square of the density, the d-state nucleons suppress the capture rate determined from Eq. (7) by a factor

$$(A_1/A_2)^2. \tag{11}$$

Here A_1 is the number of nucleons that can participate in the capture process, and A_2 is the total number of nucleons. This correction factor has been applied to the calculated fluorine, sodium, and magnesium widths listed in Table II, and we used $A_1 = 16$ and $A_2 = A$ (the mass number for the respective nucleus). This correction tends to reduce the disagreement between the calculated and measured values, but there are still some differences.

In conclusion, there is general agreement between the measured and calculated values of the π-mesic x-ray energies and widths. However, discrepancies remain in the shift and width data for nuclei with $Z > 83$ in the 4f level, and the width data for $Z > 8$ in the 1s level.

We thank Professors K. M. Crowe and E. G. Segrè for their support, Messrs Stephen Kahn, Robert Lafore, Curtis Nunnally, James Shilleto, Melvin Simmons, and Stefan Swierkowski for assistance in the measurement, Dr. Fred Goulding for supplying the detector, Professor Herb Anderson for the loan of target materials, Dr. Torlief Ericson for helpful discussions, and Mr. James Vale and the 184-Inch Cyclotron staff for their help.

REFERENCES

*Work done under the auspices of the U. S. Atomic Energy Commission.

1. D. A. Jenkins and K. M. Crowe, Phys. Rev. Letters 16, 637 (1966).
2. D. A. Jenkins, R. Kunselman, M. K. Simmons, and T. Yamazaki, Phys. Rev. Letters 17, 1 (1966).
3. S. P. Swierkowski and R. W. Lafore, Lawrence Radiation Laboratory report UCRL-16814, 1966 (unpublished).
4. A. B. Mickelwait, Ph. D. Thesis, Carnegie Institute of Technology, 1954 (unpublished); A. B. Mickelwait and H. C. Corben, Phys. Rev. 96, 1145 (1954).
5. G. E. Pustavolov, Zh. Eksperim. i. Teor. Fiz. (U. S. S. R.) 36, 1806 (1959); English translation: Soviet Physics - JETP 9, 1288 (1959).
6. E. Wichmann and N. Kroll, Phys. Rev. 101, 843 (1956).
7. A. Astbury, J. P. Deutsch, K. M. Crowe, R. E. Shafer, and R. E. Taylor, Comptes Rendus of the International Congress on Nuclear Physics, Paris, 1964, (Centre National de la Recherche Scientifique, Paris, 1964), Vol. 2, p. 225.
8. L. R. B. Elton, Nuclear Sizes (Oxford University Press, London, 1961). We used values of c and a given in Table 2. For isotopes not listed in this table, we used $c = 1.07\ A^{1/3}$ F and $a = 0.545$ F; except Li^7, $c = 0.86$ F, $a = 0.523$ F; and B^{10}, $c = 0.90$ F, $a = 0.455$ F.
9. M. Ericson and T. E. O. Ericson, Ann. Phys. 36, 323 (1966).
10. M. Ericson, Compt. Rend. 257, 3831 (1963).
11. J. Mandel, The Statistical Analysis of Experimental Data (Interscience Publishers, New York, 1964), p. 131.

12. The values for b_0 and b_1 are taken from V. K. Samaranayake and W. S. Woolcock, Phys. Rev. Letters 15, 936 (1965); and the values for c_0 and c_1 are taken from J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963).

13. We thank Dr. Torlief Ericson for giving us these numbers.

14. M. Ericson, Compt. Rend. 258, 1471 (1964).

Table I. Pi-mesic x-ray energies and widths in keV.

Isotope	Measured energy	Klein-Gordon energy	Vacuum-polarization shift	Coulomb shift	Total calculated energy, E_c	Difference ΔE	Calculated level shift	Measured natural width, w_n	Calculated width, Γ
n = 2									
S^{32}	133.2 ± 0.3	131.9	0.6	-0.0	132.5	0.7 ± 0.3	0.40	0.8 ± 0.4	0.41
K^{39}	188.6 ± 0.3	186.4	1.0	-0.0	187.4	1.2 ± 0.3	0.98	1.9 ± 0.6	1.19
Ca^{40}	209.3 ± 0.3	206.7	1.1	-0.1	207.7	1.6 ± 0.3	1.31	2.1 ± 0.6	1.52
V^{51}	278.2 ± 0.4	274.0	1.6	-0.2	275.4	2.8 ± 0.4	2.47		4.27
Cr^{52}	302.5 ± 0.5	298.5	1.8	-0.2	300.1	2.4 ± 0.5	3.04		5.01
Mn^{55}	328.5 ± 0.8	324.1	1.9	-0.3	325.7	2.8 ± 0.8	3.61		6.46
Fe^{56}	356.9 ± 1.0	350.8	2.1	-0.4	352.5	4.4 ± 1.0	4.54	6.0 ± 2.5	7.89
Co^{59}	384.6 ± 1.0	378.6	2.4	-0.5	380.5	4.1 ± 1.0	5.33		10.07
n = 3									
Y^{89}	278.2 ± 0.3	276.2	1.4	0.0	277.6	0.6 ± 0.3	0.30	0.8 ± 0.6	0.30
Nb^{93}	307.6 ± 0.3	305.4	1.6	0.0	307.0	0.6 ± 0.3	0.43	0.6 ± 0.4	0.45
Rh^{103}	370.9 ± 0.4	368.6	2.0	0.0	370.6	0.3 ± 0.4	0.86	1.2 ± 0.6	0.96
In^{115}	442.1 ± 1.1	437.8	2.5	0.0	440.3	1.8 ± 1.1	1.59		1.98
Sn^{116}	460.9 ± 0.6	456.1	2.6	0.0	458.7	2.2 ± 0.6	1.83	1.9 ± 1.2	2.24
Sn^{117}	460.4 ± 0.6	456.1	2.6	0.0	458.7	1.7 ± 0.6	1.83	2.1 ± 1.2	2.27
Sn^{118}	460.4 ± 0.6	456.1	2.6	0.0	458.7	1.7 ± 0.6	1.83	2.5 ± 1.2	2.30
Sn^{119}	460.3 ± 0.6	456.1	2.6	0.0	458.7	1.6 ± 0.6	1.83	1.9 ± 1.2	2.33
Sn^{120}	460.5 ± 0.6	456.1	2.6	0.0	458.7	1.8 ± 0.6	1.82	2.7 ± 1.2	2.37
Sn^{122}	460.3 ± 0.6	456.1	2.6	0.0	458.7	1.6 ± 0.6	1.82	2.0 ± 1.2	2.43
Sn^{124}	460.2 ± 0.6	456.1	2.6	0.0	458.7	1.5 ± 0.6	1.81	2.3 ± 1.2	2.50
I^{127}	519.1 ± 1.1	513.2	3.0	0.0	516.2	2.9 ± 1.1	2.73		3.71
Cs^{133}	560.5 ± 1.1	553.2	3.3	0.0	556.5	4.0 ± 1.1	3.51	4.2 ± 1.8	5.00
La^{139}	603.6 ± 0.9	594.8	3.6	0.0	598.4	5.2 ± 0.9	4.46		6.65
Ce^{140}	626.1 ± 2.0	616.2	3.8	0.0	620.0	6.1 ± 2.0	5.03	5.8 ± 3.8	7.47
Pr^{141}	649.5 ± 2.0	638.0	3.9	0.0	641.9	7.6 ± 2.0	5.66	6.7 ± 2.8	8.36
n = 4									
Ta^{181}	453.1 ± 0.4	450.7	2.4	0.0	453.1	0.0 ± 0.4	0.29		0.24
Au^{197}	532.5 ± 0.5	528.9	2.9	0.0	531.8	0.7 ± 0.5	0.55		0.56
Bi^{209}	589.8 ± 0.9	584.8	3.2	0.0	588.0	1.8 ± 0.9	0.86	1.7 ± 1.0	0.88
Th^{232}	698.0 ± 0.6	689.5	4.0	0.0	693.5	(4.5 ± 0.6)[a]	1.73	(6.0 ± 0.9)[a]	2.10
U^{238}	731.4 ± 1.1	721.2	4.2	0.0	725.4	(6.0 ± 1.1)[a]	2.09	(6.1 ± 1.0)[a]	2.61
Pu^{239}	766.2 ± 1.6	753.5	4.5	0.0	758.0	(8.2 ± 1.6)[a]	2.51	(9.1 ± 2.5)[a]	3.09

[a]Not used to find parameters in Eqs. (5) and (9).

Table II. Pi-mesic x-ray energies in keV, from earlier work. Data are from reference 2, except for the 3d-2p aluminum transition, from reference 14.

Isotope	Difference ΔE	Calculated level shift	Measured natural width, w_n	Calculated width, Γ
n = 1				
Li^6	-0.6 ± 0.2	-0.30	0.39 ± 0.36	0.10
Li^7	-0.8 ± 0.2	-0.52	0.57 ± 0.30	0.13
Be^9	-1.75 ± 0.2	-1.47	0.85 ± 0.28	0.55
B^{10}	-2.6 ± 0.6	-2.28	1.4 ± 0.5	1.17
B^{11}	-2.9 ± 0.7	-3.27	2.3 ± 0.5	1.35
C^{12}	-5.8 ± 0.5	-4.62	2.6 ± 0.5	2.33
N^{14}	-9.8 ± 1.1	-8.33	4.1 ± 0.4	4.59
O^{16}	-14.7 ± 1.2 [a]	-13.75	9.0 ± 2.0[a]	7.69
F^{19}	-25.8 ± 1.1	-25.68	4.6 ± 2.0[a, b]	6.58[c]
Na^{23}	-49.8 ± 1.4	-51.20	4.6 ± 3.0[a, b]	10.26[c]
Mg	-57.3 ± 1.4	-58.15		12.95[c]
n = 2				
Al^{27}	0.24 ± 0.08	0.12		0.12

[a]Revised from earlier work (see text).

[b]Not used to find parameters in Eq. (9).

[c]Including correction factor given by Eq. (11).

FIGURE LEGENDS

Fig. 1. Measured difference, ΔE, and calculated level shift from Tables I and II. The values for the tin isotopes are the largest and smallest values only. The values from the figure are to be multiplied by the factor in parentheses to obtain the values in the tables.

Fig. 2. Measured natural width, w_n, and calculated width, Γ, from Tables I and II. The values for the tin isotopes are the largest and smallest values only. The values from the figure are to be multiplied by the factor in parentheses to obtain the values in the tables.

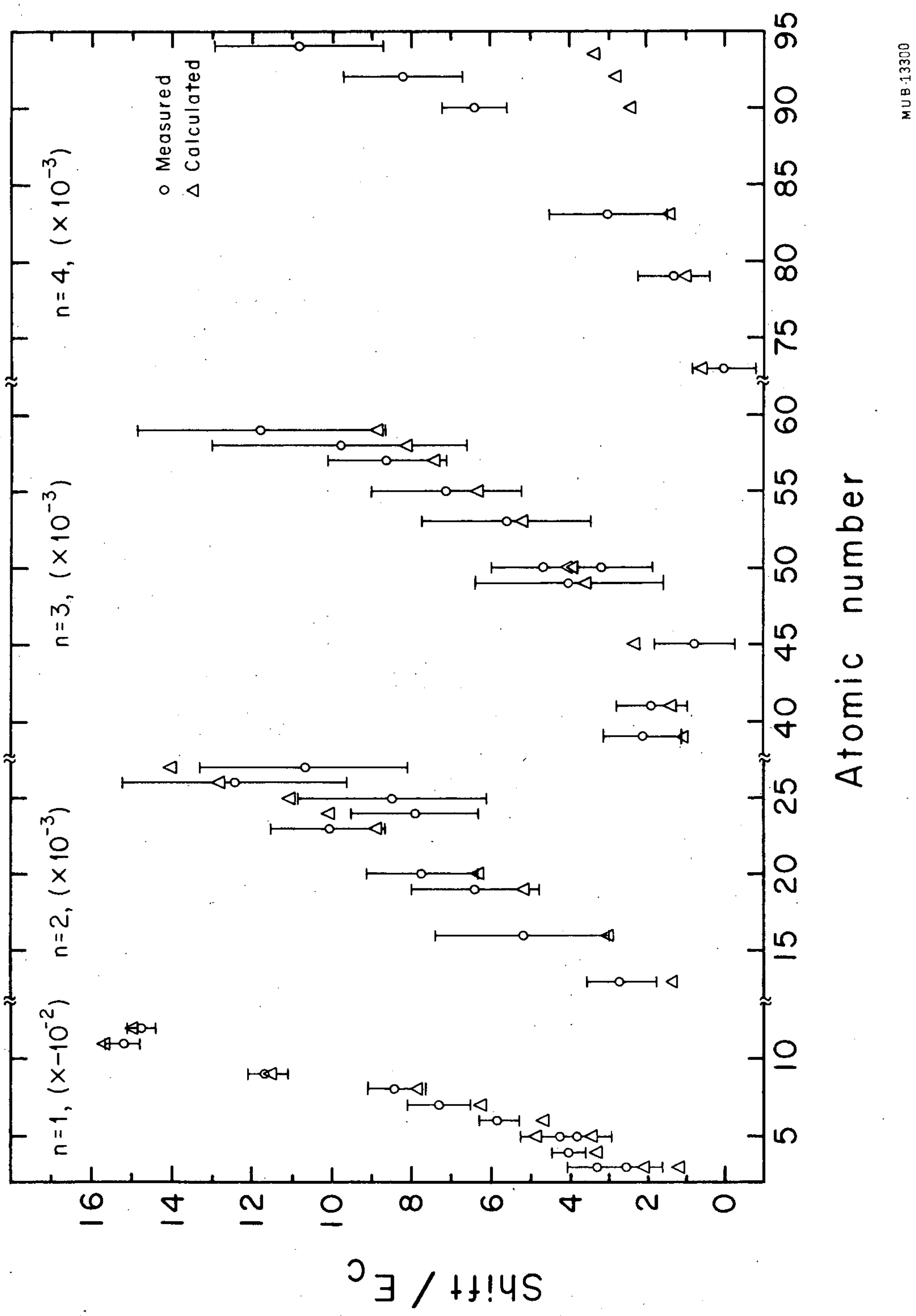
n=1, (×10-2)
n=2, (×10-3)
n=3, (×10-3)
n=4, (×10-3)
Measured
Calculated
5 10 15 20 25 40 45 50 55 60 75 80 85 90 95
Atomic number
0 2 4 6 8 10 12 14 16
Shift / Ec
MUB-13300

Fig. 1

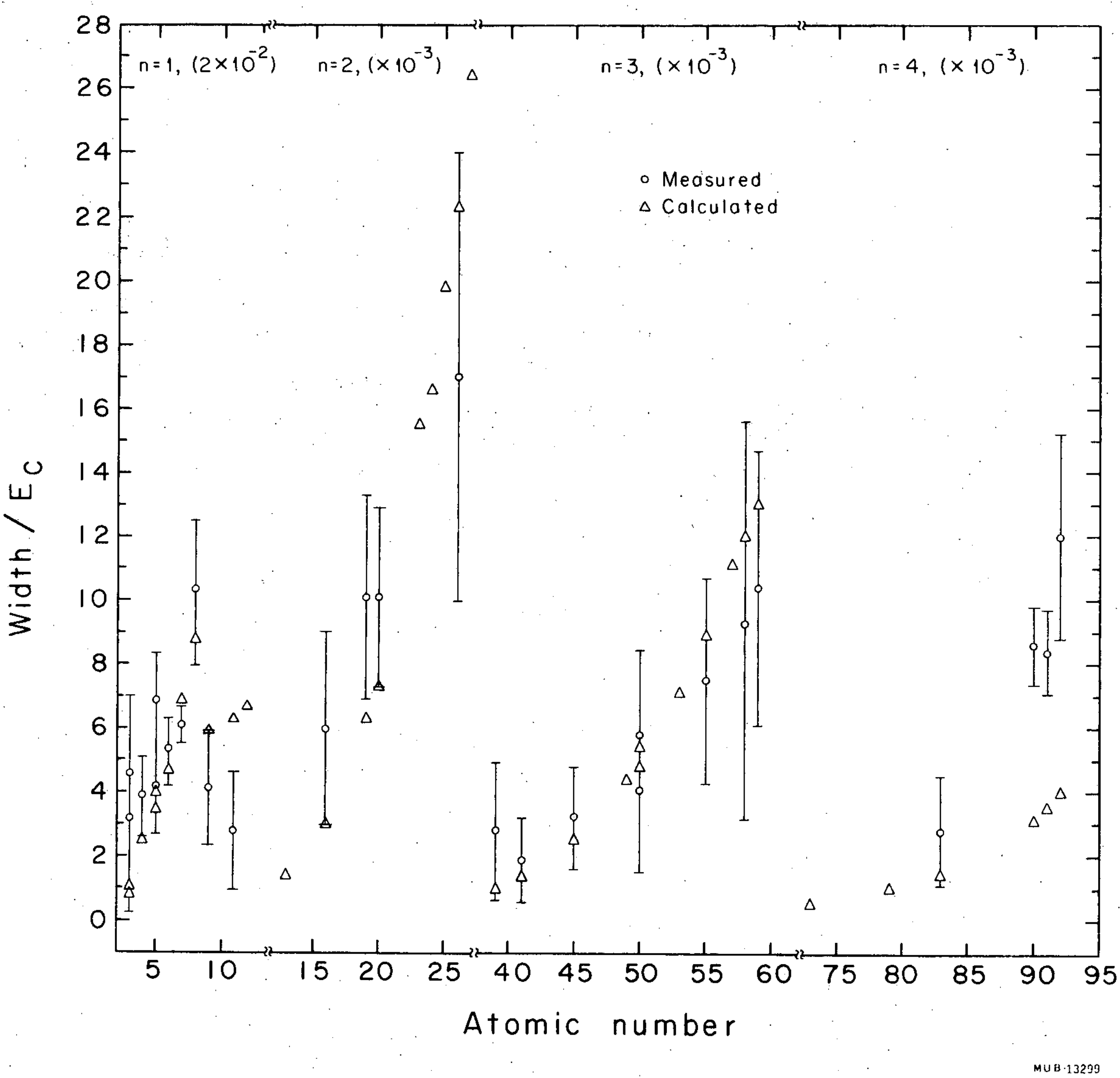
n=1, (2×10^{-2})
n=2, (×10^{-3})
n=3, (×10^{-3})
n=4, (×10^{-3})
Measured
Calculated
Width / E_c
Atomic number
28 26 24 22 20 18 16 14 12 10 8 6 4 2 0
5 10 15 20 25 40 45 50 55 60 75 80 85 90 95
MUB-13299

Fig. 2

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.