Title

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ABSTRACT

This report develops a systematic approach for solving the problem of conjunctive use of surface water and ground water in which both supply water quality and ground water quality are of major concern. The new approach utilizes a two-step nonlinear optimization. A test problem typical of a semiarid river basin with a seasonal agricultural demand and an increasing municipal and industrial demand is presented. Seasonal variations in demand, precipitation and recharge are handled by dividing each modeling year into a wet season and a dry season in the management model. Sustainable pumping and injection rates that would satisfy both the head and water quality constraints are obtained in the management model for each demand-supply scenario. An iterative technique is then used to solve the optimal pumping and injection rates within the planning horizon using the sustainable rates as upper bounds. Nonlinear programming solver MINOS is used to solve the management problem. MODFLOW and MT3D simulate the flow and transport in the ground water basin.

Key Words: Conjunctive use, Ground water management, Optimization, Contaminant transport, Ground water modeling.

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INTRODUCTION

Effective use of ground water is becoming increasingly important in the management of a water supply system in order to improve the reliability of the supply as well as to reduce its cost. However, the benefits of increasing the use of inexpensive local ground water are counterbalanced by the possible long-term deterioration of the ground water quality and the detrimental impacts of lowering of the ground water table which can cause land subsidence and increased pumping cost. Coe (1990) gave a detailed discussion of the benefits and constraints of conjunctive use of surface water and ground water in California.

Systems analysis techniques such as linear programming, dynamic programming, non-linear programming and multilevel optimization have long been used to solve the conjunctive use problem (Bredehoeft and Young, 1970 and 1983; Young and Bredehoeft, 1972; Illangasekare and Morel-Seytoux, 1986; Onta et al. 1991; Wang et al., 1995; Barlow et al. 1995). Most of these studies focus on the supply and demand aspects of the conjunctive use problem. Nonlinear constraints, if studied, are limited to head constraints in the aquifer. Recently, a few researchers have included water quality constraints in the conjunctive use management problem. Louie et al. (1984) used a constraint linear programming technique to develop tradeoff curves relating cost, water quality and overdraft. Yeh et al. (1995) used an iterative linear programming technique to solve a conjunctive use problem with supply water quality constraint for Hemet Basin, Riverside, CA. Taghavi et al. (1994) used a nonlinear programming approach to solve a ground water quality management problem associated with the application of dairy waste for a farming area in

Chino, CA. Ejaz and Peralta (1995) used a nonlinear programming approach to solve a conjunctive use problem with surface water quality constraints.

The solution of a conjunctive use problem with water quality constraints is computationally intense because of the nonlinearity of both the head constraints and the water quality constraints. In order to solve the conjunctive use problem, the ground water flow and mass transport models will need to be run numerous times that the problem may not be solvable (Taghavi et al. 1994). The other problem associated with the incorporation of ground water quality constraints in a conjunctive use problem is the slow response of ground water quality to changes in the pumping strategy. This creates a problem when the management strategy may adversely affect the ground water quality in certain parts of the aquifer, but the effect is not fully recognized until years after the implementation of the management strategy. This could mean that a pumping strategy that is feasible within the planning horizon or the modeling period of the simulation/optimization problem may not be feasible over a long period of time if the planning horizon is short compared to the time that is required for the water quality parameters to reach a steady state under the pumping strategy.

In this report, a two-step approach using nonlinear programming technique is proposed to solve a conjunctive use problem with both supply water quality constraints and ground water quality constraints. The proposed methodology assures that the pumping strategies obtained will be feasible even if the current pumping strategy is extended over a long period of time. It also significantly reduces the computational effort by decomposing the conjunctive use problem into two sub-problems.

A test problem is presented in this report to illustrate the application of the proposed methodology, and discussion and conclusion are provided at the end of this report.

METHODOLOGY

We begin the analysis of the conjunctive use problem with the water balance of a river basin (Wong et al., 1997). Figure 1 shows the cross-section of a conceptual river basin - imported water, surface water and ground water are utilized to support the local agricultural and municipal and industrial (M&I) water demand. Ground water is extracted from the aquifer to meet part of the water demand. The aquifer is replenished by infiltration of precipitation and the return of a portion of the reclaimed water. The reclaimed water may be returned as areal recharge of irrigation water or through injection wells and spreading grounds. The water balance for the aquifer can be represented by:

$$D_{agri} + D_{M\&I} = Q + S + I \qquad (1)$$

$$Q - Inj - R - Inf + \Delta G - \partial G = 0 \qquad (2)$$

where D_{agri} and $D_{M\&I}$ are the agricultural and M&I water demand, respectively [L^3/T]; Q, S and I are the ground water supply, surface water supply and imported water supply, respectively [L^3/T]; Inj is the injection rate [L^3/T]; R and Inf are areal recharge and infiltration, respectively [L^3/T]; ΔG and ∂G are increase in aquifer storage and inflow of ground water at the aquifer boundaries,

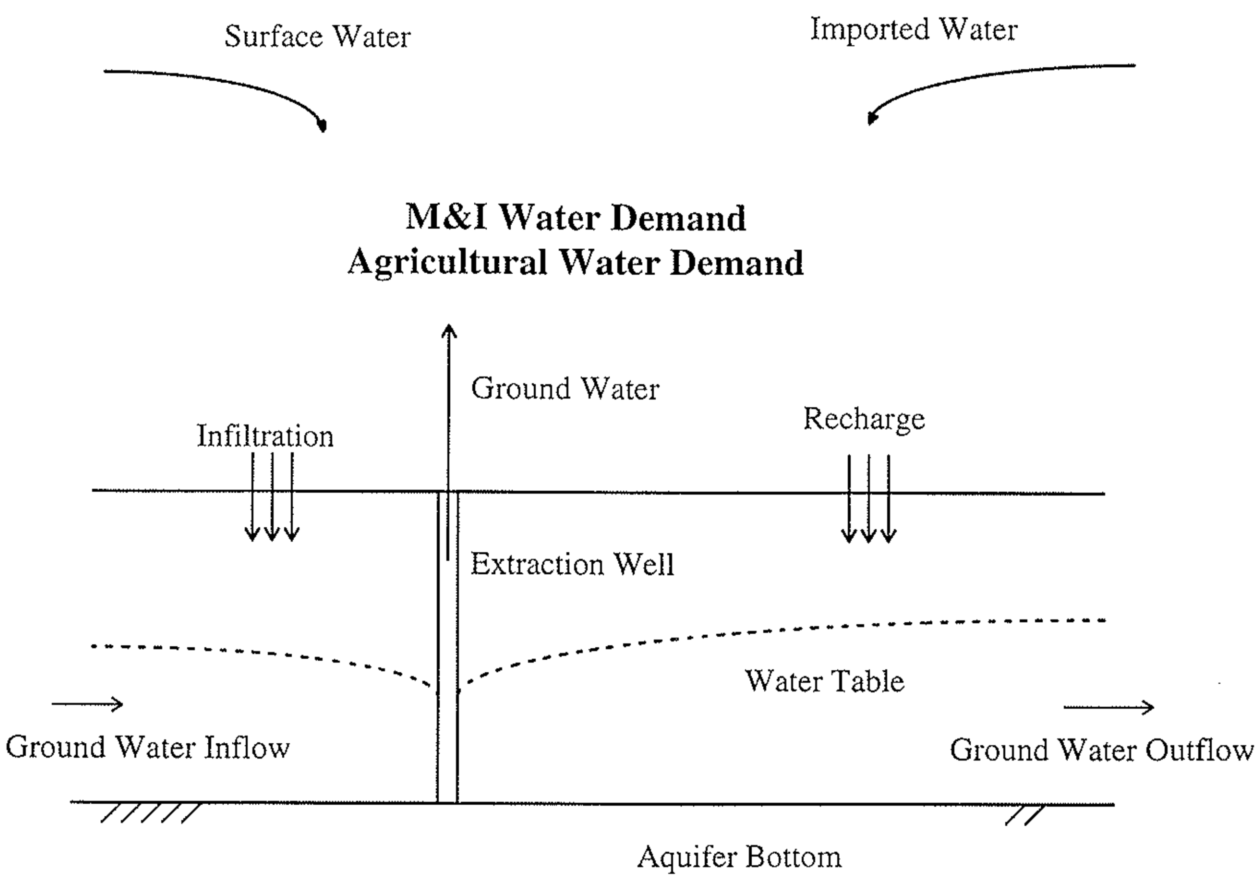


Figure 1. Cross-Section of a River Basin

respectively [L^3/T]. Evapotranspiration from the aquifer has been accounted for in the values for infiltration and recharge. Q, Inj, S and I are the decision variables in the management problem, while D_{agri}, $D_{M\&I}$, R and Inf are system inputs that determine what would be the optimal pumping strategy for a given set of constraints.

Similarly, the mass balance for the chemical of concern can be represented by:

$$QC - InjC_{inj} - RC_{rch} + \Delta G\, C - \partial G\, C_B = 0 \tag{3}$$

where C_{inj} is the concentration vector of the concentrations at the injection wells [M/L^3]; C_{rch} is the concentration in the recharge water; C is the concentration in the aquifer [M/L^3]; C_B is the concentration at the aquifer boundary [M/L^3]; ΔG and ∂G are increase in ground water storage and inflow at the aquifer boundary as defined previously. Typically, C_{inj} and C_{rch} are known. The concentration that needs to be solved by the transport model is C.

The ground water flow and mass transport simulation model is governed by two partial differential equations (Bear, 1979). For a 2-dimensional unconfined aquifer, the ground water flow equation can be written as:

$$\frac{\partial}{\partial x}\left(K_{xx} h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{yy} h \frac{\partial h}{\partial y}\right) = S_y \frac{\partial h}{\partial t} - Q' + I' + Inf' + R' \tag{4}$$

where h is the head [L]; K_{xx} and K_{yy} are the hydraulic conductivities [L/T]; and S_y [-] is the specific yield. Q', I', Inf', and R' are the per unit area sink/source terms [L/T] for the mass balance components Q, I, Inf and R as defined above. For a non-sorbing, non-reactive substance, such as total dissolved solids (TDS), the mass transport equation is:

$$\frac{\partial}{\partial x}\left(D_{xx}\frac{\partial C}{\partial x}+D_{xy}\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial y}\left(D_{yx}\frac{\partial C}{\partial x}+D_{yy}\frac{\partial C}{\partial y}\right)-\frac{\partial (V_x C)}{\partial x}-\frac{\partial (V_y C)}{\partial y}$$

$$=\frac{\partial C}{\partial t}+\frac{1}{\theta}(Q''C-Inj''C_{inj}-R''C_{rch}) \tag{5}$$

where D_{xx}, D_{yy}, D_{xy}, D_{yx} are components of the hydrodynamic dispersion coefficient tensor [M^2/T]; V_x and V_y are the pore water velocities in the x- and y-directions [L/T]; θ is the volumetric porosity [-]; and Q'', Inj'' and R'' are the volumetric flux of water per unit volume of the aquifer [1/T].

Since the agricultural demand, infiltration, and recharge are highly seasonal, the management time step for Q and Inj need to be less than one year. As a minimum, a half-year time step representing the wet season and the dry season should be used.

The objective function of the conjunctive use problem is the minimization of the overall cost of the water supply system while the constraints are demand constraints, supply water quality constraints, ground water quality constraints at the extraction wells, head constraints at the extraction and injection wells, and upper bounds imposed by the pump capacities.

Mathematically, the optimization problem can be formulated as the following:

$$\text{Minimize } Z_1 = \sum_{j=1}^{T}\sum_{i=1}^{n} x_{i,j}q_{i,j} + \sum_{j=1}^{T}\sum_{i=1}^{m} y_{i,j}p_{i,j} + \sum_{j=1}^{T}\sum_{i=1}^{l} u_{i,j}s_{i,j} + \sum_{j=1}^{T}\sum_{i=1}^{k} v_{i,j}i_{i,j}$$

(minimal cost objective) (6a)

subject to

$$\sum_{i=1}^{n} q_{i,j} + \sum_{i=1}^{l} s_{i,j} + \sum_{i=1}^{k} i_{i,j} = \left(D_{agri} + D_{M\&I}\right)_j \quad \text{(demand constraint)} \quad (6b)$$

$$\sum_{i=1}^{n} C_{i,j}q_{i,j} + \sum_{i=1}^{l} Cs_{i,j}s_j + \sum_{i=1}^{k} Ci_{i,j}i_j \le C_{supply}\left(\sum_{i=1}^{n} q_i + \sum_{i=1}^{l} s_i + \sum_{i=1}^{k} i_i\right)$$

(supply water quality constraint) (6c)

$$C_{i,j} \le C_{\max} \quad \text{(ground water quality constraint)} \quad (6d)$$

$$h_{i,j} \ge h_{\min} \quad \text{(head constraint at extraction wells)} \quad (6e)$$

$$h_{i',j} \le h_{\max} \quad \text{(head constraint at injection wells)} \quad (6f)$$

$$q_{i,j} \le q_{\max} \quad \text{(pump capacity constraints)} \quad (6g)$$

$$p_{i',j} \le p_{\max} \quad \text{(pump capacity constraints)} \quad (6h)$$

$i=1,2,3,\ldots,n; \quad i'=1,2,3,\ldots,m; \quad j=1,2,3,\ldots,T$

in which x_j, y_j, u_j and v_j are the cost functions [$\$T/L^3$]; T is the total number of management periods in the planning horizon; n, m, l and k are the number of extraction wells, injection wells, surface water sources and imported water sources; q, p, s and i are the extraction rate, injection rate, surface water supply rate and imported water rate for each individual well or sources [L^3/T];

Cs and Ci are the concentrations in the surface water and imported water. Other notations are as defined previously with subscript j denoting different management periods, subscript i denoting different water supply sources, subscript i' denoting injection wells, and subscripts $supply$ and max denoting the allowable concentration for the supply water and water at the extraction well.

All of the above constraints are nonlinear except for the demand and capacity constraints. Constraints (6c) and (6d) require the simulation of both the ground water flow and mass transport models. Constraints (6e) and (6f) require the simulation of the ground water flow model. Solving the above optimization problem is computationally intense and may not be solvable for even a relatively small problem (Taghavi et al. 1994). More importantly, the above optimization stops at time equals T, the end of the planning horizon. However, due to the slow response of the concentration distribution to changes in the pumping scheme, there is no guarantee that the optimal solution obtained for the above formulation will be sustainable over a longer time frame. We will show later that a solution that appeared to be optimal for the planning horizon may have unacceptable long-term adverse effect on the ground water quality that is not recognized by the simulation/optimization solutions. One way to address this problem is to extend the simulation period for constraint (6d) so that the final pumping policy for the management problem is extended over a longer period of time to ensure that the optimal solution is sustainable over a long period of time. However, this would further add to the computational requirements of the optimization problem. In the following, a two-step optimization approach is presented to overcome such difficulties. The two-step approach makes a number of assumptions which are typical of conjunctive use problem that warrants the consideration of both supply water quality constraints as well as ground water quality constraints. These assumptions are:

(1) cost of ground water is lower than the cost of other water supply sources,

(2) supply water has better quality than the quality of ground water,

(3) water supply sources other than ground water have better qualities than the quality requirement of the supply water,

(4) quality of the injected water is poorer than the acceptable ground water quality criterion at the extraction wells, and

(5) quality of water at any ground water inflow boundary is poorer than or the same as the quality of ground water of the aquifer.

<u>Step 1: Upper bounds of extraction and injection rates</u>

In this step, the objective is to determine the maximum pumping rate that can be achieved without violating the ground water quality constraints, the head constraints, and the capacity constraints. Demand constraints and supply water quality constraints are ignored for the moment. The optimization problem is formulated as:

$$\text{Maximize } \sum_{i=1}^{n} q_i \tag{7a}$$

subject to

$$C_i \le C_{\max} \tag{7b}$$

$$h_i \ge h_{\min} \tag{7c}$$

$$h_{i'} \le h_{\max} \tag{7d}$$

$$q_i \leq q_{\max} \tag{7e}$$

$$p_{i'} \leq p_{\max} \tag{7f}$$

Decision variables for this problem are q_i and $p_{i'}$. The simulation model is run for a sufficiently long period of time such that C_i reaches steady state. This optimization problem is solved for each management period where the water balance is different from other management periods. Solutions from this step will form an upper bound for q_i and p_i for each management period - $\overline{q}_{i,j}$ and $\overline{p}_{i',j}$, $j=1,2,3,...T$.

<u>Step 2: Optimal pumping scheme using an iterative nonlinear programming technique</u>

In this step, it is assumed that if the extraction and injection rates are less than the upper bounds established in Step 1, the ground water quality constraints will be satisfied. This assumption is reasonable since the local ground water source is assumed to be the least expensive of all the water supply sources and its quality is assumed to be the poorest. Any policy that will minimize the overall cost of the water supply will be almost the same as the one that would maximize the amount of ground water extraction. The management problem for this step is set up as follows:

$$\text{Minimize } Z_1 = \sum_{j=1}^{T}\sum_{i=1}^{n} x_{i,j}q_{i,j} + \sum_{j=1}^{T}\sum_{i=1}^{m} y_{i,j}p_{i,j} + \sum_{j=1}^{T}\sum_{i=1}^{l} u_{i,j}s_{i,j} + \sum_{j=1}^{T}\sum_{i=1}^{k} v_{i,j}i_{i,j} \tag{8a}$$

subject to

$$\sum_{i=1}^{n} q_{i,j} + \sum_{i=1}^{l} s_{i,j} + \sum_{i=1}^{k} i_{i,j} = \left(D_{agri} + D_{M\&I}\right)_j \qquad (8b)$$

$$\sum_{i=1}^{n} C_{i,j} q_{i,j} + \sum_{i=1}^{l} Cs_{i,j} s_j + \sum_{i=1}^{k} Ci_{i,j} i_j \leq C_{supply} \left(\sum_{i=1}^{n} q_i + \sum_{i=1}^{l} s_i + \sum_{i=1}^{k} i_i \right) \qquad (8c)$$

$$h_{i,j} \geq h_{\min} \qquad (8d)$$

$$h_{i',j} \leq h_{\max} \qquad (8e)$$

$$q_{i,j} \leq \overline{q}_{i,j} \qquad (8f)$$

$$p_{i',j} \leq \overline{p}_{i,j} \qquad (8g)$$

$i=1,2,3,\ldots,n; \quad i'=1,2,3,\ldots,m; \quad j=1,2,3,\ldots,T$

To handle the nonlinear supply water quality constraint, Yeh et al. (1995) linearized the constraint by substituting $C_{i,j}$ with TDS concentrations calculated from previous iterations. After an optimal set of q_i and $p_{i'}$ is obtained based on the known concentrations, $C_{q,j}$ is updated with concentrations calculated using the new optimal pumping policy. This process is repeated until convergence criterion is met. It has been found that with a reasonably good initial guess, convergence can be achieved in just two to three iterations.

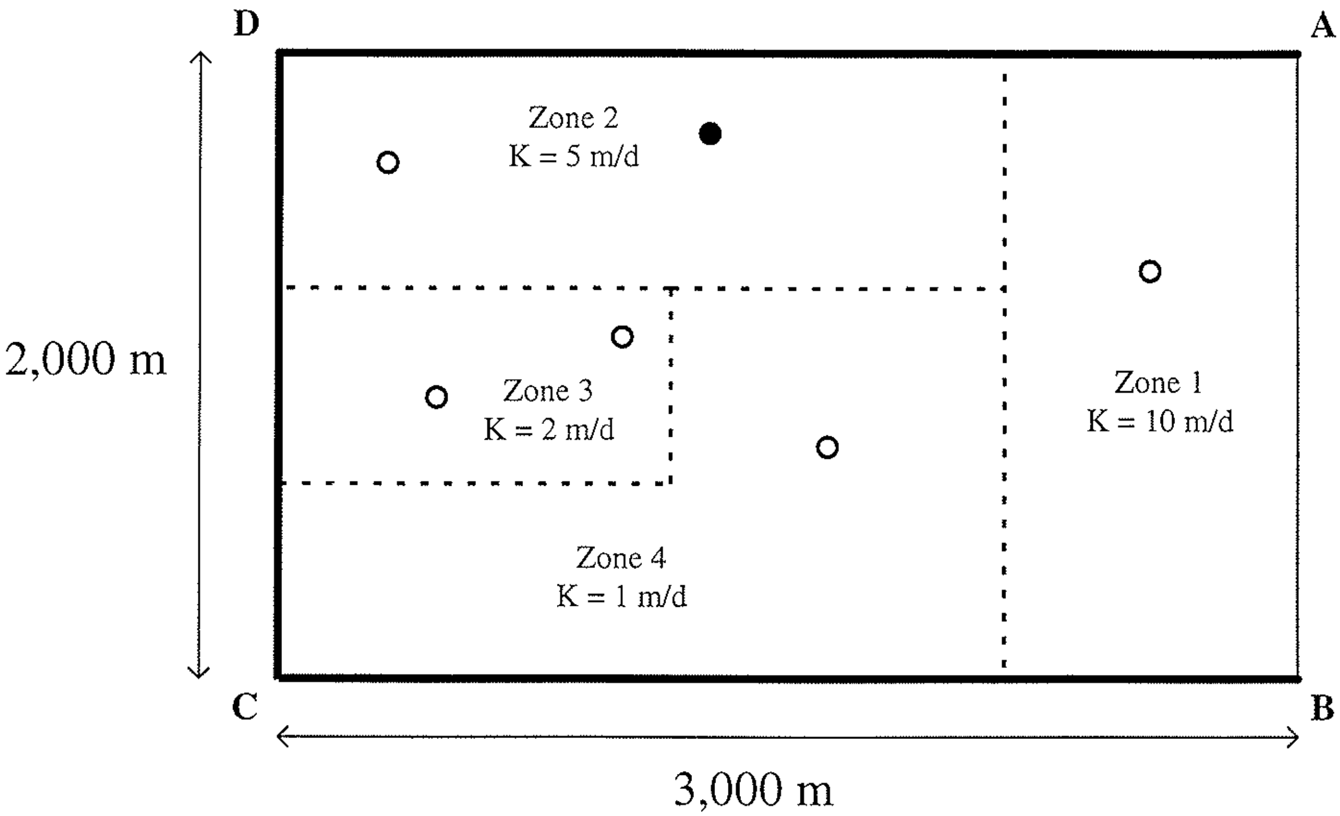
Using this iteration technique and the upper bounds obtained in Step 1, the number of nonlinear constraints in the original management problem can be reduced by more than half. Furthermore, the nonlinearity is limited to the head constraints which only requires running the ground water flow simulation model. This significantly reduces the computational requirements of the conjunctive use problem.

TEST PROBLEM

The test problem consists of an unconfined aquifer as shown in Figure 2. The aquifer has a rectangular shape with three impermeable boundaries and a constant head and constant concentration boundary. There are four zones within the aquifer with different hydraulic conductivities ranging from 10 m/d to 1 m/d. The size of the aquifer is 2,000 m wide and 3,000 m long with a flat bottom 100 m below the ground surface. Water balance of the aquifer consists of infiltration from precipitation, areal recharge of used water, injection of used water at an injection well, extraction of ground water at extraction wells and ground water flow that crosses the fixed head boundary. Areal recharge of reclaimed water is assumed to be a fraction of the agricultural and M&I water usages. Table 1 lists the aquifer characteristics and water balance components for the test problem.

Ground water and imported water are the only sources of water supply for the test problem. The agricultural water demand will remain constant throughout the 10-year planning horizon. The M&I water demand will grow at a stepping rate of 20% every two years in the next ten years.

Initially, a network of four extraction wells with pumping rates ranging from 1,000 m^3/d to 3,000 m^3/d are used to obtain a steady state in both head and TDS distributions for the specified initial conditions for the management problem. An additional extraction well and an injection well are added to the system for a total of six wells for the management problem after time zero.



○ Existing extraction wells

● New extraction well

○ New injection well

AB Constant head, constant concentratoin boundary (75 m; 400 mg/l)

BC
DC } No flow boundary
DA

Figure 2. Plan View of the Aquifer

Table 1. Model Inputs

Aquifer Dimensions	
Aquifer width	2,000 m
Aquifer length	3,000 m
Aquifer bottom	100 m below ground surface (bgs)
Specific Yield	0.001
Permeability	
Zone 1	10 m/d
Zone 2	5 m/d
Zone 3	2 m/d
Zone 4	1 m/d
Precipitation	
Wet season (Oct - Mar)	9.76 inches (4,075 m^3/d)
Dry season (Apr - Sep)	2.9 inches (875 m^3/d)
Infiltration	
Wet season (Oct - Mar)	0.976 inches (815 m^3/d)
Dry season (Apr - Sep)	0.118 inches (100 m^3/d)
Agricultural Demand (constant during planning horizon)	
Wet season	1.722 inches (1,440 m^3/d)
Dry season	14.291 inches (11,940 m^3/d)
Municipal & Industrial Demand (uniform year round)	
Year 0	6.0 inches (5,010 m^3/d)
Year 1-2	7.2 inches (6,010 m^3/d)
Year 3-4	8.4 inches (7,015 m^3/d)
Year 5-6	9.6 inches (8,020 m^3/d)
Year 7-8	10.8 inches (9,020 m^3/d)
Year 9-10	12.0 inches (10,020 m^3/d)
Recharge	
Wet season	
Year 0	1.80 inches (1,525 m^3/d)
Year 1-2	2.04 inches (1,730 m^3/d)
Year 3-4	2.28 inches (1,930 m^3/d)
Year 5-6	2.52 inches (2,135 m^3/d)
Year 7-8	5.76 inches (2,340 m^3/d)
Year 9-10	3.00 inches (2.540 m^3/d)
Dry season	
Year 0	6.20 inches (5,250 m^3/d)
Year 1-2	6.44 inches (5,455 m^3/d)
Year 3-4	6.68 inches (5,655 m^3/d)
Year 5-6	6.92 inches (5,860 m^3/d)
Year 7-8	7.16 inches (6,065 m^3/d)
Year 9-10	7.40 inches (6,265 m^3/d)
Min. and Max. water levels at well heads	65 m; 85 m
Head at river boundary	75 m
Water Quality Parameters	
TDS of infiltration	0 mg/L
TDS at river boundary	400 mg/L
TDS of M&I and agri. returned water	400 mg/L
TDS of injected water	550 mg/L
TDS of imported water	120 mg/L
Allowable TDS level in water supply	250 mg/L
Allowable TDS level at well heads	400 mg/L

The test aquifer is represented by a two-dimensional finite difference grid with a uniform grid size of 100 m x 100 m. The smallest management time step is half-year, representing the winter and summer seasons in a year. MODFLOW (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1995) and MT3D (Zheng,1990)were used to simulate the ground water flow and TDS transport in the aquifer, respectively. MINOS (Murtagh and Saunders, 1995) was used to solve the nonlinear management problem.

The optimization model for the first step is formulated as:

$$\text{Maximize} \sum_{j=1}^{2}\sum_{i=1}^{5} q_{i,j} \tag{9a}$$

subject to

$$C_{i,j} \leq 400 \tag{9b}$$

$$h_{i,j} \geq 65 \tag{9c}$$

$$h_{6,j} \leq 85 \tag{9d}$$

$$q_{i,j} \leq 5{,}000 \tag{9e}$$

$$p_j \leq 5{,}000 \tag{9e}$$

$i=1,2,3,4,5;\; j=1,2$

The objective function sums the extraction rates for both the wet season and the dry season within a year. This optimization problem is solved for all five water balance conditions corresponding to the five increases in the projected water demand in the next ten years. The simulation model is run by repeating the extraction and injection rates for the winter and summer seasons for 80 years. Heads and TDS concentrations at the extraction wells at the end of 80 years and 79-1/2 years, end of the last winter and summer seasons, are used as the constraints. The capacities of the extraction and injection wells are set at 5,000 m^3/d. Operating costs, supply water quality constraints and demands are ignored at this stage.

The solutions of the step one optimization are presented in Table 2. The amount of ground water that can be extracted increases with increasing demands, while the amount of injection decreases. This is due to the increase in the recharge as water demand increases. The injection of water to the aquifer is much higher during winters when recharge is small. Figures 3 and 4 show the water table at the end of the winter and summer seasons during the last year of the simulation. Figures 5 and 6 show the corresponding TDS distribution in the aquifer. The TDS distribution remains fairly steady even though the water table varies quite significantly between the two periods due to changes in the pumping strategy. Figure 7 shows the difference in TDS distribution between the end of winter and the end of summer in the last modeling year. The difference is highest near the edge of the TDS "plumes" as the "plume" in essence expands and contracts slightly as it goes from winter to summer and from summer to winter, respectively. Figure 8 shows the concentration versus time for one of the runs. Notice that TDS levels in the extraction wells did not reach 400 mg/L, the ground water quality limit, until it is almost sixty years into the simulation. For the wells that reached 400 mg/L, the TDS concentrations in the first ten or even twenty years are much lower than 400 mg/L and are fairly stable. If the simulation/ optimization model is only run for the length of the planning horizon, the ground water quality constraints on the pumping strategy would have little effect on the pumping strategy

Table 2. Solutions from Step One and Step Two Optimization

		Demand		Infil./Recharge		Upper Bounds (Step One)						Optimal Solution (Step Two)						
		Agri.	Domestic	Infil.	Recharge	Q1	Q2	Q3	Q4	Q5	Q6	Q1	Q2	Q3	Q4	Q5	Q6	Import
Year 1	wet	1,438	6,010	815	1,730	5,000	1,210	967	1,003	2,530	1,932	0	1,210	967	1,003	1,034	1,932	3,234
	dry	11,935	6,010	100	5,455	5,000	1,473	1,019	1,041	2,427	0	3,847	1,473	1,019	1,041	2,427	0	8,138
Year 2	wet	1,438	6,010	815	1,730	5,000	1,210	967	1,003	2,530	1,932	0	1,210	967	1,003	1,034	1,932	3,234
	dry	11,935	6,010	100	5,455	5,000	1,473	1,019	1,041	2,427	0	3,847	1,473	1,019	1,041	2,427	0	8,138
Year 3	wet	1,438	7,015	815	1,930	5,000	1,285	980	1,014	2,544	1,917	0	1,285	980	1,014	1,496	1,917	3,677
	dry	11,935	7,015	100	5,655	5,000	1,454	1,110	1,054	2,467	0	4,193	1,454	1,110	1,054	2,467	0	8,671
Year 4	wet	1,438	7,015	815	1,930	5,000	1,285	980	1,014	2,544	1,917	0	1,285	980	1,014	1,496	1,917	3,677
	dry	11,935	7,015	100	5,655	5,000	1,454	1,110	1,054	2,467	0	4,193	1,454	1,110	1,054	2,467	0	8,671
Year 5	wet	1,438	8,020	815	2,135	5,000	1,332	1,006	1,028	2,566	1,895	0	1,332	1,006	1,028	1,973	1,895	4,119
	dry	11,935	8,020	100	5,860	5,000	1,500	1,146	1,069	2,496	0	4,537	1,500	1,146	1,069	1,626	0	10,077
Year 6	wet	1,438	8,020	815	2,135	5,000	1,332	1,006	1,028	2,566	1,895	0	1,332	1,006	1,028	1,973	1,895	4,119
	dry	11,935	8,020	100	5,860	5,000	1,500	1,146	1,069	2,496	0	4,537	1,500	1,146	1,069	1,626	0	10,077
Year 7	wet	1,438	9,020	815	2,340	5,000	1,370	1,033	1,041	2,589	1,873	0	1,370	1,033	1,041	2,457	1,873	4,557
	dry	11,935	9,020	100	6,065	5,000	1,566	1,168	1,083	2,520	0	4,878	1,566	1,168	1,083	2,520	0	9,740
Year 8	wet	1,438	9,020	815	2,340	5,000	1,370	1,033	1,041	2,589	1,873	0	1,370	1,033	1,041	2,457	1,873	4,557
	dry	11,935	9,020	100	6,065	5,000	1,566	1,168	1,083	2,520	0	4,878	1,566	1,168	1,083	2,520	0	9,740
Year 9	wet	1,438	10,020	815	2,540	5,000	1,393	1,048	1,046	2,562	1,754	363	1,393	1,048	1,046	2,562	1,754	5,046
	dry	11,935	10,020	100	6,265	5,000	1,628	1,216	1,107	2,584	94	5,000	1,628	1,216	1,107	2,584	94	10,420
Year 10	wet	1,438	10,020	815	2,540	5,000	1,393	1,048	1,046	2,562	1,754	363	1,393	1,048	1,046	2,562	1,754	5,046
	dry	11,935	10,020	100	6,265	5,000	1,628	1,216	1,107	2,584	94	5,000	1,628	1,216	1,107	2,584	94	10,420

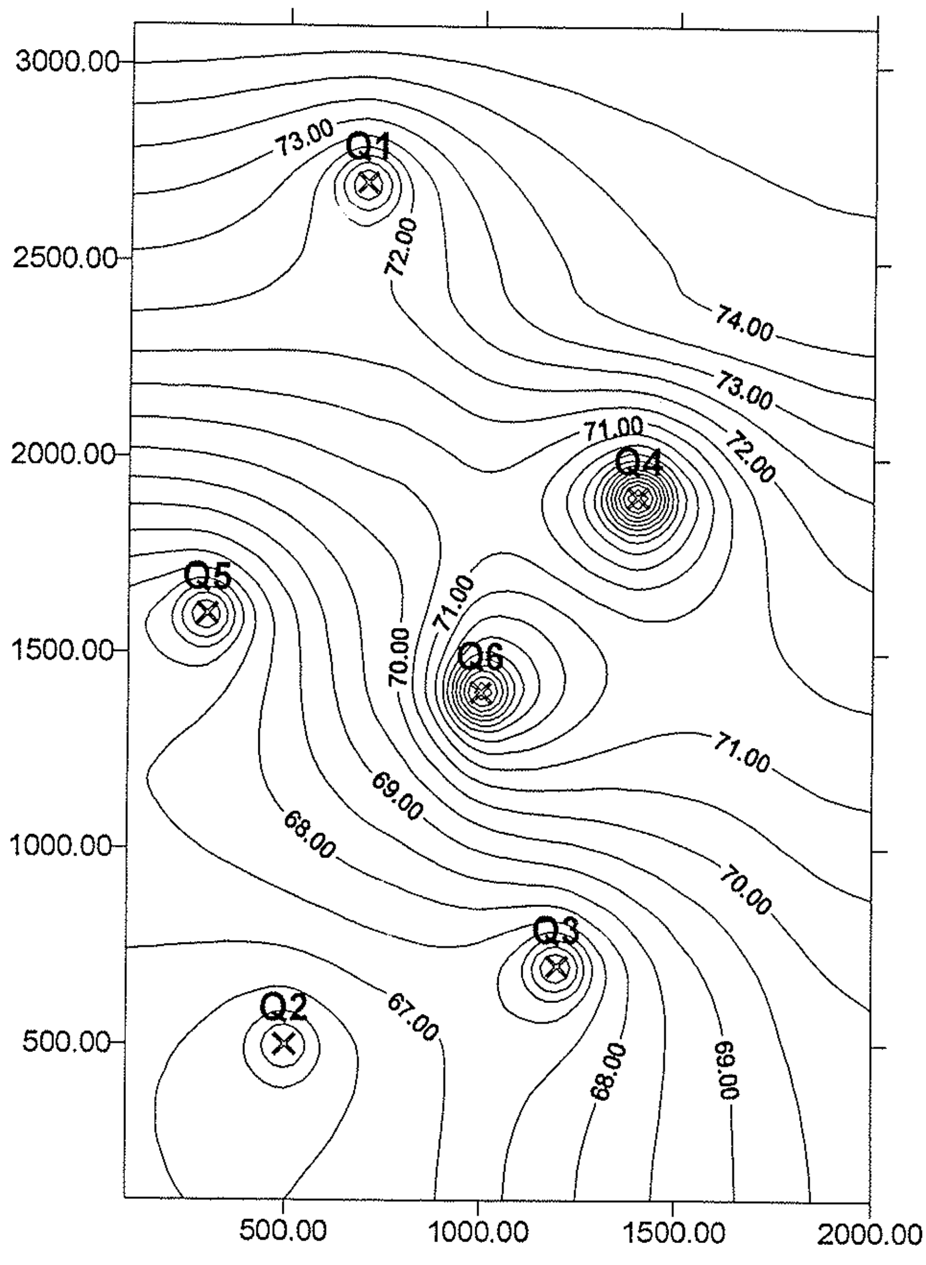


Figure 3. Ground Water Level at the End of 79-1/2 Years

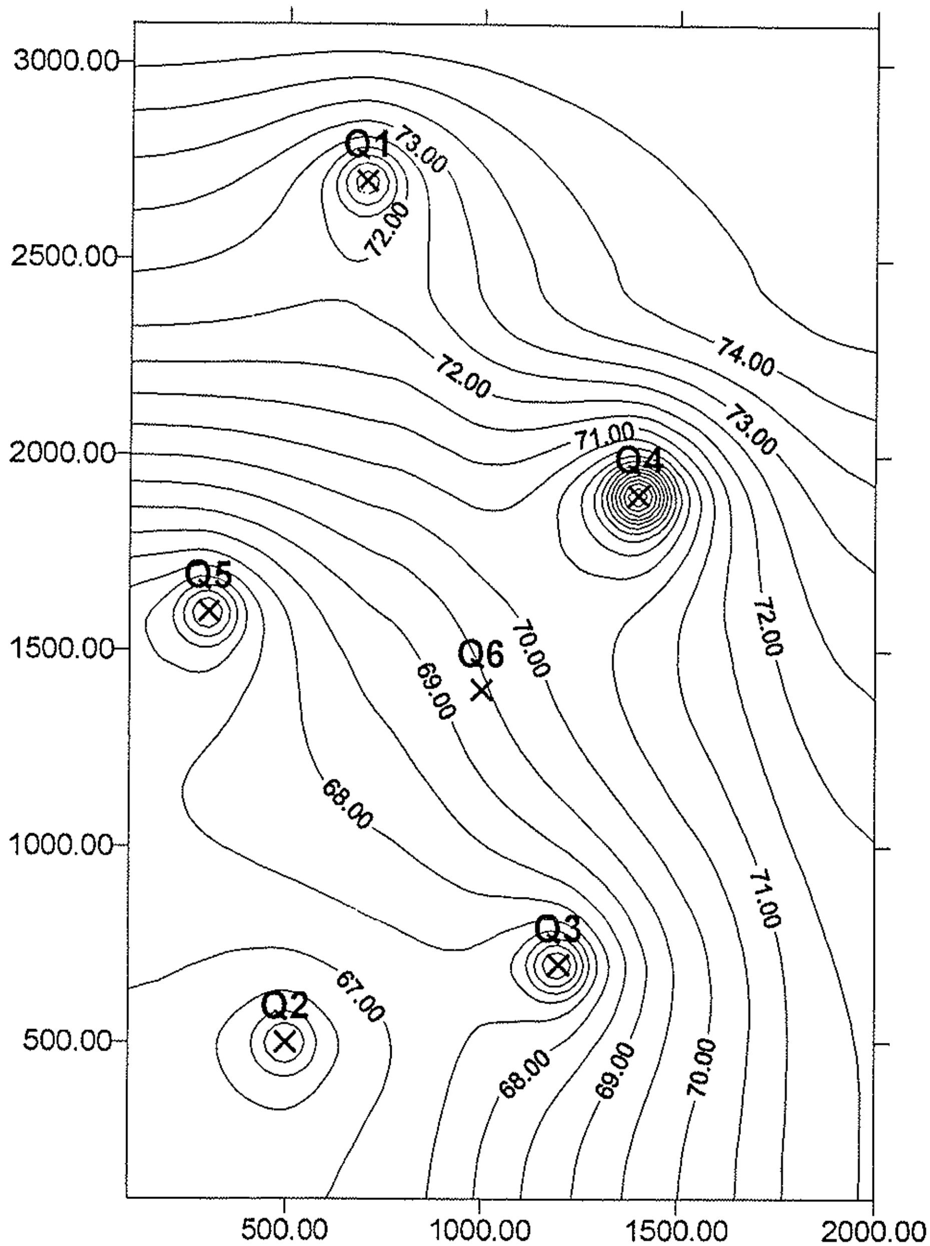


Figure 4. Ground Water Level at the End of 80 Years

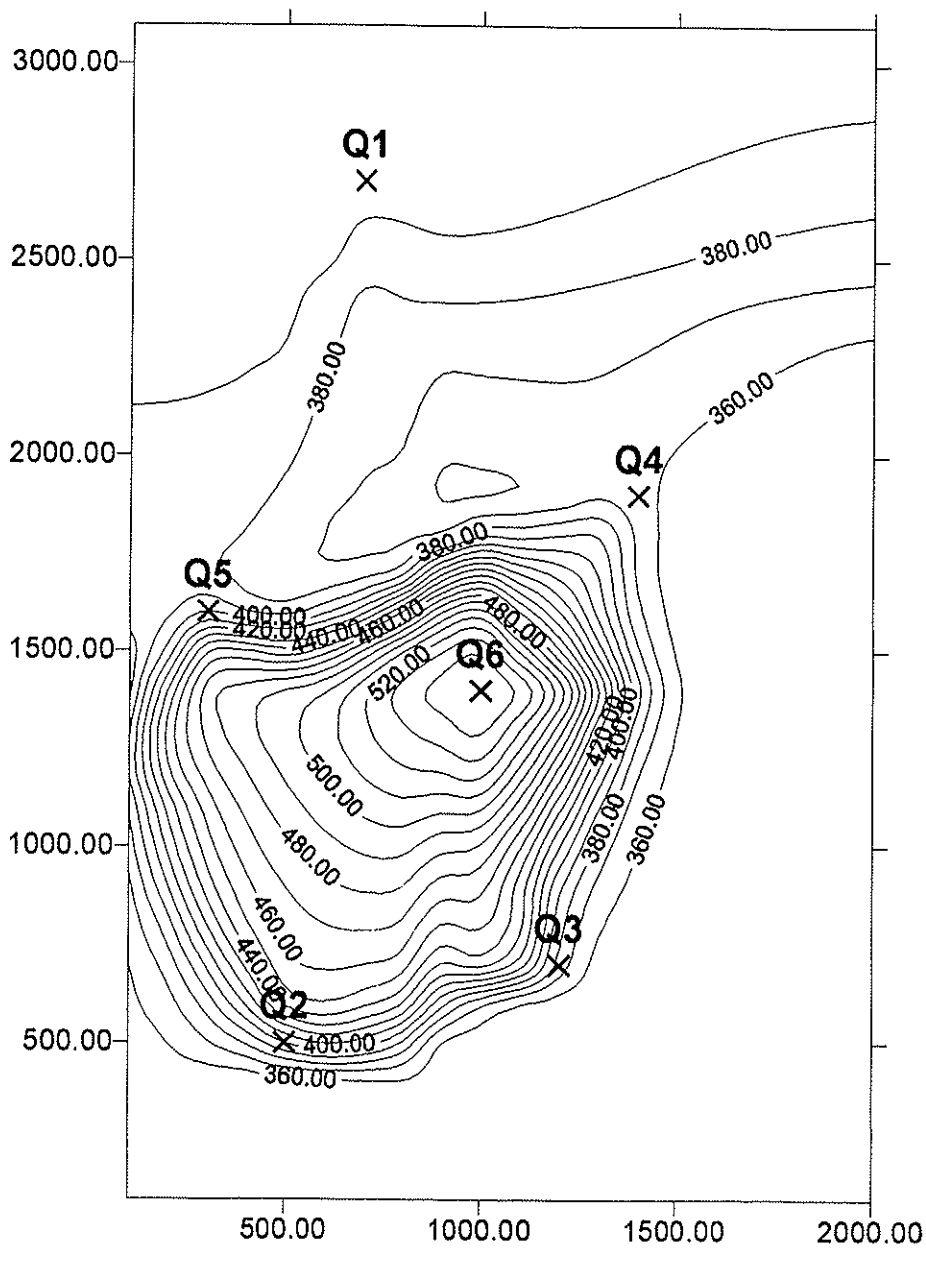


Figure 5. TDS Levels at the End of 79-1/2 Years

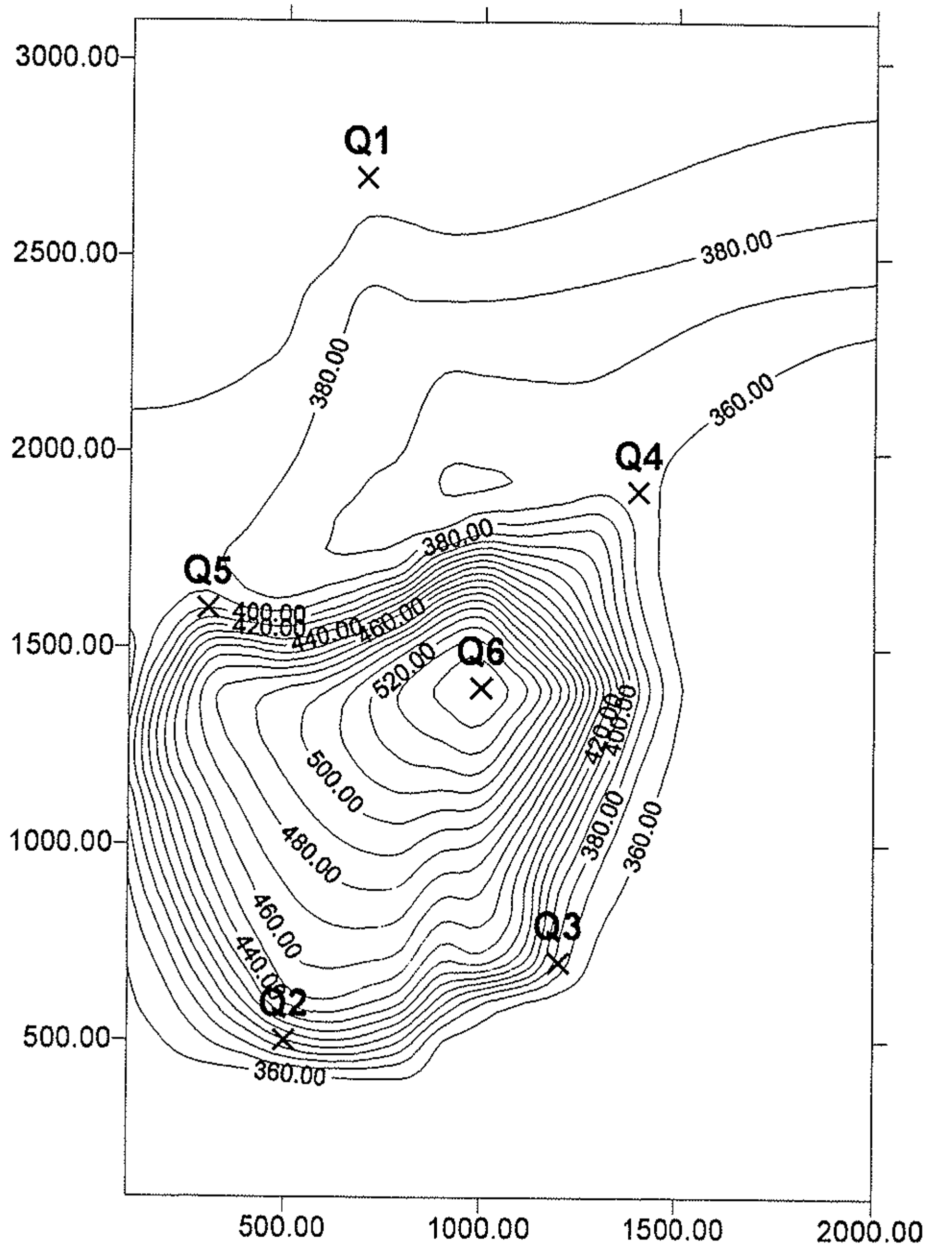


Figure 6. TDS Levels at the End of 80 Years

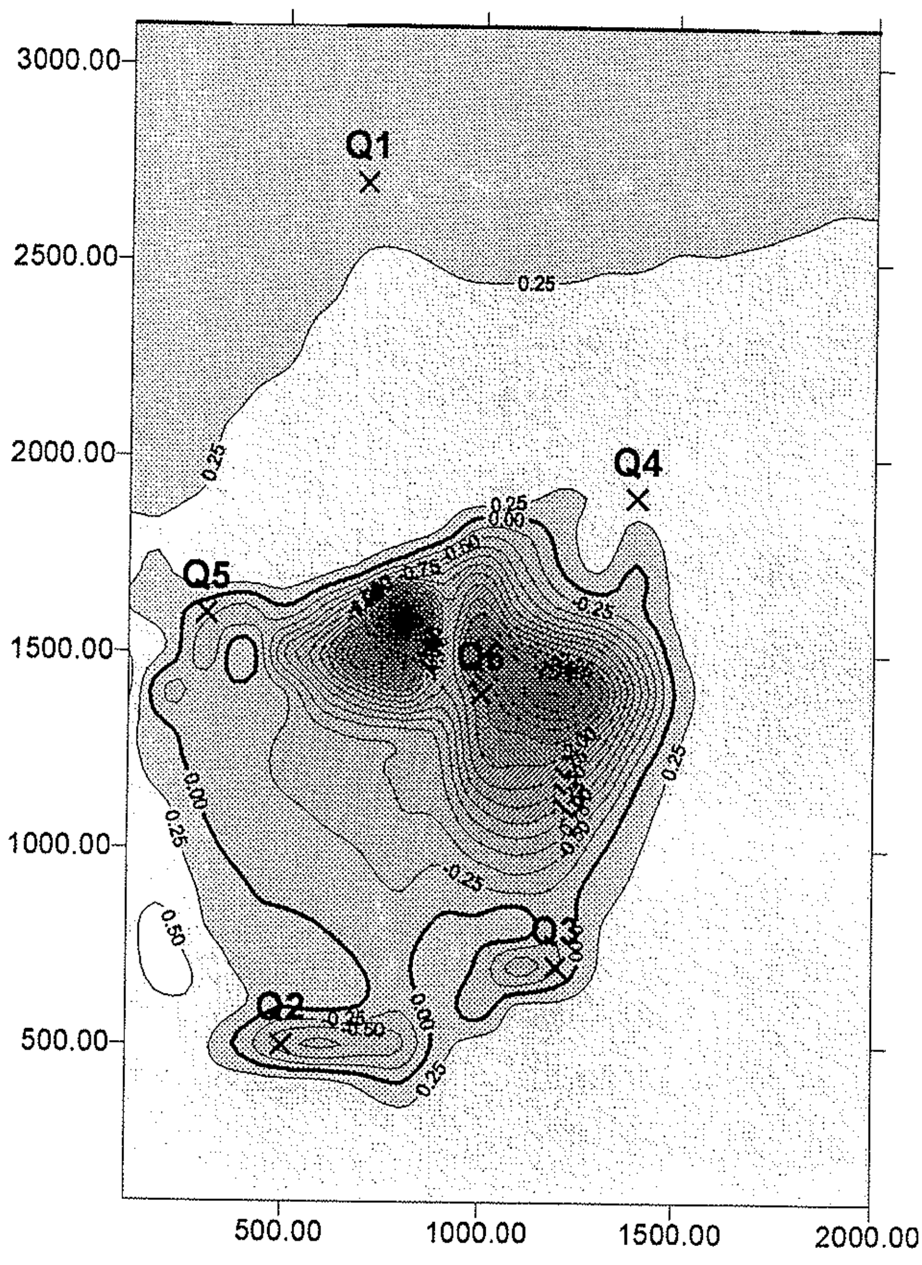


Figure 7. Change in TDS Levels Between 79-1/2 and 80 Years

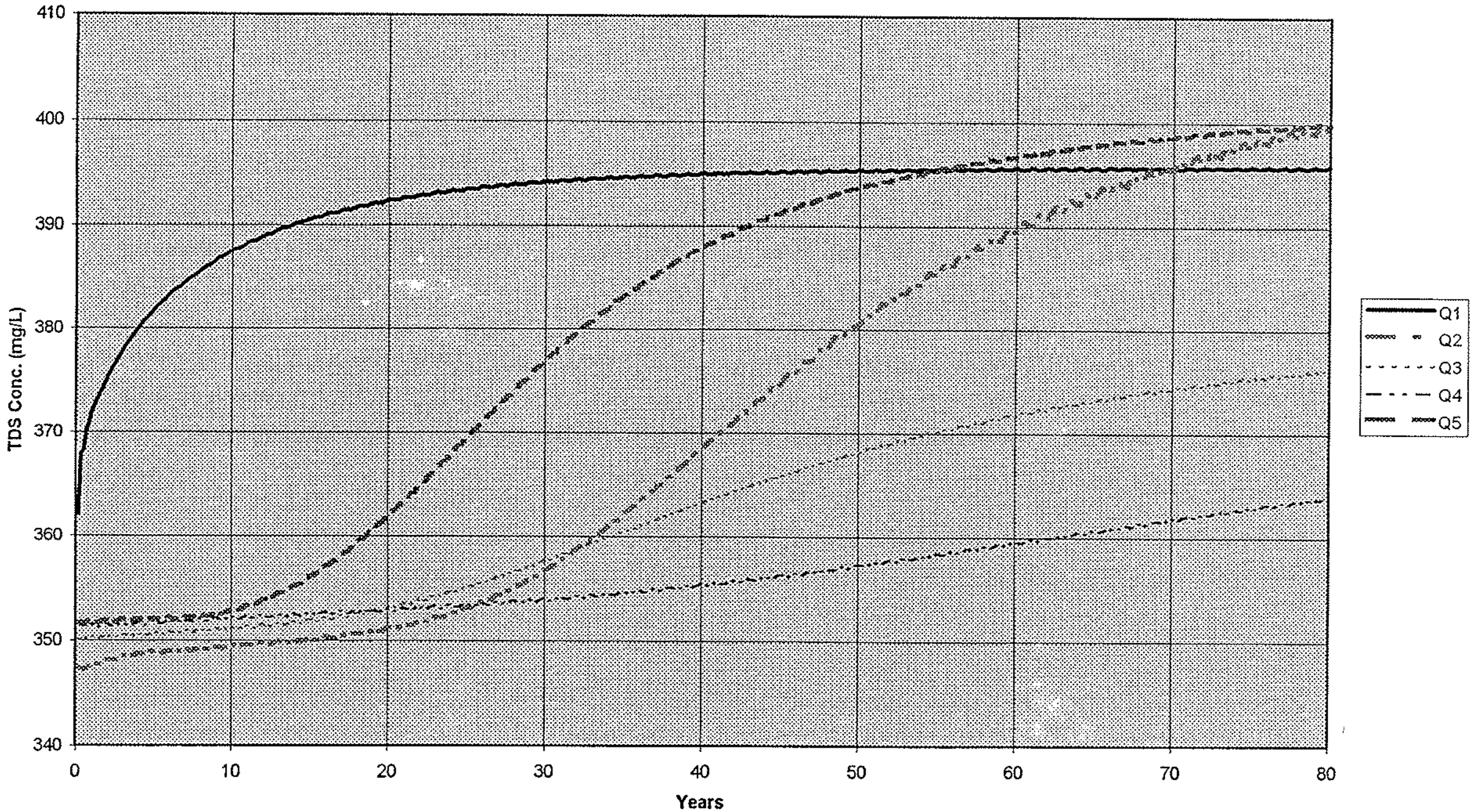


Figure 8. TDS Levels vs. Time (Upper Bounds for Q)

during the planning horizon. Therefore, the model would overestimate the optimal pumping and injection rates and the ground water quality constraints will be violated when the pumping strategy is extended for a long period of time.

The second step optimization uses the upper bounds from step one as constraints to replace the original ground water quality constraints. The objective function is the minimization of the cost over the 10-year planning horizon.

$$\text{Minimize } Z_2 = \sum_{j=1}^{20}\sum_{i=1}^{5} xq_{i,j} + \sum_{j=1}^{20} yp_j + \sum_{j=1}^{20} ui_j \tag{10a}$$

subject to

$$\sum_{i=1}^{5} q_{i,j} + p_j = \left(D_{agri} + D_{M\&I}\right)_j \tag{10b}$$

$$\sum_{i=1}^{5} q_{i,j}C^*_{i,j} + i_j C_I \le 250\left(\sum_{i=1}^{5} q_{i,j} + i_j\right) \tag{10c}$$

$$h_{i,j} \ge 65 \tag{10d}$$

$$h_{6,j} \le 85 \tag{10e}$$

$$q_{i,j} \le \overline{q}_{i,j} \tag{10f}$$

$$p_j \le \overline{p}_j \tag{10g}$$

$i=1,2,3,4,5;\qquad j=1,2,3,4,\ldots,19,20$

where x, y and u are the cost functions which are equal to \$0.05, \$0,033 and \$0.09 per cubic meter, respectively; p_j is the injection rate; $C^*_{i,j}$ are the TDS at the extraction wells from the initial policy or the previous iteration; $j=1,2,3,4,\ldots, 19,20$ represents each of the winter and summer seasons from the first year to the end of the tenth year. The solutions to the step two optimization are also presented in Table 2. Using the upper bounds for the extraction and injection rates as the initial policy, the optimal solution is obtained in three iterations. Notice that the optimal pumping strategy tends to follow the upper bounds established in step one. The exceptions are in times, mostly winters, when the demand is small and the supply water quality constraint is a limiting constraint. During these times, reduced extraction from wells with high TDS levels and more imported water are needed to satisfy the constraint. Figures 9 and 10 show the ground water table at the end of the winter and summer seasons in the tenth year, and figures 11 and 12 show the corresponding TDS levels at the two time periods. Despite the pumping policy changes significantly from winter to summer in response to the changes in demand and recharge to the aquifer, the TDS distribution in the aquifer remains fairly stable. In essence, the network of injection and extraction wells contain the TDS "plume" effectively such that high TDS levels would not reach the extraction wells.

CONCLUSIONS

The results of the test problem demonstrate that the proposed two-step optimization method can successfully solve a conjunctive use problem with supply water quality constraints and ground water quality constraints. The proposed methodology is suitable for conjunctive use problem with the following characteristics: use of ground water is preferred because of its lower cost; TDS levels in the extracted ground water are higher than drinking water quality requirement so blending with water from other sources is required; TDS levels in the injected water are higher

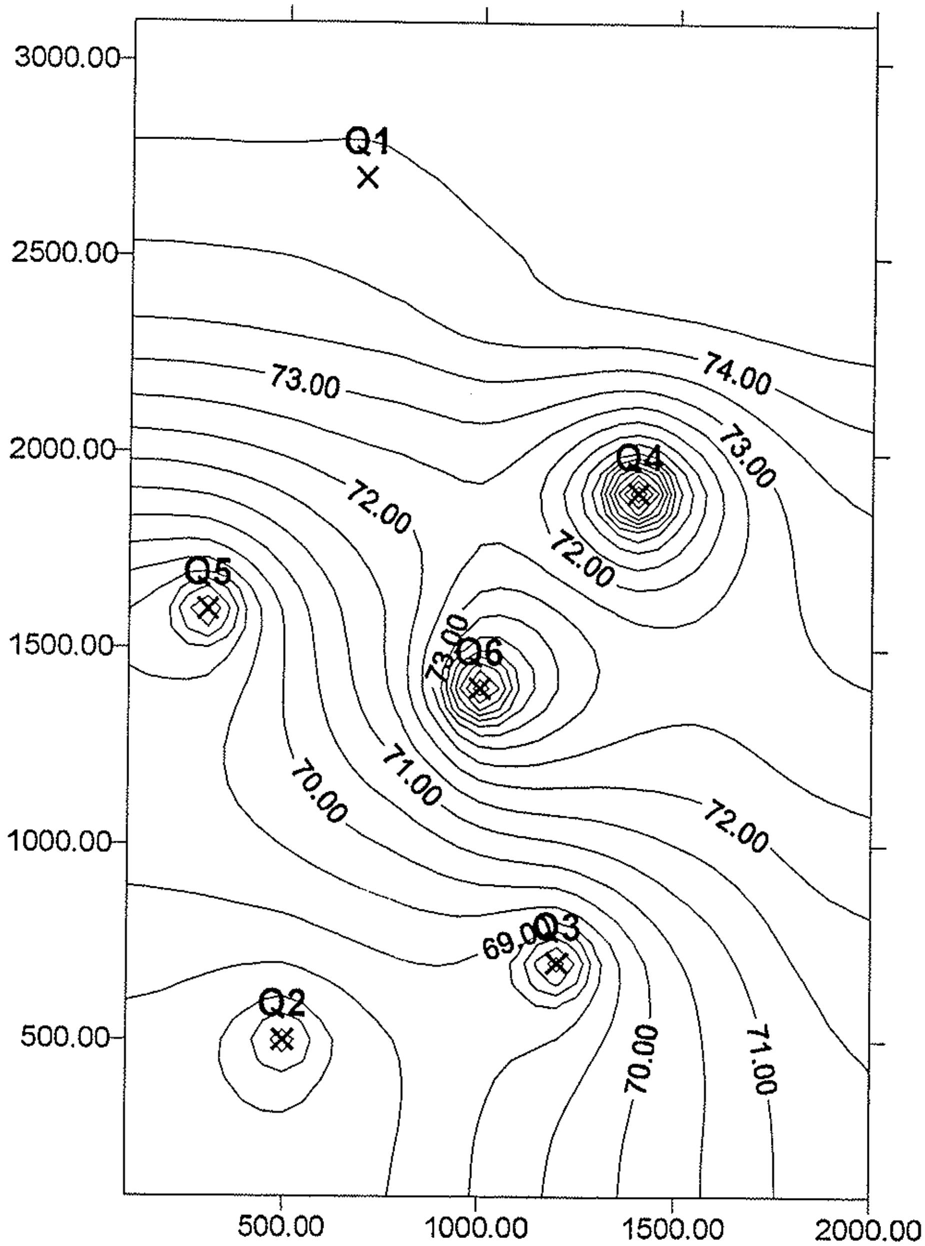


Figure 9. Ground Water Levels at the End of 9-1/2 Years

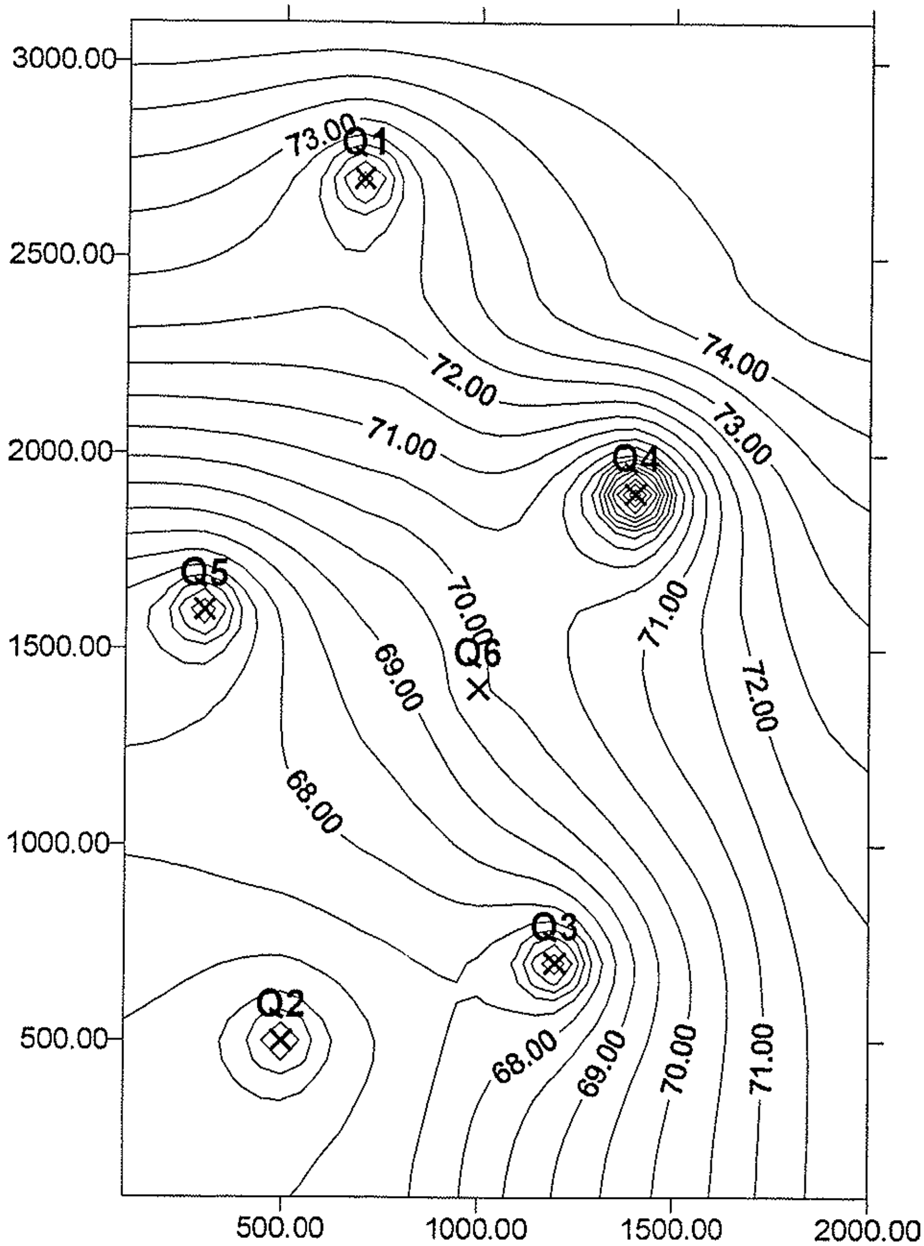


Figure 10. Ground Water Levels at the End of 10 Years

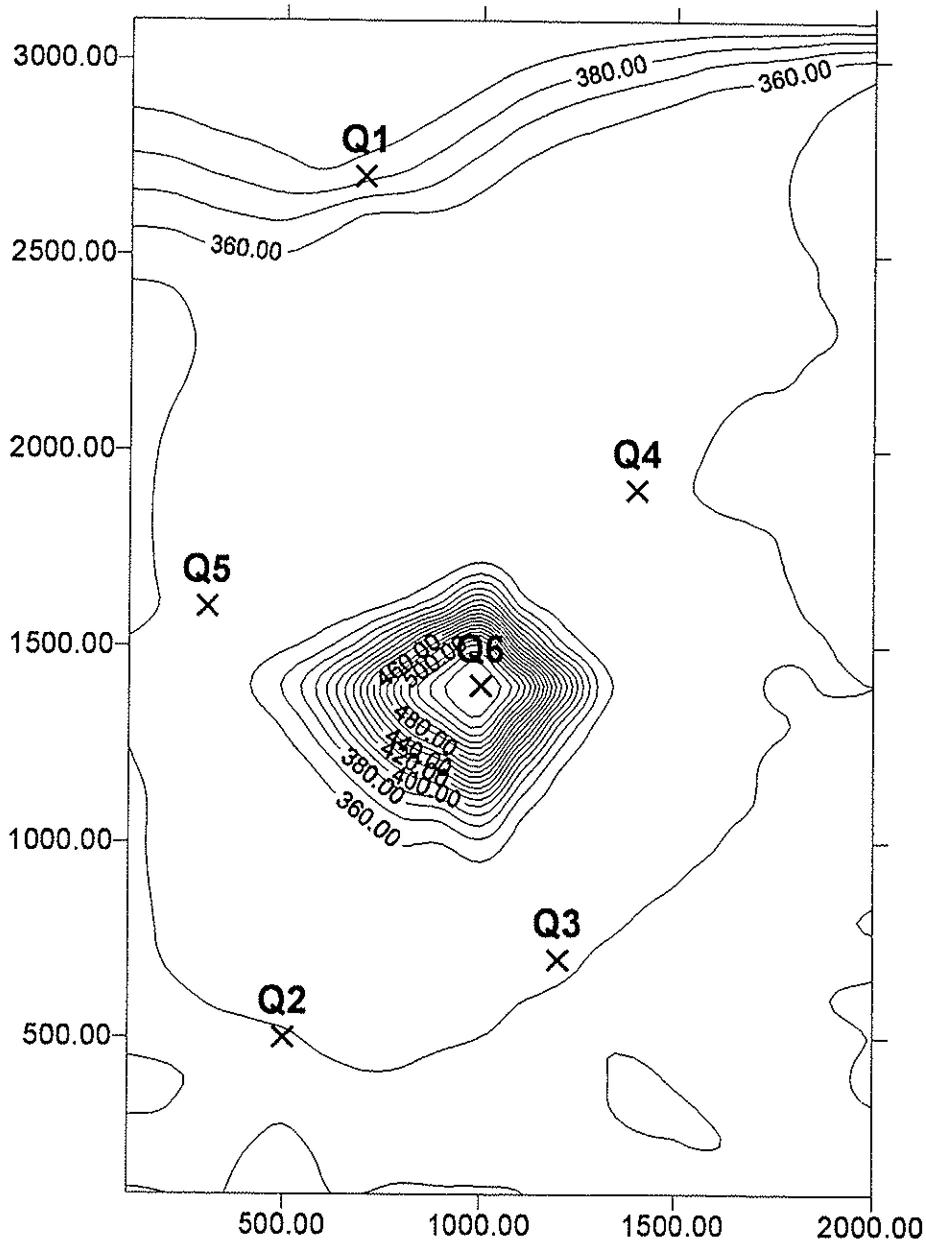


Figure 11. TDS Levels at the End of 9-1/2 Years

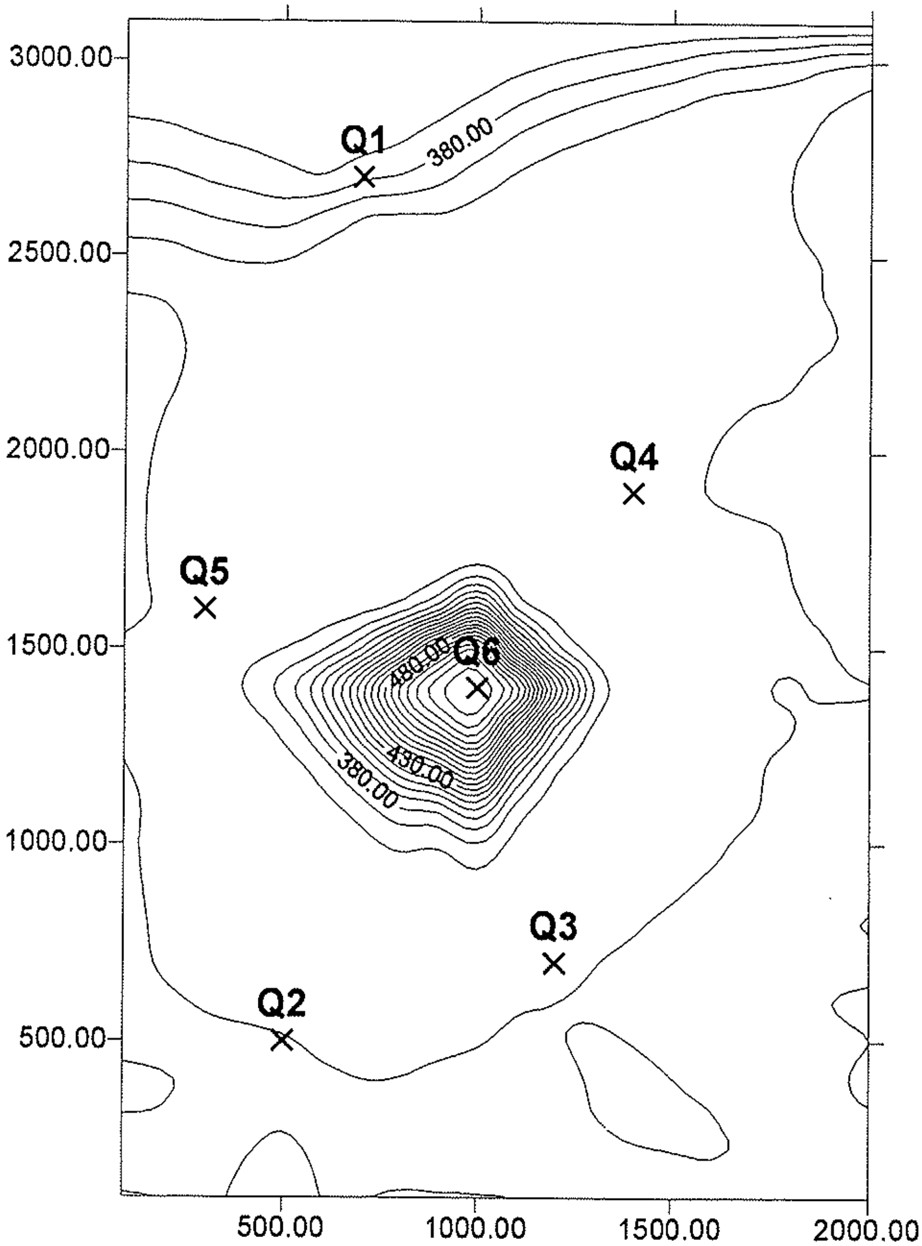


Figure 12. TDS Levels at the End of 10 Years

than the ground water quality requirement at the extraction wells. These conditions are consistent with the case when both ground water quality and supply water quality are important to the conjunctive use problem. The proposed methodology can also be used for other conjunctive use problems by modifying the optimization problem in step one. For example, a contaminated aquifer that needs to be restored by natural infiltration as well as strategic extraction and injection as part of an overall water supply policy can be solved by modifying step one in the proposed methodology to obtain the lower bounds of extraction rates and injection rates that would restore the aquifer over time and minimize the overall cost in step two. There are two major advantages of the new approach. First, the optimal solution obtained from the management model is assured to be sustainable beyond the planning horizon without an unacceptable impact on the ground water quality. Second, the computational effort of the original nonlinear programming problem can be drastically reduced. Table 3 shows the reduction of the number of decision variables and nonlinear constraints in comparison with the original conjunctive use problem. The fact that in both steps of the two-step approach, a basic optimization problem is run with only minor changes in the model inputs. This means that knowledge obtained from prior runs of the optimization model can be used very efficiently to reduce the computations for subsequent runs. For example, in step one of the above test problem, the original number of concentration and head constraints was set at twenty after careful inspection of the problem. After the first run, it was noticed that only eight constraints were at or close to the limits; the other twelve inactive constraints were then dropped from subsequent runs to reduce the computation time.

Table 3. Computational Efficiency of the Two-Step Approach

	Original problem[a]	1st step optimization	2nd step optimization
Number of decision variables	(N+K)*T	N+K	(N+K)*T
Number of non-linear constraints	2N+K+T	2N+K	N+K
Groundwater flow model	Yes	Yes	Yes
Mass transport model	Yes	Yes	No[b]
Number of runs/problems	One	T	<5[c]

[a] Assuming extraction rates and injection rates are the only decision variables.

[b] Mass transport model is not needed except in between iteration to update the linearized supply water quality constraints.

[c] With a good initial policy, convergence can be achieved in just a few iterations.

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