Title
QUARK MODEL COMPARISONS WITH p? A++ AND wA++ DATA AT 3.7 GeV/c

Author
Abrams, G.S.

Publication Date
1972-09-01

Submitted to Physical Review D

LBL-961
Preprint

QUARK MODEL COMPARISONS WITH $\rho^0\Delta^{++}$ AND $\omega\Delta^{++}$ DATA AT 3.7 GeV/c

G. S. Abrams and K. W. J. Barnham

September 19, 1972

AEC Contract No. W-7405-eng-48

LBL-961

DISCLAIMER

LBL-961

QUARK MODEL COMPARISONS WITH $\rho^{0}\Delta^{++}$ AND $\omega\Delta^{++}$ DATA AT 3.7 GeV/c[*]

G. S. Abrams and K. W. J. Barnham[†]

Department of Physics and Lawrence Berkeley Laboratory
University of California, Berkeley, California 94720

September 19, 1972

ABSTRACT

Data for the channels $\pi^{+}p \rightarrow \rho^{0}\Delta^{++}$ and $\pi^{+}p \rightarrow \omega\Delta^{++}$ at 3.7 GeV/c are compared with the quark-model constraints of Białas and Zalewski. The Class A and B predictions are found to be valid as functions of momentum transfer for both channels. The predictions of Class C are found to fail in both s and t channel coordinate systems. It is shown for both reactions that no reference frame exists for which all the Class C relations are valid.

I. INTRODUCTION

Białas and Zalewski[1] obtained constraints on the single and joint decay density matrix elements in double resonance reactions using the nonrelativistic quark model.[2] Their predictions may be divided into three classes A, B, and C, differentiated by increasing severity of theoretical assumptions necessary. The relations of Class A depend only on the additivity assumption (i.e., one writes the full scattering amplitude as a coherent sum of single quark-quark scattering terms) and parity conservation for the quark-quark scattering process. Denoting the quark-quark scattering amplitude as $(cd|ab)$ for the reaction $a + b \rightarrow c + d$, and labeling states solely by the quark spin projections (+ for +1/2, - for -1/2), we write the independent amplitudes (quantized normal to the reaction plane) as

$$f_1 = (++|++), \quad f_2 = (--|--), \quad f_3 = (+-|+-), \quad f_4 = (-+|-+),$$
$$f_5 = (+-|-+), \quad f_6 = (-+|+-), \quad f_7 = (--|++), \quad f_8 = (++|--).$$

The Class B relations follow from the equality

$$f_5 = f_6 \quad ,$$

and Class C from the equality

$$f_7 = f_8 \quad .$$

In both cases these equalities could be justified by invoking time reversal and charge conjugation invariance for free quark-quark scattering. There has been considerable controversy[3] over the validity of the assumptions made in the derivation of the Class B and C relations, especially over the applicability of a nonrelativistic formalism, though the Class A relations are accepted as theoretically valid.

The transformation properties of the relations under rotations differ for each Class. The relations of Class A are invariant under arbitrary, separate rotations of either the vector meson or Δ^{++} coordinate system about the production plane normal, while those of Class B are invariant under arbitrary equal rotations of these coordinate systems about the normal. The Class C relations have no such simple rotational properties, and are thus expected to be valid in one and only one coordinate system.

In the present Paper we test the validity of these constraints for the reactions

$$\pi^+ p \rightarrow \rho^o \Delta^{++} \tag{1}$$

and

$$\pi^+ p \rightarrow \omega \Delta^{++} \tag{2}$$

with data[4] at 3.7 GeV/c. Previous comparisons with experiment for these reactions[5] and for the reaction[6]

$$KN \rightarrow K^* \Delta \tag{3}$$

were forced by lack of data to group events within a single large interval in momentum transfer. Generally these analyses have been able to establish the validity of both the Class A and Class B predictions in both the t-channel and s-channel coordinate systems.

The experimental situation is far less clear for the Class C comparisons. While one may definitely conclude that for all of these reactions the predictions of Class C are violated in both the t-channel and s-channel coordinate systems,[5,6] these observations do not rule out the possible existence of yet a different frame in which these relations are valid. In fact the so-called "dynamic" reference frame of Donohue and Hogaasen[7] has been found by some (though not all) experiments to satisfy the Class C constraints.

In this experiment our statistics allow us to study the validity of the quark model relations for considerably finer momentum transfer intervals than previously possible. Thus we are not as sensitive to possible systematic effects in performing angular averages as were earlier experiments. In Sec.

II below we present our results for the predictions of Classes A and B, and in Section III those for Class C.

II. QUARK MODEL PREDICTIONS OF CLASSES A AND B

The quark model relationships take on their simplest form when expressed in terms of statistical tensors[8] (which are directly related to angular averages) in transversity frames; that is, frames with their quantization axis perpendicular to the scattering plane. Transversity frames are usually denoted transverse-Jackson, transverse-helicity or transverse-dynamic[7] when the (-y) axis points along the z axis of the t-channel, helicity or Donohue-Hogaasen frames, respectively. The relationships are shown in terms of statistical tensors in Table I. In Table II relation A.1 and the right-hand sides of relations A.2-A.6 have been re-expressed in terms of the more familiar, non-transversity single vertex density matrix elements.

To test the quark model relations we use the same data sample as was used in an earlier correlation study,[4] and with the same $t' = |t - t_{min}|$ binning. Our results for the constraints of Class A are shown in Fig. 1 for the $\rho\Delta$ and $\omega\Delta$ channels. In both cases the relations were evaluated in the transverse-Jackson frame. It may be seen that the overall agreement of the data with these relations is quite statisfactory for both reactions. As previously noted the Class A relationships are invariant under a rotation of either rest frame (vector meson or Δ^{++}) about the production plane normal. Therefore if they hold in the transverse-Jackson frame they should also hold in the transverse-helicity frame. The data in the transverse-helicity frame (not shown) do, in fact, show similar agreement.

For the Class B relationships we have noted that they are invariant under equal rotations of the resonance rest systems in the production plane. Since the helicity crossing angle (the angle between the t-channel and s-channel reference frames) of the vector meson differs from that of the Δ^{++}, the Class B relationships cannot hold exactly in both Jackson and helicity frames. In Fig. 2 we display our $\rho\Delta$ and $\omega\Delta$ data to test the Class B relations in the transverse-Jackson coordinate system. The agreement of the data with the model is quite good for both reactions.

A similar comparison of the data in the transverse-helicity coordinate system (not shown) gives equally impressive agreement with the Class B predictions. That we obtain agreement in both frames is not altogether surprising if one considers the vector meson and Δ^{++} crossing angles. Figure 3 shows that at 3.7 GeV/c the angles are typically within 5° of each other for $t' < 1.0\ (\mathrm{GeV}/c)^2$. Hence the rotation angle from t-channel to s-channel coordinate systems is approximately the same for the vector meson and the Δ^{++}, implying our experimental conclusion of the validity of Class B relations in both frames.

III. QUARK MODEL PREDICTIONS OF CLASS C

The Class C predictions do not possess the simple transformation properties under rotations that the Class A and B predictions do. To elucidate the behavior of these relations under rotations we use the transformation law for statistical tensors[8] specialized to rotations about the quantization axis:

$$\left(T^{J_1J_2}_{M_1M_2}\right)' = e^{-iM_1\varphi_1}\, e^{-iM_2\varphi_2}\, T^{J_1J_2}_{M_1M_2} \quad , \qquad \text{(4a)}$$

where one may have different rotation angles φ_1 and φ_2 at the vector meson and Δ^{++} vertices, respectively. In transverse frames the Class C relations 1 through 6 require the vanishing of the imaginary parts of various statistical tensors. Thus (aside from the relation C.7 involving real terms) the validity of the Class C relations implies the existence of rotation angles

φ_1 and φ_2 such that given, say, these statistical tensors in the transverse-Jackson frame, a rotation to this new frame produces purely real tensors.

In Fig. 4 we show our $\rho^o\Delta^{++}$ data testing the Class C predictions in both the transverse-Jackson and transverse-helicity coordinate systems; the corresponding $\omega\Delta^{++}$ data is shown in Fig. 5. Systematic deviations from the expected Class C predictions of zero are observed for several terms for each reaction in both coordinate systems. We conclude that the Class C relations are not valid for either the Jackson-Gottfried (t-channel) or the helicity (s-channel) coordinate system.

These relations may, however, be valid in yet another frame, rotated from the t-chanel axes by the angles φ_1 and φ_2 such that

$$\tan\,(M_1\varphi_1 + M_2\varphi_2) = \frac{\mathrm{Im}\ T^{J_1J_2}_{M_1M_2}}{\mathrm{Re}\ T^{J_1J_2}_{M_1M_2}} \quad . \qquad (4b)$$

To further investigate this point, we note that the validity of relations B.1 and B.2 requires that T^{20}_{20} has the same phase as T^{02}_{02}, so that $\varphi_1 = \varphi_2$. That is to say, if Class B is valid, then a rotation of the vector meson coordinate system by an angle θ_{D-H} about the production normal to its "dynamic reference system" also rotates the Δ^{++} into its dynamic system.[8] A further consequence of the model follows from the validity of the relations A.5 and A.6, which, together with the statement $\varphi_1 = \varphi_2$, imply that in the dynamic reference system

C.1 $\quad \mathrm{Im}\ T^{20}_{20} = 0$ $\quad$ (by definition)[7]

C.2 $\quad \mathrm{Im}\ T^{02}_{02} = 0$ $\quad$ (by B.2)

C.4 $\quad \mathrm{Im}\ T^{22}_{20} = 0$ $\quad$ (by A.5)

C.6 $\quad \mathrm{Im}\, T^{22}_{02} = 0 \quad$ (by A.6 and B.2) .

Therefore four of the seven Class C relations are automatically valid in the dynamic reference system if the Class A and B relations are valid (in either the transverse-Jackson or transverse-helicity frames).

To investigate the validity of the Class C relations, we therefore consider rotations about the production normal which lead to the conditions

C.1 $\quad \mathrm{Im}\, T^{20}_{20} = 0$

C.3 $\quad \mathrm{Im}\, T^{22}_{22} = 0$

C.5 $\quad \mathrm{Im}\, T^{22}_{11} = 0$.

The angular rotations required to transform these complex statistical tensors in the transverse-Jackson frame to purely real terms can be found using Eq. (4b) and are shown in Figs. 6(a) and 6(b) for $\rho^o\Delta^{++}$ and $\omega\Delta^{++}$, respectively.

From Fig. 6(a) we see that the phase relations predicted by the quark model are consistent with the $\rho^o\Delta^{++}$ data, since in each t' bin the same rotation angle will satisfy relations C.1, C.3 and C.5. In marked contrast the $\omega\Delta^{++}$ data of Fig. 6(b) show a significant departure from the Class C quark model predictions. We conclude from Fig. 6(b) that a rotation which leads to the validity of relation C.1 for $\omega\Delta^{++}$ at 3.7 GeV/c necessarily implies that relations C.3 and C.5 will fail to hold. Further, the smooth behavior of the data of Fig. 6(b) as a function of momentum transfer suggests that improper averaging over too large a momentum transfer interval is not the source of the discrepancies observed.

As we have shown above, for $\rho^o\Delta^{++}$ the phase relations do appear to be valid. We may then perform a further test of the model using the remaining Class C relation C.7, which may be written as

$$T^{22}_{22} = \frac{1}{\sqrt{6}} - T^{22}_{2-2} - T^{22}_{00} \quad . \qquad (5)$$

We find that this equality is badly violated for our data. To illustrate the breakdown of relation (5) we compare in Fig. 7 the magnitudes of the left- (lhs) and right-hand sides (rhs) of this equation for our $\rho^o\Delta^{++}$ data. While the rhs of Eq. (5) is rotationally invariant for equal angles φ_1 and φ_2 [see Eqs. (4)], the observed values for $|T^{22}_{22}|$ are systematically smaller than the observed values of the rhs. Thus we are led to conclude that no real rotation about the production normal exists that will lead to the satisfying of relation C.7 for our $\rho^o\Delta^{++}$ data. To our knowledge this result has not been observed previously. However we should caution that our analysis does not consider the influence of the s-wave $\pi\pi$ background beneath the ρ^o. Therefore interpretation of this violation of relation C.7 may not be unambiguous.

IV. SUMMARY AND CONCLUSIONS

The quark model relations of Classes A, B and C have been tested as a function of momentum transfer for the reactions $\pi^+p \to \rho^o\Delta^{++}$ and $\pi^+p \to \omega\Delta^{++}$. The Class A and B predictions are found to be a generally correct description of the data. The Class C relations are not found to be simultaneously satisfied in any reference frame for either $\rho^o\Delta^{++}$ or $\omega\Delta^{++}$. This conclusion for $\rho^o\Delta^{++}$ should be tempered with the observations that the violation occurs for only one relationship (C.7), and that the effect of the s-wave background under the ρ is not presently understood. However the violation for the $\omega\Delta^{++}$ reaction is statistically significant and systematic for several t' bins. We are led to conclude that earlier contradictory results[5] probably did not have sufficient data to resolve this question.

The question of the extent to which angular momentum conservation in the forward direction could be responsible for the agreement of the quark model predictions is discussed in Ref. 9. A more general discussion of the implications of the validity of Class A and B relations for non-quark models is in preparation.[10]

ACKNOWLEDGMENTS

We would like to acknowledge essential contributions to earlier phases of this experiment made by our colleagues, in particular Gerson Goldhaber, W. Ralph Butler, Donald G. Coyne, Bronwyn H. Hall, and Jimmy MacNaughton.

REFERENCES

*Work supported by the U. S. Atomic Energy Commission.

†Present address: Department of Physics, Imperial College, London SW 7, England.

1. A. Białas and K. Zalewski, Nuclear Physics B6, 465 (1968).
2. C. Itzykson and M. Jacob, Nuovo Cimento 48A, 909 (1967), and J. L. Friar and J. S. Trefil, Nuovo Cimento 49A, 642 (1967).
3. A. Krzywicki and A. Le Yaouanc, Nuclear Physics B14, 246 (1969), and H. J. Lipkin, Nuclear Physics B20, 652 (1970).
4. K. W. J. Barnham et al., $\rho^o\Delta^{++}$ and $\omega\Delta^{++}$ Joint Decay Correlations at 3.7 GeV/c, LBL-960 (June 1972), submitted to Phys. Rev. D.
5. K. Böckmann et al., Phys. Letters 28B, 72 (1968), and M. Aderholz et al. Nuclear Physics B8, 485 (1968).
6. W. De Baere et al., Nuovo Cimento 61A, 397 (1969); J. Friedman and R. R. Ross, Phys. Rev. Letters 22, 152 (1969); B. Haber et al., Nuclear Physics B17, 289 (1970); and G. S. Abrams et al., Phys. Rev. D1, 2433 (1970).
7. J. T. Donohue and H. Hogaasen, Phys. Letters 25B, 554 (1967).
8. A. Kotanski and K. Zalewski, Nuclear Physics B4, 559 (1968).
9. K. W. J. Barnham, Joint Decay Correlations in $\pi^+p \rightarrow V^o\Delta^{++}$ Reactions at 3.7 GeV/c, Proceedings of the Seventh Rencontre de Moriond, Vol. I, Ed. by J. Tran Thanh Van (CNRS, Paris).
10. G. S. Abrams, K. W. J. Barnham, and J. J. Bisognano, Dipole Regge Exchange and the Quark Model, LBL-962, to be submitted to Phys. Rev. D.

Table I. Białas and Zalewski quark model relationships in transversity frames.

Class A	A.1	$T^{20}_{00} = \sqrt{2}\, T^{02}_{00}$
	A.2	$\mathrm{Re}\, T^{22}_{20} = \frac{1}{2}\, \mathrm{Re}\, T^{20}_{20}$
	A.3	$\mathrm{Re}\, T^{22}_{02} = \frac{1}{\sqrt{2}}\, \mathrm{Re}\, T^{02}_{02}$
	A.4	$T^{22}_{00} = \frac{1}{2\sqrt{6}} - \frac{1}{\sqrt{2}}\, T^{02}_{00}$
	A.5	$\mathrm{Im}\, T^{22}_{20} = \frac{1}{2}\, \mathrm{Im}\, T^{20}_{20}$
	A.6	$\mathrm{Im}\, T^{22}_{02} = \frac{1}{\sqrt{2}}\, \mathrm{Im}\, T^{02}_{02}$
Class B	B.1	$\mathrm{Re}\, T^{20}_{20} = \sqrt{2}\; \mathrm{Re}\, T^{02}_{02}$
	B.2	$\mathrm{Im}\, T^{20}_{20} = \sqrt{2}\; \mathrm{Im}\, T^{02}_{02}$
	B.3	$\mathrm{Re}\, T^{22}_{20} = \mathrm{Re}\, T^{22}_{02}$
	B.4	$\mathrm{Im}\, T^{22}_{20} = \mathrm{Im}\, T^{22}_{02}$
	B.5	$\mathrm{Im}\, T^{22}_{2-2} = 0$
	B.6	$\mathrm{Im}\, T^{22}_{1-1} = 0$
Class C	C.1	$\mathrm{Im}\, T^{20}_{20} = 0$
	C.2	$\mathrm{Im}\, T^{02}_{02} = 0$
	C.3	$\mathrm{Im}\, T^{22}_{22} = 0$
	C.4	$\mathrm{Im}\, T^{22}_{20} = 0$
	C.5	$\mathrm{Im}\, T^{22}_{11} = 0$
	C.6	$\mathrm{Im}\, T^{22}_{02} = 0$
	C.7	$\frac{1}{\sqrt{6}} - (T^{22}_{22} + T^{22}_{2-2} + T^{22}_{00}) = 0.$

Table II. Białas and Zalewski
Class A relationships
with single vertex statistical tensors
re-expressed in terms of non-transversity
frame ρ_{nm}.[a]

A.1	$\rho^{1,1} + \rho^{1,-1} = \frac{4}{3}\rho_{3,3} + \frac{4}{\sqrt{3}}\,\mathrm{Re}\,\rho_{3,-1}$
A.2	$\mathrm{Re}\,T^{22}_{20} = \frac{1}{8}(\rho^{1,-1} - 3\rho^{1,1} + 1)$
A.3	$\mathrm{Re}\,T^{22}_{02} = \frac{1}{8}(\frac{4}{\sqrt{3}}\,\mathrm{Re}\,\rho_{3,-1} - 4\rho_{3,3} + 1)$
A.4	$T^{22}_{00} = \frac{1}{\sqrt{6}}(\frac{1}{4} + \rho_{3,3} + \sqrt{3}\,\mathrm{Re}\,\rho_{3,-1})$
A.5	$\mathrm{Im}\,T^{22}_{20} = -\frac{1}{2\sqrt{2}}\,\mathrm{Re}\,\rho^{1,0}$
A.6	$\mathrm{Im}\,T^{22}_{02} = -\frac{1}{\sqrt{3}}\,\mathrm{Re}\,\rho_{3,1}$

[a] $\rho^{n,n'}$ denotes a vector meson density matrix element, and $\rho_{2m,2m'}$ a Δ^{++} matrix element.

FIGURE CAPTIONS

Fig. 1. Comparison of the quark model Class A predictions with $\rho\Delta$ and $\omega\Delta$ data in the transverse-Jackson coordinate system. (□: right-hand side; ▲: left-hand side.) For $\rho\Delta$: (a) A.1, (b) A.2, (c) A.3, (d) A.4, (e) A.5, (f) A.6; for $\omega\Delta$: (g) A.1, (h) A.2, (i) A.3, (j) A.4, (k) A.5, (ℓ) A.6.

Fig. 2. Comparison of the quark model Class B predictions with $\rho\Delta$ and $\omega\Delta$ data in the transverse-Jackson coordinate system. (□: right-hand side; ▲: left-hand side.) For $\rho\Delta$: (a) B.1, (b) B.2, (c) B.3, (d) B.4, (e) B.5, (f) B.6; for $\omega\Delta$: (g) B.1, (h) B.2, (i) B.3, (j) B.4, (k) B.5, (ℓ) B.6. For relations B.5 and B.6 the quark model prediction is that the right-hand side be zero.

Fig. 3. Helicity crossing angles χ_V (for the vector meson taken to have a mass of 780 MeV/c^2) and χ_Δ (for a Δ^{++} of mass 1240 MeV/c^2) at 3.7 GeV/c as a function of t'.

Fig. 4. Class C quark model predictions for $\rho^o\Delta^{++}$. If the constraints were valid the data would have an expected value of zero for each relation. Transverse-Jackson frame: (a) C.1, (b) C.2, (c) C.3, (d) C.4, (e) C.5, (f) C.6, (g) C.7; transverse-helicity frame: (h) C.1, (i) C.2, (j) C.3, (k) C.4, (ℓ) C.5, (m) C.6, (n) C.7.

Fig. 5. Class C quark model predictions for $\omega\Delta^{++}$ (in the same format as in Fig. 14).

Fig. 6. Rotation angle required to satisfy the Class C relations, calculated in the transverse-Jackson reference frame. ɸ C.1, ◆ C.3 and ▲ C.5 (a) $\rho^o\Delta^{++}$; (b) $\omega\Delta^{++}$.

Fig. 7. Comparison of right- (●) and left- (◆) hand sides of Eq. (5) for the $\rho^o\Delta^{++}$ channel. Note that we here compare the magnitudes of the (in general) complex tensors.

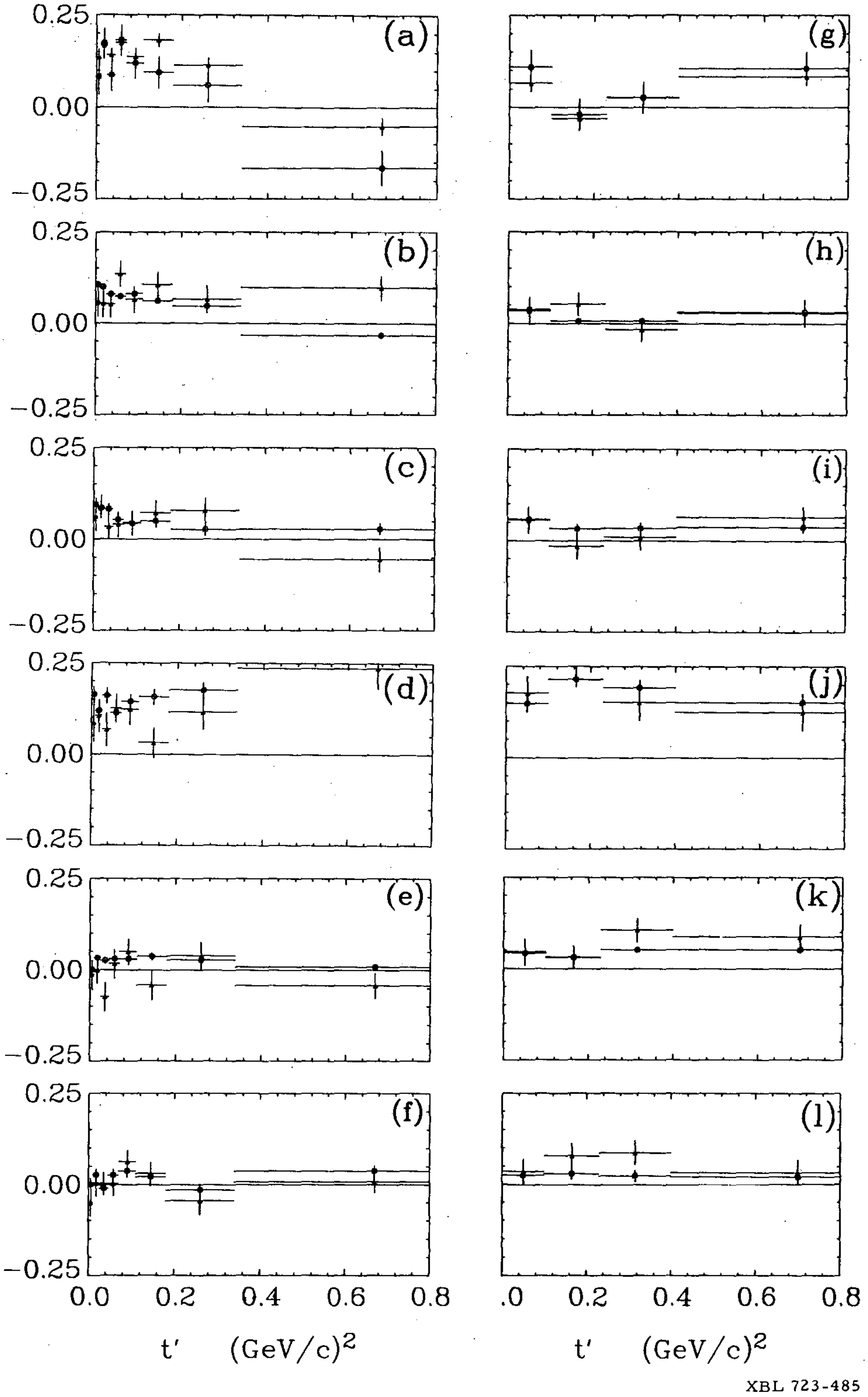


Fig. 1

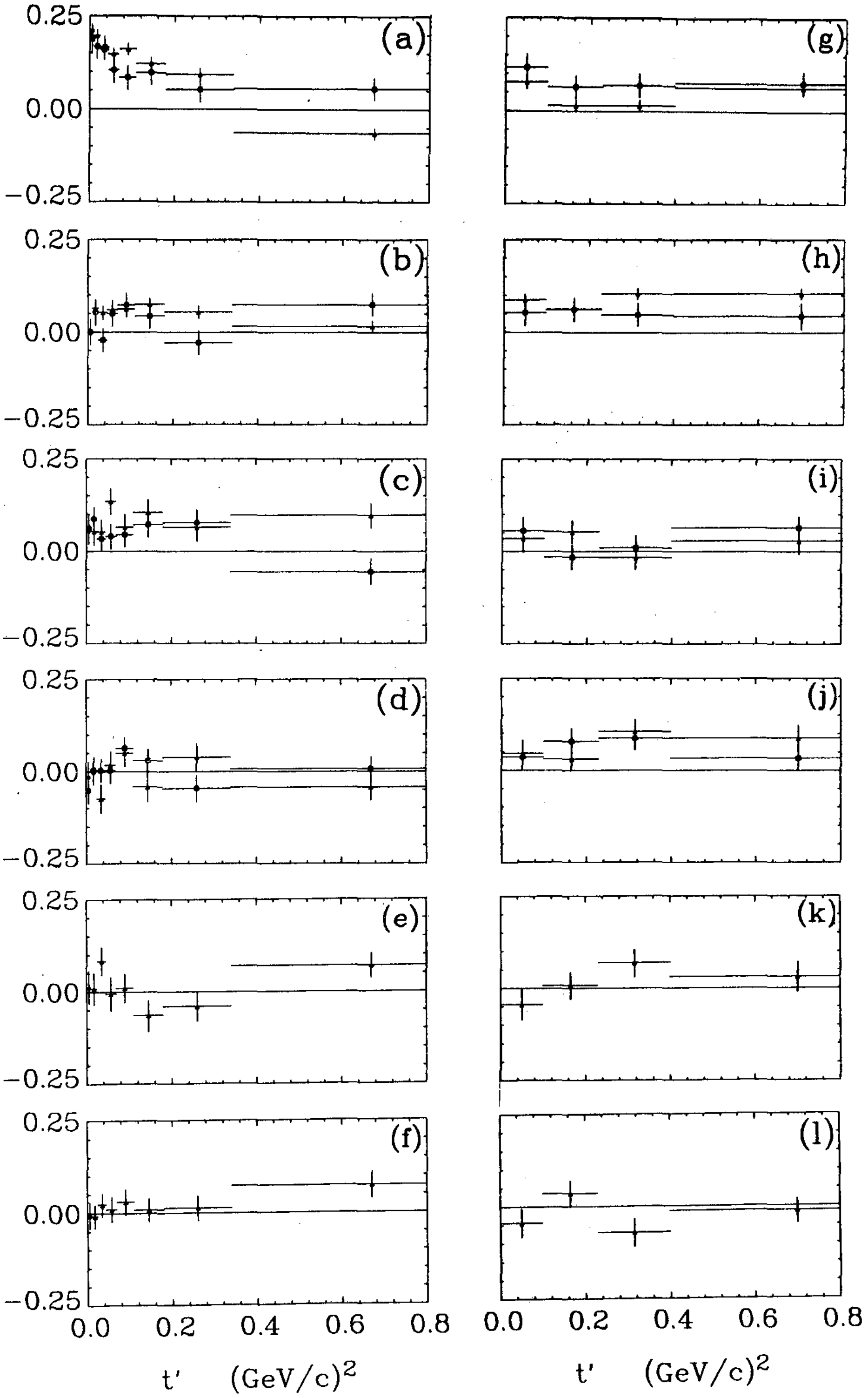
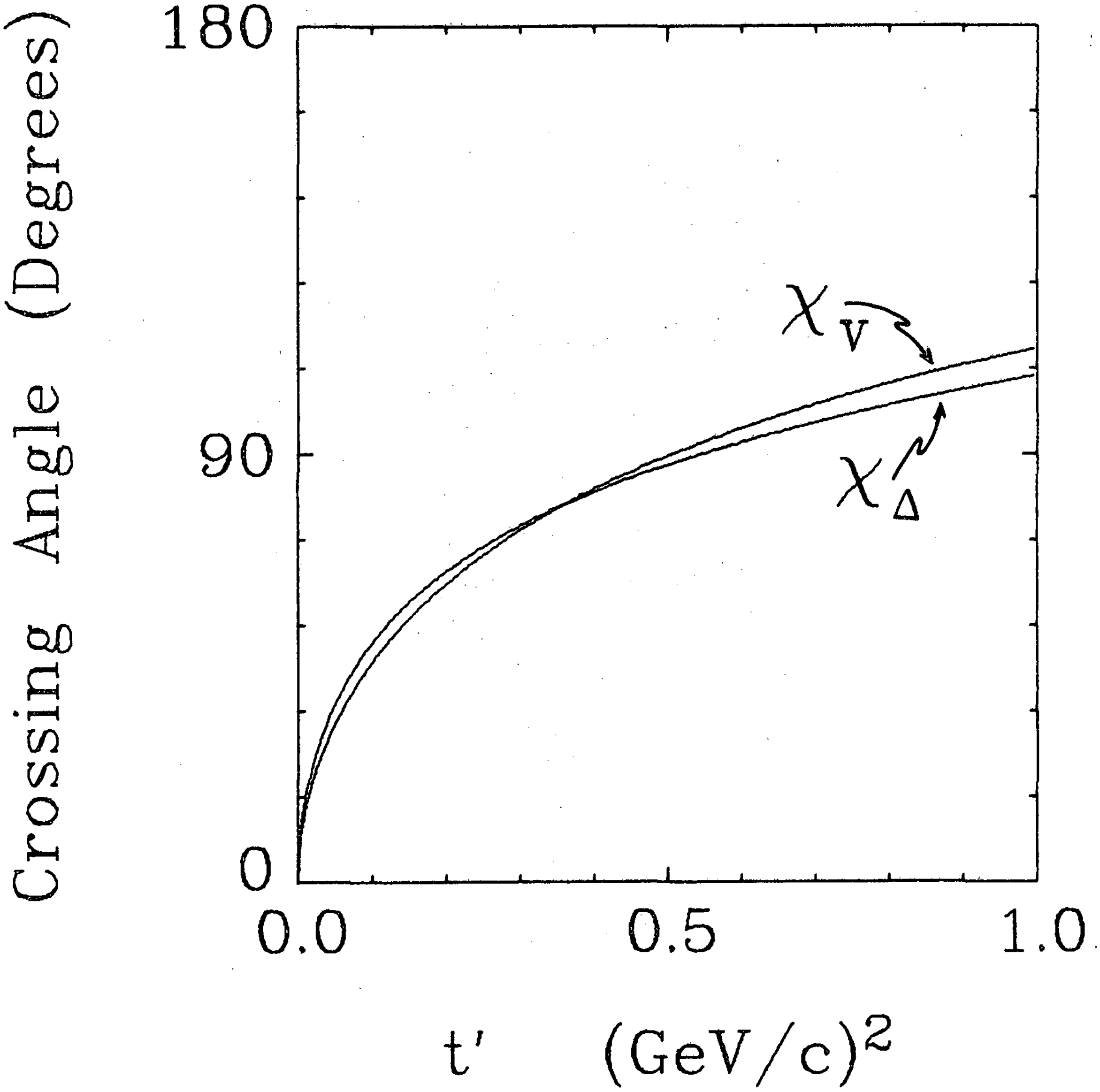


Fig. 2



XBL 723-480

Fig. 3

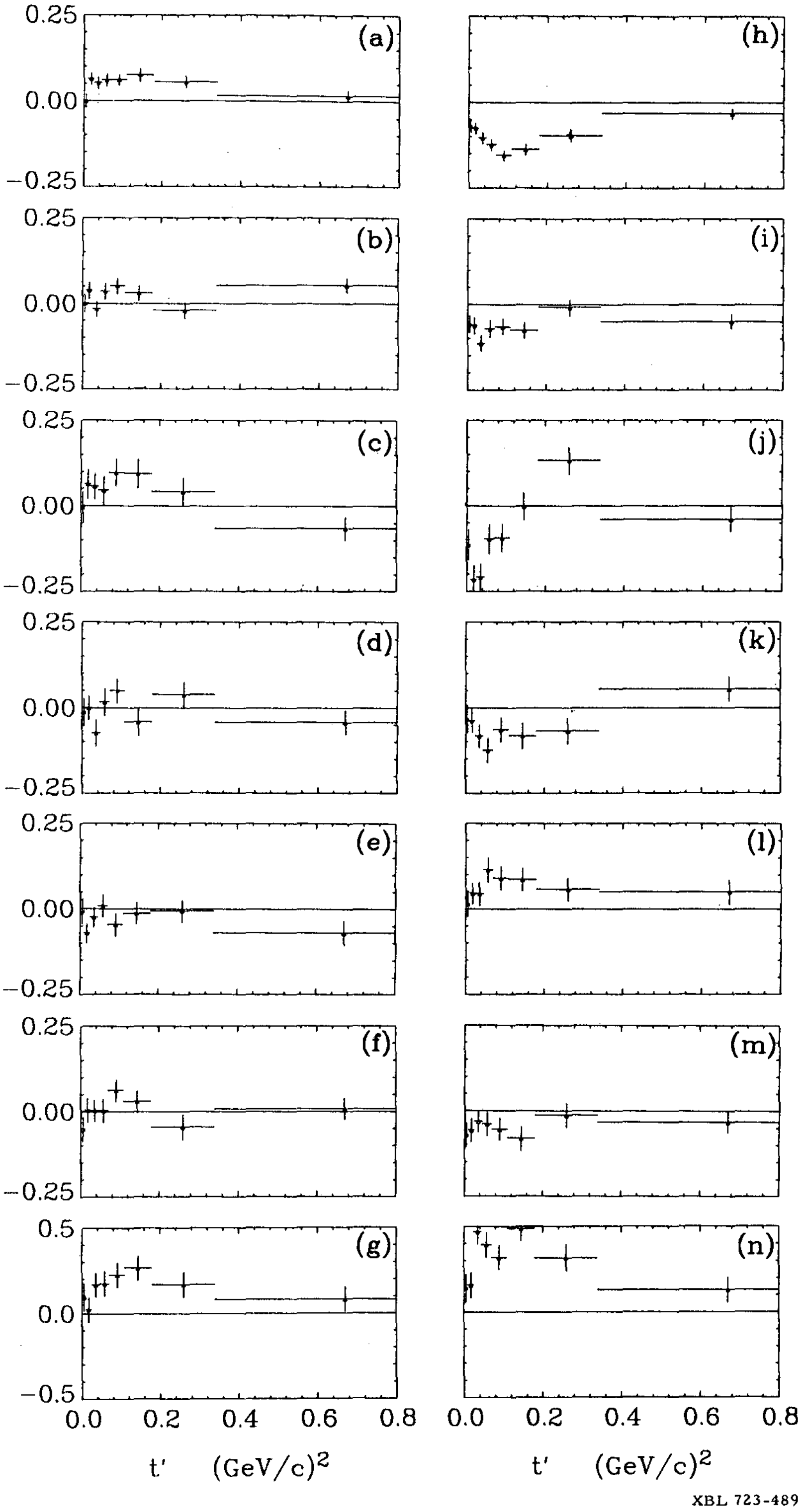

0.25
0.00
−0.25
0.5
0.0
−0.5
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)
(k)
(l)
(m)
(n)
0.0
0.2
0.4
0.6
0.8
t′ (GeV/c)^2
XBL 723-489

Fig. 4

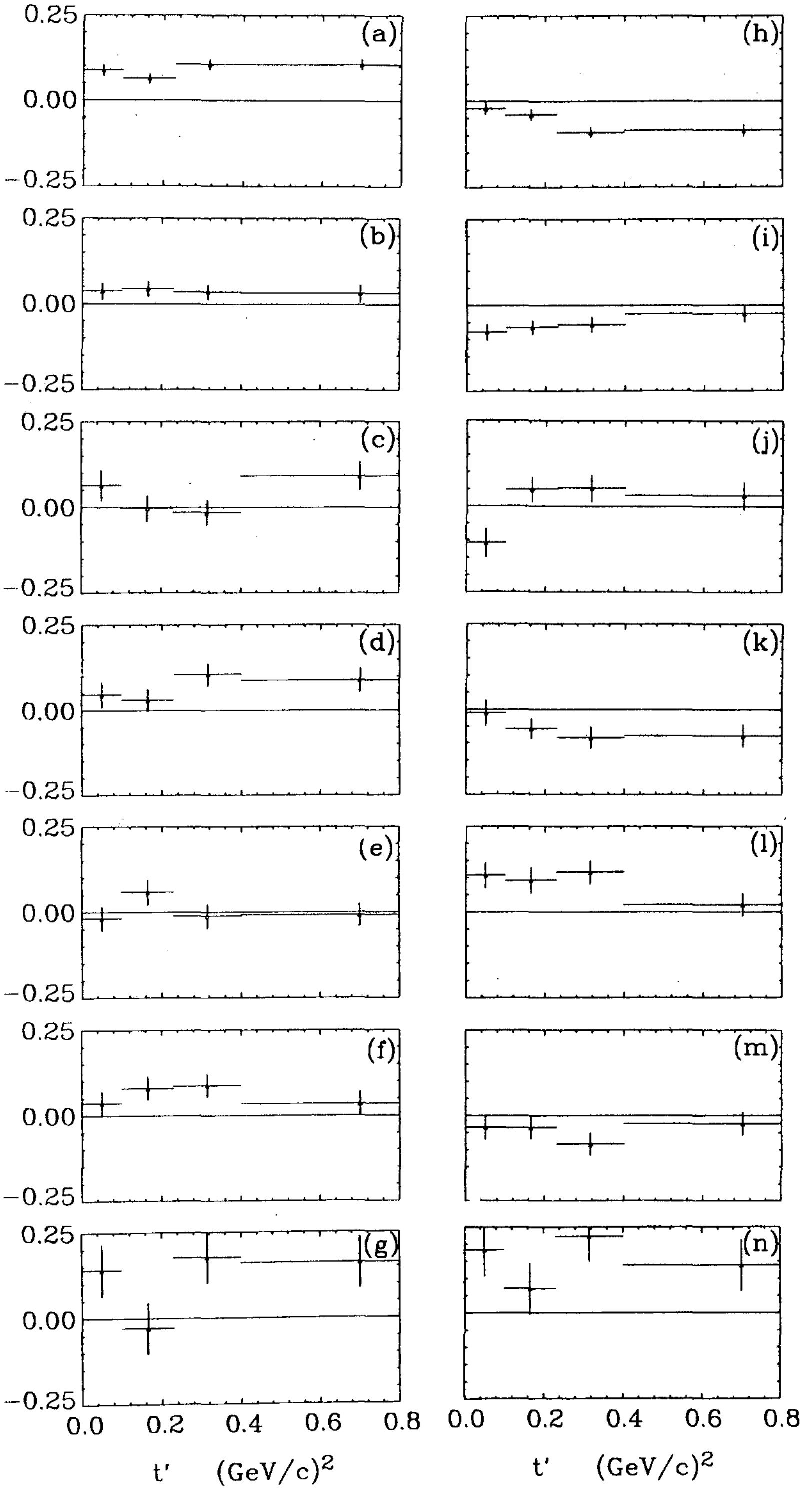
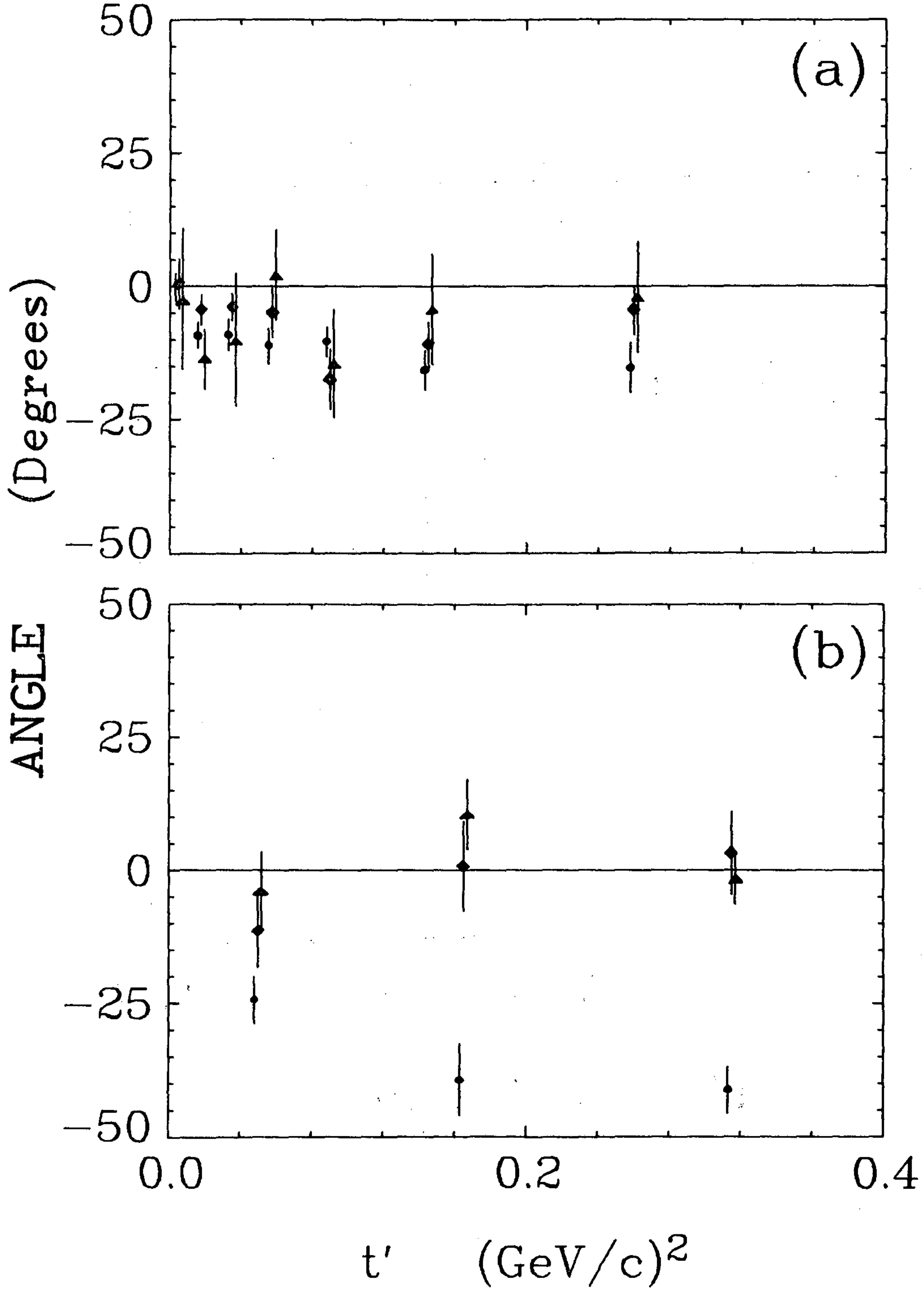
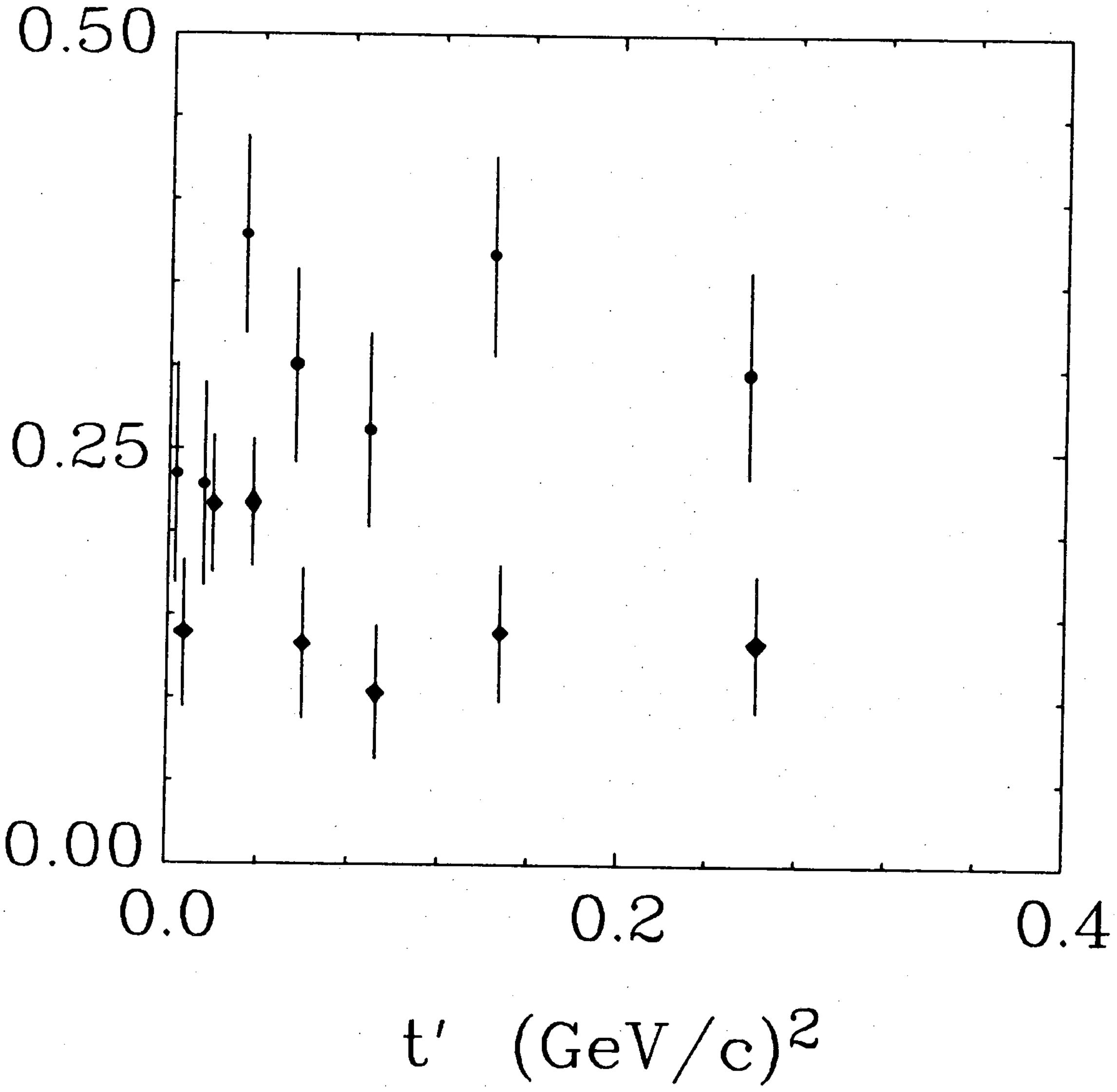


Fig. 5



XBL 725-839

Fig. 6



XBL 725-837

Fig. 7

TECHNICAL INFORMATION DIVISION
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720